Άλγεβρα IV

Τριώνυμο

Βαγγέλης Ψύχας

Τριώνυμο Ι

- $f(x) = ax^2 + bx + c \qquad a, b, c \in \mathbb{R} \;\; a \neq 0.$

- $f(x) = a\left(x^2 + \frac{b}{a}x + \frac{c}{a}\right) = a\left(x^2 + 2\frac{b}{2a}x + \frac{c}{a}\right) =$

$$= a\left(x^2 + 2\frac{b}{2a}x + \frac{b^2}{4a^2} - \frac{b^2}{4a^2} + \frac{c}{a}\right) =$$

$$= a\left(\left(x + \frac{b}{2a}\right)^2 - \frac{b^2 - 4ac}{4a^2}\right) = \dashrightarrow$$

Τριώνυμο ΙΙ

$$\dashrightarrow = a\left(\left(x+\frac{b}{2a}\right)^2 - \frac{\overbrace{b^2-4ac}^{\Delta}}{4a^2}\right)$$

✧ Διακρίνουσα $\Delta = b^2 - 4ac$.

Τριώνυμο $(\Delta = b^2 - 4ac > 0)$ (I)

✧ $f(x) = ax^2 + bx + c \quad a, b, c \in \mathbb{R} \ \ a \neq 0.$

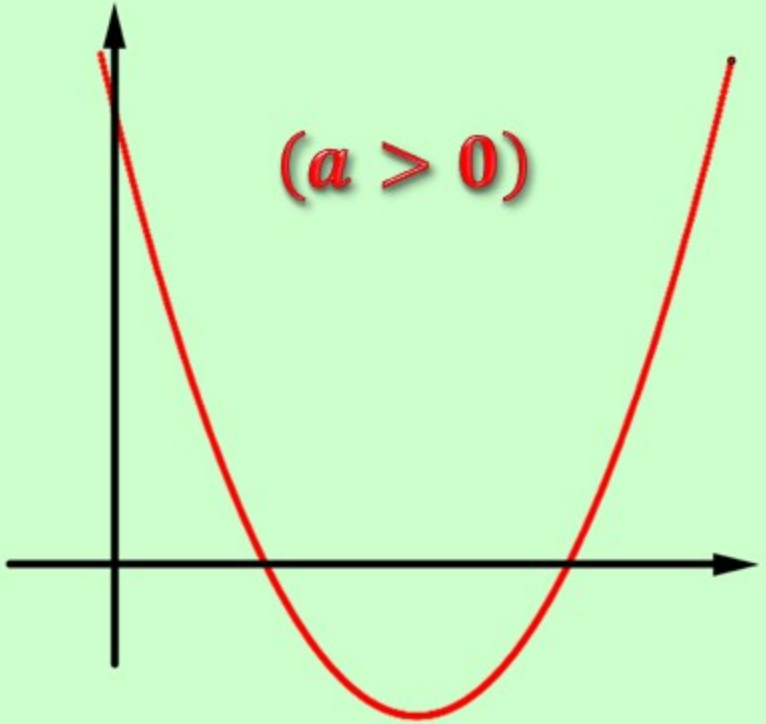
✧ $\rho_1 = \dfrac{-b + \sqrt{\Delta}}{2a}$ ✧ $\rho_2 = \dfrac{-b - \sqrt{\Delta}}{2a}$

