A Brain-Friendly Guide

Head First Physics

A learner's companion to mechanics and practical physics

Conserve your energy by spotting patterns

Think like a physicist

Understand how stuff really works

Try experiments, and solve dozens of puzzles and exercises

Deal with pressure without being under it

O'REILLY®

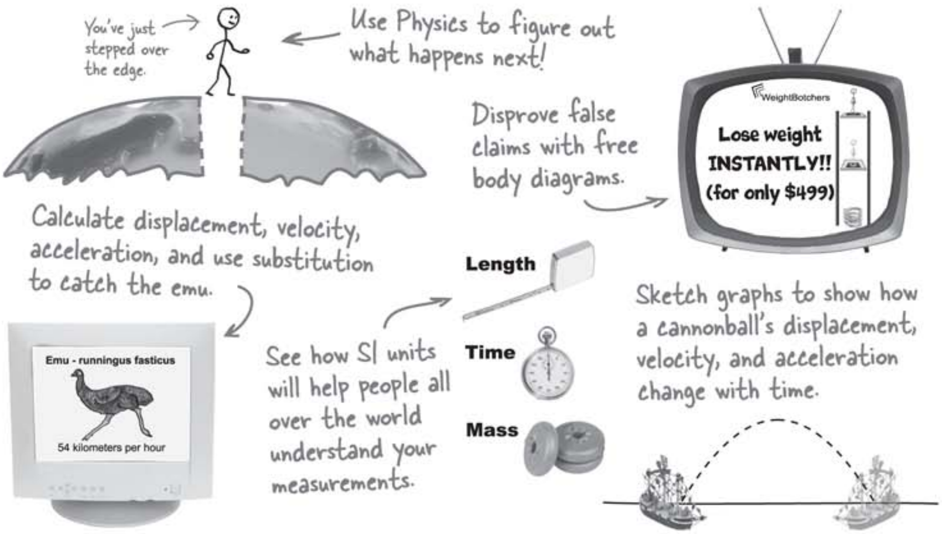
Heather Lang, Ph.D.

Physics/Science

What will you learn from this book?

Are you struggling to pass the AP Physics B exam? Is your college physics class giving you a headache? Do you wish you understood how the world around you really worked? Then check out *Head First Physics*, a complete guide to algebra-based mechanics and practical physics.

Physics is all about the world we live in, encompassing everything from conservation of energy to orbital behavior. You'll master scientific notation, SI units, vectors, Newton's laws, and circular and simple harmonic motion.



Why does this book look so different?

We think your time is too valuable to spend struggling with new concepts. Using the latest research in cognitive science and learning theory to craft a multi-sensory learning experience, *Head First Physics* uses a visually rich format designed for the way your brain works, not a text-heavy approach that puts you to sleep.

"It can be difficult to break students of the habit of memorizing, but the Head First approach encourages a deeper understanding of the material and gives students the tools they need to break down complex problems into simpler ones. Definitely a help in solving AP Exam problems."

Diane Jaquith, High School Physics Teacher

"This is a truly remarkable book. The physics is taught clearly and without too much mathematics by looking at a series of well-chosen real-life or comedy tasks. If math really doesn't turn you on, this is a great way to learn Physics! I didn't think it was possible to do some of this stuff without calculus, but *Head First Physics* has done it."

John Allister, M.S. [Cambridge University]

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Advance Praise for *Head First Physics*

"If you want to learn some physics, but you think it's too difficult, buy this book! It will probably help, and if it doesn't, you can always use it as a doorstop or hamster bedding or something. I wish I had a copy of this book when I was teaching physics."

— John Allister, physics teacher

"*Head First Physics* has achieved the impossible - a serious textbook that makes physics fun. Students all over will be thinking like a physicist!"

— Georgia Gale Grant, freelance science writer, communicator and broadcaster

"Great graphics, clear explanations and some crazy real world problems to solve! This text is full of strategies and tips to attack problems. It encourages a team approach that's so essential in today's work world."

— Diane Jaquith, high school physics, chemistry and physical science teacher

"This is an outstandingly good teacher masquerading as a physics book! You never feel phased if you don't quite understand something the first time because you know it will be explained again in a different way and then repeated and reinforced. "

— Marion Lang, teacher

"This book takes you by the hand and guides you through the world of physics."

— Catriona Lang, teacher

"*Head First Physics* really rocks - I never thought it was possible to enjoy learning physics so much! This book is about understanding and not about rote learning, so you can get to grips with the physics and remember it much better as a result."

— Alice Pitt-Pitts

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"Head First Statistics is by far the most entertaining, attention-catching study guide on the market. By presenting the material in an engaging manner, it provides students with a comfortable way to learn an otherwise cumbersome subject. The explanation of the topics is presented in a manner comprehensible to students of all levels."

— Ariana Anderson, Teaching Fellow/PhD candidate in Statistics, UCLA

"Head First is an intuitive way to understand statistics using simple, real-life examples that make learning fun and natural."

— Michael Prerau, computational neuroscientist and statistics instructor, Boston University

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— Danielle Levitt

"Down with dull statistics books! Even my cat liked this one."

— Cary Collett

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— Bill Sawyer, ATG Curriculum Manager, Oracle

"Elegant design is at the core of every chapter here, each concept conveyed with equal doses of pragmatism and wit."

— Ken Goldstein, Executive Vice President, Disney Online

"I feel like a thousand pounds of books have just been lifted off of my head."

—Ward Cunningham, inventor of the Wiki and founder of the Hillside Group

"This book's admirable clarity, humor and substantial doses of clever make it the sort of book that helps even non-programmers think well about problem-solving."

— Cory Doctorow, co-editor of Boing Boing
Author, *Down and Out in the Magic Kingdom*
and *Someone Comes to Town, Someone Leaves Town*

"It's fast, irreverent, fun, and engaging. Be careful—you might actually learn something!"

—Ken Arnold, former Senior Engineer at Sun Microsystems
Co-author (with James Gosling, creator of Java), *The Java Programming Language*

"I received the book yesterday and started to read it...and I couldn't stop. This is definitely très 'cool.' It is fun, but they cover a lot of ground and they are right to the point. I'm really impressed."

— Erich Gamma, IBM Distinguished Engineer, and co-author of *Design Patterns*

"One of the funniest and smartest books on software design I've ever read."

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Head First Physics

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No pizza delivery guys were harmed in the making of this book.

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[M]

This book is dedicated to... anyone who made me laugh while writing it!

Author of Head First Physics

Heather Lang

Heather studied physics in Manchester, gaining a first class honours degree. She likes explaining how stuff works and persuading people to send her chocolate in the post. Her first foray into science communication was via the BaBar Particle Physics Teaching Package. She followed this up with a Ph.D. in the grey area between physics and biochemistry, but got fed up of sharing a fridge with petri dishes and moved on from the lab into education and Head First Physics.

When not explaining how stuff works, Heather likes to play extreme sports such as chess and cricket, play with sliders on a sound desk, or play the fool while running school chess clubs (in the name of teaching of course).

Table of Contents (Summary)

Table of Contents (the real thing)

Intro

Your brain on Physics. Here *you* are trying to *learn* something, while here your *brain* is doing you a favor by making sure the learning doesn't *stick*. Your brain's thinking, "Better leave room for more important things, like which wild animals to avoid and whether naked snowboarding is a bad idea." So how *do* you trick your brain into thinking that your life depends on knowing physics?

1 think like a physicist

In the beginning ...

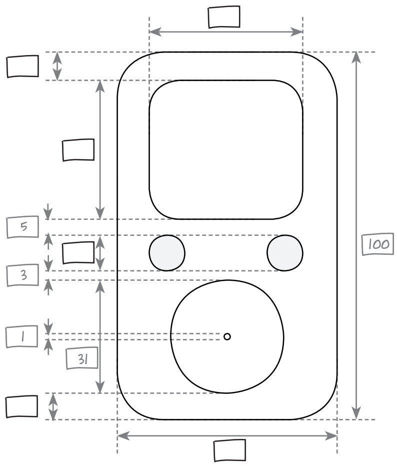
Physics is about the world around you and how everything in it works. As you go about your daily life, you're **doing** physics all the time! But the thought of actually **learning** physics may sometimes **feel** like falling into a bottomless pit with no escape! Don't worry... this chapter introduces how to **think like a physicist**. You'll learn to step into problems and to use your **intuition** to spot **patterns** and '**special points**' that make things much easier. By **being** part of the problem, you're one step closer to getting to the solution...

making it all MEAN something

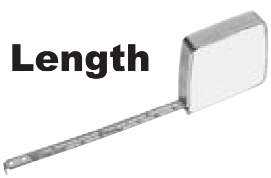
Units and measurements

2

How long is a piece of string? Physics is based on making **measurements** that tell you about **size**. In this chapter, you'll learn how to use **units** and **rounding** to avoid making mistakes - and also why **errors** are OK. By the time you're through, you'll know when something is **significant** and have an opinion on whether size really **is** everything.



Length



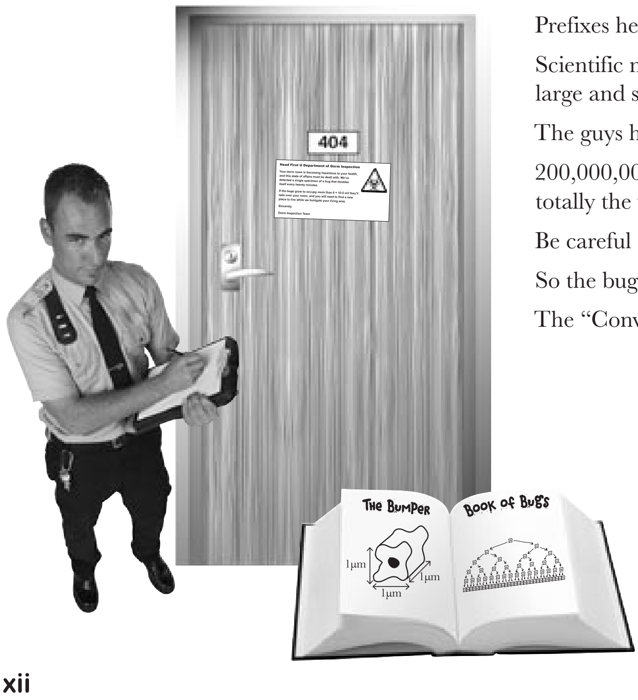
Time

Mass

3 scientific notation, area, and volume

All numbers great and small

In the real world, you have to deal with all kinds of numbers, not just the ones that are easier to work with. In this chapter, you'll be taking control of unwieldy numbers using scientific notation and discovering why rounding a large number doesn't mean having to write a zillion zeros at the end. You'll also use your new superpowers to deal with units of area and volume - which is where scientific notation will save you lots of grief (and time) in the future!

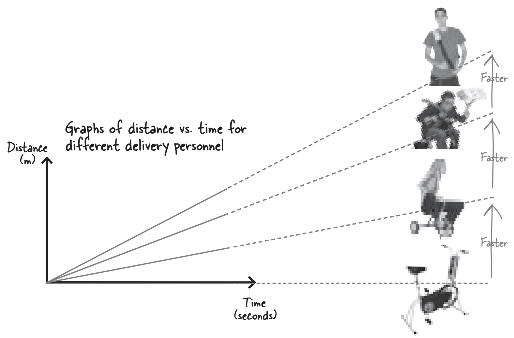


equations and graphs

Learning the lingo

4

Communication is vital. You're already off to a good start in your journey to truly think like a physicist, but now you need to **communicate your thoughts**. In this chapter, you're going to take your first steps in two universal languages - **graphs** and **equations** - pictures you can use to *speak a thousand words* about experiments you do and the physics concepts you're learning. *Seeing is believing*.

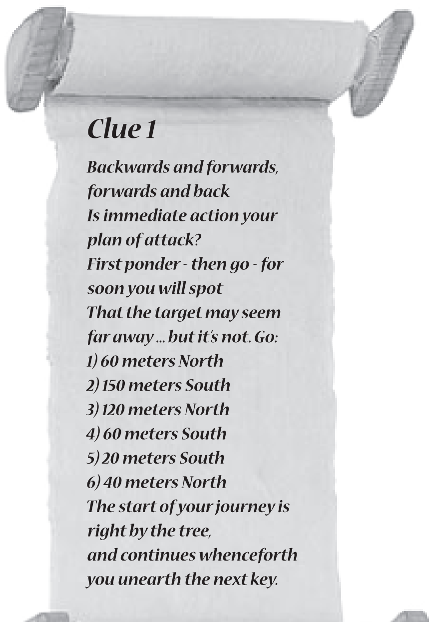


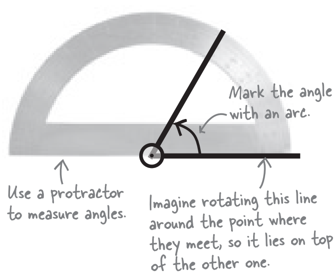
5 dealing with directions

Vectors

Time, speed, and distance are all well and good, but you really need DIRECTION too if you want to get on in life.

You now have multiple physics superpowers: You've mastered graphs and equations, and you can estimate how big your answer will be. But **size** isn't everything. In this chapter, you'll be learning about **vectors**, which give **direction** to your answers and help you to find **easier shortcuts** through complicated-looking problems.





6 Displacement, Velocity, and Acceleration

What's going on?

It's hard to keep track of more than one thing at a time.

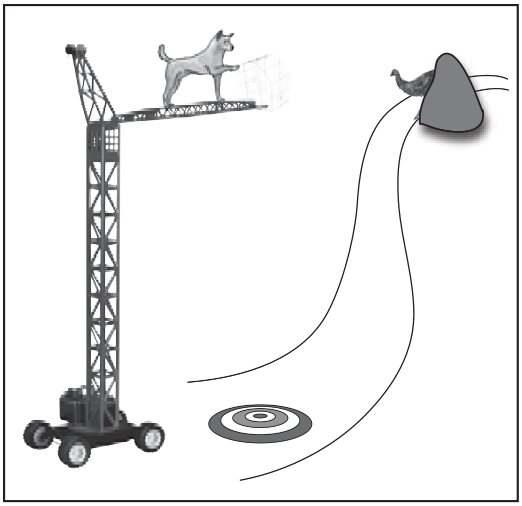
When something falls, its displacement, velocity, and acceleration are all important at the same time. So how can you pay attention to all three without missing anything? In this chapter, you'll increase your experiment, graph, and slope superpowers in preparation for bringing everything together with an equation or two.

7 Equations of motion (part 1)

Playing With Equations

It's time to take things to another level.

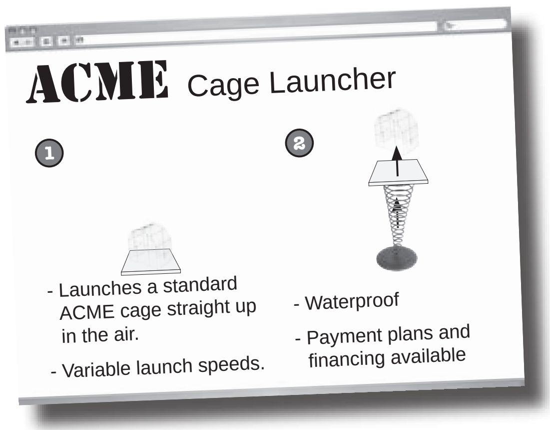
So far, you've done experiments, drawn graphs of their results, and worked out equations from them. But there's only so far you can go since sometimes your graph isn't a straight line. In this chapter, you'll expand your math skills by making **substitutions** to work out a key **equation of motion** for a curved displacement-time graph of a falling object. And you'll also learn that **checking** your GUT reaction to an answer can be a good thing.

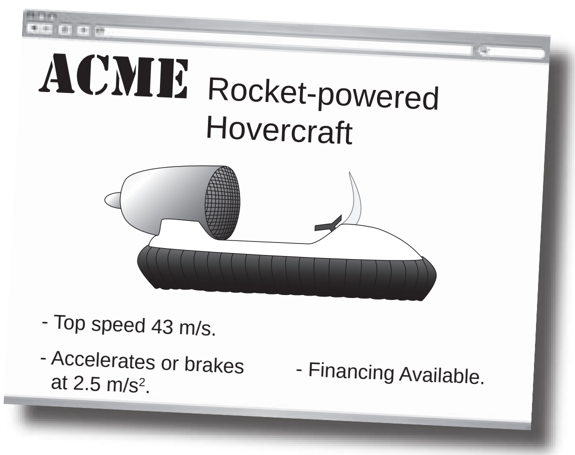


8 equations of motion (part 2)

Up, up, and... back down

What goes up must come down. You already know how to deal with things that are falling **down**, which is great. But what about the other half of the bargain - when something's launched **up** into the air? In this chapter, you'll add a third key **equation of motion** to your armory which will enable you to **deal with** (just about) **anything**! You'll also learn how looking for a little **symmetry** can turn impossible tasks into manageable ones.



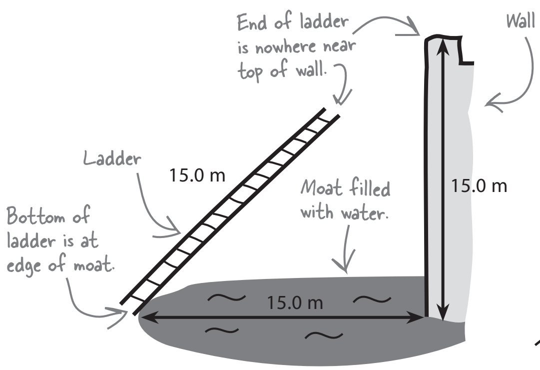


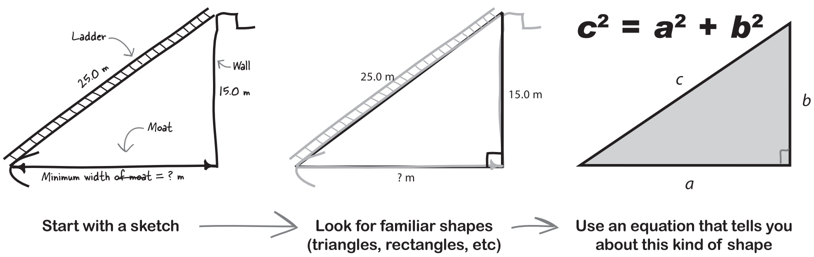
triangles, trig and trajectories

Going two-dimensional

9

So you can deal with one dimension. But what about real life? Real things don't just go up or down - they go sideways too! But never fear - you're about to gain a whole new bunch of **trigonometry** superpowers that'll see you spotting **right-angled triangles** wherever you go and using them to **reduce complicated-looking problems into simpler ones that you can already do**.

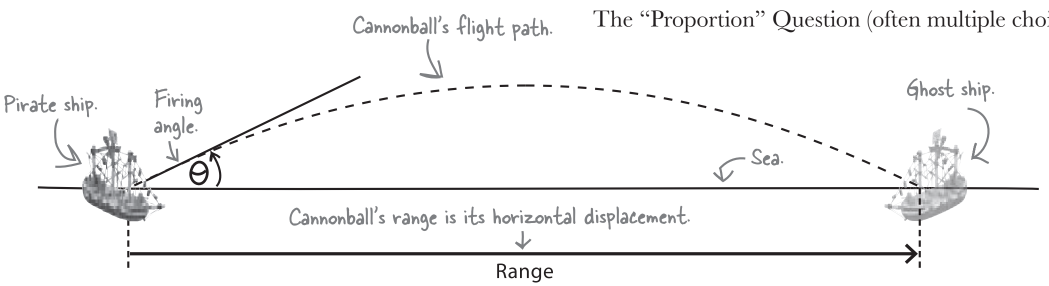




10 momentum conservation

What Newton Did

No one likes to be a pushover. So far, you've learned to deal with objects that are already moving. But what makes them go in the first place? You know that something will move if you push it - but *how* will it move? In this chapter, you'll overcome **inertia** as you get acquainted with some of **Newton's Laws**. You'll also learn about **momentum**, why it's **conserved**, and how you can use it to solve problems.

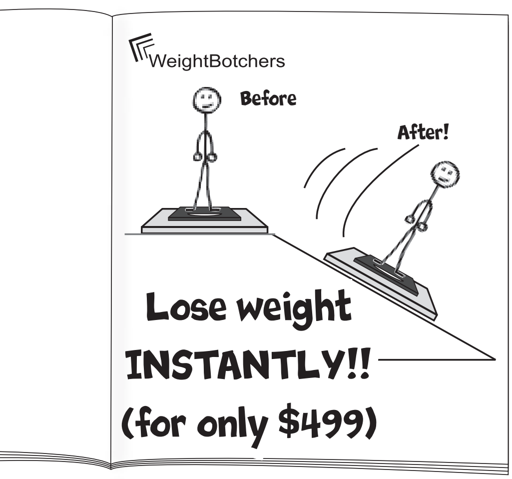


11 weight and the normal force

Forces for courses

Sometimes you have to make a statement forcefully.

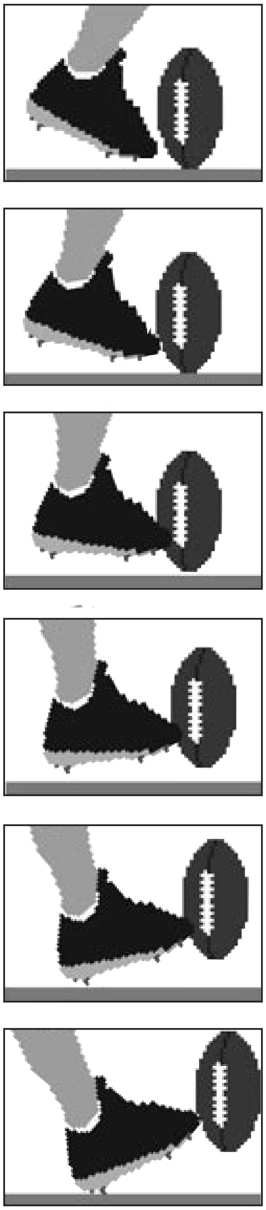
In this chapter, you'll work out **Newton's 2nd Law** from what you already know about momentum conservation to wind up with the key equation, $\mathbf{F}_{net} = m\mathbf{a}$. Once you combine this with spotting **Newton's 3rd Law force pairs**, and drawing **free body diagrams**, you'll be able to deal with (just about) anything. You'll also learn about why mass and **weight** aren't the same thing, and get used to using the **normal force** to support your arguments.



12 using forces, momentum, friction and impulse

Getting on with it

It's no good memorizing lots of theory if you can't apply it. You already know about equations of motion, component vectors, momentum conservation, free body diagrams and Newton's Laws. In this chapter, you'll learn how to fit all of these things together and apply them to solve a much **wider range** of physics problems. Often, you'll spot when a problem is **like** something you've seen before. You'll also add more realism by learning to deal with **friction** - and will see why it's sometimes appropriate to act on **impulse**.

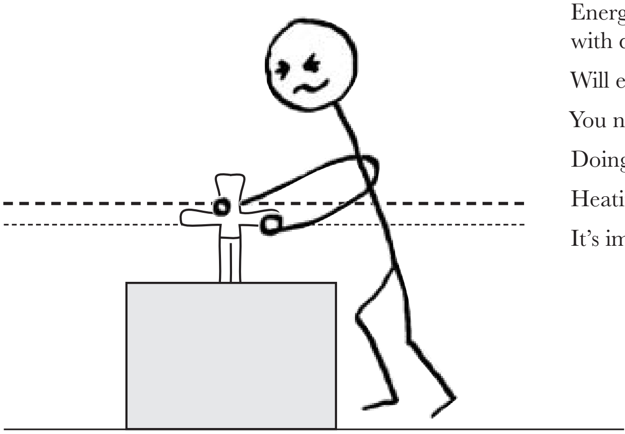


13 torque and work

Getting a lift

You can use your physics knowledge to do superhuman feats.

In this chapter, you'll learn how to harness torque to perform amazing displays of strength, by using a lever to exert a much larger force than you could on your own. However, you can't get something for nothing - **energy** is always **conserved** and the amount of **work** you do to give something **gravitational potential energy** by lifting it doesn't change.

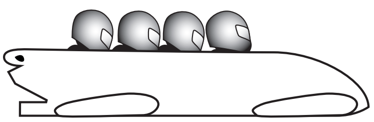


14 energy conservation

Making your life easier

Why do things the hard way when there's an easier way?

So far, you've been solving problems using equations of motion, forces and component vectors. And that's great - except that it sometimes takes a while to crunch through the math. In this chapter, you'll learn to spot where you can use **energy conservation** as a shortcut that lets you solve complicated-looking problems with relative ease.

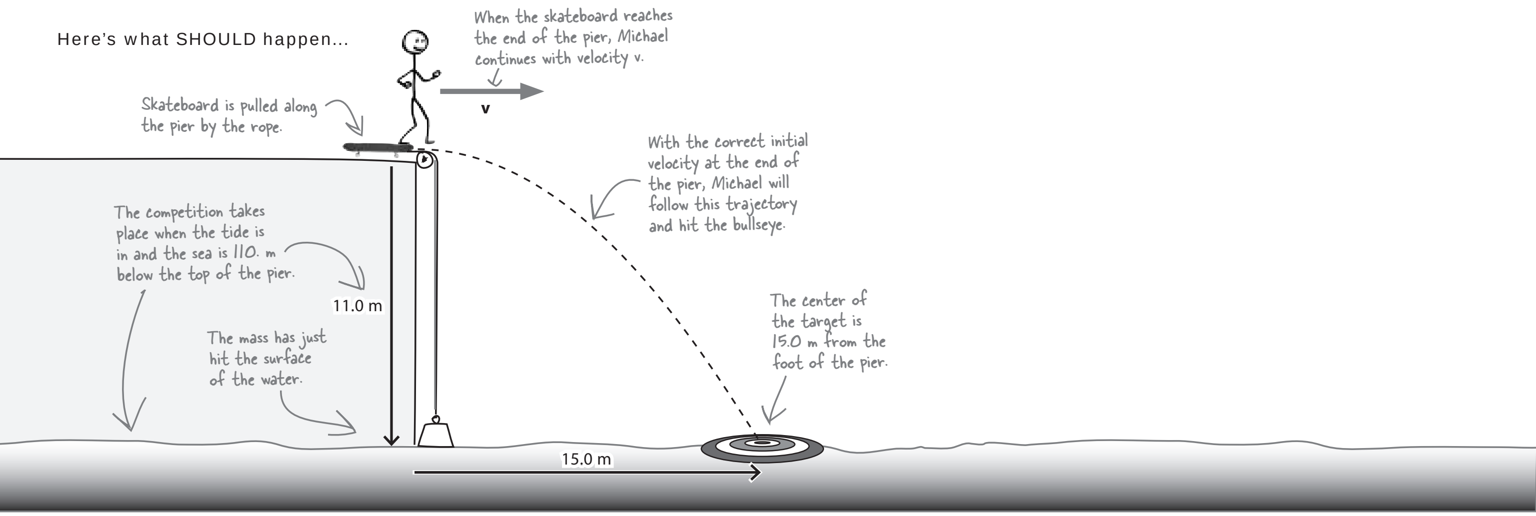


15 tension, pulleys and problem solving

Changing direction

Sometimes you need to deal with the tension in a situation

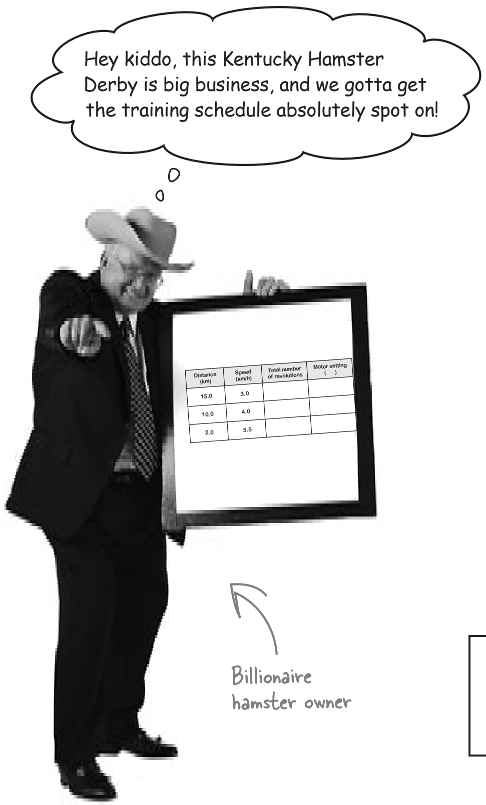
So far, you've been using forces, free body diagrams and energy conservation to solve problems. In this chapter you'll take that further as you deal with ropes, **pulleys**, and yes, **tension**. Along the way, you'll also practise looking for familiar signposts to help navigate your way through complicated situations.

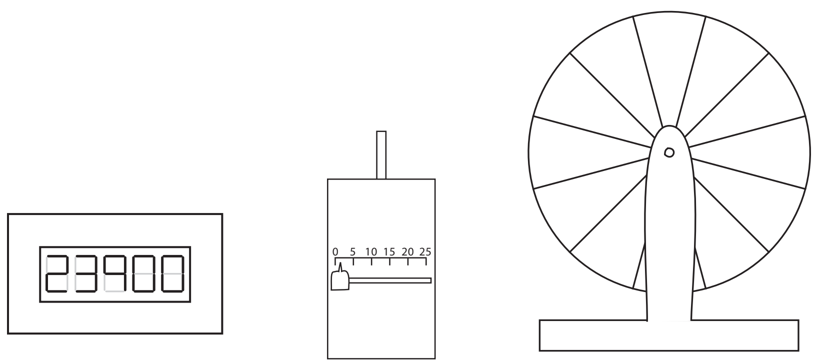


16 circular motion (part 1)

From α to ω

You say you want a revolution? In this chapter, you'll learn how to deal with **circular motion** with a crash course in **circle** anatomy, including what the **radius** and **circumference** have to do with pies (or should that be πs). After dealing with **frequency** and **period**, you'll need to switch from the **linear** to the **angular**. But once you've learned to use **radians** to measure angles, you'll know it's gonna be alright.

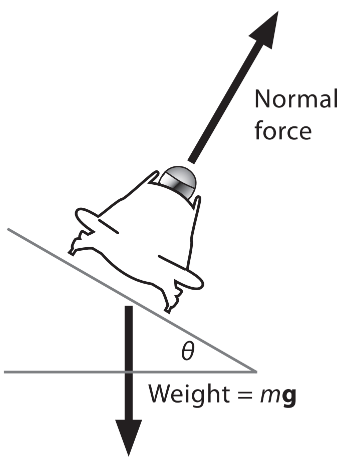




17 circular motion (part 2)

Staying on track

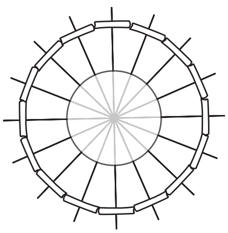
Ever feel like someone's gone off at a tangent? That's exactly what happens when you try to move an object along a **circular** path when there's not enough **centripetal force** to enable this to happen. In this chapter, you'll learn exactly what centripetal force is and how it can keep you on track. Along the way, you'll even solve some pretty serious problems with a certain Head First space station. So what are you waiting for? Turn the page, and let's get started.

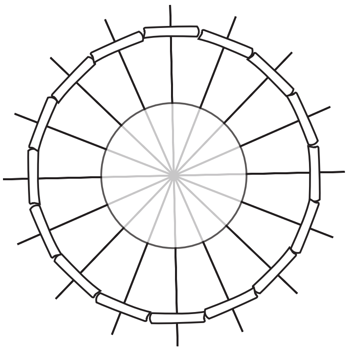


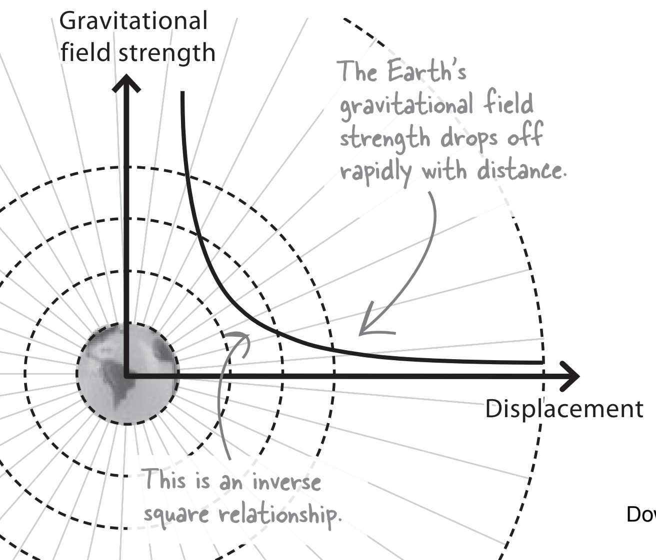
18 gravitation and orbits

Getting away from it all

So far, you've been up close and personal with gravity - but what happens to the attraction as your feet leave the ground? In this chapter, you'll learn that gravitation is an **inverse square law**, and harness the power of **gravitational potential** to take a trip to **infinity**... and beyond. Closer to home, you'll learn how to deal with **orbits** - and learn how they can revolutionize your communication skills.



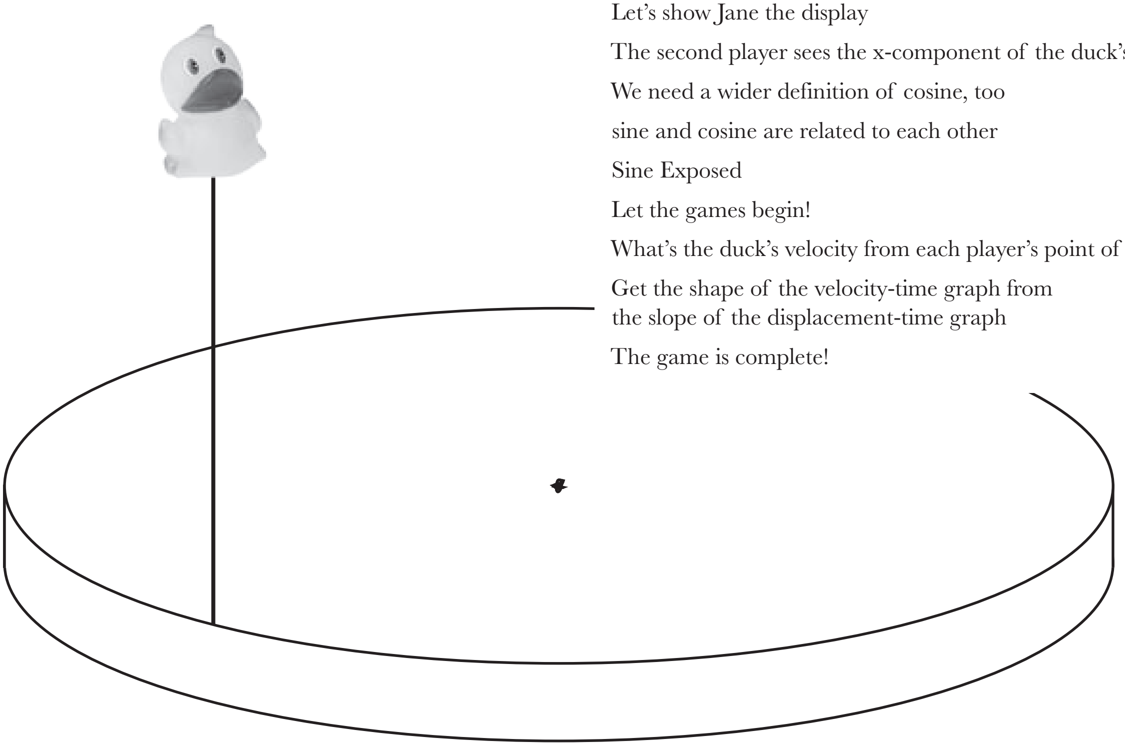




19 Oscillations (part 1)

Round and round

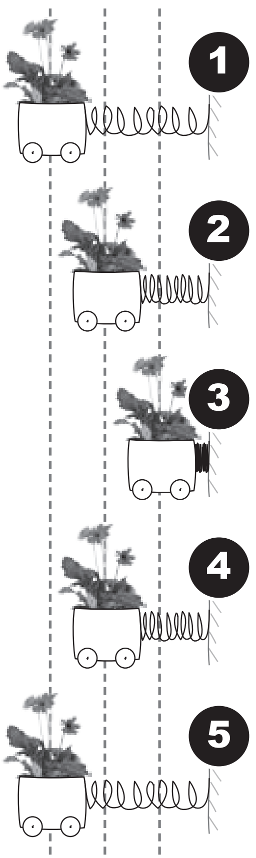
Things can look very different when you see them from another angle. So far you've been looking at circular motion from above - but what does it look like from the side? In this chapter, you'll tie together your **circular motion** and **trigonometry** superpowers as you learn extended definitions of **sine** and **cosine**. Once you're done, you'll be able to deal with anything that's moving around a circle - whichever way you look at it.



20 Oscillations (part 2)

Springs 'n' swings

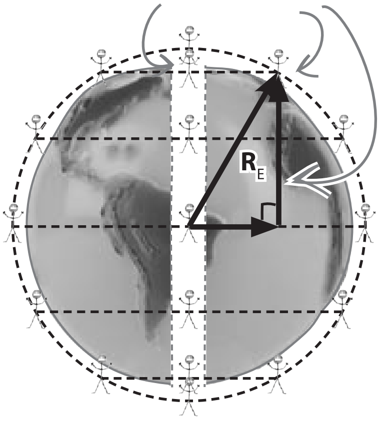
What do you do when something just happens over and over? This chapter is about dealing with **oscillations**, and helps you see the big picture. You'll put together what you know about graphs, equations, forces, energy conservation and periodic motion as you tackle springs and pendulums that move with **simple harmonic motion** to get the ultimate "I rule" experience ... without having to repeat yourself too much.



21 think like a physicist

It's the final chapter

It's time to hit the ground running. Throughout this book, you've been learning to **relate** physics to **everyday life** and have absorbed **problem solving** skills as you've gone along. In this final chapter, you'll use your new set of **physics tools** to dig into the problem we started off with - the bottomless pit through the center of the earth. The key is the question: "How can I use what I know to work out what I don't know (yet)?"



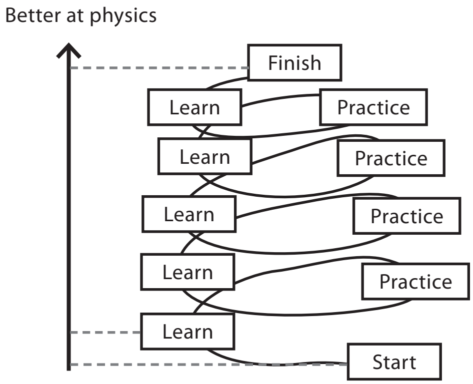
leftovers

The Top Six Things (we didn't cover)

i

No book can ever tell you everything about everything.

We've covered a lot of ground, and given you some great thinking skills and physics knowledge that will help you in the future, whether you're taking an exam or are just curious about how the world works. We had to make some really tough choices about what to include and what to leave out. Here are some topics that we didn't look at as we went along, but are still **important** and **useful**.



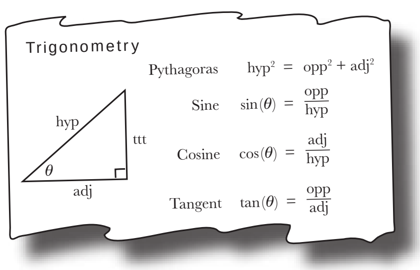
equation table

Point of Reference

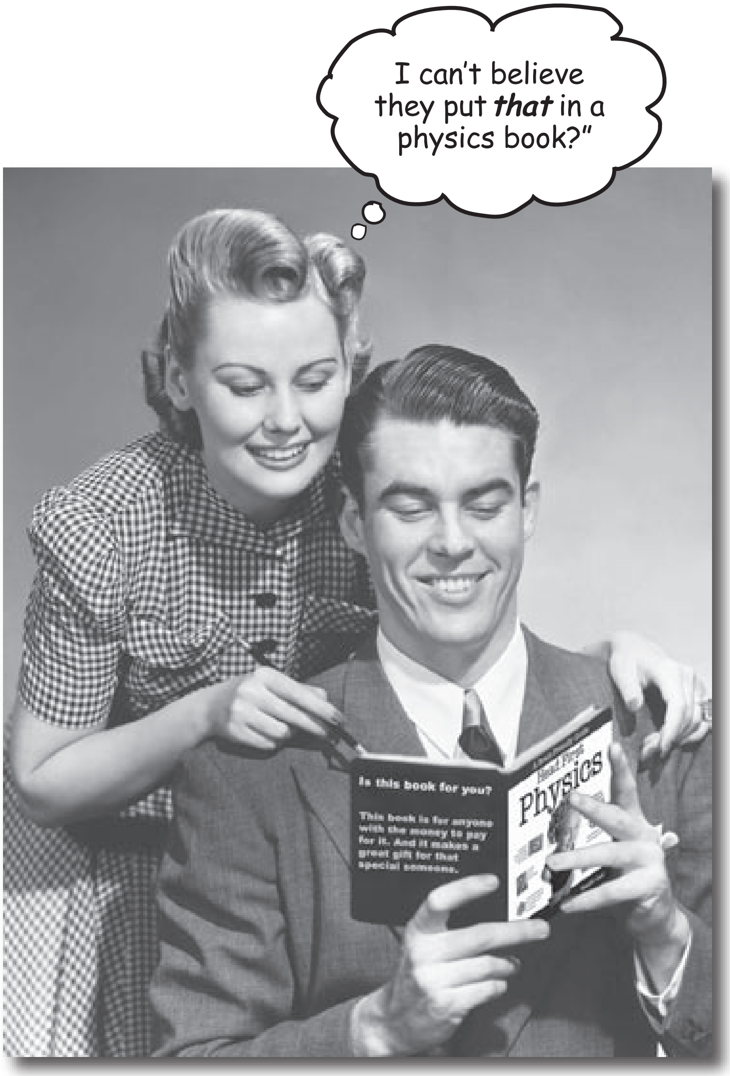
ii

It's difficult to remember something when you've only seen it once.

Equations are a major way of describing what's going on in physics. Every time you use equations to help **solve a problem**, you naturally start to become familiar with them without the need to spend time doing rote memorization. But before you get to that stage, it's good to have a place you can **look up** the equation you want to use. That's what this **equation table** appendix is for - it's a point of reference that you can turn to at any time.



Intro



In this section we answer the burning question:
"So why DID they put that in a physics book?"

Who is this book for?

If you can answer "yes" to all of these:

Don't worry if you're missing the calculator – they only cost a few dollars.

1. Do you have access to a **pen** and a **scientific calculator**?

2. Do you want to **learn** and **understand physics** by **doing**, rather than by reading, whether you need to **pass an exam at the end** or not?

3. Do you prefer **chatting with friends about interesting things** to **dry**, **dull**, **academic lectures**?

this book is for you.

Who should probably back away from this book?

If you can answer "yes" to any of these:

1. **Are you someone who's never studied basic algebra?**

   (You don't need to be advanced, but you should be able to add, subtract, multiply and divide. We'll cover everything else you need to know about math and physics.)

2. Are you a physics ninja looking for a ***reference* book**?

3. Are you **afraid to try something different**? Would you rather have a root canal than mix stripes with plaid? Do you believe that a physics book can't be serious if it involves implementing a training schedule for thoroughbred hamster racing?

this book is not for you.

[Note from marketing: this book is for anyone with the cash to buy it.]

We know what you're thinking

"How can *this* be a serious physics book?"

"What's with all the graphics?"

"Can I actually *learn* it this way?"

We know what your *brain* is thinking

Your brain craves novelty. It's always searching, scanning, *waiting* for something unusual. It was built that way, and it helps you stay alive.

So what does your brain do with all the routine, ordinary, normal things you encounter? Everything it *can* to stop them from interfering with the brain's *real* job—recording things that *matter*. It doesn't bother saving the boring things; they never make it past the "this is obviously not important" filter.

How does your brain *know* what's important? Suppose you're out for a day hike and a tiger jumps in front of you, what happens inside your head and body?

Neurons fire. Emotions crank up. *Chemicals surge*.

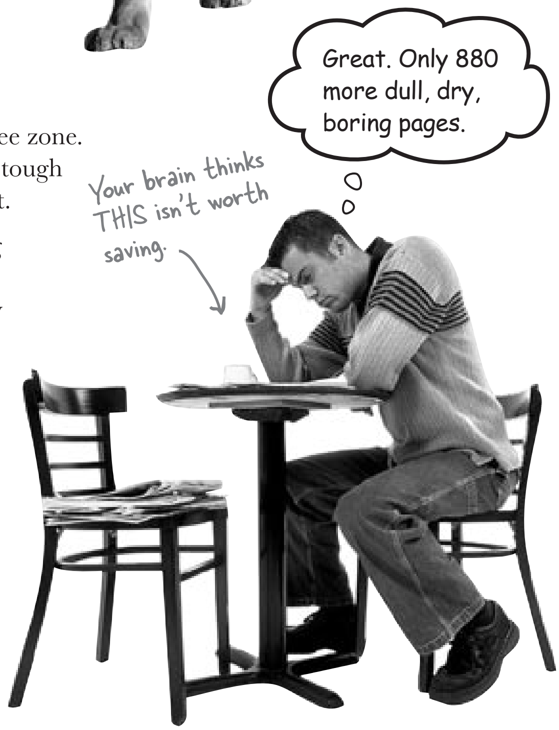
And that's how your brain knows...

This must be important! Don't forget it!

But imagine you're at home, or in a library. It's a safe, warm, tiger-free zone. You're studying. Getting ready for an exam. Or trying to learn some tough technical topic your boss thinks will take a week, ten days at the most.

Just one problem. Your brain's trying to do you a big favor. It's trying to make sure that this *obviously* non-important content doesn't clutter up scarce resources. Resources that are better spent storing the really *big* things. Like tigers. Like the danger of fire. Like how you should never have posted those photos on your Facebook page.

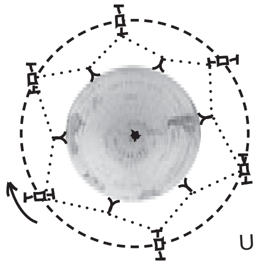
And there's no simple way to tell your brain, "Hey brain, thank you very much, but no matter how dull this book is, and how little I'm registering on the emotional Richter scale right now, I really *do* want you to keep this stuff around."



We think of a "Head First" reader as a learner.

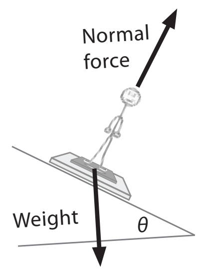
So what does it take to *learn* something? First, you have to *get* it, then make sure you don't *forget* it. It's not about pushing facts into your head. Based on the latest research in cognitive science, neurobiology, and educational psychology, *learning* takes a lot more than text on a page. We know what turns your brain on.

Some of the Head First learning principles:



Make it visual. Images are far more memorable than words alone, and make learning much more effective (up to 89% improvement in recall and transfer studies). It also makes things more understandable. Put the words within or near the graphics they relate to, rather than on the bottom or on another page, and learners will be up to *twice* as likely to solve problems related to the content.

Use a conversational and personalized style. In recent studies, students performed up to 40% better on post-learning tests if the content spoke directly to the reader, using a first-person, conversational style rather than taking a formal tone. Tell stories instead of lecturing. Use casual language. Don't take yourself too seriously. Which would *you* pay more attention to: a stimulating dinner party companion, or a lecture?



Get the learner to think more deeply. In other words, unless you actively flex your neurons, nothing much happens in your head. A reader has to be motivated, engaged, curious, and inspired to solve problems, draw conclusions, and generate new knowledge. And for that, you need challenges, exercises, and thought-provoking questions, and activities that involve both sides of the brain and multiple senses.

Get—and keep—the reader's attention. We've all had the "I really want to learn this but I can't stay awake past page one" experience. Your brain pays attention to things that are out of the ordinary, interesting, strange, eye-catching, unexpected. Learning a new, tough, technical topic doesn't have to be boring. Your brain will learn much more quickly if it's not.

Touch their emotions. We now know that your ability to remember something is largely dependent on its emotional content. You remember what you care about. You remember when you *feel* something. No, we're not talking heart-wrenching stories about a boy and his dog. We're talking emotions like surprise, curiosity, fun, "what the...?" , and the feeling of "I Rule!" that comes when you solve a puzzle, learn something everybody else thinks is hard, or realize you know something that "I'm more technical than thou" Bob from engineering *doesn't*.

Metacognition: thinking about thinking

If you really want to learn, and you want to learn more quickly and more deeply, pay attention to how you pay attention. Think about how you think. Learn how you learn.

Most of us did not take courses on metacognition or learning theory when we were growing up. We were *expected* to learn, but rarely *taught* to learn.

But we assume that if you're holding this book, you really want to learn how to do physics. And you probably don't want to spend a lot of time. If you want to use what you read in this book, you need to *remember* what you read. And for that, you've got to *understand* it. To get the most from this book, or *any* book or learning experience, take responsibility for your brain. Your brain on *this* content.

The trick is to get your brain to see the new material you're learning as Really Important. Crucial to your well-being. As important as a tiger. Otherwise, you're in for a constant battle, with your brain doing its best to keep the new content from sticking.

So just how *DO* you get your brain to treat physics like it was a hungry tiger?

There's the slow, tedious way, or the faster, more effective way. The slow way is about sheer repetition. You obviously know that you *are* able to learn and remember even the dullest of topics if you keep pounding the same thing into your brain. With enough repetition, your brain says, "This doesn't *feel* important to him, but he keeps looking at the same thing *over* and *over* and *over*, so I suppose it must be."

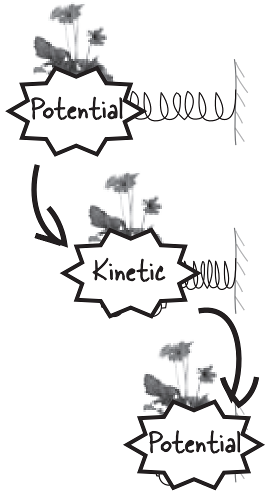
The faster way is to do ***anything that increases brain activity,*** especially different *types* of brain activity. The things on the previous page are a big part of the solution, and they're all things that have been proven to help your brain work in your favor. For example, studies show that putting words *within* the pictures they describe (as opposed to somewhere else in the page, like a caption or in the body text) causes your brain to try to makes sense of how the words and picture relate, and this causes more neurons to fire. More neurons firing = more chances for your brain to *get* that this is something worth paying attention to, and possibly recording.

A conversational style helps because people tend to pay more attention when they perceive that they're in a conversation, since they're expected to follow along and hold up their end. The amazing thing is, your brain doesn't necessarily *care* that the "conversation" is between you and a book! On the other hand, if the writing style is formal and dry, your brain perceives it the same way you experience being lectured to while sitting in a roomful of passive attendees. No need to stay awake.

But pictures and conversational style are just the beginning…

Here's what WE did:

We used ***pictures***, because your brain is tuned for visuals, not text. As far as your brain's concerned, a picture really *is* worth a thousand words. And when text and pictures work together, we embedded the text *in* the pictures because your brain works more effectively when the text is *within* the thing the text refers to, as opposed to in a caption or buried in the text somewhere.



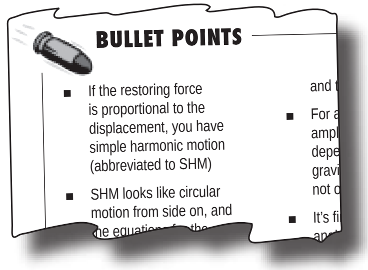
We used ***redundancy***, saying the same thing in *different* ways and with different media types, and *multiple senses*, to increase the chance that the content gets coded into more than one area of your brain.

We used concepts and pictures in ***unexpected*** ways because your brain is tuned for novelty, and we used pictures and ideas with at least *some* ***emotional*** *content*, because your brain is tuned to pay attention to the biochemistry of emotions. That which causes you to *feel* something is more likely to be remembered, even if that feeling is nothing more than a little ***humor***, ***surprise***, or ***interest.***

We used a personalized, ***conversational style***, because your brain is tuned to pay more attention when it believes you're in a conversation than if it thinks you're passively listening to a presentation. Your brain does this even when you're *reading*.

We included more than 80 ***activities***, because your brain is tuned to learn and remember more when you ***do*** things than when you *read* about things. And we made the exercises challenging-yet-do-able, because that's what most people prefer.

We used ***multiple learning styles***, because *you* might prefer step-by-step procedures, while someone else wants to understand the big picture first, and someone else just wants to see an example. But regardless of your own learning preference, *everyone* benefits from seeing the same content represented in multiple ways.

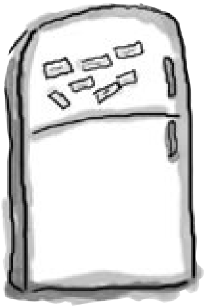


We include content for ***both sides of your brain***, because the more of your brain you engage, the more likely you are to learn and remember, and the longer you can stay focused. Since working one side of the brain often means giving the other side a chance to rest, you can be more productive at learning for a longer period of time.

And we included ***stories*** and exercises that present ***more than one point of view,*** because your brain is tuned to learn more deeply when it's forced to make evaluations and judgments.

We included ***challenges***, with exercises, and by asking ***questions*** that don't always have a straight answer, because your brain is tuned to learn and remember when it has to *work* at something. Think about it—you can't get your *body* in shape just by *watching* people at the gym. But we did our best to make sure that when you're working hard, it's on the *right* things. That ***you're not spending one extra dendrite*** processing a hard-to-understand example, or parsing difficult, jargon-laden, or overly terse text.

We used ***people***. In stories, examples, pictures, etc., because, well, because *you're* a person. And your brain pays more attention to *people* than it does to *things*.



Here's what YOU can do to bend your brain into submission

So, we did our part. The rest is up to you. These tips are a starting point; listen to your brain and figure out what works for you and what doesn't. Try new things.

1. **Slow down. The more you understand, the less you have to memorize.**

   Don't just *read*. Stop and think. When the book asks you a question, don't just skip to the answer. Imagine that someone really *is* asking the question. The more deeply you force your brain to think, the better chance you have of learning and remembering.

2. **Do the exercises. Write your own notes.**

   We put them in, but if we did them for you, that would be like having someone else do your workouts for you. And don't just *look* at the exercises. **Use a pencil.** There's plenty of evidence that physical activity *while* learning can increase the learning.

3. **Read the "There are No Dumb Questions"**

   That means all of them. They're not optional sidebars—***they're part of the core content!*** Don't skip them.

4. **Make this the last thing you read before bed. Or at least the last challenging thing.**

   Part of the learning (especially the transfer to long-term memory) happens *after* you put the book down. Your brain needs time on its own, to do more processing. If you put in something new during that processing time, some of what you just learned will be lost.

5. **Drink water. Lots of it.**

   Your brain works best in a nice bath of fluid. Dehydration (which can happen before you ever feel thirsty) decreases cognitive function.

6. **Talk about it. Out loud.**

   Speaking activates a different part of the brain. If you're trying to understand something, or increase your chance of remembering it later, say it out loud. Better still, try to explain it out loud to someone else. You'll learn more quickly, and you might uncover ideas you hadn't known were there when you were reading about it.

7. **Listen to your brain.**

   Pay attention to whether your brain is getting overloaded. If you find yourself starting to skim the surface or forget what you just read, it's time for a break. Once you go past a certain point, you won't learn faster by trying to shove more in, and you might even hurt the process.

8. **Feel something.**

   Your brain needs to know that this *matters*. Get involved with the stories. Make up your own captions for the photos. Groaning over a bad joke is *still* better than feeling nothing at all.

9. **Do lots of physics!**

   The main way to learn how to do physics is by... doing physics. And that's what you're going to do throughout this book. We're going to give you a lot of practice: every chapter has exercises that pose problems for you to solve. Don't just skip over them—a lot of the learning happens when you solve the exercises. We included a solution to each exercise—don't be afraid to **peek at the solution** if you get stuck! Look at the first couple of lines, then turn back and take it from there yourself! But try to solve the problem before you look at the solution. And definitely make sure you understand the solution before you move on to the next part of the book.

Read Me

This is a learning experience, not a reference book. We deliberately stripped out everything that might get in the way of learning whatever it is we're working on at that point in the book. And the first time through, you need to begin at the beginning, because the book makes assumptions about what you've already seen and learned.

We begin with experiments, measurements, graphs and equations, then move on to forces and energy conservation, and then more advanced topics such as gravitation and simple harmonic motion.

It's important to start with a firm foundation. We start out with the building blocks and tools of physics – experiments, measurements, graphs, equations – and most importantly, how to approach problems by thinking like a physicist. But this is no dry, theoretical introduction. Right from the word go, you'll be picking up these important skills by *solving problems yourself*. As the book goes on, your brain is freed up to learn new concepts such as Newton's Laws and energy conservation because you've already absorbed and practiced the fundamentals. By the time you reach the end of the book, you'll even be sending people into space. We teach you what you need to know at the point where it becomes important, as that's when it has the most value. Yes - even the math!

We cover the same general set of topics that are in the mechanics sections of the AP Physics B and A Level curriculums

While we focus on the overall learning experience rather than exam preparation, we provide good coverage of the mechanics sections of the AP Physics B and A Level curriculums, as well as the practical side of experiments and data analysis in physics. This means that as you work your way through the topics, you gain a deeper understanding that will help you get a good grade in whatever exam you're taking. You'll also learn how to break down complicated problems into simpler ones that you already know how to do. This is a far more effective way of learning physics than rote memorization, as you'll feel confident about tackling *any* problem even when you haven't seen one exactly like it before.

We help you out with online resources.

Our readers tell us that sometimes you need a bit of extra help, so we provide online resources, right at your fingertips. We give you an online forum where you can go to seek help, and other resources too. The starting point is

http://www.headfirstlabs.com/books/hfphy/

The activities are NOT optional.

The exercises and activities are not add-ons; they're part of the core content of the book. Some of them are to help with memory, some are for understanding, and some will help you apply what you've learned. ***Don't skip the exercises.*** The crossword puzzles are the only thing you don't *have* to do, but they're good for giving your brain a chance to think about the words and terms you've been learning in a different context.

The redundancy is intentional and important.

One distinct difference in a Head First book is that we want you to *really* get it. And we want you to finish the book remembering what you've learned. Most reference books don't have retention and recall as a goal, but this book is about *learning*, so you'll see some of the same concepts come up more than once.

The Brain Power exercises don't have answers.

For some of them, there is no right answer, and for others, part of the learning experience of the Brain Power activities is for you to decide if and when your answers are right. In some of the Brain Power exercises, you will find hints to point you in the right direction.

The technical review team

John Allister

Scott Donaldson

Georgia Gale Grant

Diane Jaquith

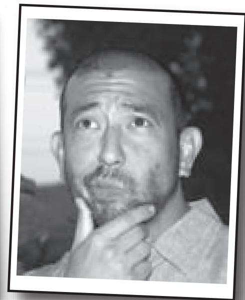
Not pictured
(but just as awesome):

Philip Kromer
Janet Painter
Don Wilke

Marion Lang

Catriona Lang

Michael Lew



Bill Mietelski

Alice Pitt-Pitts

Technical Reviewers:

John Allister has degrees from both Oxford and Cambridge universities, including a Master's in Experimental and Theoretical Physics. He taught physics for 5 years and is currently training for ordination in the Church of England.

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Georgia Gale Grant is a freelance science writer, communicator and broadcaster. She read Chemistry at Oxford University before an MSc in Science Communication at Imperial College London.

Diane Jaquith earned her Master's degree in Physics at Wesleyan University. She taught physics, chemistry, and physical science at Durham High School, Durham, CT. She later taught chemistry at Notre Dame College and Pinkerton Academy in Derry, NH.

Catriona Lang studied singing at Birmingham Conservatoire. She currently earns her living as a singing teacher.

Marion Lang is a Classics graduate from St Andrews University who now works as a Nursery Teacher and also runs NYCoS Mini Music Makers classes. She is a member of Stirling Gaelic Choir.

Bill Mietelski is a Software Engineer and a huge Head First & Kathy Sierra fan. He plans on putting the things he learned in Head First Physics to good use while improving his golf game.

Michael Lew is an AP* Physics and Computer Science teacher at Loyola High School in Los Angeles, CA and has been teaching since 1991. In his spare time, he enjoys spending time with his wife, Britt, and his three children, Mike, Jade, and Dane.

Alice Pitt-Pitts enjoyed being a guinea pig reviewer for *Head First Physics*. She also likes reading, cycling and ice cream and now knows that all of these involve energy conservation!

*AP is a registered trademark of the College Board

Acknowledgments

Catherine Nolan

The editors:

Thanks go to **Catherine Nolan** and **Brett McLaughlin**, who were editors on this project at various points, and coped admirably with the US-UK time difference. Thanks also to **Mike Loukides** for starting the whole thing off as far as Head First Physics was concerned.

Brett McLaughlin

Lou Barr

The O'Reilly team:

Thanks go to **Lou Barr** for turning my "wouldn't it be dreamy if..." thoughts into reality with any artwork that's more complicated than a line drawing. And also to **Brittany Smith**, who pulled off the impossible in the final stages of production. Plus **Laurie Petrycki**, **Caitrin McCullough, Sanders Kleinfeld, Julie Hawks, Karen Shaner** and **Keith McNamara**.

The reviewers:

Thanks to everyone on the opposite page. In particular, I'd like to mention **Donald Wilke** for his extremely detailed physics-specific comments and **John Allister** for a physics educator point of view that spanned the whole book. A lot of improvements post tech review were down to the comments of physics guinea-pigs **Marion Lang**, **Catriona Lang** and **Alice Pitt-Pitts**, who did a sterling job of pointing out where things could be clearer.

Like distributed computing, but with a physics book.

The distributed.physics project:

Between them, these heros and heroines got through a draft of the entire book in a single day...

As well as to say: "thanks," this is an experiment to test the theory that everyone mentioned in a book will buy a copy.

Alice Pitt-Pitts, Andrew Lynn, Brian Widdas, Catriona Lang, Emma Simmons, Gareth Poulton, Graham Wood, Hazel Rostron-Wood, Jason Williams, John Vinall, Marion Lang, Peter Scandrett, Robin Lang, Roger Thetford, Stephen Swain, Tim Bannister, Tim Dickinson and **Will Burt**.

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1 think like a physicist

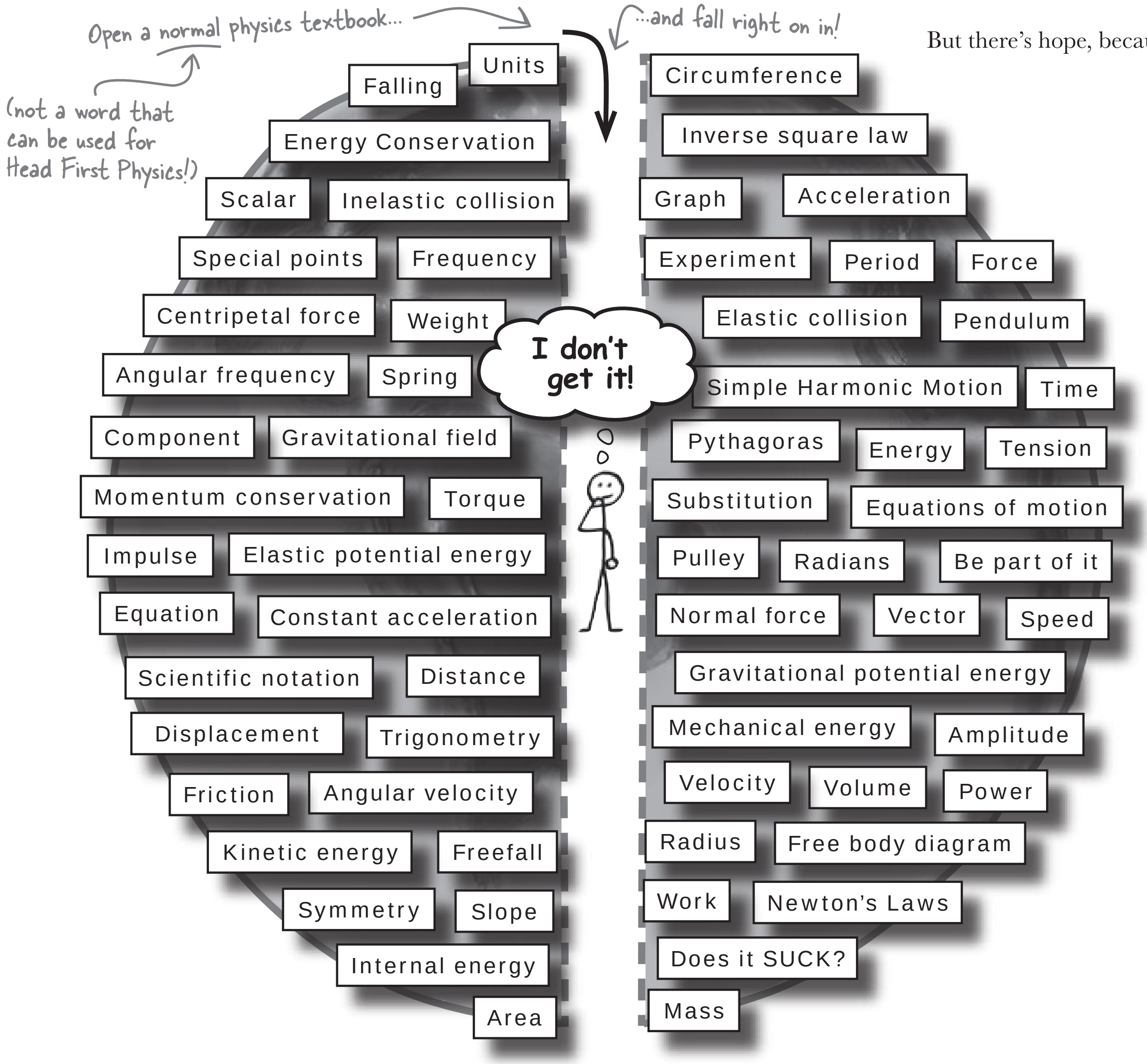
In the beginning ...

Physics is about the world around you and how everything in it works. As you go about your daily life, you're **doing** physics all the time! But the thought of actually **learning** physics may sometimes **feel** like falling into a bottomless pit with no escape! Don't worry... this chapter introduces how to **think like a physicist**. You'll learn to step into problems and to use your **intuition** to spot **patterns** and '**special points**' that make things much easier. By **being** part of the problem, you're one step closer to getting to the solution...

Physics is the world around you

Physics is about **the world around you** and how stuff in the world **actually works**. How do you aim a cannon with no direct line of sight? How can a satellite orbit the earth without falling back down? Will you win a prize shooting ducks at the fairground? Will the Dingo catch the Emu...

All of this **should** be really interesting... except that opening a normal physics textbook can make you feel rather like you've just fallen into a bottomless pit....



But there's hope, because...

You already know more than you think you do!

Honest!

Imagine you're part of a physics problem. What would you feel?

- [x] You can get a feel for what's going on by being a part of it.

Places where important or interesting things happen.

- [x] You can use your intuition to spot special points.

Where have you seen or experienced something like this before?

- [x] You can use your life experience to spot what things are like.

You don't pass physics by memorizing things. You pass physics by learning how to think about it.

This book is all about learning to think like a physicist.

You can get a feel for what's happening by being a part of it

The best way to get started with any kind of physics is to imagine that you're there, in the middle of it. Maybe you're a block, or a car, or a racing driver. Then ask yourself, "**What would I feel?**"

Which direction am I moving in?

Am I speeding up or slowing down?

Is there anything pushing or pulling me?

(and so on...)

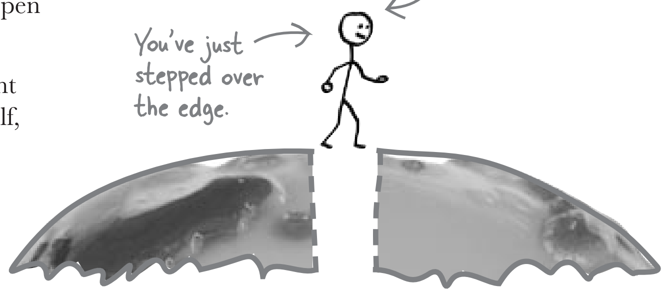
Be a part of it!

Go to the start of the problem then BE a part of it!

So - could you ever escape from the bottomless pit?

Suppose you really **are** falling into a bottomless pit that runs from one side of the world to the other. What do you think would happen (assuming that the earth isn't hot and molten inside)?

Not sure where to start? That's okay ... break it down and go right back to the beginning. **Be a part of the problem!** Ask yourself, "What would I feel just after I step into the tunnel?"



BE part of it

Your job is to imagine you just stepped out over a bottomless pit. What would you feel if you were part of the scenario?

Which direction are you moving in? Are you speeding up or slowing down?

WHY are you feeling that?

DIRECTION: ....

....

SPEED: ....

....

WHY: ....

....

Ask yourself, "What would I FEEL if I was part of the scenario?"

BE part of it - Solution

Your job is to imagine you just stepped out over a bottomless pit. What would you feel if you were part of the scenario? Which direction are you moving in? Are you speeding up or slowing down?

WHY are you feeling that?

DIRECTION: I fall down, into the tunnel.

SPEED: I get faster as I fall.

WHY: Gravity attracts me into the earth.

Normally the solutions aren't visible when you're working on the problem... but we're just getting started.

Relax

Don't worry if you wrote down something a bit different.

This is what we wrote. Your answers should be similar, but maybe not identical.

there are no Dumb Questions

Q: But all I've done is write down what I already knew and what was really obvious! I haven't worked out what happens inside the earth at all!

A: Physics is about being able to put yourself into a problem and asking "What would I feel?" When you're doing this, you need to start at the start - with what's initially going on.

Q: Why? It hasn't helped me get a final answer!

A: Starting off a question with 'obvious' things gives your brain time to calm down and settle. It's the first step towards solving a more complicated problem. Once you've made a start, you can build on it by using your intuition and experience to spot 'special points' (where important or interesting things happen) and similarities to problems you've seen before.

Q: What if I start out OK then get stuck or make a mistake? Surely I've completely failed if I don't get the right answer at the end?

A: Usually people grading exams are more interested in whether you **understand** the physics, even though the math part is important too. So, if you're able to start off in the right direction and show that you understand the important physics principles, you'll get credit for it even if you get stuck or make a mistake later on.

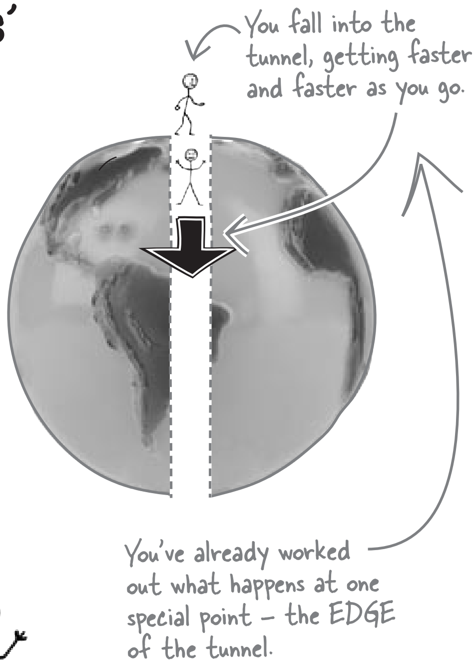
Q: But I still have no idea what happens next here!

A: You've already realized that **gravity** is important. And that you'll fall **faster** as you fall into the tunnel. That's a great start for you to build on.

Use your intuition to look for 'special points'

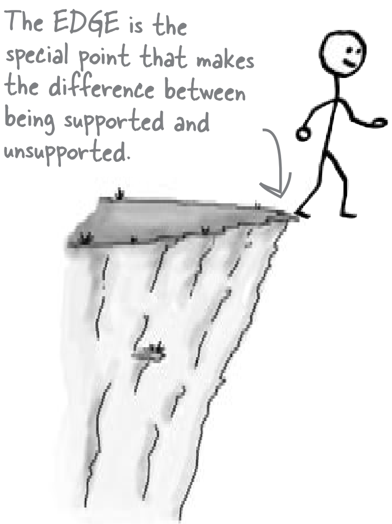
You've just started off the bottomless pit problem by **being a part of it**. You started at the start, imagined you'd just stepped over the edge, and asked yourself "What would I feel?" And you know that you'll fall into the tunnel, getting faster as you go.

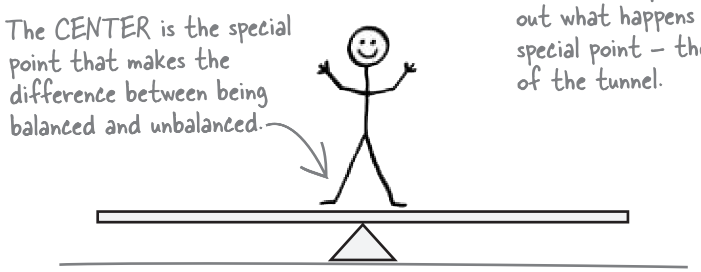
But what happens next?



The key is to use your intuition to look for **'special points**' – places where important or interesting things happen.

For example, the **edge** of a cliff is a special point because that's where you change from being supported by the ground to being unsupported. And the **center** of a seesaw is a 'special point' because it's the only place on the seesaw that one person can stand without either side going up or down.





Spotting special points then asking "**What would I feel** if I was there?" helps you to understand what's going on.

You already did that for one special point - at the **edge** of the tunnel. Now it's time to look out for more special points in this problem, so you can think about what's going on there.

Once you know what's happening at each special point you can play connect the dots and work out what's happening in between too.

In physics, the 'special points' where things happen are usually at the <u>EDGES</u> and in the <u>CENTER</u>.

BRAIN POWER

The **edge** of the tunnel is a special point. Can you spot any other special points in this problem?

So we just worked out that you fall into the tunnel from one special point - the edge. And now we're supposed to look for other special points.

Jill: I have a hunch that the **center** of the earth must be important - it just looks like it must be!

Frank: Yeah. Even though we're assuming the center of the earth isn't hot (because we're dealing with an Earth which has lots of physics words written on it), the center still looks really important!

Jill: But what's gonna happen there?

Frank: I'm not sure. I guess that either you stop, or you keep on falling. But that's not narrowing it down all that much!

Jill: Maybe we can use **what we already figured out**? Didn't we say that when you're at the surface and step over the edge of the tunnel, you fall down into the tunnel because of **gravity**?

Frank: Yeah, that's right. I guess that the earth attracts you because it's so big. Gravity is the stuff the earth's made of and the stuff you're made of attracting each other, right?

Jill: And when you're at the surface - at the edge of the tunnel - the whole of the earth is under you. So gravity attracts you downwards.

Frank: Yeah, that makes sense. So what's going on in the center? The whole Earth isn't under you any more - it's kinda all around you. There's the same amount of Earth around you in all directions!

Jill: Then you must get pulled in all directions at once. Ouch! Sounds like you'd get torn apart or something!

Frank: Hmm. The Earth's gravity isn't strong enough to pull my atoms apart when I'm standing on the surface. I have a feeling it'll be more **like** standing in the exact center of a seesaw.

Jill: You mean, kinda like a **balance** point? I guess if you were at either **end** of the seesaw - or at either end of the tunnel - you'd move. But if you're in the center of the seesaw - or the center of the earth, you're balanced.

Frank: Yeah. With the seesaw, it's like you have one foot 'pulling' you an equal amount each way, so you stay balanced. And in the center of the earth, you have half the earth on one side and half the earth on the other side. That's balanced too.

Jill: So you must stop when you reach the center of the earth if you balance there. We solved the problem - you never get out!

Frank: Hmm... but didn't we say before that you're **already** moving **very fast** by the time you reach the center?

Once you've found a special point, ask yourself: "What would I FEEL there?" and "What is being there LIKE?"

What's it SIMILAR to?

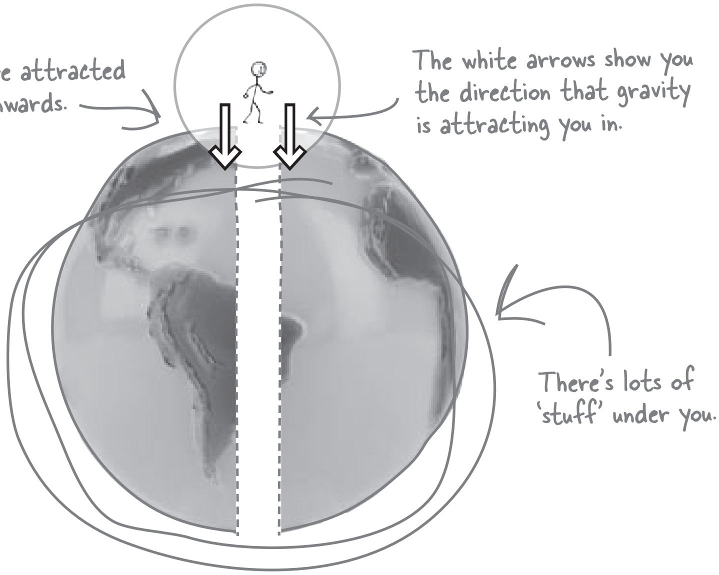
The center of the earth is a special point

As Frank and Jill have worked out, the **center** of the earth is a special point where important or interesting things might happen. Maybe your eye was drawn to the center because of the **symmetry**, as there's the same amount of Earth around you in all directions.

So - be a part of it! Imagine yourself at the center of the earth. What would I feel there? What is being there **like**?

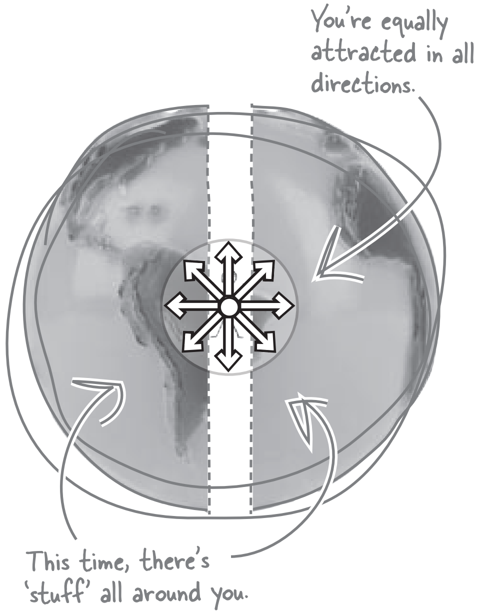
At the **first special point**—the **edge** of the tunnel—you're pulled downwards into the tunnel by gravity. This is because of the attraction between the "stuff" the earth's made of and the "stuff" you're made of.

(Stuff isn't a particularly technical term - we're basically using it to mean all of your atoms.)



At the **second special point**, in the **center** of the earth, all of your atoms are pulled equally in all directions. You aren't attracted in any direction more strongly that you are in any other direction.

Being attracted in all directions at once may not sound like fun - but you're made of stronger stuff. The Earth's gravity isn't strong enough to pull your atoms apart. This means that all of the attractions **balance** each other out.



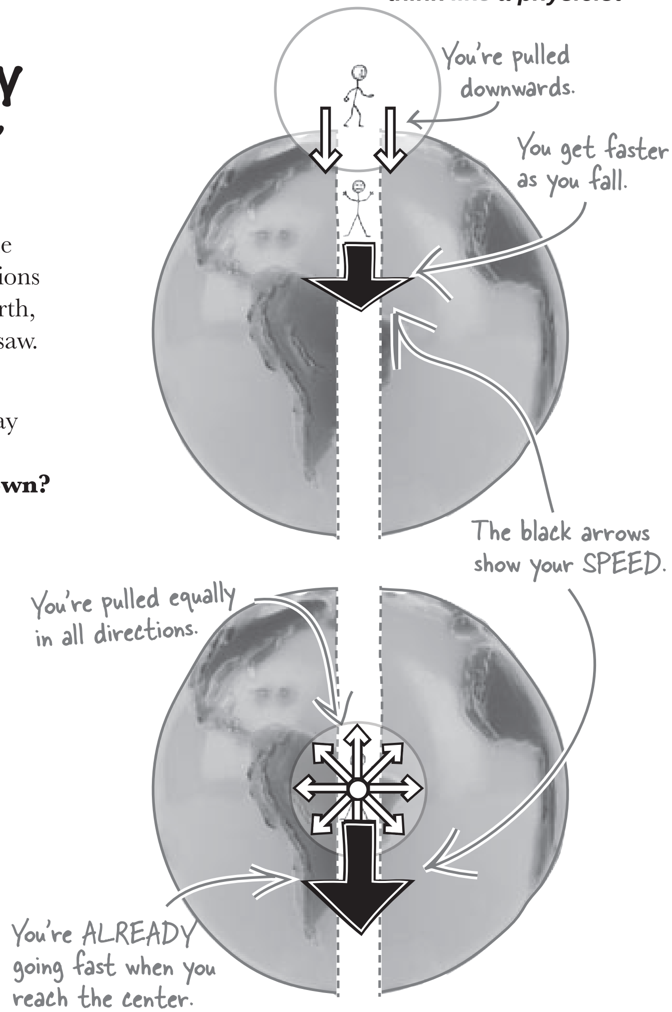
At the center, you're equally attracted in all directions - so all the gravitational attractions balance out.

Ask yourself "What am I ALREADY doing as I reach the special point?"

But Frank and Jill nearly made a big mistake when they were thinking about out what it would **feel like** in the center of the earth. At first, they thought that if all the gravitational attractions **balance** out, then you'll be stationary in the center of the earth, like you are when you balance yourself on the center of a seesaw.

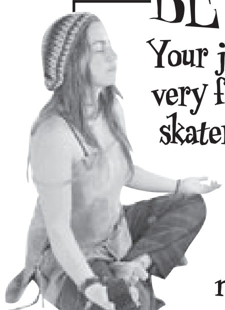
But is that really true? You're **already** going very fast when you reach the center of the earth. You've fallen a very long way to get there, moving faster and faster all the time. **If all the attractions balance out, what is there to slow you down?**

What is it **like** to be already going fast when there's nothing pulling or pushing you?



When you put yourself in a problem, try to imagine what you're ALREADY doing when you reach the special point before going on to think about what happens next.

BE part of it



Your job is to imagine that you're going very fast. Maybe you're a car or a speed skater. What it is LIKE to be going very fast when nothing can pull or push you, and you can't pull or push on anything either? That means no brakes and no grabbing on to something to slow down. Does this give you any clues about what it will be LIKE at the center of the earth when all the attractions balance out?

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BE part of it - Solution

Your job is to imagine that you're going very fast. Maybe you're a car or a speed skater. What it is LIKE to be going very fast when nothing can pull or push you, and you can't pull or push on anything either? That means no brakes and no grabbing on to something to slow down. Does this give you any clues about what it will be LIKE at the center of the earth when all the attractions balance out?

If I can't brake or grab onto anything, then I can't slow down. I'll just keep on going really fast.
I think the same thing will happen in the center of the earth. None of the directions of attraction will "win", and I'll just keep on going at the same speed.

You might have said that it was LIKE something else. That's OK. The main thing is that you keep on going at the SAME SPEED if there's nothing pushing or pulling on you.

Isn't this all a bit ridiculous? You wouldn't go at the same speed-you'd slow down from air resistance or bumping into the sides of the tunnel...or something like that... especially as the earth turns!

That's right - we've made some assumptions to turn the problem into a simpler version

We already made an **assumption** back on page 4 that (for this problem) the earth is solid and isn't hot in the middle, as getting fried isn't helpful.

And quite right - we're also assuming that air resistance doesn't slow you down and that the pit goes between the North and South poles, so you don't hit the sides as the earth turns.

In physics, the way of solving a complex problem is often to make approximations or assumptions to turn it into a simpler problem. That's OK, as you can ask yourself later on what the difference would be if you hadn't made the assumption. But only once you've got to come to grips with the simpler version.

Sometimes, including all the hard stuff right from the start makes the problem impossible to solve.

In physics, you sometimes make approximations or assumptions to turn a complex problem into a simpler version.

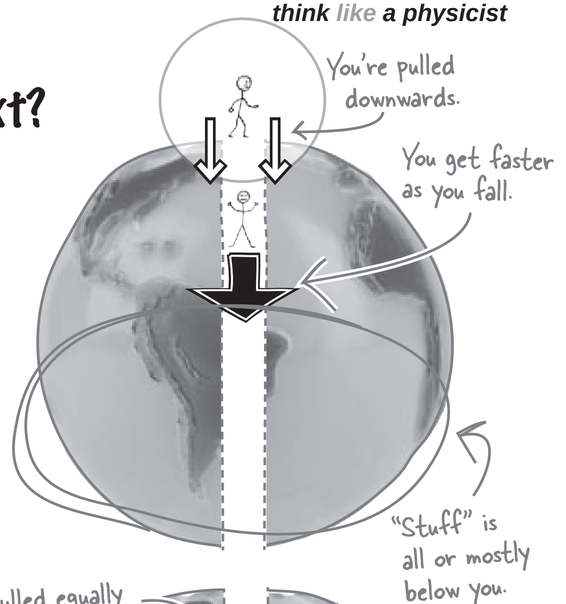
Understanding the simpler version helps you with the complex version.

Where you're at - and what happens next?

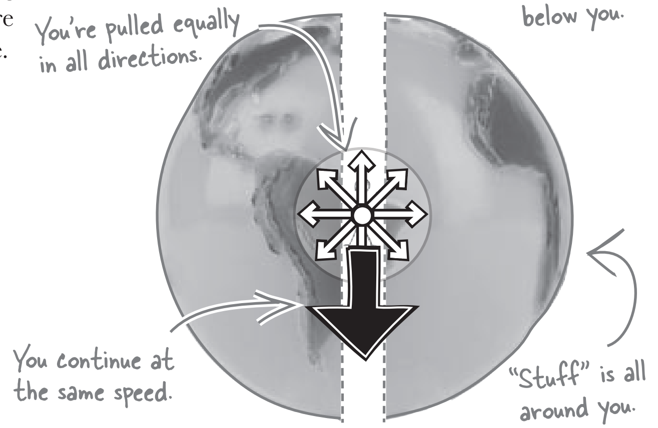
You've learned to step into the problem, so you can **be a part of it** and ask "**What would I feel**" and "**What's it like.**" This is a good way to start off and helps you see what the important things in the problem are. Here, you realized that **gravity** is important, and that you fall towards the center of the earth, getting faster and faster as you go.

You've also used your intuition to spot '**special points.**' You've spotted that the **center** of the earth is a place where the gravitational attraction between the stuff you're made of and the stuff the earth's made of is the same in all directions.

And you worked out that this means you just keep on going at the **same speed** you were already going at as you pass through the center because there's nothing to slow you down! So you're going quickly, but you aren't getting faster and faster anymore.



But now you're through the center, **what happens next**? What would you feel? What's it like? You already know that gravity is important and that its influence depends on where you and the earth are compared to each other. So - what happens next?



Sharpen your pencil

What do you think happens after you pass through the center of the earth? Do you continue at the same speed? Do you start falling faster? Do you slow down? How far do you think you keep falling? Or do you think something else happens?

Draw a picture then write down any ideas you have.

Write things on your sketch to help explain!

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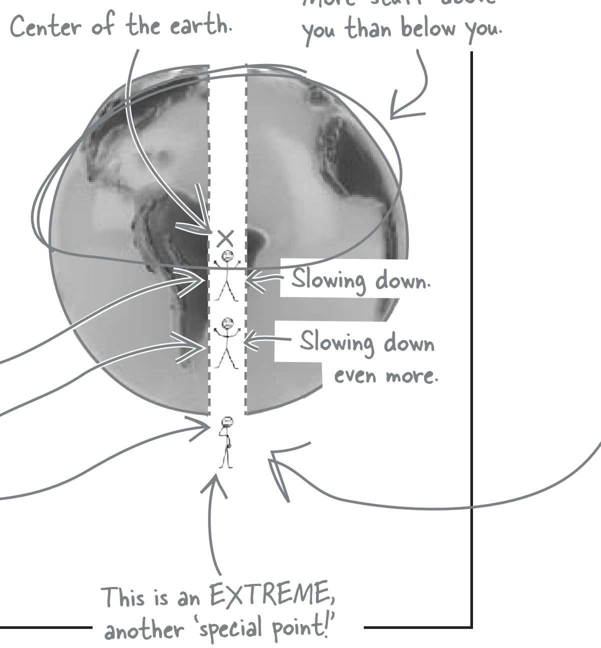
Hint: Think about where the majority of the earth's "stuff" is when you're at various points in the tunnel.

Sharpen your pencil Solution

What do you think happens after you pass through the center of the earth? Do you continue at the same speed? Do you start falling faster? Do you slow down? How far do you think you keep falling? Or do you think something else happens?

Draw a picture then write down any ideas you have.

I think that after passing through the center there starts to be more Earth above you than there is below you. This acts a bit like brakes – you're moving away from the center but gravity's attracting you back in. The further from the center you are, the more Earth's above you, so the more you slow down. I think you'll be moving slower and slower until you reach the other side of the tunnel.



You should never be afraid to ask questions!

there are no Dumb Questions

Q: I thought that gravity always speeds you up when you fall. Now you're saying it can slow you down?

A: Things are always attracted towards each other by gravity. Whether you're already moving away from the earth or moving towards it, you'll always be attracted towards the center of the earth.

Q: But that doesn't say anything about speeding up or slowing down!

A: You need to think about the speed and direction you're already traveling in. If you throw a ball up, it's moving away from the center of the earth, and it gets slower. When it comes back down again, it's moving towards the center of the earth, and it gets faster.

It looks like you're **always** attracted towards the center, right?

You're always attracted towards the center unless you're already in the center.

Right. When you're on the surface at the start, there's a lot more Earth under you than there is on top of you, and you're attracted towards the center. This makes you speed up.

When you're in the center, the attractions all cancel each other out, and you keep going at the same speed.

As you move through the center towards the other side, there's more and more Earth above you than there is below you. So you start being attracted back towards the center, which slows you down.

Now put it all together

Back on page 4, you wondered if you could ever escape from the bottomless pit. Being able to step out at the other side of the earth doesn't really count as escaping, as you'd be a very long way from where you started!

So - are you going to be able to get back home again, or are you doomed to hang out at the other end of the bottomless pit forever?

So - could you ever escape from the bottomless pit

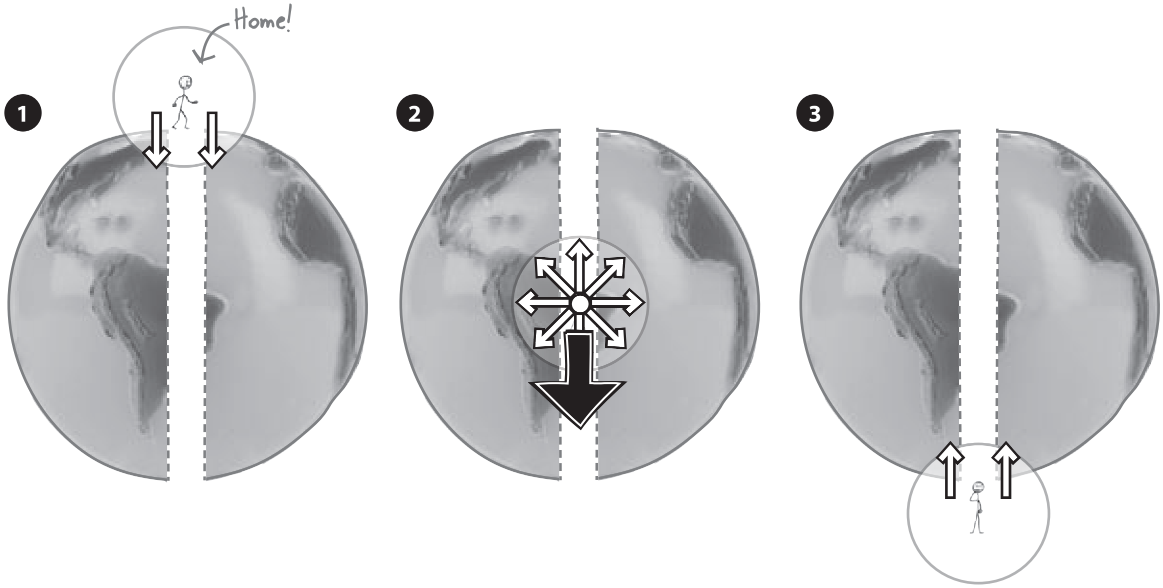
What if the bottomless pit on page 4 was real? Would you fall out the other end? Or get stuck? Or what?

That's a pretty hard thing to work out, so let's break it down and go right back to the start. Be... yo...

Sharpen your pencil

Could you ever get back home again - back to where you started before you fell in??

The pictures show what you've worked out for three special points so far. Use them to explain whether you think you'll ever be able to make it back home again.



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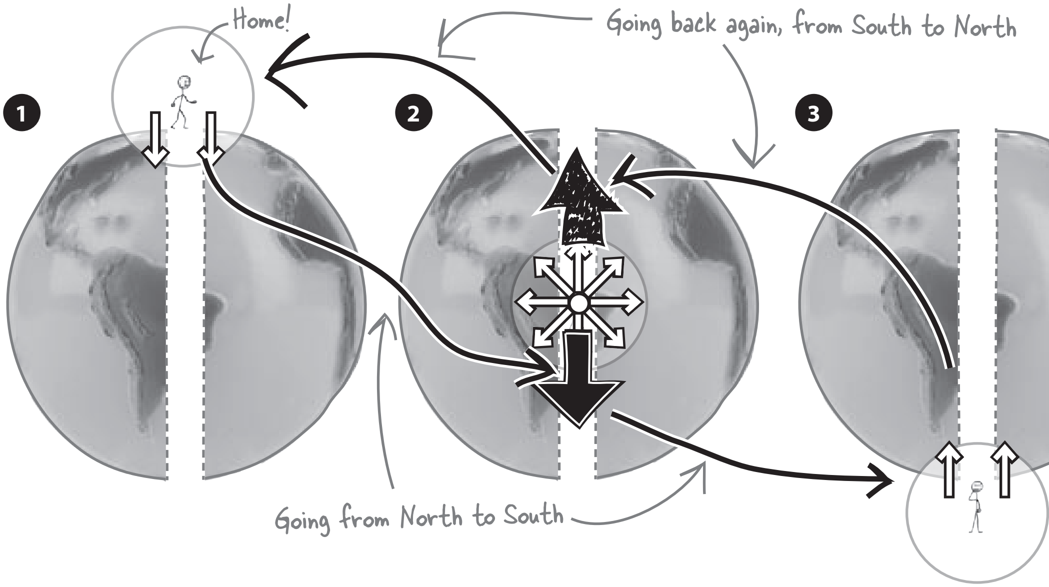
Hint: Turn the book upside-down. What's it LIKE?

Sharpen your pencil Solution

Could you ever get back home again - back to where you started before you fell in??

The pictures show what you've worked out for three special points so far. Use them to explain whether you think you'll ever be able to make it back home again.

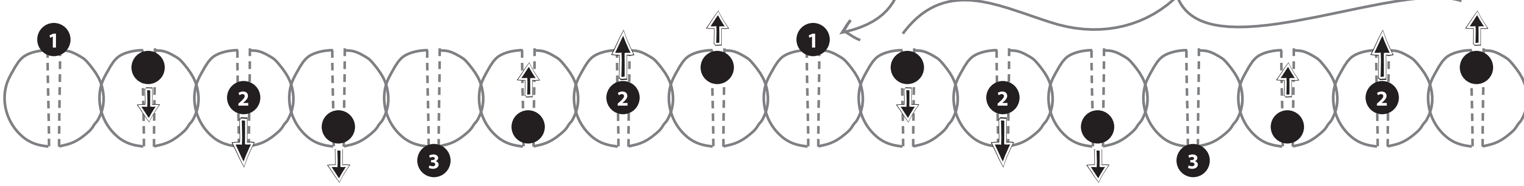


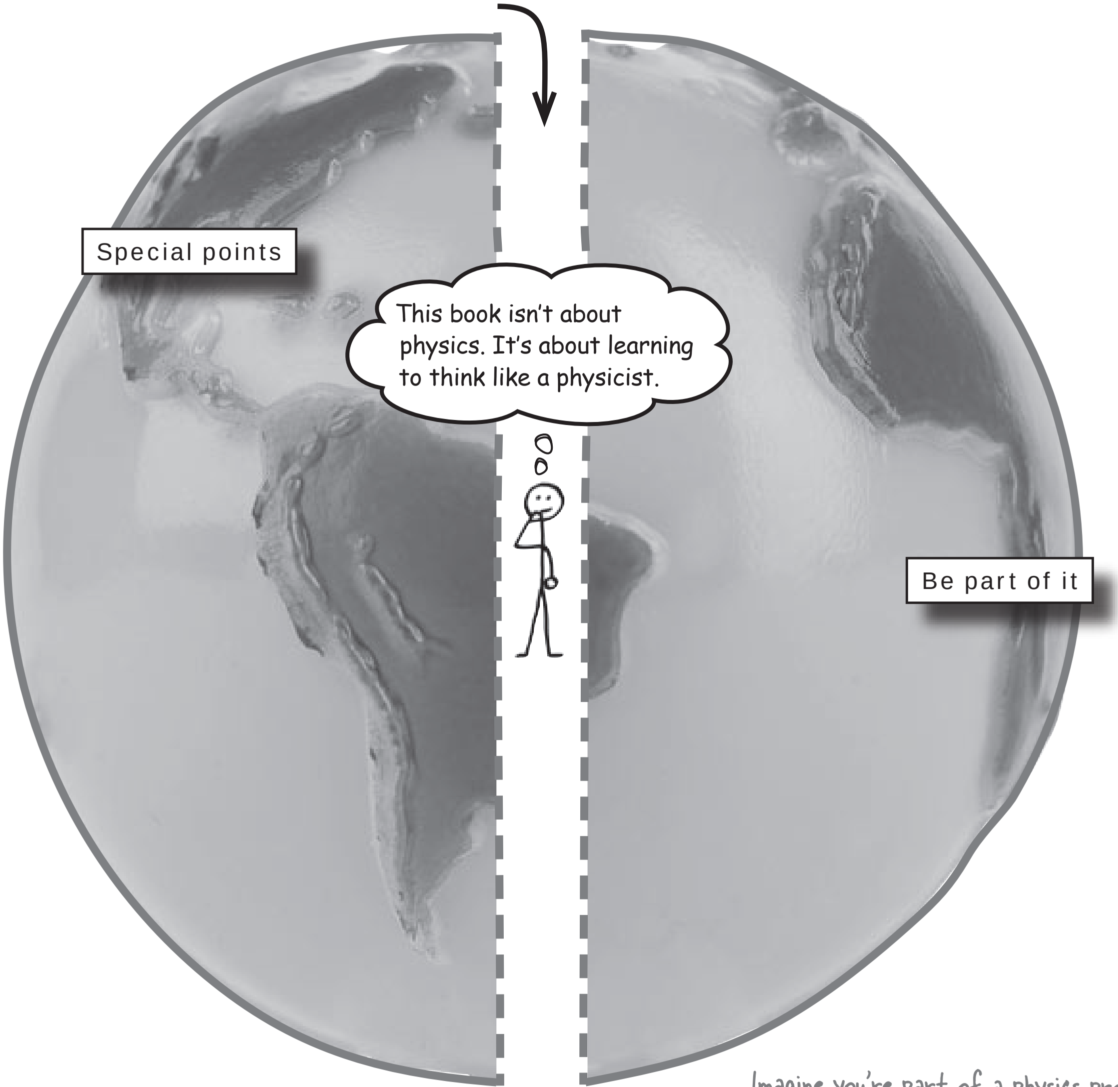
If you're at the other end of the tunnel, you can just step back in again and do the entire journey in the opposite direction! It's all exactly the same as your original trip through the earth - speed up, through center, slow down, emerge - except you're going back the way you came.

Though do be careful - if you forget to step off the Earth Express at the other end, you'll fall back into the tunnel again and keep on falling to and fro through it!

Not only can you escape, you end up on top of the world!

(As long as you remember to step off here!)





Imagine you're part of a physics problem. What would you feel?

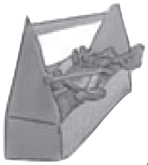
- You can get a feel for what's going on by being a part of it.

Places where important or interesting things happen.

- You can use your intuition to spot special points.

Where have you seen or experienced something like this before?

- You can use your life experience to spot what things are like.



Your Physics Toolbox

You've got Chapter 1 under your belt, and you've added some problem-solving concepts to your toolbox.

Be Part of It

Putting yourself in the heart of the problem often gives you clues about what might be happening.

You can draw on your experience because physics is all about how the world works, and you've got plenty of experience there. Imagine yourself in the scenario and ask "What would I feel?"

What's it LIKE?

You can get a long way with physics problems by asking yourself what the situation is LIKE.

It may look very different on the surface – but if you can spot an analogy or pattern and can connect the situation to something you already know how to deal with, you're sorted.

Special Points

Special points are extremes, places where important or interesting things happen.

If you work out what's going on at "special points," you can go on to "connect-the-dots" and work out what's going on in between as well.

Be visual!

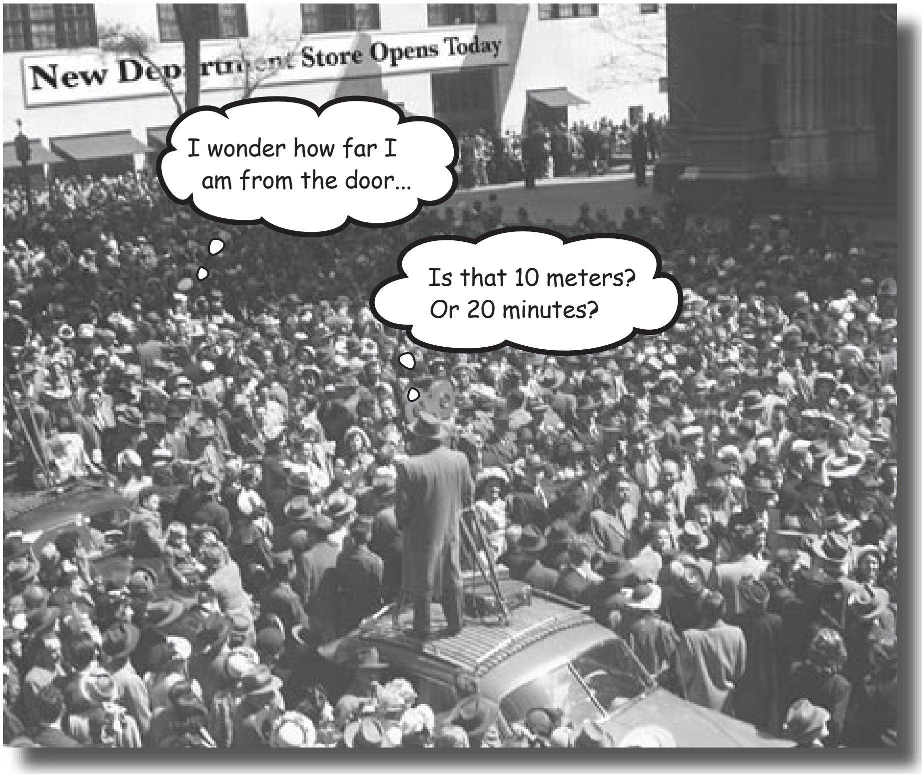
Engage the whole of your brain by "thinking aloud" in pictures as well as in words. Drawing the scenario and then annotating it with what you know is one of the most powerful tools in your toolbox.

You already know more than you think you do

Above all else, don't panic or worry about physics. You already know more than you think you do from your real life experience. And by the end of this book you'll know – and understand – a whole lot more.

2 making it all MEAN something

Units and measurements



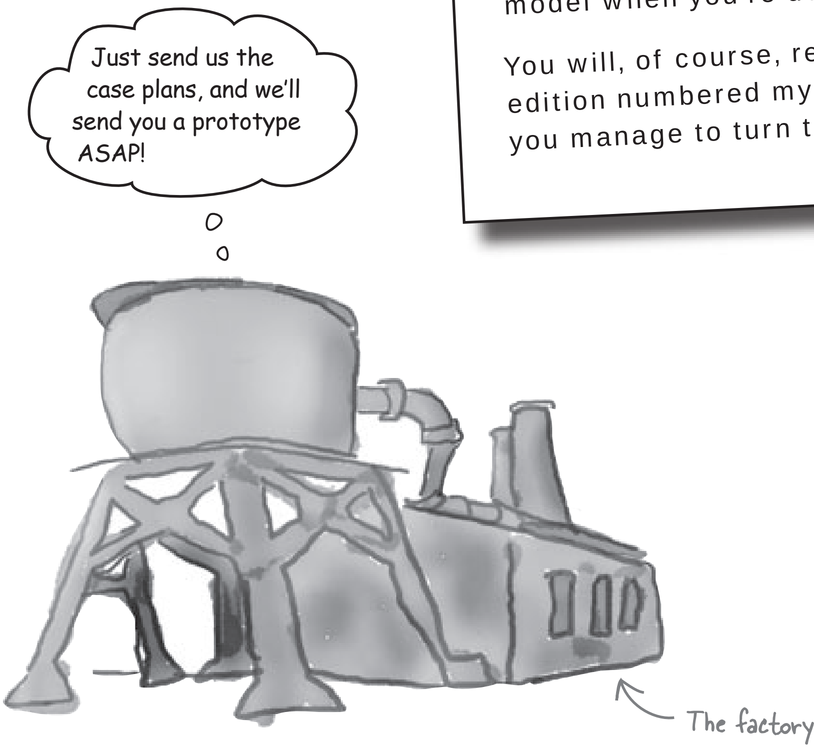
How long is a piece of string? Physics is based on making **measurements** that tell you about **size**. In this chapter, you'll learn how to use **units** and **rounding** to avoid making mistakes - and also why **errors** are OK. By the time you're through, you'll know when something is **significant** and have an opinion on whether size really **is** everything.

It's the best music player ever, and you're part of the team!

Introducing the myPod - a revolution in portable music players!

Your design team has just finished the final case prototype. Now you need to draw up the blueprints to be sent to the factory that's manufacturing the cases.

> MEMO
>
> From: myPod Case Design Team
>
> We've just sent over the latest, and hopefully final, model myPod case design.
>
> Could you draw up the plans and send them to factory where the cases are being manufactured? And send us back the myPod model when you're done.
>
> You will, of course, receive one of the limited edition numbered myPods for your troubles if you manage to turn this around quickly!

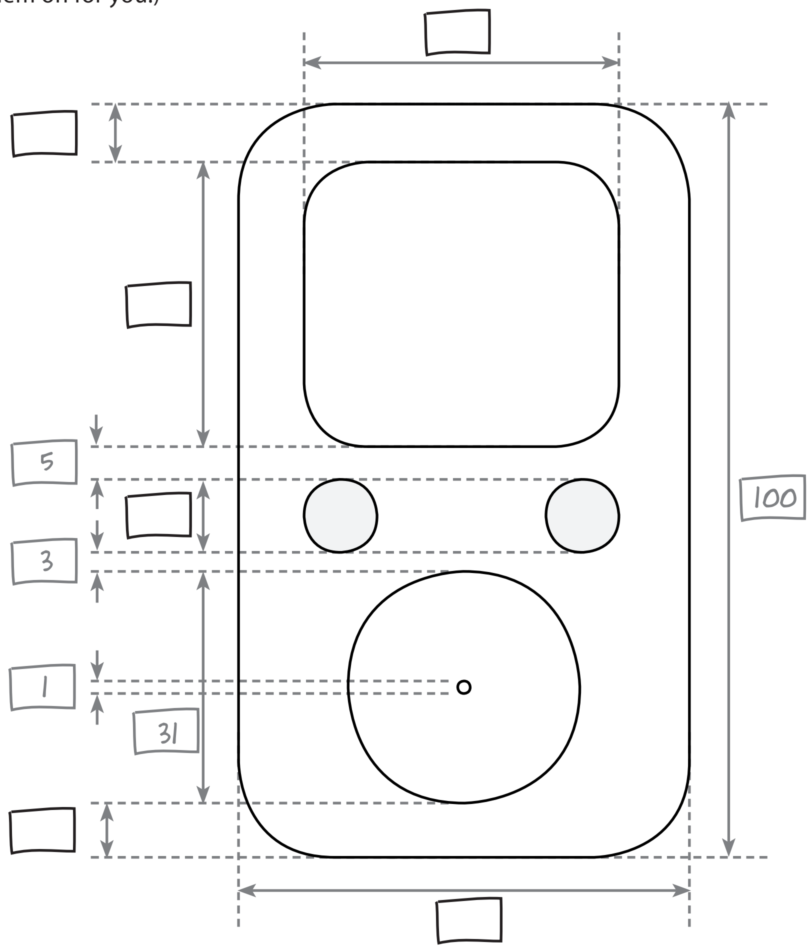


So you get on with measuring the myPod case

The quicker the factory gets the plans, the better.

Sharpen your pencil

Here's the myPod case with various lengths marked out that you'll need to measure. Cut out the ruler (or just use your own that looks similar to ours) and write in the lengths. (The myPod design team already started writing them on for you.)

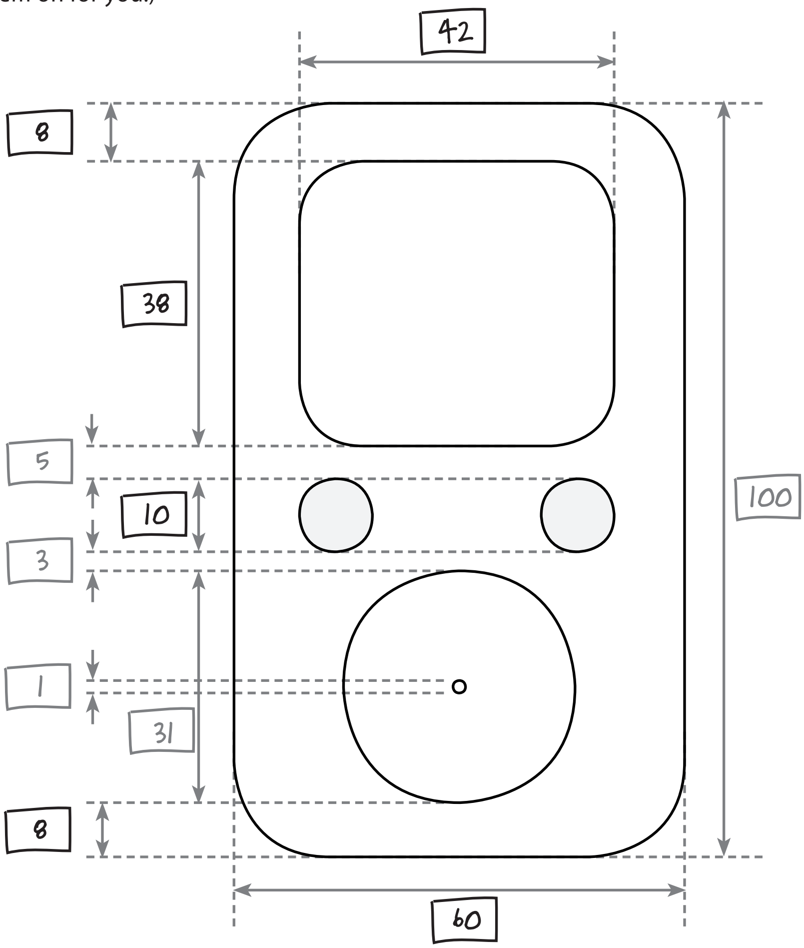


When the myPod case comes back from the factory...

After a lightning-quick turnaround, the myPod case comes back from the factory. But there's a problem.

Sharpen your pencil Solution

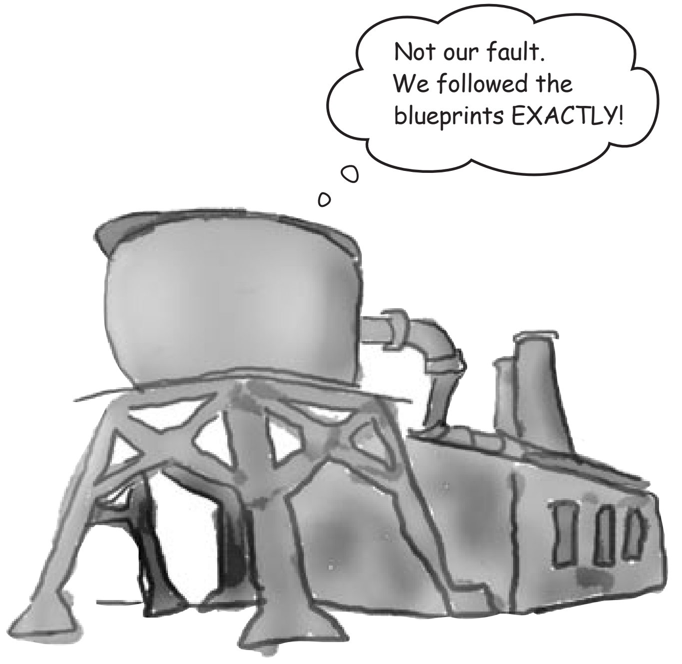
Here's the myPod case with various lengths marked out that you'll need to measure. Cut out the ruler (or just use your own that looks similar to ours) and write in the lengths. (The myPod design team already started writing them on for you.)



... it's waaay too big!

The myPod case is huge. Massive. Rocket-sized, not pocket-sized.

But when you give the factory a call, they say they followed your instructions exactly.



BRAIN POWER

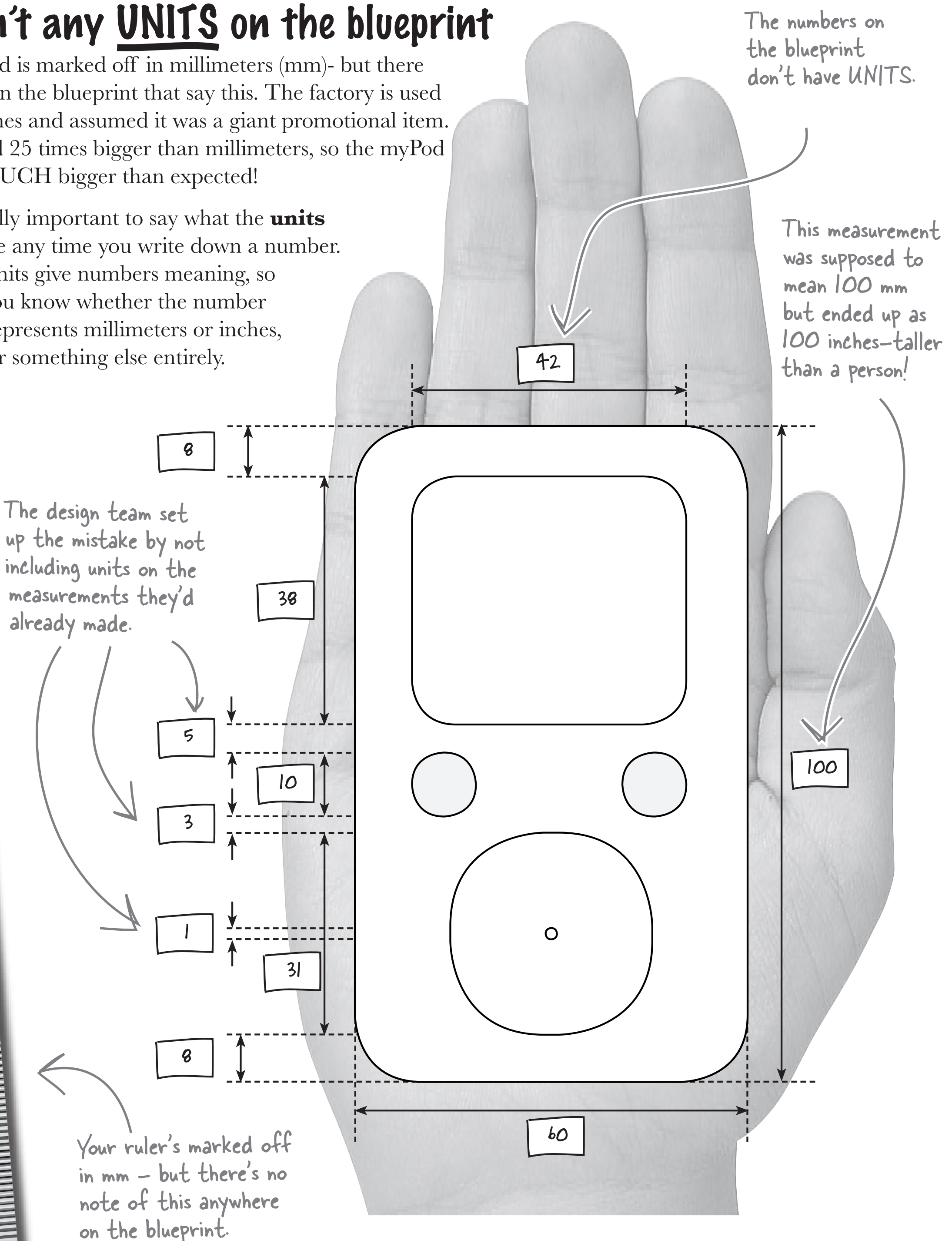
Something's obviously gone very wrong. But what?!

Have another look at your blueprint, and see if it could be interpreted differently.

There aren't any UNITS on the blueprint

The ruler you used is marked off in millimeters (mm)- but there aren't any notes on the blueprint that say this. The factory is used to working in inches and assumed it was a giant promotional item. Inches are around 25 times bigger than millimeters, so the myPod has come back MUCH bigger than expected!

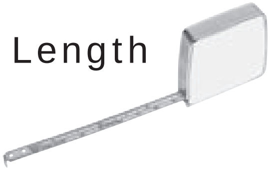
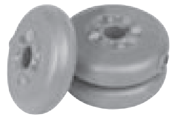
In physics, it's really important to say what the **units** are any time you write down a number. Units give numbers meaning, so you know whether the number represents millimeters or inches, or something else entirely.



A number without any units is meaningless.

Units Magnets

Throughout the book, you'll be attaching lots of different units to numbers to give them meaning. Your job is to match the units with the kind of quantity they measure. You might not have heard of all of these, but give it a shot.

Length	Time	Mass
		

Use these spaces to draw the magnets in the right columns.

inches

yards

milliseconds

milligrams

years

meters

feet

minutes

tonnes

millimeters

kilometers

hours

seconds

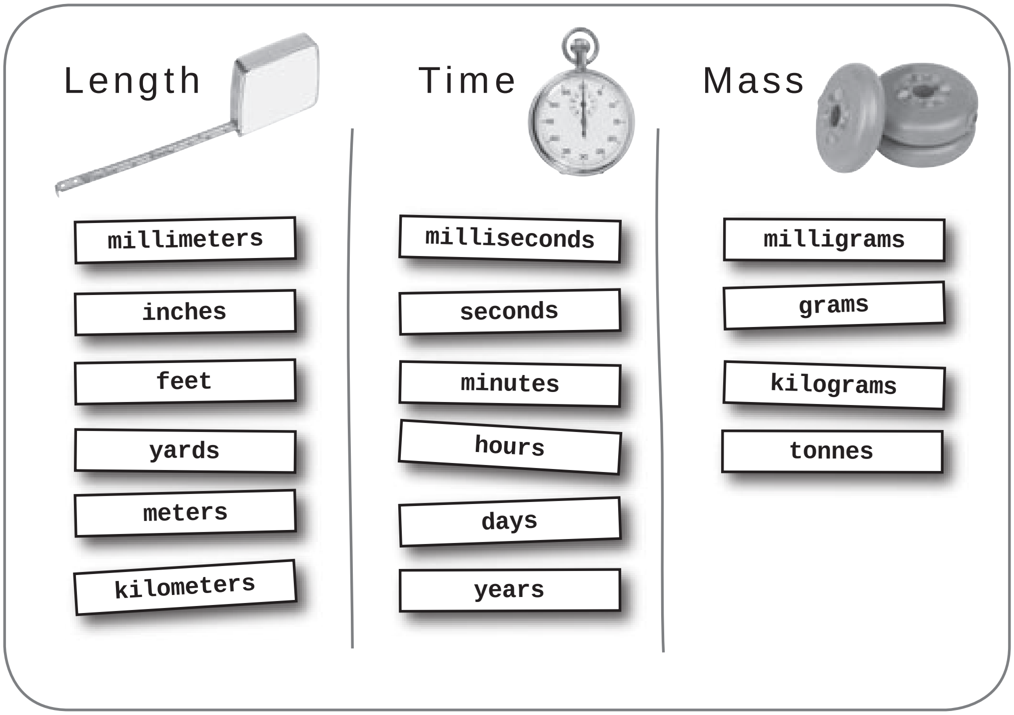
grams

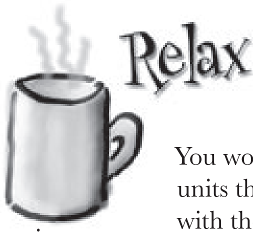
kilograms

days

Units Magnets Solution

Check your work; were you able to match these up correctly?





Don't worry if you're not familiar with all of these units just quite yet.

You won't have to work with all of these unfamiliar units throughout the book! Instead, you'll be sticking with the system used worldwide, which is what the next couple of pages are all about!

Plus you can always look up unfamiliar units.

You'll use SI units in this book (and in your class)

AP Physics B or UK A Level

The system of units used in physics worldwide is called SI (short for Système Internationale). They're much easier to use since they go up in multiples of 1000 for each 'step.'

If you're working with **lengths**, instead of having to do calculations using 12 inches in a foot, 3 feet in a yard, and 1760 yards in a mile, you have 1000 millimeters in a meter, 1000 meters in a kilometer, and so on and so forth.

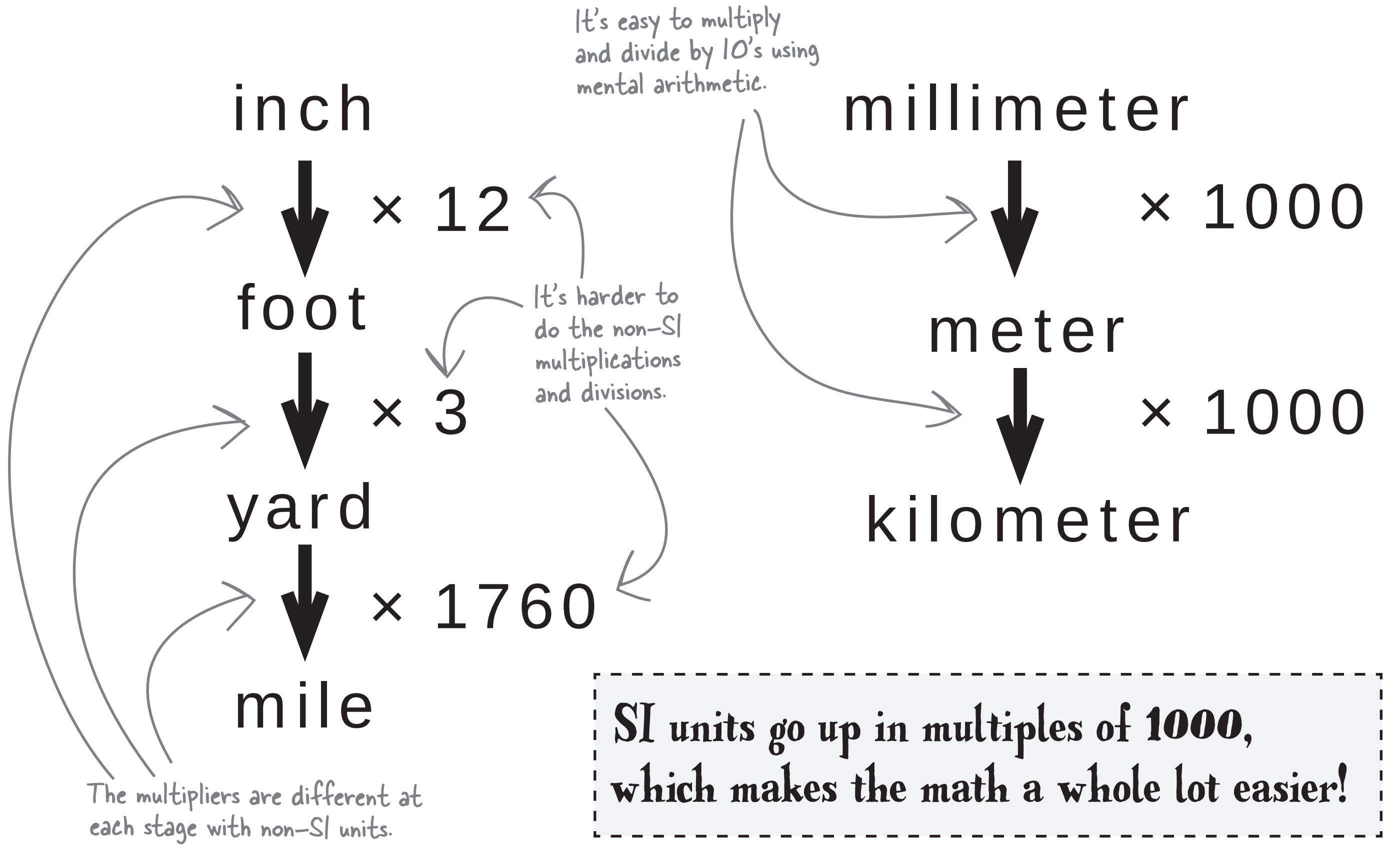
Working with lengths is much easier in SI.

And with **masses**, instead of having to remember that there are 16 ounces in a pound and 2000 pounds in a ton, you have 1000 milligrams in a gram, 1000 grams in a kilogram (about the equivalent of three cans of soda), and so on. The only SI unit which doesn't follow this convention is **time**.

And working with masses is easier too.

But time is so widely agreed on; it'd be silly to reinvent it!

Multiplying and dividing by 1000 is more straightforward mental arithmetic, so calculations involving SI units are quicker and easier than calculations with other unit systems. If you're converting meters to kilometers, you divide by 1000 (easier), but going from yards to miles involves dividing by 1760 (not straightforward, and definitely not mental arithmetic!).



SI units go up in multiples of 1000, which makes the math a whole lot easier!

there are no Dumb Questions

Q: Run it past me again - why am I being forced to use SI units when I'm more used to yards and miles? I really have no idea how much a kilogram is!

A: SI units have been used throughout the world as the basic standard in physics since 1960. SI units are an agreed worldwide standard and make sure that everyone is using the same words and definitions when they make measurements.

Q: But I don't see why I can't just use the units I'm more familiar with. Surely I'm less likely to make mistakes in calculations if I use units I'm used to?

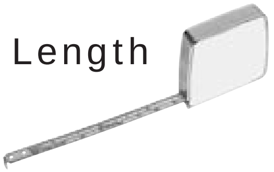
A: SI units actually make calculations easier. Instead of having to use all sorts of weird ratios to move between units (like inches, feet, yards, miles), you'll use tens. So even if they're less familiar at first, they'll be quicker and easier in the long run.

Q: But I'm not at all familiar with SI units at the moment. What kinds of units am I going to come across?

A: It's funny you should ask ...

Here are the SI units you'll use the most

Length



The SI unit of length is the **meter**.

Other related units are the **millimeter** (1000th of a meter), **centimeter** (100th of a meter), and **kilometer** (1000 meters).

Time

The SI unit of time is the **second**.

To work with time units, you'll just use common sense. There are 60 seconds in a **minute**, 60 minutes in an **hour**, 24 hours in a **day**, and 365 days in a **year**.

Mass

The SI unit of mass is the **kilogram**.

Other related units are the **gram** (1000th of a kilogram) and the **milligram** (1000th of a gram).

If you use SI units, people all over the world will understand your measurements.

there are no Dumb Questions

Q: It's a real pain to have to write out 'millimeters' or whatever every time. At least the units I'm used to have abbreviations - like lb for pounds.

A: SI units have abbreviations too! Generally, you just use the first letter of the unit - m for meters, s for seconds, and so on.

Q: OK, but what about things that start with the same letter - meters and minutes, for example?

A: The main SI unit takes precedence. The main unit for length is the meter, so it gets abbreviated to 'm'. The main SI unit for time is the second - the minute is defined as 60 seconds, so it isn't as important and usually gets abbreviated as 'min'.

Q: OK, so what about kilometers and kilograms. They start with the same FOUR letters!

A: The 'kilo' is a **prefix** that goes in front of the unit. A kilogram is 1000 times more than a gram; a kilometer is 1000 times further than a meter. The abbreviation includes the prefix as well, so kilograms are 'kg' and kilometers are 'km'.

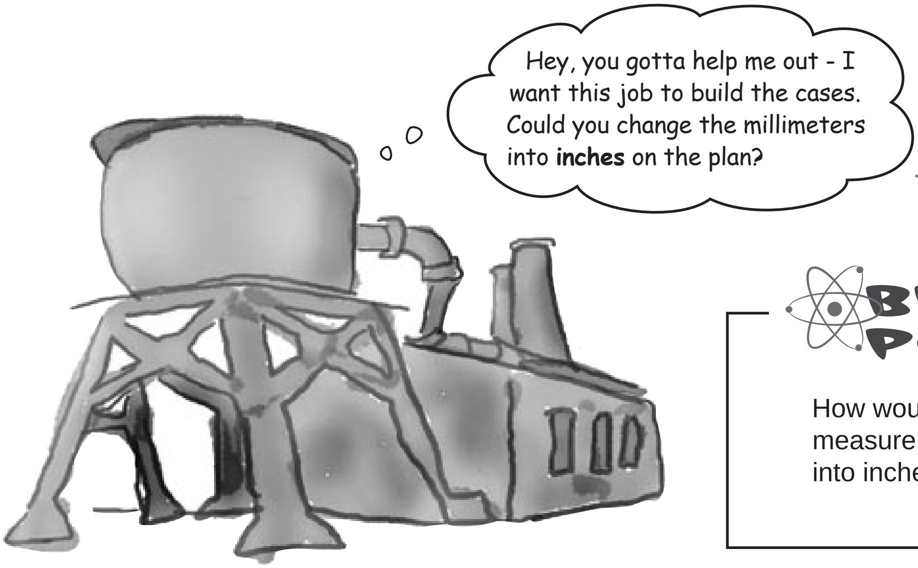
It's easier when everyone uses SI units.

Q: So "kilo" means 1000, right? But what does 'milli' mean, then? It sure meant 1000 when the millennium came around, but a millimeter and kilometer are different things, right?!

A: Great observation! Kilo is Greek for 1000, and milli is Latin for 1000. In the SI system, 'kilo' in front of a unit means it's 1000 times as big - so a kilogram is 1000 grams. And 'milli' in front of a unit means it's 1000 times smaller - so a millimeter is 1/1000th of a meter.

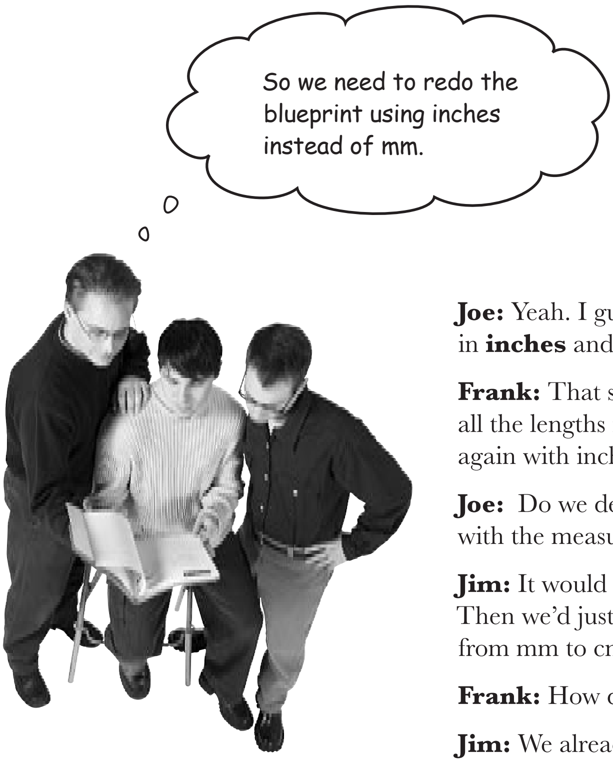
Q: I was kind of wondering something. The meter is the main SI unit, and it doesn't have a prefix before the unit. So why is the kilogram the main SI unit and not the gram? That's plain weird!

A: Most everyday physics things like cars, people, and such have masses that are a nice manageable number of kilograms, but thousands, or even millions, of grams. It was a convention that everyone ended up using from 1960 onwards. It's easier when everyone does the same thing!



BRAIN POWER

How would you convert the measurements you've already made into inches **without remeasuring**?



Joe: Yeah. I guess we can remeasure the myPod using a ruler marked off in **inches** and make a new blueprint.

Frank: That sounds like an awful lot of work. It took ages to measure all the lengths in the first place, and I can't face having to do it all over again with inches instead of mm.

Joe: Do we definitely have to remeasure though? Can we do something with the measurements we already made instead?

Jim: It would be nice if they wanted the blueprint in centimeters instead. Then we'd just have to multiply each measurement by 0.1 to **convert** it from mm to cm.

Frank: How does that work?

Jim: We already know that there are 10 mm in 1 cm, which means that 1 mm = 0.1 cm. For every mm, you have 0.1 cm. So if you multiply the number of mm by 0.1, you get the number of cm.

Frank: You mean if the measurement is 23 mm, you multiply the number of mm in the measurement by the number of cm that's equivalent to 1 mm. So 23 × 0.1 = 2.3 cm. But what about the blueprint? That needs to be in **inches**, not cm, right?

Joe: What if we find out **how may inches 1 mm is**? Can't we do exactly the same thing we just did going from mm to cm?

Jim: Hmm ... Yes, I think we could.

Frank: So we'd multiply the length in mm by the number of inches that's equivalent to 1 mm. It's the same thing that we did to **convert** a measurement from mm to cm, but it's more useful, as it's what we're actually supposed to be doing!

Joe: So we can just use a calculator to do the new plans without remeasuring. That rocks!

Jim: Let's get to it!

If you know how many inches 1 mm is, you can CONVERT your measurements from mm to inches.

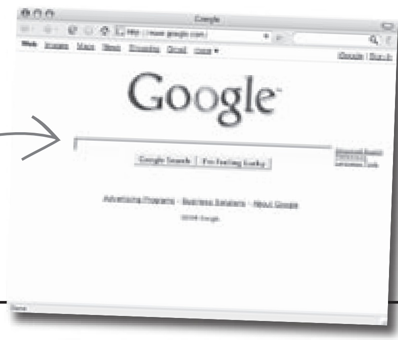
You use conversion factors to change units

At the moment you have a myPod blueprint measured in millimeters that you want to convert to inches. Although the **numbers** you've written down on the blueprint will change, converting the **units** of a measurement doesn't change its **size**. The myPod still fits comfortably in your pocket whether its size is described using millimeters or inches!

A meaningful measurement consists of both a number and its units. A **conversion factor** is the number you need to multiply your measurement by to convert it from one set of units to another. For example, if you want to convert measurements from mm to cm, you multiply by 0.1, as 1 mm = 0.1 cm.

Most physics books have a table you can use to look up less obvious conversion factors (for example, to convert non-SI units to SI units). Google Calculator can also do the same job. If you don't have access to a computer, we've also included some conversion factors in Appendix B.

"Just Google it"



Type what you want to know into the search box on the Google homepage, www.google.com.

If you're at home or in class, the quickest way of looking up a conversion factor is to use Google! You can type things like 1 mm in inches or 1 kilogram in pounds into the search box, and it automatically runs Google Calculator for you and gives you an answer!

Sharpen your pencil

Time to work out a conversion factor to change the myPod blueprint from mm to inches!

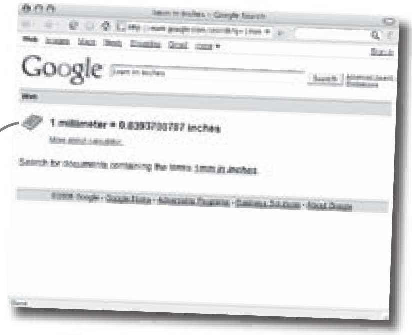
Type <u>1 mm in inches</u> into Google (or look it up in a book).

Write how many inches 1 mm is here:

....

Make sure you don't put "quotes" around this, or Google will look for web pages containing that phrase instead of looking up the conversion factor.

Remember to give your answer MEANING and CONTEXT by mentioning its UNITS.



Sharpen your pencil Solution

Time to work out a conversion factor to change the myPod blueprint from mm to inches!

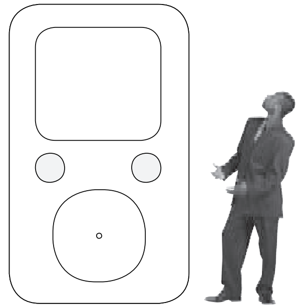
Type 1 mm in inches into Google (or look it up in a book).

Write how many inches 1 mm is here:

1 millimeter is the same as 0.0393700787 inches.

This is how you should answer exam questions– give UNITS and a CONTEXT so that your answer has MEANING.

Never ever give just a number... or this happens!



You can write a conversion factor as a fraction

Now that you know that 1 millimeter is the same as 0.0393700787 inches, you need to do some math to convert the other myPod measurements to inches.

The key is writing your conversion factor as a **fraction** so that the top and bottom of the fraction are both the same size:

These two lengths are equivalent.

$$\frac{0.0393700787 \text{ inches}}{1 \text{ millimeter}}$$

This fraction = 1, as the top and bottom are both the same size.

You can then multiply your measurement by the conversion factor fraction. Since the top and bottom of the fraction are the same size (or length), multiplying by the fraction is the same as multiplying by 1, and the **size** of your measurement doesn't change.

But multiplying by the conversion factor will change the **units** of your measurement, which is what you want to do!

If you multiply a measurement by a conversion factor, you don't change its size, but DO change its UNITS.

I guess the conversion factor fraction could be written either way- with the inches on the top or the millimeters on the top. So I need to think about which way to write it.

With fractions, you can divide out units like you'd divide out numbers.

You can write the conversion factor fraction either way up:

The top and bottom are both the same size.

$$\frac{1 \text{ millimeter}}{0.0393700787 \text{ inches}} = \frac{0.0393700787 \text{ inches}}{1 \text{ millimeter}} = 1$$

To work out the width of the player (60 mm) in inches, we want to end up with units in inches at the end, so use the conversion factor fraction that makes the millimeters divide out.

You have millimeters on the top and millimeters on the bottom, so they divide out.

$$60 \text{ millimeters in inches} = 60 \cancel{\text{millimeters}} \times \frac{0.0393700787 \text{ inches}}{1 \cancel{\text{millimeter}}}$$

$$= 2.362204722 \text{ inches}$$

The only units you're left with are inches – which is what you want.

Think about the SIZE of your answer. You're expecting a number less than 60, so this looks OK.

there are no Dumb Questions

Q: I don't get the "conversion factor is equal to 1" thing. How can that be when there are different numbers on the top and bottom?

A: There aren't just different numbers on the top and bottom of the fraction - there are also different units.

The top and bottom of the fraction are exactly the same size, so the fraction equals 1. The numbers are different because they're expressed in different units.

Q: And being able to write the fraction in two different ways isn't a problem because I can work out which one to use from the units?

A: Yes. You want the old units to divide out so that you're just left with the new units. Set up your fraction in a way that ensures this will happen.

Or you can think, "Do I expect the answer to be bigger or smaller than the number I started with?" That works too.

Q: So far, we've wanted to go from millimeters to inches. But if I wanted to go FROM inches TO millimeters, would I just turn the fraction the other way up?

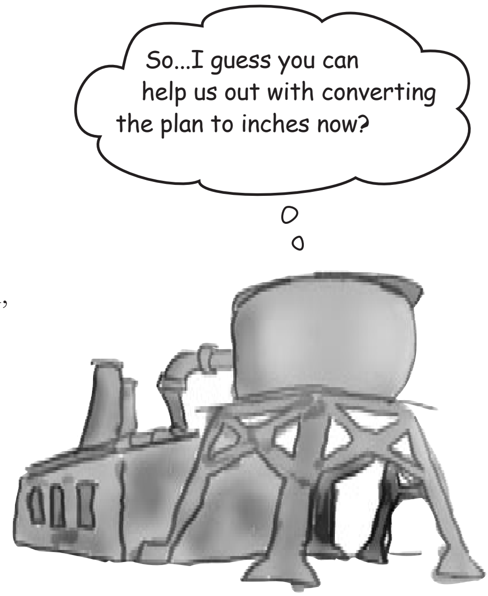
A: Absolutely! Though always do an error-check just in case. Ask yourself if the units are going to divide out, and check that the answer's around the size you expected.

Units help you keep track of trickier problems.

Here you're only converting one set of units. But sooner than you think, you'll be asked to work out how many seconds are in a year, which will involve you converting to minutes, then hours, then days, and finally years - a calculation that uses five different units in total!

So it's best to practice simpler problems using the same techniques you'd have to use for more difficult problems. It's like practicing individual tennis shots over and over to perfect your technique. Then when you face a difficult opponent, the shot is totally second nature, and your brain is free to think about the match situation.

There's also the fact that **examiners will reward you for showing your work**, even if you get the final answer wrong! In your exam, you get rewarded for demonstrating that you **understand the physics** - and that means you must show the examiners how you worked out your answer.



Now you can use the conversion factor to update the blueprint

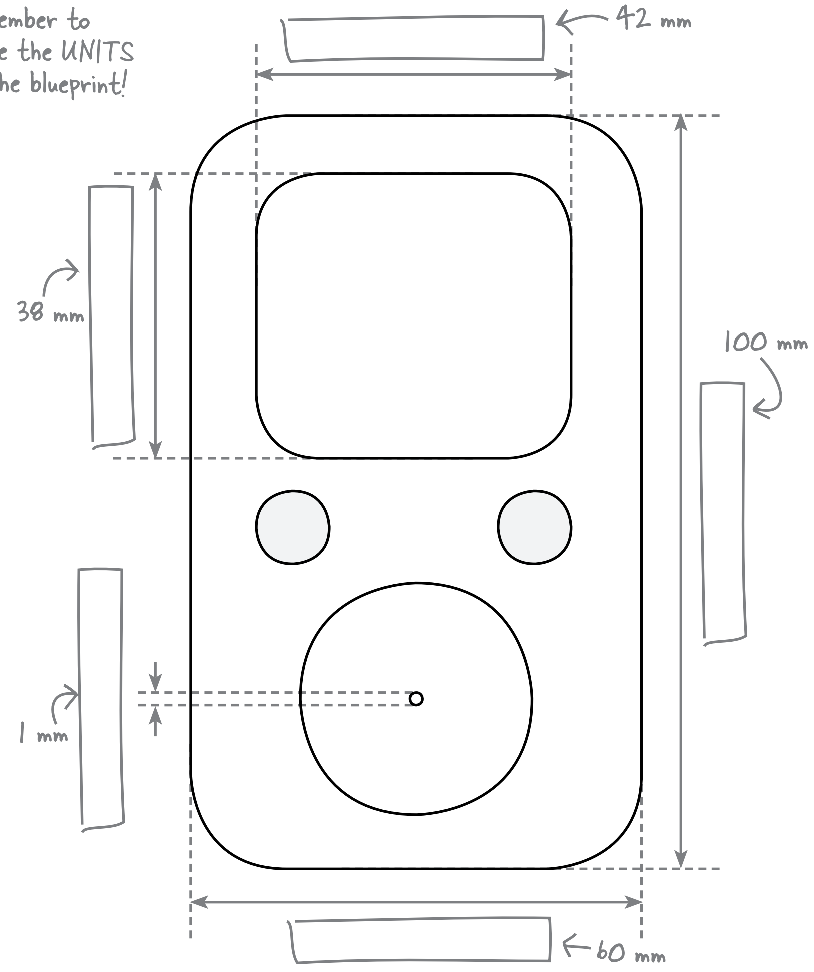
OK, so you used Google Calculator to find out that **1 mm = 0.0393700787 inches**. Now it's time to modify the blueprint so that it uses inches instead of millimeters, and the factory can cope with it.

Sharpen your pencil

Here's the blueprint with some of the millimeter lengths marked. **Convert** them to inches and earn that limited edition player! Remember to **show your work** (there's space down the right for that).

There's some space over here for your work.

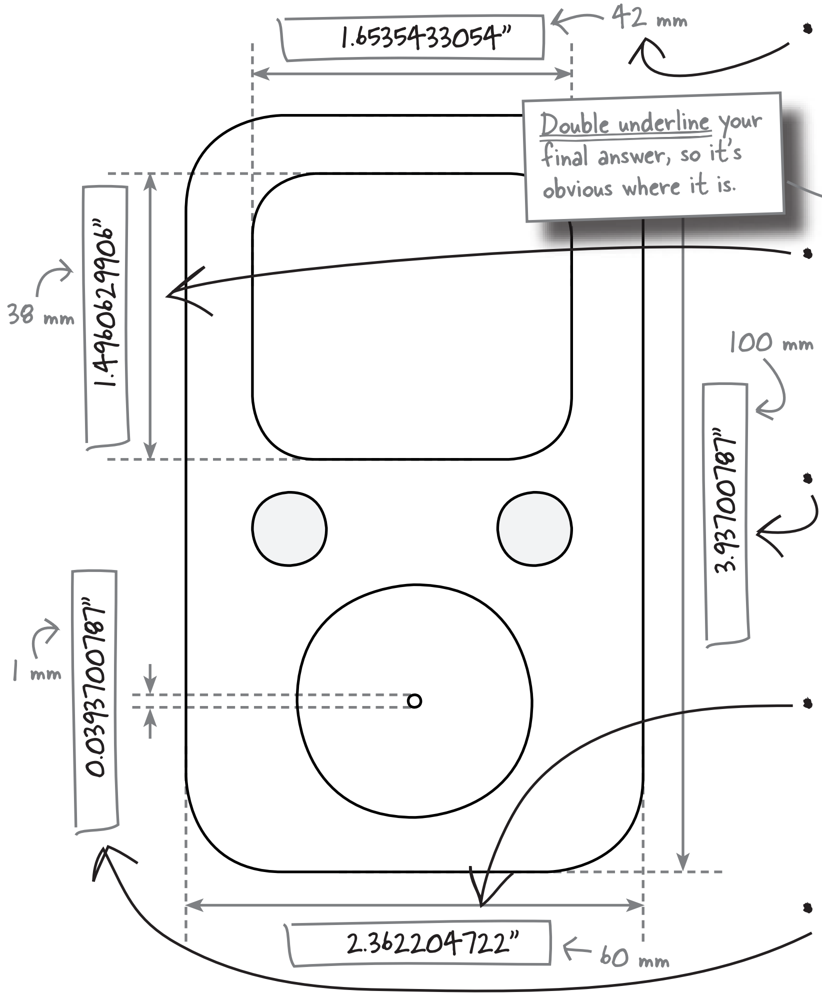
Remember to write the UNITS on the blueprint!



Sharpen your pencil Solution

Make sure you lay out your work, so it's obvious which bit of the problem you're doing.

Here's the blueprint with some of the millimeter lengths marked. **Convert** them to inches and earn that limited edition player! Remember to **show your work** (there's space down the right for that).



- 42 mm in inches

  $= 42 \text{ mm} \times \frac{0.0393700787"}{1 \text{ mm}}$

  = <u>1.6535433054"</u>

- 38 mm in inches

  $= 38 \text{ mm} \times \frac{0.0393700787"}{1 \text{ mm}}$

  = <u>1.4960629906"</u>

- 100 mm in inches

  $= 100 \text{ mm} \times \frac{0.0393700787"}{1 \text{ mm}}$

  = <u>3.93700787"</u>

- 60 mm in inches

  $= 60 \text{ mm} \times \frac{0.0393700787"}{1 \text{ mm}}$

  = <u>2.362204722"</u>

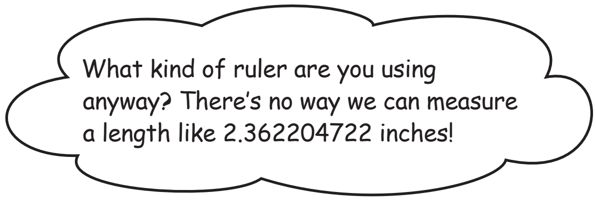
- 1 mm in inches – already know that 1 mm = <u>0.0393700787"</u>

If you need to change units during a problem, look up a conversion factor to help, and show your work when you do it!

You just converted the units for the entire blueprint!

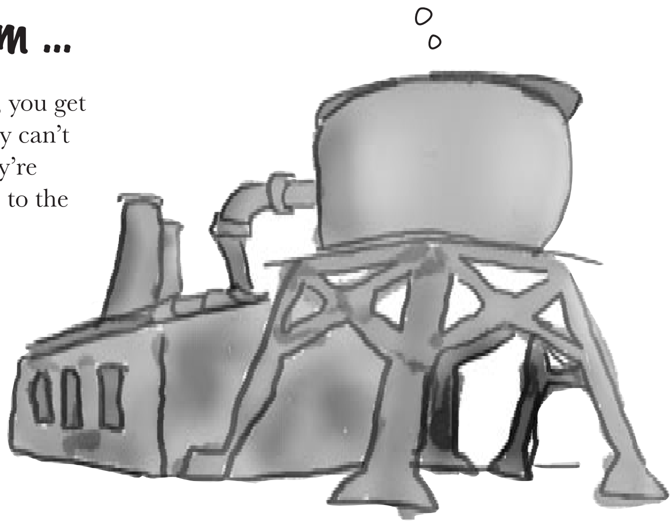
After **converting** all measurements from mm to inches and checking that the **units** are written at the end of **every** number, you send the blueprints off to the factory and dream of your limited edition myPod ...

It's important to make sure you include the units EVERY time you write down a number as a final answer.



But there's STILL a problem ...

A couple of hours after mailing the blueprints, you get a call from the factory. They're saying that they can't follow the instructions on the blueprint, as they're not capable of manufacturing the myPod case to the nearest 0.0000000001 inch!



BRAIN POWER

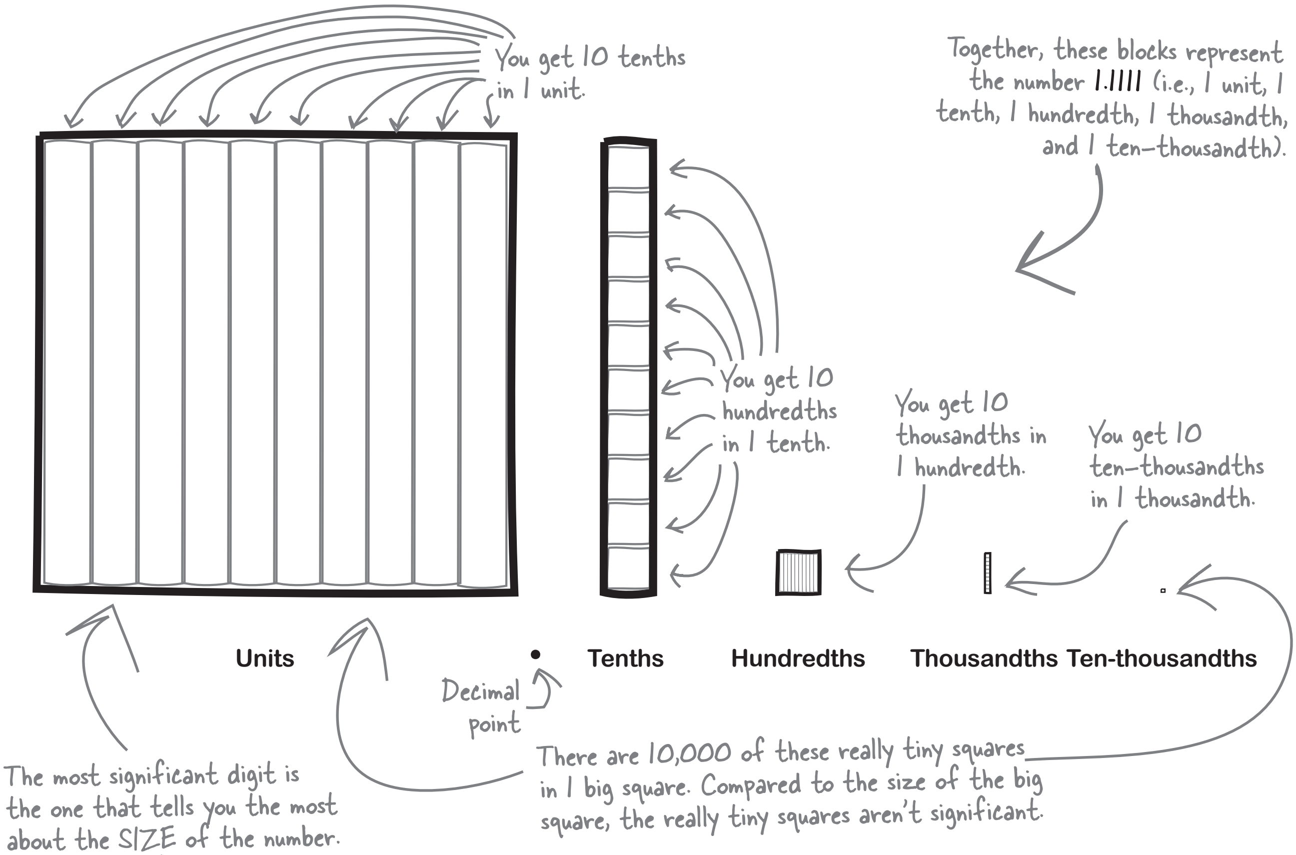
What could have caused this new problem? And how can you fix it?

What to do with numbers that have waaaay too many digits to be usable

Right now your problem is that the numbers on the blueprint have **too many digits**. The blueprint says that the myPod is 2.362204722 inches wide, which implies that you measured it to the nearest 0.000000001 inch. But unless you have a ruler that can measure individual atoms, you didn't!

You have to decide how many of the digits in your answers are **significant**. A number's most significant digit is the one that tells you the most about **how big the number is** - usually the **first non-zero digit**. The next-most significant digit is the next one along, and so on.

Don't worry if you've heard this called 'significant figures' in the past. Significant digits and significant figures are two different names for exactly the same thing.



If your number is 0.0022, then it's the thousandths digit that's the most significant.

The most significant digit is the one that tells you the most about the SIZE of the number.
So the <u>first non-zero digit</u> is the most significant.

How many digits of your measurements look significant?

Since the numbers on your blueprint have too many digits, it makes sense to **round** them to get rid of some of the less significant digits. For instance, if you round 2.362204722 inches to one significant digit, it's 2 inches (since it's closer to 2 than it is to 3); to two significant digits it would be 2.4 inches (since it's closer to 2.4 than it is to 2.3).

Don't worry too much about this – you'll be looking at rounding numbers on the next couple of pages.

But **how many significant digits** should you round your measurements to?

Sharpen your pencil

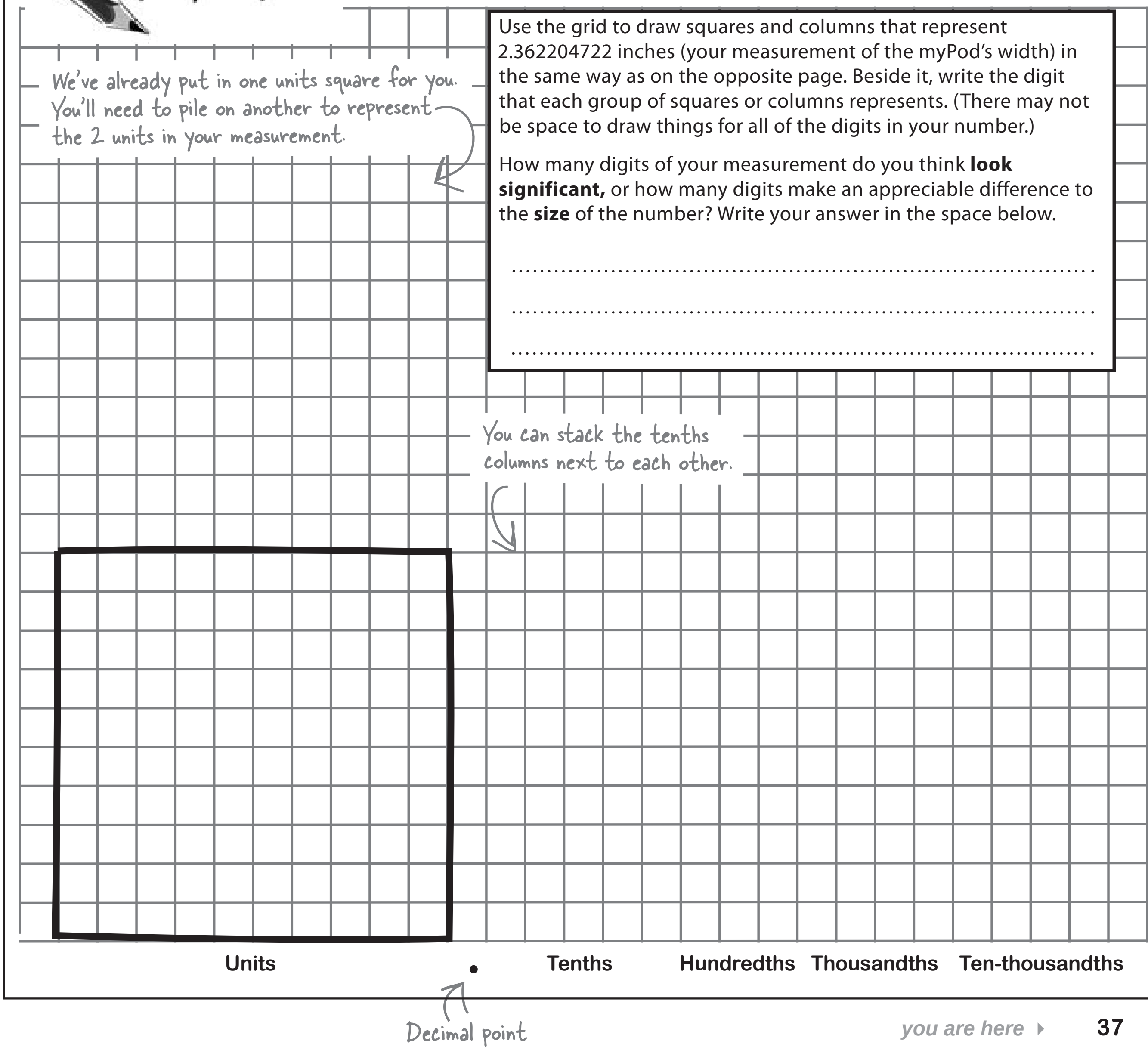
Use the grid to draw squares and columns that represent 2.362204722 inches (your measurement of the myPod's width) in the same way as on the opposite page. Beside it, write the digit that each group of squares or columns represents. (There may not be space to draw things for all of the digits in your number.)

How many digits of your measurement do you think **look significant,** or how many digits make an appreciable difference to the **size** of the number? Write your answer in the space below.

....

....

....

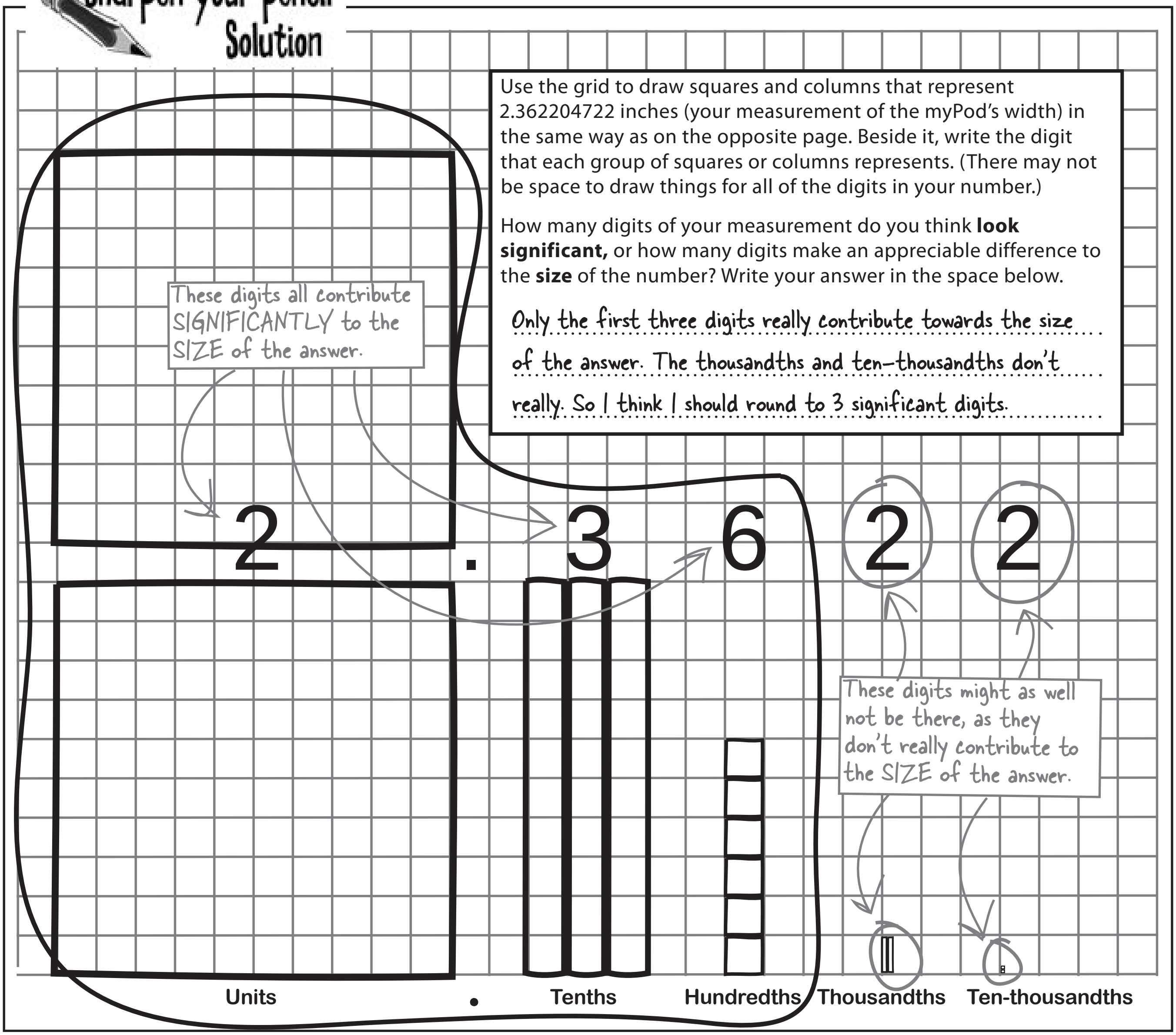


Sharpen your pencil Solution

Use the grid to draw squares and columns that represent 2.362204722 inches (your measurement of the myPod's width) in the same way as on the opposite page. Beside it, write the digit that each group of squares or columns represents. (There may not be space to draw things for all of the digits in your number.)

How many digits of your measurement do you think **look significant,** or how many digits make an appreciable difference to the **size** of the number? Write your answer in the space below.