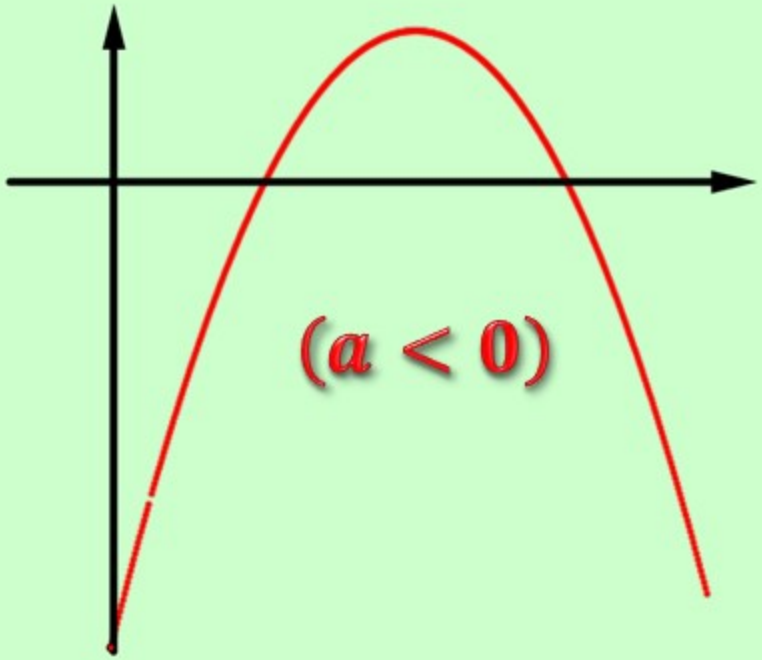
✧ $f(x) = a(x - \rho_1)(x - \rho_2)$

Τριώνυμο $(\Delta = b^2 - 4ac > 0)$ (II)

♦ $f(x) = ax^2 + bx + c \quad a, b, c \in \mathbb{R} \ \ a \neq 0.$

♦ $\rho_1 = \dfrac{-b + \sqrt{\Delta}}{2a}$ ♦ $\rho_2 = \dfrac{-b - \sqrt{\Delta}}{2a}$ ♦ $f(x) = a(x - \rho_1)(x - \rho_2)$





Τριώνυμο $(\Delta = b^2 - 4ac > 0)$ (III)

♦ $f(x) = ax^2 + bx + c \quad a, b, c \in \mathbb{R} \ \ a \neq 0.$

♦ $\rho_1 = \dfrac{-b + \sqrt{\Delta}}{2a}$ ♦ $\rho_2 = \dfrac{-b - \sqrt{\Delta}}{2a}$ ♦ $f(x) = a(x - \rho_1)(x - \rho_2)$

$(a > 0)$

♦ $f(x) < 0 \Leftrightarrow x \in (\rho_1, \rho_2)$

♦ $f(x) > 0 \Leftrightarrow$

$\Leftrightarrow x \in (-\infty, \rho_1) \cup (\rho_2, +\infty)$

$(a < 0)$

♦ $f(x) > 0 \Leftrightarrow x \in (\rho_1, \rho_2)$

♦ $f(x) < 0 \Leftrightarrow$

$\Leftrightarrow x \in (-\infty, \rho_1) \cup (\rho_2, +\infty)$

Τριώνυμο $(\Delta = b^2 - 4ac = 0)$

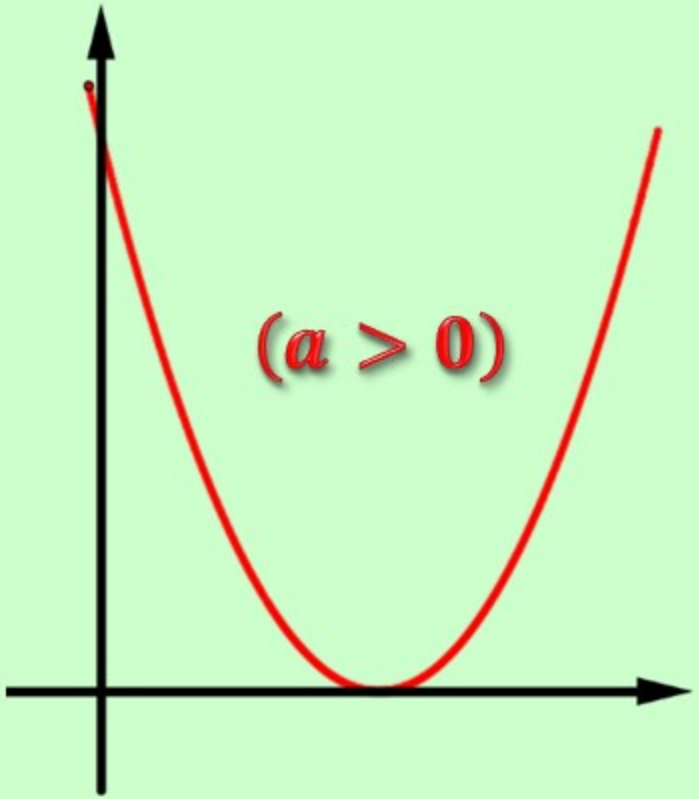
✧ $f(x) = ax^2 + bx + c \quad a, b, c \in \mathbb{R} \ \ a \neq 0.$