Only the first three digits really contribute towards the size of the answer. The thousandths and ten-thousandths don't really. So I think I should round to 3 significant digits.



The first THREE digits of a number are the most significant. The other digits don't contribute much to the SIZE of the number.

Generally, you should round your answers to three significant digits

Unless you have extra information you can use to help, you should **round** your final answers for any calculation to **three significant digits**. The other digits in the number don't really contribute towards the size of the number.

You need to follow certain rules when you're rounding answers

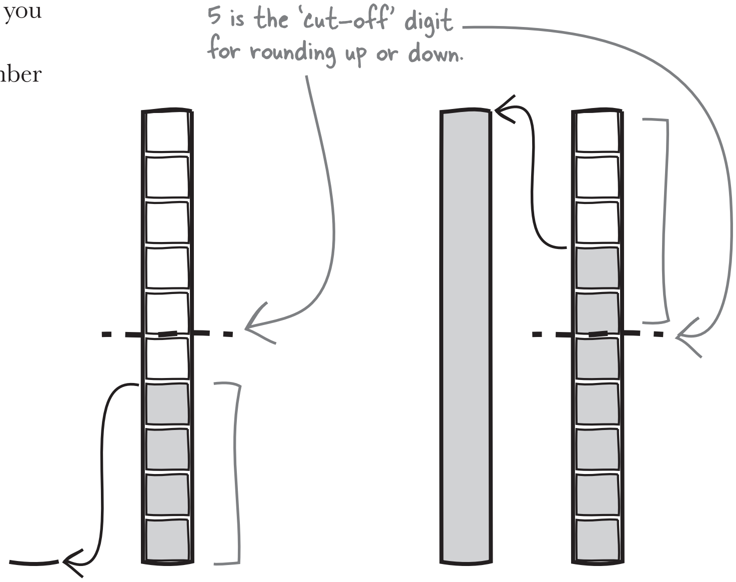
When you're rounding a number, look at the digit to the **right** of the final significant digit that you want to round to. If this digit is 4 or less, then round down. If it's 6 or more, then round up.

So you would **round down** the width of the myPod, 2.3622... inches, to 2.36 inches, as it's closer to 2.36 than it is to 2.37. But you'd **round up** a measurement of 4.5874... inches to 4.59 inches, as it's closer to 4.59 than it is to 4.58.

If the digit to the right of your final significant digit is a 5, then look to see if there's another digit after the 5. If there is, round up. If there isn't, round upor down to the nearest even digit. So 2.365 would round down to 2.36, and 2.375 would round up to 2.38.

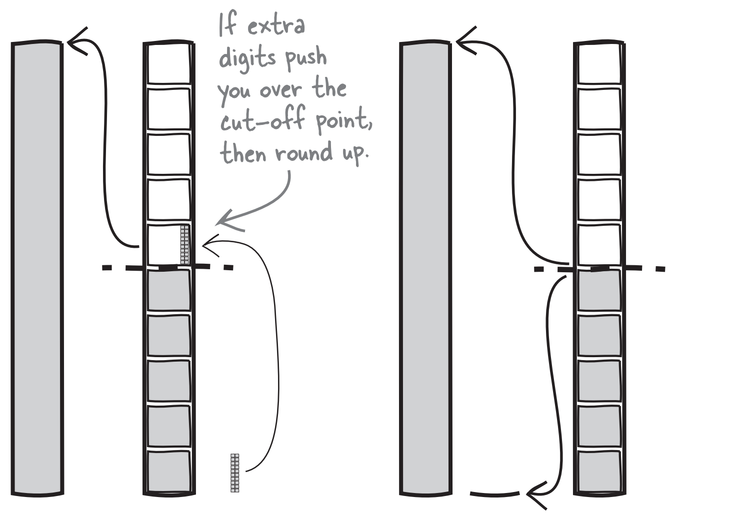
Once you've rounded your answer, always say how many significant digits you've rounded to, for example, by writing 2.36 (3 sd).

sd stands for "significant digits."



If the digit to the right of your last significant digit is 4 or less, round down.

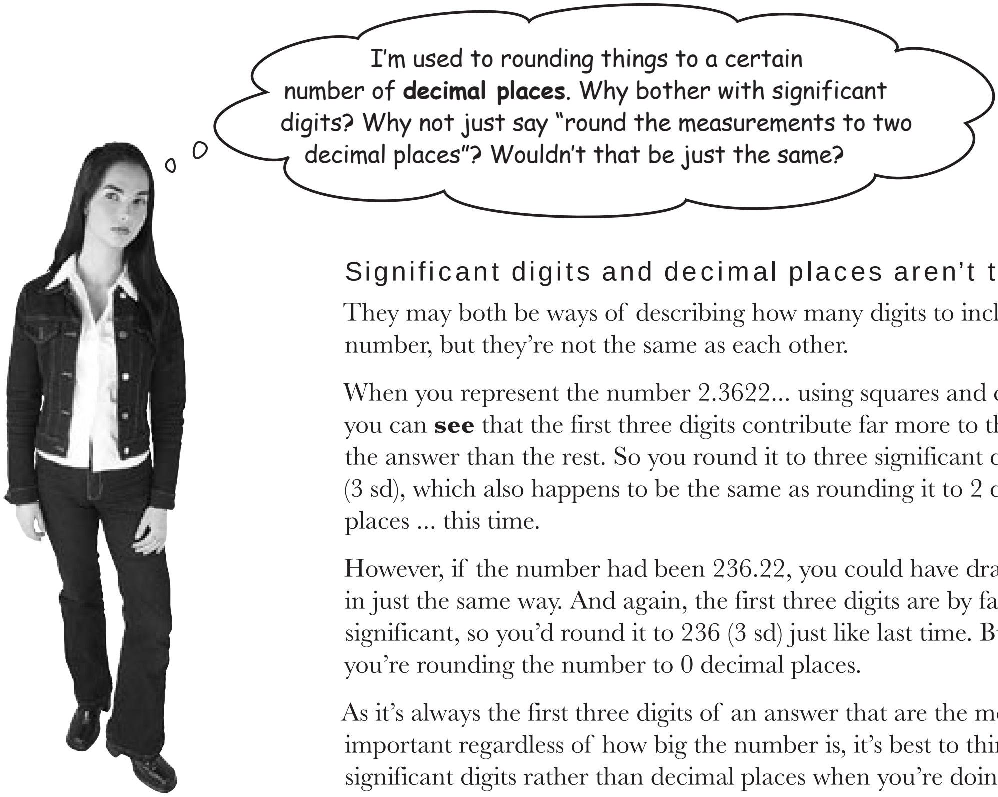
If the digit to the right of your last significant digit is 6 or more, round up.



If the digit to the right of your last significant digit is a 5 and there are other digits after it, round up.

If the digit to the right of your last significant digit is 5 and there aren't any more digits, round up or down to the nearest even number.

When you're rounding a number, the digit to the right of your final significant digit tells you what to do.



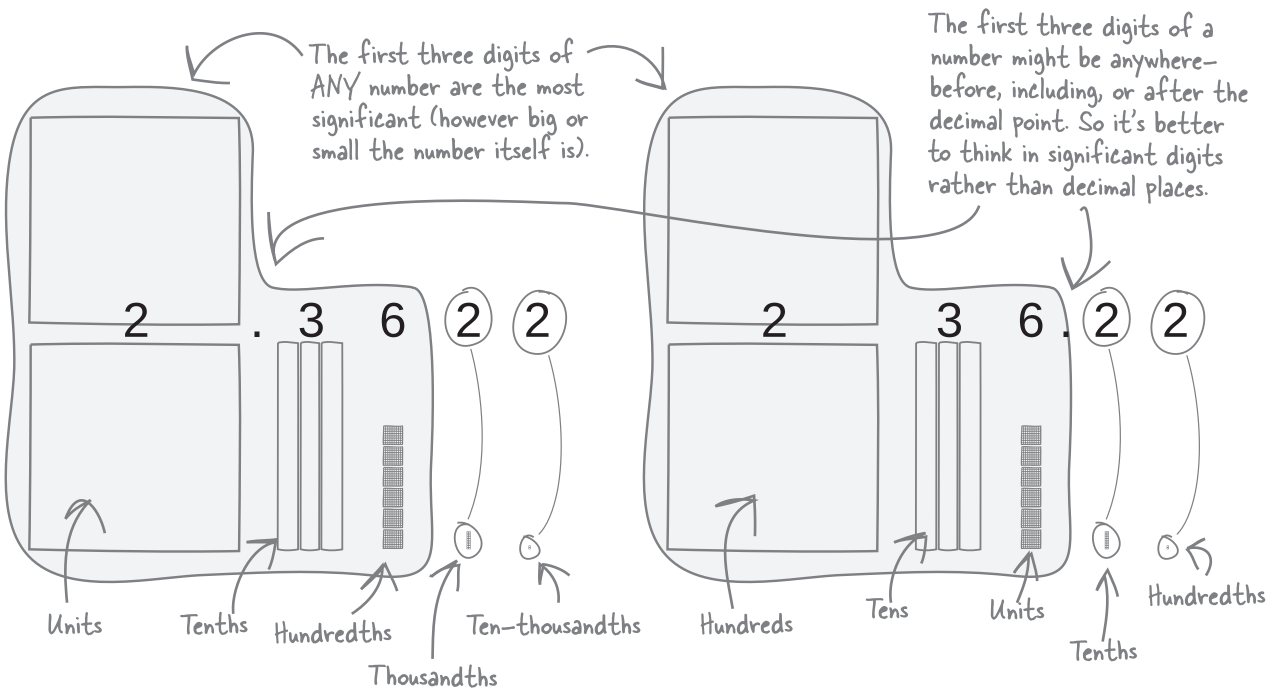
Significant digits and decimal places aren't the same

They may both be ways of describing how many digits to include in a number, but they're not the same as each other.

When you represent the number 2.3622... using squares and columns, you can **see** that the first three digits contribute far more to the size of the answer than the rest. So you round it to three significant digits, 2.36 (3 sd), which also happens to be the same as rounding it to 2 decimal places ... this time.

However, if the number had been 236.22, you could have drawn it out in just the same way. And again, the first three digits are by far the most significant, so you'd round it to 236 (3 sd) just like last time. But this time, you're rounding the number to 0 decimal places.

As it's always the first three digits of an answer that are the most important regardless of how big the number is, it's best to think about significant digits rather than decimal places when you're doing physics.



Is it OK to round the myPod blueprint to three significant digits?

You've already come a long way towards earning that free limited edition myPod! First, you measured the myPod case with your ruler and produced a blueprint so that the factory could produce the cases.

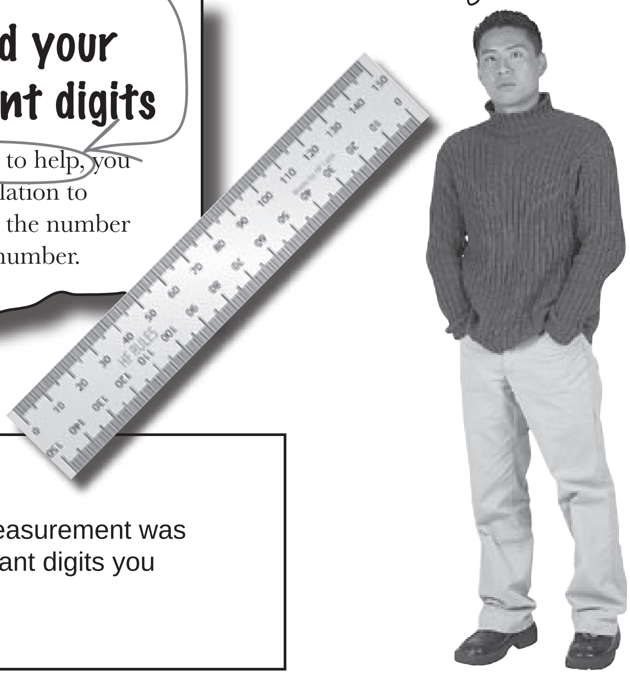
After a blip where the case came back 25 times too big (because there weren't any **units** on the blueprint, the factory assumed it was a giant promotional item), you came storming back and learned that with **conversion factors**, you can change the mm to inches without remeasuring the myPod.

Then, the factory pointed out that the converted measurements had **too many digits** in them - don't believe everything your calculator tells you! But you realized that some digits are more **significant** than others, as they tell you the most about the **size** of the number. And you learned that, in general, you should round calculations to **three significant digits**. So ... are you set to go?

I guess that rounding answers to three significant digits is the **general** rule in physics. But what if I **know more** about how a **specific** measurement was made. Does that affect what I should do?

Generally, you should round your answers to three significant digits

Unless you have extra information you can use to help, you should **round** your final answers for any calculation to **three significant digits**. The other digits in the number don't really contribute towards the size of the number.



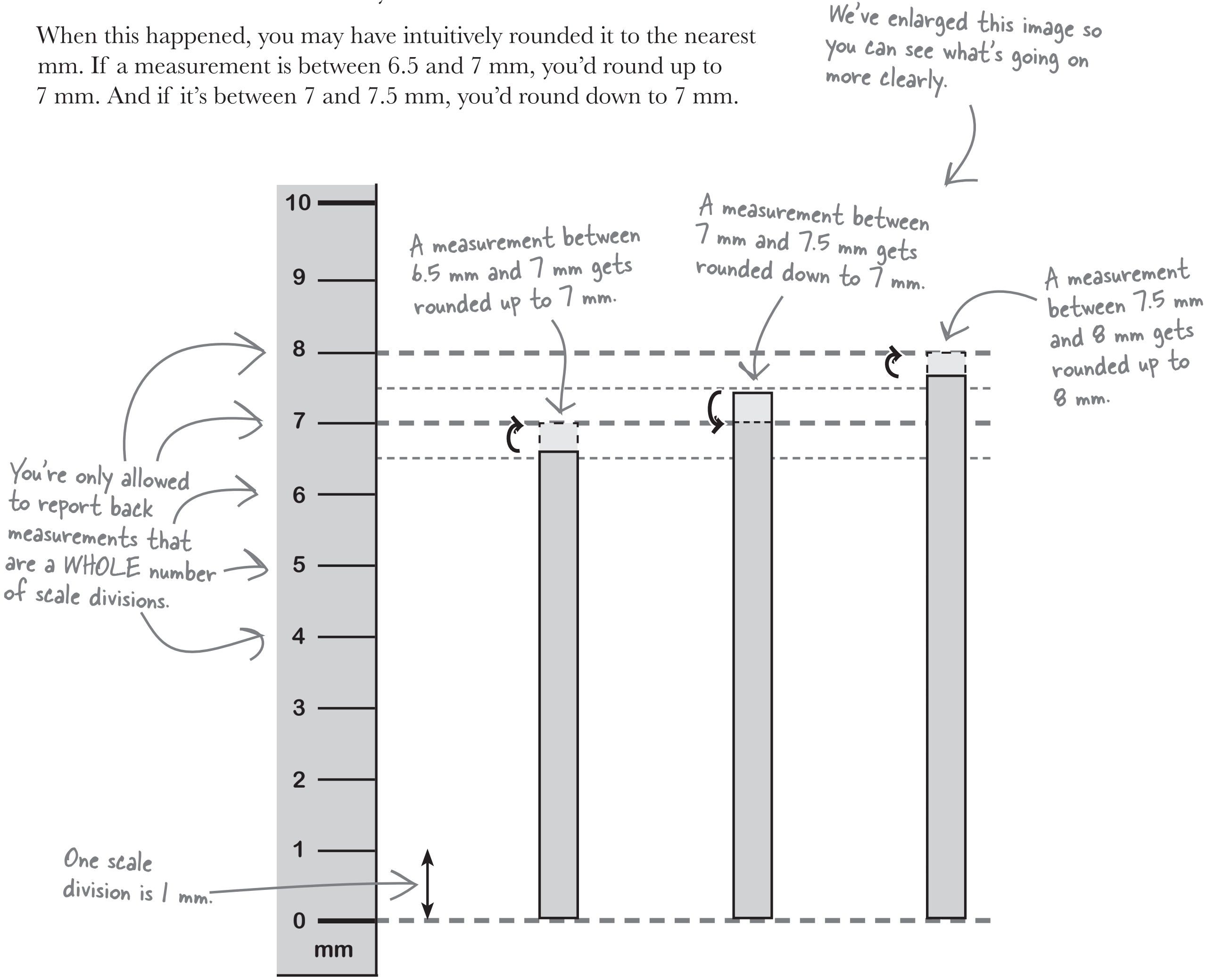
BRAIN POWER

How might the way the original measurement was made affect the number of significant digits you round your converted answer to?

You ALREADY intuitively rounded your original myPod measurements!

When you were measuring the myPod to draw your original plan, you probably found that some of the dimensions weren't a whole number of millimeters long. The thing you were measuring didn't **exactly** line up with one of the scale divisions on your ruler.

When this happened, you may have intuitively rounded it to the nearest mm. If a measurement is between 6.5 and 7 mm, you'd round up to 7 mm. And if it's between 7 and 7.5 mm, you'd round down to 7 mm.



You should always round your measurement to the nearest scale division on your measuring apparatus.

Any measurement you make has an error (or uncertainty) associated with it

Any time you make a measurement, you intuitively **round** it to the nearest scale division on your measuring apparatus. But this means that a measurement you make with your ruler and write down as "7 mm" could actually **range** from just over 6.5 mm to just under 7.5 mm.

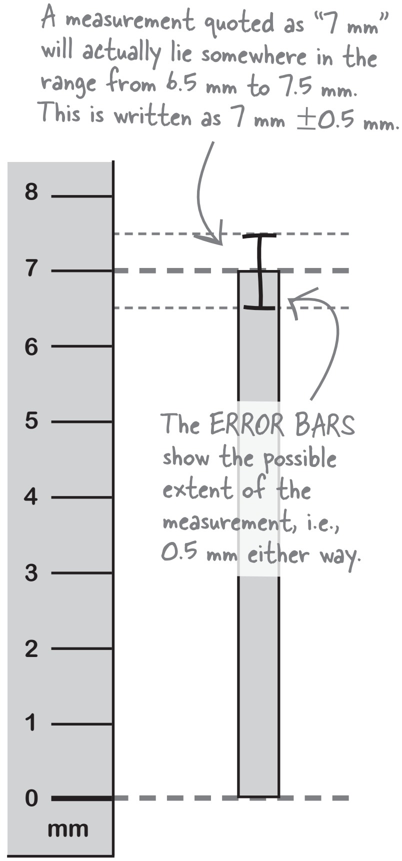
If you make a measurement, it's important that you (and any others using it) know how much **uncertainty** or **error** is associated with it. Was the 7 mm length measured using a ruler with a scale division of 1 mm, or with a micrometer with a scale division of 0.001 mm?

If you make a drawing of the thing you're measuring, you can show the range that the measurement may lie in using **error bars**, which mark the range's extremes.

If you write down your measurement, you can show the margin of error using numbers. The 7 mm measurement made using the ruler might be up to 0.5 mm larger (and rounded down to 7 mm) or 0.5 mm smaller (and rounded up to 7 mm). You write this as 7.0 mm ± 0.5 mm.

Because you round your measurements to the nearest scale division, the associated error is always **± half a scale division**.

You say this as "plus or minus."



there are no Dumb Questions

Q: If my measurement has an error, does that mean I did something wrong?

A: No! In this context, "error" is another word for "uncertainty" - the range that your measurement might fall into.

Q: I can eliminate the error on a measurement completely if I have a good enough measuring device, right?

A: Not really. You can reduce it, like by using a micrometer where the error is ±0.0005 mm instead of a ruler where it's ±0.5 mm. But no surface is ever perfectly smooth at the atomic level, so you'd never be able to completely eliminate uncertainty.

Q: So is trying to reduce errors associated with measurements a good thing, or is that just being a perfectionist?

A: The smaller the error on your measurements, the more certain you can be of your results.

Q: OK. But a couple of pages ago, we decided we should round calculations to 3 significant digits. But now we're making measurements like 7.0 mm ± 0.5 mm. Neither the measurement nor the error have three significant digits!

A: That (general) rule was for calculations, not for raw measurements.

Q: But if we know how the measurements were originally made, surely that affects how I round the converted values on the blueprint?

A: Yes. Rounding to three significant digits is a general rule when you don't know how the measurements you used in your calculation were made. But when you have more information about the error, you can propagate that through the unit conversion.

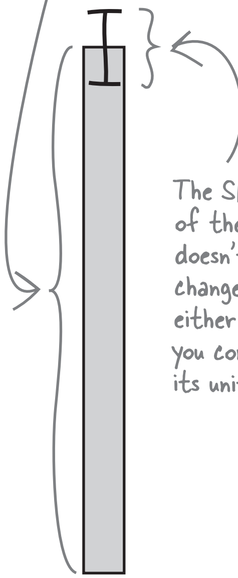
Q: That kinda makes sense, but how do I actually DO it?

A: It's funny you should ask - that's what we're just getting on to now.

The error on your original measurements should propagate through to your converted blueprint

You already used your ruler to round the myPod measurements to the nearest mm. This gives you the **error** associated with each measurement.

To work out the equivalent to rounding like this using inches, you need to **convert your error** from mm to inches. Then you can write your final answers in inches, along with their errors in inches, so the factory knows how well they have to measure.



But the problem is that the converted **measurements** have too many digits. If I convert the error from mm to inches, I'm just shifting the problem over - now my converted **errors** have too many digits! How's anyone supposed to know how much of the converted error is significant?!

Round converted errors to ONE significant digit.

Once you've converted your error, it's conventional to round it to one significant digit. Then you round the measurement to the last digit affected by the error (which is the same as saying "round to the same number of decimal places as the error").

This is just an example – it's not what your myPod error converts to!

So if a converted error becomes ± 0.061842375 inches, you'd round it to one significant digit: ± 0.06 inches (1 sd). So a measurement of 27 mm, which converts to 1.0106299213 inches, would be rounded to 1.01 inches ± 0.06 inches since the hundredths digit of the answer is the last to be affected by the error.

It's conventional to round converted errors to ONE significant digit, then round converted measurements to the last digit affected by the error.

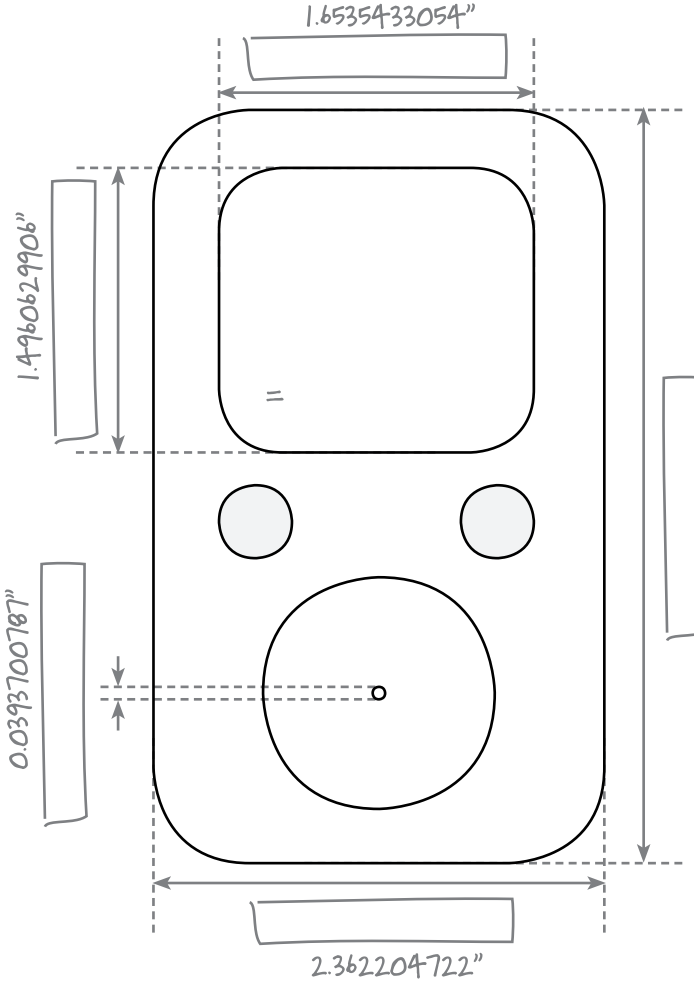
Right! Time to attack the blueprint again!

Now that you know how to convert measurements and how many significant digits to include, it's time to attack the blueprint again!

Sharpen your pencil

Use the page opposite to work out how many significant digits to quote the measurements to. Write your rounded answers in the boxes! Remember, you originally measured the myPod to the nearest mm, and Google Calculator told you that 1 mm = 0.0393700787 inches.

Remember to state the **error** on your measurements.

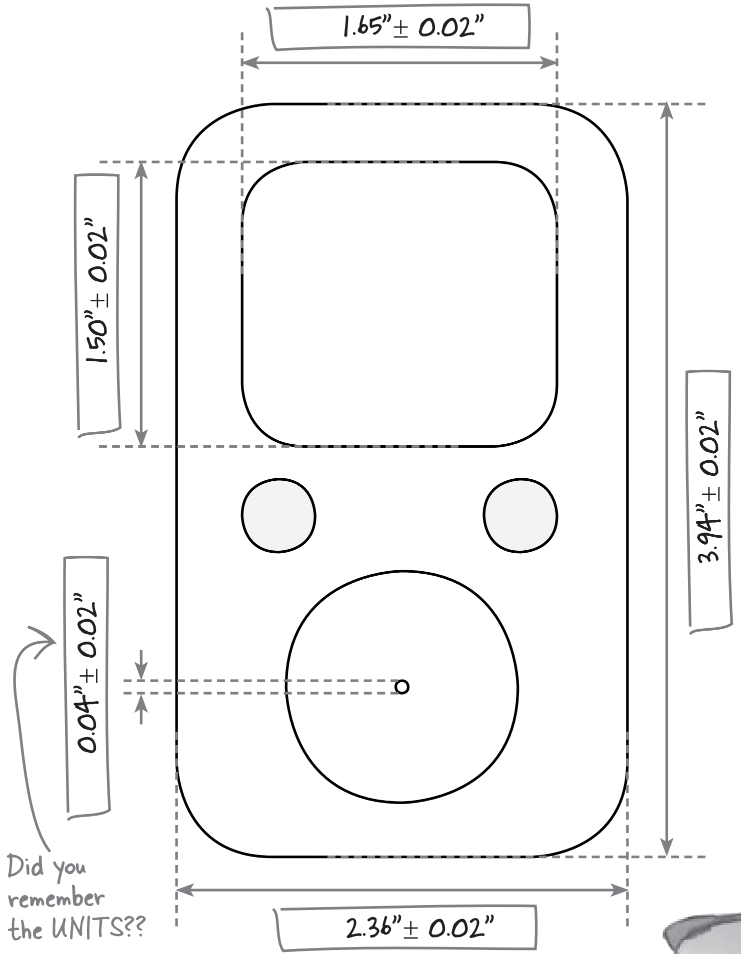


There's some space down here for you to work out the error on your measurements. Make sure you explain the stages you're going through.

Sharpen your pencil Solution

Use the page opposite to work out how many significant digits to quote the measurements to. Write your rounded answers in the boxes! Remember, you originally measured the myPod to the nearest mm, and Google Calculator told you that 1 mm = 0.0393700787 inches.

Remember to state the **error** on your measurements.



There's some space down here for you to work out the error on your measurements. Make sure you explain the stages you're going through.

Scale division on original ruler is 1 mm, so error on measurement is ± 0.5 mm.

Convert error: 0.5 mm in inches

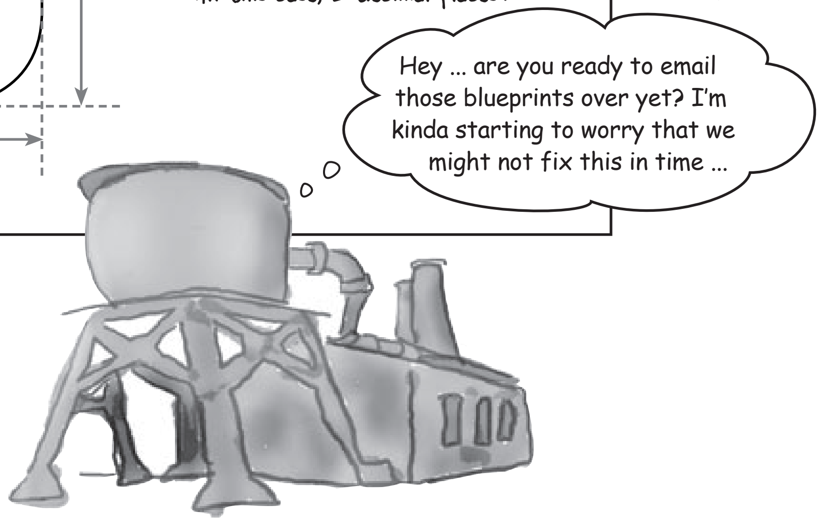
$$= 0.5\ \text{mm} \times \frac{0.0393700787''}{1\ \text{mm}}$$

$$= 0.01968503935''$$

Round error to 1 sd:

Error = ± 0.02" (1 sd)

So quote myPod measurements to the same number of decimal places. (In this case, 2 decimal places.)



STOP!! Before you hit send, do your answers SUCK?!

What's happened so far has probably convinced you that it's a good idea to **check over your answers** before you turn them in. So - **does your answer SUCK**?

S is for Size– How big/small are you expecting the answer to be?

U is for Units– Does the answer have units, and are they what was asked for?

C is for Calculations– Check them over and look out for silly mistakes!

K is for 'K'ontext– Go back to the big picture - what are you trying to do, and is it the same as what you actually did to get your answer?

Always check your answers before moving on.

Sharpen your pencil

Fill in the sections to see if your myPod blueprint **SUCK**s!

This is so you don't lose points in exams for doing 'silly things' that you could have avoided.

S

SIZE– Are the answers the size you're expecting?

....

....

U

UNITS– Do they have units, and are they what you were asked for?

....

....

C

CALCULATIONS– Did you do the math right?

....

....

K

"K'ONTEXT– What are you trying to do, and is it the same as what you actually did?

....

....

Sharpen your pencil Solution

This is an EXTREMELY useful way of checking if your answer is plausible.

Fill in the sections to see if your answer SUCKs.

S

SIZE– Are the answers the size you're expecting?

Well, inches are bigger than millimeters, so the inches measurements will be smaller numbers than the millimeter measurements. They seem about right for a music player.

U

UNITS– Do they have units, and are they what you were asked for?

The factory need inches, and I converted the lengths to inches. I also used the right number of significant digits.

C

CALCULATIONS– Did you do the math right?

I think so. The conversion factor is the right way up (so the units divide out) and the sizes already checked out OK.

K

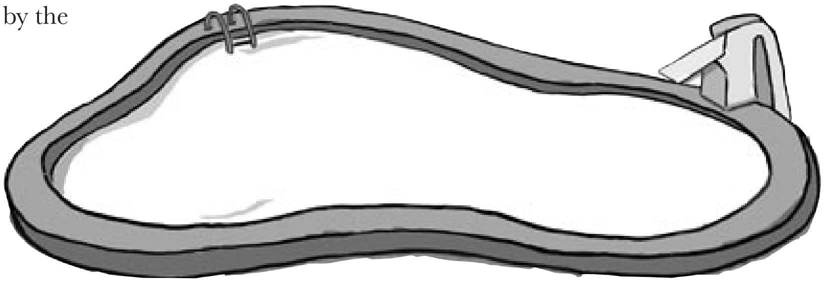
"K'ONTEXT– What are you trying to do, and is it the same as what you actually did?

I want to convert measurements from mm to inches using the correct number of significant digits (based on the error associated with the original measurements).

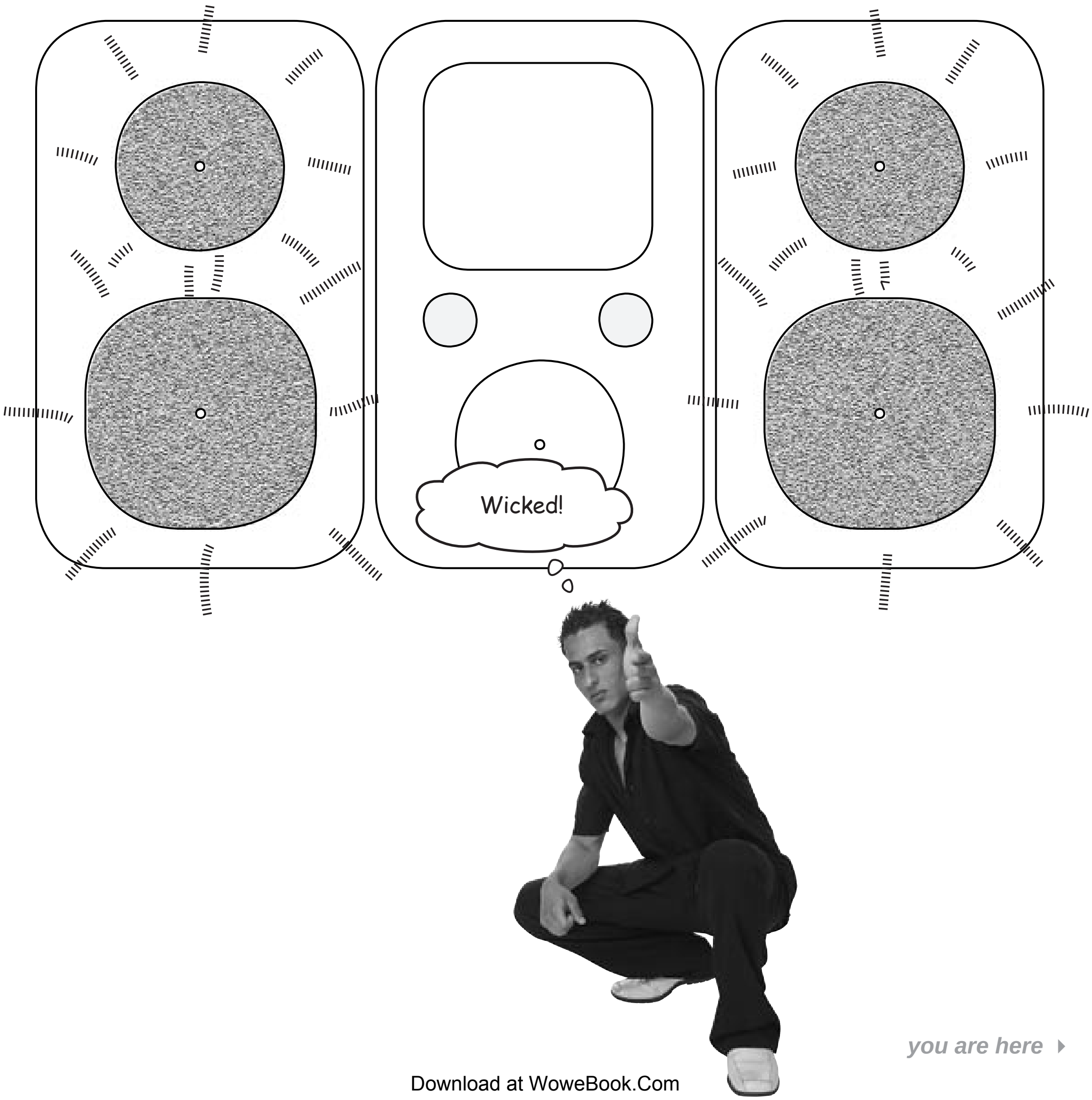
ALWAYS ask yourself: "Does my answer SUCK?" before you move on to something else

You nailed it!

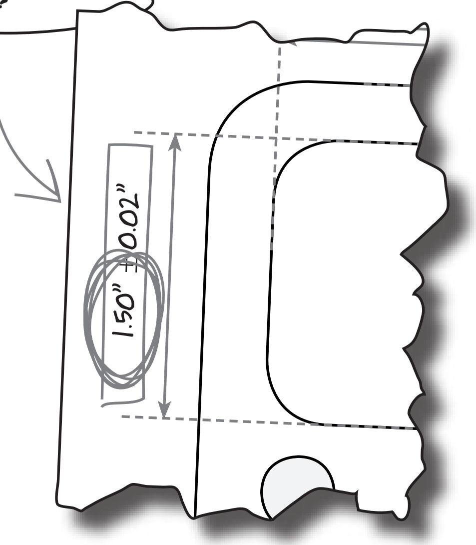
The blueprints are right at last, and the factory is happy! Before you know it, you're relaxing by the pool with your limited edition myPod.



But what about the giant myPod case the mailman delivered? It was used for an even more limited edition supergiant version that sold for thousands of dollars in an online auction, with massive publicity.



Not so fast! I think I've spotted something. I put 1.5" for the height of the screen instead of 1.50". The 0 doesn't add any information to the measurement, and there's no point in wasting ink, right? So why's it in the blueprint?



The zero gives you extra information.

The number of significant digits you include in an answer implies the size of the error. The final digit you include is the one that is uncertain.

Here, your error is ± 0.02 inches - the length could be up to two hundredths of an inch either way. Writing down the measurement as 1.50 inches correctly implies that the hundredths digit is uncertain. But writing down 1.5 inches implies that the tenths digit is uncertain, as it's the last digit in the answer.

What do we do if we don't know anything about the errors of the measurement we are using in a calculation?

Answers should have the same number of significant digits that you were provided with in the question.

If you don't know what the errors are, use the same number of significant digits in the problem.

The first three digits of a number are the ones that are significant enough to be worth keeping when you round an answer. This means that most of the numbers you are given to work with are usually rounded to three significant digits.

So when you come up with an answer, you should also round it to three significant digits to preserve what was originally done to the number before you were given it.

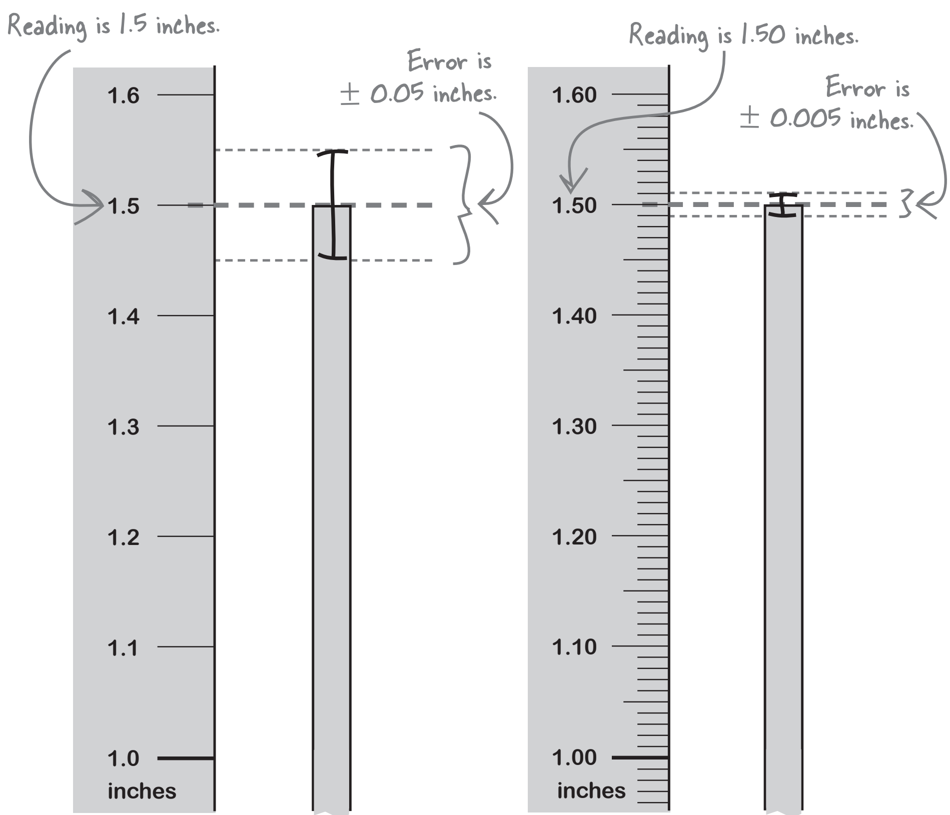
When you write down a measurement, you need the right number of significant digits

If you write down a measurement of 1.5 inches, you're implying that it was measured to the nearest 0.1 inch (at best), i.e., the error is ± 0.05 inches.

If you write down a measurement of 1.50 inches, you're implying that it was measured to the nearest 0.01 inch (at best), i.e., the error is ± 0.005 inches.

So if you write down 1.5 inches when you should have written down 1.50 inches, you're implying that the measurement is **TEN times worse** than it actually is.

We've enlarged this image, so you can see what's going on more clearly.



If you leave out a zero that would be affected by the error, you're implying that your measurement is TEN times worse than it actually is!

there are no Dumb Questions

Q: In some physics books, there are tables of constants with lots of significant digits, like the speed of light = 29979245.8 meters per second. That implies an error of ± 0.05 meters per second. Am I really supposed to write down NINE significant digits when the error propagates through to my answer?

A: When you're asked to work with numbers like that, don't round anything until you reach the end of your calculation. It's normal then to round your **final answer** to **three** significant digits.

Q: I'm a bit confused about zeros now. At first I thought they were just placeholders, but now you're saying they're sometimes significant. The zeros in the number 0.005 aren't significant, right? So how can I tell whether a zero is significant or not?

A: We're just going to interview a zero to get it all straightened out ...

Zeros Exposed

This week's interview:
Hero or Zero?

Head First: Now, onto today's special guest - a very well known figure. As one of the papers asked recently - hero or a lot of fuss about nothing? So, Zero, what's your take on this debate about your importance?

Zero: Well, the short answer to your question is that my significance kinda depends on where I am in a situation.

Head First: How do you mean? Surely you'll be equally significant - or might that be insignificant - to anyone doing math all around the world? What's location gotta do with it?

Zero: I don't mean where in the world you find me. I mean **where in the number** you find me!

Head First: So, you reckon your significance depends on where in a number you are. But surely zero is zero, whether it's zero units, zero tenths, or so forth?

Zero: Well, yes, that's why I was originally invented - so you wouldn't lose your place in a number.

Head First: So you're just a placeholder, right?

Zero: Oh no, not at all! Sometimes I'm a placeholder, but sometimes I'm really significant!

Head First: You're gonna have to help me here ...

Zero: Well, I'm a placeholder if I'm at the left of a decimal point in a number that's less than 1. Like in 0.00123, the zeros at the start of the number are just padding so that the rest of the digits fall into the right place.

Head First: And when aren't you just padding?

Zero: If I'm not at the start of the number, then I'm significant. Especially if I'm part of a measurement.

Head First: Why are measurements special?

Zero: Measurements are quoted to the same number of significant digits as your measuring device. So if your ruler's in mm and you measure something 5 mm long, then you write 5 mm ...

Head First: ... but there's no zero there ...

Zero: ... and if your ruler was marked off in tenths of a mm, you'd write 5.0 mm.

Headfirst: But why write the extra zero at the end when it's just the same thing? It's still 5 mm long!

Zero: Because you've measured it to a tenth of a millimeter this time. So you need to have a figure in there to say how many tenths there are. The number of tenths is highly significant!

Head First: But there weren't any tenths! So 5 mm and 5.0 mm are just the same number, aren't they?

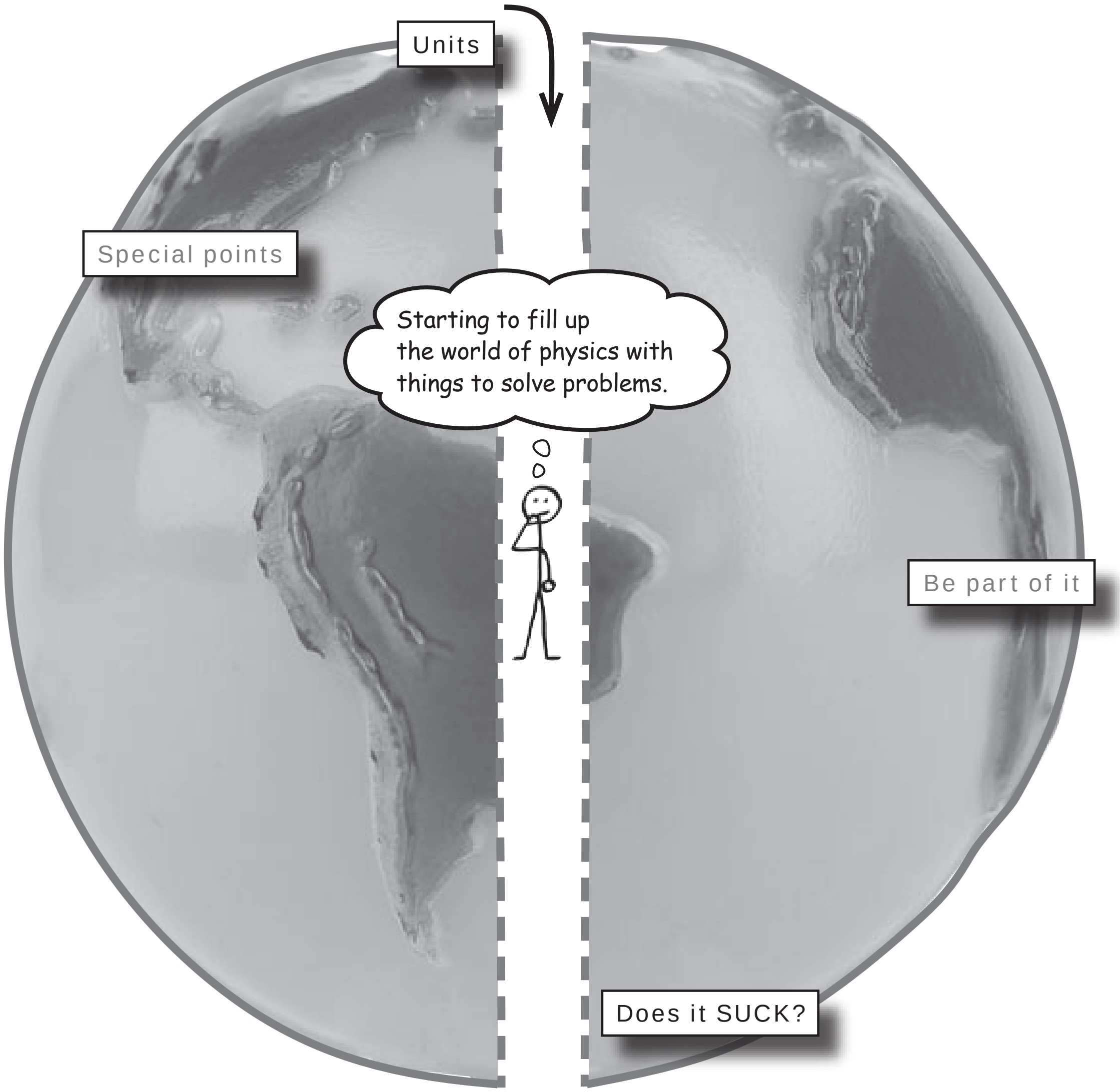
Zero: But they don't have the same meaning. **Measurements have meaning!** If it has a decimal point, then the last figure of the measurement tells you about the size of the error. So the last figure is always significant - even if it's a zero!

Head First: And if there's no decimal point, say in a number like 1000?!

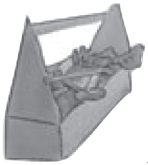
Zero: Then it's ambiguous - you don't know exactly where it was rounded. Was the 1000 originally 501 rounded to the nearest thousand (1 sd), or 1000.1 rounded to the nearest unit (4 sd)? That's why you should always mention the number of sd when you make a measurement or give an answer.

Head First: Well, thank you, Zero, for coming in today and explaining what you do.

Measurements have MEANING! Zeros are significant when they show you the error on a measurement.



☑	Units	Reference standards for measurements. For example, meters for distance and seconds for time.
☑	Does it SUCK?	Check the Size, Units, Calculations, and Kontext of your answers to see if they make sense.



Your Physics Toolbox

You've got Chapter 2 under your belt, and you've added some terminology and answer-checking skills to your tool box.

Units

A number needs to have units for it to mean something.

You're only allowed to add things together if they have the same units.

Errors

Any measurement you make has an error associated with it that reflects the uncertainly in the measurement.

Don't worry – they're not called errors because you did something wrong!

Converting units

To convert an answer from one unit to another you need to multiply it by a conversion factor.

This is a fraction where the top and bottom are both equal sizes – but are expressed in different units.

Arrange things so that the units you don't want divide out when you multiply your answer by the conversion factor.

Also, think about whether you expect the number part of your converted answer to be bigger or smaller than what you currently have.

Significant digits

Any time your calculator gives you an answer, you'll need to round it.

Round your answer to the same number of significant digits as the least precise number you were given to work with.

(Usually this will be 3 significant digits.)

Does it SUCK?

Memory aid to see if your answer makes sense.

Size – How big did you expect your answer to be?

Units – Does your answer have the correct units?

Calculations – Did you do the math right?

'K'ontext – What are you trying to do – and is that what you actually did?

3 scientific notation, area, and volume

All numbers great and small

In the real world, you have to deal with all kinds of numbers, not just the ones that are easier to work with. In this chapter, you'll be taking control of unwieldy numbers using **scientific notation** and discovering why rounding a large number doesn't mean having to write a zillion zeros at the end. You'll also use your new superpowers to deal with units of **area** and **volume -** which is where scientific notation will save you lots of grief (and time) in the future!

A messy college dorm room

Well, actually a particularly filthy college dorm room - Matt and Kyle probably wouldn't know one end of a vacuum cleaner from the other, and the idea of cleaning has never entered their heads.

But the Dorm Inspector has had enough...

Head First U Department of Dorm Inspection

Your dorm room is becoming hazardous to your health, and this state of affairs must be dealt with. We've detected a single specimen of a bug that doubles in number every twenty minutes.

If the bugs grow to occupy more than 6×10^{-5} m^3, they'll take over your room, and you will need to find a new place to live while we fumigate your living area.

Sincerely,

Dorm Inspection Team

So how long before things go really bad?

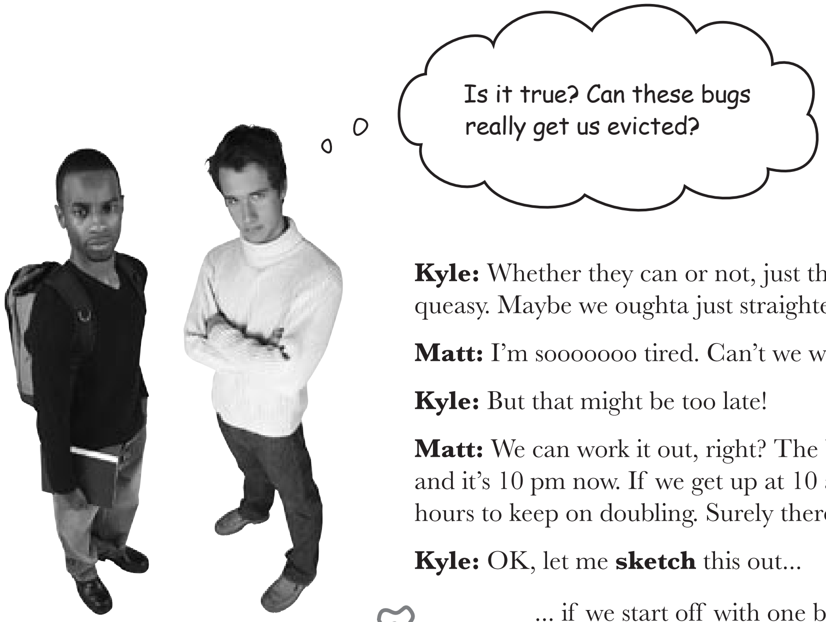
Do we have to clean up tonight, or can we just wait until tomorrow?

Yeah, how serious can these little bugs be?

BRAIN POWER

Every 20 minutes, the bugs will divide in two. So the total number of bugs will double every 20 minutes.

How many do you think there will be by tomorrow (12 hours later) - and how might you work that out?



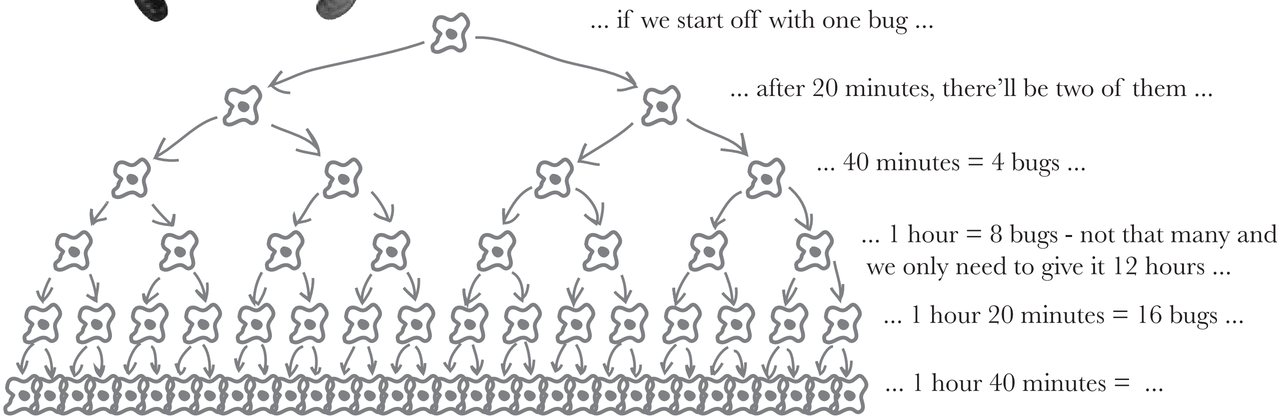
Kyle: Whether they can or not, just the thought of it makes me queasy. Maybe we oughta just straighten this place up now.

Matt: I'm sooooooo tired. Can't we wait until tomorrow?

Kyle: But that might be too late!

Matt: We can work it out, right? The bug doubles every 20 minutes, and it's 10 pm now. If we get up at 10 am, we've given the bug 12 hours to keep on doubling. Surely there can't be that many by then?

Kyle: OK, let me **sketch** this out...



Kyle: ... hmmm, I'm not sure - my drawing's getting messy!

Matt: Yeah, the drawing will take forever. There's gotta be a **math** way to figure out how many bugs there'll be after 12 hours.

Kyle: Yeah, OK.

Matt: Hmmm. I can't think of an equation for "the bugs double every 20 minutes," but we could just make a table to keep track of things and keep on doubling until 12 hours are up. Then we'll know how many bugs there'll be by the morning.

Kyle: I think that'll work, but there's still that funny phrase in the note, "If the bugs grow to occupy more than 6×10^{-5} m^3." I don't know what that is, but it sure ain't a number of bugs.

Matt: Why don't we worry about that later, once we know **how many** bugs there'll be...

Sharpen your pencil

Matt and Kyle have drawn up the table below and started doubling the bugs. Your job is to finish off the table to see how many bugs there'll be after 12 hours.

You start off with 1 bug. After 20 minutes, it's doubled once, and there are 2 bugs.

This is as far as they got with their sketch.

Number of doublings	Elapsed time	Number of bugs
1	20 min	2
2	40 min	4
3	1 h	8
4	1 h 20 min	16
5	1 h 40 min	

Number of doublings	Elapsed time	Number of bugs

There are a lot of doublings in 12 hours, so we've given you space to continue the table.

Sharpen your pencil Solution

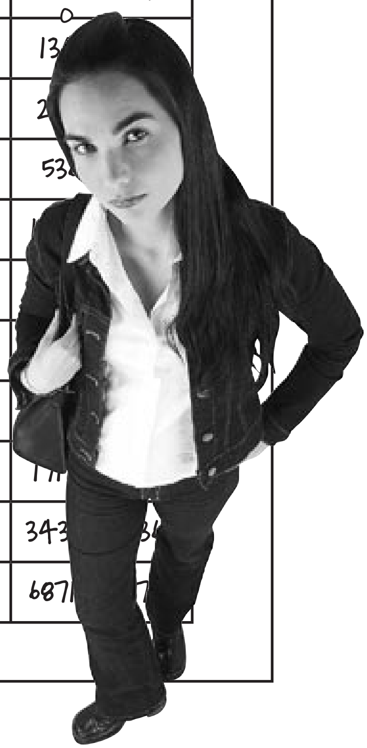
Matt and Kyle have drawn up the table below and started doubling the bugs. Your job is to finish off the table, to see how many bugs there'll be after 12 hours.

You start off with 1 bug. After 20 minutes, it's doubled once, and there are 2 bugs.

This is as far as they got with their sketch.

Number of doublings	Elapsed time	Number of bugs
1	20 min	2
2	40 min	4
3	1 h	8
4	1 h 20 min	16
5	1 h 40 min	32
6	2 h	64
7	2 h 20 min	128
8	2 h 40 min	256
9	3 h	512
10	3 h 20 min	1024
11	3 h 40 min	2048
12	4 h	4096
13	4 h 20 min	8192
14	4 h 40 min	16384
15	5 h	32768
16	5 h 20 min	65536
17	5 h 40 min	131072
18	6 h	262144

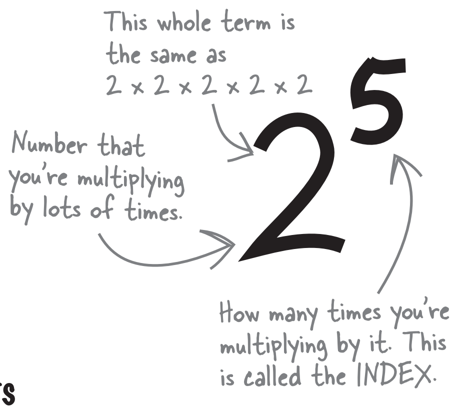
Number of doublings	Elapsed time	Number of bugs
19	6 h 20 min	524288
20	6 h 40 min	1048576
21	7 h	2097152
22	7 h 20 min	4194304
23	7 h [illegible]	[illegible]
24	[illegible]	[illegible]
25	[illegible]	[illegible]
26	8 h 40 min	67108864
27	9 h	13[illegible]
28	9 h 20 min	2[illegible]
29	9 h 40 min	53[illegible]
30	10 h	[illegible]
31	10 h 20 min	[illegible]
32	10 h 40 min	[illegible]
33	11 h	[illegible]
34	11 h 20 min	[illegible]
35	11 h 40 min	343[illegible]
36	12 h	687[illegible]



There are a lot of doublings in 12 hours, so we've given you space to continue the table.

Power notation helps you multiply by the same number over and over

If you want to multiply by the same number several times over, you can write it down using **power notation**. This means that $2 \times 2 \times 2 \times 2 \times 2$ becomes 2^5, as there are five instances of 2. When you say 2^5 out loud, you say "two to the power of five" or sometimes just "two to the five." The five part is called the **index**.

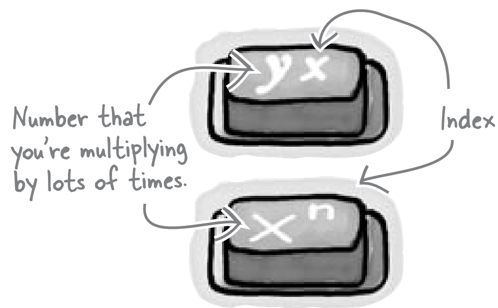


Your calculator's power button gives you superpowers

You can use the **power button** on your calculator to multiply by the same number lots of times without having to type it all out. Usually, you type in the number you want to multiply by, then press the power button, then type the number of times you want to multiply by it.

Watch out though - different calculators have different things written on the power button! Make sure you know what yours looks like and how it works before you try to use it!

If your calculator doesn't have a power button, then you'll need to get a scientific calculator. It'll help you out in the long run as you move onto solving more sophisticated and complicated physics problems.



There's space here to explain what you're doing.

Sharpen your pencil

(a) The number of bugs doubles every 20 minutes. How many times do you need to multiply by 2 to get the total number of bugs after 12 hours?

(b) How many bugs will there be after 12 hours?

Power notation makes multiplying by the same number over and over less prone to mistakes.

Take time to jot down what you're doing and why. It helps you to stay on track..

Sharpen your pencil Solution

(a) The number of bugs doubles every 20 minutes. How many times do you need to multiply by 2 to get the total number of bugs after 12 hours?

(b) How many bugs will there be after 12 hours?

(a) There are 3 lots of 20 minutes in an hour. In 12 hours there are 3 x 12 = 36 periods of 20 minutes, so they double 36 times.

(b) Number of bugs after 12 h = number at the start x 36 groups of 2.

Number $= 1 \times 2^{36} =$ ☐

Write what YOUR calculator said in here.

Don't just copy answers down and move on.

It's important to understand the answers your calculator gives you.

Your calculator displays big numbers using scientific notation

Sometimes, an answer has too many digits to fit on your calculator's screen. When that happens, your calculator displays it using **scientific notation**. Scientific notation is an efficient and shorter way of writing very long numbers.

In math, scientific notation is often called standard form. Don't worry, they're the same thing.

The value of 2^{36} has 11 digits in it, but a calculator doesn't have enough space to display all of the digits. So instead, they've **rounded** the answer to the number of **significant digits** that they can fit on the screen.

The **first** part of the number on the screen is for the part that starts 6.87...

Answers written in scientific notation have two parts.

But there were already 8 bugs after an hour. 8 is more than 6.87, so how can the answer to 2^{36} possibly be that small?!

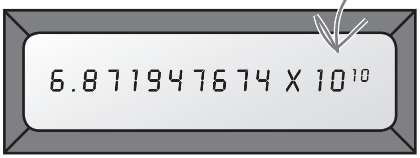
The second part of the number tells you the size of the first part.

Numbers written in scientific notation have two parts.

The **first** part is a number with **one significant digit before the decimal point** and the rest of the number after the decimal point.

The **second** part tells you the **number of 10's you have to multiply the first part by** to make your answer the correct **size**.

This part tells you how many 10's to multiply the first part by.



The first calculator's given an answer of $6.871947674 \times 10^{10}$. It's given you 10 significant digits, and the number is the same as writing $6.871947674 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10$, which is 68719476740.

The second calculator has displayed 6.871947E10, which is 68719470000- it's rounded the answer to seven significant digit and used an E to indicate the second part of the number because of the limits of its display.

Scientific notation uses powers of 10 to write down long numbers

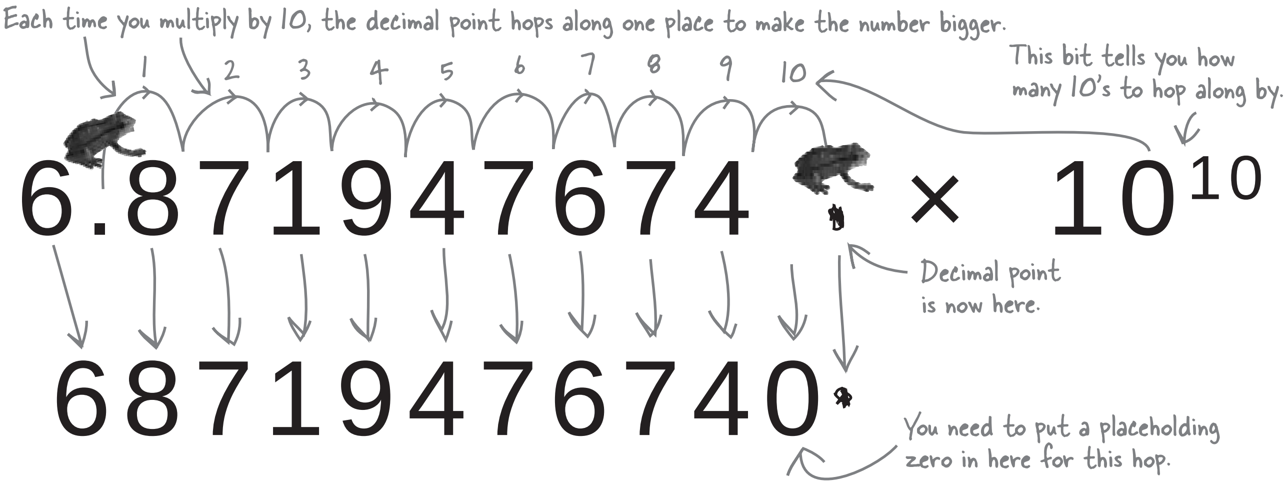
So you don't have to write them out the long way if they have 28 digits or something!

You calculator's given you the answer $2^{36} = 6.871947674 \times 10^{10}$. You know that to get this into the form you're used to, you need to multiply the first part of the number by ten groups of 10.

Each time you multiply by 10, the number's digits **shift along one place to the left** so that each digit is worth 10 times more than it was before.

But it's quite hard for you to draw that, so practically speaking, you can get to the same place by **hopping the decimal point** the correct number of times to the right. Then the number becomes 68719476740.

You can work out where the number's digits should lie by 'hopping' the decimal point.



You should round your answers to **three significant digits**, like you did in Chapter 2.

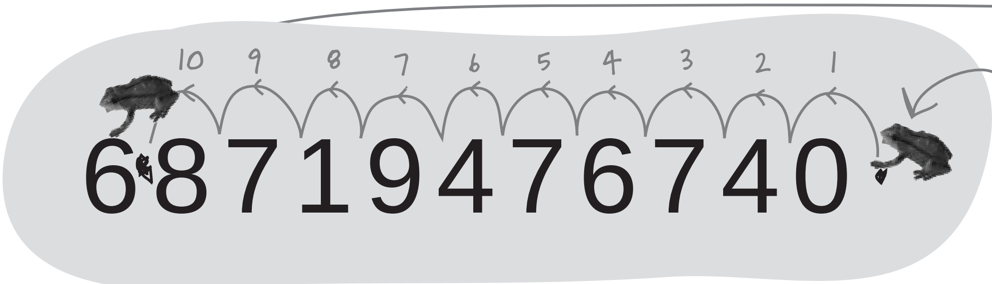
Does writing our answers with scientific notation really help us keep track of the digits?

Scientific notation helps you round your answers.

If you have to round a long number to 3 significant digits (sd) as a final answer, then the start is OK, but putting in the right number of zeros is a real pain.

It's a lot easier in scientific notation, as the tens are spelled out at the end of the number. This lets you rewrite $6.871947674 \times 10^{10}$ as 6.87×10^{10} without hopping the decimal point along.

Strictly speaking, it's the digits that move, not the decimal point, but that's much harder for you to draw!



To convert a normally-written number into scientific notation, count how many hops until only one digit is left in front of the decimal point, and multiply the number by that many 10's.

You can just start here, as this is the answer your calculator gave you.

$$6.871947674 \times 10^{10}$$

3 significant digits

The less significant digits – round to get rid of them.

$$6.87 \times 10^{10} \text{ (3 sd)}$$

You don't need placeholding zeros after a decimal point.

Or however many sd is appropriate for your answer.

Scientific notation helps you to round your answers to 3 significant digits without making mistakes.

there are no Dumb Questions

Q: I thought what you're calling 'scientific notation' is actually called 'standard form.' What gives?

A: They're both the same thing. Scientists use the term 'scientific notation' and mathematicians 'standard form.'

Q: Why should I bother with scientific notation when I'm really careful about how I type numbers into my calculator?

A: Your calculator screen might not be big enough to display an answer that's either really big or really small. So you need to understand scientific notation, or it won't make sense.

Q: But I have a super duper flashy calculator that'll display lots and lots of digits on its humongous screen. So if I'm careful, why would I ever need scientific notation?

A: You could be given a number in scientific notation to work with - in an exam question or when you look something up to find out how big it is.

Q: How big are we talking about?

A: Well, the earth's mass is 5.97×10^{24} kilograms. That's a very big number with a lot of zeros at the end if you write it out longhand.

Q: OK, I can see why I might not be happy handling over 20 zeros at the end. But why would I ever want to write an answer I've worked out myself in scientific notation?

A: If you're rounding your answer to 3 significant digits (like you'll do in your exam), then it's much easier to use scientific notation than it is to scrawl a whole lot of zeros across your page.

You can just take the number your calculator gives you, for example, $6.871947674 \times 10^{10}$, and write 6.87 × 1010 without having to do anything else to it?

Q: So are you saying that scientific notation isn't just there because my calculator's screen isn't big enough - it helps me as well?

A: Yep, scientific notation helps you to write and round very long numbers in a much shorter form. So it's not just about calculators - it's about making your life easier.

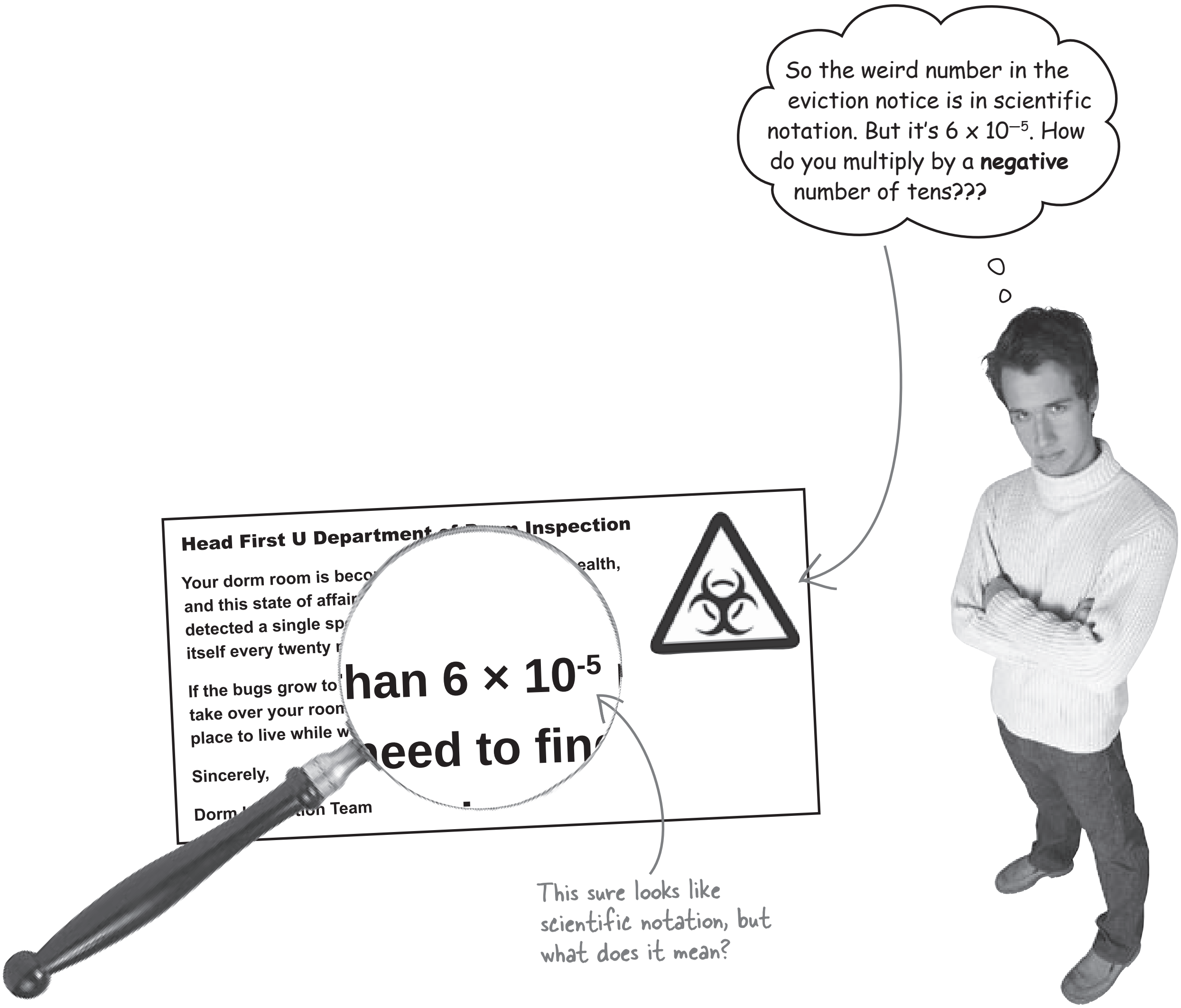
Q: So which came first - small calculator screens or scientific notation.

A: Scientific notation came first by several hundred years!

Q: I have one more question. Are numbers in scientific notation always written with one digit in front of the decimal point? Couldn't you equally write 6.87×10^{10} as 687×10^{8}?

A: Conventionally, they're written with one digit in front of the decimal point. Your brain will soon get used to estimating the size of the number from the tens part, so sticking to the convention is best.

Scientific notation helps you to handle very long numbers that would otherwise have many digits, even when you've round them.



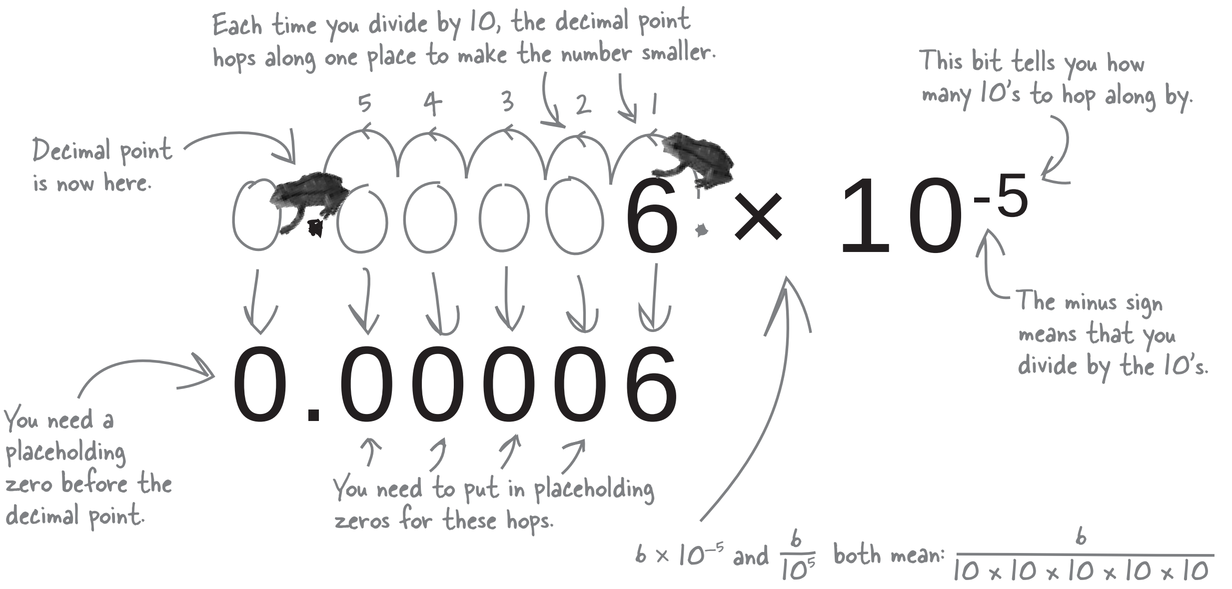
So the weird number in the eviction notice is in scientific notation. But it's 6×10^{-5}. How do you multiply by a **negative** number of tens???
Head First U Department
Inspection
Your dorm room is
ealth,
and this state of
detected a single
itself every twenty
If the bugs grow to
take over your room
place to live while
Sincerely,
Dorm
Team
han 6×10^{-5}
eed to
This sure looks like scientific notation, but what does it mean?

Scientific notation helps you with small numbers as well

The number in the Dorm Inspection note is 6×10^{-5}. It's scientific notation alright - but there's a **negative** power of 10. This is the conventional way of showing you that you should **divide** by the 10's instead of multiplying.

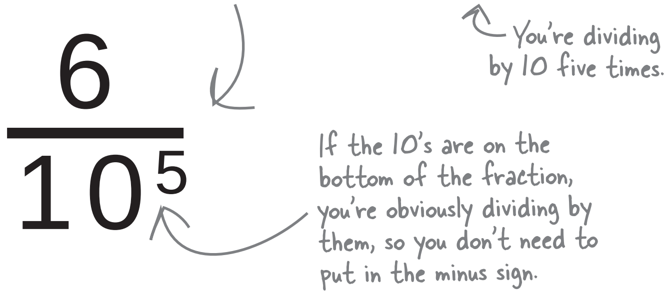
Every time you divide by 10, the number's digits shift along one place to the right, so each digit is worth 10 times less than it was before.

It's difficult for you to draw that, but practically speaking, you can get to the same place if you hop the decimal point the correct number of times to the left. So 6×10^{-5} works out as 0.00006.



Another way of showing this is as a **fraction**, by writing $\frac{6}{10^5}$.

This makes it more obvious that you're dividing by all the 10's, as they appear on the bottom of a fraction.



The key thing with scientific notation is to rewrite the number with ONE digit before the decimal point, then multiply or divide by the correct number of 10's.

But that's dumb. Why write 10^{-5} when you can use a fraction that makes it obvious what's going on? And why use a minus sign anyway?

The minus sign is part of a pattern

The convention of using a minus (or negative) sign in the index to say you're dividing by that number of 10's comes from a pattern that you're about to discover for yourself...

Exercise

In this exercise, spot the pattern behind the convention of using a negative index (6×10^{-5}) to represent dividing by a series of 10's.

Original number	In scientific notation	Number of 10's you're multiplying or dividing by.
10000	10^4	4 (multiplying)
1000		
100		
10		
1		
0.1		
0.01	10^{-2}	2 (dividing)
0.001		
0.0001		

(Between each row: Divide by 10.)

This is a negative index, like in the Dorm Inspector's note.

Write down any patterns you spot.

....

....

Is there anything in your table that looks a bit unusual?

....

....

In this exercise, you're going to spot patterns that'll help you write very small numbers in scientific notation even more efficiently than we've done over there.

Original number	In scientific notation	Number of 10's you're multiplying or dividing by.
10000	10^4	4 (multiplying)
1000	10^3	3 (multiplying)
100	10^2	2 (multiplying)
10	10^1	1 (multiplying)
1	10^0	0
0.1	10^{-1}	1 (dividing)
0.01	10^{-2}	2 (dividing)
0.001	10^{-3}	3 (dividing)
0.0001	10^{-4}	4 (dividing)

Divide by 10. (between each row and the next)

UNUSUAL: Any number to the power of 0 is 1. This is a math convention, but the table should help you see why it's sensible.

The PATTERN is that every time you divide by 10 to make the number smaller, the index of the scientific notation version decreases by 1.

So 10^{-4} is a more compact way of writing $\frac{1}{10^4}$, right?

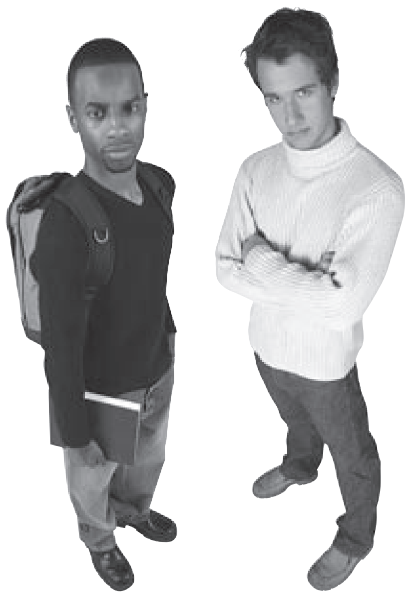
Right. And it helps you estimate the size of the number too.

If the number in scientific notation is **greater** than 1, then it'll be **multiplied** by 10's, and if it's **less** than 1, it'll be **divided** by 10's.

So you can instantly see whether a number is larger or smaller than 1 by looking at the **sign** of the index. And you can see how large or small it is by looking at the **size** of the index, that is, the number of 10's that are doing the multiplying or dividing.

You can estimate the size of a number written in scientific notation by looking at the sign and size of the index.

So now we know that the number in the note is written in scientific notation. 6 × 10 - 5 is the same as 0.00006. That's a relief.



Kyle: But 6×10^{-5} what?? It can't be 6×10^{-5} bugs - the index is negative so that's much less than the 1 bug we started with!

Matt: There's that funny m^3 thing after the number though. It kinda looks like it might be meters - but what's the little 3 for? Typo for MP3?

Kyle: I don't see where music comes into it. Hey, maybe that's scientific notation as well. Could m^3 be meters × meters × meters?

Matt: But when would you ever want to multiply units together?

Kyle: Hmm, good point. But the meters × meters × meters thing reminds me of doing stuff with length × width × height years ago. That's three lengths multiplied by each other.

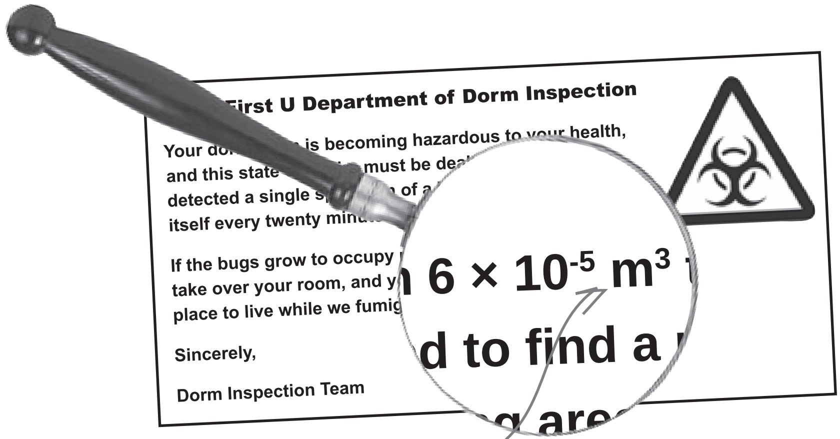
Matt: And that would make a **volume**! Cubic meters!

Kyle: Like how **area**'s measured in square meters? m^2?

Matt: Yeah. So the note's saying that if the bugs grow to occupy more than that volume, we wind up in trouble. But how do we find out the volume that all the bugs take up? We know how many of them there are after 12 hours, but that doesn't say how big they are all together.

Kyle: I wonder if there's anywhere we can **look up** the volume of one bug? Then we can multiply that by the number of bugs to get the total volume.

Matt: Sweet!



m^3 is m × m × m i.e., length × width × height, so represents a VOLUME.

You can write units in scientific notation, e.g. area (m^2) or volume (m^3).

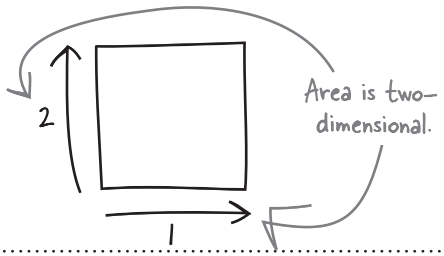
Square meters

Cubic meters

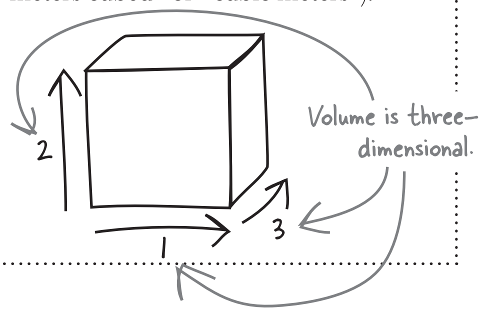
You'll often need to work with area or volume

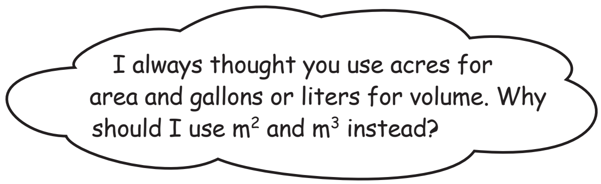
The guys have decided that if they can find out the **volume** of one bug, they can multiply that by the total number of bugs to get the total volume.

Area is the amount of space something occupies in **two dimensions**. In SI units, it's measured in m^2 (if you say it out loud, this is "meters squared" or "square meters").



Volume is the amount of space something occupies in **three dimensions**. In SI units, it's measured in m^3 (if you say it out loud, this is "meters cubed" or "cubic meters").





Area and volume units based on length help you visualize how big things are.

There are lots of other units that people can use for areas and volumes. But in your physics course, it makes the most sense to use units based on **length**.

For a start, this makes it easier to **visualize** how big an area or volume is. If you can imagine a meter, then you can also imagine a square meter or cubic meter. And if you know a football field is approximately 90 m long and 50 m wide, then you have a good idea of what $90 \times 50 = 4500\ m^2$ looks like!

Also, in physics you'll sometimes need to work out the area or volume of something when you know how long it is in different directions. It's far easier to use volume units based on the lengths you already know than it is to introduce even more conversion factors.

Look up facts in a book (or table of information)

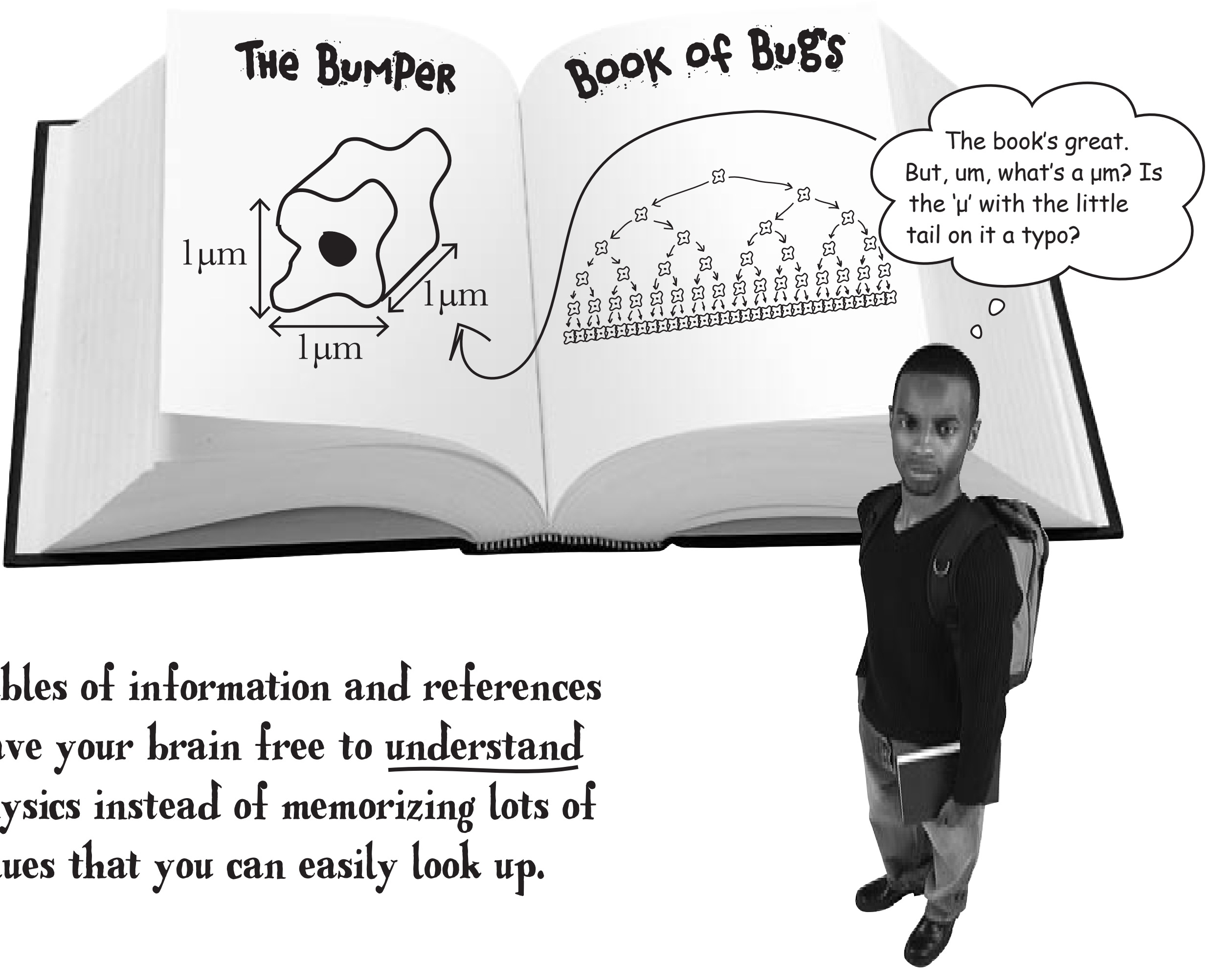
If you're taking a test, you're not completely on your own. In both the multiple choice and the free response sections of the AP Physics B exam, you'll get a **table of information**.

You'll have a table of information in your exam. There's one a bit like it in Appendix B, but it's best for you to go to the AP Central website to download and practice with the one you'll actually have, so it's familiar.

The stuff in the table is there to prevent you from having to **memorize** lots of values (like the mass of an electron) that are vital for certain parts of a question, but don't help you **understand** the physics.

The table of information doesn't have anything in it about bugs, but Matt and Kyle find something that does. The Big Book of Bugs says that the particular strain of bug the Inspector found in their room is about 1 μm long by 1 μm wide by 1 μm high.

The only problem is, what on earth is a μm?



Tables of information and references leave your brain free to understand physics instead of memorizing lots of values that you can easily look up.

Prefixes help with numbers outside your comfort zone

Back in Chapter 2, you learned that you can put **prefixes** in front of units to show how big or small the units are. A kilometer is 1000 meters (10^3 meters), a millimeter is 0.001 of a meter (10^{-3} meters), and so forth.

There are several other SI unit prefixes. The µ prefix in the bug book is the Greek letter 'mu,' and is short for 'micro.' One µm is one millionth of a meter (10^{-6} meters).

That's not exactly the size of number you're used to working with from day to day! Your brain is happier with numbers closer to 1, so it feels better to say that the bug is 1 µm long instead of 0.000001 m long (or 10^{-6} m long).

You usually get a new prefix when something becomes 1000 times bigger, at 10^3, 10^6, 10^9, etc. The exception is 'centi,' which means 10^{-2} and is commonly used in the context of centimeters.

These are the SI prefixes you'll come across most often. (There are more of them, but there's no point in knowing what they all are if you'll never use them).

Power of 10	SI Prefix	Symbol
10^{12}	tera	T
10^9	giga	G
10^6	mega	M
10^3	kilo	k
10^{-2}	centi	c
10^{-3}	milli	m
10^{-6}	micro	µ
10^{-9}	nano	n
10^{-12}	pico	p

The little tail is very important, as this is µ, not u.

You usually get a new prefix every 10^3.

there are no Dumb Questions

Q: So why bother with scientific notation at all when you can just incorporate all the 10's into the prefix, and say µm instead of 1×10^{-6} m?

A: There are equations you'll come across later on that only work when you're measuring distance in meters, mass in kilograms, and so on. So you end up converting everything into meters (usually using scientific notation) anyway.

Scientific notation also makes calculations easier - as you'll see in a moment ...

Q: So why bother with µm and all those prefixes then?

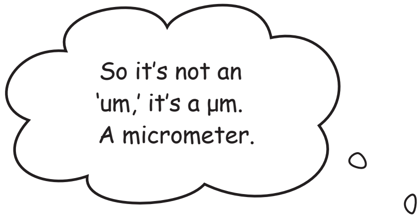
A: Mainly ease of use. In everyday life, it's far easier to talk about "1 millimeter" or "10 kilometers" than it is to talk about "1×10^{-3} meters" or "1×10^4 meters."

Q: But why is that easier?

A: People usually have a better feel for numbers that are similar to the numbers of objects they can count.

Q: So that's why you get things like 'nanotechnology' - it's easier to talk about numbers that are close to everyday counting numbers than it is to say 'thousand-millionth technology'!

A: Yes. When speaking out loud, a physicist will prefer to say that something is 100 nm long rather than 0.0000001 m or 1×10^{-7} m. Once you get used to how big a nanometer is, your brain is happy to use it as a starting point.



Kyle: Yeah, a millionth of a meter. That sure is tiny - I can't even picture it in my head!

Matt: So if one bug is so small, maybe we can get away with cleaning even later! There's a game on tomorrow - If we figure out how many bugs there will be after 16 hours, we could maybe catch that before cleaning.

Kyle: 16 hours is $16 \times 3 = 48$ groups of 20 minutes, so they double in number 48 times. My calculator says that $2^{48} = 2.81 \times 10^{14}$ (to 3 significant digits). So there'd be 2.81×10^{14} bugs.

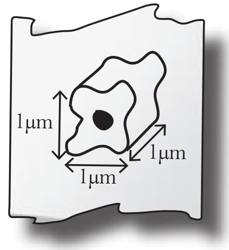
Matt: And after 12 hours, there were 6.87×10^{10} bugs. So what we really want to do is work out what **volume** these bugs take up and compare it with the volume in the Dorm Inspector's note.

Kyle: That's pretty simple. One bug = 1 μm^3, right? So 6.87×10^{10} bugs is 6.87×10^{10} μm^3, and 2.81×10^{14} bugs is 2.81×10^{14} μm^3.

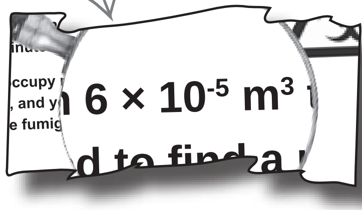
Matt: Uhhh, not so fast. The volume in the note is measured in m^3, not μm^3. So we have to convert the **units** of our answers to m^3 to be able to compare them.

Kyle: Ah, good point. It looks like we're gonna have to do some **calculations** using numbers in scientific notation, keeping in mind that 1 m is the same as 1×10^6 µm. I've no idea how to do that.

Matt: Me neither.



Number of bugs × volume of 1 bug must be less than the volume mentioned in the Dorm Inspector's note.



BRAIN POWER

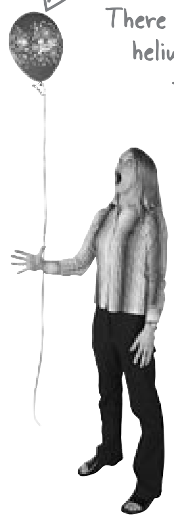
How might scientific notation help you to multiply a very large number and a very small number together?

Scientific notation helps you to do calculations with large and small numbers

Writing down numbers in scientific notation really helps you do calculations that involve big numbers, small numbers, or both.

Suppose someone asks you to work out the mass of all the atoms in a balloon. They tell you that there are 6×10^{22} atoms in the balloon, and each atom has a mass of 6.65×10^{-27} kg.

If you don't use scientific notation, you end up with a horrendous bout of calculator-bashing:



There are 6×10^{22} helium atoms in the balloon.

One helium atom has a mass of 6.65×10^{-27} kg.

The mass of all the atoms is the number of atoms times the mass of one atom :

Mass of all atoms = 6000000000000000000000 × 0.0000000000000000000000000665 kg

This is a horrible-looking calculation!

Powers of 10 make calculations easier.

If you're multiplying together two numbers written in power notation, you can **add the indices**.

Add the indices: 5 + 3 = 8

$$10^5 \times 10^3 = 10^8$$

Here you're multiplying by five lots of 10, and then multiplying by another three lots of 10. So you're multiplying by eight lots of 10 in total, or 10^8.

Add the indices: 5 + (–3) = 2

$$10^5 \times 10^{-3} = 10^2$$

Here you're multiplying by five lots of 10, and dividing by three lots of 10. This is the same as multiplying by two lots of 10 in total, or 10^2.

5 + (–3) is the same as 5 – 3.

$\frac{1}{10^{-3}}$ is the same as 10^3

$$\frac{10^5}{10^{-3}} = 10^8$$

If there's a division sign in your calculation, it's easiest to rewrite it as a multiplication. You can rewrite this as $10^5 \times 10^3 = 10^8$.

$\frac{1}{10^3}$ is the same as 10^{-3}.

$$\frac{10^5}{10^3} = 10^2$$

You can rewrite this as $10^5 \times 10^{-3} = 10^2$.

Pool Puzzle - Powers of 10

Here's the chance to practice working with powers of 10 with some calculations. Your **job** is to take numbers from the pool and place them into the boxes in these statements. You may **not** use the same number more than once, and you won't need to use all the numbers.

$\frac{1}{10^{-24}}$ is the same as 10^{24}.

$\frac{10^{-12}}{10^{-24}} = \square$

Rewrite this as a multiplication (rather than a division) before you work out the answer.

$10^4 \times 10^{-16} = \square$

The vast majority of people have $\square$ nose.

One divided by a million = $\square$

$\frac{10^{-12}}{10^{24}} = \square$

$\frac{1}{10^{24}}$ is the same as 10^{-24}.

$10^5 \times 10^{-13} \times 10^4 = \square$

The two biggest numbers in the pool are $\square$ and $\square$.

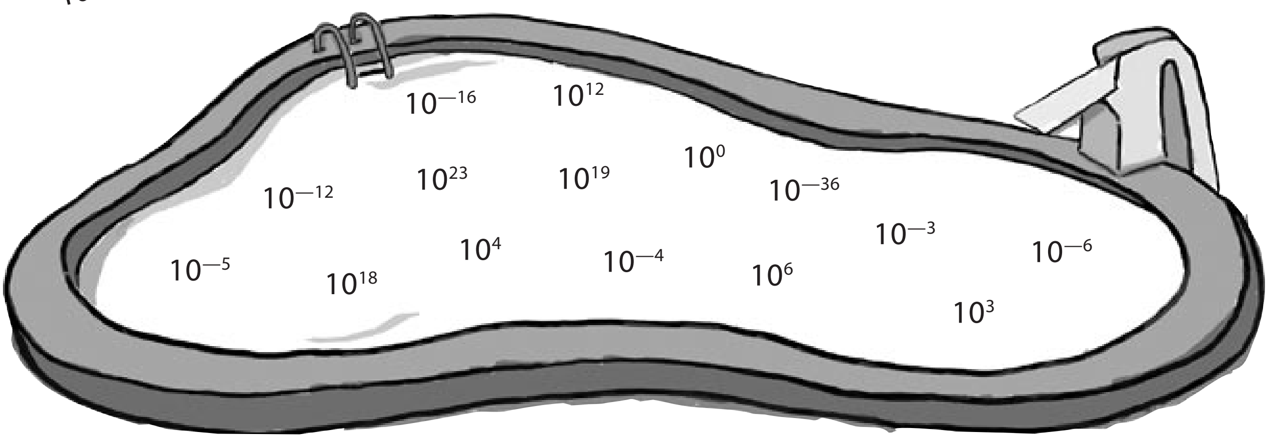
10000 is the same as $\square$.

A millionaire is likely to have at least \$ $\square$ dollars in the bank.

There are $\square$ meters in a kilometer.

$\frac{10^{14}}{10^{-3}} \times 10^{-6} \times 10^7 = \square$

0.000001 is the same as $\square$.



Pool Puzzle - Powers of 10 - SOLUTION

Here's the chance to practice working with powers of 10 with some calculations. Your **job** is to take numbers from the pool and place them into the boxes in these statements. You may **not** use the same number more than once, and you won't need to use all the numbers.

$\frac{1}{10^{-24}}$ is the same as 10^{24}.

$\frac{10^{-12}}{10^{-24}} = \boxed{10^{12}}$

This is the same as $10^{-12} \times 10^{24}$.

$10^{4} \times 10^{-16} = \boxed{10^{-12}}$

The vast majority of people have $\boxed{10^{0}}$ nose.

One divided by a million = $\boxed{10^{-6}}$

$\frac{10^{-12}}{10^{24}} = \boxed{10^{-36}}$

This is the same as $10^{-12} \times 10^{-24}$.

$10^{5} \times 10^{-13} \times 10^{4} = \boxed{10^{-4}}$

The two biggest numbers in the pool are $\boxed{10^{23}}$ and $\boxed{10^{19}}$.

10000 is the same as $\boxed{10^{4}}$.

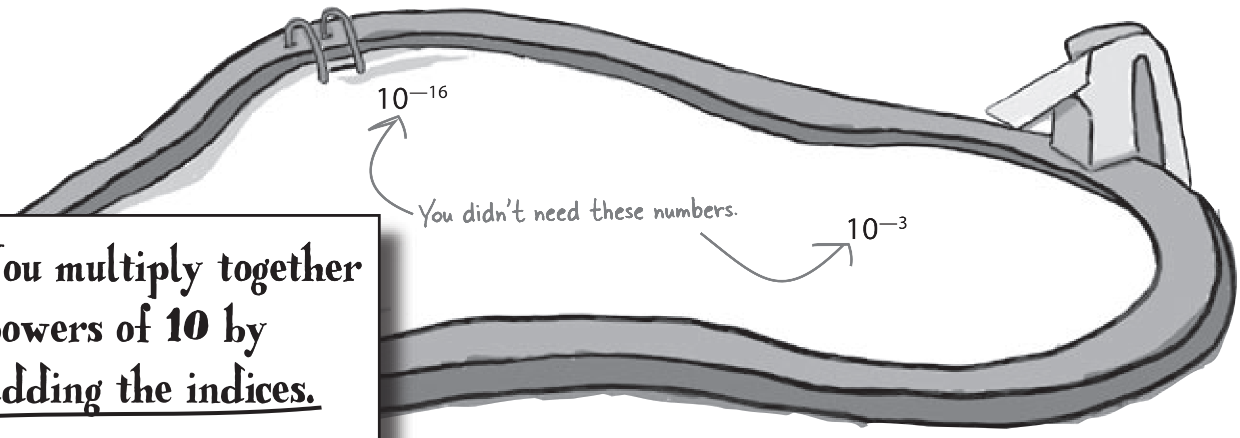
A millionaire is likely to have at least \$ $\boxed{10^{6}}$ dollars in the bank.

There are $\boxed{10^{3}}$ meters in a kilometer.

This is the same as $10^{14} \times 10^{3} \times 10^{-6} \times 10^{7}$.

$\frac{10^{14}}{10^{-3}} \times 10^{-6} \times 10^{7} = \boxed{10^{18}}$

0.00001 is the same as $\boxed{10^{-5}}$



You multiply together powers of 10 by adding the indices.

But numbers written in scientific notation have the bit at the start as well as the power of 10 at the end!

Once you can handle powers of 10, you can do calculations using scientific notation.

A number written in scientific notation has two parts. The first part has one digit before the decimal point, and the second part is a power of 10.

If you're multiplying together two numbers written in scientific notation, it's easiest to deal with the decimal point parts and powers of 10 parts separately before putting them back together.

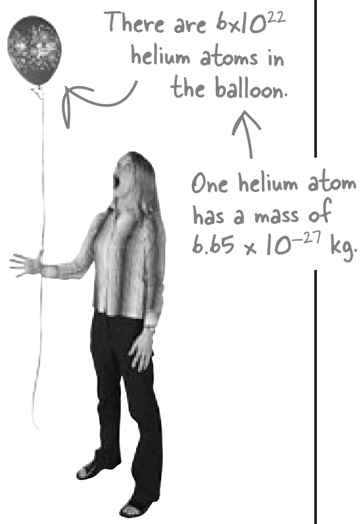
So if you're doing $2 \times 10^3 \times 4 \times 10^2$, you can change the order of the things you're multiplying together:

$2 \times 4 \times \underline{10^3 \times 10^2} = 8 \times 10^5$, which is the correct answer!

Rewrite the order of the parts in the multiplication so that the powers of 10 are all together.

Exercise

Spend some time going through the balloon multiplication step by step before going back to the bug problem.



a. There are 6×10^{22} helium atoms in the balloon. 1 helium atom has a mass of 6.65×10^{-27} kg. Write down the multiplication you'd do to find the mass of all the atoms in the balloon.

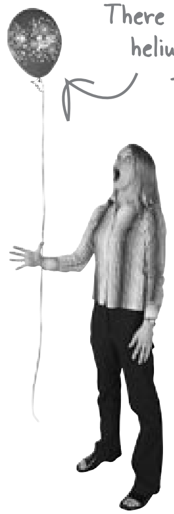
b. You are allowed to multiply numbers together in any order. Change the order of the things you're multiplying together so that the powers of 10 are next to each other.

c. Multiply together the two 'decimal point' parts. Then multiply together the two 'powers of 10' parts. This should give you an answer with one decimal point part and one power of 10 part.

d. Rewrite your answer from part c. so that the decimal point part has one digit in front of the decimal point. (You'll need to adjust the power of 10 part.)

Exercise Solution

Spend some time going through the balloon multiplication step by step before going back to the bug problem.



a. There are 6×10^{22} helium atoms in the balloon. 1 helium atom has a mass of 6.65×10^{-27} kg. Write down the multiplication you'd do to find the mass of all the atoms in the balloon.

Mass of all atoms $= 6 \times 10^{22} \times 6.65 \times 10^{-27}$ kg

b. You are allowed to multiply numbers together in any order. Change the order of the things you're multiplying together so that the powers of 10 are next to each other.

Mass of all atoms $= 6 \times 6.65 \times 10^{22} \times 10^{-27}$ kg

c. Multiply together the two 'decimal point' parts. Then multiply together the two 'powers of 10' parts. This should give you an answer with one decimal point part and one power of 10 part.

Mass of all atoms $= 39.9 \times 10^{-5}$ kg

d. Rewrite your answer from part c. so that the decimal point part has one digit in front of the decimal point. (You'll need to adjust the power of 10 part.)

Mass of all atoms $= 3.99 \times 10^{-4}$ kg

Your answer needs to stay the same size.

If you wind up with the answer 39.9×10^{-5}, you need to rewrite it in scientific notation as a number that has a part with **one digit in front of the decimal point** and a power of 10 part.

You need to divide the 39.9 part of the number by 10 to get 3.99. But the number must remain the same size- so you need to multiply the second part by 10.

$$39.9 \times 10^{-1} \times 10^{-5} \times 10^{1} = 3.99 \times 10^{-4}$$

You need to divide the first part by 10 (same as multiplying by 10^{-1}) to get one digit in front of the decimal point.

So you need to multiply the second part by 10 to keep your answer the same size.

there are no Dumb Questions

Q: My calculator has a button that lets me input numbers in scientific notation. Why can't I just use that?

A: Sometimes you don't get to use a calculator, like in the multiple choice part of the AP exam!

Plus doing the calculation like this helps you to see the size of answer you're expecting, and you're more likely to spot silly mistakes when you ask yourself if your answer SUCKs.

Q: But when will I ever see numbers like this in my exam?

A: There are lots of very big or very small numbers in physics - the size of an electron, the mass of the earth, etc.

If you're doing a calculation with numbers written in scientific notation, you should separate out the powers of 10 to add the indices before recombining them with the rest of the numbers.

The guys have it all worked out

Well, they hope they do...

Matt and Kyle have had a go at working out the volume of the bugs using scientific notation. But have they done it correctly?

Sharpen your pencil

Matt and Kyle have had a go at working out the volume of bugs in m^3 that there'll be after 12 and 16 hours. Your job is to see if you agree with their calculations, and decide whether their final answer SUCKs (Size, Units, Calculations, 'K'ontext).

Look at the **Size** of their answer - if it feels wrong, then look to see if there's a mistake with the **Units** or **Calculations**.

There are 10^6 μm in 1 m.

We'll use this as the conversion factor for the volume.

After 12 hours:

$$\text{Volume of bugs} = 6.87 \times 10^{10}\ \mu m^3$$

$$= 6.87 \times 10^{10}\ \cancel{\mu m^3} \times \frac{1\ m^3}{10^6\ \cancel{\mu m^3}}$$

$$= 6.87 \times 10^{10} \times 10^{-6}\ m^3$$

$$= \underline{\underline{6.87 \times 10^{4}\ m^3}}$$

After 16 hours:

$$\text{Volume of bugs} = 2.81 \times 10^{14}\ \mu m^3$$

$$= 2.81 \times 10^{14}\ \cancel{\mu m^3} \times \frac{1\ m^3}{10^6\ \cancel{\mu m^3}}$$

$$= 2.81 \times 10^{14} \times 10^{-6}\ m^3$$

$$= \underline{\underline{2.81 \times 10^{8}\ m^3}}$$

Sharpen your pencil Solution

Matt and Kyle have had a go at working out the volume of bugs in m^3 that there'll be after 12 and 16 hours. Your job is to see if you agree with their calculations, and decide whether their final answer SUCKs (Size, Units, Calculations, 'K'ontext).

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After 12 hours:

$$\text{Volume of bugs} = 6.87 \times 10^{10}\ \mu m^3$$

$$= 6.87 \times 10^{10}\ \cancel{\mu m^3} \quad \frac{1\ m^3}{10^6\ \cancel{\mu m^3}}$$

$$= 6.87 \times 10^{10} \times 10^{-6}\ m^3$$

$$= 6.87 \times 10^{4}\ m^3$$

After 16 hours:

$$\text{Volume of bugs} = 2.81 \times 10^{14}\ \mu m^3$$

$$= 2.81 \times 10^{14}\ \cancel{\mu m^3} \times \frac{1\ m^3}{10^6\ \cancel{\mu m^3}}$$

$$= 2.81 \times 10^{14} \times 10^{-6}\ m^3$$

$$= 2.81 \times 10^{8}\ m^3$$

These answers are totally the wrong SIZE!

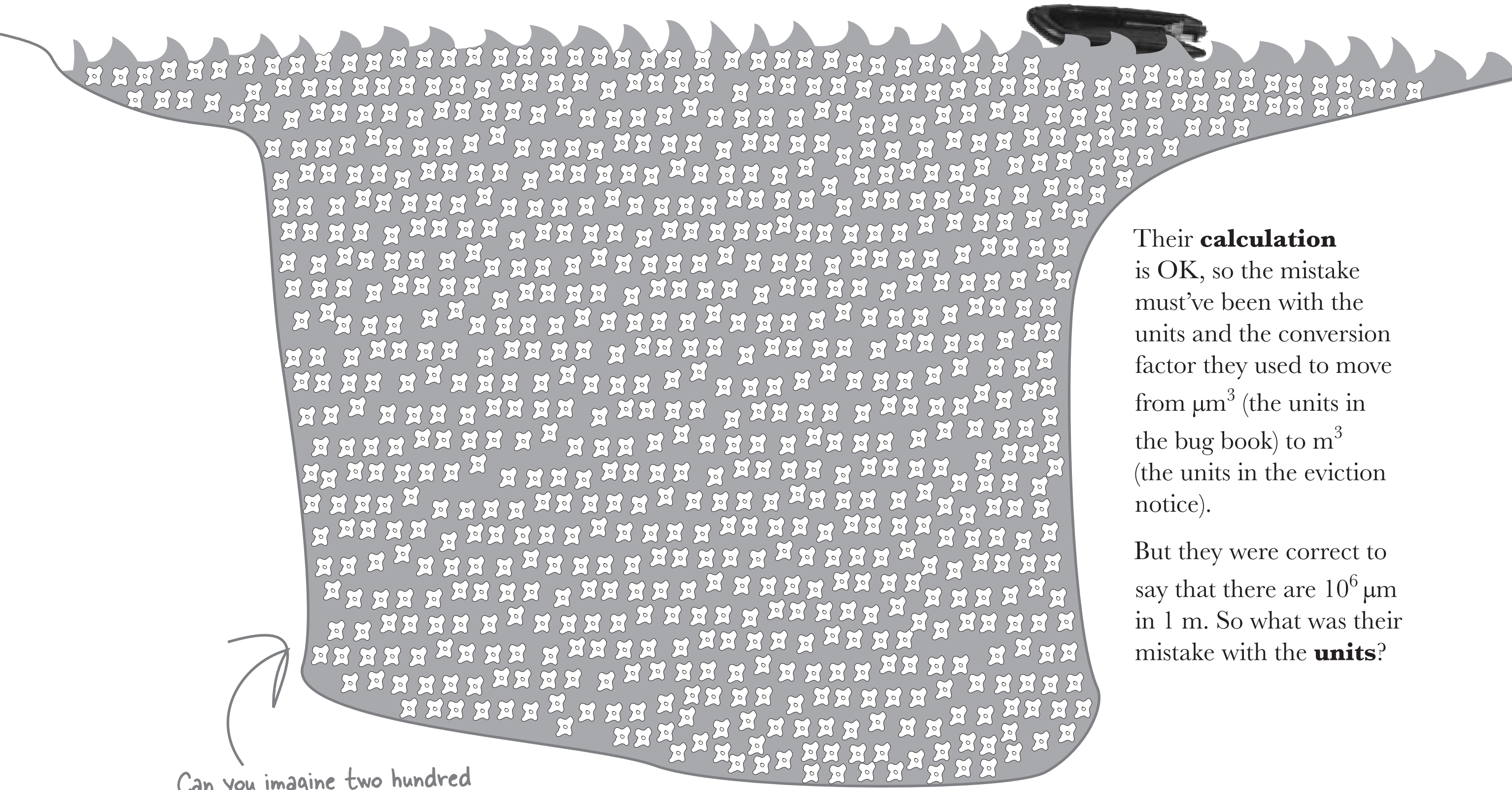
$2.81 \times 10^8\ m^3$ is the same volume as 2000 giant football stadiums!

This is the problem! There are 10^6 μm in 1 m, but that's LENGTH, not VOLUME. You can't use it as a conversion factor for volume.

Volume is length × length × length and has units of m^3, not m. Converting between μm^3 and m^3 is different from converting between μm and m.

200,000,000 meters cubed bugs after only 16 hours is totally the wrong size of answer!

The **size** of Matt and Kyle's answer is waaay off - if you spotted that, well done! It just doesn't make sense to say that the bugs will occupy a volume of two hundred million cubic meters after only 16 hours - **especially if you can visualize the size of just one cubic meter**!



Their **calculation** is OK, so the mistake must've been with the units and the conversion factor they used to move from μm^3 (the units in the bug book) to m^3 (the units in the eviction notice).

But they were correct to say that there are 10^6 μm in 1 m. So what was their mistake with the **units**?

Can you imagine two hundred million cubic meters of bugs? Think of an enormous sea of bugs....eww.

BRAIN POWER

There **are** 10^6 μm in a m - that part is correct.

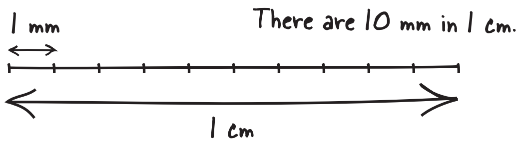
So why has there been such a problem converting the units from μm^3 to m^3?

Be careful converting units of area or volume

Although there are 1×10^6 µm in a m, there aren't 1×10^6 μm^3 in a m^3. This is difficult to see because the numbers are so big - and it's what the guys missed. They've ended up saying that the bugs will occupy a volume of two hundred million cubic meters after only 16 hours, which must be nonsense!

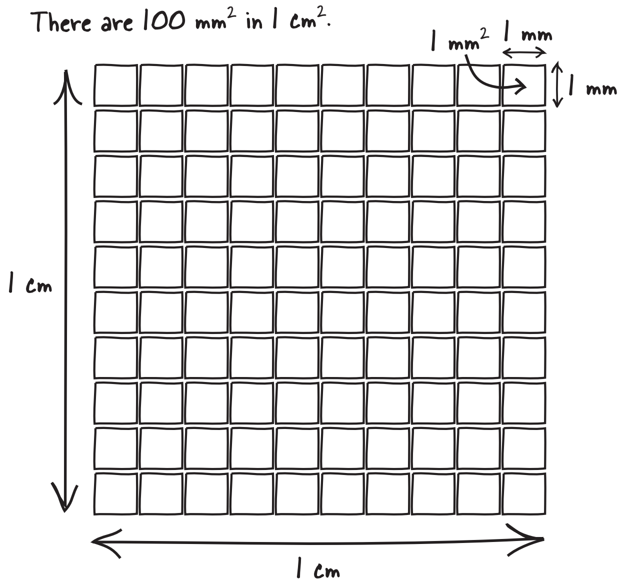
A µm is so small; it's easier to visualize how this happened using mm and cm to measure area and volume.

Length is **one-dimensional**.
There are 10 mm in 1 cm.



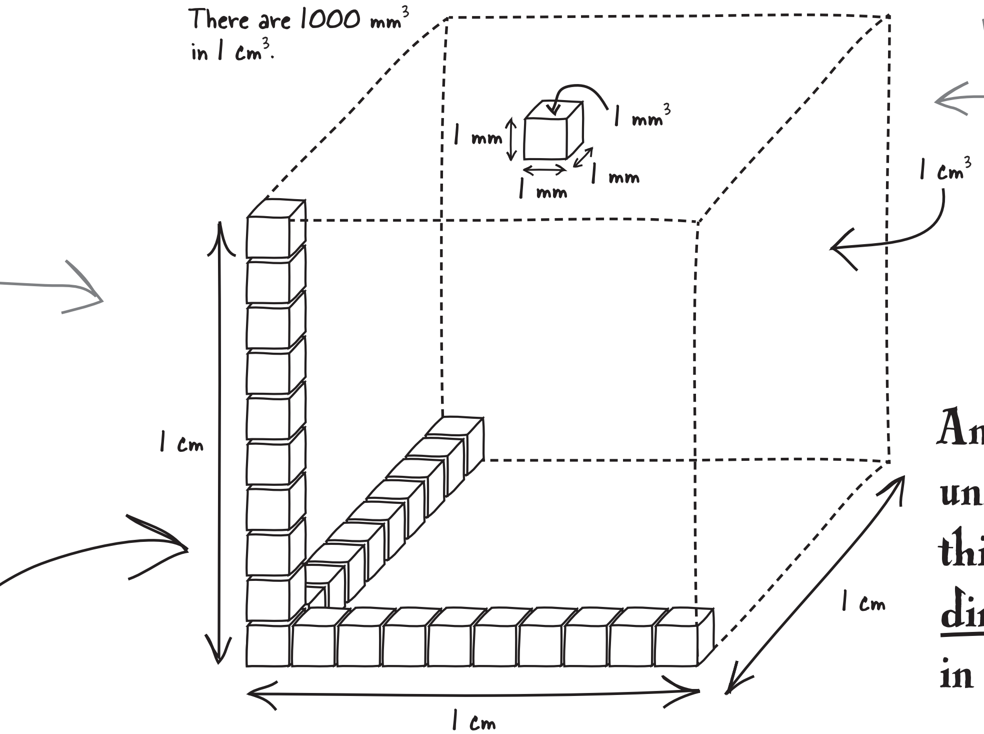
All these pictures are magnified from actual size!

Area is **two-dimensional**, i.e., length × width.
So there are $10 \times 10 = 100$ mm^2 in 1 cm^2.
Or in scientific notation: $10^1 \times 10^1 = 10^2$ mm^2 in 1 cm^2.



Area and volume are NOT the same as length.

Volume is **three-dimensional**, i.e., length × width × height.
So there are $10 \times 10 \times 10 = 1000$ mm^3 in 1 cm^3.
Or in scientific notation: $10^1 \times 10^1 \times 10^1 = 10^3$ mm^3 in 1 cm^3.



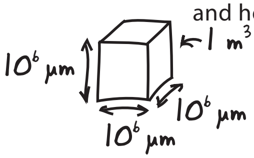
Any time you convert units of area or volume, think about how many dimensions you're working in before you get going!

Sharpen your pencil

The bugs occupy 6.87×10^{10} μm^3 after 12 h and 2.81×10^{14} μm^3 after 16 h. How many m^3 is that, and how does this compare with the 6×10^{-5} m^3 mentioned in the note? (There are 10^6 μm in 1 m.)

Sharpen your pencil Solution

The bugs occupy 6.87×10^{10} µm^3 after 12 h and 2.81×10^{14} µm^3 after 16 h. How many m^3 is that, and how does this compare with the 6×10^{-5} m^3 mentioned in the note? (There are 10^6 µm in 1 m.)



There are 10^6 µm in 1 m. Volume is three-dimensional.

So there are $10^6 \times 10^6 \times 10^6 = 10^{18}$ µm^3 in 1 m^3.

After 12 hours:

$$\text{Volume of bugs} = 6.87 \times 10^{10}\ \mu m^3 = 6.87 \times 10^{10}\ \mu m^3 \times \frac{1\ m^3}{10^{18}\ \mu m^3} = 6.87 \times 10^{10} \times 10^{-18}\ m^3 = \underline{\underline{6.87 \times 10^{-8}\ m^3}}$$

This is less than 6×10^{-5} m^3.

After 16 hours:

$$\text{Volume of bugs} = 2.81 \times 10^{14}\ \mu m^3 = 2.81 \times 10^{14}\ \mu m^3 \times \frac{1\ m^3}{10^{18}\ \mu m^3} = 2.81 \times 10^{14} \times 10^{-18}\ m^3 = \underline{\underline{2.81 \times 10^{-4}\ m^3}}$$

This is more than 6×10^{-5} m^3.

So the bugs won't take over ... unless the guys sleep in!

You just worked out that if the guys remember to set their alarms for 12 hours from now, the bugs won't reach the tipping point and get the guys evicted.

But if the guys try waiting 16 hours to catch the football game, the Dorm Inspector will turf them out. Unfortunately...

Question Clinic: The "Converting units of area or volume" Question

In the course of doing another question, you'll often find yourself having to **convert units of area or volume**. As soon as you see areas or volumes mentioned, think of the bugs occupying 2000 football stadiums - you don't want to majorly mess up like that! Keep your cool, and think about what area and volume actually **are** to convert the units.

These are all LENGTHS, and they're all in mm.

But you're asked for a VOLUME in m^3, not mm^3.

4. A treasure chest is 800 mm long, 400 mm wide, and 500 mm high.

   a. What is its volume in m^3?

   b. How many gold blocks measuring 20 cm by 10 cm by 5 cm could fit in the chest? (Assume that you're allowed to melt down the gold so that it will fit exactly.)

It's probably best to work through the whole question in m^3, as this is the unit you're asked to give your answers in.

Here's yet another way of measuring length (cm) that needs to be converted into a volume.

With volume questions, you sometimes have to think of how many blocks you could stack in each direction before you reach the side of the container. This bit lets you know that you don't need to allow for that.

It's often easiest to convert the lengths to m first and work out the volume in m^3. (Though you could also work out the area in cm^3 and convert that to m^3 at the end.)

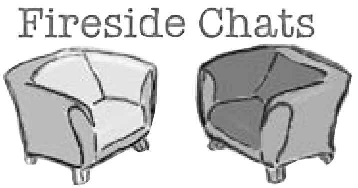
When you have to convert units of area or volume, draw a little **sketch** to make sure the powers of 10 work out OK in your conversion factor.

1 m^2 (square of side 1000 mm by 1000 mm)

$1\ m^2 = 1000\ mm \times 1000\ mm$

$= 10^3\ mm \times 10^3\ mm$

$= 10^6\ mm^2$



Tonight's talk: **A normal number and one written in scientific notation go head-to-head. Who will be the last one standing when the revolution comes.**

Normal number: | **Scientific notation number:**

Well, well, Mr. Obscurity. Lovely to meet you at last!

Less of that! I know most people go 16+ years before they meet me, but I'm well worth the wait!

You hide for decades while I do all the counting, shopping, algebra, and so on. Then, suddenly you show up and expect to take center stage. It's just plain rude!

You can still do all the stuff you've always done – I'm not trying to steal your thunder. But the fact is that I am naturally better at some things.

Yeah? Show me one thing you can do that I can't!

OK, what about writing out Bill Gates' fortune nicely? 5×10^{10} dollars.

No way! "50 billion dollars" gives you a much better idea of size than these silly little numbers by the ten.

Well, my way is really obviously "5" with ten zeros after it – what's so difficult about that?

It's not what people are used to! They've been doing it my way all their lives, and you've totally failed to show that your way is better.

Well then, Mister "They're used to me," how would you talk about the mass of a proton?

Um, let me look up my table of information. That'd be 0.00000000000000000000000000167 kilograms.

When have people ever seen silly numbers like that? They're not used to them at all! They're better off writing it my way – 1.67×10^{-27} kilograms.

But they can still count up my zeros to get the size.

Well, yeah, but who's going to bother with that?

Um, OK. I guess you are the master of showing big or small numbers. So ... do you plan to take over completely?

Normal number

Scientific notation number:

Not completely. People still prefer doing things your way if they can! They've even invented a whole load of units so they don't have to go around saying "blah times ten to the power of whatever" all the time. Like nanometers, kilograms, and such.

I told you they like me more!

Actually, all I've said is that people like numbers they feel comfortable with. But when it comes to doing **calculations** with the numbers, that's when I shine.

Yeah, right. Every calculator manufacturer has a different way to input you, especially your 10 to the power of negative thingamajiggers.

Ah, but the humans don't have to do that if they're multiplying or dividing numbers like me. They only type the first bit into their calculators, the 1.67 or whatever. Then they do the tens part separately, on paper (which is just simple adding).

Sorry? You can multiply by adding? Sounds a bit suspicious to a purist like me.

Yes, if you're multiplying lots of tens parts together, all you need to do is add the indices. $10^2 \times 10^3 = 10^5$ can be broken down into "$10^2 = 10 \times 10$" and "$10^3 = 10 \times 10 \times 10$." Together, they make "$10 \times 10 \times 10 \times 10 \times 10$." Five lots of 10 is 10^5.

So you're saying that the first bit of you is easy to deal with (but only because people have practiced with me), and the second bit is easy because they're used to adding sums (again, because of me)?

I suppose you could put it like that. Maybe we need each other more than we realized in the beginning.

The giant who came for breakfast

Once upon a time, long long ago, a refugee from the land of the giants came to the king's palace. He was hungry. Now, when I say hungry, I don't just mean the kind of hunger you feel in between meals. The giant was twice as tall as a normal person...and starving.

The king sighed with relief when he realized that the giant wasn't about to gobble him up, but instead was far more interested in a hearty meal of sausage, bacon, and eggs.

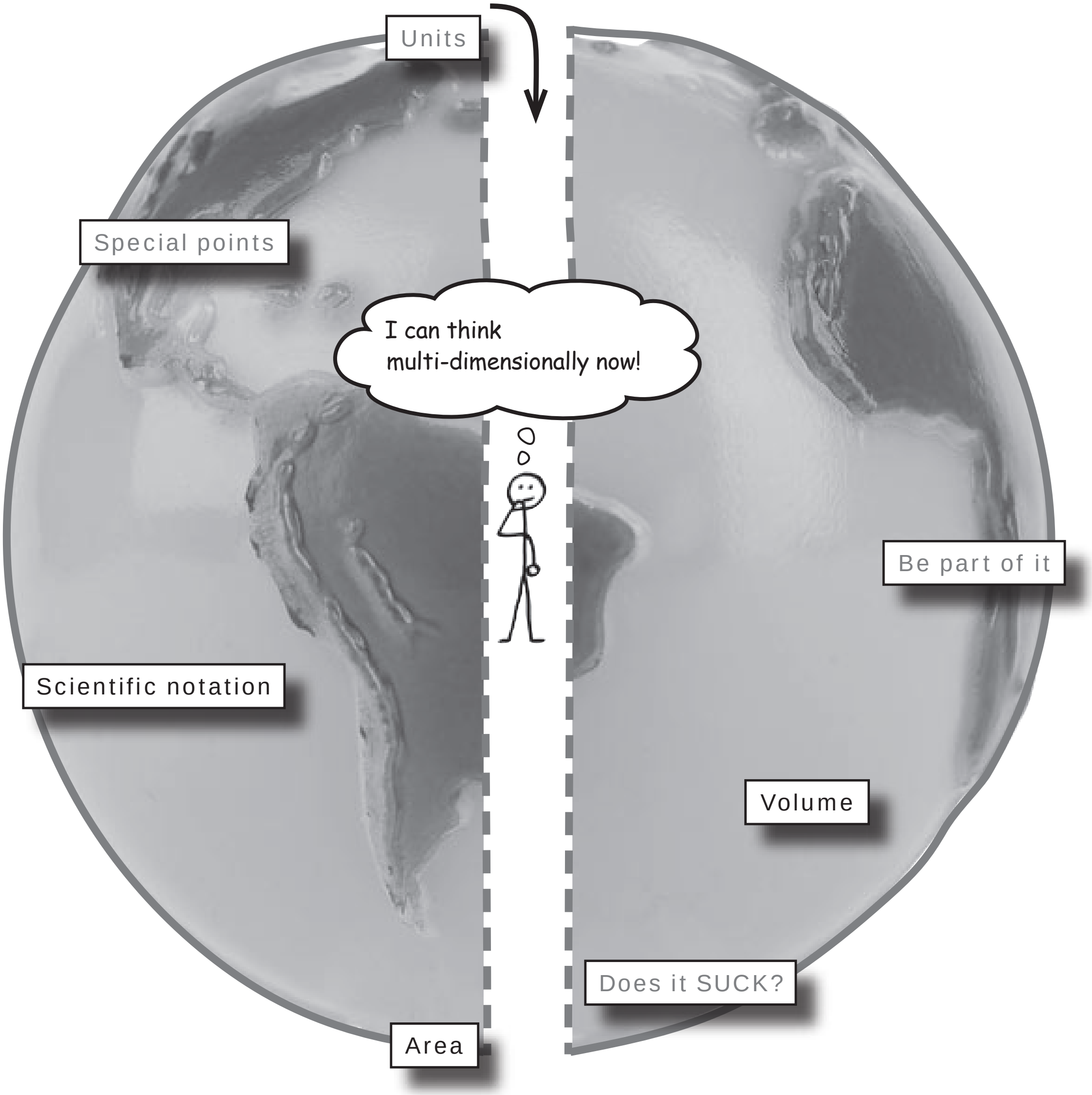
"But if it's paltry food you bring, I'll eat the servants, then the king!" he concluded.

The smile froze on the king's lips, becoming the kind of grimace you might muster when face-to-face with a hungry giant who'd just threatened to turn you into a tasty snack.

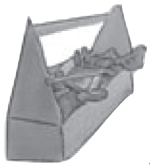
The king gathered his advisers around him to ask them for advice (which advisers generally provide).

"Your highness," they began. "Since the giant is twice a tall as a normal person and perfectly in proportion, we should serve him twice as much breakfast...maybe with a couple of extra pieces of toast just in case."

Should the king listen, and agree to feed the giant twice as much as a normal person because he's twice as tall?



☑	Scientific notation	A method of representing long numbers using powers of 10.
☑	Area	Two-dimensional space.
☑	Volume	Three-dimensional space.



Your Physics Toolbox

You've got Chapter 3 under your belt, and you've added some terminology and answer-checking skills to your toolbox.

Power notation

Power notation is a way of showing that you're multiplying or dividing by the same number over and over again.

For example, if you have 10^6, it means you're multiplying by 10 six times over.

And if you're dividing by the same number several times, you can show this using a negative index, e.g., 10^{-7}.

Multiplying powers of 10 by each other

Multiply together different powers of 10 by adding the indices.

For example, $10^5 \times 10^{-2} = 10^3$ because $5 + (-2) = 3$

Dividing powers of 10 by each other

If you're dividing by a power of 10, it's easiest to rewrite this as a power of 10 you're multiplying by, then add the indices as before.

For example:

$$\frac{10^5}{10^2} = 10^5 \times 10^{-2} = 10^3$$

Scientific notation

Scientific notation is a way of writing really long numbers using two parts multiplied together.

The first part is written as a number with one digit in front of the decimal point.

The second part is a power of 10.

For example, $5 \times 10^3 = 5000$

Calculations using scientific notation

If you have to multiply together several numbers written in scientific notation, it's easiest to treat the powers of 10 separately before putting the number back together.

Converting units of area and volume

Areas are based on lengths – you can imagine what a m^2 or a m^3 looks like and see if your answer makes sense.

Start any units conversion of area or volume with a sketch to make sure you use the correct conversion factor.

This will often involve multiplying or dividing powers of 10 by each other.

The giant who came for breakfast

Should the king listen, and feed the giant twice as much because he's twice as tall as a normal person?

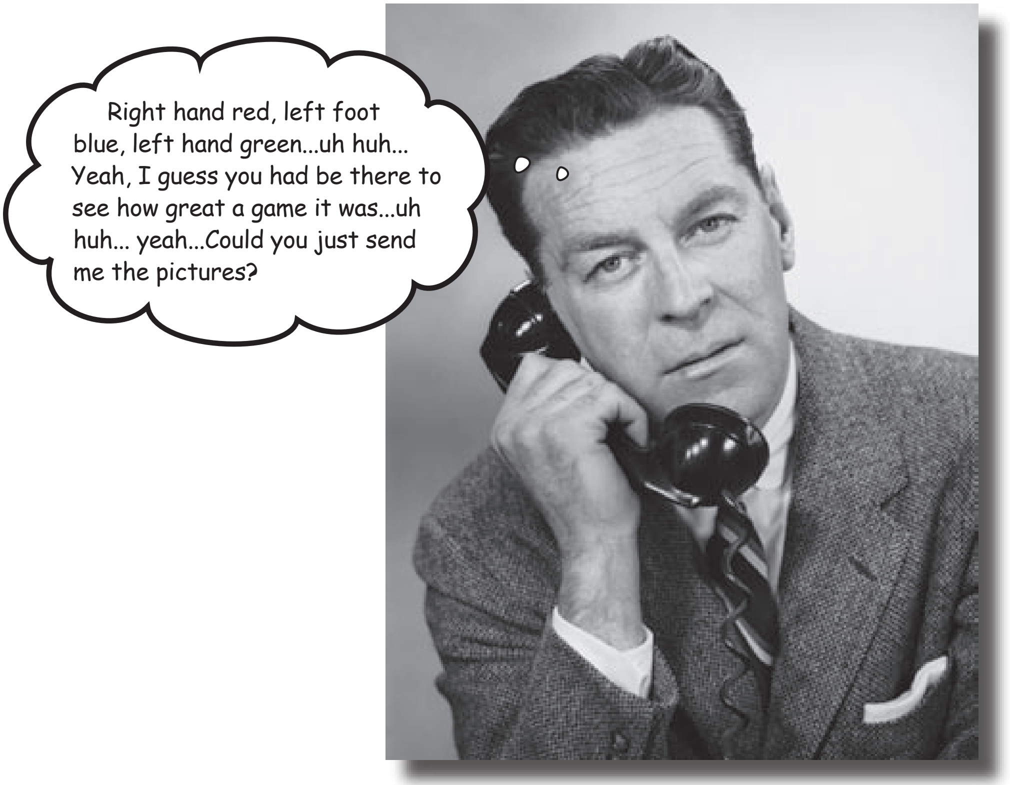
The king needs to make sure that the giant's stomach ends up full of food, so there's no room left in there for him or his advisors!

If the giant's twice as tall as a normal man and in proportion, then he's also twice as wide from side to side, and twice as deep from front to back.

So the giant's stomach is twice as high, twice as wide and twice as deep as a normal man's and is $2 \times 2 \times 2 = 8$ times larger -so the king should order **eight** breakfasts for the giant to be on the safe side.

4 equations and graphs

Learning the lingo

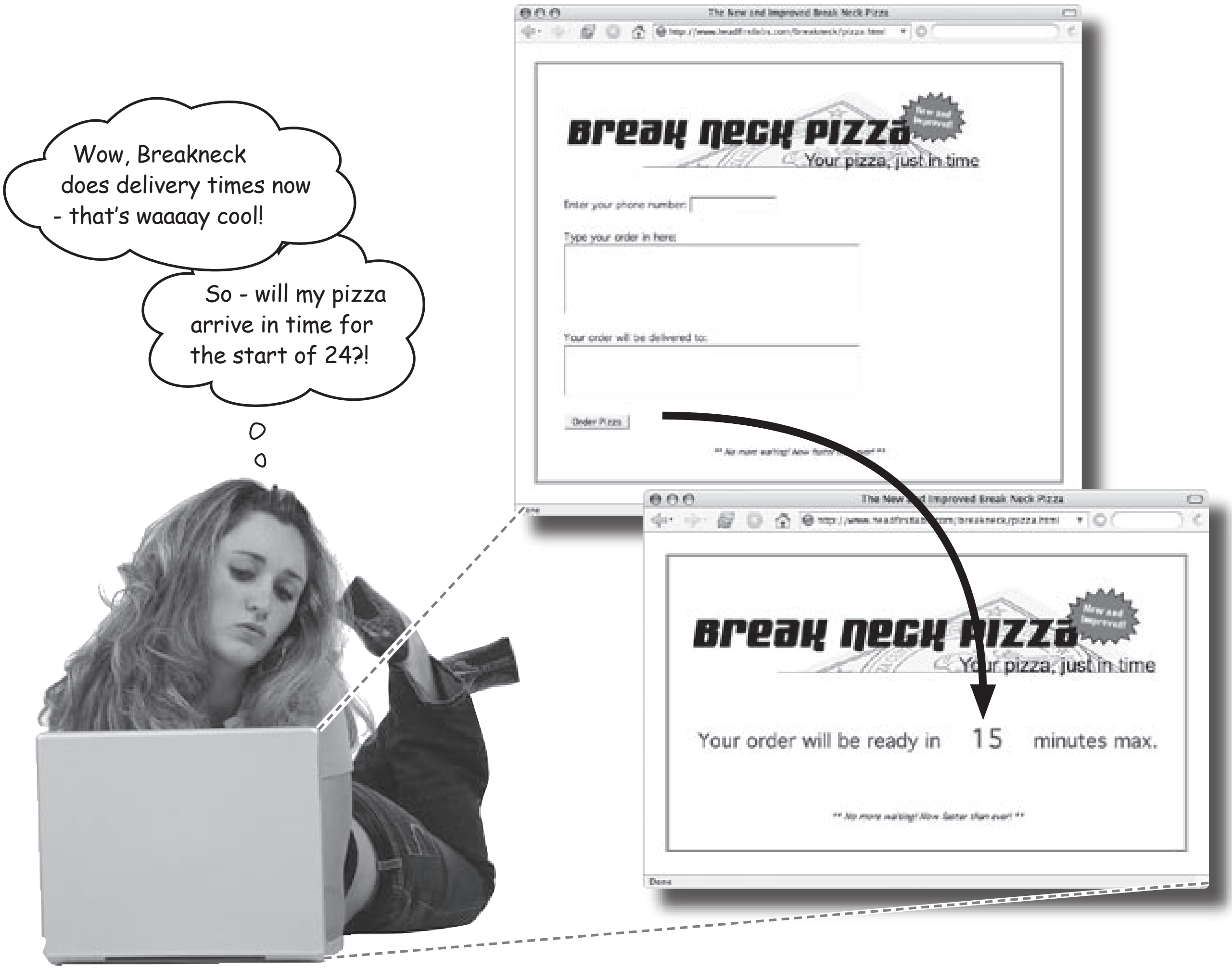


Communication is vital. You're already off to a good start in your journey to truly think like a physicist, but now you need to **communicate your thoughts**. In this chapter, you're going to take your first steps in two universal languages - **graphs** and **equations** - pictures you can use to *speak a thousand words* about experiments you do and the physics concepts you're learning. *Seeing is believing*.

The new version of the Break Neck Pizza website is nearly ready to go live ...

Break Neck Pizza already revolutionized pizza delivery through its patented "just in time" cooking process and its large fleet of delivery bicycles.

But now it's even better! The award-winning Break Neck website has just been upgraded to give each customer a **delivery time** for their order.



... but you need to work out how to give the customer their delivery time

You've been called in to figure out what time the website should quote each customer. The web programmers are happy to implement your solution, and you also have Alex, the top Break Neck delivery guy, on hand to help. He knows how long it takes him to get to some houses, but you'll need another way to work out the delivery time to new, unknown neighborhoods.

Alex, Break Neck's top delivery guy.

Sharpen your pencil

Write down everything you can think of that might affect the delivery time the website should give to a customer.

How fast Alex can cycle ....

....

....

....

....

....

....

Sharpen your pencil Solution

Write down everything you can think of that might affect the delivery time the website should give to a customer.

- Alex's cycling speed
- Distance from Break Neck to the customer
- Can all the pizzas be cooked at once?
- Deep pan or Italian base?
- How long it takes the website to process the order
- Whether the customer enters their address correctly

These both have to do with the time it takes Alex to reach the house.

Both of these affect the time it takes for the pizza to cook.

It's fine if you had some other answers – like the amount or type of topping – that we haven't jotted down here.

It's OK to assume that the web guys have already dealt with these. You only need to think about the physics!

If you write the delivery time as an equation, you can see what's going on

The total delivery time is the time Alex spends cycling to the customer's house plus the cooking time for the pizza.

It takes a lot of words to describe this, and it's difficult to tell what's going on without reading the whole thing. This is why in physics, people use **equations** to describe how the world works.

You can use **letters** with subscripts as symbols to represent each of the times:

t_{total} for the total delivery time.

t_{cyc} for Alex's cycling time.

t_{cook} for the cooking time.

Use 't' to show that a symbol represents a time, and use subscripts to indicate which time, e.g., t_{cook}.

Then you can write down the equation $t_{total} = t_{cyc} + t_{cook}$. This says exactly the same thing as "The total delivery time is equal to the time Alex spends cycling to the customer's house plus the cooking time for the pizza." Except the equation lets you see that at a glance.

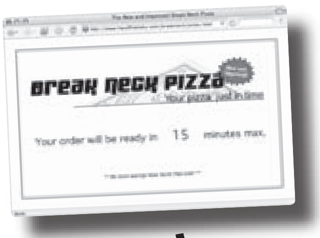
We're using italics here to represent variables. You don't have to do this when you're writing them down!

Equations let you represent the real world symbolically.

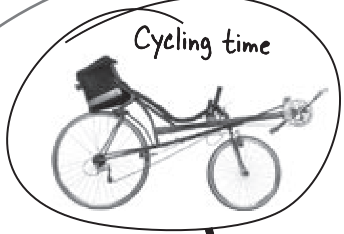
Use variables to keep your equation general

So you can reuse it for ANY customer who orders pizza.

An equation like $t_{total} = t_{cyc} + t_{cook}$ is **general** because it isn't tied to any particular values for the cycling or cooking times. This means you can use the same equation for any order. Any quantity represented by a letter rather than a number is called a **variable**. Here it's good for t_{total}, t_{cyc}, and t_{cook} to be variables because their values will vary for each customer.



These are all VARIABLES, as they're different for each customer.



$$t_{total} = t_{cyc} + t_{cook}$$

The order you add in doesn't matter. You could also write $t_{total} = t_{cook} + t_{cyc}$.

there are no Dumb Questions

Q: Why bother with an equation when a description will do just as well?

A: Descriptions are good because if you can explain something in words, you know you've got it. But equations are also good because they let you be short and sweet when you're explaining things.

Q: But the equations use letters in them. Surely that makes things harder because you have to explain what each of the letters means before anyone else can understand what the equation says.

A: That's true - but once you know what the letters represent, they're much quicker to write down and take in than a load of words.

Q: But why use letters in the equation at all? Surely it would be easier to get the delivery time if we write numbers in there from the start?

A: You're right, that would work - but only for one particular house! You'd need to start over again for each new delivery.

Q: Can't I use different letters to represent each thing instead of using *t* all the time?

A: Each of the variables in your equation is a time. It's conventional to use the letter *t* to represent a time, and use subscripts to indicate which time that variable represents.

Q: But why use subscripts rather than using two letters next to each other? Surely using initials like 'dt' for delivery time would be clearer?

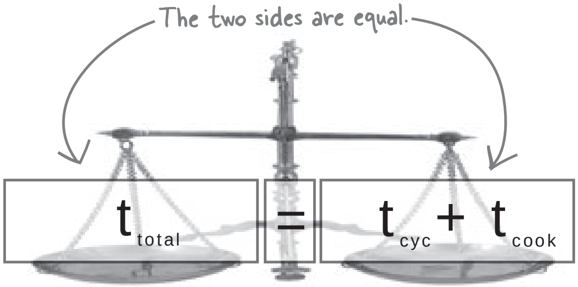
A: One reason is that in physics and math, you indicate that you're multiplying two variables together by writing them next to each other. So you write $a \times b$ as ab. The second is that '*dt*' is already reserved for something else (which you'll come across later on in the book).

Q: Oh yeah, I kinda remember that from math. Why is it useful?!

A: It helps you to see the building blocks of your equation more easily. We're just getting to that ...

Equations Up Close

An equation uses symbols to show you that two things are **equal** to each other.



$$t_{total} = t_{cyc} + t_{cook}$$

An equation must have an equal sign!

If it doesn't, it's not an EQUAtion!

In an equation, each of the blocks you add together or subtract from each other is called a **term**. A term can be a number, a single variable, or several numbers and variables multiplied or divided by each other.

$$\boxed{t_{total}} = \boxed{t_{cyc}} + \boxed{t_{cook}}$$

These are terms.

A TERM is one of the building blocks that you add or subtract.

You can break down equations by thinking about them **one term at a time**. If a term is more complicated than a single variable, you need to do all the calculations (multiplying, dividing, and so forth) within a term before you add it to or subtract it from other terms. So that you can spot terms easily, it's usual to use a shorthand for multiplication, like *bz* instead of *b* × *z* since this groups together all the variables in each term.

$$\boxed{x} = \boxed{y} + \boxed{bz}$$

Think about equations one term at a time

This term is two variables multiplied together. If you're putting numbers into your equation, you need to work out bz before you add the term to the others.

You need to work out Alex's cycling time

Your equation for the delivery time is $t_{total} = t_{cyc} + t_{cook}$. There are two terms on the right hand side: t_{cyc} for the cycling time, and t_{cook} for the cooking time. You can make the job of working out the total time, t_{total}, easier for yourself by thinking about t_{cyc} and t_{cook} one at a time.

You already made great progress with t_{cyc} (Alex's cycling **time**) when you intuitively realized that t_{cyc} must depend on the **speed** at which Alex cycles and the **distance** the customer's house is from Break Neck. But **how** do the speed and distance affect the cycling time?

What do you expect to happen for **extremes**, like a short distance or a high speed?

Sharpen your pencil Solution

Write down everything you can think of that migh
time the website should give to a customer.

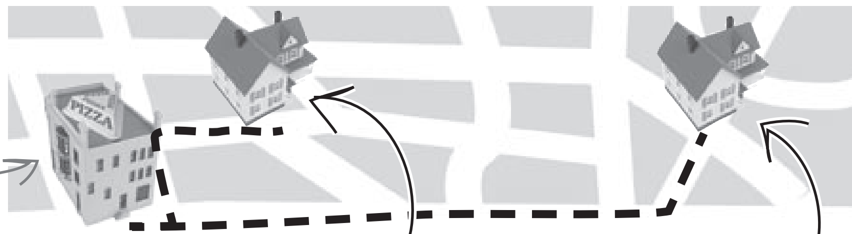
How fast Alex can cycle

Distance from Break Neck to the customer

Part of your Sharpen answer from page 98.

Pizza Delivery Magnets

Your job is to use the magnets to explain **how** the **distance** to the house and the **speed** at which Alex cycles affect the **time** it takes him to get there. There are only two words on the magnets, but they all have a place on the pictures.

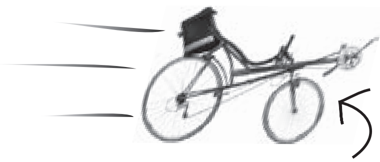


Break Neck Pizza HQ

Remember – t_{cyc} is the time it takes Alex to cycle there.

If the distance is ____
then t_{cyc} is ____.

If the distance is ____
then t_{cyc} is ____.



If the speed is ____
then t_{cyc} is ____.

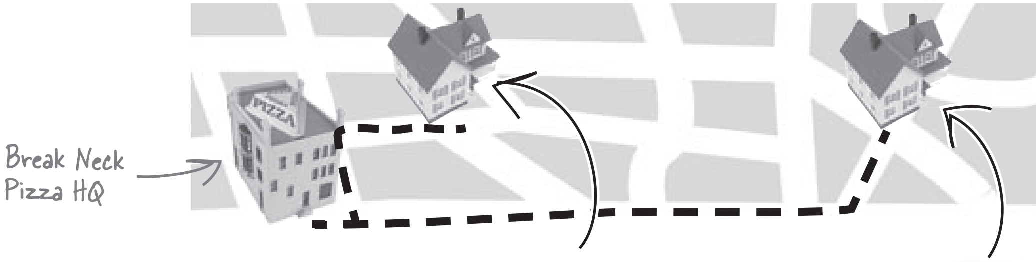
If the speed is ____
then t_{cyc} is ____.

higher | higher | lower | higher

lower | lower | higher | lower

Pizza Delivery Magnets Solution

Your job is to use the magnets to explain how the **distance** to the house and the **speed** at which Alex cycles affect the **time** it takes him to get there. There are only two words on the magnets, but they all have a place on the pictures.



If the distance is **lower**

then t_{cyc} is **lower**.

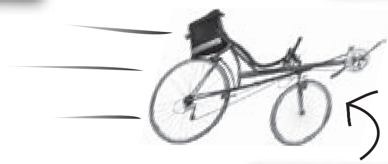
If the distance is **higher**

then t_{cyc} is **higher**.

Remember – t is the time it takes Alex to cycle there.

If the speed is **lower**

then t_{cyc} is **higher**.



If the speed is **higher**

then t_{cyc} is **lower**.

I don't see the point of doing this when I don't know the **values** for the distance or speed yet. Our customers want actual times, not general trends!

When you start, it helps to be a part of it and think about what happens at different extremes.

Before you start trying to work out what's going on with numbers and actual measurements, it's always a good idea to **be a part of it**. Use your intuition to think about what you expect to happen to the cycling time at certain **extremes**, like a long distance or a high speed.

If you already have a good idea about how the distance and speed affect the time, you're more likely to spot any little mistakes you make in your math.

OK, so we know that houses that are further away will take longer to reach. But how much longer? How are we supposed to know the time it's going to take Alex to get to a whole load of houses that are different distances away when he hasn't been there before?

Joe: Well, I guess we could get him to cycle to each potential customer's house in his area, one house at a time, and set up a database. So when the customer types in their address, the website can look it up in the database and give out a delivery time.

Frank: Think of the overtime that'd cost us! It'd take forever for him to do all that. And he wouldn't even be delivering pizzas; he'd just be recording times to the houses of people who might never order a pizza...that doesn't sound like a good idea at all.

Joe: OK. Maybe we could work out an **equation** for Alex's cycling time. We already know that t depends on his speed and the distance, so we'd be angling for something that looks like "t = something to do with speed and distance."

Frank: Ooh, actually, working out the distance to each house is fine. The web guys can already do that with their online mapping gizmo.

Joe: We now need to work out Alex's speed, so we can try to use it in an equation. I know this sounds weird ... but we might try **being** Alex. Putting yourself in someone else's shoes is supposed to help you solve problems.

Frank: Like "Alex pushes the pedals, and the bike goes forward"? OK, let's brainstorm. So say I'm Alex, out delivering pizzas all evening. I don't want to ride too fast, or I'll get tired.

Joe: But I also don't want to ride too slow, or I won't deliver that many pizzas and won't get as many tips.

Frank: But I'm the top delivery guy, so I have the experience to start off riding at the best speed and to keep up that speed for the whole evening.

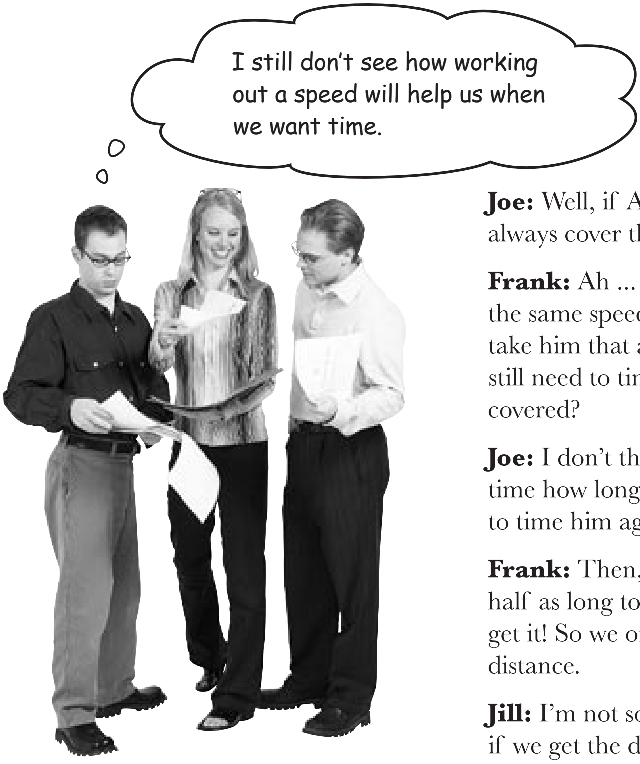
Jill: So Alex always rides at the **same speed**. Hmm, if we can work out his speed, could we use that and the **distances** from the mapping gizmo to work out the **time** it takes him reach a house?

Frank: I think you might be right. But how on earth are we supposed to work out what speed Alex rides at? And how do we use that to get a delivery time?

It usually helps to BE someone (or something), so you see what the physics looks like from inside the problem.

BRAIN POWER

How might it be possible to **measure** Alex's speed, so you can use it to work out cycling times for deliveries?



Joe: Well, if Alex always rides at the **same speed**, he should always cover the **same distance** in the **same amount of time**.

Frank: Ah ... I think I see what you mean. If Alex always goes at the same speed, and we **measure** his time for 1 km, it'll always take him that amount of time to cycle 1 km. But wouldn't we still need to time him over all possible distances to keep our bases covered?

Joe: I don't think we need to time Alex over lots of distances. If we time how long it takes him to cycle 1 km, we know, without having to time him again, that it'll take him twice as long to go 2 km.

Frank: Then, we can take his time for 1 km and say it'll take him half as long to go 0.5 km, and three times as long to go 3 km. I get it! So we only need to time him once to get his time for **any** distance.

Jill: I'm not so sure about only timing him once. A lot hangs on this- if we get the delivery times wrong, the customers get free pizza, and that's very expensive. Why don't we time him more than once over 1 km and take an **average** of all his times in case there's a bit of fluctuation between one run and the next?

Joe: A bit of **uncertainty**, you mean? That makes sense. And why don't we cover our bases by timing Alex over a variety of distances to make sure he does always ride at the same speed?

Jill: Sounds good to me.

Frank: Aren't we giving ourselves a harder calculation to do at the end though? If we just time Alex once over 1 km, it's easy to scale the time for a different distance - 2 km takes twice as long, 0.5 km takes half as long, and so forth. If we time him more than once over a variety of distances, how can we work them out?

Jill: If we work out Alex's speed in meters per second, we can use that to work out the time. So suppose he goes at 10 m/s; he'd cover 100 m in 10 s, 1 km in 100 s, and so on. We can estimate his time for any distance if it follows the same pattern.

Frank: OK, I think you've managed to convince me - especially since the town is relatively flat, so there aren't any hills to mess things up. Let's go **design** an experiment!

When you do an experiment, you should make multiple measurements so you get the best average possible.

I'm still not convinced about needing to make more than one measurement. Why would taking an average help me?

Making multiple measurements gives you an idea of how widely your results are spread out.

At the moment, you're assuming that Alex always rides at the same speed. But what if he doesn't, or if his speed varies between trips or at different distances?

If you only time Alex once, you have no idea how consistent his speed is. If you time him more than once, and his results have a wide **spread**, then you'll probably need to think again. But if his times are all similar to each other and close to the margin of **error** of your stopwatch, then you're good to go with their **average**.

The margin of error for a measuring device is ± half a scale division.

When you design an experiment, think about what might go wrong!

Making more than one measurement is just one way of **improving** your experiment. Everything in your experiment - Alex, the road, your tape measure, and your stopwatch - could cause you problems! If you don't think about the worst that could happen before you start, you could end up with useless results and no chance to repeat the experiment.

Would you really ask Alex to do it all again?

Sharpen your pencil

Think about what might go wrong with all the things in your experiment - and how you would make sure the worst didn't happen.

Item involved in experiment	Potential source(s) of error	How to deal with this
Alex and his bike	Alex's speed is inconsistent.	Time him more than once over a distance, and take an average.
The road		
Tape measure		
Stopwatch	The watch doesn't start at the right time.	

There might be more than one thing that could go wrong with each item.

Sharpen your pencil Solution

Think about what might go wrong with all the things in your experiment - and how you would make sure the worst didn't happen.

Item involved in experiment	Potential source(s) of error	How to deal with this
Alex and his bike	Alex's speed is inconsistent. Alex gets tired as he cycles longer.	Time him more than once over a distance and take an average. Time him over both long and short distances.
The road	The road might not be flat. Alex would be slower uphill and faster downhill.	Make sure you do the experiment on a flat piece of road.
Tape measure	The tape measure might have stretched with use and not be accurate.	Test it against something you know is OK before you use it.
Stopwatch	The watch doesn't start at the right time. The watch doesn't stop at the right time.	Make sure you and Alex agree on a timing protocol before you start.

Don't worry if you said some other things, or if you didn't come up with every single one of these possibilities.

There are two different types of errors- random and systematic.

You already realized that if Alex's speed isn't 100% consistent, his times will be **spread** around. This is similar to the range that a measurement may fall into due to your measuring device having scale divisions (like your myPod measurements). If Alex's times aren't too different for each ride, you can reduce the **random error** by making **several measurements** and taking an **average**.

Some sources of error are built into the system. If you time Alex going downhill, he will be faster than he would around the town, which is mostly flat. If the tape measure has stretched, the actual distance will be shorter than you think it is. Errors like these are called **systematic errors,** and they **bias** the results in one particular direction. Taking averages doesn't help reduce bias, as all the results will be higher (or lower) in the direction of the bias. You need to spot them in advance and **plan ahead** to minimize them.

Random errors involve SPREAD. Minimize them by AVERAGING.

Systematic errors involve BIAS. Minimize them by PLANNING ahead.

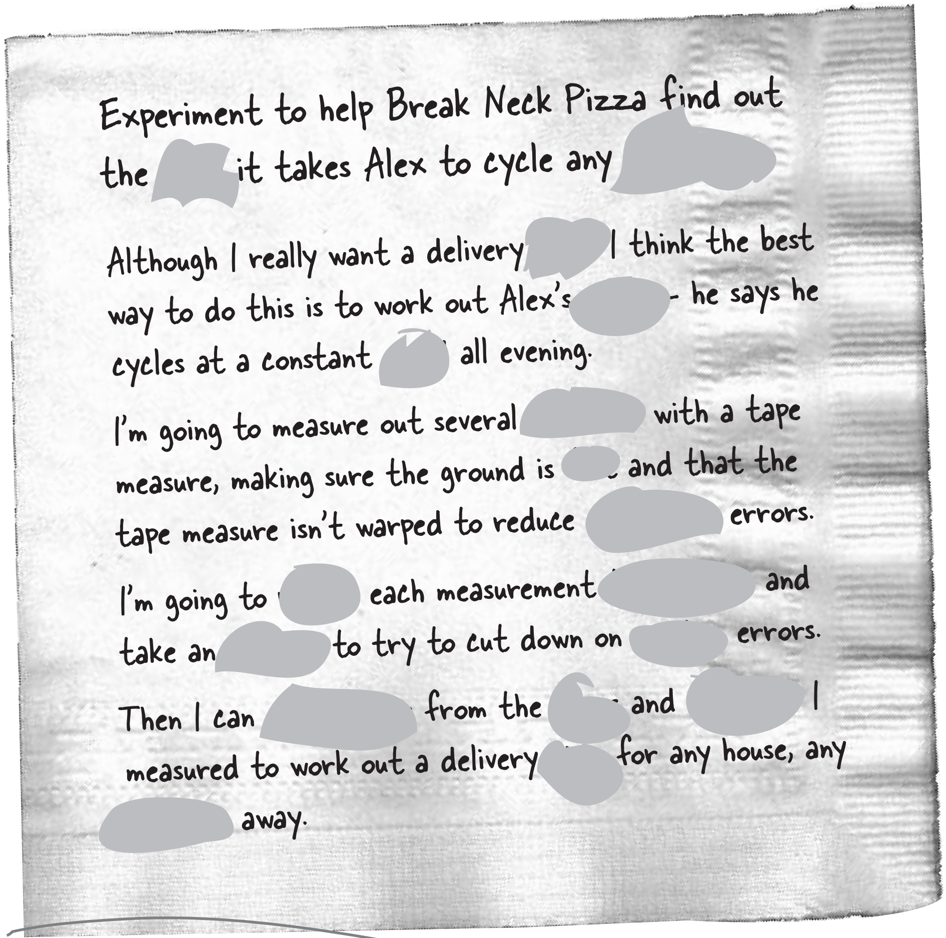
OK - time to recap where you're at...

Exercise

You're designing an experiment to help Break Neck Pizza with their website, on a spare napkin (as all good physicists do), but unfortunately, some of the words have succumbed to pizza grease.

Fill in the blanks using the words at the bottom of the box.

You might not use all of the words, and you may use some more than once!



This means use the results you already have for some values to work out what the result would be for any value.

Missing words: speed, three times, extrapolate, direction, systematic, distance, once, repeat, average, time, distances, random, flat, times

Conduct an experiment to find out Alex's speed

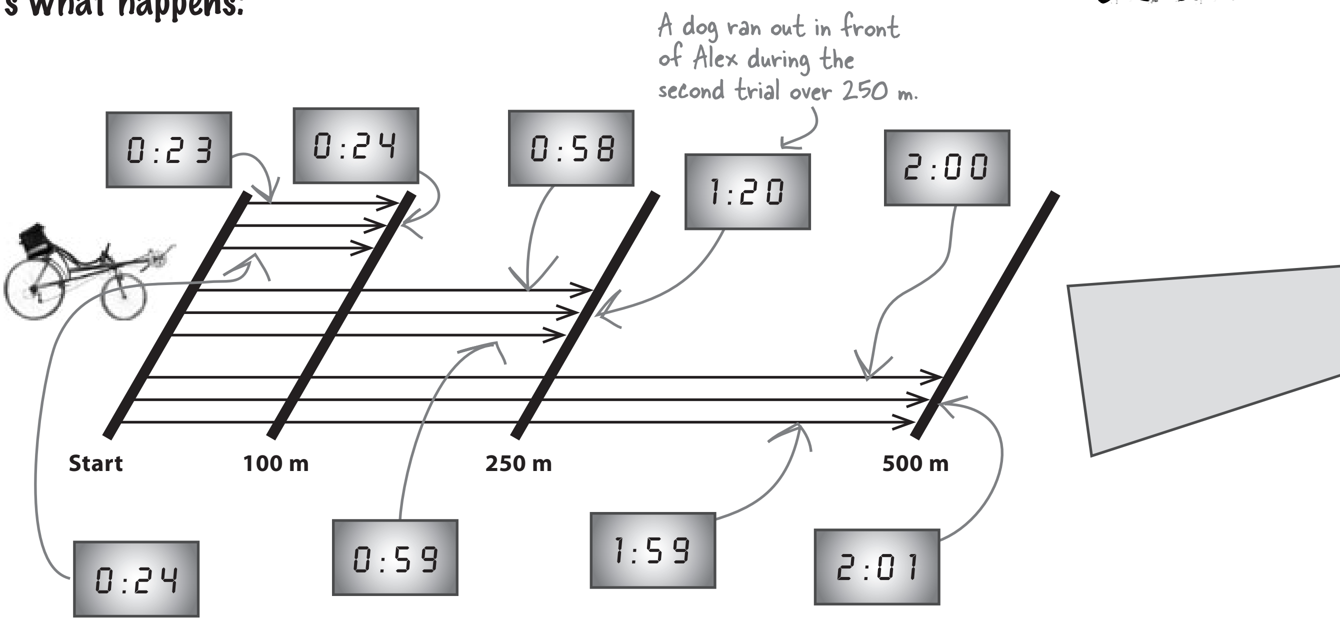
You need to give Break Neck Pizza's customers a delivery **time**. It's important not to get this wrong since you'll have to give out free pizza if you do!

You've decided to do an **experiment** to find out Alex's speed when he's out delivering pizzas. Then you can use that to work out the **time** it takes him to cover any **distance**.