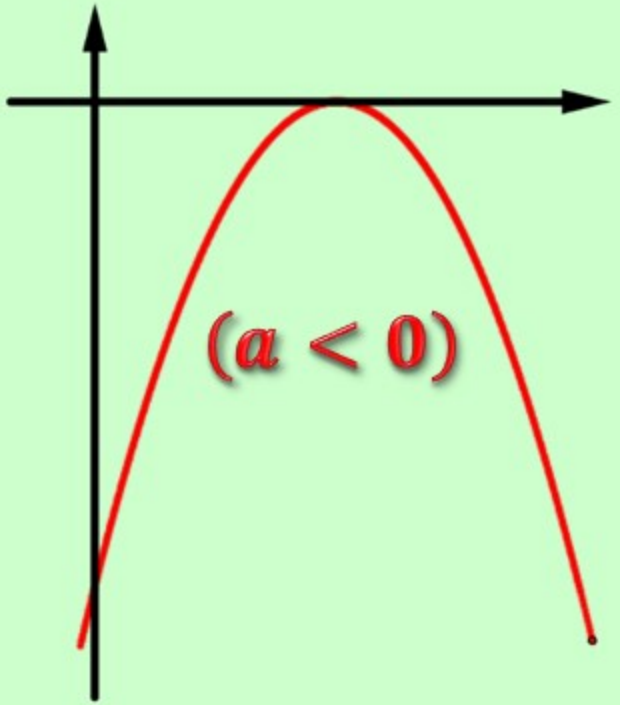
✧ $\rho_1 = \rho_2 = \rho = \dfrac{-b}{2a}$

✧ $f(x) = a(x - \rho)^2$

Τριώνυμο $(\Delta = b^2 - 4ac = 0)$

- $f(x) = ax^2 + bx + c \quad a, b, c \in \mathbb{R} \ \ a \neq 0.$
- $\rho_1 = \rho_2 = \rho = \dfrac{-b}{2a}$
- $f(x) = a(x - \rho)^2$





Τριώνυμο $(\Delta = b^2 - 4ac = 0)$

- $f(x) = ax^2 + bx + c \quad a, b, c \in \mathbb{R} \ \ a \neq 0.$
- $\rho_1 = \rho_2 = \rho = \dfrac{-b}{2a}$
- $f(x) = a(x - \rho)^2$

$(a > 0)$

- $f(x) \geq 0 \quad \forall x \in \mathbb{R}$

$(a < 0)$

- $f(x) \leq 0 \quad \forall x \in \mathbb{R}$

Τριώνυμο $(\Delta = b^2 - 4ac < 0)$

✧ $f(x) = ax^2 + bx + c \quad a, b, c \in \mathbb{R} \ \ a \neq 0.$

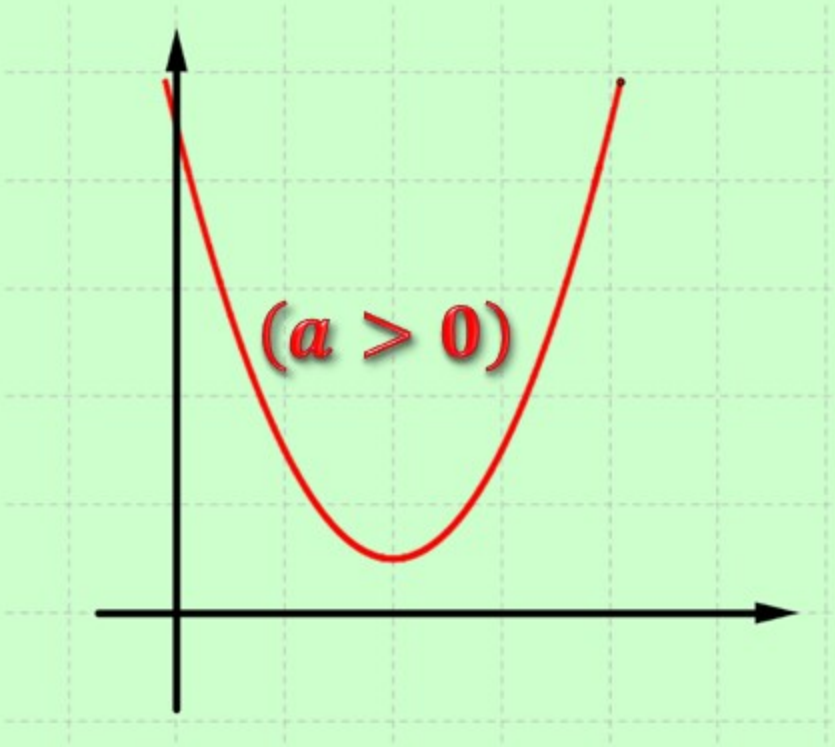
✧ $z_1 = \dfrac{-b + i\sqrt{-\Delta}}{2a}$ ✧ $z_2 = \dfrac{-b - i\sqrt{-\Delta}}{2a}$