You're timing Alex over three different distances, three times for each distance. That way you'll be able to spot how consistent Alex's speed is and smooth over **random errors**. These crop up because you can't recreate exactly the same conditions each time, and because your measuring devices can't possibly be accurate to the nearest fraction of an atom! You've also thought of possible sources of **systematic** error and made sure your equipment is up to scratch.

> Experiment to help Break Neck Pizza find out the time it takes Alex to cycle any distance.
>
> Although I really want a delivery time, I think the best way to do this is to work out Alex's speed – he says he cycles at a constant speed all evening.
>
> I'm going to measure out several distances with a tape measure, making sure the ground is flat and that the tape measure isn't warped to reduce systematic errors.
>
> I'm going to repeat each measurement three times and take an average to try to cut down on random errors.
>
> Then I can extrapolate from the times and distances I measured to work out a delivery time for any house, any distance away.

Here's what happens:



Write down your results... in a table

As you're doing your experiment, write your results down in a **table.** This keeps all your related pieces of information next to each other, in rows or columns. You're less likely to make a mistake writing down a measurement if you use a table. It's also much easier to see what's going on and **spot patterns** in your experiment

The **headings** for your table should go in the top row, with the thing you're changing on the left, and the thing(s) you're measuring to the right. You should put the **units** of each column in the headings.

As you make measurements for your experiment, write them down in a TABLE.

Sharpen your pencil

Put units in the headings so that the table just has numbers in it.

Fill in the table using the results shown on the opposite page.
You'll need to add some extra information to the table headers as well.

Put your headings in the top row.

Distance Alex cycles ( )	Time 1 (s)	Time 2 ( )	Time 3 ( )	Average time ( )
500	120			

Your values go in these rows.

Put the thing you're varying in the left hand column, in order of size.

Anything you're measuring should go in columns to the right.

If you're averaging your measurements, this should be to the right of the measurements themselves.

Transfer your experimental results into the table.

Tables help you keep your results ordered and make it easier for you to spot patterns.

Design experiment | Carry out experiment | Look at results

Sharpen your pencil Solution

Is it fair to include the 80 seconds in this average, considering a dog got in his way and slowed him down?

Fill in the table using the results shown on the opposite page.
You'll need to add some extra information to the table headers as well.

Remember to say what UNITS you're using.

Distance Alex cycles (m)	Time 1 (s)	Time 2 (s)	Time 3 (s)	Average time (s)
100	23	24	24	23.7 (3 sd)
250	58	80	59	~~65.7 (3 sd)~~ 58.5
500	120	121	119	120 (3 sd)

You'll usually lose points every time you don't mention units. That adds up pretty quickly!

If you don't include the off measurement, then the average time for 250 m becomes 58.5 s.

Look out for measurements that don't fit.

One of the reasons for timing Alex more then once over each distance was to assess the **spread** (or **scatter**) of his results. If he can't ride at a consistent speed, you'll have to think again about your whole approach.

But if you don't have a good reason to discard it, then it's gotta stay in there!

In this experiment, all of the times for each distance are close together, with a spread similar to your stopwatch's margin of error. However, there's one time that's way off. The 80 s measurement for Alex going 250 m sticks out, and you need to think about why. Is it because Alex isn't very consistent - or did something go wrong just that one time?

Thinking about where your outliers may have come from will help you to improve your experimental setup and reduce errors.

If the measurements are spread out because Alex is **inconsistent**, you should probably make more and more measurements at each distance, so you can get a better idea of the spread and take a better **average**.

If there's a good **reason** for the outlying data (which there is here because a dog ran out in front of him), then it's OK to **discard** the measurement since it's not representative.

If a measurement doesn't fit, think about WHY it doesn't before deciding what to do with it.

Use the table of distances and times to work out Alex's speed

Now that you have a table of distances and times, you can work out Alex's speed. Speed is measured in miles per hour, kilometers per hour, or meters per second. "Per" means "divided by," so whatever the **units**, the **dimensions** of speed are **distance divided by time**. And the speed itself is the change in the distance Alex has gone since he started divided by the change in the time that's gone by since he started.

Don't worry too much about the 'change in' bit for now. You'll see why it's really important later on in the chapter.

$$\text{speed} = \frac{\text{(change in) distance}}{\text{(change in) time}}$$

kilometers
per
hour

You already knew this equation because you already knew the units!

Once you've worked out Alex's speed, you can give the customer a delivery time for any distance.

Sharpen your pencil

Using the equation above, work out Alex's speed for each of the distances in the table.

Distance Alex cycles (m)	Average time (s)	Average speed (meters per second)
100	23.7	
250	58.5	
500	120.0	

There's space down here for your WORK (doing the speed calculations and converting the units).

You can work out the equation for speed using its UNITS - "miles per hour," "kilometers per hour," and so on. Speed is distance divided by time.

Strictly speaking, this reasoning only works for simple equations. But here you intuitively know it's right.

Sharpen your pencil Solution

Using the equation, work out Alex's speed for each of the distances in the table:

$$\text{speed} = \frac{\text{(change in) distance}}{\text{(change in) time}}$$

Distance Alex cycles (m)	Average time (s)	Average speed (meters per second)
100	23.7	4.22 (3 sd)
250	58.5	4.27 (3 sd)
500	120.0	4.17 (3 sd)

3 sd means "3 significant digits."

Remember to EXPLAIN what you're doing.

For the 100 m distance:

$$\text{speed} = \frac{\text{distance}}{\text{time}} = \frac{100\text{ m}}{23.7\text{ s}} = 4.22 \text{ meters per second}$$

For the 250 m distance:

$$\text{speed} = \frac{\text{distance}}{\text{time}} = \frac{250\text{ m}}{58.5\text{ s}} = 4.27 \text{ meters per second}$$

For the 500 m distance:

$$\text{speed} = \frac{\text{distance}}{\text{time}} = \frac{500\text{ m}}{120.0\text{ s}} = 4.17 \text{ meters per second}$$

It's OK - there will be some spread in your experimental results due to random errors.

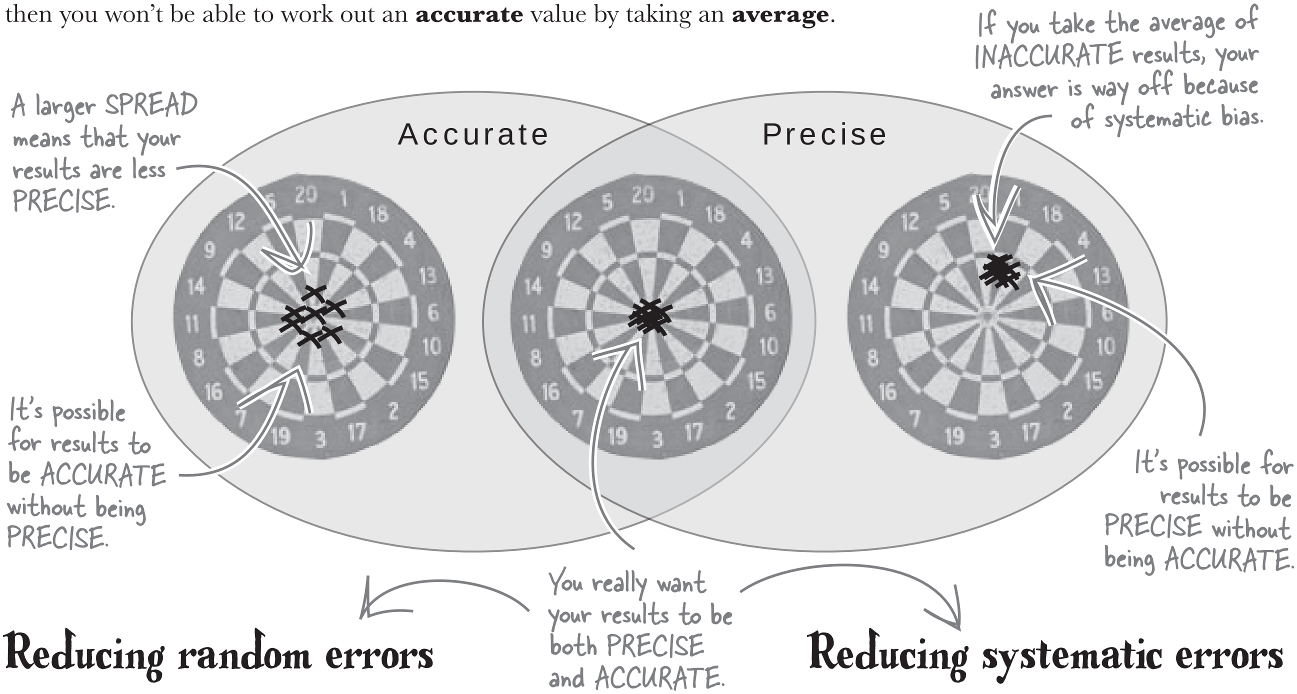
For a start, the error on each of your measurements is ± half a scale division. Then there are random fluctuations in things you don't have control over - tiny changes in wind speed, the exact state of Alex's tires, tiny changes in the surface of the road, and so on.

When you measure the same thing more than once, you shouldn't expect exactly the same answer each time.

Random errors mean that results will be spread out

Random errors mean that your experimental results will be **spread** around an **average**. If they're not spread out very much, your results are said to be **precise**.

However, if you have an underlying **systematic** error that's biasing your results in one particular direction (like if the stopwatch is consistently started at the wrong time), then you won't be able to work out an **accurate** value by taking an **average**.



Reducing random errors improves your precision.

Reducing systematic errors improves your accuracy.

there are no Dumb Questions

Q: So I can reduce random errors by choosing a more accurate measuring device?

A: You mean a more precise measuring device. Smaller scale divisions mean a smaller spread and lead to greater precision. A more accurate device would be one that reads true values.

Q: So could I get perfect precision as long as I use a fine enough scale division?

A: Not quite. At very small scale divisions, you get random fluctuations (errors), e.g., the distance read by a micrometer is affected by surface imperfections, and the reading on a balance by tiny air currents.

Q: OK, but the spread in Alex's speeds isn't because of things like that, is it?

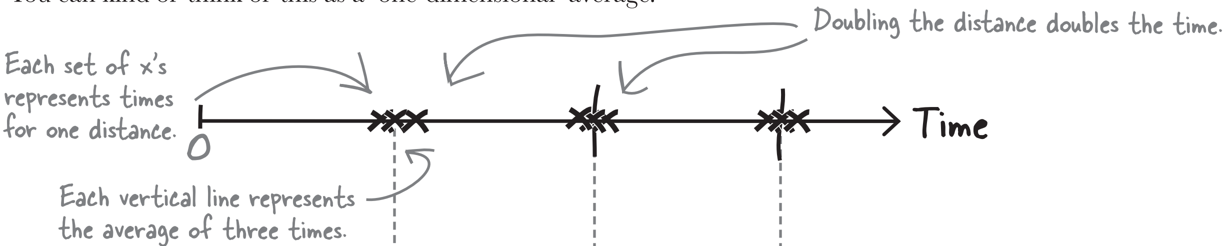
A: It's impossible to set up your experiment with exactly the same initial conditions, right down to the position of every atom, each time. Your results will always have some kind of spread whatever you do.

Q: That's annoying. I worked out an average speed over three distances, and it came out different each time. How do I decide what the best average is?

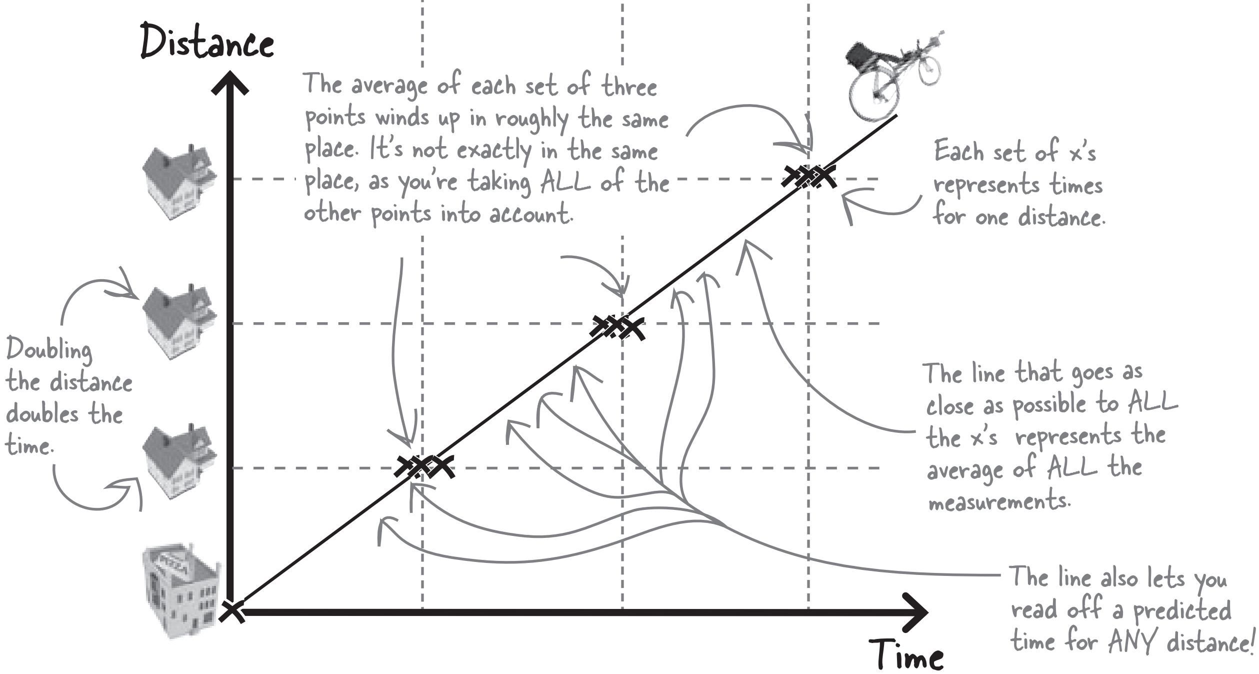
A: You really need a better way of taking averages. We're just getting on to that now ...

A graph is the best way of taking an average of ALL your results

So far you've been taking the average of one set of results at a time - but the average times (and, therefore, average speeds) for each distance are slightly different. You can kind of think of this as a 'one-dimensional' average.



The best way to work out the average speed across **all** of your results is to draw a **graph**. Since you're expecting the time to double every time you double the distance, triple every time you triple the distance, and so on, your measurements will lie along a **straight line**. You can kind of think of this as taking a 'two-dimensional' average, as you're using two axes for your plot and are able to include measurements made for different distances on the same graph.



This also has the advantage of giving you a way of reading off the time it should take to cover any distance.

As is often the case in physics, a picture speaks a thousand words!

A Graph Up Close

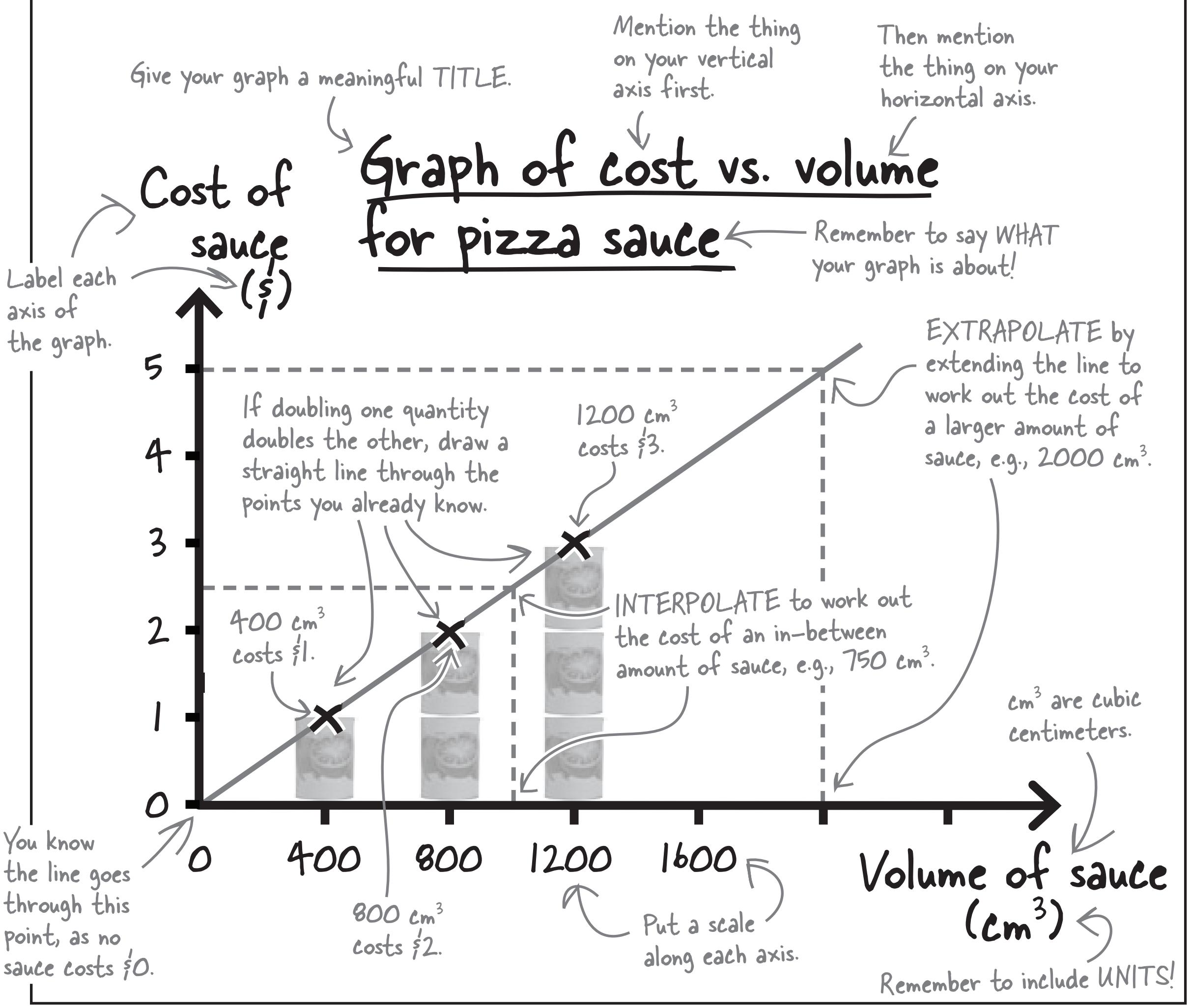
You've probably drawn graphs before, so here's a quick refresher.

The most important thing is to make it clear **what** your graph is representing! That means you need a **title**, **labels** for each of your axes, and **units** along each axis.

Plot your points by putting the center of an '**x**' exactly where your measurement is.

If you're expecting each thing you've plotted to double when you double the other thing, draw the best **straight line** you can through **all the points** on your graph. This takes the best **average** possible of all your data and allows you to read off extra values by **interpolating** (for values that lie between your measurements) or **extrapolating** (for points that lie beyond the range of your measurements).

This graph isn't of the experiment you did with Alex - that's your job on the next page! It's another graph where the doubling thing happens - the cost vs. volume of pizza sauce!



We were kinda expecting Alex's average speed to be the same for each distance. But it wasn't - we got a different answer each time. How's drawing a graph supposed to help??

...e (s)	Average speed (meters per second)
	4.22 (3 sd)
	4.27 (3 sd)
	4.17 (3 sd)

Plotting your measurements on a graph helps you take a better average than before.

The first time around, you took a separate average of Alex's times for each distance. But you got a different answer for each distance - not a great result.

By doing multiple measurements over more than one distance in the first place, you increase the precision of your result by **reducing random errors**. But if you don't somehow combine all these results together to get one answer, you're not answering the question completely.

If you plot the points on a **graph** and draw the best-fitting straight line you can, then you're using **all** of them to take an average - a much better way of doing things.

And because it's **visual**, you'll be able to **see** if any of your points are outliers.

Fitting a line to a graph is like taking an 'informed average.'

Graph-drawing Tips

- If you're plotting how something changes with time, then **time always goes along the horizontal axis.**
- Look at the **extremes** of your data - the largest and smallest numbers. Choose a scale that allows you to use most of the paper.
- Remember to mention your **units** on the axes!
- Plot points using a small **x** (not a dot), so you can still see the points after you have joined them up.
- Join the points freehand with a **smooth line** unless you know that a straight line is more appropriate (e.g., if someone went at a steady speed).
- Drawing a straight line through your points is like taking an **average**, except in a better, more visual way than you did before in your table.
- Give your graph a meaningful **title** that makes its **context** clear.

Use these tips to help you with this exercise.

Use a graph to show Alex's time for ANY distance

Alex is good at sticking to the same speed. This means that if you double the distance, then the time he takes to cover the new distance will double accordingly.

In other words, the distance and time points should lie along a **straight line** when you plot them on a graph (give or take experimental error). And once you've plotted a graph, you can read off Alex's time for **any** distance.

Just like the cost and amount of sauce did

Sharpen your pencil

Distance Alex cycles (m)	Time 1 (s)	Time 2 (s)	Time 3 (s)
100	23	24	24
250	58	80	59
500	120	121	119

a) Plot the measurements from the table as points on the graph (you can skip the outliers).

b) Then draw a **best fit straight line** through the points so that you can read off the time it'll take Alex to travel **any** distance.

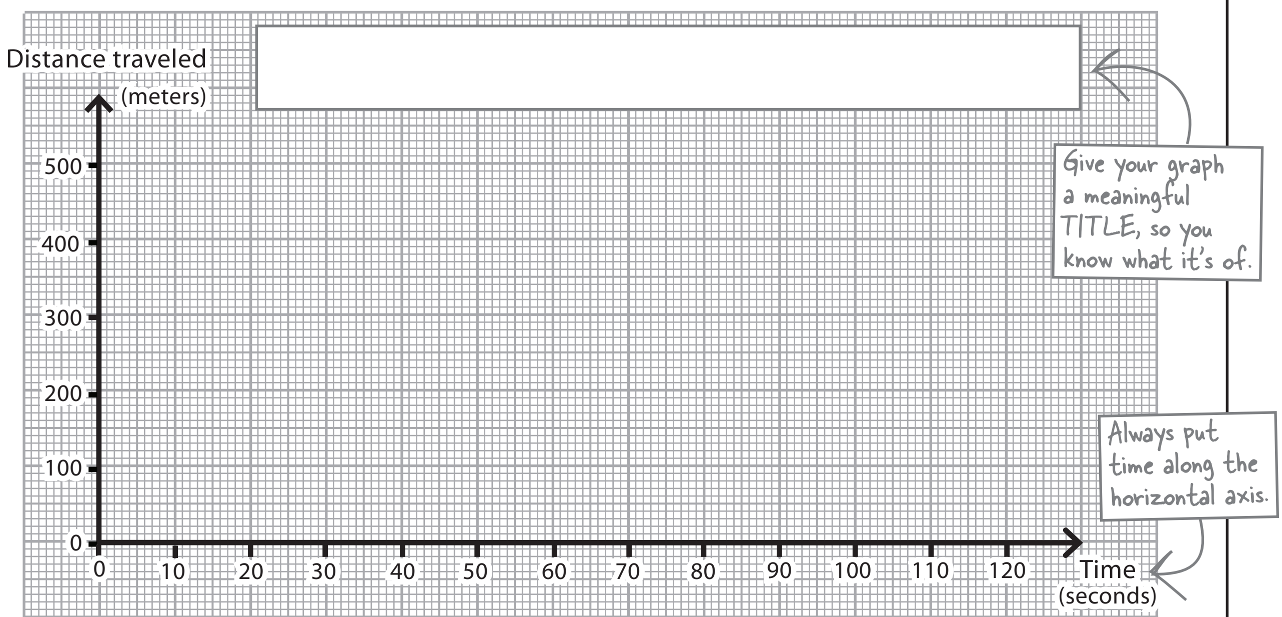
c) How long do you think it'll take Alex to travel 400 m?

....

d) And how would you work out his time for a house 1040 m away?

....

....

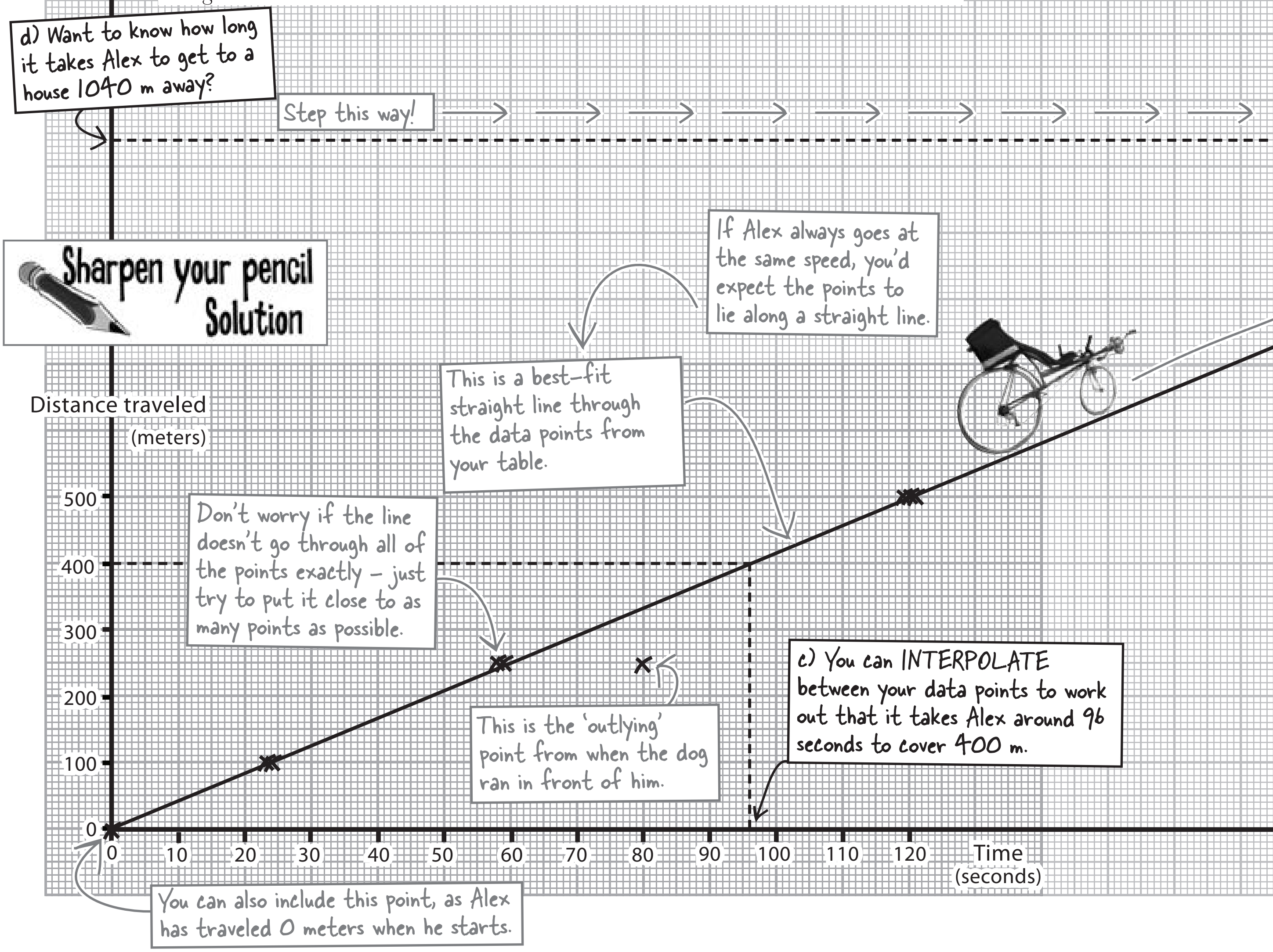


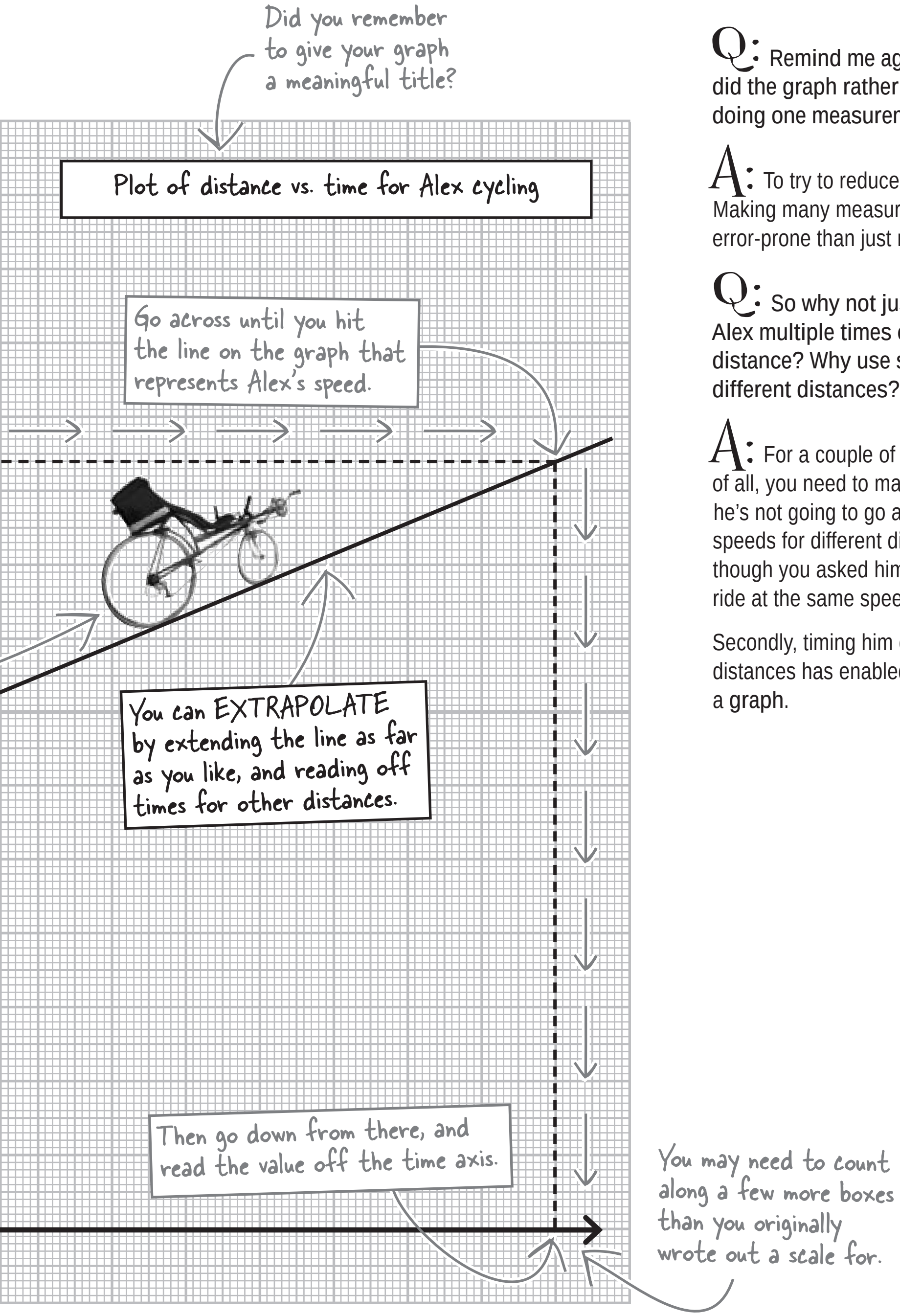
The line on the graph is your best estimate for how long Alex takes to cycle ANY distance

Now that you've drawn a graph, you can use it to read off ANY distance Alex may have to travel by extending the straight line you drew, and estimating the pattern or **extrapolating** from that.

So for example, if a house is 1040 m away (much further than any of the distances we got Alex to bike in the experiment), you can extend the graph and straight line to find out that it would take him 250 seconds to get there. The graph works!

There's only one problem - how to get the graph "into" the Break Neck website, so it can give the time to the customer.





there are no Dumb Questions

Q: Remind me again why we did the graph rather than just doing one measurement?

A: To try to reduce the errors. Making many measurements is less error-prone than just making one.

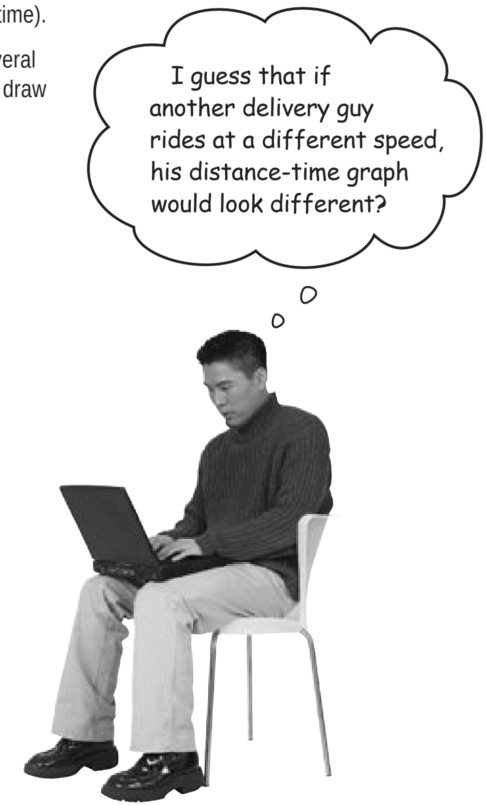
Q: So why not just measure Alex multiple times over one distance? Why use several different distances?

A: For a couple of reasons. First of all, you need to make sure that he's not going to go at different speeds for different distances (even though you asked him to try and ride at the same speed each time).

Secondly, timing him over several distances has enabled you to draw a graph.

Q: So why not be like a spreadsheet program and join the points properly? Why draw a line that doesn't even go through some of the points?

A: Doing a best-fit straight line effectively takes an **average** of your measurements. If Alex goes at the same speed the whole way, you'd expect the points to all lie along a straight line. Errors mean that they don't. You're trying to 'find' the line that the errors have obscured. A graph allows you to see where **outlying** points are. You can then make them count less by not going so near them with your line.



You can see Alex's speed from the steepness of the distance-time graph

If you SEE it, you get it!

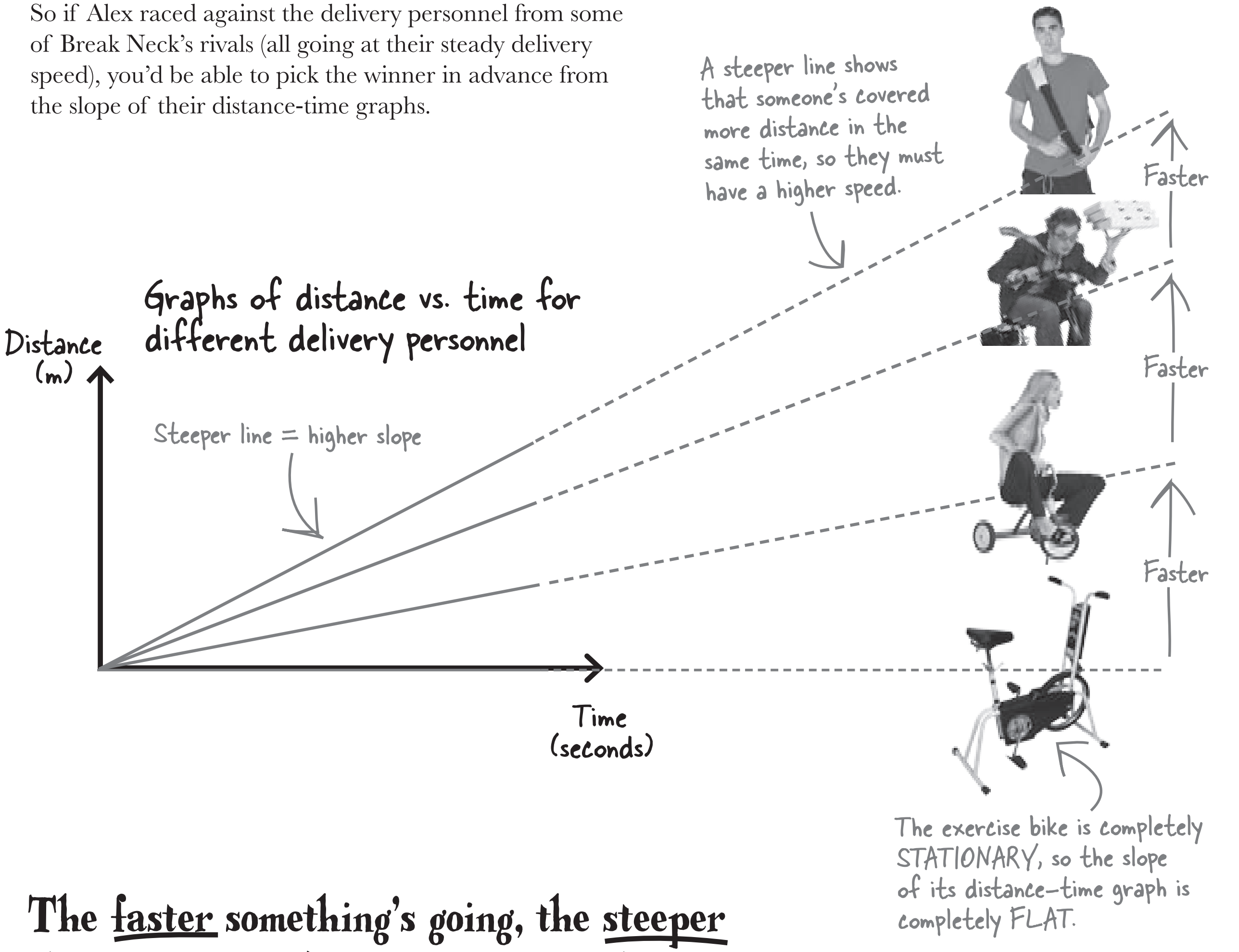
The distance-time graph not only lets you read off values, but it also lets you **see** how fast something's going at a glance.

The faster someone is, the greater the distance they go in a set time. When you plot their distance-time graph, you'll see its **slope** (or **gradient**) is steeper compared to someone who's slower. The **higher** the speed, the **steeper** the slope of the distance-time graph.

So if Alex raced against the delivery personnel from some of Break Neck's rivals (all going at their steady delivery speed), you'd be able to pick the winner in advance from the slope of their distance-time graphs.

In physics, the steepness of a graph is called its SLOPE.

The slope can also be called the gradient – they're the same thing.



The faster something's going, the steeper the slope of its distance-time graph.

So you can only compare the slopes of graphs if they're all drawn on the same set of axes?

This is being qualitative, like saying "this one's faster" or "this one's slower" but without mentioning any numbers.

You can only compare the slopes of graphs by eye if they have the same scale.

All the graphs on the opposite page have the same **scale**. This means that you can just **see** who's going the fastest, as their graph has the steepest slope.

But if you have one graph where 1 cm on paper = 200 m in real life and another where 1 cm = 10 m, then a line representing the same thing will look very different.

However, a delivery person will still cover the same distance in the same time no matter which scale you use for their graph. So the distance on their graph will still change by the same number of meters in the same time. Whatever scale it's drawn at, the numbers don't change, and you can calculate a value for the slope.

Which means to do things with numbers so that the website can give the customer a delivery time

Will calculating a value for the slope help us work out Alex's speed? We can't really expect the customer to read delivery times off a graph!

This means doing things with numbers (so that the website can give the customer a delivery time).

To be quantitative, you have to calculate the slope of your graph using numbers.

If you calculate the slope of your distance-time graph, it'll give you an **equation** which will show you how the distance and time relate to each other. And as Alex's **speed** is (change in) distance divided by (change in) time, you'll be able to work that out. This is what you really want for the Break Neck Pizza website!

When you calculate the slope of a graph, you get an equation which shows you how the two things you plotted on the graph relate to each other.

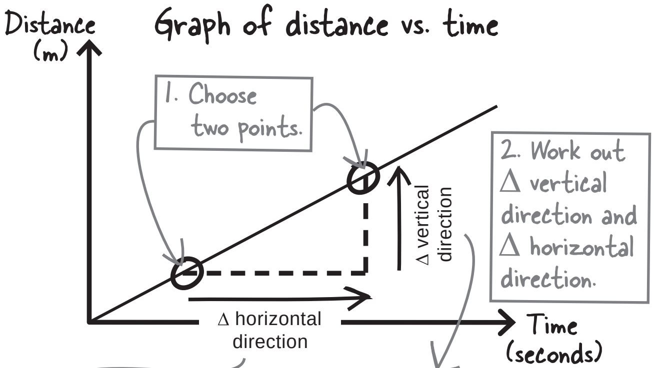
Alex's speed is the slope of the distance-time graph

To calculate the slope of a straight line graph, pick two points on it, and work out the **change in the vertical direction** divided by the **change in the horizontal direction**. The steeper the graph, the larger the slope.

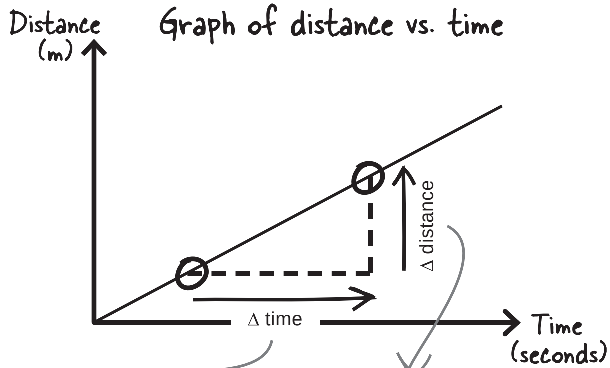
On your graph, the vertical axis is distance, and the horizontal axis is time. So the slope of your graph is **change in distance** divided by **change in time** - exactly the same as your equation for Alex's **speed**.

To save time, you can use shorthand 'Δ' which means "change in." So "change in distance" is Δ distance.

> second. "Per" means "divided by", so whatever the **units**, the **dimensions** of speed are **distance divided by time**. And the speed itself is the change in the distance Alex has gone since he started, divided by the change in the time that's gone by since he started.
>
> $$\text{speed} = \frac{\text{(change in) distance}}{\text{(change in) time}}$$
>
> kilometers per hour
>
> You already knew this equation, because you already knew the units!



$$\text{Slope} = \frac{\Delta\ \text{vertical direction}}{\Delta\ \text{horizontal direction}}$$



$$\text{Speed} = \frac{\Delta\ \text{distance}}{\Delta\ \text{time}}$$

Slope-calculating tips

- You need to choose two points on your line to calculate its slope.
- Try to find points where your line crosses the gridlines on the graph paper so that the numbers you use are whole numbers of gridline divisions.
- Choose two points that are far apart from each other - this makes your calculation more accurate.
- The slope is the change in the vertical direction (between your two points) divided by the change in the horizontal direction (between your two points).
- Remember to write down the equation for the slope before putting the numbers in.
- Always subtract the coordinates of the left-most point from the coordinates of the right-most point.
- When you've calculated the slope, look back at the graph to see if your answer SUCKs. A slope of 2 would mean that the graph goes up 2 for every 1 you go across. Does your graph look the same as your numerical answer?
- Remember to include units in your answer.

Use these tips to help you work out the slope of Alex's distance-time graph.

Now work out Alex's average speed from your graph

Sharpen your pencil

a. Choose two points on the line, and calculate the **slope** of the distance-time graph. Use the tips on the opposite page to help you.

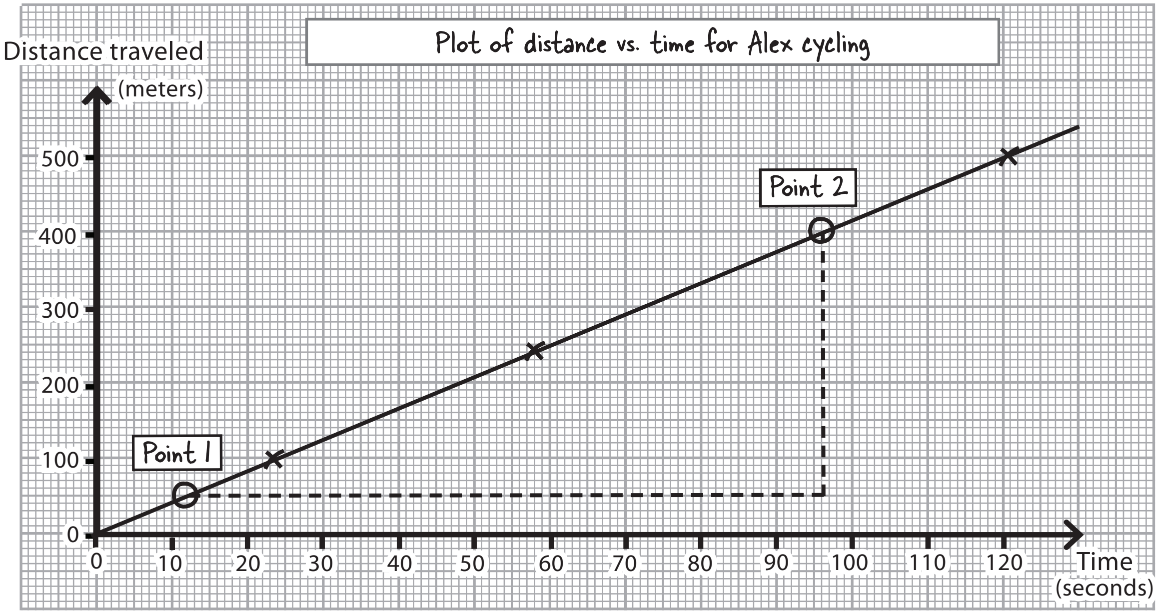
b. What do you notice about the **units** of your answer?

This may just have something to do with Alex's speed ...

Sharpen your pencil Solution

a. Choose two points on the line, and calculate the **slope** of the distance-time graph.

b. What do you notice about the **units** of your answer?



a. Point 1 is (12 s , 50 m), Point 2 is (96 s , 400 m)

Slope is $\frac{\Delta \text{ vertical}}{\Delta \text{ horizontal}} = \frac{\Delta \text{ distance}}{\Delta \text{ time}}$ ← This is the same as your equation for speed!

(Putting numbers in) $= \frac{400 \text{ m} - 50 \text{ m}}{96 \text{ s} - 12 \text{ s}} = \frac{350 \text{ m}}{84 \text{ s}} =$ 4.17 meters per second (3 sd)

Don't worry if you picked different points or got a slightly different answer.

b. The units of the slope (meters per second) are the same as the units for speed – distance divided by time.

This makes sense, as the equation for slope of the graph is the same as working out the distance covered per amount of time. The slope of the distance-time graph is the speed!

The slope of the distance-time graph is the speed.

there are no Dumb Questions

Q: Way back on page 112, I worked out Alex's speed for each of the measurements we made, and it always worked out somewhere close to 4.17 meters per second (the answer I just got from the graph). So what's the point of doing a graph when it just gives me close to the same answer again?

A: The main reason is that by plotting the points and drawing a best-fit straight line, you take a better and more informed **average** of your experimental results than you'd be able to otherwise. The speed you got from doing the graph is more **accurate** than what you'd get from a single measurement.

A graph helps you **see** what's going on. If you see it, you get it.

Q: I got a slightly different answer from you. Is that OK?

A: You probably drew a slightly different 'best-fit' line through your points. That's fine.

Q: But there must be an actual best-fit line?

A: Yes, if you use special statistical software to draw the line, you always get the same answer. But you don't have that on hand during an exam!

Q: OK. But what about Alex's speed and the slope of the graph? How can you say they're the same thing?

A: Alex's average speed is Δ distance divided by Δ time - e.g., meters per second. The slope of any graph is Δ vertical direction divided by Δ horizontal direction.

So if you draw a graph of Alex's distances and times with distance on the vertical axis and time on the horizontal axis, then its slope is the same as Alex's average speed.

You need an equation for Alex's time to give to the web guys

You've used the results of your experiment to draw a distance-time graph and calculate its slope, which is the same as Alex's speed. That's fantastic!

But the web guys need to be able to use Alex's speed and the distance to a house to work out the **time** it takes Alex to get there. And they're asking you for an **equation** that they can plug the distance and speed into to get the time.

You already have an equation that involves speed, distance, and time, which you worked out from the units of speed:

$$\text{speed} = \frac{\Delta\ \text{distance}}{\Delta\ \text{time}}$$

meters
per
second

This gives you the average speed at which something travels between two points. But the equation says "speed = " on the left hand side, allowing you to work out a speed if you know a distance and a time. What the web guys want is an equation that says "**Δ time =** " on the left hand side, so they can work out a time from the map distance and Alex's speed.

How are you going to get an equation that the web guys can use?

Rearrange the equation to say "Δtime = something"

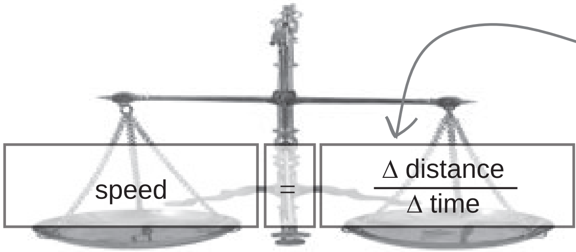
If you have an equation where the thing you want isn't on the left hand side all by itself, you'll have to **rearrange** the equation so that it is.

The top thing to remember when rearranging equations is that you must do the same thing to each term on both sides of the equation or else the two sides won't be equal (or balance) anymore. The huge advantage of this is that you don't have to remember lots and lots of equations, as you can rearrange the ones you know to figure out what you need.

For time, speed, and distance, you only need ONE equation, which you ALREADY know (from the units of speed).

You can rearrange your equation to say "Δ time = something" like this:

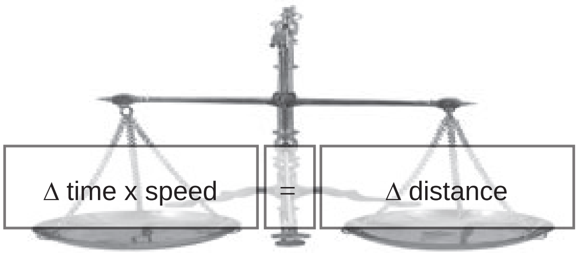
1



Δ is short for 'change in.' It's a Greek letter called 'delta'.

At the moment you're dividing by Δ time. You want an equation that says "Δ time = something." So multiply both sides by Δ time to get it off the bottom.

2

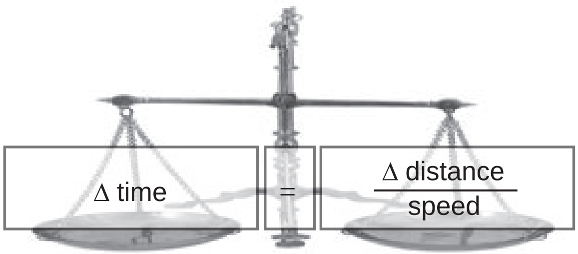


If you're adding or subtracting, that's a whole new term that you need to add/subtract from each side.

If you're multiplying or dividing, you need to do the same thing to each TERM on both sides.

You now have Δ time where you want it, but at the moment it's being multiplied by speed. So divide both sides by speed to sort that out.

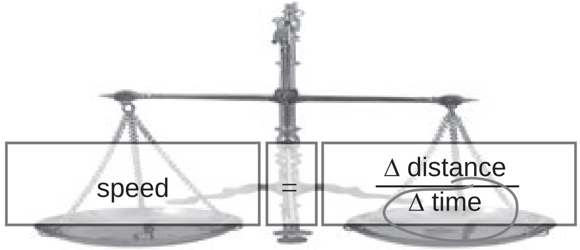
3



When you rearrange an equation, always make sure you do the same thing to each side so that the equation remains BALANCED

An equation that says "**Δ time = something**" that you can use for the Break Neck website. Sorted!

1



At the moment you're dividing by Δ time. You want an equation that says "Δ time = something." So multiply both sides by Δ time to get it off the bottom.

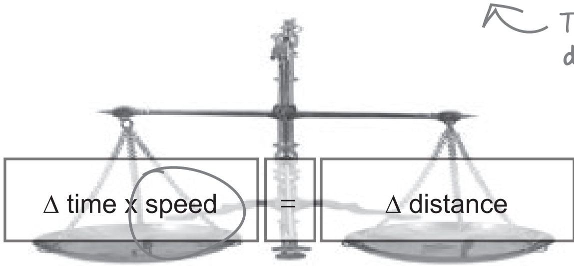
Multiply both sides by Δ time.

$$\Delta\text{ time} \times \text{speed} = \frac{\Delta\text{ distance} \times \Delta\text{ time}}{\Delta\text{ time}}$$

$$\Delta\text{ time} \times \text{speed} = \frac{\Delta\text{ distance} \times \cancel{\Delta\text{ time}}}{\cancel{\Delta\text{ time}}}$$

These Δ times divide out.

2



You now have Δ time where you want it, but at the moment it's being multiplied by speed. So divide both sides by speed to sort that out.

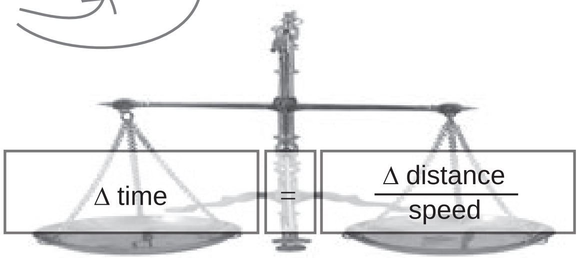
$$\frac{\Delta\text{ time x speed}}{\text{speed}} = \frac{\Delta\text{ distance}}{\text{speed}}$$

Divide both sides by speed.

$$\frac{\Delta\text{ time x } \cancel{\text{speed}}}{\cancel{\text{speed}}} = \frac{\Delta\text{ distance}}{\text{speed}}$$

These speeds divide out.

3



When you rearrange an equation, show your work ...

An equation that says "Δ time = something" that you can use for the Break Neck website.

Math is the main tool you use to communicate ideas and make predictions in physics.

A large part of physics is **understanding** the physical principles behind things. And to an extent you're right - you can do reasonably well in high school physics by understanding some principles and rote-learning a few equations.

But when you step up a level to AP Physics, there are far too many potentially useful equations and relationships for you to be able to memorize them all, never mind every single way they can be rearranged. And you need to be able to use the principles to make concrete **predictions** - for example, the time it'll take Alex to cycle a particular distance. And that has to involve **math**.

Or the UK equivalent – A level

This chapter is all about learning to use graphs and equations, two universal **languages** of physics. Like any language, math can be difficult to get used to at first. But with practice, you'll get used to communicating the physics principles with it. More than that - as you move on, the math enables you to understand and **visualize** difficult physical principles and concepts.

At AP level, being able to rearrange equations yourself is a must, so stick at it. The **rewards** are massive, as you'll only need to learn relatively few fundamental equations, which you can then rearrange to get any other equation you might need. Thinking like a physicist isn't about rote memorization. So hang in there - it will get easier!

That's a promise!

In physics, math is a vital tool for making predictions and communicating principles.

Use your equation to work out the time it takes Alex to reach each house

You now have an **equation** that lets you **predict** how long it takes Alex to get to each of Break Neck's potential customers!

First, you designed and carried out an **experiment**. After looking at the results to see how spread out they were, you plotted the results on a **graph**, drew a best-fit straight line, and calculated the **slope** of the line to find Alex's **speed** in terms of the change in distance and change in time. Phew!

And you've just **rearranged** the speed equation so that you can put in a speed and distance to calculate the time it takes Alex to travel that distance at that speed:

$$\Delta \text{ time} = \frac{\Delta \text{ distance}}{\text{speed}}$$

So now you can do a **test run** with some random addresses to catch any potential problems before the site goes live.

This involves putting in some numbers to see if what comes out is sensible.

A 'test run' is an important part of seeing if your answer's correct.

Sharpen your pencil

Use Alex's speed (4.17 meters per second) and the equation you worked out to give these Break Neck customers a delivery time.

Customer's address	Distance (m)	Calculation	Time (s)
57 Mt. Pleasant Street	1000		
710 Ashbury	3500		
29 Acacia Road	5100		

Sharpen your pencil Solution

Use Alex's speed (4.17 meters per second) and the equation you worked out to give these Break Neck customers a delivery time.

Customer's address	Distance (m)	Calculation	Time (s)
57 Mt. Pleasant Street	1000	$\Delta \text{time} = \frac{\Delta \text{distance}}{\text{speed}} = \frac{1000}{4.17}$	240 (3 sd)
710 Ashbury	3500	$\Delta \text{time} = \frac{\Delta \text{distance}}{\text{speed}} = \frac{3500}{4.17}$	840 (3 sd)
29 Acacia Road	5100	$\Delta \text{time} = \frac{\Delta \text{distance}}{\text{speed}} = \frac{5100}{4.17}$	1220 (3 sd)

So you do a test run with the website ...

... but there's a problem. The programmers set the website up to display delivery times in units of **minutes**. Your calculation used **seconds**, but the units weren't **converted** before being displayed.

Doing the question right, but giving the wrong UNITS in your answer is a common, but avoidable, error.

Design experiment | Carry out experiment | Look at results | Plot results on graph | Calculate slope | Rearrange equation | ~~Use equation~~ | CHECK EQUATION!

So just convert the units, and you're all set...right?

Not so fast! Any time you think you're ready to roll with an important website (or even an answer on your exam), take a look from 20,000 feet away and ask yourself: Does this **SUCK**?

You're never finished until you've asked: "Does it SUCK?"

Sharpen your pencil

Change the units for this customer's delivery time to minutes, then step back to see if it SUCKs.

Customer's address	Time (s)	Calculation	Time (minutes)
57 Mt. Pleasant Street	240		

S **SIZE - Are the answers the size you're expecting?**

....

....

U **UNITS - Do they have units, and are they what you were asked for?**

....

....

C **CALCULATIONS - Did you do the math right?**

....

....

K **"K'ONTEXT - What are you trying to do, and is it the same as what you actually did?**

....

....

Sharpen your pencil Solution

Change the units for this customer's delivery time to minutes, then step back to see if it SUCKs.

Customer's address	Time (s)	Calculation	Time (minutes)
57 Mt. Pleasant Street	240	$240\text{ s} = 240\text{ s} \times \frac{1\text{ min}}{60\text{ s}}$ ← conversion factor	4

S

SIZE - Are the answers the size you're expecting?

Well, 4 minutes is plausible for pizza delivery in a way that 240 minutes isn't!

U

UNITS - Do they have units, and are they what you were asked for?

They asked for minutes, and they've got minutes. (Well, they do now!)

C

CALCULATIONS - Did you do the math right?

The first equation I wrote down was $t_{tot} = t_{cyc} + t_{cook}$, where t_{cyc} was the cycling time, and t_{cook} the cooking time. OH NO, I FORGOT ABOUT THE COOKING TIME!!

K

"K'ONTEXT - What are you trying to do, and is it the same as what you actually did?

Apart from using the wrong units originally, and forgetting the cooking time, I think the rest is OK.

Doing part of the problem right, then writing the 'interim' answer to that part as your final answer by mistake is another common (and very avoidable) error!

Design experiment | Carry out experiment | Look at results | Plot results on graph | Calculate slope | Rearrange equation | ~~Use equation~~ | CHECK EQUATION! | Use equation

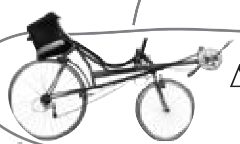
Include the cooking time in your equation

The original equation included both a delivery time **and** a cooking time. You did a fantastic job of working out the hard part - the delivery time - but forgot to add on the cooking time at the end.

Total delivery time — Cycling time — Cooking time

$$t_{total} = t_{cyc} + t_{cook}$$

This is the total delivery time the website should give the customer.



$$\Delta\,\text{time} = \frac{\Delta\,\text{distance}}{\text{speed}}$$

From the online maps

4.17 meters per second

An Italian pizza takes 10 minutes to cook, and a deep pan pizza takes 13 minutes. I can cook them all at the same time.

Sharpen your pencil

Work out the cooking time and total delivery time for each of the orders.

Work out the cooking time from what the chef tells you.

Customer's address	Distance (m)	Order	Road time (s)	Road time (min)	Cooking time (min)	Delivery time (min)
57 Mt. Pleasant Street	1000	1 Italian	240	4		
710 Ashbury	3503	2 deep pan	840			
29 Acacia Road	5100	1 Italian 1 deep pan	1220			

Sharpen your pencil Solution

Work out the cooking time and total delivery time for each of the orders.

Customer's address	Distance (m)	Order	Road time (s)	Road time (min)	Cooking time (min)	Delivery time (min)
57 Mt. Pleasant St.	1000	1 Italian	240	4	10	14
710 Ashbury	3500	2 deep pan	840	14	13	27
29 Acacia Road	5100	1 Italian 1 deep pan	1220	20 m 20 s	13	33 m 20 s

This answer works out at 20.3333333... minutes. The fraction at the end is a third of a minute, which is 20 seconds (not 33 seconds). Don't forget that there are 60 seconds in a minute (and not 100), so you need to convert the fraction of a minute into seconds at the end.

The Break Neck website goes live, and the customers love it!

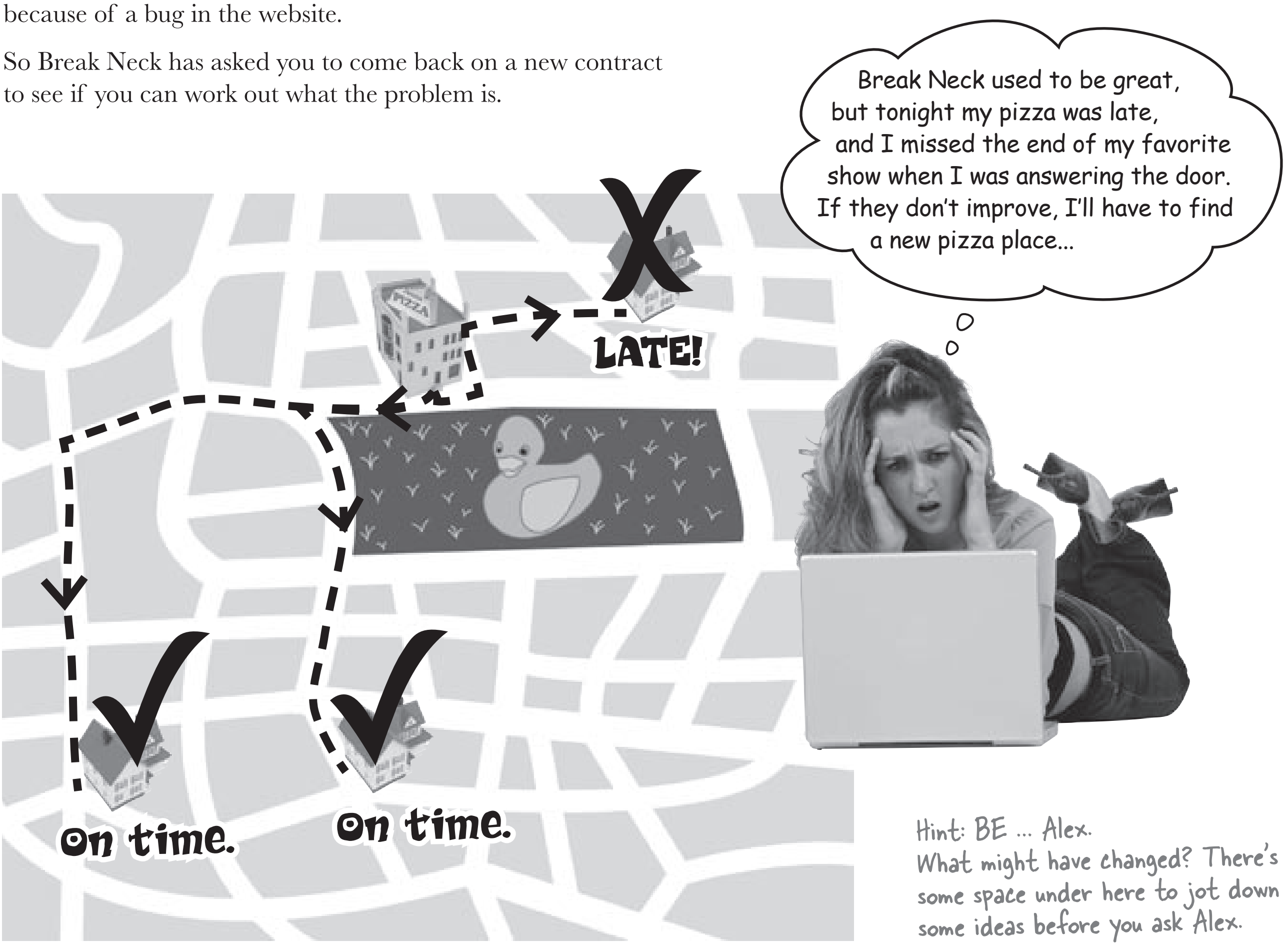
You've done it! Break Neck have a shiny new website thanks to your experiment, graph, equation, and units conversion. Every time a customer makes an order, they're given a delivery time that they can plan their evening around.

The customers are delighted, and word's starting to get around. Alex is pleased too. He's getting more tips and has nearly saved up enough for the new myPod he has his eye on.

A few weeks later, you hear from Break Neck again

Most of the pizzas are still arriving on time ... but some of the pizzas are **late,** and the customers are starting to get impatient with Break Neck. The worst thing that could happen would be customers switching to one of Break Neck's competitors because of a bug in the website.

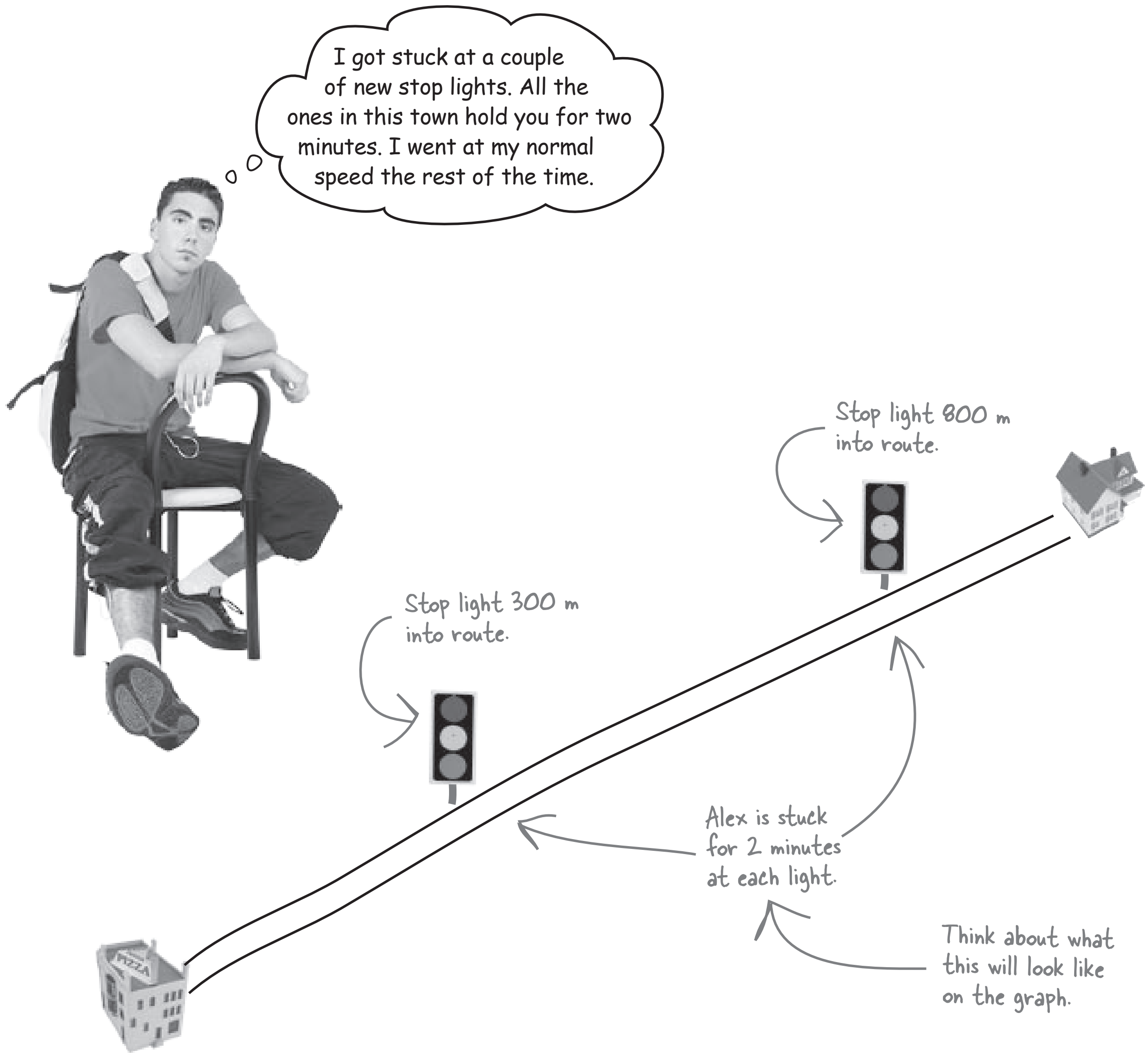
So Break Neck has asked you to come back on a new contract to see if you can work out what the problem is.



Hint: BE ... Alex.
What might have changed? There's some space under here to jot down some ideas before you ask Alex.

BRAIN POWER

But why are only **some** pizzas now arriving late when the rest are still arriving on time?


I got stuck at a couple of new stop lights. All the ones in this town hold you for two minutes. I went at my normal speed the rest of the time.
Stop light 800 m into route.
Stop light 300 m into route.
Alex is stuck for 2 minutes at each light.
Think about what this will look like on the graph.

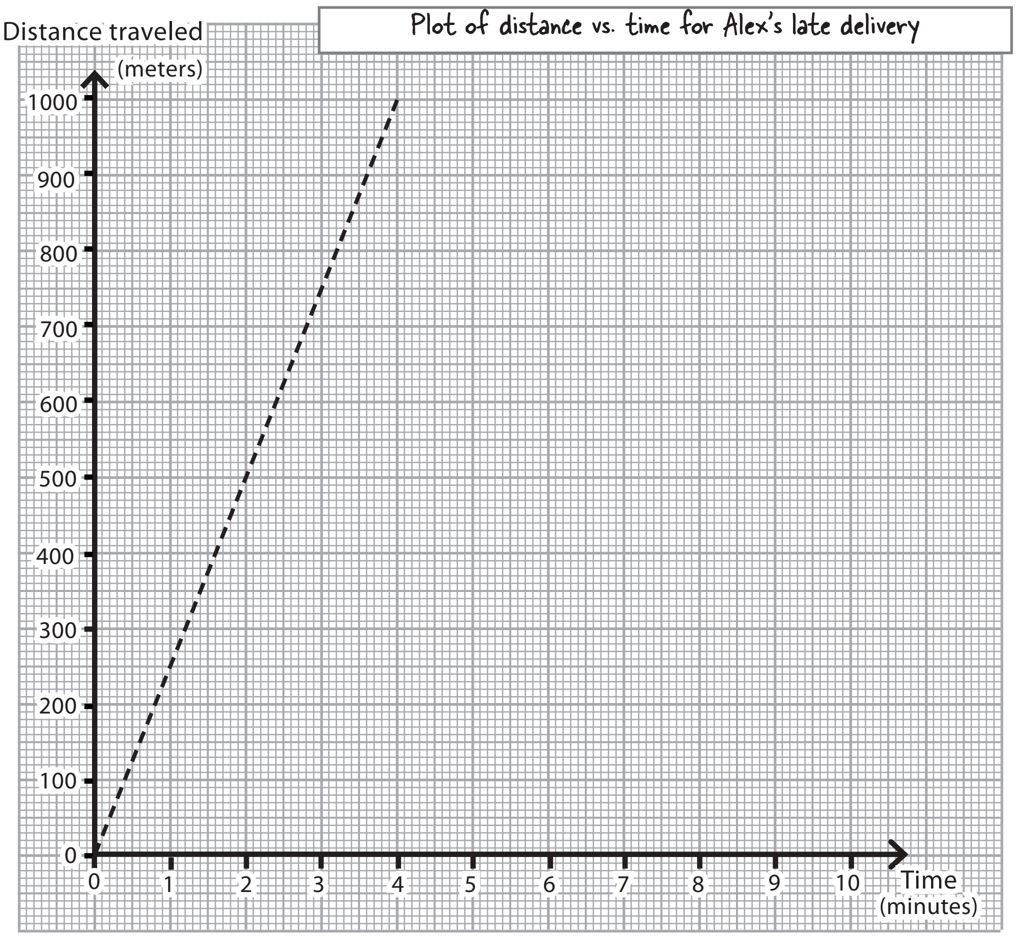
A graph lets you see the difference the stop lights made

If you can **see the difference** the stop lights made to Alex's journey, you're more likely to be able to work out what to do about it.

Sharpen your pencil

The line already on the graph represents what we expected Alex to do - travel at a constant speed without stopping, so he reaches the house in four minutes.

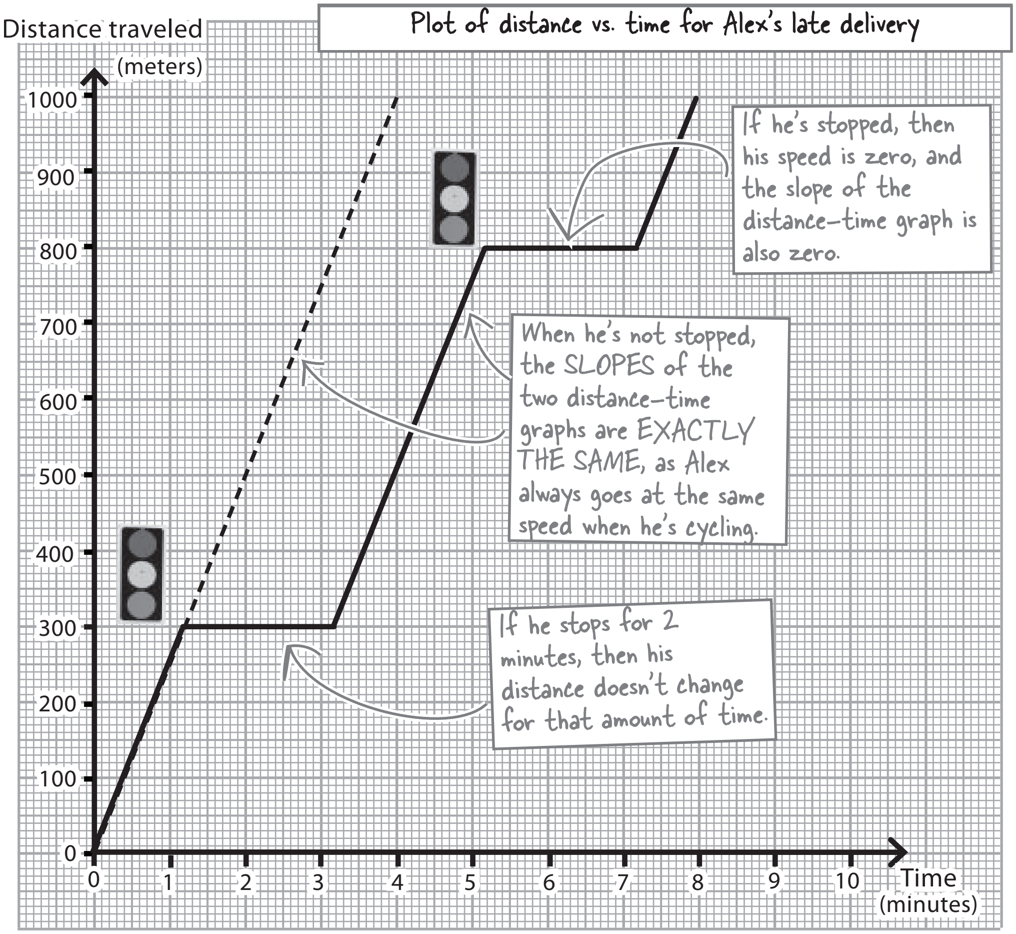
Draw Alex's actual journey (which he describes over there on page 136) on the graph to **show** and compare the difference that the stop lights made.



Sharpen your pencil Solution

The line already on the graph represents what we expected Alex to do - travel at a constant speed without stopping, so he reaches the house in four minutes.

Draw Alex's actual journey (which he describes over there on page 136) on the graph to **show** and compare the difference that the stop lights made.



It's easy to tell when something's sitting still, as the slope of its distance-time graph is zero (or a flat line).

The stop lights change Alex's average speed

The **average** speed for a trip is the constant speed at which you could have traveled to cover the same **total** distance in the same total time. So Alex's average speed is the **slope** of a line between his start and end points on his distance-time graph since the slope of a distance-time graph is the speed.

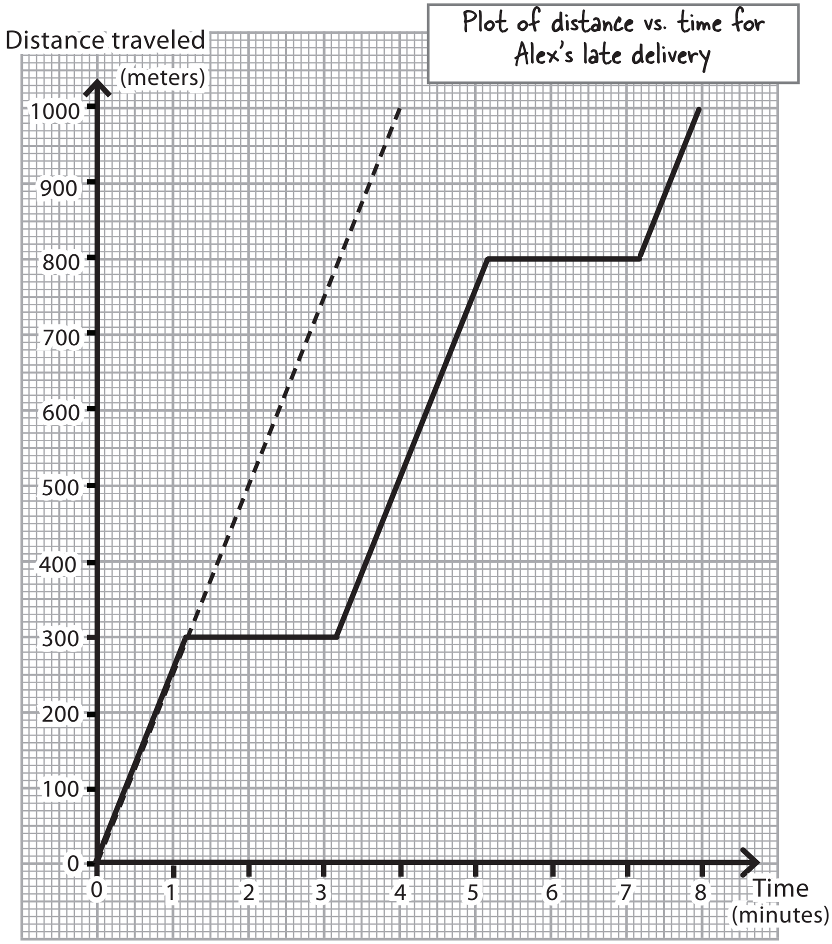
Sharpen your pencil

a. Draw a line on the graph to represent Alex's average speed.

b. Calculate Alex's **average** speed for the delivery in meters per second (show your work down the side).

c. Does Alex ever actually travel at his average speed?

Space for part b work.



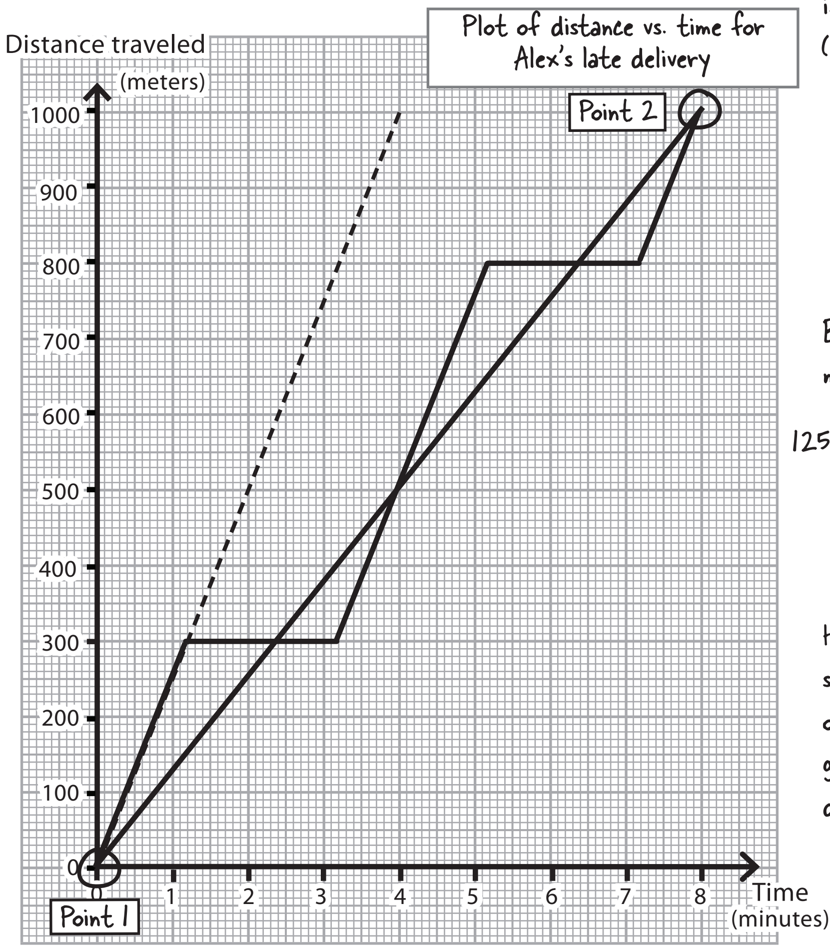
Space for part c answer.