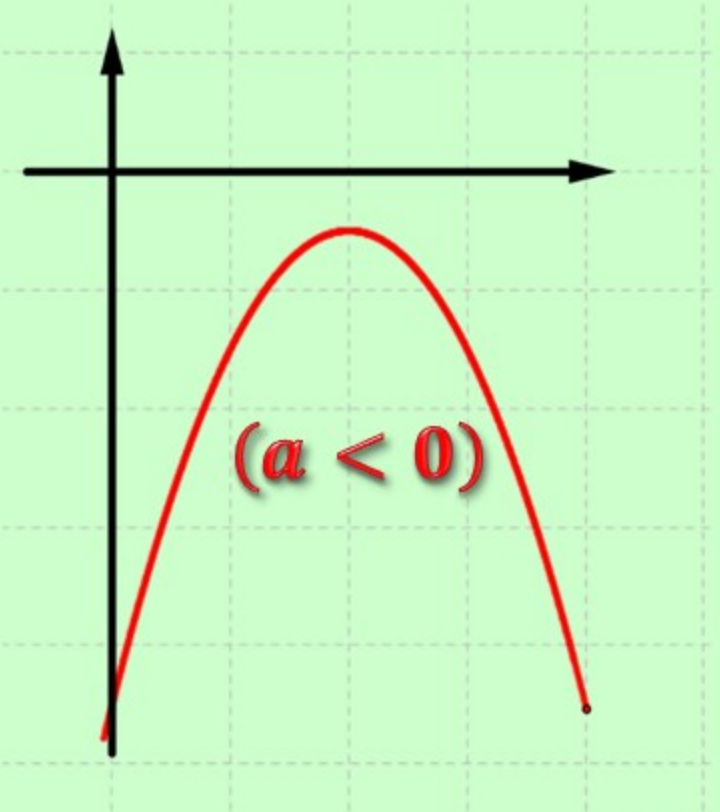
✧ $f(x) = a(x - z_1)(x - z_2)$

Τριώνυμο $(\Delta = b^2 - 4ac < 0)$

♦ $f(x) = ax^2 + bx + c \quad a, b, c \in \mathbb{R} \ \ a \neq 0.$

♦ $z_1 = \dfrac{-b + i\sqrt{-\Delta}}{2a}$ ♦ $z_2 = \dfrac{-b - i\sqrt{-\Delta}}{2a}$ ♦ $f(x) = a(x - z_1)(x - z_2)$





Τριώνυμο $(\Delta = b^2 - 4ac < 0)$

♦ $f(x) = ax^2 + bx + c \quad a, b, c \in \mathbb{R} \ \ a \neq 0.$

♦ $z_1 = \dfrac{-b + i\sqrt{-\Delta}}{2a}$ ♦ $z_2 = \dfrac{-b - i\sqrt{-\Delta}}{2a}$ ♦ $f(x) = a(x - z_1)(x - z_2)$

$(a > 0)$ $(a < 0)$

♦ $f(x) > 0 \quad \forall x \in \mathbb{R}$ ♦ $f(x) < 0 \quad \forall x \in \mathbb{R}$

Τριώνυμο (Vieta)

- $f(x) = ax^2 + bx + c \qquad a, b, c \in \mathbb{R} \;\; a \neq 0.$

- $S = \rho_1 + \rho_2 = z_1 + z_2 = \dfrac{-b}{a}$

- $P = \rho_1 \cdot \rho_2 = z_1 \cdot z_2 = \dfrac{c}{a}$

- $x^2 + \dfrac{b}{a}x + \dfrac{c}{a} = 0 \Leftrightarrow x^2 - Sx + P = 0$

Τύποι Vieta

- $f(x) = ax^3 + bx^2 + cx + d \qquad a, b, c, d \in \mathbb{R} \;\; a \neq 0.$

- $\rho_1 + \rho_2 + \rho_3 = \dfrac{-b}{a}$

- $\rho_1 \cdot \rho_2 + \rho_1 \cdot \rho_3 + \rho_2 \cdot \rho_3 = \dfrac{c}{a}$

- $\rho_1 \cdot \rho_2 \cdot \rho_3 = -\dfrac{d}{a}$