Sharpen your pencil Solution

a. Draw a line on the graph to represent Alex's average speed.

b. Calculate Alex's **average** speed for the delivery in meters per second (show your work down the side).

c. Does Alex ever actually travel at his average speed?



Space for part b work.

Choose point 1 to be the start of the run, and point 2 the end.

i.e., (0 min , 0 m)

(8 min , 1000 m)

$$\text{speed} = \frac{\Delta \text{ distance}}{\Delta \text{ time}} = \frac{1000 \text{ m}}{8 \text{ min}} = 125 \text{ meters per min}$$

But was asked for speed in meters per second:

$$125 \frac{\text{meters}}{\text{min}} = 125 \frac{\text{meters}}{\text{min}} \times \frac{1 \text{ min}}{60 \text{ s}} = 2.08 \text{ meters per second (3 sd)}$$

Space for part c answer.

He never travels at the average speed for any significant length of time (though he must briefly go at it as he speeds up/slows down for each light).

With an AVERAGE speed, it's the changes in total distance and total time that are important.

Add on two minutes per stop light to give the customer a maximum delivery time ...

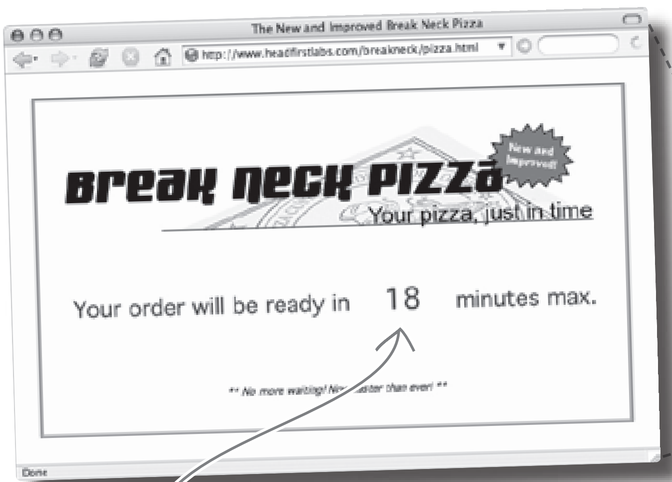
Trying to do something with the average speed will be too difficult at this stage; this is simpler.

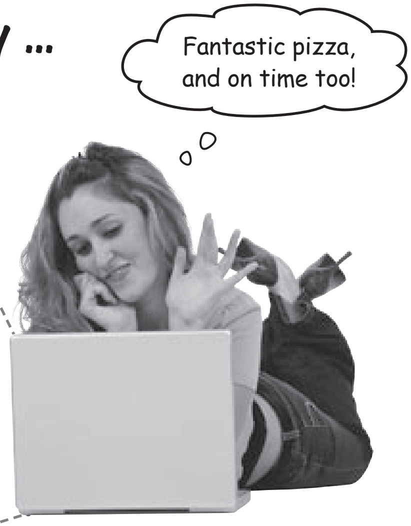
In physics, simpler is usually clearer – and, therefore, better!

So you - and Alex - have reached the end of the road. You realized that the stop lights slow him down. But you were able to get the online map to consider the number of stop lights on his route when giving the customer a maximum delivery time.

If the lights are red, that's OK because you planned for that to happen. If the lights are green, and Alex isn't slowed down, then the pizza arrives early, and the customer is still happy.

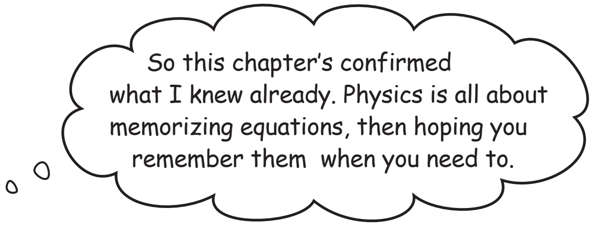
... the customers are extremely happy ...





The 14 minutes you calculated before plus up to 2 minutes for each of the 2 stop lights.

... and you're invited to the Pizza Party



If you understand physics, you don't have to memorize equations!

Think like a physicist!

Physics isn't about learning equations. Physics is fundamentally about the world around you.

When you're trying to work stuff out with physics, it usually involves making measurements, then drawing graphs and/or writing down equations that show you how varying one thing (like the distance to a house) changes something else (like the delivery time for a pizza).

Yes, there'll be some equations that you'll have to learn to get by - but only a few. Most of the ones in this book are equations that you can work out from the **general physics principles** you're learning. Remember - put yourself into the heart of the problem, then use what you already know! Use your intuition and graphs to see whether you expect something to get bigger or smaller when you change another thing.

In physics, memorizing is the opposite of understanding!

If you see it, you get it!

I guess there are a few equations that are kind of common sense, so you don't need to memorize them if you just think about **where** they come from?

You <u>already</u> knew the two equations you've used in this chapter!

The first equation you used was:

$$\text{speed} = \frac{\Delta\ \text{distance}}{\Delta\ \text{time}}$$

meters per second

But that was **something you already knew**! Every time you say something has a speed of "70 miles per hour," you're quoting **units** that you can use to write down that equation straight out of your head!

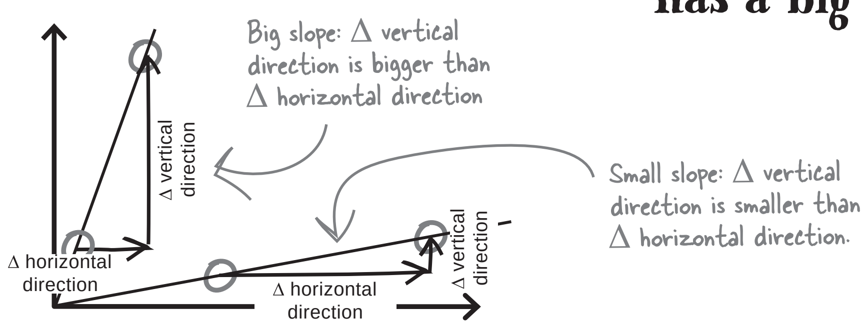
Then you can rearrange it to get what you want.

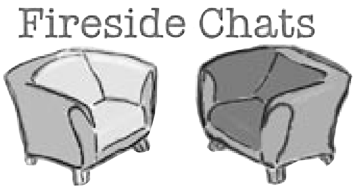
The second equation you used was:

$$\text{slope} = \frac{\Delta\ \text{vertical direction}}{\Delta\ \text{horizontal direction}}$$

But you already knew that one too! **Steep hills have big slopes**. So you can use your intuition to work out that Δ vertical must be on the top of the fraction so that you get a big value for a big slope.

A steep hill has a big slope.





Tonight's talk: **Graph and Equation debate who's best at getting the point across.**

Graph: You've heard it said - "A picture speaks 1000 words," and "If you see it, you get it." Which kinda makes me wonder - what on earth are you doing here in my chapter, Equation?

Equation: Well, that's a nice way to draw me into the conversation considering the chapter title actually names me first - "equations and graphs," it says there.

Graph: Obviously one that got past the editor! I mean, in a chapter that's mostly about visualizing things, what role do you have to play?

Equation: I'm just a different way of visualizing things. With me, it's easy to see, at a glance, when two things are equal to each other - you can't do that.

Graph: But with me, you can see how two things relate to each other-how one thing responds when the other thing changes.

Equation: But I can do that too. All you need to do is think what'll happen to the stuff on the other side of the equation when you change the value of one of the variables.

Graph: Huh? Variables, which you have to look up before you make any sense whatsoever! All those symbols and letters, whereas I'm easily accessible from the start!

Equation: I disagree- you only make sense once someone's gotten used to working with graphs! Anything strange can become familiar if you use it a lot. Once you're used to my kind of shorthand, you'll be able to use it to convey a lot of information in a very small space and short amount of time.

Graph

And random letters are easy to get used to, are they?

Equation

They're not random, actually. Physicists always choose the same letter to mean the same one thing in equations. So once you've learned the lingo, it's not as hard as you make it out to be.

Graph: But can you do slopes?! Can you display experimental results? Do people plant little kisses on you, like this ✕ ? In short, are you loved?

Equation: Has it ever occurred to you that a graph and an equation are just different ways of saying these same things?

Graph: Uhhhhh - how do you mean?

Equation: Take that distance-time graph from earlier on in the chapter. It's the same as the equation Δdistance = speed × Δtime except drawn out. (But it's not as long-winded!)

Graph: I think not! I have a slope, and you have ... well, you have nothing of great interest.

Equation: Oh, but this equation tells you all about the slope of the graph. You have to work out your slope using an equation in the first place, don't you? And if you've plotted a distance-time graph, then it's the same equation as I already said.

Graph: No, it's not! The slope is Δ vertical direction divided by Δ horizontal direction, and there's no division sign in your equation!

Equation: Ah, but equations can be rearranged to give you anything you want (as long as it was in or related to the equation in the first place).

Graph: Well, I guess you might just be a teeny bit more versatile. But I'm still the best for visualizing things!

Equation: I'm not denying that - just pointing out that I'm not entirely useless, you know!

Question Clinic: The "Did you do what they asked you" Question

Some questions will give you one **unit** (like minutes) - but ask for your answer in another unit (like seconds). And some questions have some harder intermediate steps before a more simple step to give the **final** answer. Always check that you've done what they asked you! Otherwise you'll lose credit for 'little' mistakes on questions that you mostly did right!

This question contains two different units of distance.

It also contains two different units of time.

2. A runner jogs one kilometer to a 400 m track, **where he starts doing laps. For the first half hour, he passes the finish line every 90 seconds. After** this, he does four more laps in 75 seconds each.

a. How far does he run in the first 30 minutes of his training session?

b. If he started his training session at 1:00 pm, what time does he finish at?

c. If he takes the bus home, how far has he run in total that day?

This distance is written in words rather than figures, which makes it more difficult to spot!

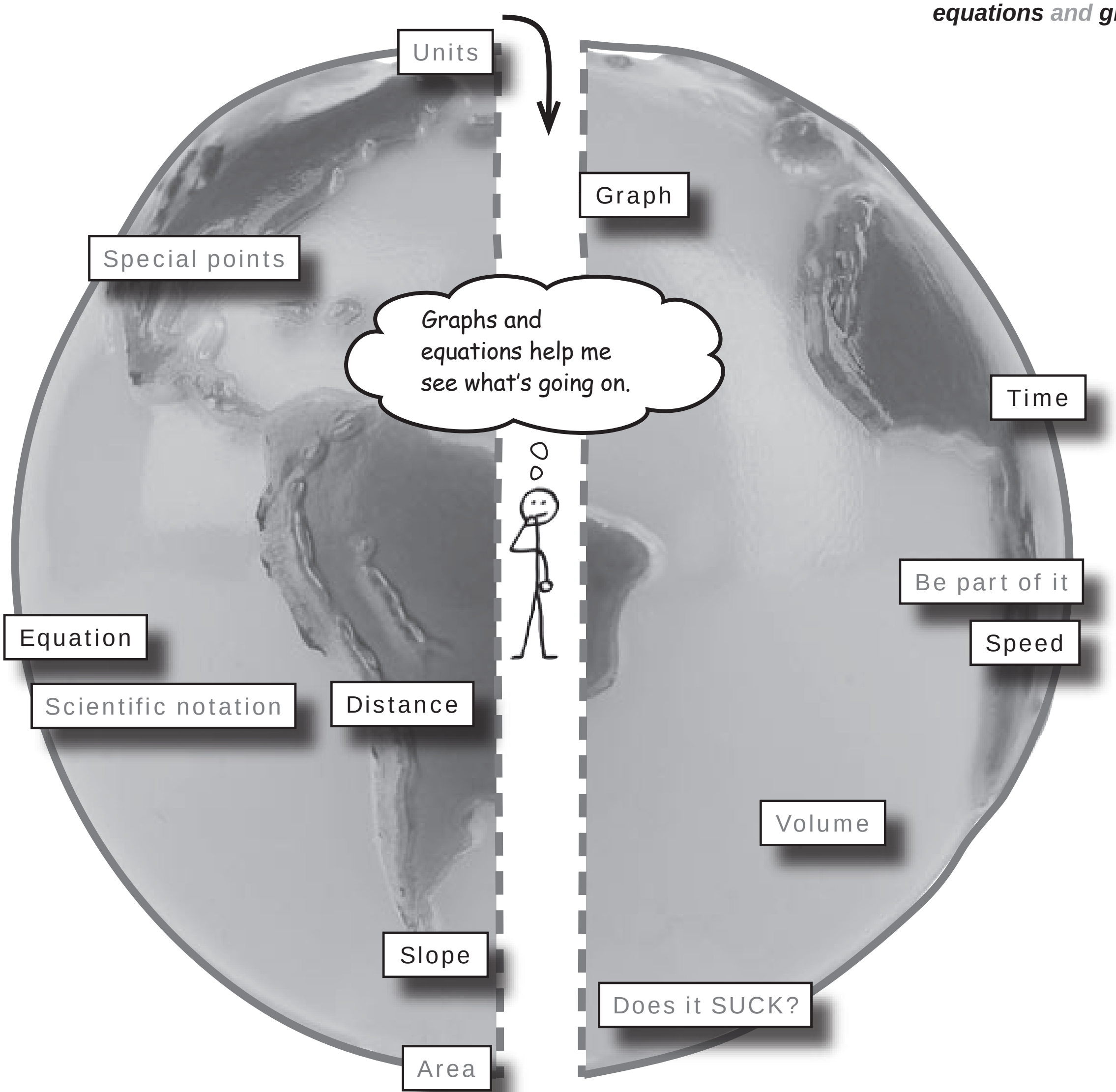
And here's yet another unit of time!

And another way of writing down a time!

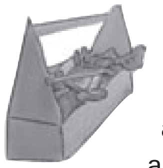
One more unit of time to finish off with!

By the time you've done the whole question, have you forgotten about the 1 km he jogged to get to the track in the first place?

The first type of "Did you do what they asked you?" question is there to make sure you **think about the physics** (and **units**) involved rather than just plugging numbers into a formula. The other type makes sure you **read the whole question** and can follow things through to the very end.



☑	Distance	A length; the number of meters (or miles, or so forth) you cover when you take a particular route between two points.
☑	Time	How long something takes; how many seconds (or minutes, and so on) that elapse between two moments you're interested in.
☑	Speed	How fast something's going - the rate of change of distance with time.
☑	Graph	A visual representation of how two variables depend on each other.
☑	Equation	A mathematical representation of how variables depend on each other.
☑	Slope	The steepness of a graph; the change in the vertical direction divided by the change in the horizontal direction.



Your Physics Toolbox

You've got Chapter 4 under your belt, and you've added some terminology and answer-checking skills to your toolbox.

Do an experiment

An experiment enables you to find out how two variables depend on each other – like distance and time in this chapter.

Think about errors

Think about sources of experimental error at the design stage.

Reduce systematic errors by planning ahead.

Reduce random errors by making multiple measurements and averaging (either mathematically or by drawing a graph).

Draw a graph

A graph visually shows you how two variables relate to each other.

You'll usually want to know how something varies as time goes on.

ALWAYS put time on the horizontal axis of your graph.

The slope of a graph

You can use the equation

$$\text{slope} = \frac{\Delta \text{ vertical direction}}{\Delta \text{ horizontal direction}}$$

to compare the two variables you've plotted on your graph to each other.

Work out an equation

An equation shows you how variables relate to each other mathematically.

Use the same letter for the same 'type' of thing, e.g., 't' for a time.

Use subscripts to represent different things of the same 'type', like you did with t_{cook}, t_{total}, and t_{cyc}.

Rates and slopes

When you've plotted time along the horizontal axis, the slope of the graph gives you the rate at which the variable on the vertical axis changes with time.

So the slope of a distance – time graph gives you the speed, as speed is rate of change of distance with time.

Rearrange your equation

If the equation you come up with doesn't have the variable you want to work out on its own on the left hand side, you have to rearrange your equation.

Make sure you always do the same thing to both sides at each stage so that your equation stays balanced.

It's safer to show more work than it is to show no work at all.

5 dealing with directions

Vectors

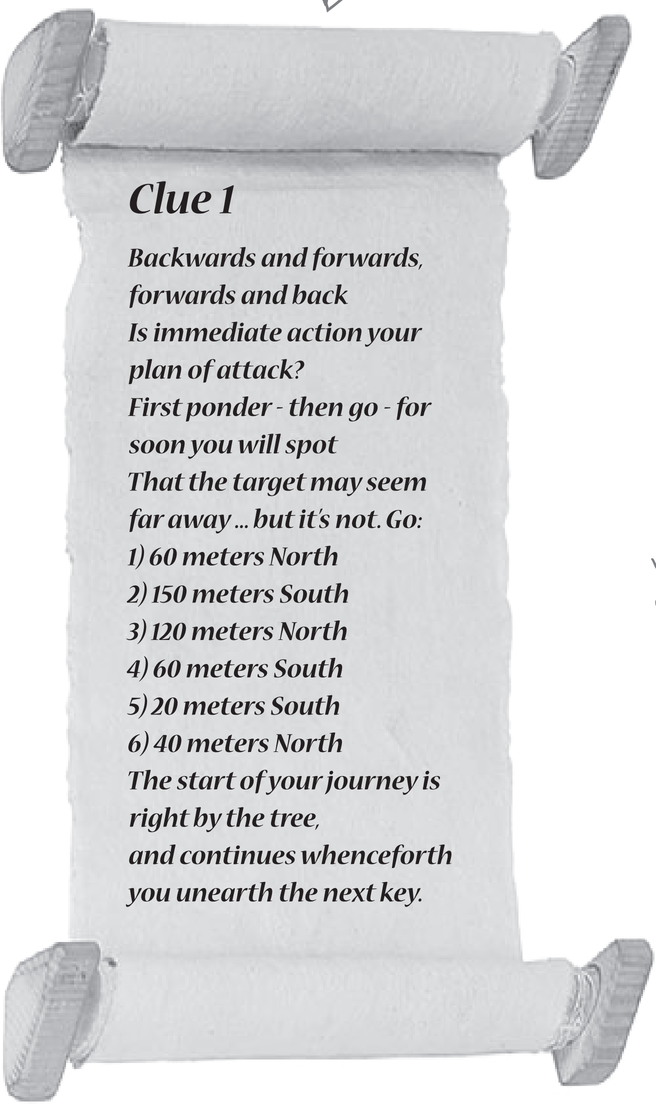
Time, speed, and distance are all well and good, but you really need DIRECTION too if you want to get on in life.

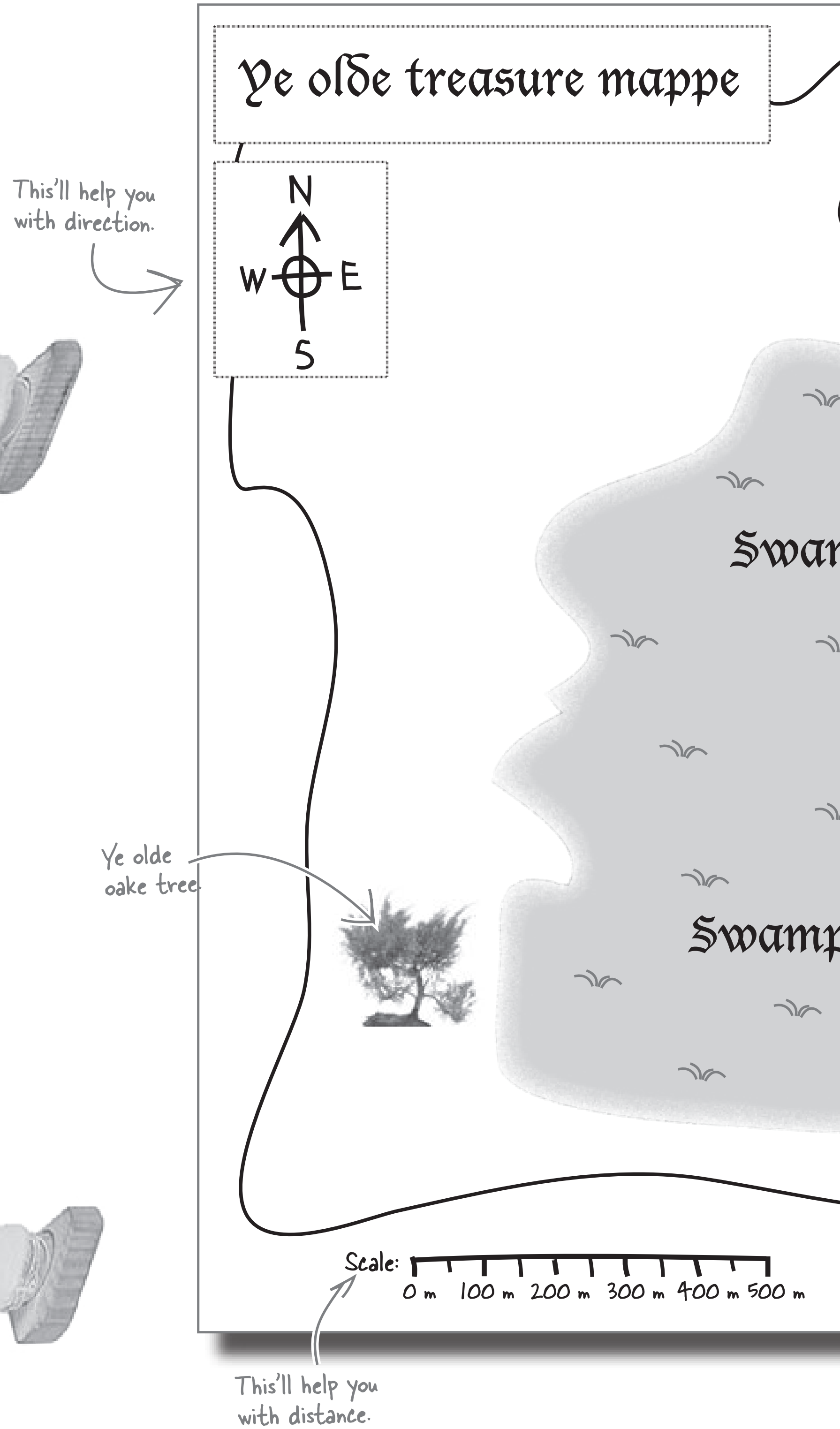
You now have multiple physics superpowers: You've mastered graphs and equations, and you can estimate how big your answer will be. But **size** isn't everything. In this chapter, you'll be learning about **vectors**, which give **direction** to your answers and help you to find **easier shortcuts** through complicated-looking problems.

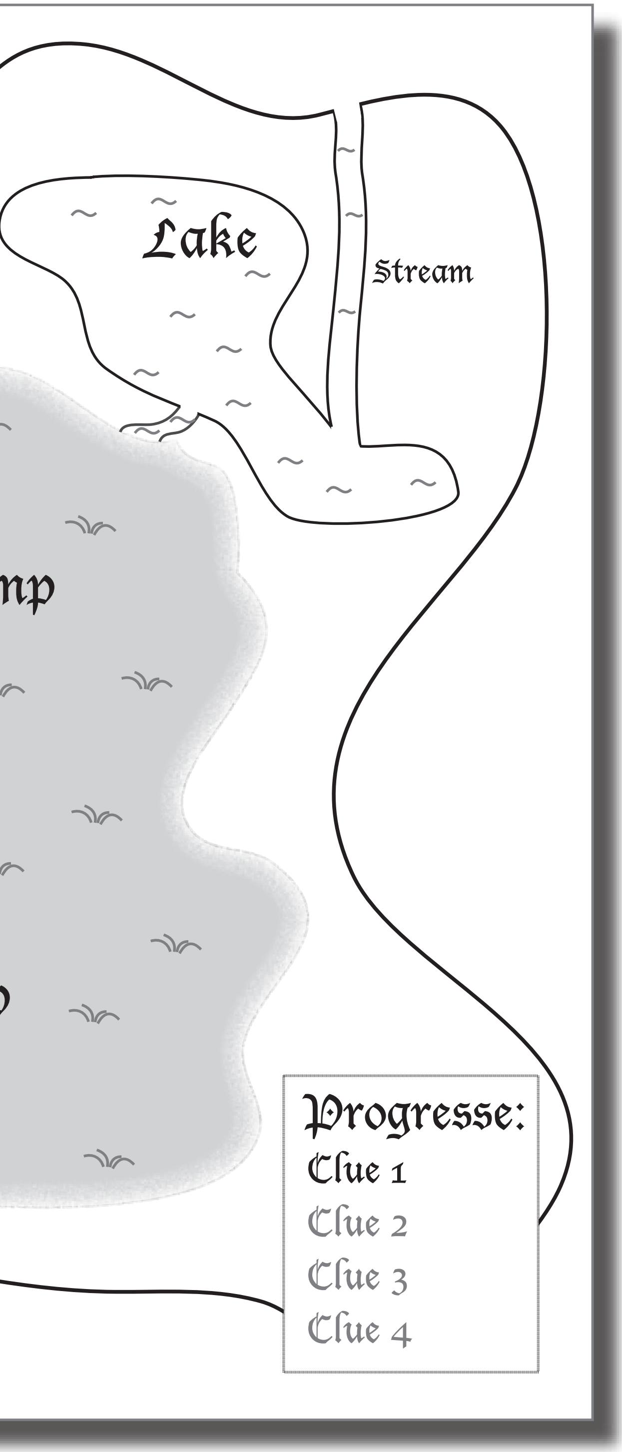
The treasure hunt

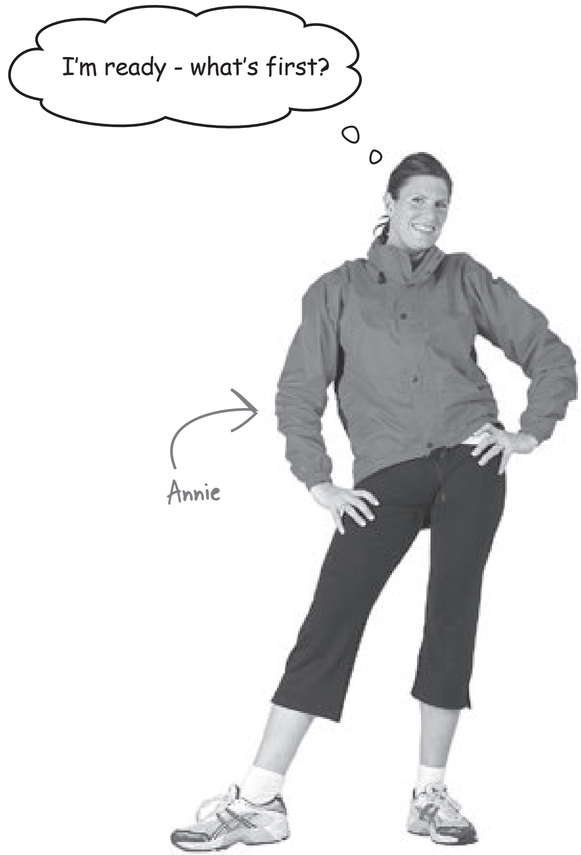
It's treasure time. You and your teammate, Annie, are part of a scavenger hunt. To be the first team to reach the prize at the end of the game, you have to follow four clues.

Here's the first clue ...









BRAIN POWER

What do you think is the best way to guide Annie to the next clue **as quickly as possible**?

Joe: I say we just tell Annie to get going and follow the directions as quickly as possible!

Mary: Hang on, the clue says to "first ponder - then go."

Joe: Hmmm?

Mary: I mean, we should think first rather than rushing into it. The directions do seem to be a bit...uh... repetitive. It's silly to do the same thing over and over again if we don't have to.

Joe: Oh yeah, I see what you mean. The first instruction sends us off to the North - and then the next one makes us retrace our steps back to the South again!

Mary: All of the directions in the clue are either North or South. So we're just running up and down the same line until we get to the end of the directions.

Joe: So following the instructions exactly isn't the quickest way after all.

Mary: What if we try to imagine the directions first - that'll be quicker than doing all that running backwards and forwards. So that's 60 m North, then 150 m South, then ...

Joe: Isn't it better to sketch them out? It'll be much easier to see what's going on than trying to hold onto all these directions in our heads.

Mary: I guess so - let's get to work!

ALWAYS start with a sketch!

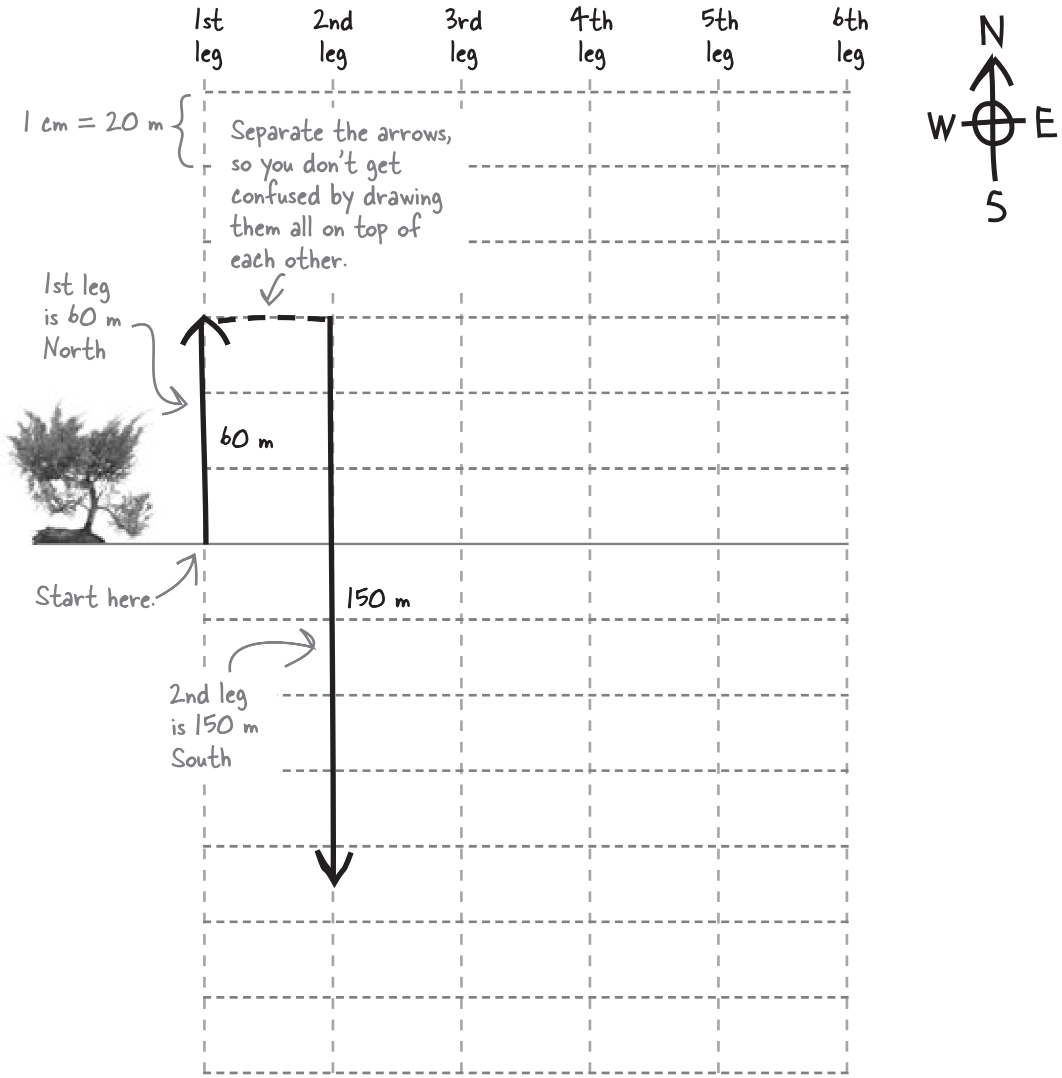
Sketching things out on paper leaves room in your brain to think about physics.

Sharpen your pencil

Your team already started a sketch of the instructions in clue 1, but they haven't managed to finish it off yet.

That's your job.

They've decided to represent each leg of the instructions using an arrow so that 1 cm represents 20 m. They've also decided to spread the arrows out a bit, so they aren't all drawn on top of each other.



Sharpen your pencil Solution

Your team already started a sketch of the instructions in clue 1, but they haven't managed to finish it off yet.

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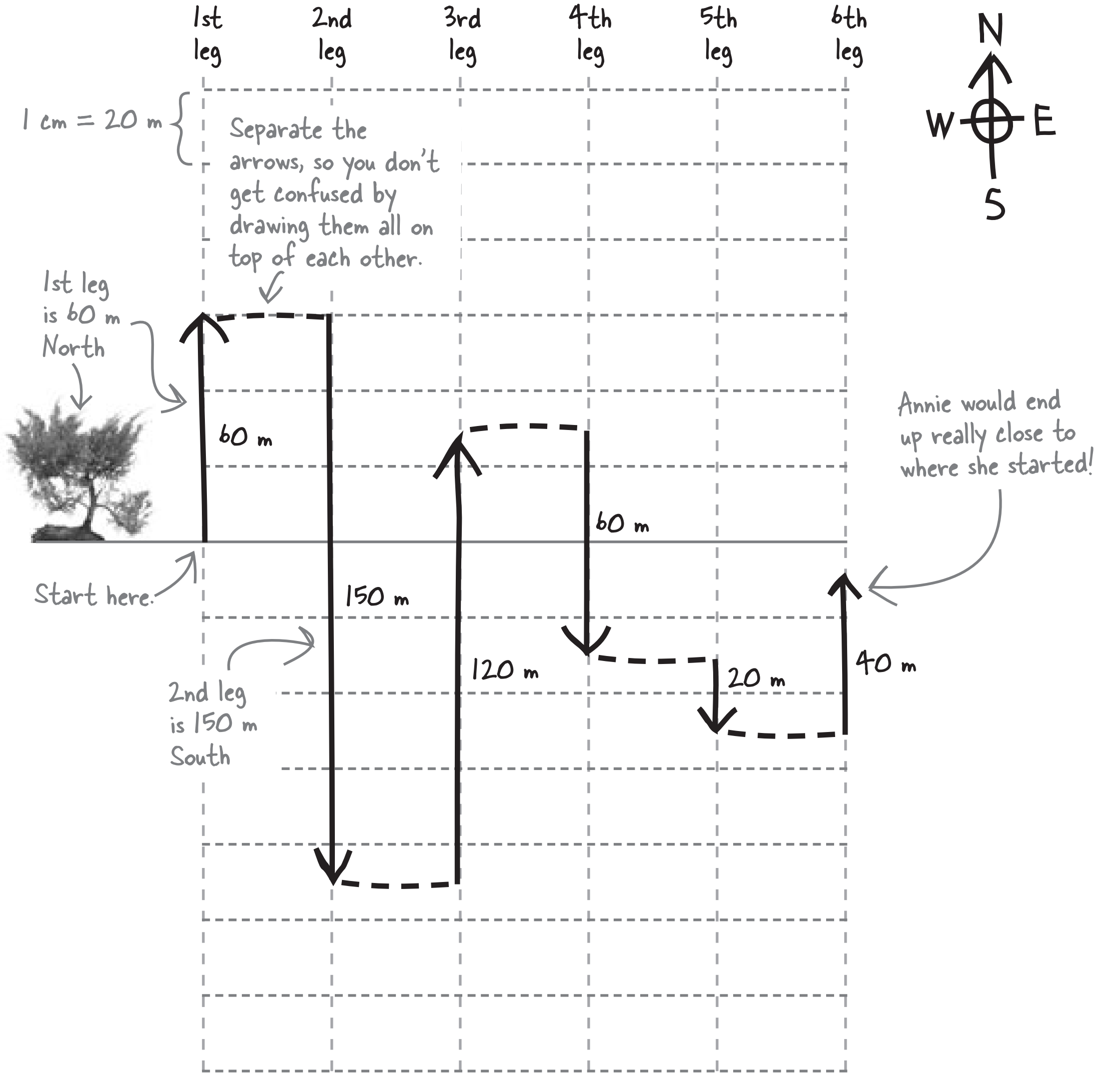


Displacement is different from distance

You've just worked out that Annie will end up very close to the tree if she follows the directions in the clue line by line. This illustrates the **difference between distance and displacement.**

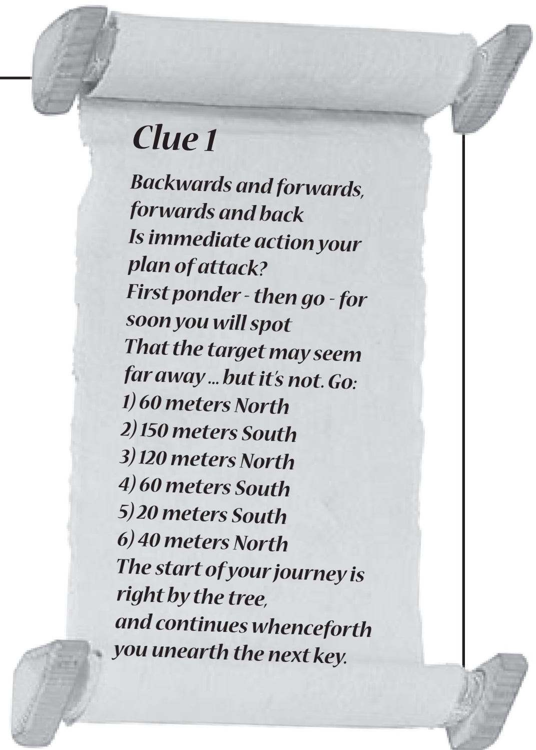
Distance is the actual total distance traveled. If you walk 70 m North, then 30 m South, you travel a **total distance** of 100 m.

This is just a number with units – a size with no indication of direction.

But **displacement** is the change in **position** between two points regardless of the route you take to get there. If you walk 70 m North, then 30 m South, you wind up 40 m North of where you started. So your displacement is 40 m North.

This has both a size and a direction.

Sharpen your pencil



a. Work out the **distance** Annie would travel if she followed the instructions in the clue exactly.

b. Work out Annie's **displacement** - the size and direction of the change in position between her start and finish points.

Sharpen your pencil Solution

a. Work out the **distance** Annie would travel if she followed the instructions in the clue exactly.

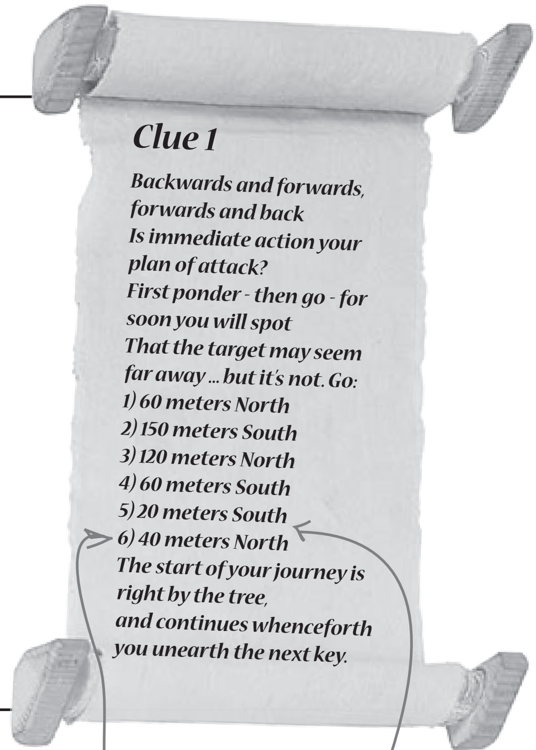
Distance = 60 + 150 + 120 + 60 + 20 + 40

= 450 m

She'd travel 450 m if she followed the instructions in the clue exactly.

b. Work out Annie's **displacement** - the size and direction of the change in position between her start and finish points.

The picture with the arrows on it shows that she ends up 10 m South of where she started.



Each of the instructions in the clue is a displacement – with a SIZE and a DIRECTION.

there are no Dumb Questions

Q: So Annie ends up 10 m South of where she started! Isn't the clue a lot of fuss about nothing?!

A: The point of the clue is thinking before you act. If you set out instantly to try and save time, you actually end up running much further.

Q: We've done a lot with units in the past. Are the units of distance and displacement the same?

A: Distance and displacement are both measured in meters (or other units of length).

Q: How can distance and displacement be different things when they both have the same units?

A: Displacement has a direction attached to it - distance doesn't.

Q: Doesn't a direction have units too?

A: No. North, South, left, right, horizontal, vertical ... etc. None of them have units.

Q: OK, I think I'm getting it now. Distance and displacement are different because distance is just a size - but with displacement there's also a direction.

A: That's right ... and we're just getting to that now.

Distance has a size.

Displacement has a size and a direction.

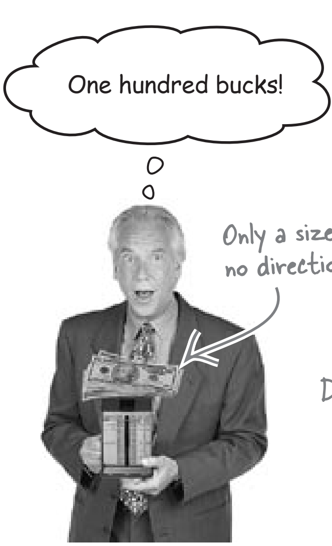
Distance is a scalar; displacement is a vector

Distance is an example of a **scalar** quantity in physics. Scalars only have a **size**, like 10 meters.

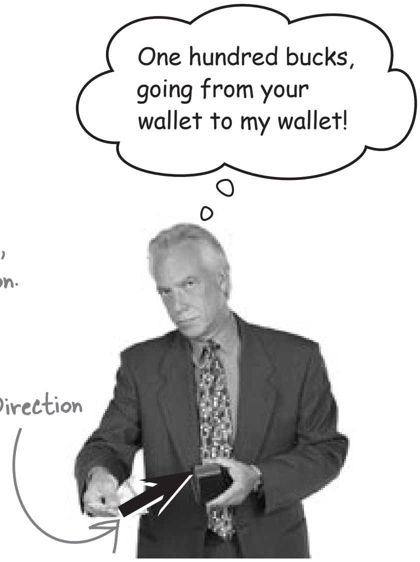
Displacement is an example of a **vector** quantity in physics.

Vectors have a **size** and a **direction**, for instance 10 meters South.

The instructions in the clue are all vectors, as they have a size and a direction. The route you take to get to the next clue isn't important - all you're really interested in is the change in position between the start and end points.



Scalars only have a SIZE.



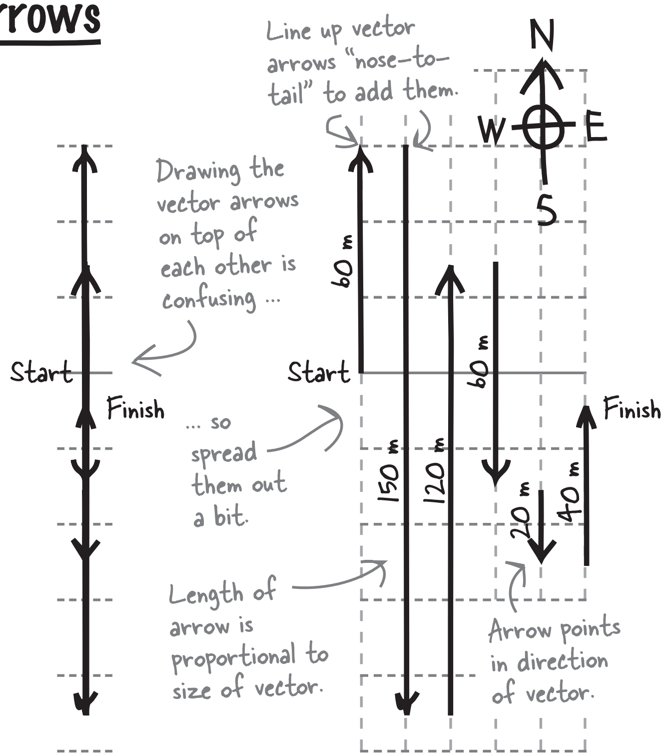
Vectors have a SIZE and a DIRECTION.

You can represent vectors using arrows

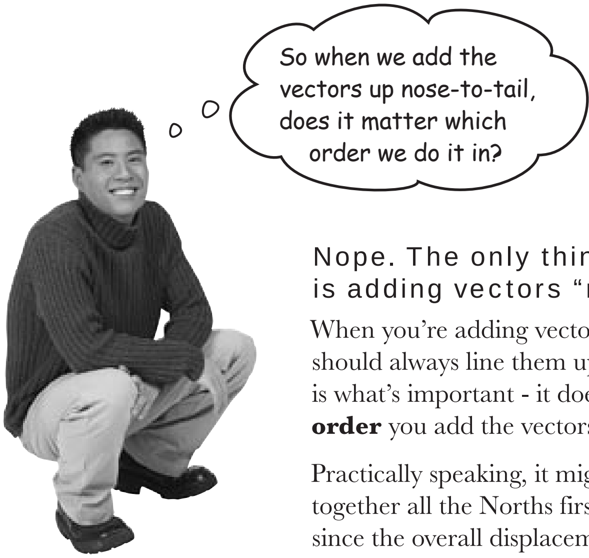
You can represent a vector quantity using an **arrow**, where the length of the arrow is proportional to the vector's size, and the arrow points in the vector's direction. You already did this when you were solving the clue.

You also intuitively added the vectors correctly, lining them up "**nose-to-tail**" by putting the tail of the next vector by the nose of the previous one.

If the vectors all lie along a straight line (like the ones here) it can be confusing just to draw them all on top of each other. It's easy to lose track of where you are. So sometimes it's appropriate to line them up next to each other (like you already did) with the understanding that they're actually all on top of each other.



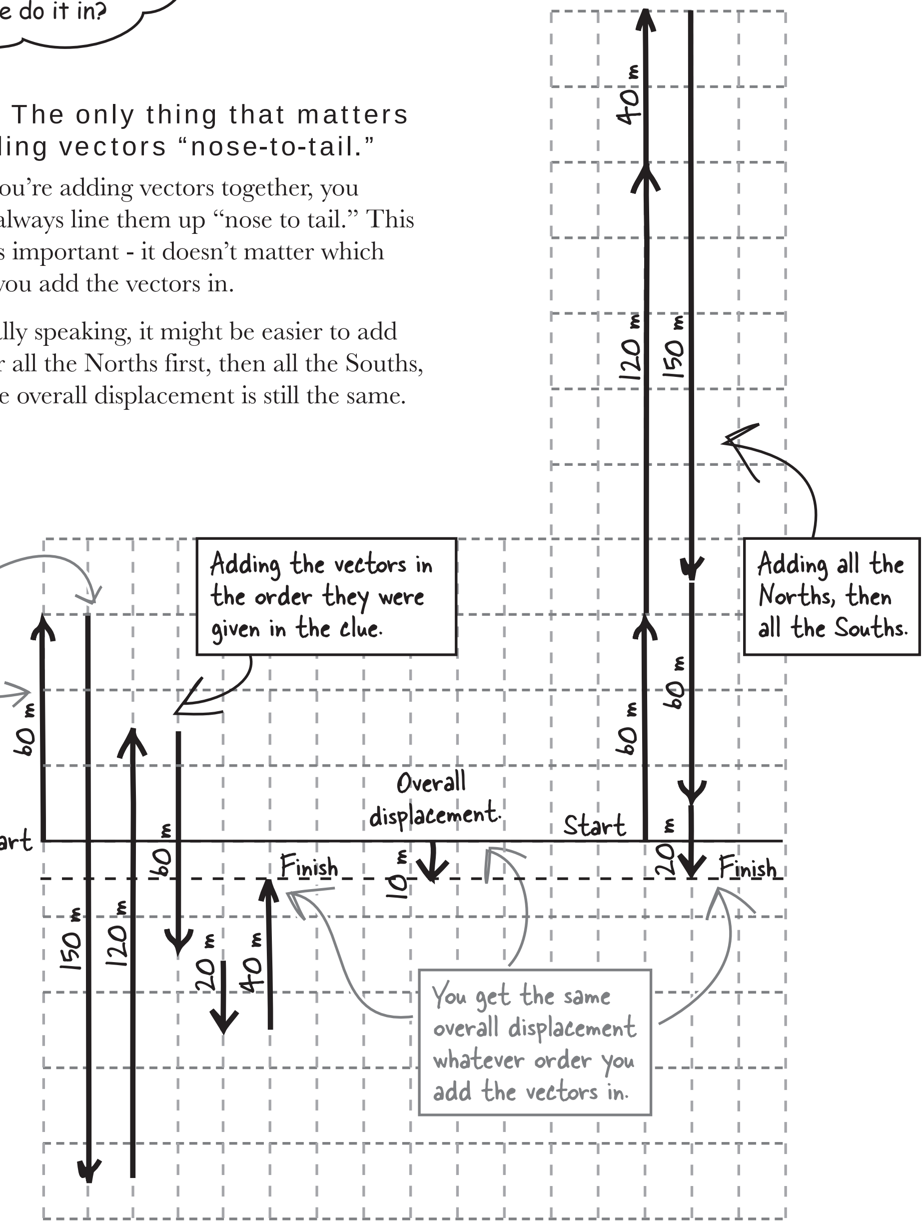
You can add vector arrows by lining them up "nose-to-tail."



Nope. The only thing that matters is adding vectors "nose-to-tail."

When you're adding vectors together, you should always line them up "nose to tail." This is what's important - it doesn't matter which **order** you add the vectors in.

Practically speaking, it might be easier to add together all the Norths first, then all the Souths, since the overall displacement is still the same.



Adding the vectors in ANY ORDER will result in the same overall displacement.

You can add vectors quickly using math.

If you go 60 m North, and then 60 m South, your displacement is zero since you're back where you started.

North and South are **opposite directions** - so you can use **opposite signs** to represent them mathematically. Suppose you decide that traveling North is positive, and traveling South is negative. So 60 m North, then 60 m South is a displacement of 60 - 60 = 0 m (or 60 + -60 = 0).

When you were working out the displacement, you might already have intuitively done something like this (by making North the positive direction), like 60 - 150 + 120 - 60 - 20 + 40 = -10 m, which is the same as 10 m South of where you started. Or you may have added together all the Norths, then all the Souths, like the vector diagram on the other page.

You can use opposite signs to mean opposite directions.

This ONLY works if you have two opposite directions, like North-South, up-down, left-right, and so forth.

there are no Dumb Questions

Q: If vectors add "nose-to-tail," then how do scalars add?

A: The same way they always have - you just add the numbers together.

Q: Are there any other vector quantities apart from displacement?

A: Yes - we'll meet some others soon ...

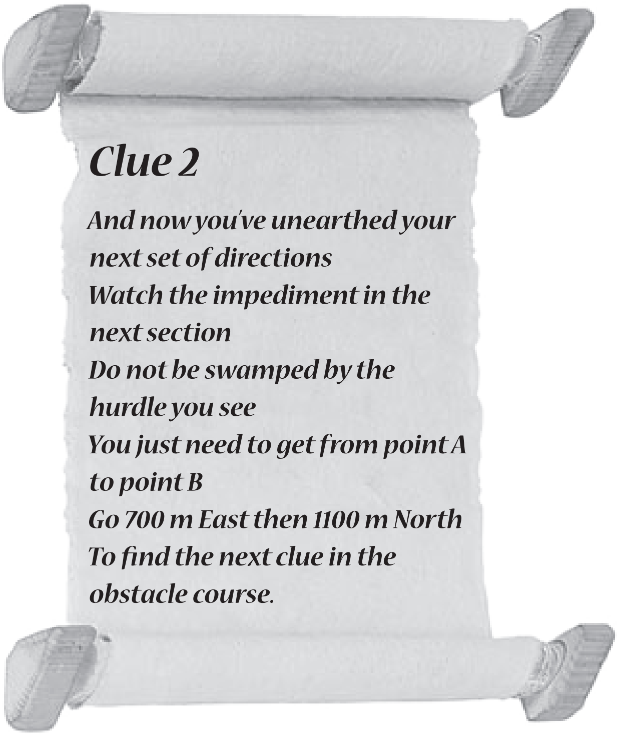
Q: Don't you need to define a starting point before you add your vectors?

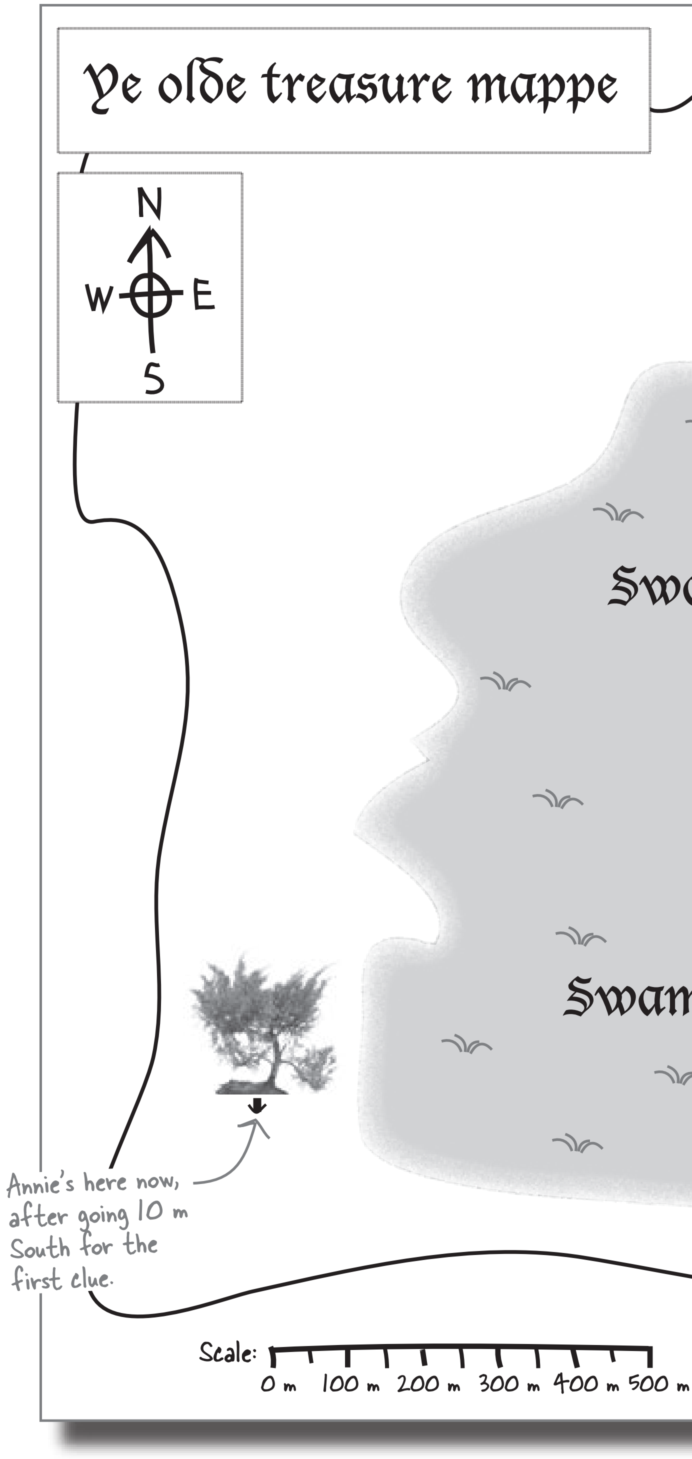
A: Yes, that's right. Sometimes there'll be an obvious starting point - like a tree! Sometimes you'll need to define one. For example, if you're describing heights, it's conventional to make 0 m equal to sea level and measure everything else in reference to that.

Q: How do you decide which way is positive and which way is negative?

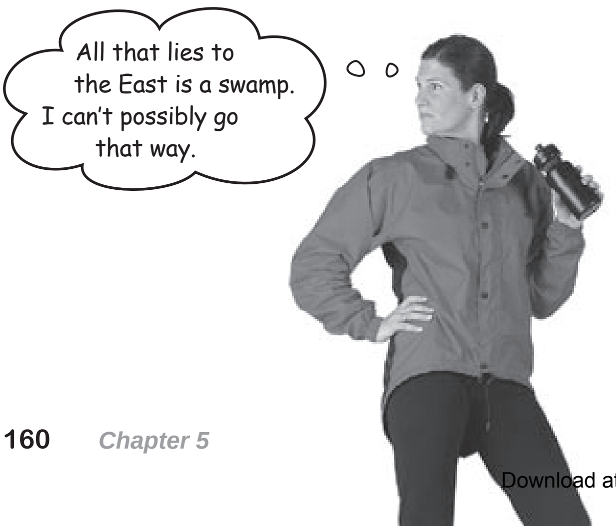
A: It's up to you - as long as you choose a direction and stick with it, the math will work out the same. If you make North positive and your answer is -10 m, it means 10 m South. And if you make South positive and get the answer 10 m, that means 10 m South as well. You just need to remember how to **interpret the sign** of your answer at the end.

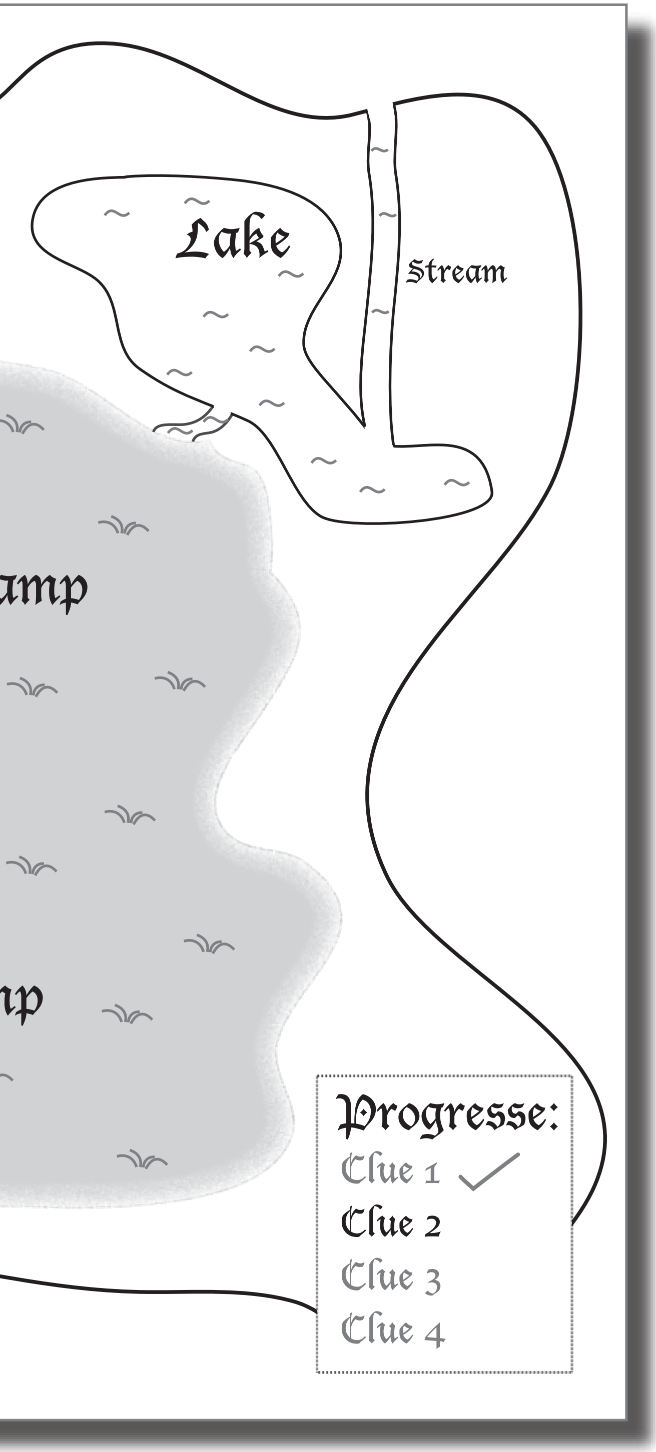
You found the next clue...





But there's a problem ...





Sharpen your pencil

a. Draw the instructions from the clue on the map using vector arrows.

b. "Do not be swamped" is an important part of the clue. Write down your ideas about how you might achieve this.

....

....

....

....

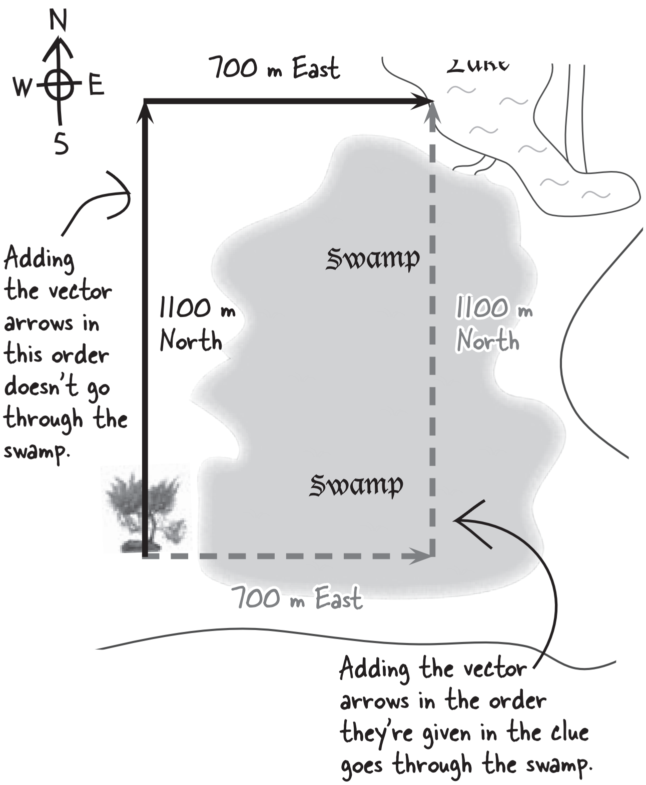
....

Sharpen your pencil Solution

a. Draw the instructions from the clue on the map using vector arrows.

b. "Do not be swamped" is an important part of the clue. Write down your ideas about how you might achieve this.

Going East then North is impossible because of the swamp. But you can get to exactly the same place by going 1100 m North, then 700 m East. – you can follow the instructions either way around.



You can add vectors in any order

Even when vectors aren't pointing along the same line, you can still add them together by lining them up "nose-to-tail."

Whichever order you add them in, you always end up with the same difference in position between the start and finish points.

So you can send Annie 1100 m North first, then 700 m East. This way, she'll find the next clue without getting swamped.

there are no Dumb Questions

Q: So you're saying that even if I have hundreds of vectors, it doesn't matter what order I add them in?

A: Exactly! As long as you line the vectors up "nose-to-tail," you'll always end up with the same resultant vector at the end.

Q: Wait, what's a resultant vector?!

A: It's just another way of saying "answer vector." A resultant vector is what you get when you add together other vectors.

Q: So if I add together vectors, do I always get a resultant vector as my answer?

A: Yes. If you're adding vectors, your answer must also be a vector.

Q: Can a vector equal zero? Is that still a vector?

A: Yep, in the context of vector addition, you get a special vector called the "zero vector."

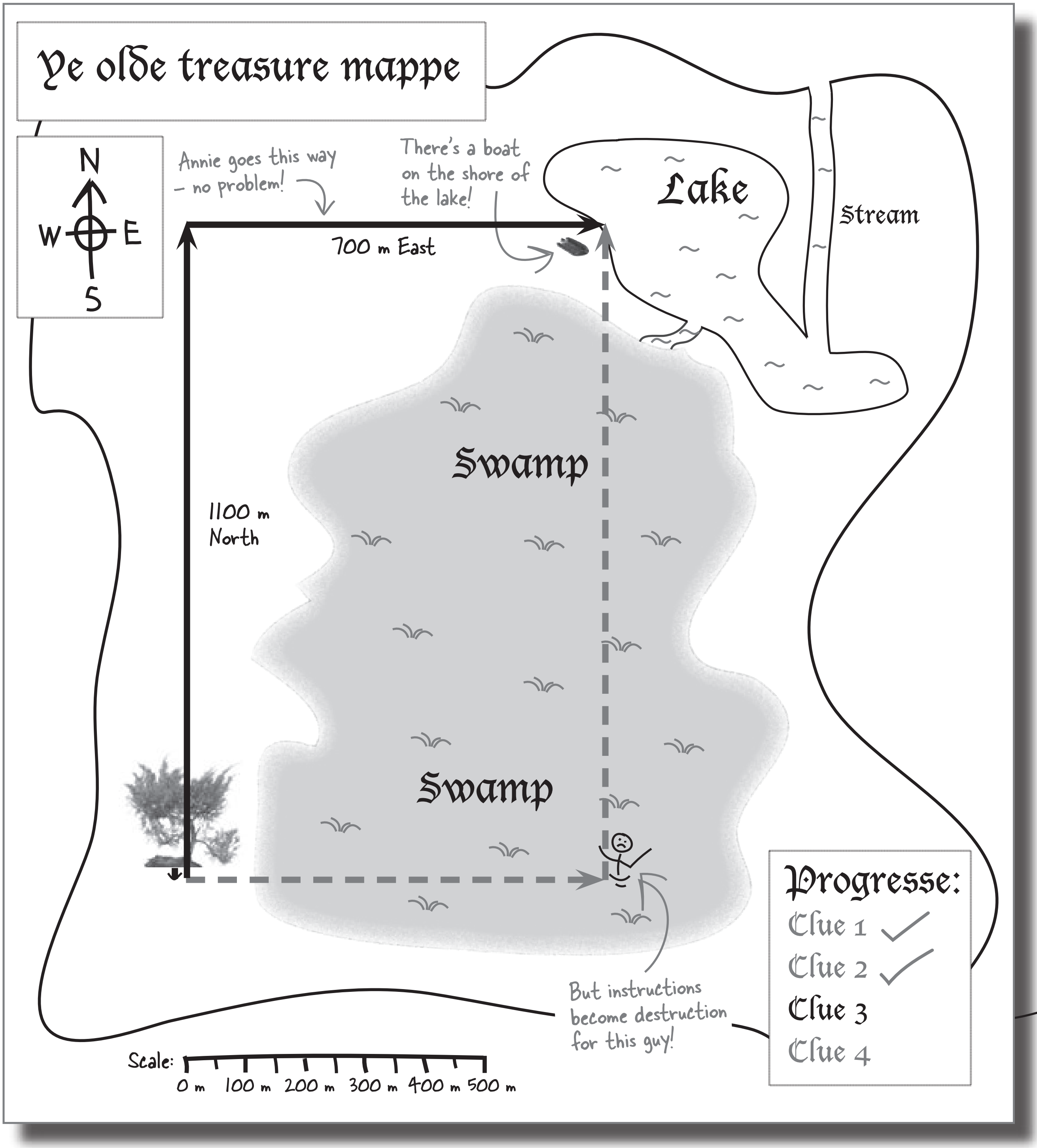
Q: It's OK to use a different sign for two directions when they're total opposites, like North and South, isn't it? But what about when they're not opposites - like North and East in this clue. How do I add the vectors using math then?

A: Great question - and something you'll learn all about in chapter 9.

When you add vectors together, your answer can be called the resultant vector.

BULLET POINTS

- Scalars have size. Distance is an example of a scalar.
- Vectors have both size and direction. Displacement is an example of a vector.
- You can represent vectors using arrows.
- The length of a vector arrow is equal to the size of the distance.
- The direction of a vector arrow is equal to direction of movement.
- You add vectors by lining them up "nose-to-tail" and following the arrows from start to finish.
- If your vectors all lie along a straight line, you can add them quickly using math by defining one direction as positive and the other direction as negative.
- If your vectors point in different directions, you can still add them by lining them up nose-to-tail, and following the arrows.
- You can add vectors in any order regardless of which direction they point in.


Ye olde treasure mappe
N
W
E
S
Annie goes this way – no problem!
There's a boat on the shore of the lake!
700 m East
Lake
Stream
Swamp
1100 m North
Swamp
But instructions become destruction for this guy!
Progresse:
Clue 1
Clue 2
Clue 3
Clue 4
Scale:
0 m 100 m 200 m 300 m 400 m 500 m

Well done - you've found the third clue!

And along with the third clue at the shore of the lake is a motor boat. There's a problem though - the clue has a lot of numbers and technical jargon in it.

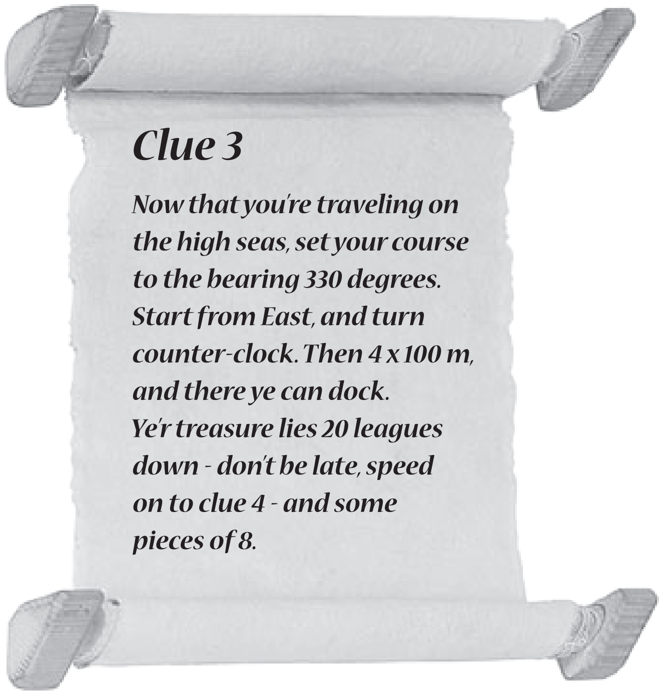
To pick your way through it, first, work out **what** you're supposed to do. Once that's clear, think about **how** you might do it.

First what, then how.

Sharpen your pencil

Think - first **what**, then **how**.

Underline the parts of the clue that give you the "what," then use your own words to write down what you're supposed to do - and also how you might do it.



Question Clinic: The "Wheat from the chaff" Question

Sometimes a question will have a lot of unfamiliar words or jargon in it. Don't panic! Often a complicated-looking build-up leads into a straightforward question to test your ability to sort out the stuff that actually matters. The question may give you more information than you need to actually solve the problem, so don't worry if you don't use everything you're given.

Parts of the question contain irrelevant information and unfamiliar words. Don't let that stress you out or make you think you can't do it.

Here's some relevant information!

Clue 3

Now that you're traveling on the high seas. Set your course to the bearing 330 degrees. Start from East, and turn counter-clock. Then 4 x 100 m, and there ye can dock. Ye'r treasure lies 20 leagues down - don't be late, Speed on to clue 4 - and some pieces of 8.

Sometimes there'll be a line break in the middle of something relevant. Don't let that put you off!

As long as you can sort out the wheat from the chaff – or the important information from the irrelevant padding – you'll be fine.

Here are some numbers that you don't actually need to use to get your answer.

Look out for direction as well as size.

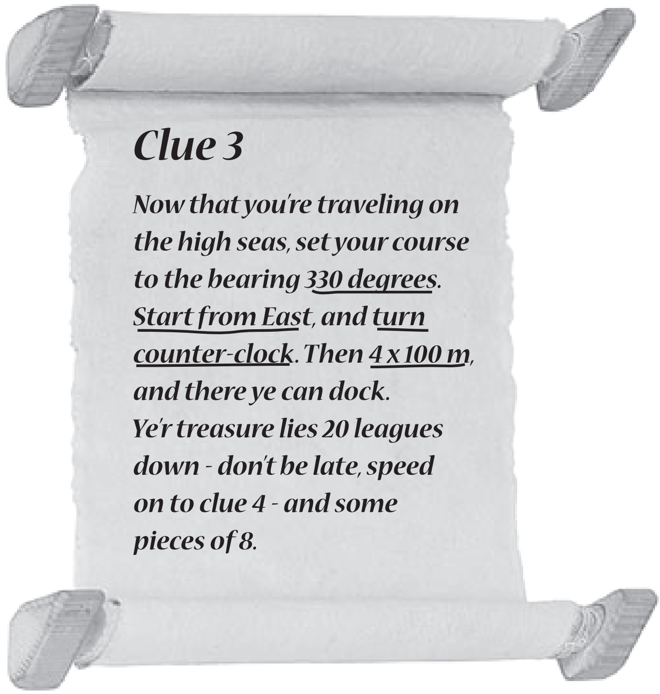
First what - then how! Start off by looking at the end of the question to see what you're actually being asked. Sometimes extra numbers or information are included in a simple factual question to test your understanding of the physics.

Look out for questions on 'hard topics' (like particle physics) that you don't think you know much about. Often they're just questions about simpler stuff disguised to make them look 'difficult.' You may already know how to do them, so don't get psyched out at the start.

Sharpen your pencil Solution

Think - first **what**, then **how**.

Underline the parts of the clue that give you the "what," then use your own words to write down what you're supposed to do - and also how you might do it.



WHAT – 4 x 100 m is 400 m
A bearing of 330 degrees sounds like an angle.
Measure it counter-clockwise from the East.

HOW – Protractor measures 330° counter-clockwise from the East, and a ruler plus the scale on the map measures the 400 m.

Don't worry if you've forgotten a lot about angles. We'll spend a while going over them on the next few pages.

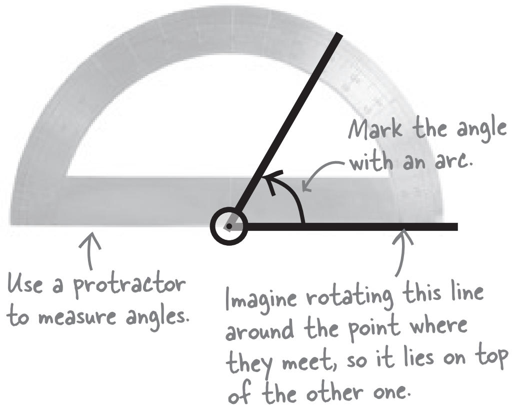
First WHAT - then HOW!

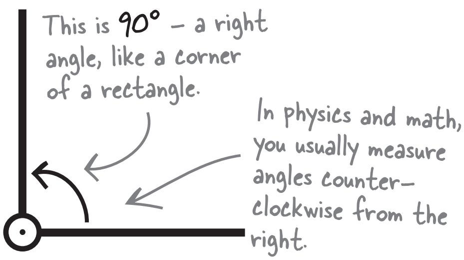
Angles measure rotations

Angles measure rotations, how far you have to rotate a line so that it lies on top of another line.

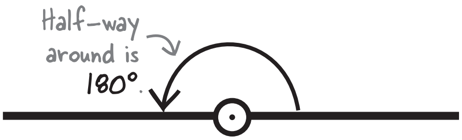
You've probably already come across a common way of measuring angles - in **degrees** (symbol: °).

You measure angles with a **protractor**. It's marked off in degrees, so you can line it up and read off the size of the angle just like you would with a ruler.

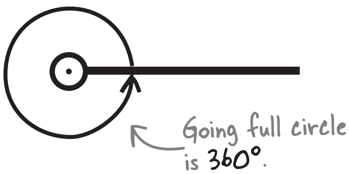




An angle that comes up a lot in physics is the **right-angle**, which is **90°**. It's a quarter of a revolution, and it's the angle you find in the corners of a rectangle. So you see right-angles everywhere you go - between the ground and anything standing on it (chair, table, building, and so forth).

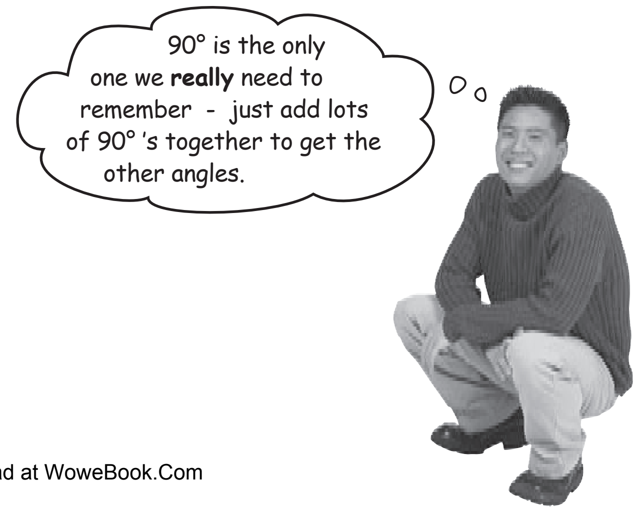


Half a revolution is **180°**, and it's called a straight angle. So anything that does a U-turn to go in the opposite direction has rotated 180°. It looks like a straight line.



There are 360° in a complete revolution - if you go all the way around, so the line ends up back on top of itself, that's an angle of 360°.

90° = a right angle
180° = halfway around
360° = a full revolution



there are no Dumb Questions

Q: Why 360° in a full rotation? That seems a bit random. I mean - why not a nicer number like 100° or 1000°? Doesn't that fit in better with SI units?

A: The rotation originated from ancient civilizations thousands of years ago. But the most practical reason is that 360 divides exactly by a lot of useful numbers.

Q: But that works just as well if there are 100 degrees in a circle. Half a rotation would be 50, and a quarter would be 25 ...

A: Yes, we were just getting to that. With 360° in a rotation, a third of a rotation is 120°, and a sixth is 60°. Now, try doing that with 100 degrees in a circle- 33.33333333... degrees in a third of a rotation, anyone?

Q: Can you get angles bigger than 360° if something keeps on going round and round? Or does the angle 'reset' itself to 0° when you get back to where you started?

A: It depends on what you're trying to do - sometimes talking about the total rotation is useful; other times 'resetting' the angle when you get back to the start is useful.

Q: Is an angle a scalar or a vector?

A: Good question! It can be either, depending on whether you're talking about the total rotation (scalar) or the rotation distance between the start and finish points.

Q: Vectors are represented by straight arrows, where the length is proportional to the size. But how do you use a straight vector arrow to give the direction of an angle that's kinda curved?!

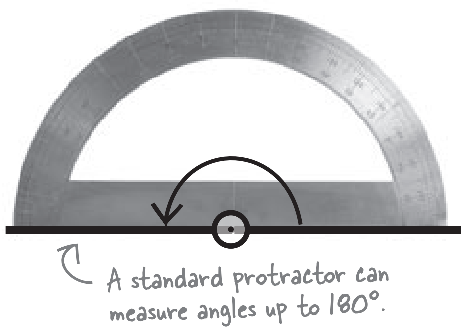
A: Yes - you can't directly represent an angle using a vector arrow. It's a valid question, but you don't have to worry about it for the moment, as we'll get on to representing angular quantities with vector arrows in chapter 12.

Now you can get on with clue 3!

You've worked out that you need to start off facing East, and rotate counter-clockwise through an angle of 330°. Once you've done that, you'll travel 400 m in the direction you're now facing to reach clue 4.

400 m on a bearing of 330 counter-clockwise from East is a displacement, not a distance, as it has both a size (400 m) and a direction (330° counter-clockwise from the East).

You can use an angle to indicate direction.



BRAIN POWER

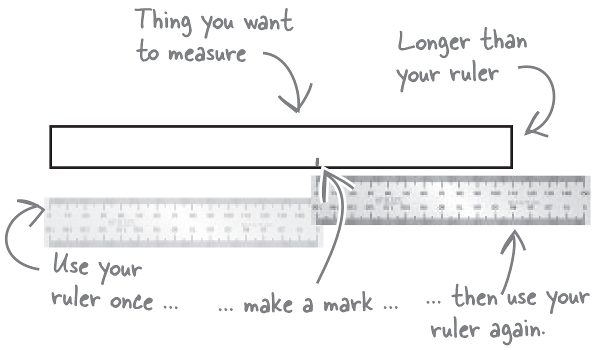
How can **you** measure an angle of 330° when your protractor only goes up to 180°?

If you can't deal with something big, break it down into smaller parts

If you're measuring something with your ruler, and it's too long, you can start off by measuring along as far as you can. Then, you can make a mark, move the ruler along, and measure the next part. And so on.

The underlying principle is one that's essential for physics - especially when it gets more complicated! If something's too big for you to deal with it all at once, **break it down into smaller parts** that you can deal with.

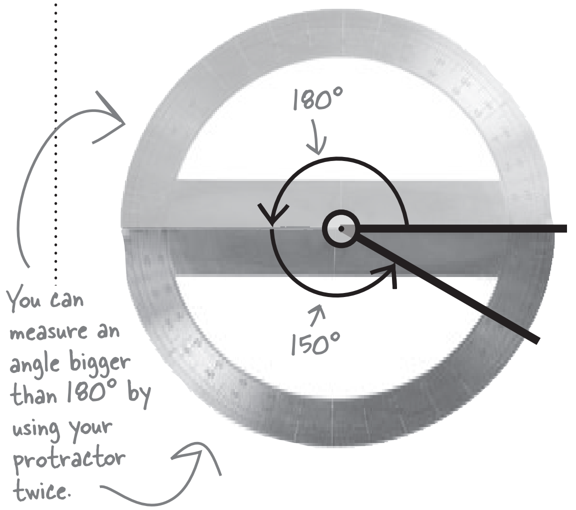
There are two ways you can do this with a standard 180° protractor - do whatever you're most comfortable with:



You can use your protractor twice to break down the angle

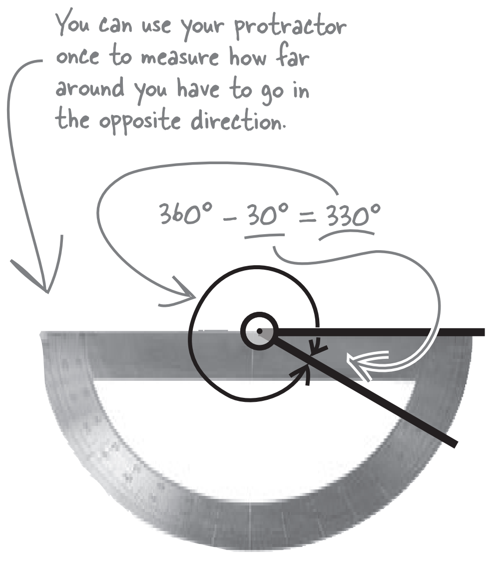
You can do exactly the same as you would with a ruler, and use your protractor twice. It goes up to 180°, so measure that far first.

To work out how much further to go, you can say 330° - 180° = 150°, then measure around another 150° from where you stopped.



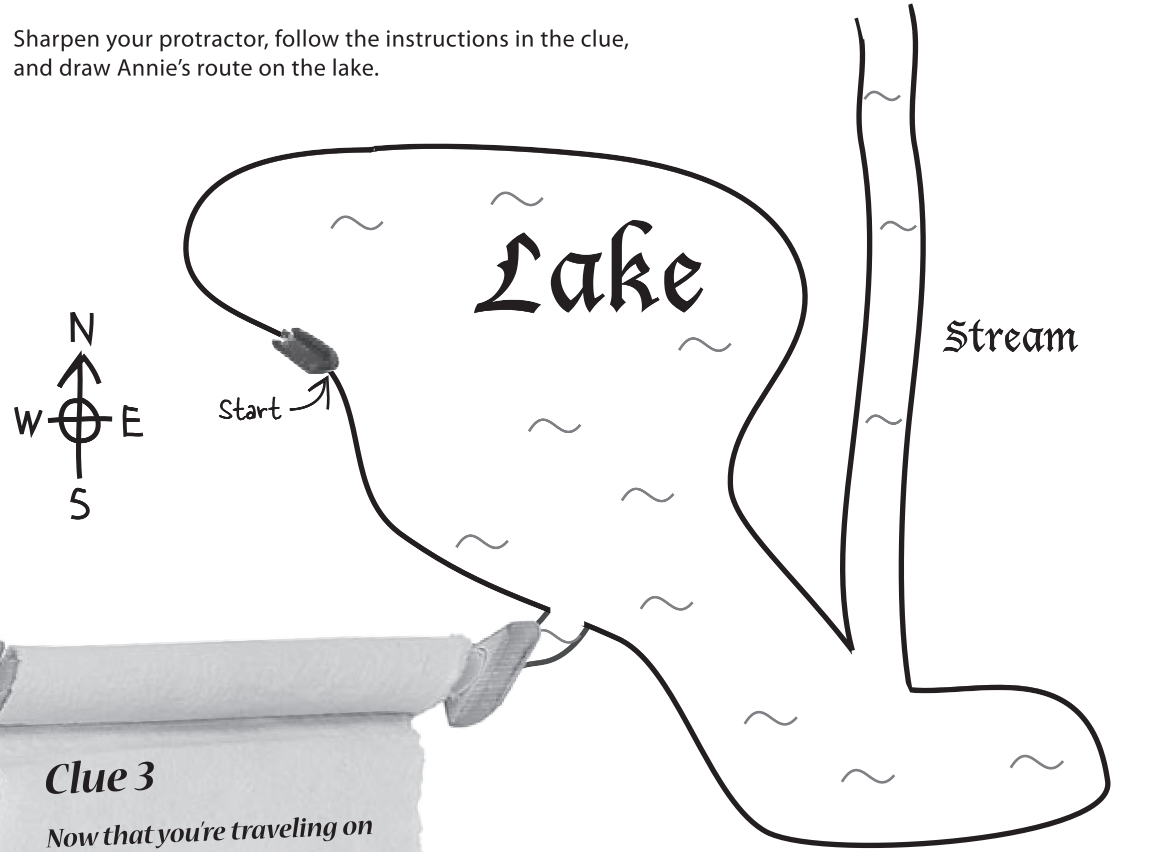
You can measure your angle the other way around

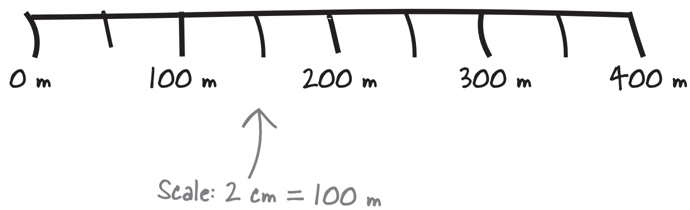
You can also say, "330° is only 30° short of being a full 360° circle." So turning 330° counterclockwise is the same as turning 30° clockwise.

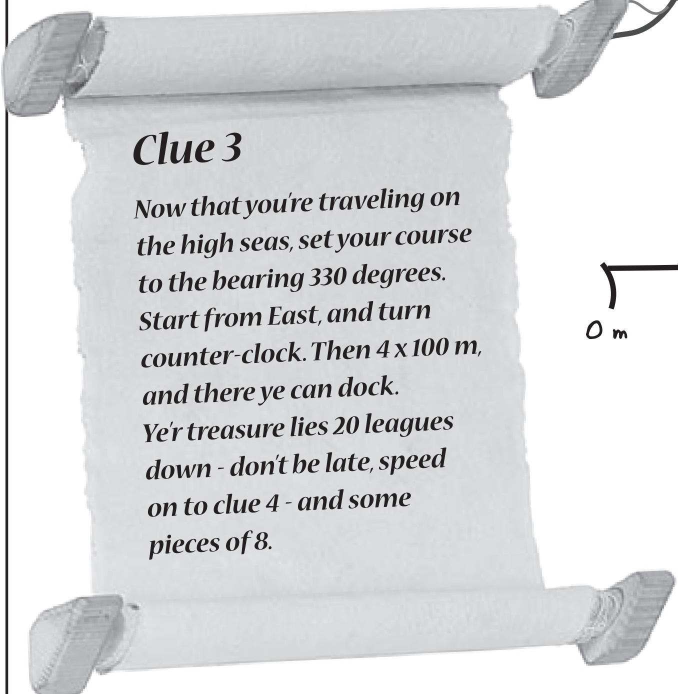


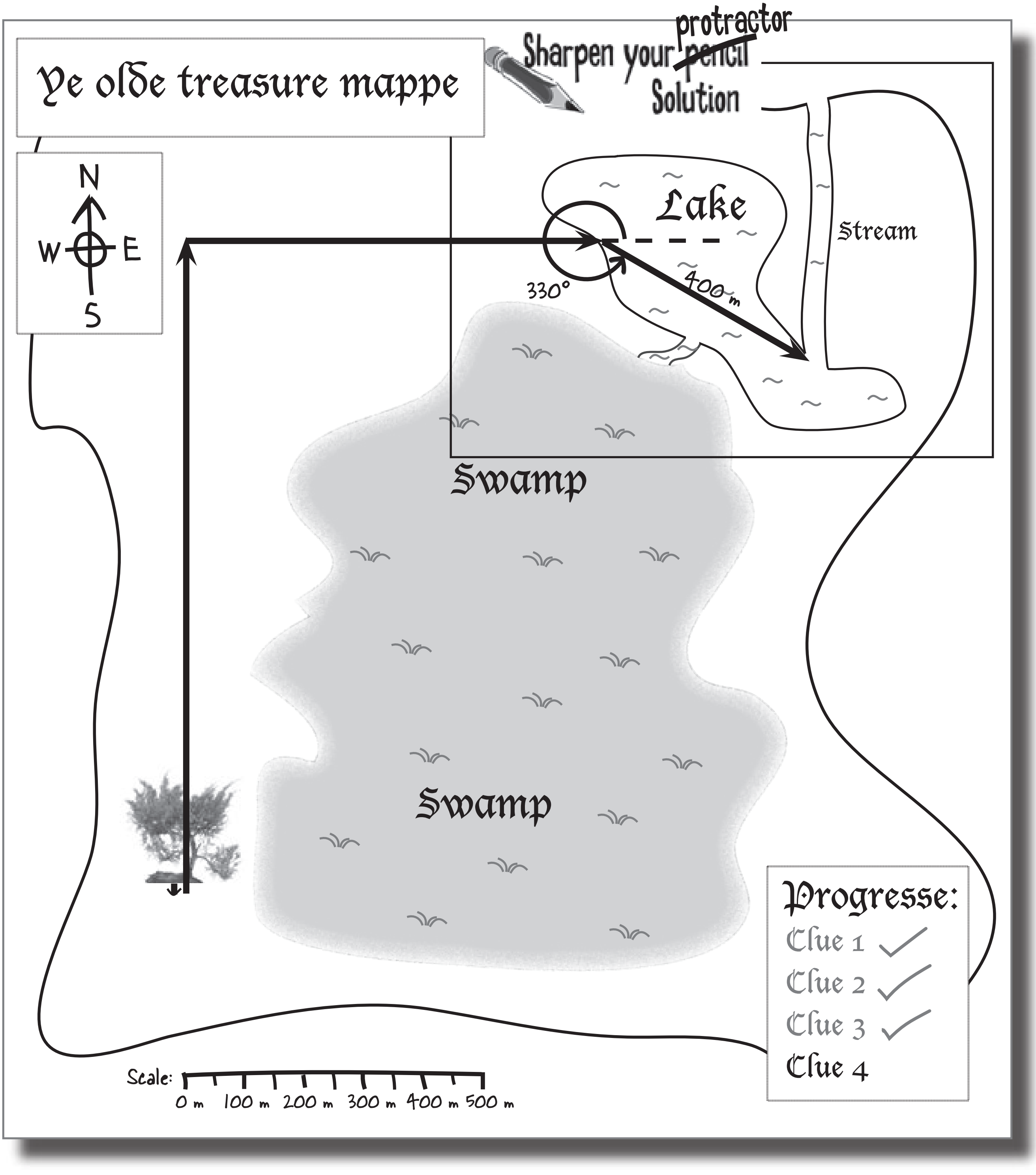
Sharpen your ~~pencil~~ protractor

Sharpen your protractor, follow the instructions in the clue, and draw Annie's route on the lake.








Ye olde treasure mappe
Sharpen your ~~pencil~~ protractor
Solution
N
W
E
S
Lake
Stream
330°
400 m
Swamp
Swamp
Progresse:
Clue 1
Clue 2
Clue 3
Clue 4
Scale:
0 m
100 m
200 m
300 m
400 m
500 m

You move onto the fourth clue...

You got the angle right because Annie's found a sign sticking out of the water with the fourth clue on it!

A word you've not seen yet in this book

Hmmm...... velocity is a new term... is it okay to guess what it means?

You can work out what things are from their context.

Sometimes an unfamiliar word comes up. Whether it's completely new to you or something you saw once before then forgot about, the important thing is **not to panic.**

Often you'll be able to work out what it means from the context. What is the rest of the sentence or paragraph about? Are there any units mentioned? What is the rest of the question about? What are they asking you to do?

Sharpen your pencil

See if you can work out what a velocity might be from the context.

If you already know what a velocity is, then write down how someone who doesn't could work it out from the context.

....

....

....

....

Sharpen your pencil Solution

See if you can work out what a velocity might be from the context.

If you already know what a velocity is, then write down how someone who doesn't could work it out from the context!

Velocity has UNITS of m/s, which looks like meters divided by seconds.

That's the same units as speed.

There's also a DIRECTION mentioned—North.

I think that velocity might be a vector version of speed, with both a size and a direction.

Just like displacement is the vector version of distance.

Velocity is the 'vector version' of speed

Velocity is measured in meters per second - which is exactly the same units as speed. Velocity is the 'vector version' of speed - it has a **direction** as well as a **size**.

Speed is a scalar - "I'm traveling at 1.5 m/s."

Velocity is a vector - "I'm traveling North at 1.5 m/s."

Same units

Direction

Size

So the clue is asking you to go North at 1.5 m/s for a minute.

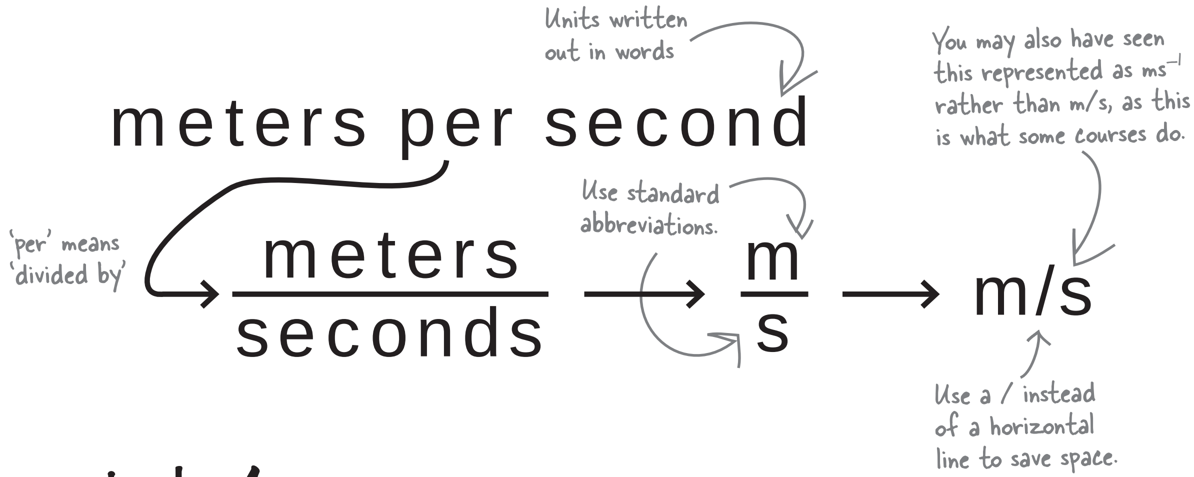
Scalar → **Speed is rate of change of distance.** ← Scalar

Vector → **Velocity is rate of change of displacement.** ← Vector

Write units using shorthand

You've spotted that the units of velocity are written as m/s in the clue. This is a more concise way of writing meters per second. The '/' means 'per' or 'divided by.'

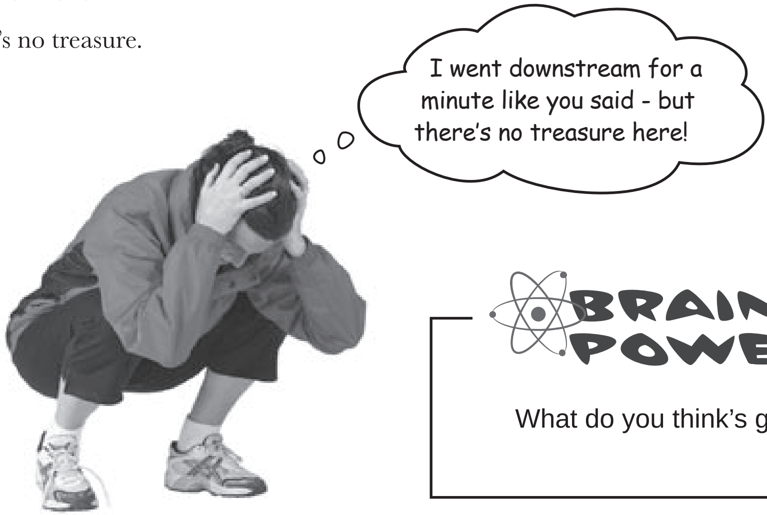
Meters per second is the same as meters divided by seconds. So when you use the standard letters to abbreviate the units, you get m/s.



So, on to clue 4 ...

Now that you know that velocity is the vector version of speed, you tell Annie to point the boat North, set the controls to 1.5 m/s, and travel for a minute before dropping the anchor.

But when she arrives, there's no treasure.

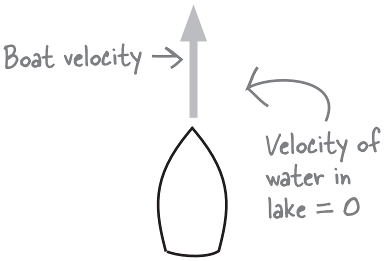


BRAIN POWER

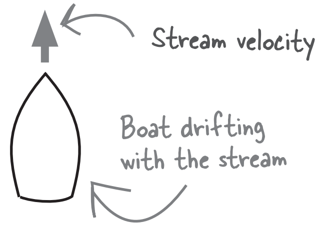
What do you think's gone wrong?

You need to allow for the stream's velocity too!

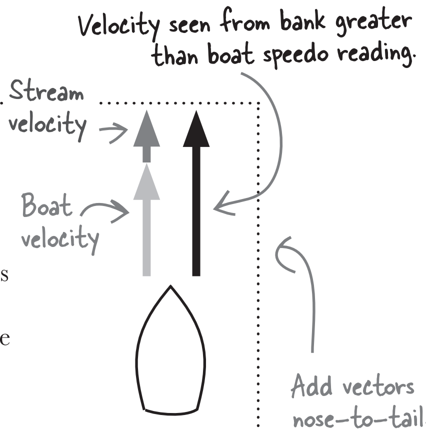
Clue 3 took place entirely on a still lake. So when the boat was set to go at a certain velocity relative to the water, that was the velocity it went at.



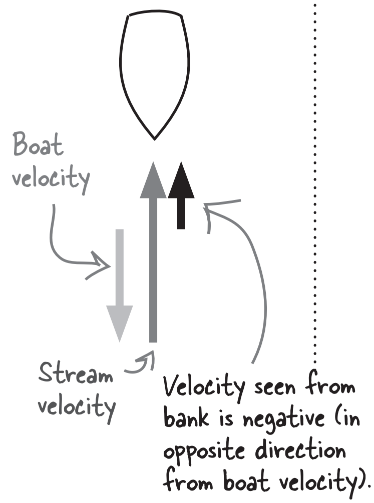
Going North from the Clue 4 sign involves going downstream, towards the sea. If Annie didn't start the motor, the boat would move North with the same velocity as the stream.



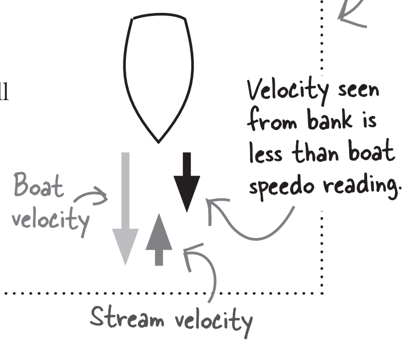
But Annie did start the motor - and the boat moved North with a reading of 1.5 m/s on its speedometer. But the speedometer tells you how fast the boat's going **relative** to the water. This means that Annie went faster than the clue said she should, at 1.5 m/s plus the velocity of the stream.



If the upstream current was really, really fast, the boat would go backwards even if the reading on its speedometer said it was going forwards!



If Annie had been going South, upstream, the boat and stream velocity vectors would point in different directions. So the overall velocity of the boat as seen from the bank would be less than the reading on its speedometer.

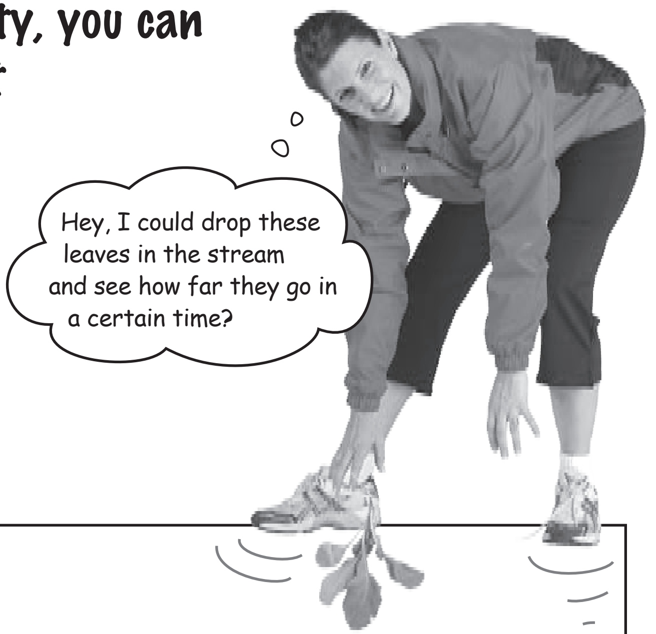


If you can find the stream's velocity, you can figure out the velocity for the boat

It's possible to add velocity vectors by lining them up nose-to-tail in the same way you added up the displacement vectors earlier on.

You know that the overall velocity of the boat needs to be 1.5 m/s to the North. And Annie works out a way of measuring the velocity of the stream by dropping leaves in it.

Now, you only need to do the calculation, send Annie on her way, and the treasure is yours ...



Sharpen your pencil

The boat needs to go North with an overall velocity of 1.5 m/s for a minute.

a. If the leaves Annie throws into the stream travel 10 m North in 20 s, what's the velocity of the stream?

b. What speed should Annie set on the boat's speedometer to solve the clue and find the treasure? (You may find it helpful to use vector arrows to visualize this.)

Sharpen your pencil Solution

The boat needs to go North with an overall velocity of 1.5 m/s for a minute.

a. If the leaves Annie throws into the stream travel 10 m North in 20 s, what's the velocity of the stream?

b. What speed should Annie set on the boat's speedometer to solve the clue and find the treasure? (You may find it helpful to use vector arrows to visualize this.)

a. Leaves travel 10 m North in 20 s.

$$\text{Velocity} = \frac{\text{Change in displacement}}{\text{Change in time}}$$

$$= \frac{10\text{ m}}{20\text{ s}} = \underline{\underline{0.5\text{ m/s North}}}$$

Displacement = 10 m North

t = 20 s

b. Annie wants to go North at 1.5 m/s.

Stream will 'provide' 0.5 m/s of this velocity.

So the boat should go **North at 1.0 m/s** (relative to the stream).

Want overall velocity of 1.5 m/s North

Stream velocity 0.5 m/s North

Required boat velocity (relative to stream)

If you're asked for a vector quantity, remember to give a DIRECTION as well as a size.

A sketch makes everything clearer!

there are no Dumb Questions

Q: So basically a vector is something I can draw as an arrow, right?

A: Yes - the length of the arrow represents the size, and the direction of the arrow represents the direction.

Q: I was fine with displacement, where the length of the arrow represents an ACTUAL length. But I'm kinda finding it hard to visualize a velocity.

A: Something's overall velocity vector points in the direction that it's currently moving in. If it's going fast, you draw a long arrow, and if it's going slow, a shorter arrow.

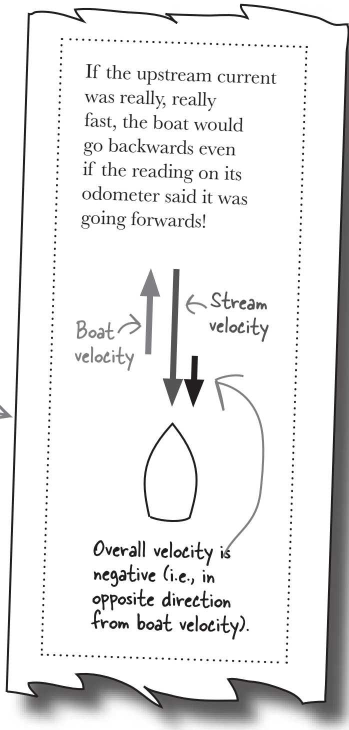
Q: Right - so something going fast to the North would have a long arrow pointing North, and something moving slowly to the East would have a short arrow pointing East.

A: Exactly.

The overall velocity vector points in the direction the thing's currently moving in.

I get that - but I'm still not sure about what happens when the boat's trying to fight against a really fast current. How can the boat go backwards when its speedometer says it's going forwards.

A speedometer gives the velocity RELATIVE to the water. But you're interested in the velocity RELATIVE to the bank



Here's what would happen if the upstream current was really fast.

Q: How do you mean? Surely the boat can only have one velocity?

A: The boat's speedometer tells you how fast it's going relative to the water. But if the water is also moving, then the boat's velocity vector, as seen from the bank of the stream, will be different. It points in the direction that the boat is currently moving in. The velocity of the boat as seen from the bank is equal to the boat's velocity relative to the water plus the water's velocity.

Q: But that doesn't make sense.

A: You know the moving walkway at the airport that helps you travel long distances more quickly? What happens if you turn around and start walking in the opposite direction of the walkway's motion? You start moving backwards relative to the rest of the building, but you're actually moving forwards relative to the walkway. The walkway moves you backwards more quickly than you're walking forwards.

It's the same with the boat and the stream.

So Annie goes North at 1.0 m/s for 1 minute ...

... but doesn't find any treasure this time either.

Are we ever gonna find it?!

BRAIN POWER

Annie did exactly what you asked her to, but it hasn't worked out again. Any ideas why?

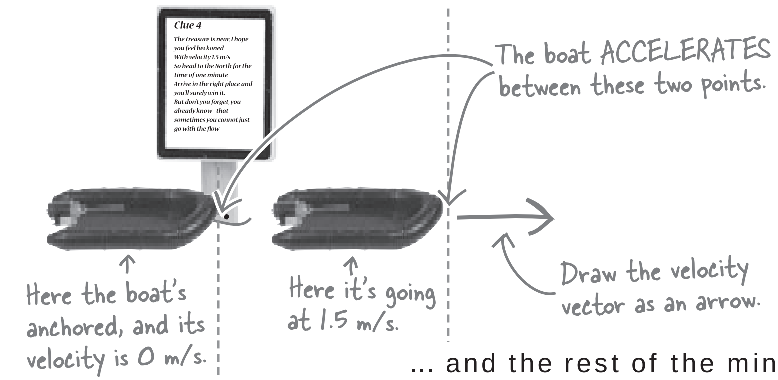
It takes the boat time to accelerate from a standing start

It's OK if you didn't think of the boat accelerating – this one was pretty tricky!

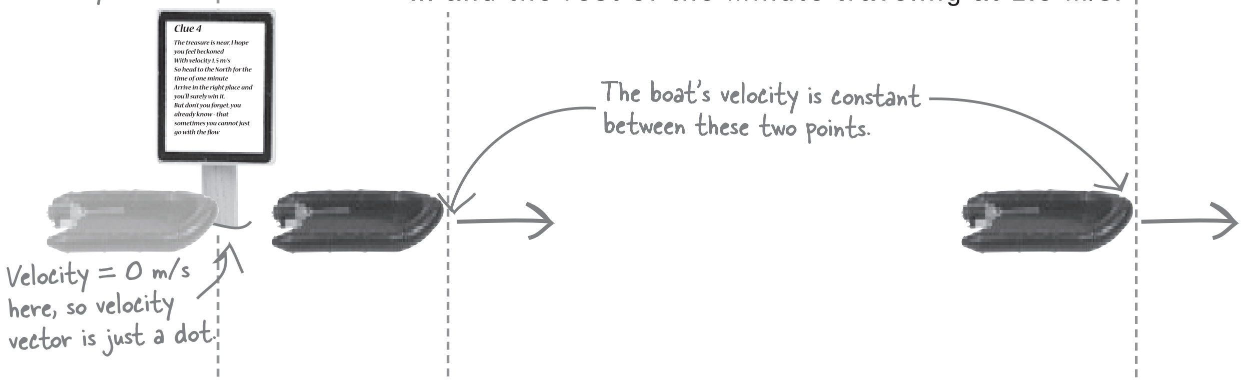
Although Annie pointed the boat North and set the controls for 1.5 m/s, the boat takes time to get up to that speed from standing still. It takes time to **accelerate** from 0 m/s to 1.5 m/s.

Annie started timing the boat before it had accelerated up to 1.5 m/s. So it didn't go at 1.5 m/s for the whole minute - and didn't go as far as it should have.

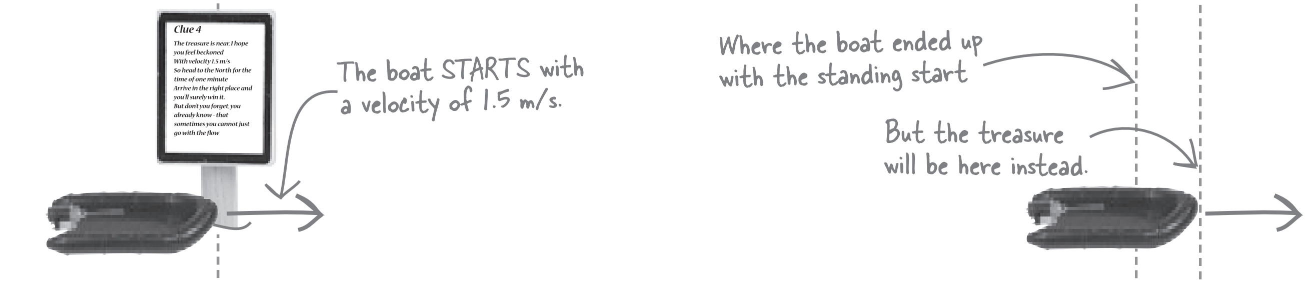
The boat spent part of the minute accelerating from 0 m/s to 1.5 m/s ...



... and the rest of the minute traveling at 1.5 m/s.



If it had spent the WHOLE MINUTE traveling at 1.5 m/s, it would've gone even further.

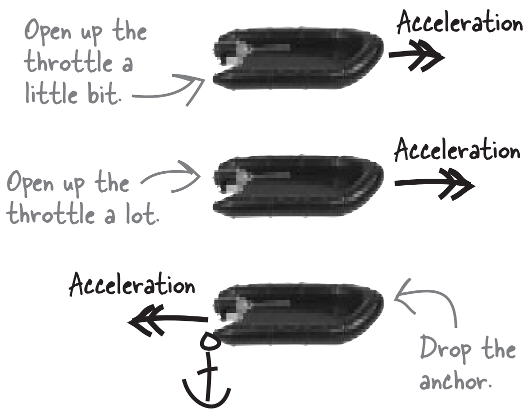


How do you deal with acceleration?

Acceleration is the rate of change of velocity - if something is accelerating, it means that its velocity is changing. Acceleration is a vector with a size and a direction.

If the boat is going forwards, and you open up the throttle, it accelerates in the same direction you're already going in. Its acceleration vector points forwards.

If a boat's going forwards, and you drop the anchor, it decelerates. You can think of this as acceleration in the opposite direction from the one it's already traveling in, so its acceleration vector points backwards.



Acceleration is the rate of change of velocity.

Draw the directions of the velocity vector and acceleration vector for each of these things, showing what they'll look like at the very moment when the acceleration that's described starts to happen.

We gave the acceleration vector a second arrowhead, so you can tell the difference between it and velocity or displacement vectors.

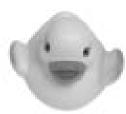
The boat is traveling forwards, and you open up the throttle.

The boat is drifting backwards, and you open up the throttle.

The boat is going forwards, and you drop the anchor.

The boat is going backwards, and you drop the anchor.

The duck is going from right to left, and it just hit the edge.



The duck is going forwards and is pushed by a current coming from the right.

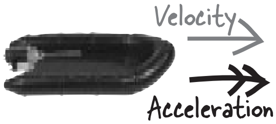
Your right, as you look straight at the duck, that is.

Sharpen your pencil Solution

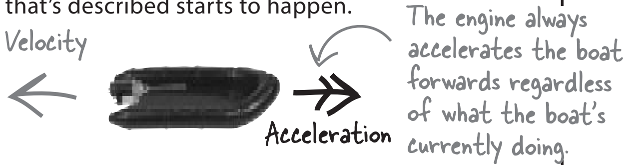
Velocity vector is gray.

Acceleration vector is black with two arrowheads.

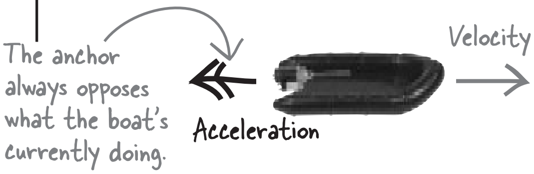
Draw the directions of the velocity vector and acceleration vector for each of these things, showing what they'll look like at the very moment when the acceleration that's described starts to happen.



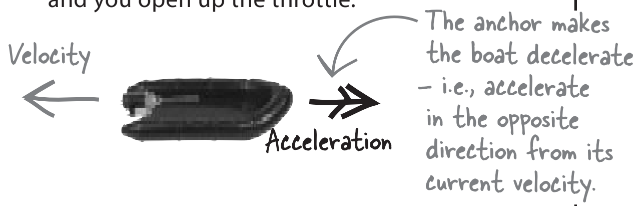
The boat it traveling forwards, and you open up the throttle.



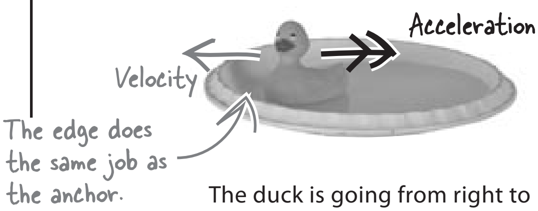
The boat is drifting backwards, and you open up the throttle.



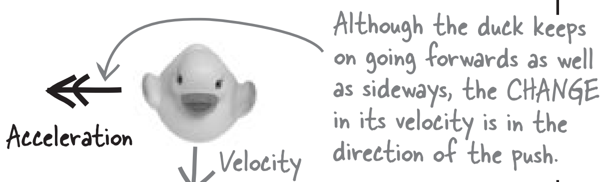
The boat is going forwards, and you drop the anchor.



The boat is going backwards, and you drop the anchor.



The duck is going from right to left, and it has just hit the edge.



The duck is going forwards and is pushed by a current coming from the right.

there are no Dumb Questions

Q: Isn't it confusing to represent the displacement, velocity, and acceleration all with arrows?

A: It's only confusing if you don't label your sketches with what's going on.

Q: How can the anchor make the boat accelerate when it slows it down?

A: The anchor changes the velocity of the boat. Acceleration is rate of change of velocity. So even slowing down is still an acceleration.

Q: But wouldn't that be a DEceleration then?!

A: The 'de' means the acceleration and velocity are in **opposite directions**. So when you do math, the acceleration and velocity vectors will have **opposite signs**.

Q: Ah math, lovely. Shouldn't I know the units of acceleration to do that?

A: Acceleration is the rate of change of velocity. You'll work out the units in chapter 6; right now, we're just dealing with the concept.

So it's back to the boat ...

You told Annie to start at the Clue 4 sign, set the controls for 1.0 m/s, and go North for a minute. But because the boat took time to accelerate from a standing start, it didn't go the whole way at 1.0 m/s, and Annie ended up in the wrong place.

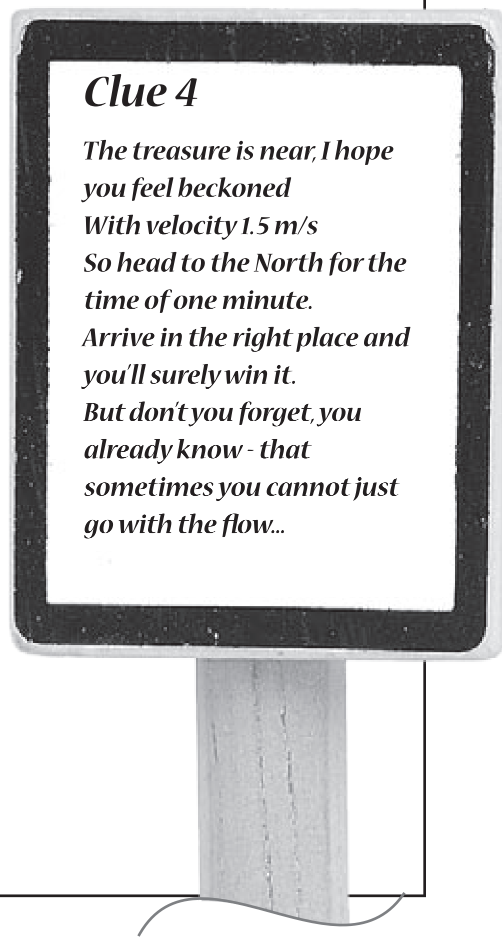
Now that you know that the boat's acceleration could be a problem, how are you going to guide Annie to the treasure?

Sharpen your pencil

Can you think of how to make sure Annie ends up where the treasure is?

Draw / write / explain / calculate in the space below.

We're not telling you which of these you'll have to do – that's up to you! The main thing is to explain your idea as clearly as possible.



Sharpen your pencil Solution

Can you think of how to make sure Annie ends up where the treasure is?

Draw / write / explain / calculate in the space below.

We're not telling you which of these you'll have to do – that's up to you! The main thing is to explain your idea as clearly as possible.

There are TWO ways you could have done this problem.

<u>You can do this by working out the DISPLACEMENT.</u>

The boat needs to travel at 1.5 m/s for 60 s to reach the treasure.

In 1 s, the boat travels 1.5 m.

So in 60 s, the boat travels $1.5 \times 60 = 90$ m

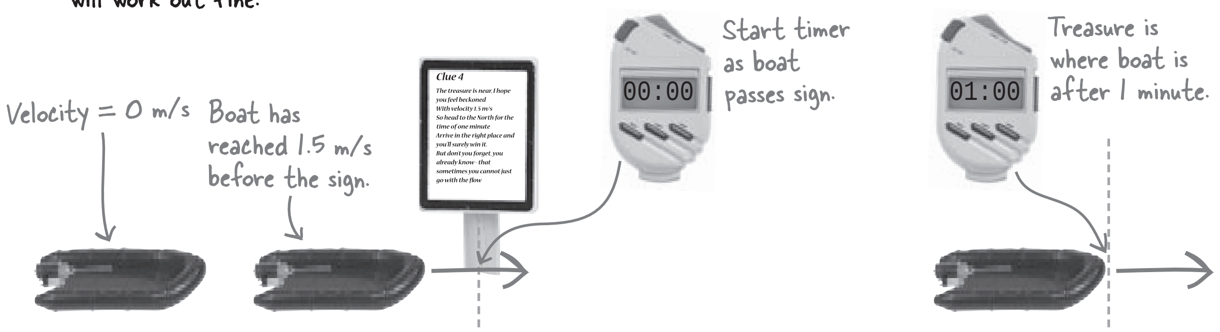
So tell Annie to go <u>90 m North</u> of the clue 4 sign.

The principle for velocity and displacement is exactly the same as for speed and distance.

If you forget a formula, you'll often be able to work it out using common sense, like this. So don't panic!

<u>You can also do this with a 'rolling start.'</u>

If the boat's ALREADY going at the correct velocity when it passes the clue 4 sign, then going for a minute, as per the clue, will work out fine.



That's right - there may be more than one way of doing something.

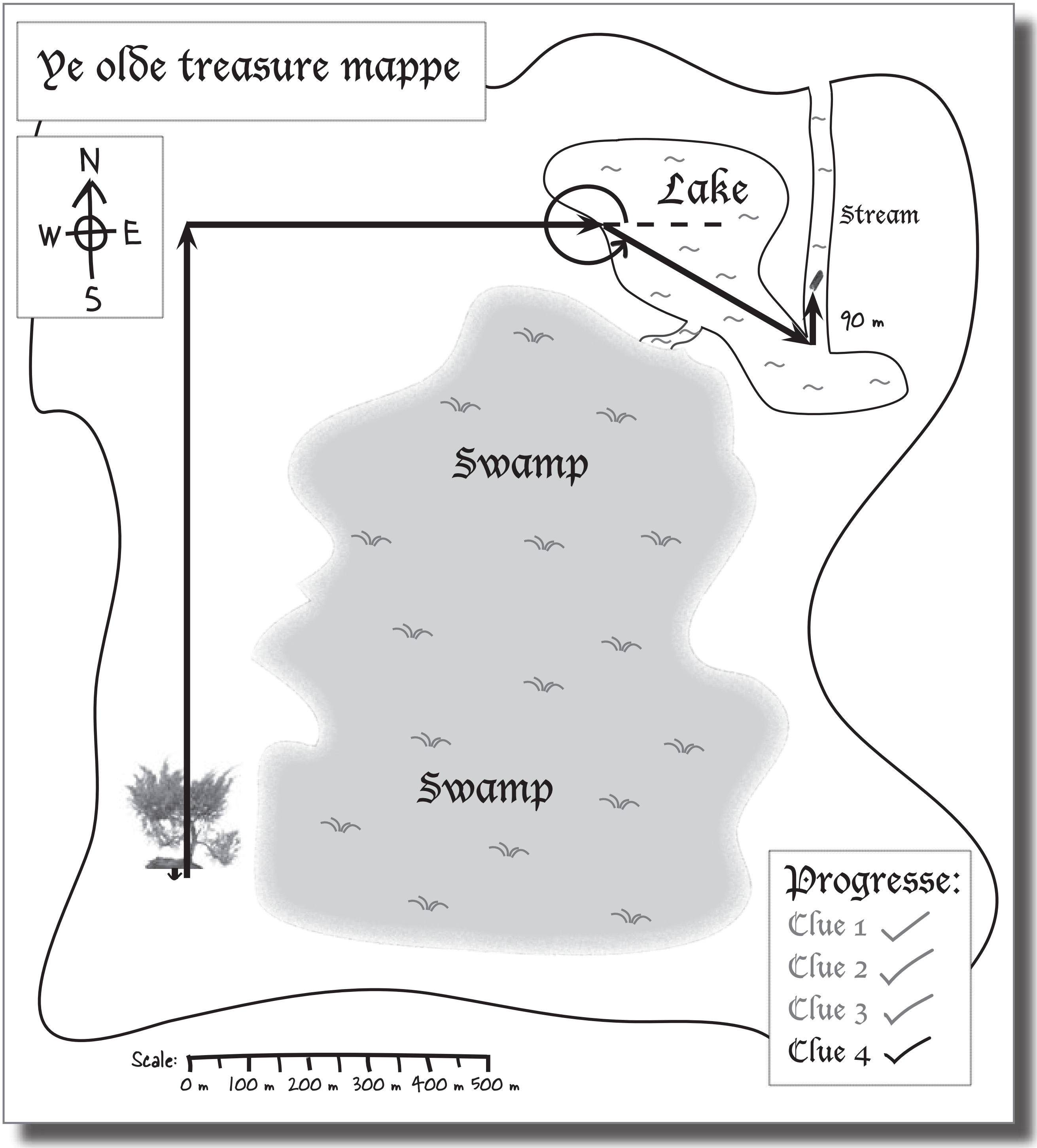
Sometimes there's more than one approach to a problem, and each approach will lead you to the same answer.

This is the case for mathematical problems where there are multiple equations you can use to solve them, or there may be a shortcut you can take that'll save you time.

This is especially true of the problems that ask you to design an experiment and allow you to use a lot of equipment to do it. There will usually be several different set-ups that'll work, and it's up to you to design, draw, and describe what you want to do.

Like you just did here!

Problem Solving 101: Understand WHAT's going on, then work out HOW to do it.


Ye olde treasure mappe
N
W
E
S
Lake
Stream
90 m
Swamp
Swamp
Progresse:
Clue 1
Clue 2
Clue 3
Clue 4
Scale:
0 m 100 m 200 m 300 m 400 m 500 m

Vector, Angle, Velocity, Acceleration = WINNER!!!

Clue 1

Direction is important.

Displacement is the 'vector version' of distance. It has both a size and a direction.

You can represent opposite directions using vector arrows or math signs.

Clue 2

You can add vectors that don't lie along a straight line by drawing them out and adding 'nose-to-tail.'

It doesn't matter which order you add vectors in as long as you line them up 'nose-to-tail.'

Clue 3

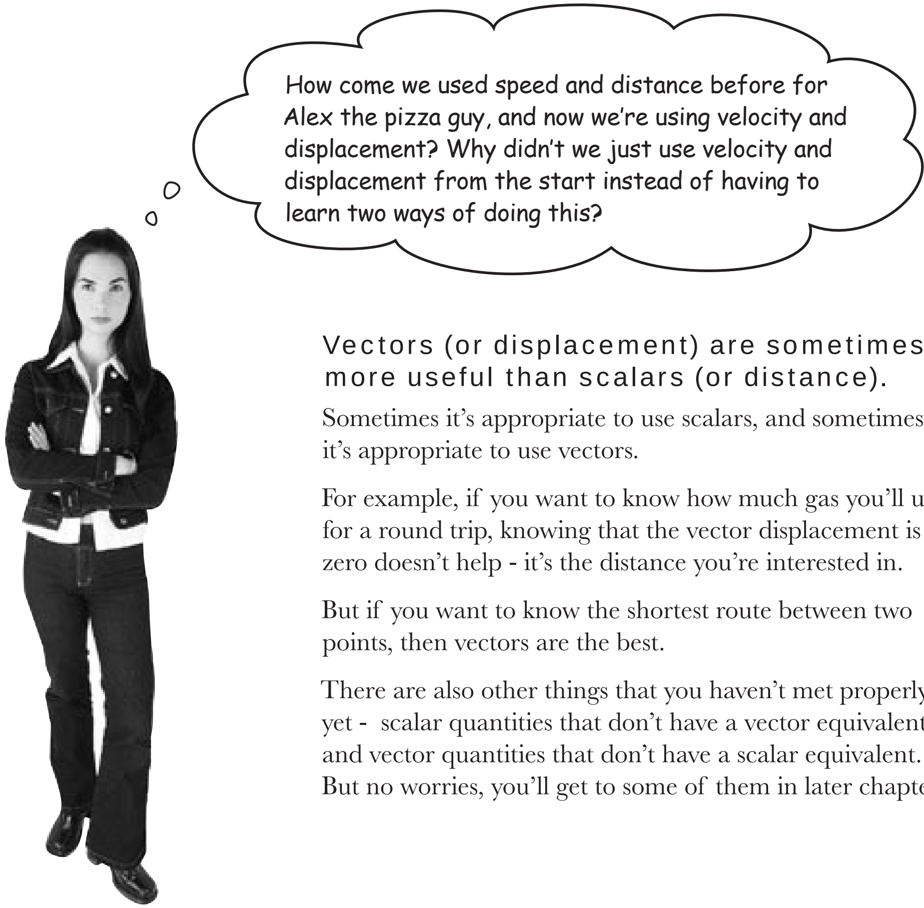
In physics, you measure angles counter-clockwise from the horizontal.

Clue 4

Velocity is the 'vector version' of speed. It has both a size and a direction.

You can add velocity vectors by lining them up 'nose-to-tail.'

Be careful about what a velocity is relative to; for example, a boat's velocity could be relative to the stream or the bank.



Vectors (or displacement) are sometimes more useful than scalars (or distance).

Sometimes it's appropriate to use scalars, and sometimes it's appropriate to use vectors.

For example, if you want to know how much gas you'll use for a round trip, knowing that the vector displacement is zero doesn't help - it's the distance you're interested in.

But if you want to know the shortest route between two points, then vectors are the best.

There are also other things that you haven't met properly yet - scalar quantities that don't have a vector equivalent and vector quantities that don't have a scalar equivalent. But no worries, you'll get to some of them in later chapters.

It's up to you to decide which is best for any situation.

Sometimes it's appropriate to use vectors.
Sometimes it's appropriate to use scalars.

Sharpen your pencil

Here's a map of one of Alex's pizza deliveries from the pizza shop to a customer's house.

a. Draw his route to show his overall distance, and draw a vector arrow to show his overall displacement.

b. Draw vector arrows to represent his velocity at each of the X's on the road.

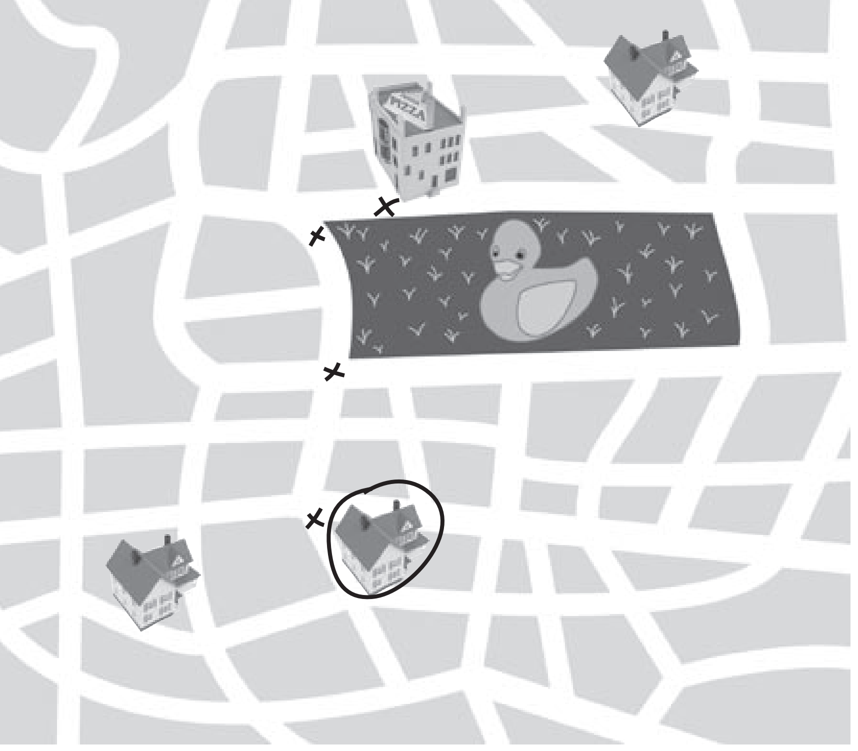
c. Explain why it was more appropriate to use distance and speed to deal with Alex rather than displacement and velocity.

....

....

....

....



Sharpen your pencil Solution

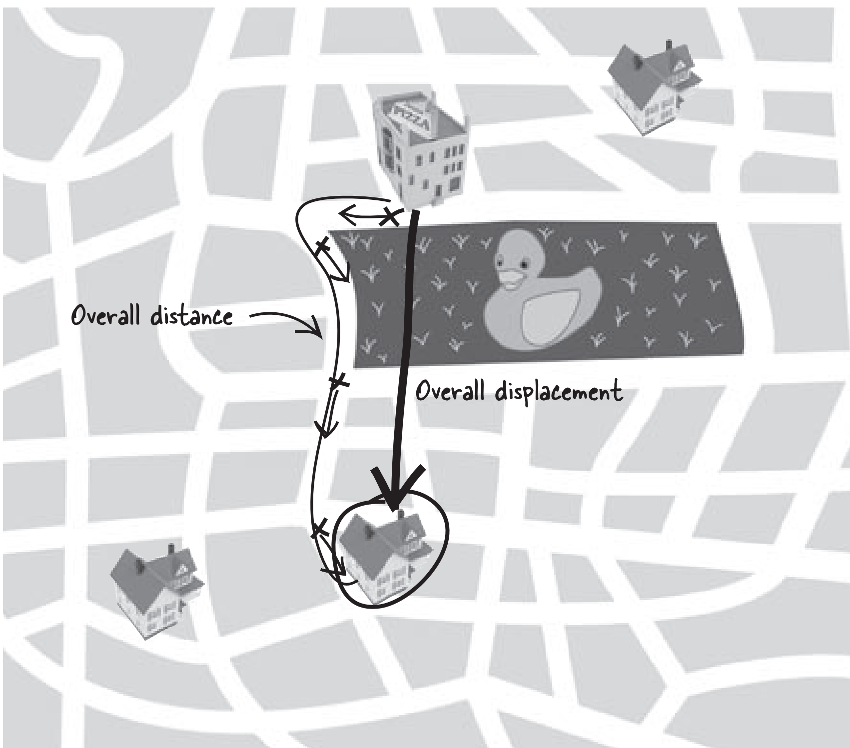
Here's a map of one of Alex's pizza deliveries from chapter 3.

a. Draw his route to show his overall distance, and draw a vector arrow to show his overall displacement.

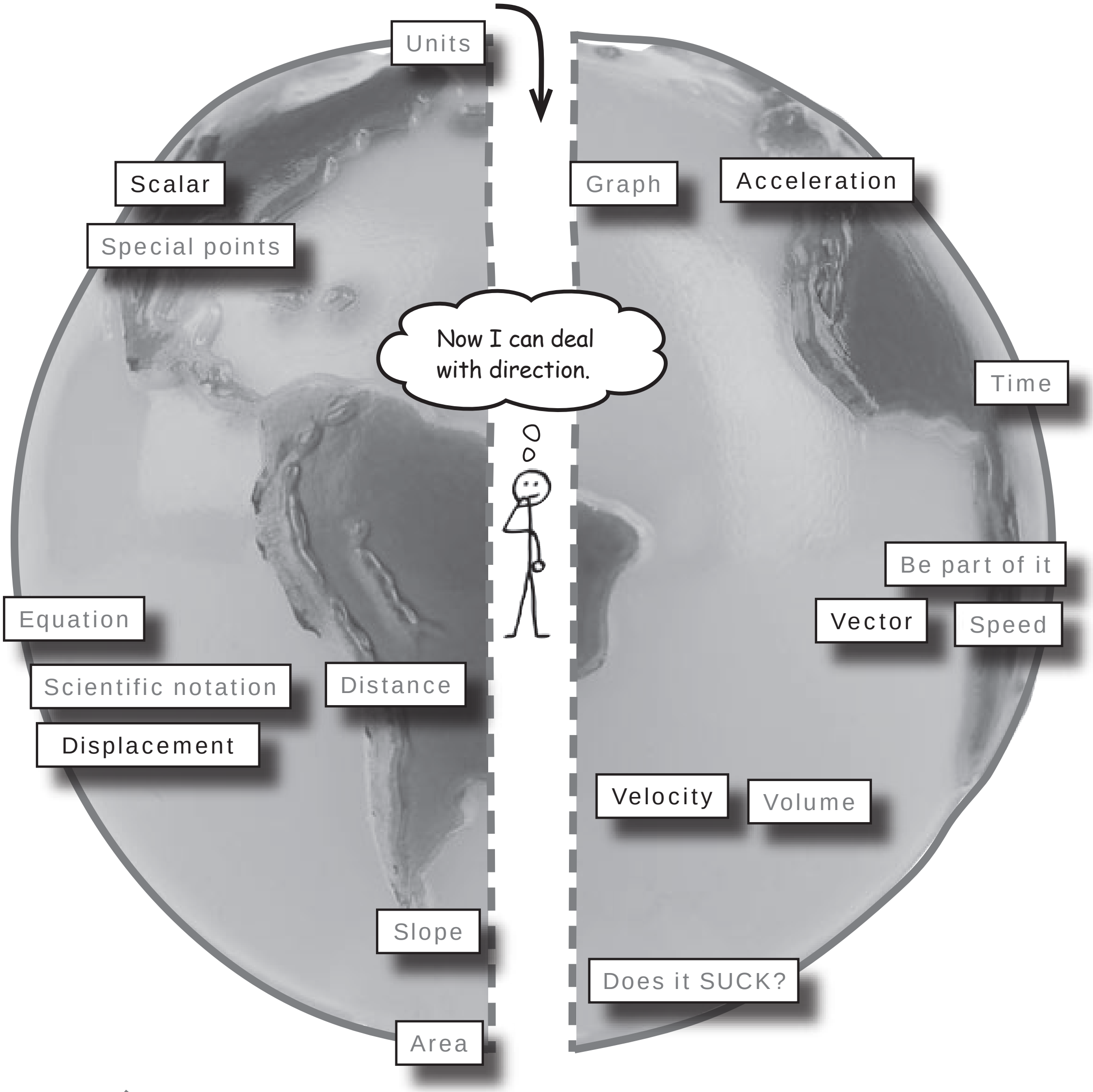
b. Draw vector arrows to represent his velocity at each of the X's on the road.

c. Explain why it was more appropriate to use distance and speed to deal with Alex rather than displacement and velocity.

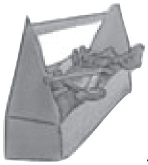
Alex can't go directly through buildings or duckponds works – but the displacement vector does. So it would be silly to talk about displacement instead of distance. And he always covers the distance at the same speed even though the direction of his velocity changes. So there's no point in talking about velocity either.



You only use distance and speed if direction isn't important.



☑	Scalar	A quantity that just has a size.
☑	Vector	A quantity with both a size and a direction.
☑	Displacement	The "vector version" of distance. The change in position between two points.
☑	Velocity	The "vector version" of speed. The rate of change of displacement.
☑	Acceleration	The rate of change of velocity.



Your Physics Toolbox

You've got Chapter 5 under your belt and added some terminology and math skills to your toolbox.

First what, then how

Before you start to work on a problem, work out what it is you're supposed to be doing.

Go through the question and underline the important parts.

Only think about how you'll do the problem once you've worked out what they're looking for.

Start with a sketch

Start every physics problem with a sketch. No, really – start every physics problem with a sketch!

A sketch gathers everything you know into one place in a visual way, so your brain is free to think about physics.

Math with vectors

Add vector arrows by lining them up "nose-to-tail."

OR

If your vectors all lie along the same line, define one direction as positive and the other as negative.

Add the sizes of the vectors, making sure you get the signs correct.

Is direction important?

Ask yourself, "Is direction important?"

Then you'll know whether to use scalars (like distance and speed) or vectors (like displacement and velocity).

Measuring angles

In physics, you measure angles counter-clockwise from the horizontal.

You can measure angles greater than 180° with your protractor either by thinking of how much greater than 180° the angle is, or how much less than 360° it is.

Direction of velocity and acceleration vectors

An object's velocity vector points in the direction it's currently moving in.

An object's acceleration vector points in the direction the velocity is currently changing in. If the velocity is being changed by a push, the acceleration vector points in the direction of the push.

Hey ... I thought physics was supposed to be about **labs**! Like, when do we actually start doing experiments and stuff...

You're right. Experiments are valuable tools for observation.

The next few pages will show you an experimental design set-up…but put yourself in the problem, and try it on your own first. Don't worry, you already know more than you think you do.

Try it

An electromagnet is a magnet that can be switched on and off using electricity.

You have a steel ball-bearing, a tape measure, a timer, and an electromagnet that you can rig to switch off when the timer starts. Your challenge is to design an experiment which will enable you to draw a graph of displacement vs. time for a falling object.

a. List any additional equipment you would like to use.

Don't be afraid to try this! The next few pages will take you through it ... but you already know more than you think you do!

b. Draw and label a diagram of your experimental design.

c. Briefly describe how you would carry out your experiment. You should mention what measurements you will make, and how you will use them to draw graphs that will show you a value for the displacement at any time.

Question Clinic: The "Design an experiment" Question

Physics is based on experimental results, so it's important to be able to design an experiment yourself. There are often several good ways to do this. Some problems will give you a list of equipment, but you may not need to use it all, depending on what you decide to do.

The AP Physics B free response paper ALWAYS includes at least one question that asks you to design or analyze an experiment.

These types of questions always give you a list of available equipment, and may say that additional items are available if you can think of any you'd like to use.

6. You have a steel ball-bearing, a tape measure, a timer and an electromagnet that you can rig to switch off when the timer starts. Your challenge is to design an experiment which will enable you to draw a graph of displacement vs. time for a falling object.

a. List any additional equipment you would like to use.

b. Draw and label a diagram of your experimental design.

c. Briefly describe how you would carry out your experiment. You should mention what measurements you will make, and how you will use them to draw a graph that shows you a value for the displacement at any time.

If you're plotting any graph involving time, then time should be on the horizontal axis.

These types of questions also contain buzzwords that tell you exactly what you're supposed to do. So be careful – if it says 'draw' and you don't, you'll automatically lose points!

When you see a **"design an experiment" question**, make sure you **underline everything you're supposed to do** to separate out the wheat from the chaff and clarify the task in hand.

Look out for **buzzwords** in the question that tell you exactly what you're supposed to do!

Who Does What?

Match each **term** to its description, which says what you **have to do** when you answer the question.

Term

design

describe how you would carry out ...

draw a graph

label

measurement

a value for ...

draw a diagram

Description that says what you have to do

A number with units. The reason you're doing your experiment - you can work it out from your results.

Words that say how you set up your experiment.

Annotation with arrow describing part of a drawing.

Make a picture of how the equipment you're using works together.

Make a plot of one set of measurements against another set of measurements.

Words that say what you do with the equipment you've set up.

A number, with units, that you read from a scale or a meter in the course of your experiment.

WHO DOES WHAT?

Match each **term** with its description, which says what you **have to do** when you answer the question.

Buzzword	Description that says what you have to do
design	A number, with units. The reason you're doing your experiment - you can work it out from your results.
describe how you would carry out ...	Words that say how you set up your experiment.
draw a graph	Annotation with arrow describing part of a drawing.
label	Make a picture of how the equipment you're using works together.
measurement	Make a plot of one set of measurements against another set of measurements.
a value for ...	Words that say what you do with the equipment you've set up.
draw a diagram	A number, with units, that you read from a scale or a meter in the course of your experiment.

First up, you'll find it helpful to jot down what you can **measure** or **do** with the equipment you have available, and write down any relationships between them. Also write down what you're being asked to do!

6. You have a steel ball-bearing, a tape measure, a timer, and an electromagnet that you can rig to switch off when the timer starts. Your challenge is to design an experiment which will enable you to draw a graph of displacement vs. time for a falling object.

Sharpen your pencil

Here are all the pieces of equipment mentioned in the question. Your job is to write down what quantity you can **measure** with it, and what you can **do** with it in the experimental setup.

You should also write down what it is you're being asked to find by doing your experiment, plus any relationships between the **units** of what you can measure and what you're being asked to work out.

Piece of equipment	What can you measure / do with it?
Steel ball-bearing	
Tape measure	
Timer	
Electromagnet	

What I'm supposed to do:

Relationships between what the items in the table can measure and what I'm being asked to work out:

Any other equipment you'll need to be able to do the experiment:

If you have a list of equipment, ask yourself: "What can I DO with this stuff?" and "How can these items work together?"

Sharpen your pencil Solution

You wouldn't draw out a table like this in an exam – just annotate the equipment list you're given!

Here are all the pieces of equipment mentioned in the question. Your job is to write down what quantity you can **measure** with it, and what you can **do** with it in the experimental setup.

You should also write down what it is you're being asked to find by doing your experiment, plus any relationships between the **units** of what you can measure and what you're being asked to work out.

Piece of equipment	What can you measure / do with it?
Steel ball-bearing	I can drop it so that it falls downwards.
Tape measure	I can measure the height that the ball-bearing falls from.
Timer	I can time how long the ball-bearing takes to fall.
Electromagnet	It can hold the ball-bearing then drop it when it switches off. The timer starts when the magnet switches off.

What I'm supposed to do

Draw a graph of displacement vs. time.

Relationships between what the items in the table can measure and what I'm being asked to work out:

I can measure displacement (m) with the tape measure and time (s) with the timer.

Any other equipment you'll need to be able to do the experiment:

Something to hold the electromagnet. And a way to stop the timer when the ball lands.

Look at the UNITS of what you can measure and what you're being asked to work out. How do they relate to each other?

This particular question asks if you'd like to use any other equipment. So do the 'ideal world' test - in an ideal world, what would you need to measure the values as accurately as possible? Then, you actually need to design the experiment in your head before you can describe, draw, or label it!

You need extra equipment to stop the timer and to hold the electromagnet

You can measure displacement and time using the tape measure and timer. And the question says that the electromagnet can be rigged up to release the ball-bearing when the timer starts.

All you need then is to stop the timer when the ball lands - which will need an extra piece of equipment, for instance a switch plate rigged up to stop the timer when the ball lands on it.

Sharpen your pencil

Is the experimental setup you now have in mind similar to what you drew at the start, or is it different?

If it's different, draw and label a diagram of your new experimental setup, and explain how you'll use it to make measurements and draw a graph that shows you a value for the displacement at any time.

You might not have any changes, and if you don't...that's okay too...

Try it

You have a steel ball-bearing, a tape measure, a timer, and an electromagnet that you can rig to switch off when the timer starts. Your challenge is to design an experiment which will enable you to draw a graph of displacement vs. time for a falling object.

a. List any additional equipment you would like to use.

b. Draw and label a diagram of your experimental design.

c. Briefly describe how you would carry out your experiment. You should mention what measurements you will make, and how you will use them to draw graphs that will show you a value for the displacement at any time.

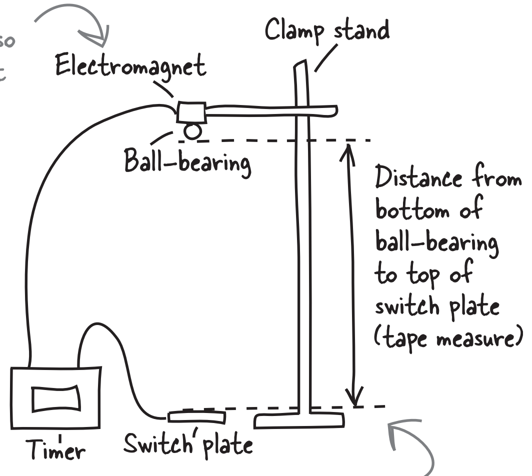
Sharpen your pencil Solution

Is the experimental setup you now have in mind similar to what you drew at the start, or is it different?

If it's different, draw and label a diagram of your new experimental setup, and explain how you'll use it to make measurements and draw a graph that shows you a value for the displacement at any time.

You might not have any changes, and if you don't...that's okay too...

Make sure you include labels so it's clear what everything is.

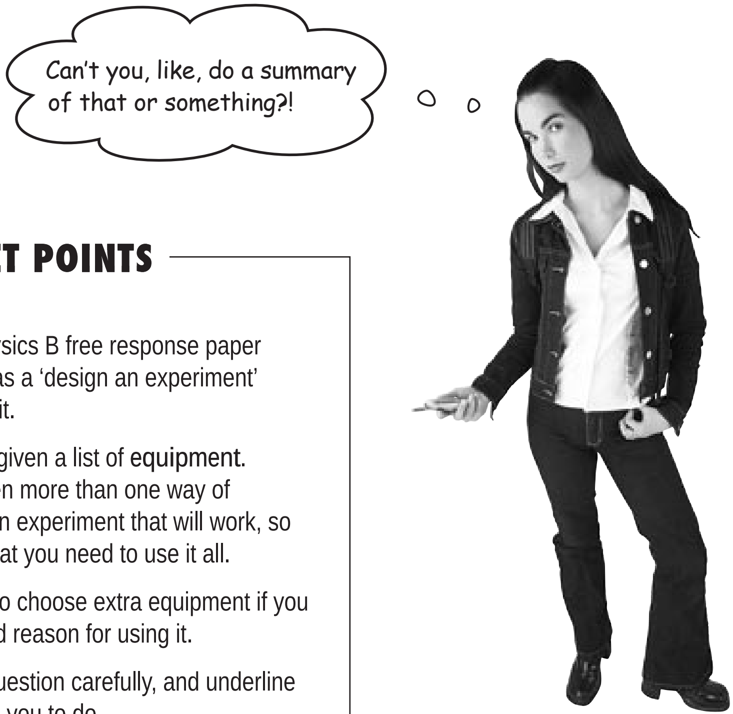


Use the clamp stand and the tape measure to set the height of the ball-bearing. Time how long it takes to fall from that height using the timer, electromagnet and switch plate. Use a range of heights, from the smallest the timer can measure to the height of the ceiling, and several heights in between as well. And time each height two or three times to reduce random errors.

Then, plot a graph with the time along the horizontal axis and the distance up the vertical axis. Draw a smooth line through the data points. The graph lets you read off the time it'll take for the ball-bearing to fall any distance.

Don't spend too much time making your diagram look pretty.

On your graphs, time should always be along the horizontal axis.



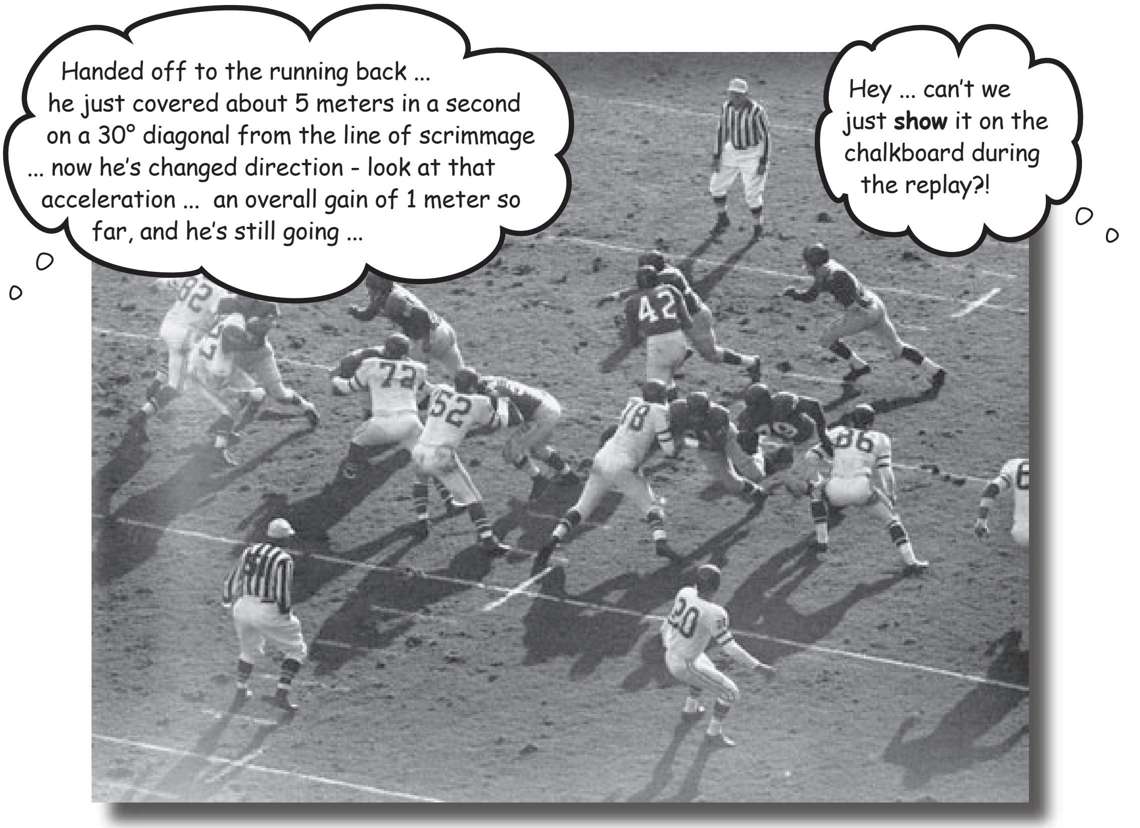
BULLET POINTS

- The AP Physics B free response paper ALWAYS has a 'design an experiment' question in it.
- You will be given a list of **equipment.** There's often more than one way of designing an experiment that will work, so don't feel that you need to use it all.
- You can also choose extra equipment if you have a good reason for using it.
- **Read** the question carefully, and underline what it asks you to do.
- If the question asks you to **design**, **describe**, **draw**, **label** or **explain**, that's what you get points for!
- You may find it useful to jot down what you can **measure** or **do** with each piece of equipment to get your creative juices flowing.
- Remember to explain clearly **what** you'll do in your experiment - **how** you'll use your equipment, and what you'll plot on a graph.
- If you're plotting a graph, **time** always goes along the **horizontal axis.**

Remember to explain clearly what you'll do in your experiment!

6 Displacement, Velocity, and Acceleration

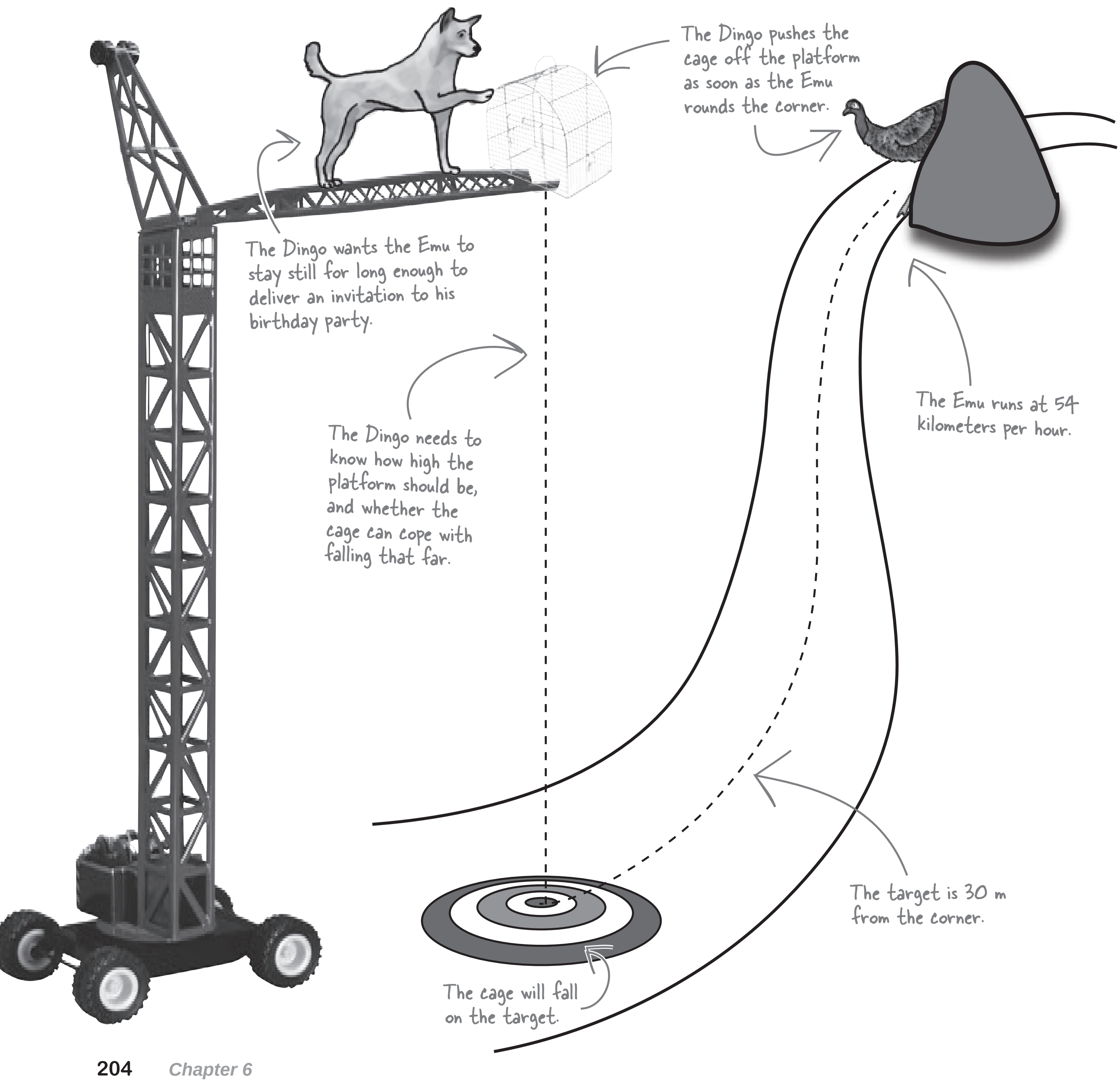
What's going on?



It's hard to keep track of more than one thing at a time.

When something **falls**, its **displacement**, **velocity,** and **acceleration** are all important at the same **time**. So how can you pay attention to all three without missing anything? In this chapter, you'll increase your **experiment**, **graph,** and **slope** superpowers in preparation for bringing everything together with an **equation** or two.

Just another day in the desert ...



... and another Dingo-Emu moment!

Every year it's the same. The Dingo wants to invite the Emu to his birthday party - but the daft bird won't stop running for long enough for him to deliver the invitation. So this year, the Dingo's decided that extending a paw of friendship needs drastic measures. He's hired a crane, and wants to push a cage off the platform the moment the Emu rounds the bend. But is this practical? **What height does the platform need to be, and will the cage be able to handle hitting the ground at a high speed?**

So the Dingo calls the crane company's customer service department to ask some questions ...

Crane Company Magnets

The crane company gets to work on the problem. But we accidentally dropped their memo and some of the words fell off. Your job is to put them back in the right places. You might use some magnets more than once, and some not at all.

Also, **underline the most important parts** in the memo to separate the important stuff from the fluff - the wheat from the chaff.

To: Dingo   Re: Cage

Hmmm, falling cage landing exactly on running ______, tricky! The Emu's ______ is on the computer up there - ______ - and we set up the crane and target ______ past the corner. The ______ falls at the same time as the ______ rounds the corner. If we work out the ______ the cage falls in the ______ it takes the Emu to run ______, we can set the crane to that ______ and take home a fat ______. Be careful - the ______ is only guaranteed if it hits the ground at less than 25 m/s.

30 m

54 m/s

time

height

commission

cage

velocity

Emu

54 km/h

speed

distance

30 km

Crane Company Magnets - Solution

The crane company gets to work on the problem. But we accidentally dropped their memo and some of the words fell off. Your job is to put them back in the right places. You might use some magnets more than once, and some not at all.

Also, **underline the most important parts** in the memo to separate the important stuff from the fluff - the wheat from the chaff.

To: Dingo   Re: Cage

Hmmm, falling cage landing exactly on running Emu, tricky! The Emu's speed is on the computer up there - 54 km/h - and we set up the crane and target 30 m past the corner. The cage falls at the same time as the Emu rounds the corner. If we work out the distance the cage falls in the time it takes the Emu to run 30 m, we can set the crane to that height and take home a fat commission. Be careful - the cage is only guaranteed if it hits the ground at less than 25 m/s.

[30 km]

[54 m/s]

These didn't get used because the units are wrong.

[velocity]

The Emu's speed, rather than his velocity, is important, as the road is curved.

NOTES

- What time does the cage fall for?
- What height should the crane be?
- Will the cage be going faster than 25 m/s when it hits the ground?

BRAIN POWER

Which of these would you try to work out first?

How can you use what you know?

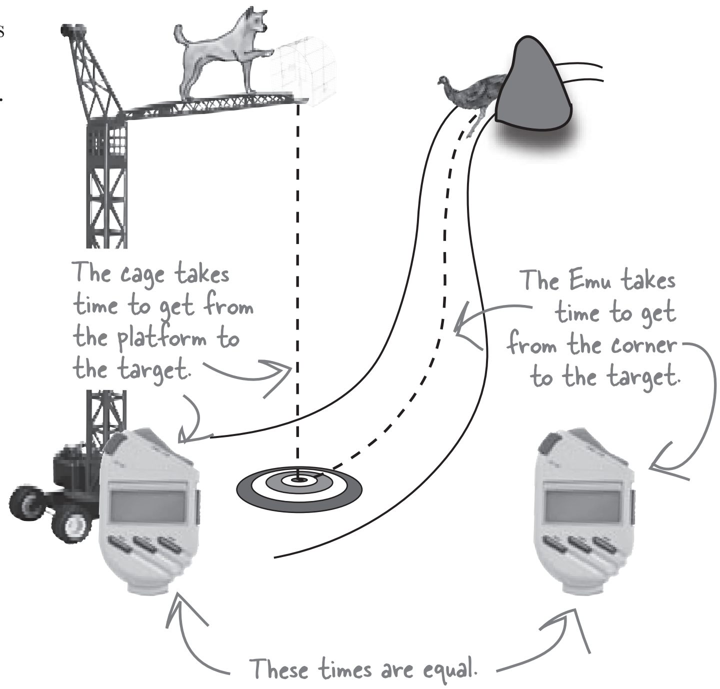
The Dingo drops the cage as soon as the Emu rounds the corner. Then, the cage falling and the Emu running both take the same **time** to reach the target.

The **time** that the Emu takes to arrive depends on the **speed** he runs at and the **distance** he covers from the corner to the target. As the Emu always runs with a **constant speed**, you already know an **equation** you can use to do this.

Once you know the time it takes the Emu to arrive, you'll have to figure out how far the cage **falls** during that time. This will give the Dingo the height that he needs to set the platform at.

However, if the cage travels faster than 25 m/s in the time it takes for the Emu to reach the target, this plan won't work because the cage will hit the ground and be destroyed upon impact.

You haven't dealt with falling things yet – but don't worry, that's what this chapter's about!



Sharpen your pencil

First things first. Work out the time it takes the Emu to cover 30 m from the corner to the target at a speed of 54 km/h.

Hint: You'll need to convert units.

Sharpen your pencil Solution

Work out the time it takes the Emu to cover 30 m from the corner to the target at a speed of 54 km/h.

Convert units: km/h to m/s

$$54 \text{ km/h in m/s} = 54 \frac{\text{km}}{\text{hours}} \times \frac{1000 \text{ m}}{1 \text{ km}} \times \frac{1 \text{ hour}}{60 \text{ mins}} \times \frac{1 \text{ min}}{60 \text{ s}}$$

$$= 15 \text{ m/s}$$

After stringing together conversion factors, you're left with meters on the top and seconds on the bottom – m/s.

If you don't feel so confident about stringing them together, you can do the units conversion one step at a time. That's fine too.

Work out the time it takes:

$$\text{speed} = \frac{\Delta \text{ distance}}{\Delta \text{ time}}$$

Equation comes from the units of speed. Meters per second is a distance divided by a time.

This symbol means 'implies that'. You can use it going from one line to the next as you rearrange an equation.

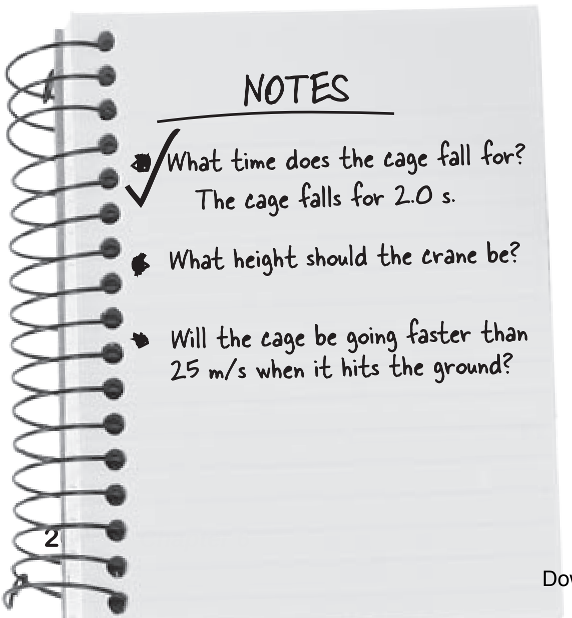
$$\Rightarrow \text{speed } \Delta \text{ time} = \Delta \text{ distance}$$

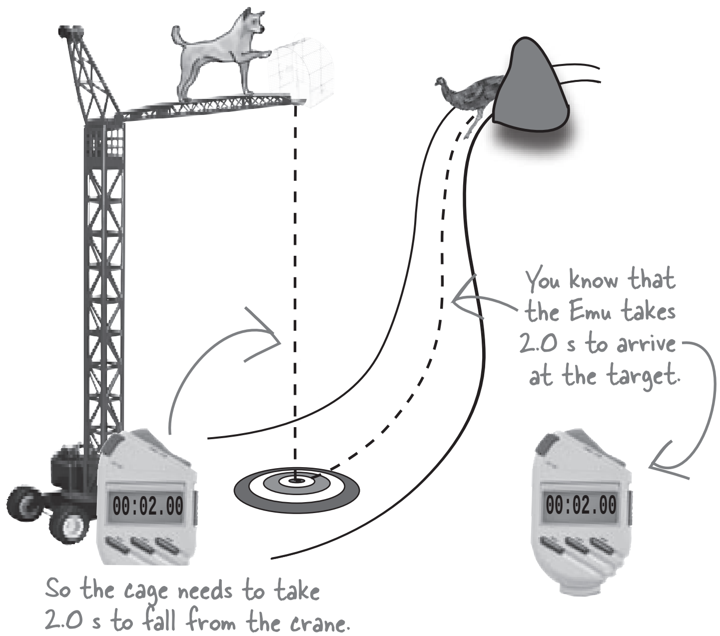
$$\Rightarrow \Delta \text{ time} = \frac{\Delta \text{ distance}}{\text{speed}} = \frac{30 \text{ m}}{15 \text{ m/s}} = \underline{\underline{2.0 \text{ seconds}}} \text{ (2 sd)}$$

Rearrange equation to get Δ time = ...

The problem gave numbers with 2 significant digits to work with, so your answer should have 2 sd.

The Emu takes 2.0 seconds to reach the target - so the cage needs to take 2.0 seconds to reach the target as well.





So I have the time figured out, but uh ... I still don't know how high the crane should be, or how fast the cage is going when it hits the ground. Isn't that the point?

Don't be afraid to start out doing a question, even if you're not quite sure what direction it's going to take.

Don't worry - you've already made progress.

When you started out, you knew a couple of facts about the Emu's speed and the distance he covers - but nothing at all about the cage or the crane platform.

Now we need to figure out how fast the cage is going when it hits the ground after 2.0 s and the distance it falls in that time.

BE the cage

Your job is to imagine that you're the cage. What do you feel at each of the points in the picture? Which direction are you moving in? Are you speeding up or slowing down? Why are you moving like this?

At Point 1:..

....

At Point 2:..

....

At Point 3:..

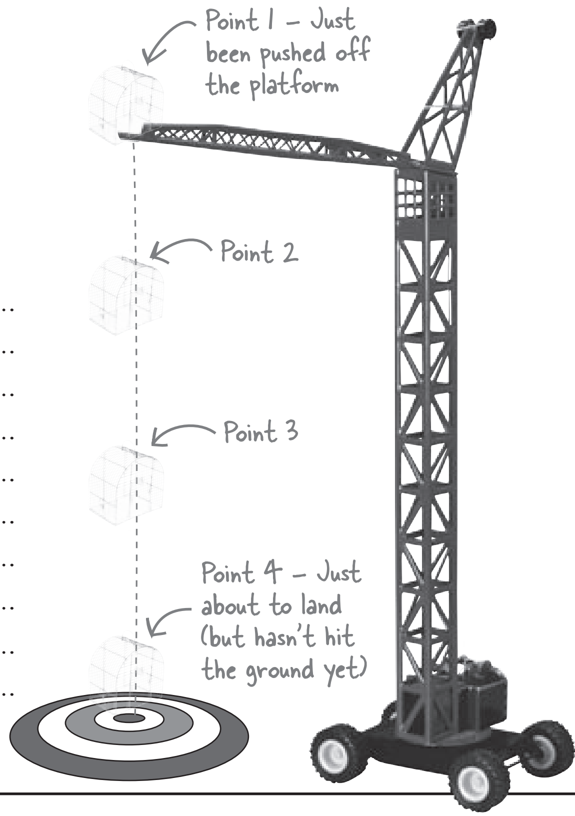
....

At Point 4:..

....

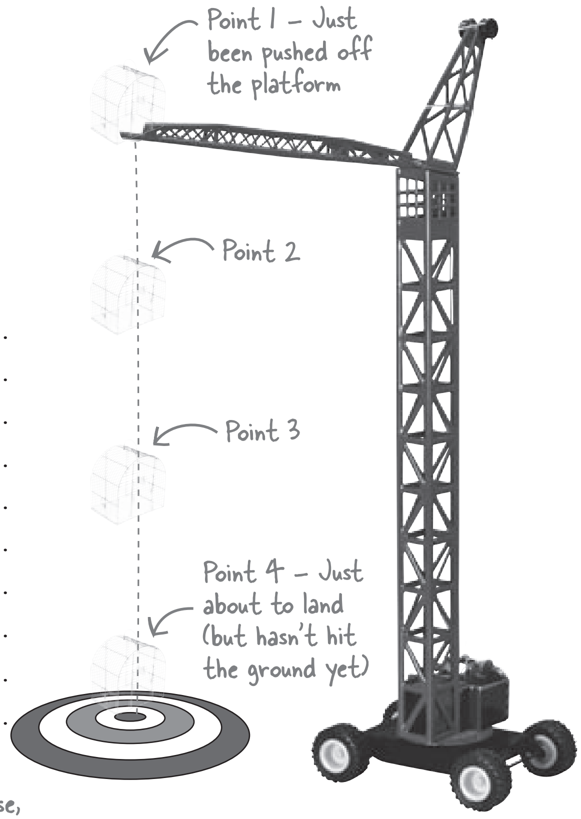
Why:..

....



BE the cage - SOLUTION

Your job is to imagine that you're the cage. What do you feel at each of the points in the picture? Which direction are you moving in? Are you speeding up or slowing down? Why are you moving like this??



At Point 1: A 'special point', as I'm suddenly going from standing still to starting to move downwards.

At Point 2: Falling down faster than I was at point 1.

At Point 3: Falling down even faster than I was at point 2.

At Point 4: This is the fastest I'll be going before I hit the ground (I'll be here after 2.0 seconds if the height is right).

Why: Gravity's accelerating me downwards.

In this problem, we gave you headings to use, but it's always a good idea to make it clear which part of the problem you're answering at each stage!

The cage accelerates as it falls

You've spotted that the cage **accelerates** as it falls. Acceleration is the **rate of change of velocity**. You can tell that the cage is accelerating because its velocity is continually changing. It starts off with zero velocity, then gets faster and faster until it hits the ground.

We're going to talk about the cage's displacement and velocity, as the DIRECTION is starting to become important – the cage isn't being launched up into the air, just dropped!

With that in mind, it's on to working out the cage's **velocity** after 2.0 seconds and its **displacement** in that time so that the Dingo knows whether the idea's a starter - and if so, how high to make the platform.

You know that something is accelerating if its velocity is changing.

Hey ... what's with this talk of displacement and velocity? I was quite happy with distance and speed.

Displacement and velocity will be more useful to you in the long term.

As the cage is always falling in the **same direction** - straight down - you could use either distance and speed or displacement and velocity to describe its motion.

But soon you're going to be dealing with situations where **direction** is crucially important, and you must use vectors. As you practice using displacement, velocity, and acceleration for the cage, you'll soon get comfortable with them, which will stand you in good stead in the future.

Displacement, Velocity, and Acceleration Up Close

Displacement is the 'vector version' of distance and is represented by the letter **x** in equations (or the letter **s** in some courses).

Velocity is rate of change of displacement - the 'vector version' of speed. It is represented by the letter **v** in equations.

Acceleration is rate of change of velocity, represented by **a**, and doesn't have a scalar equivalent. If an object's velocity is changing, you need to know which **direction** the velocity is changing in for the statement to have meaning. Otherwise, you don't know if the object's speeding up, slowing down, or changing direction - which are all ways that an object's velocity can change.

'Vectorize' your equation

You've already used the equation $\text{speed} = \frac{\Delta \text{ distance}}{\Delta \text{ time}}$ to work out that it takes the Emu 2.0 seconds to reach the target.

The 'vector version' of this equation is

$$\text{velocity} = \frac{\Delta\text{displacement}}{\Delta \text{ time}}, \text{ or } \quad \mathbf{v} = \frac{\Delta\mathbf{x}}{\Delta t}$$

Δ means 'change in'

It's fundamentally the same, except that it involves velocity and displacement instead of speed and distance.

$$\mathbf{v} = \frac{\Delta \mathbf{x}}{\Delta t}$$

velocity

change in displacement

change in time

We're using bold letters, like **x** and **v**, to represent vectors and italic letters, like *t*, to represent scalars.

So we need to work out the **displacement** of the cage after 2.0 seconds. That doesn't sound too bad.

Jim: We also need to work out what its **velocity** will be when it hits the ground. If that's more than 25 m/s, then the cage will shatter.

Joe: Why don't we work out the velocity first? That way, if it turns out that the cage is going too fast after it's been falling for 2.0 seconds, we won't have to bother working out the displacement as well.

Frank: Sounds good. I'm all for spotting shortcuts!

Jim: Well, we've done something similar before with that cyclist who rode everywhere at the same speed. Can't we use the **equation** $\mathbf{v} = \frac{\Delta \mathbf{x}}{\Delta t}$ to work out the cage's velocity

Frank: Yeah, let's just use that equation! We want to know the velocity, and that equation says "**v** =" on the left hand side. **v** for velocity. It's perfect!

Joe: Um, I'm not so sure. The cage doesn't have the same velocity all the time - it accelerates as it falls.

Jim: But we can still use that equation, right? If we work out the displacement, we can divide it by the time to get the velocity.

Joe: I don't think so. If the cage always had the same velocity, then, fair enough, that would work. But the cage's velocity is always changing because it's **accelerating** - it isn't **constant**. We want to know what its velocity is at the very end, as it hits the ground.

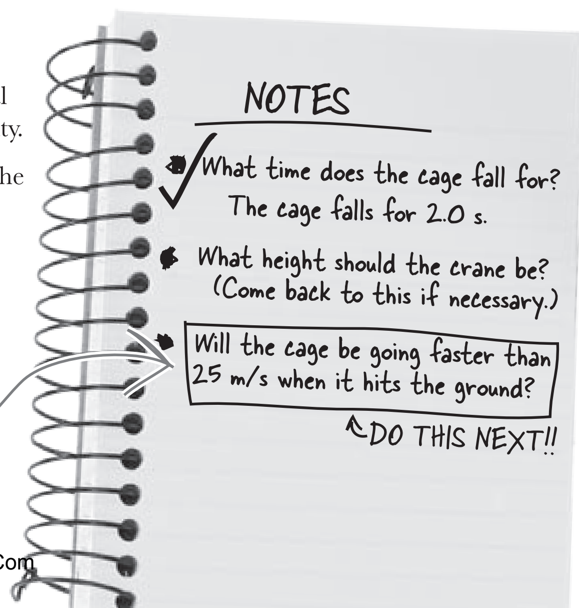
Frank: Oh ... and when it hits the ground, it's only been traveling at that velocity for a split second.

Jim: Yeah, as it gets closer to the ground its velocity increases, so it covers more and more meters per second. If we divided the total displacement by the total time, we'd get the cage's **average** velocity.

Joe: But we need to know what the velocity is the **instant** it hits the ground. An average velocity's no good to us.

Frank: I guess we need to do something different ...

NEVER blindly stick numbers into an equation. Always ask yourself "What does this equation MEAN?"

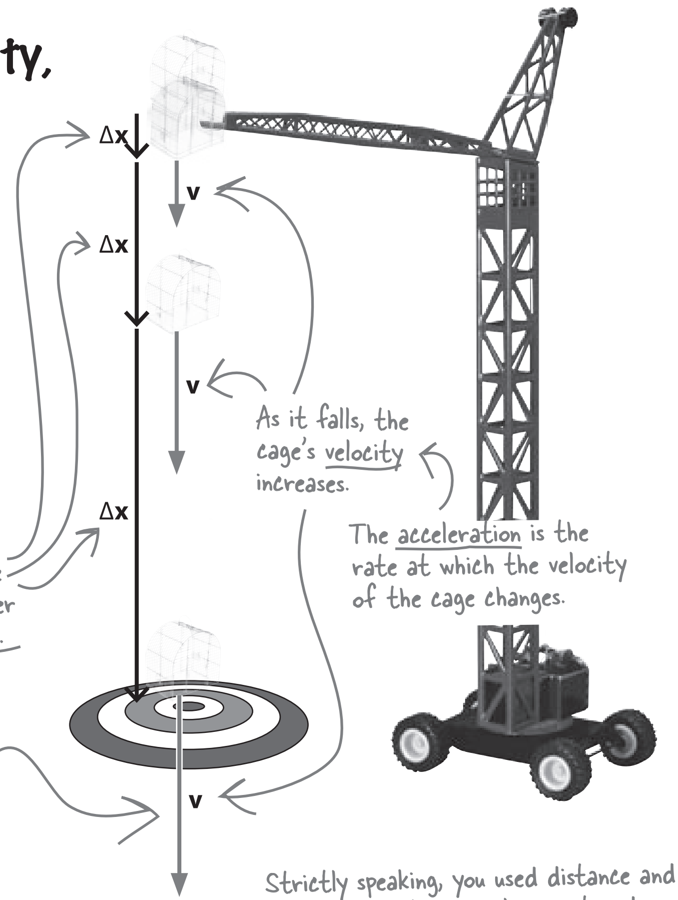


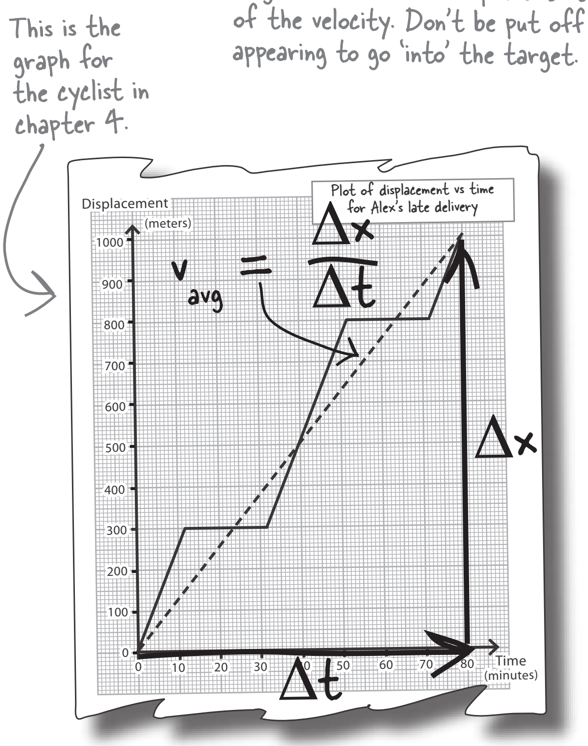
If you calculate the cage's velocity first, you won't have to bother calculating its displacement if it turns out that the cage will break.

You want an instantaneous velocity, not an average velocity

The equation $\mathbf{v} = \frac{\Delta \mathbf{x}}{\Delta t}$ works fine if you have something traveling at a constant velocity. But the cage gets faster and faster as it falls - and you want to know what its velocity is the **instant** it hits the ground.

The best you can do with the equation is to work out the cage's **average** velocity, which is the **constant** velocity it would need to travel with to cover that displacement in that time. But since the cage isn't traveling with a constant velocity, this value won't help you out.

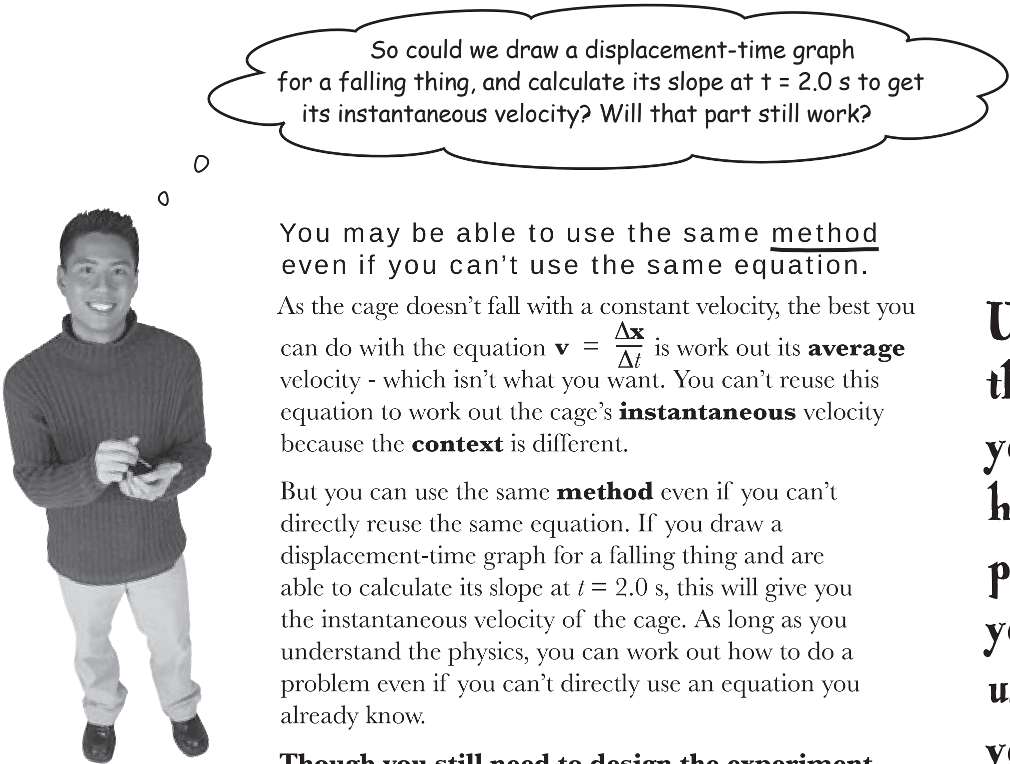




You've previously used the equation $\mathbf{v} = \frac{\Delta \mathbf{x}}{\Delta t}$ to work out the **average** velocity of a cyclist who was slowed down by stop lights, and it gave you the slope of a straight line between the start and end points of his displacement-time graph. Using the **slope** of his displacement-time graph at that point, you were also able to work out his **instantaneous** velocity at any point.

BRAIN POWER

How might you try to work out a value for the **instantaneous** velocity of the cage just before it hits the ground.



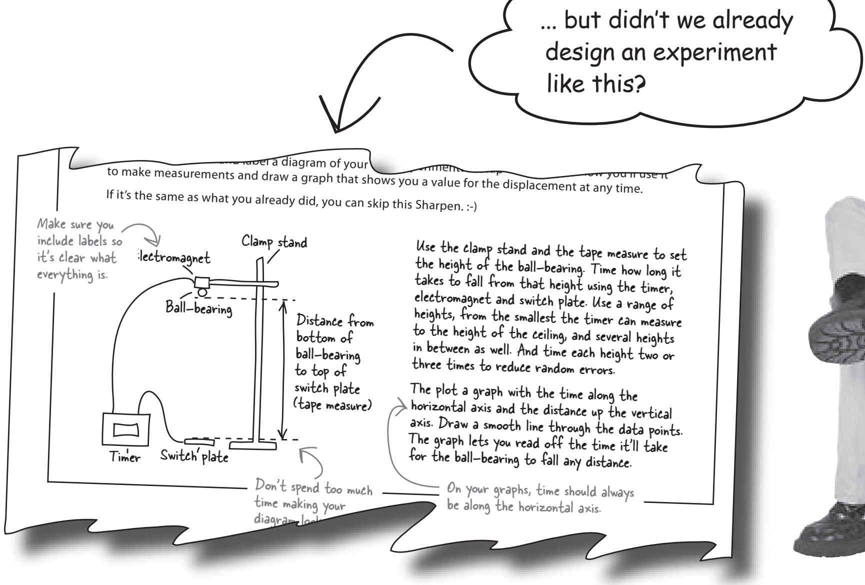
You may be able to use the same method even if you can't use the same equation.

As the cage doesn't fall with a constant velocity, the best you can do with the equation $\mathbf{v} = \frac{\Delta \mathbf{x}}{\Delta t}$ is work out its **average** velocity - which isn't what you want. You can't reuse this equation to work out the cage's **instantaneous** velocity because the **context** is different.

But you can use the same **method** even if you can't directly reuse the same equation. If you draw a displacement-time graph for a falling thing and are able to calculate its slope at $t = 2.0$ s, this will give you the instantaneous velocity of the cage. As long as you understand the physics, you can work out how to do a problem even if you can't directly use an equation you already know.

Though you still need to design the experiment...

Understanding the physics helps you to work out how to solve a problem even if you can't directly use an equation you already know.



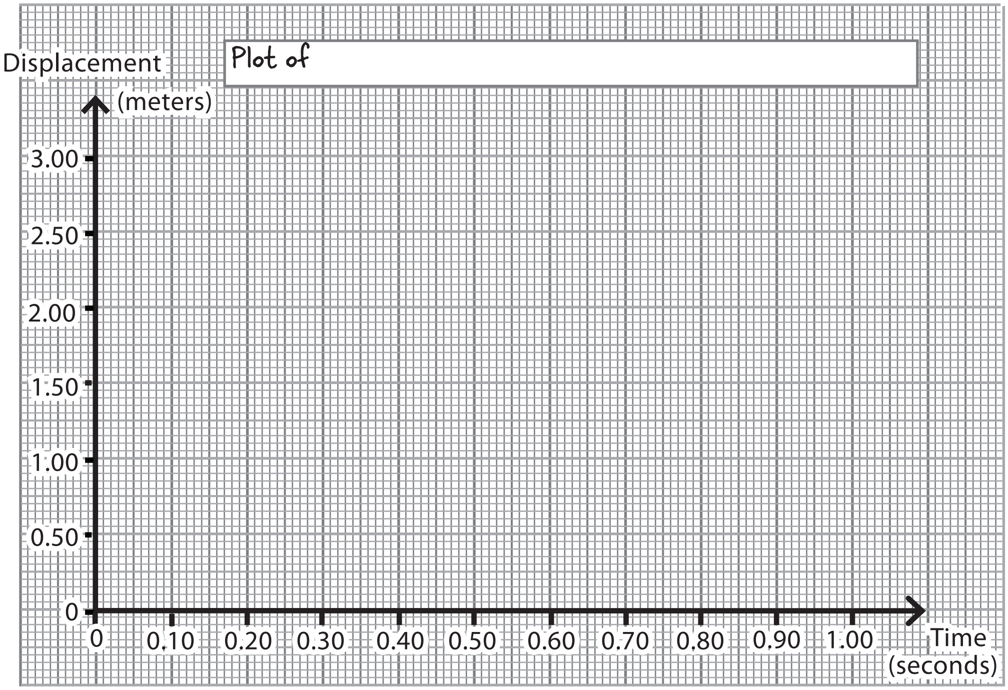
Sharpen your pencil

Displacement of ball (m)	Time 1 (s)	Time 2 (s)
0.10	0.142	0.150
0.25	0.228	0.224
0.50	0.316	0.319
0.75	0.387	0.390
1.00	0.456	0.451
1.50	0.552	0.556
2.00	0.639	0.637
2.50	0.712	0.712
3.00	0.779	0.782

When you carry out the experiment with the falling ball-bearing, electromagnet, and timer that you designed earlier, you get the results shown in the table.

Use these measurements to draw a displacement-time graph for the falling ball-bearing.

(Don't worry about calculating the slope of your graph for now - you'll do that next.)



Sharpen your pencil Solution

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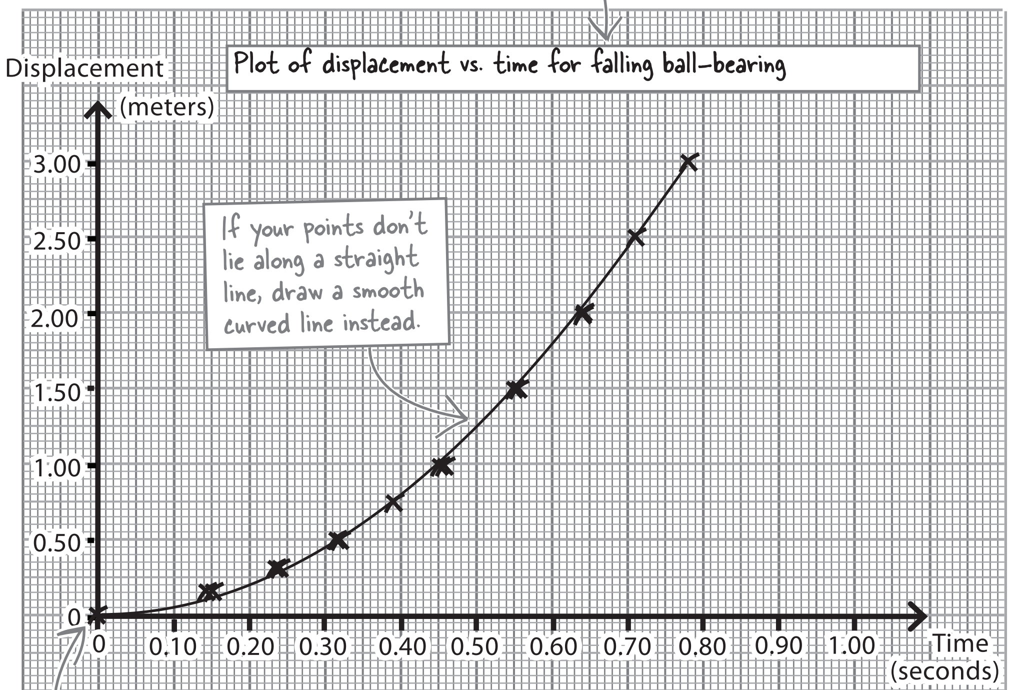
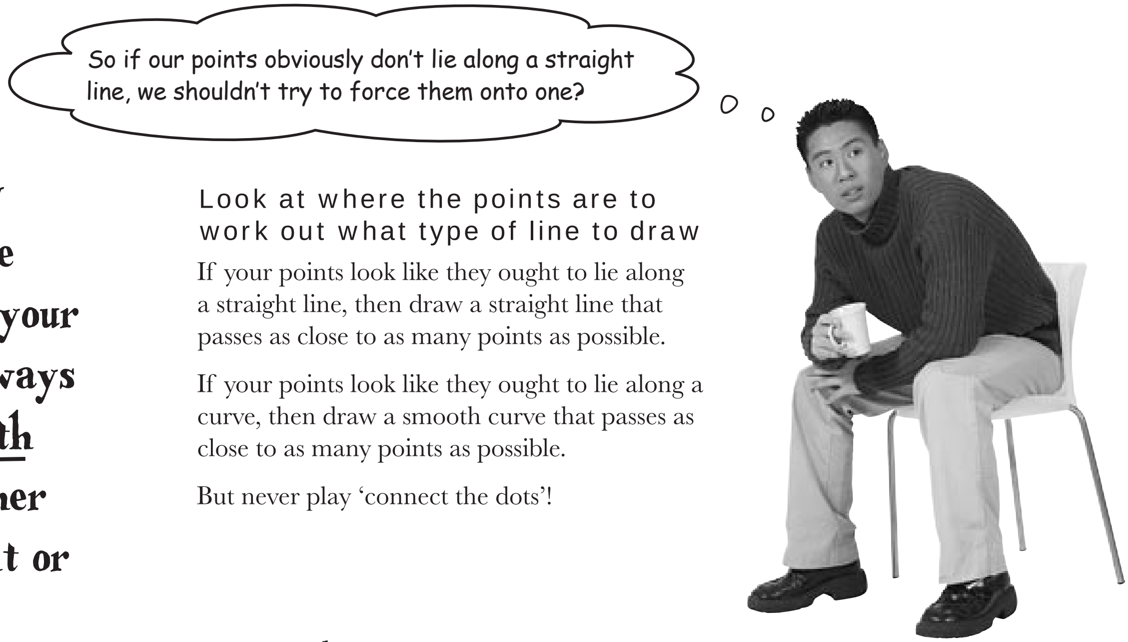
2.50	0.712	0.712
3.00	0.779	0.782

It's important to mention that this object is falling.



Remember to include the point (0, 0) – the ball-bearing has zero displacement at t = 0.



Never play 'connect the dots' with your points. Always use a smooth line (whether it's straight or curved).

Look at where the points are to work out what type of line to draw

If your points look like they ought to lie along a straight line, then draw a straight line that passes as close to as many points as possible.

If your points look like they ought to lie along a curve, then draw a smooth curve that passes as close to as many points as possible.

But never play 'connect the dots'!

there are no Dumb Questions

Q: So ... why am I drawing a displacement-time graph when I want to know the cage's velocity after 2.0 s?

A: You can use the displacement-time graph to get the cage's velocity after 2.0 s.

Q: But what does velocity have to do with displacement?

A: Velocity is rate of change of displacement. That means that the slope at a point on a displacement-time graph is the same as the velocity at that point.

Q: Velocity and displacement are vectors, right? Do I have to write them in bold letters like in the book?

A: Not if you're just handwriting them. We've made the vectors bold so you get used to thinking of them the right way, but you don't have to do that in your solutions.

Q: Why does the slope of a graph matter? How does it help me?

A: The slope of a graph is the change in the vertical direction divided by the change in the horizontal direction. On a displacement-time graph, displacement is on the vertical axis, and time is on the horizontal axis.

So the equation for the slope gives you change in displacement divided by change in time - which is the same as the equation for velocity.

Q: OK, so I see why the displacement-time graph is important. But why haven't I drawn a straight line on it this time?

A: Last time, the points on your graph lay along a straight line. But this time it's obvious that they don't.

Q: So should I mimic a spreadsheet program, using my ruler to draw straight lines from point to point?

A: No, not in physics. The cage doesn't move jerkily from point to point - it moves smoothly. So you should draw a **smooth** line that goes as close to as many of the points as possible.

Q: OK, so the displacement-time graph is curved. I can work out the slope of a straight line graph, it's $\frac{\Delta x}{\Delta t}$.

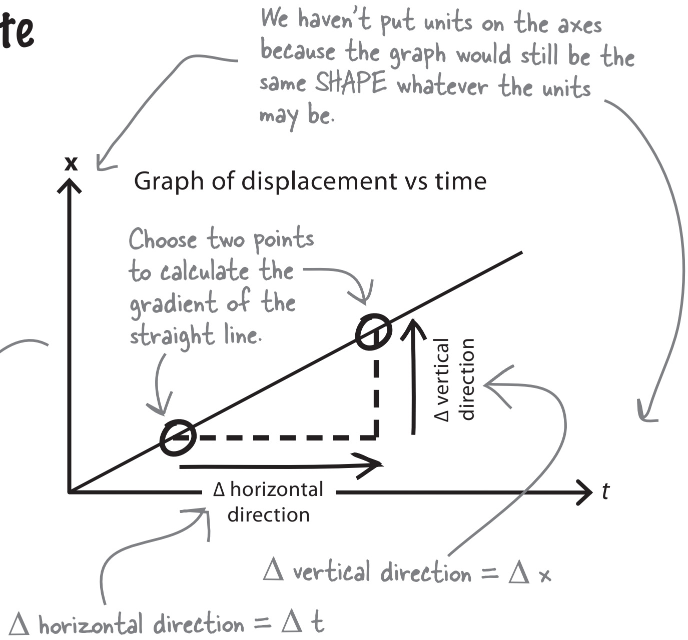
But how can I work out the slope of the curved graph when it won't "sit still" for long enough for me to work out $\frac{\Delta x}{\Delta t}$ for a straight portion?

A: Funny you should ask ...

You already know how to calculate the slope of a straight line...

In Chapter 4, you calculated the slope of a straight line graph by picking two points on it, and calculating $\frac{\Delta \text{ vertical direction}}{\Delta \text{ horizontal direction}}$.

If the graph is plotted with displacement, **x** , on the vertical axis and time, t , on the horizontal axis, this expression for the slope becomes $\frac{\Delta \mathbf{x}}{\Delta t}$, which is the velocity.



$$\text{Slope} = \frac{\Delta \text{ vertical direction}}{\Delta \text{ horizontal direction}}$$

$$\text{Slope} = \frac{\Delta x}{\Delta t}$$

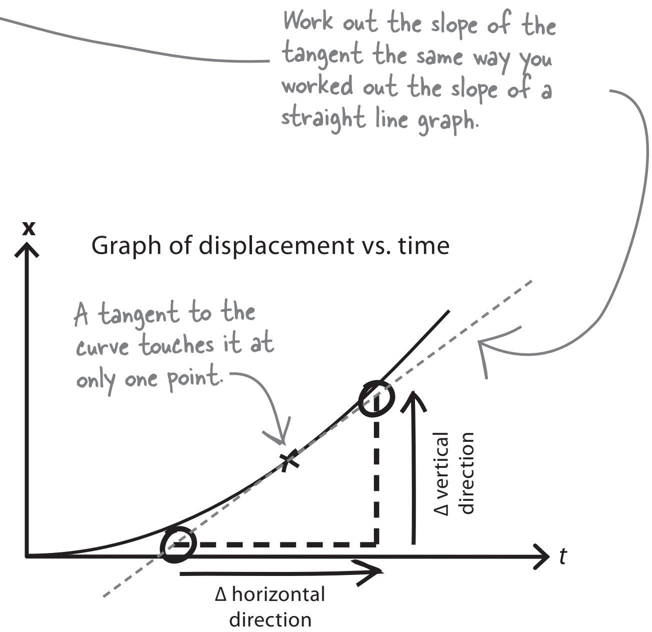
$$\text{Slope} = \text{Velocity}$$

Velocity is rate of change of displacement with time.

A point on a curved line has the same slope as its tangent

You can calculate the slope at a point on a curved graph by drawing a **tangent**. This is a straight line that touches a curve at only that point without crossing the curve.

This means that the tangent has the same slope as the curve at that point - so calculating the slope of the tangent tells you the slope of the curve at that point.



A tangent is a line that touches a curve at one point without crossing it.

there are no Dumb Questions

Q: Isn't a tangent something to do with circles?

A: In this context, a tangent is a straight line that touches a curve at one point but doesn't intersect it. The tangent has the same slope as the curve at that point. You can also talk about a tangent to a circle - a straight line that touches it at one point.

Q: Aren't there tangents in trigonometry as well?

A: 'Tangent' means something different in the context of trigonometry. The definition there is related to this one. You'll learn more about the other meaning in chapter 9.

Q: So here, I can draw a tangent that touches my curve at a point and use it to work out the velocity at that point?

A: Yes - the slope of a displacement-time graph gives you the rate of change of distance with time, which is the same as the velocity.

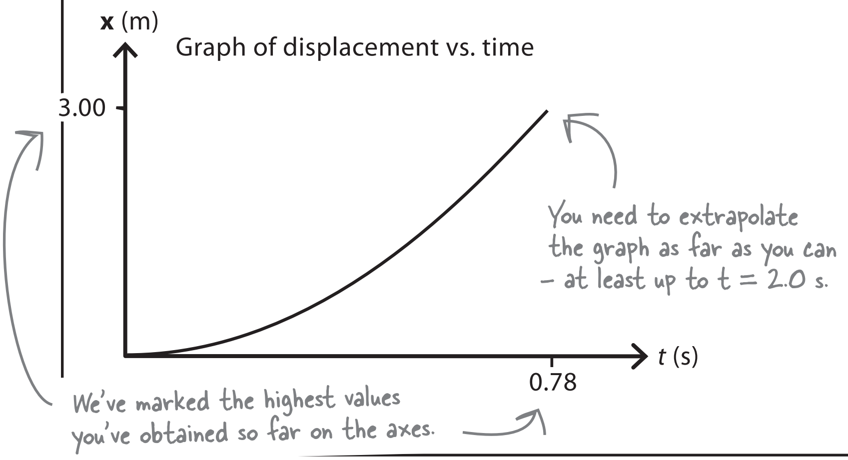
Q: But the graph I've drawn only goes up to 0.78 s, and I want to know the velocity after 2.0 s. Do I have to extrapolate my graph out to 2.0 s or something?

A: Let's try it ...

Sharpen your pencil

When you drew the displacement-time graph for the cyclist, you were able to **extrapolate** it further than the measurements you'd originally made.

Now you can extrapolate your graph for the ball bearing experiment. Your set of measurements goes up to 0.78 s, but you're interested in what's going on after it's been falling for 2.0 s. We've redrawn it to give you more space.



Sharpen your pencil Solution

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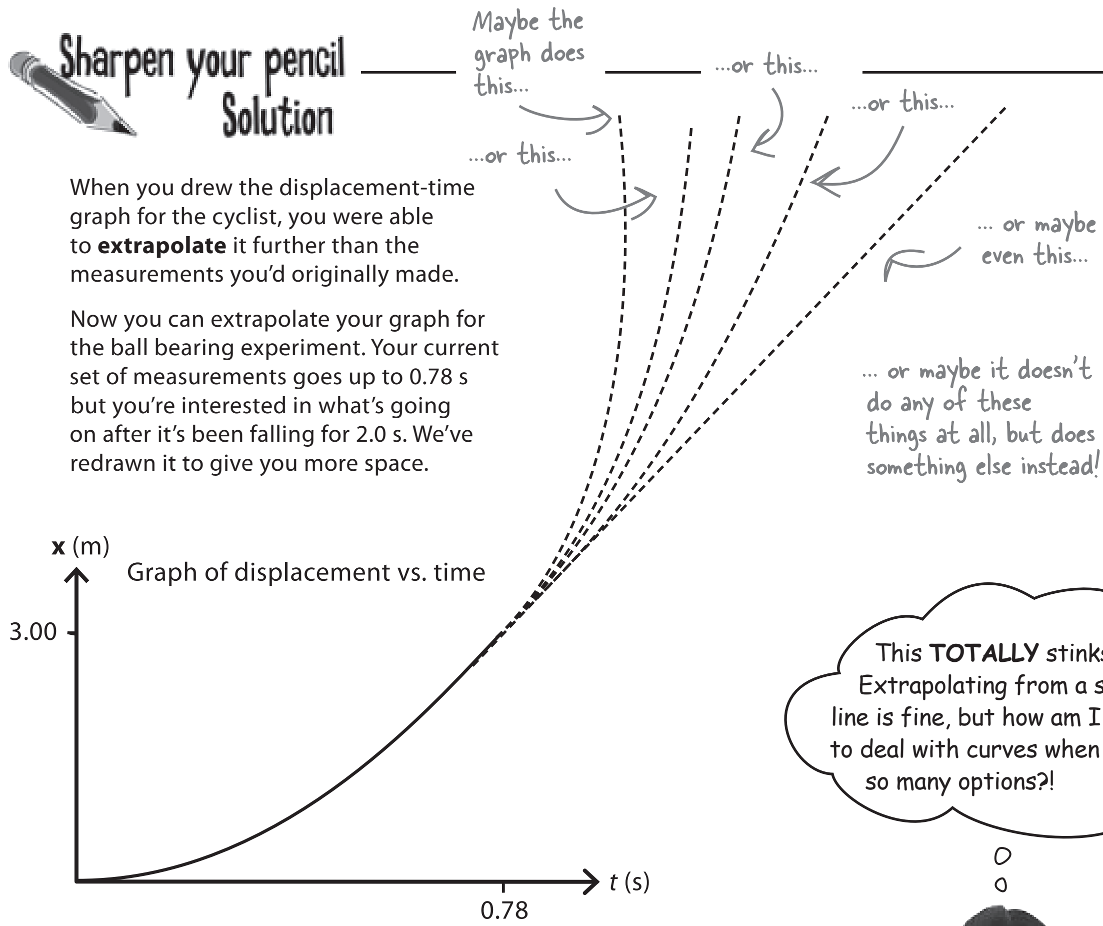
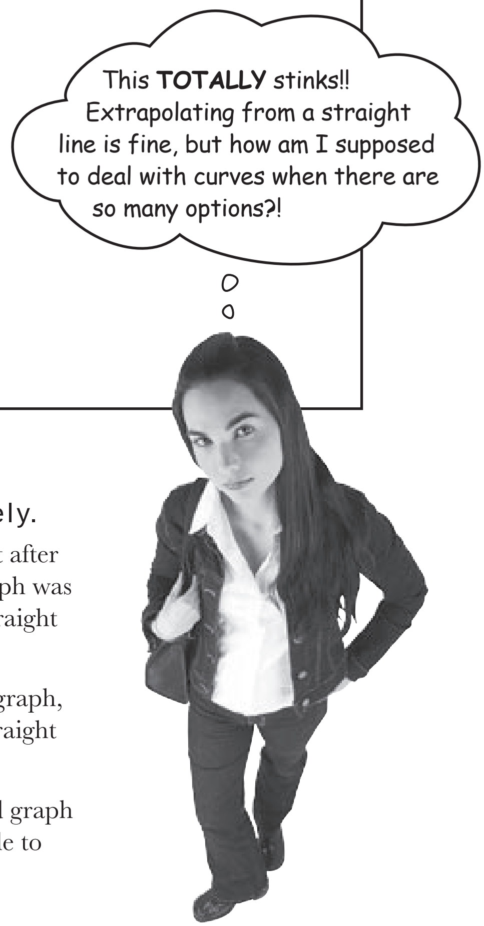




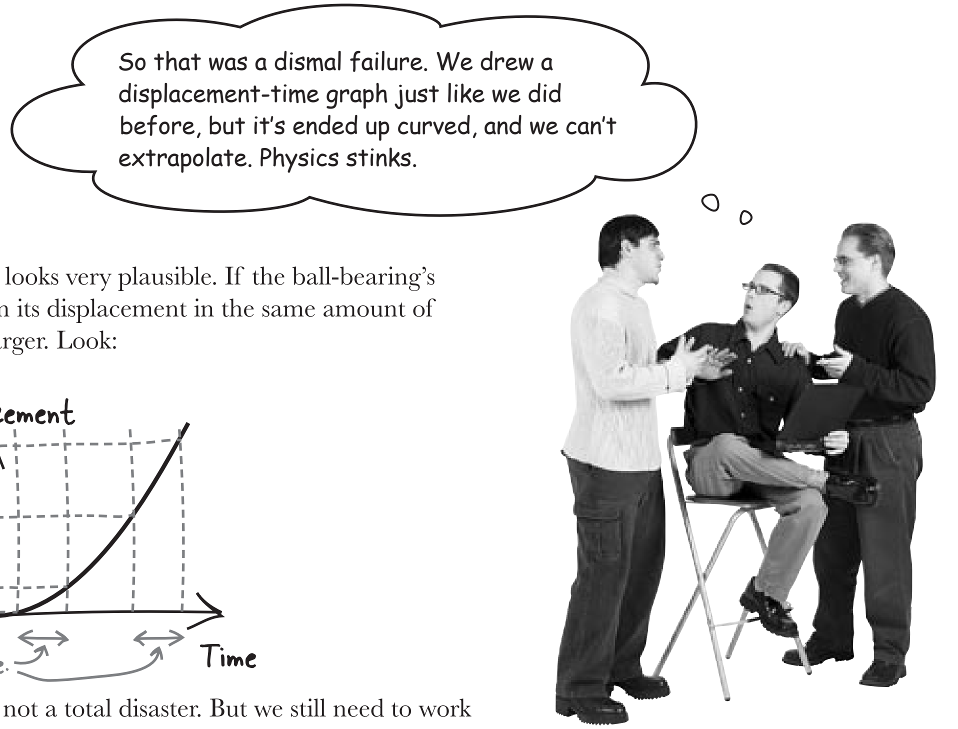
It's only meaningful to extrapolate a graph if its points lie along a straight line.

It's nearly impossible to extrapolate a curve accurately.

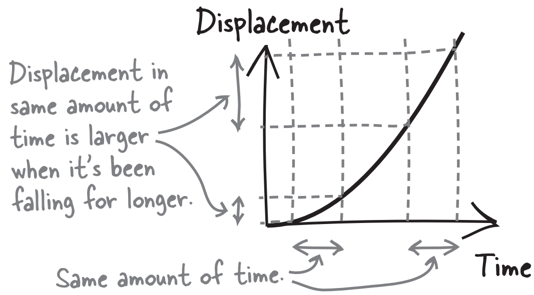
It turns out that this method isn't so hot after all. Drawing the displacement-time graph was fine, but this time, instead of being a straight line, it's a curve.

With a straight line displacement-time graph, its easy to use a ruler to continue the straight line as far as you need to.

But you can't extrapolate from a curved graph in the same way, as it's almost impossible to tell exactly how the curve will continue.



Joe: But our curved graph looks very plausible. If the ball-bearing's getting faster as it falls, then its displacement in the same amount of time will keep on getting larger. Look:



Frank: OK, so maybe it's not a total disaster. But we still need to work out the cage's **velocity** after two seconds. I don't see how we can do that - without dropping the actual cage from a distance high enough to make it fall for 2.0 s. That sounds tough. Even in a room with a 3 meter ceiling, we didn't get the ball-bearing to fall for more than 0.78 seconds.

Joe: We can't keep on dropping the cage - we might break it, which is what we're trying to avoid! Plus there'd be the repair bill for the road.

Jim: Hmmm. When we were drawing a displacement-time graph for a cyclist, we didn't ever need him to ride long distances, just short ones.

Frank: That's because we were able to **extrapolate** his displacement-time graph. But we already said we can't do that here!

Jim: The last time we drew a displacement versus time graph, we calculated its **slope** to work out the pizza guy's **velocity**. And then we used that in an **equation** to work out his time for any distance.

Frank: But here the velocity's changing - it doesn't have one single value. We already said we can't use the equation we worked out then.

Joe: But what if we use the slope of our displacement-time graph at various points to plot a **velocity-time graph**? If it's a nice shape, we might be able to extrapolate it and use it to get the velocity after 2.0 s

Jim: Yeah, if we draw some **tangents** on our displacement - time graph, we could do that. It might just work ...

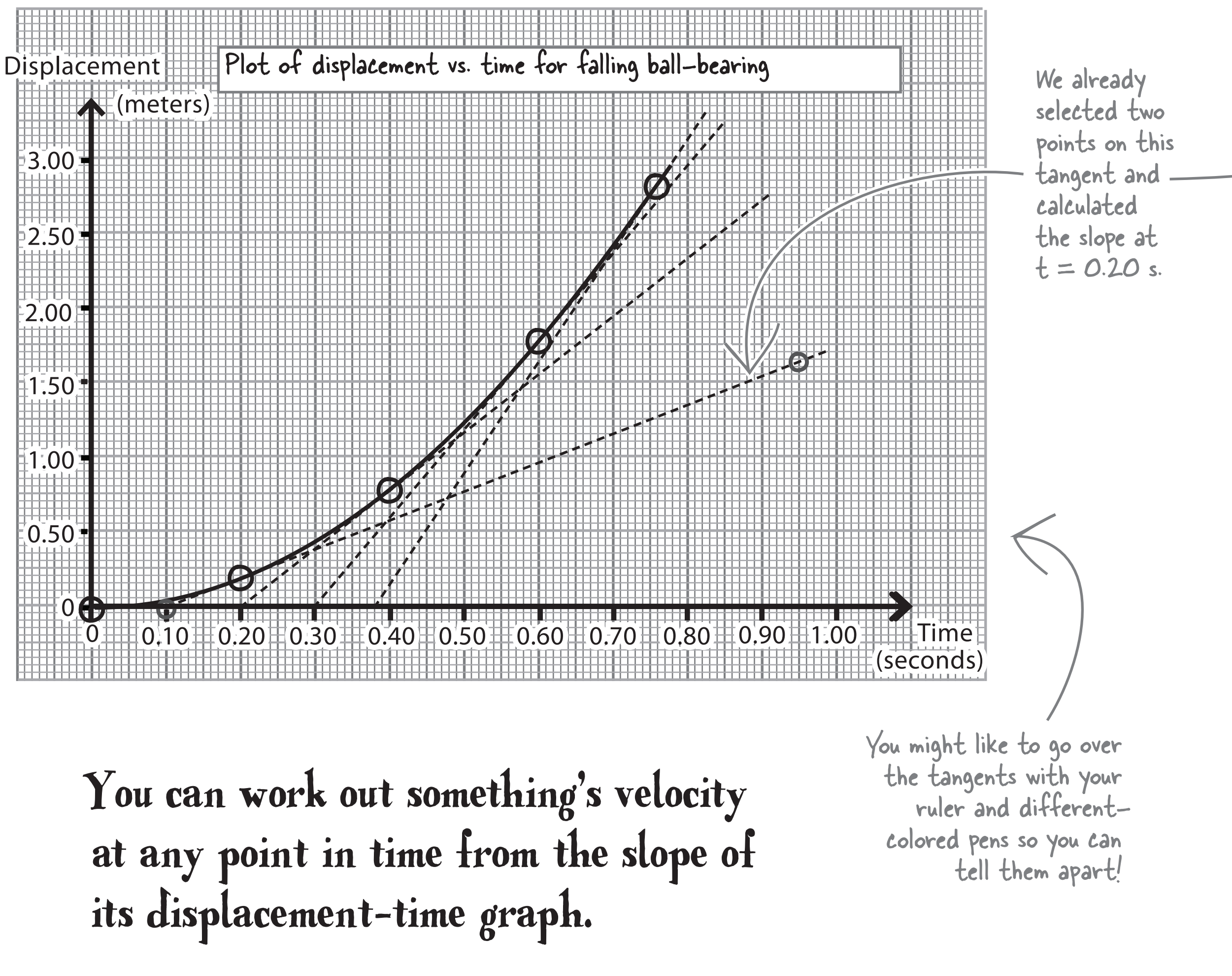
Usually when you do an experiment, you'll use the results to draw a graph - then use the graph to work out an EQUATION.

Sharpen your pencil

You want to plot the velocity-time graph for the ball-bearing. You can get values for its velocity at various points in time from the **slope** of your displacement-time graph at each point in time. As this is a curved graph, we've already selected some regularly-spaced points on it and sketched in their **tangents** for you.

a. Fill in the table by choosing two points on each tangent and working out its slope - and, therefore, the velocity of the ball-bearing at each point.

b. Use the velocities you've calculated to plot the velocity-time graph for the ball-bearing. You'll need to write in all the labels yourself and choose your own scale for the vertical axis.



You can work out something's velocity at any point in time from the slope of its displacement-time graph.

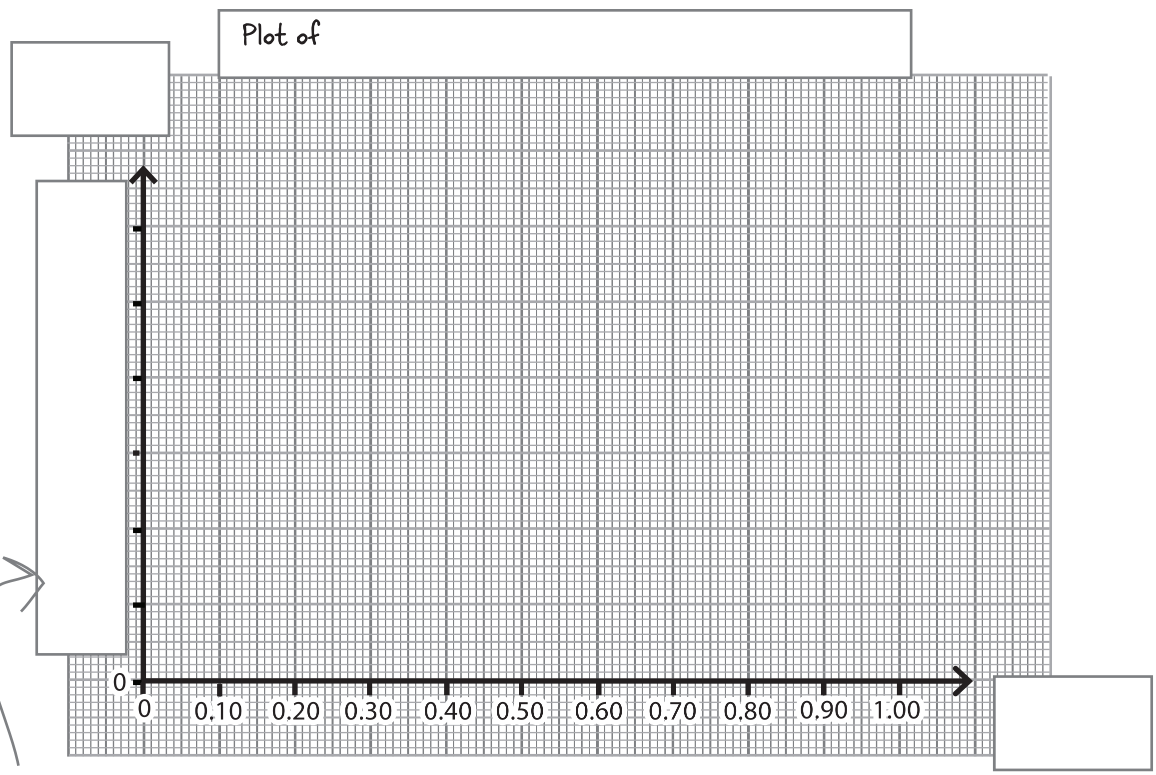
This is Δx between two points you pick on the tangent.

This is Δt between two points you pick on the tangent.

a.

Time at point (s)	Δx (m)	Δt (s)	Velocity = $\frac{\Delta x}{\Delta t}$ (m/s)
0.00			
0.20	1.65 - 0.00 = 1.65	0.95 - 0.10 = 0.85	$\frac{1.65}{0.85}$ = 1.94 (3 sd)
0.40			
0.60			
0.76			

b.

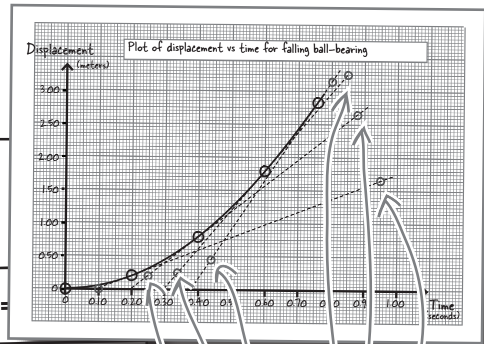


You'll need to pick a suitable scale for your vertical axis.

Sharpen your pencil Solution

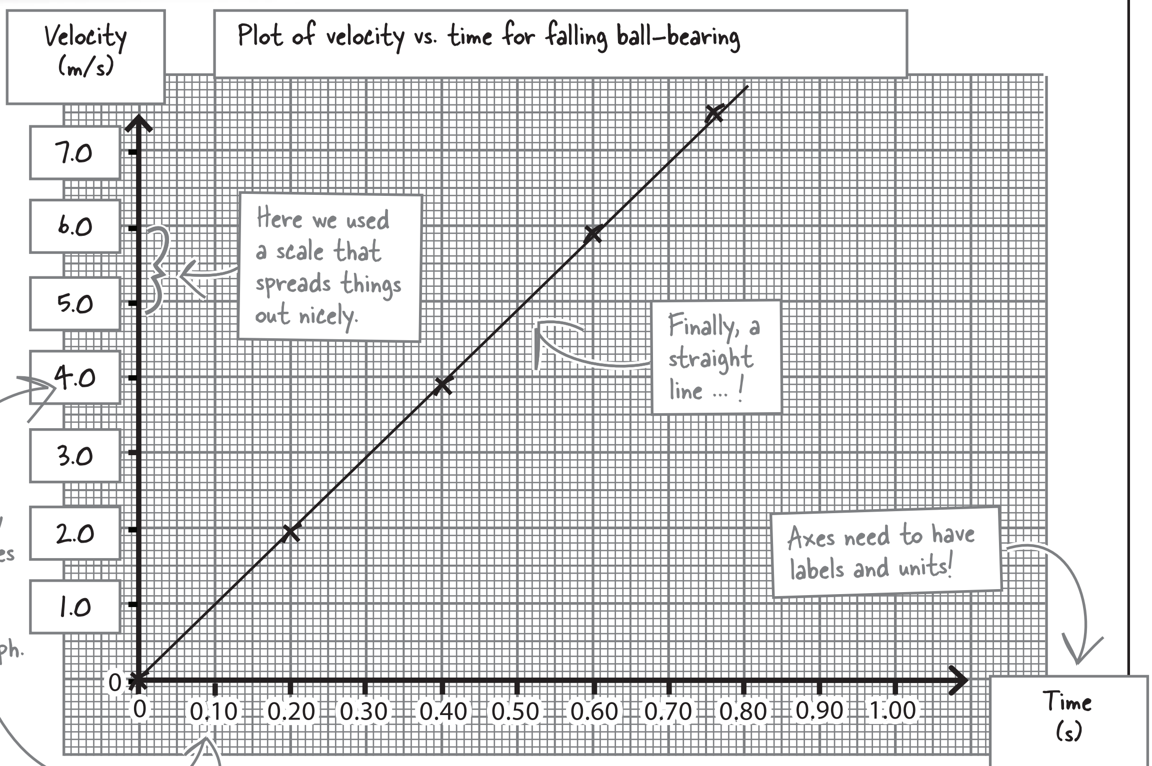
a.

Time at point (s)	Δx (m)	Δ*t* (s)	Velocity =
0.00	0	1	$\frac{0}{1} = 0$
0.20	1.65 - 0.00 = 1.65	0.95 - 0.10 = 0.85	$\frac{1.65}{0.85}$ = 1.94 (3 sd)
0.40	2.65 – 0.20 = 2.45	0.88 – 0.25 = 0.63	$\frac{2.45}{0.63}$ = 3.89 (3 sd)
0.60	3.25 – 0.25 = 3.00	0.85 – 0.34 = 0.51	$\frac{3.00}{0.51}$ = 5.88 (3 sd)
0.76	3.15 – 0.45 = 2.70	0.80 – 0.44 = 0.36	$\frac{2.70}{0.36}$ = 7.50 (3 sd)

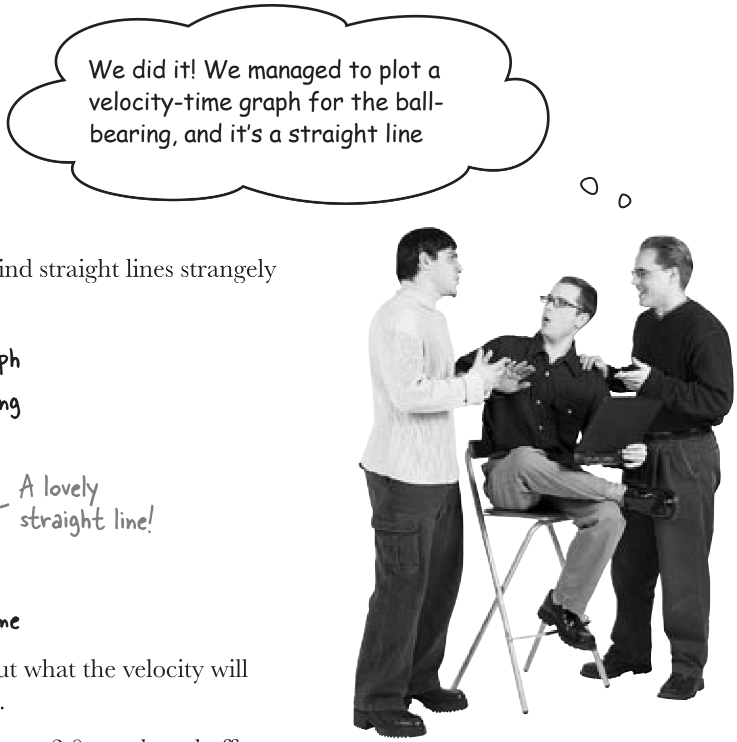


These are the points we chose to use. If you chose slightly different points or got slightly different values for the velocities, then don't worry as long as you were close!

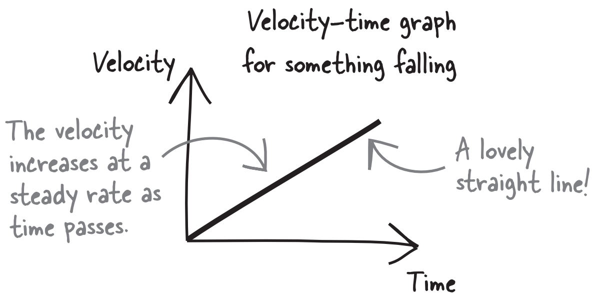
b.



Only give as many significant figures on your scale as you are able to plot on your graph.



Jim: That looks, hmmm, nice. I'm starting to find straight lines strangely comforting, like they're somehow meant to be.



Frank: But what now? We still need to work out what the velocity will be after 2.0 s. Our graph only goes up to 0.78 s.

Jim: We can always **extrapolate** the graph out to 2.0 s and read off the velocity. It's OK to extrapolate straight line graphs!

Joe: Yeah ... but it would be kinda nice to come up with an **equation** if we can, so we can quickly work out the velocity at any time. Like, what if the Dingo wants to put the crane somewhere else?

Frank: With the cyclist, we used the slope of his displacement-time graph to work out the equation $\mathbf{v} = \frac{\Delta \mathbf{x}}{\Delta t}$.

Jim: The cyclist's displacement-time graph was a lovely straight line, like this one. Except this is a velocity-time graph. I wonder if we can use it to work out an equation in a similar way.

Joe: Well, the slope of the velocity-time graph will be $\frac{\Delta \mathbf{v}}{\Delta t}$ because velocity is on the vertical axis this time. So the slope would be the rate of change of velocity ...

Frank: Hey! Didn't we say before that the acceleration is the rate of change of velocity? So we can use the slope to get the equation $\mathbf{a} = \frac{\Delta \mathbf{v}}{\Delta t}$.

Jim: Ooh - because it's a lovely straight line graph, the slope is constant. So the acceleration must be constant. Which means we can use the graph to calculate the acceleration - then use the acceleration to work out the velocity after any time!

The slope of a 'something'-time graph tells you the rate at which the 'something' changes with time.

Velocity is rate of change of displacement.

$$v = \frac{\Delta x}{\Delta t}$$

Acceleration is rate of change of velocity.

$$a = \frac{\Delta v}{\Delta t}$$

The slope of something's velocity-time graph lets you work out its acceleration

Acceleration is rate of change of velocity with time. So the slope of a velocity-time graph is the acceleration.

The pizza guy goes at a **constant velocity**, so his displacement-time graph has a **constant slope**. His velocity-time graph is a flat line, as his velocity is constant at all times. The slope of his velocity-time graph is zero, so his acceleration is zero.

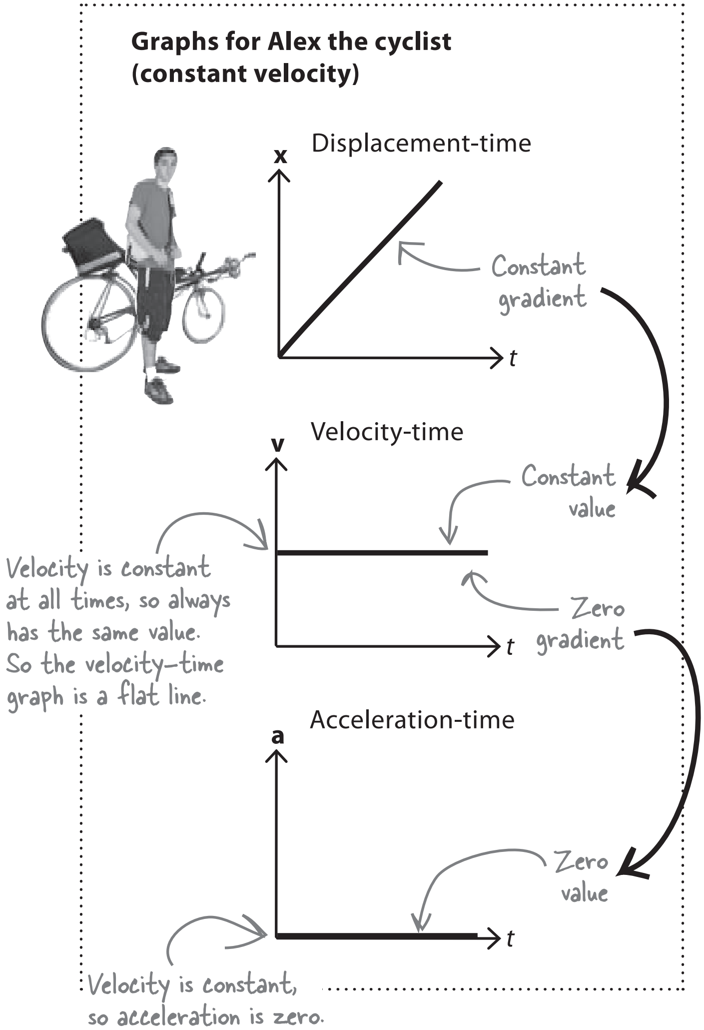
The falling thing's velocity-time graph is a straight line with a **constant slope**. As acceleration is rate of change of velocity, this means that it has a **constant acceleration**, equal to the slope of the velocity-time graph $\mathbf{a} = \frac{\Delta \mathbf{v}}{\Delta t}$.

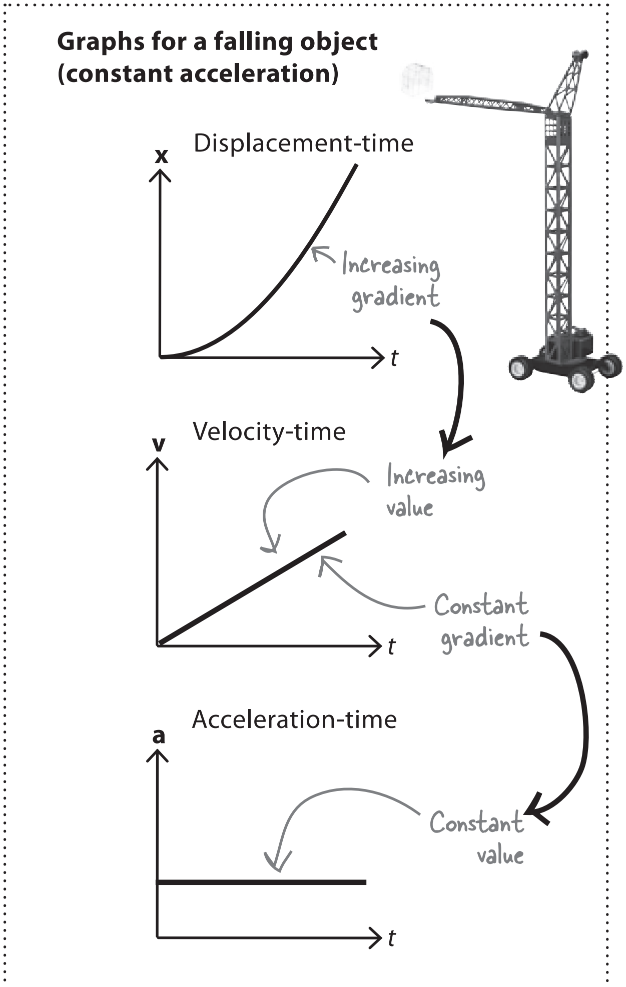
The slope of a velocity-time graph is the acceleration.

Acceleration

$$a = \frac{\Delta v}{\Delta t}$$

Slope of velocity-time graph





there are no Dumb Questions

Q: Why do the graphs over there on the opposite page not have any numbers or units on them?

A: Because these are 'sketch graphs' where the shape is the most important thing. The x-*t*, v-*t* and a-*t* graphs for something with a constant velocity will always have the same shape, no matter what that constant velocity is. Same goes for anything with constant acceleration.

You'll sometimes see graphs like this on an exam - these allow you to show that you understand physics principles by choosing, drawing, or explaining the shape of a graph.

Q: I don't get why Alex's acceleration-time graph is a flat line.

A: Alex cycles with a constant velocity. So at each point in time, his velocity is always the same. So the graph doesn't go up or down - the value stays the same.

Q: And how do you get from that to 'zero value' for the acceleration?

A: Acceleration is rate of change of velocity. The velocity-time graph shows that his velocity isn't changing. So his acceleration must be zero.

Q: OK, I think I get Alex's graphs now. But I'm still puzzled about how the falling thing's displacement-time graph turned into a straight line velocity-time graph.

A: Velocity is the rate of change of displacement. As the curve gets steeper, its slope gets larger. So you know that the velocity increases as time goes on, and the velocity-time graph you just drew showed that the increases form a straight line.

Q: So, what are the units of acceleration, anyway? Surely I need to know that to be able to do calculations?

A: Funny you should ask ...

Work out the units of acceleration

Velocity is rate of change of displacement, in other words how something's displacement varies with time. You already worked out that its units are meters per second (m/s).

Acceleration is rate of change of velocity, or how something's velocity varies with time. Although you've met acceleration as a concept before, you now need to deal with acceleration in calculations. Which means that you need to know about its **units**.

Sharpen your pencil

Hint: It's easiest to write the units as fractions to work them out before changing them to the 'inline' style of m/s at the very end.

Fill in the blanks in the table to work out the units of acceleration.

Quantity	is rate of change of	Units of the changing thing	Units of time	Units of quantity
Velocity	Displacement	m	s	$\frac{m}{s}$ = m/s
Acceleration				

Sharpen your pencil Solution

Fill in the blanks in the table to work out the units of acceleration.

Quantity	is rate of change of	Units of the changing thing	Units of time	Units of quantity
Velocity	Displacement	m	s	$\frac{m}{s}$ = m/s
Acceleration	Velocity	m/s	s	$\frac{\frac{m}{s}}{s} = \frac{m}{s^2} = m/s^2$

You're dividing by seconds twice, so it works out as m/s^2.

s^2 is a weird unit. I guess I should think of them as something other than 'square seconds,' right?

Think of m/s^2 as (meters per second) per second

Velocity is the rate of change of displacement, so its units are meters per second, or m/s.

Acceleration is the rate of change of velocity, so its units are [velocity] per second, or meters per second per second, or m/s^2.

This might seem weird at first, as m^2 is a visible area in 'square meters,' but there's no such thing as a 'square second'! But if you instead think of the units as (meters per second) per second, it makes a lot more sense.

Square brackets around something is shorthand for 'units of'. So [velocity] means 'units of velocity'.

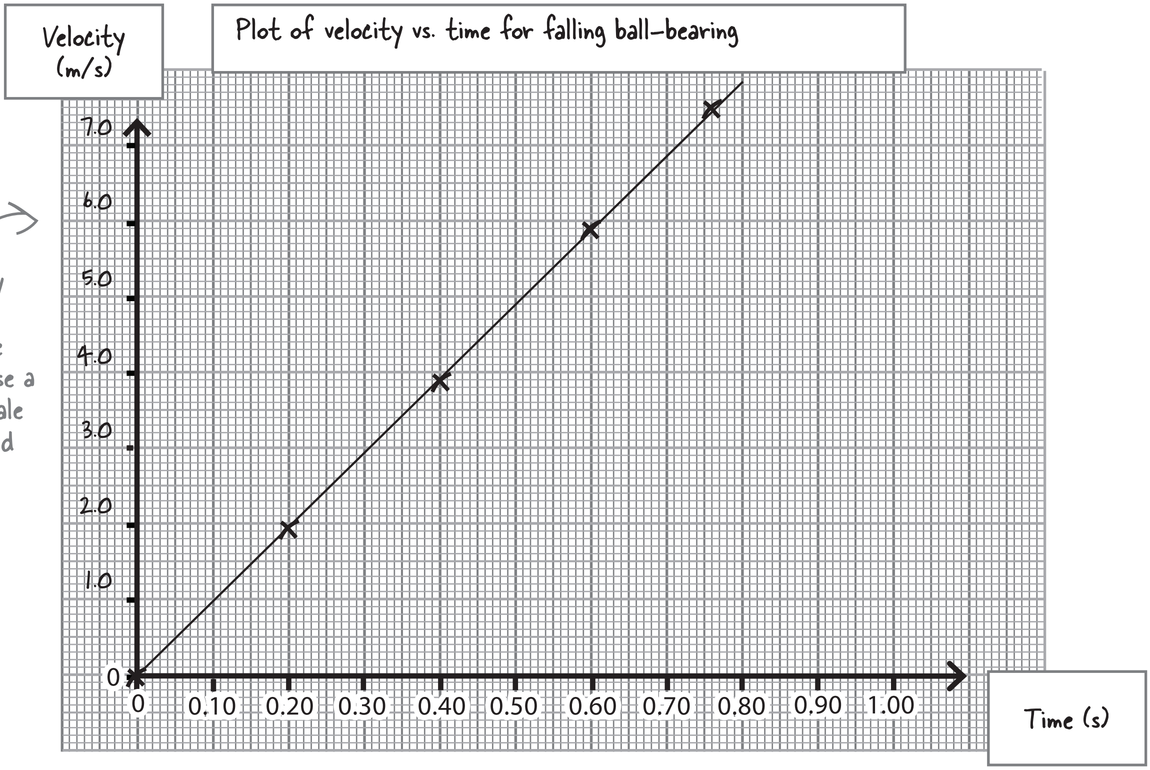
The units of acceleration are m/s^2, or (meters per second) per second.

'per' means 'divided by'

Sharpen your pencil

a. Use your velocity-time graph to get a value for the acceleration of a falling object.

b. Use that to work out the cage's velocity when it's been falling for 2.0 s. Is this less than 25 m/s?



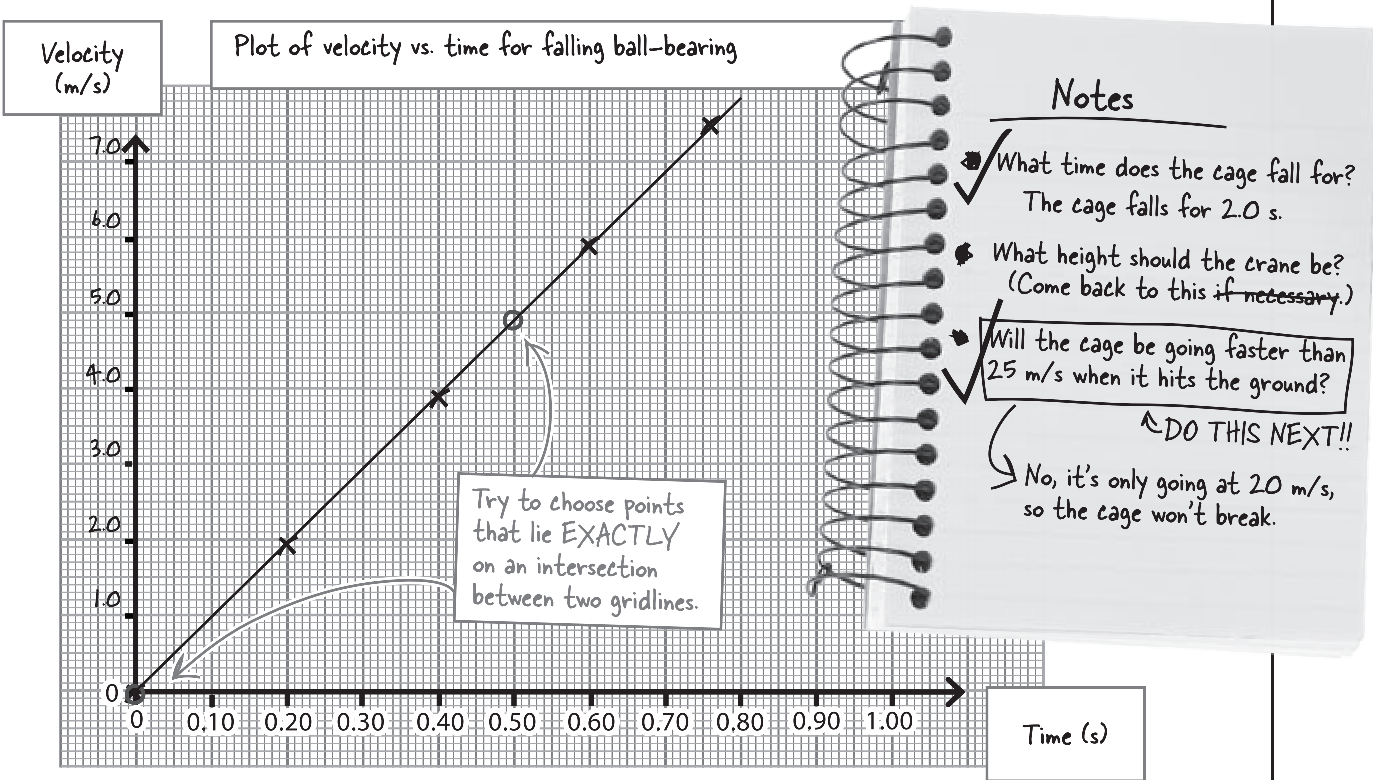
You already titled and labeled the graph, chose a vertical scale and plotted the points.

There's space down here for you to show your work.

Sharpen your pencil Solution

a. Use your velocity-time graph to get a value for the acceleration of a falling object.

b. Use that to work out the cage's velocity when it's been falling for 2.0 s. Is this less than 25 m/s?



a. Acceleration = Rate of change of velocity

Remember to write down what it is you're actually doing!

$$a = \frac{\Delta v}{\Delta t}$$

$$a = \frac{4.9 - 0.0}{0.50 - 0.00}$$

These were the points we chose – if you chose slightly different points and got a slightly different answer, that's OK.

$$a = 9.8 \text{ m/s}^2 \text{ (2 sd)}$$

You can only quote the answer to 2 sd as the velocities are only plotted on the graph to 2 sd.

b. Velocity after 2.0 s:

$$a = \frac{\Delta v}{\Delta t}$$

The cage starts off with zero velocity. So a 20 m/s change in velocity means it ends up going at 20 m/s.

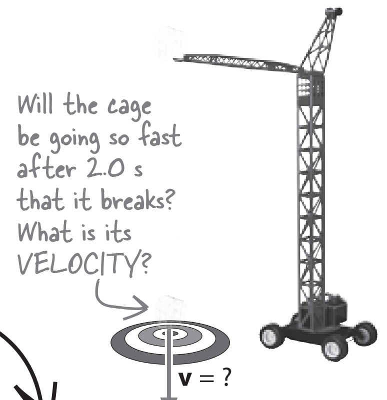
$$\Rightarrow \Delta v = a\Delta t = 9.8 \times 2.0 = 19.6 = 20 \text{ m/s (2 sd)}$$

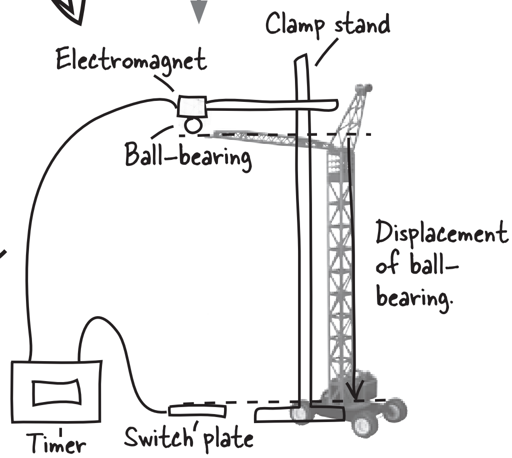
Your final answer should have 2 sd, as the numbers you were working with only have 2 sd.

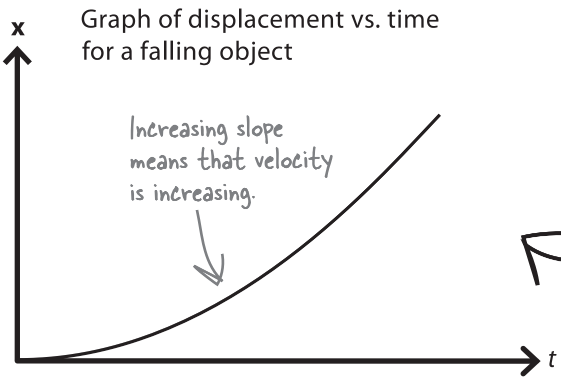
This is less than 25 m/s, so the cage won't break, and the plan is a go.

Success! You worked out the velocity after 2.0 s - and the cage won't break!

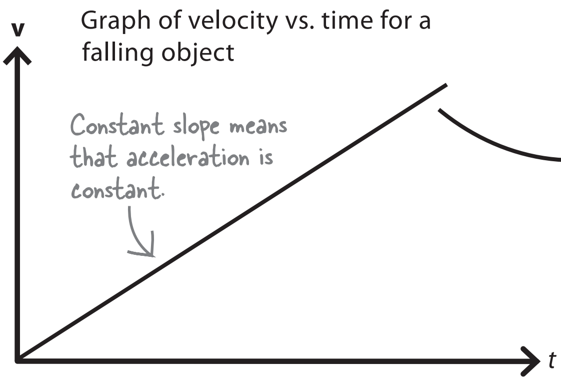
You've just worked out that the cage will be going at 20 m/s after 2.0 s, so the cage won't break. - and the Dingo will be able to stop the Emu for long enough to deliver his party invitation! You got here by designing an **experiment,** which let you draw the **displacement-time graph** for a falling object.







Then, you used the **slope** at various points of the displacement-time graph to draw a **velocity-time graph**.



Then, you used the slope of the velocity-time graph to calculate a value for the **acceleration due to gravity, 9.8 m/s²**.

Finally you used that value in the **equation** $\mathbf{a} = \frac{\Delta \mathbf{v}}{\Delta t}$, which you rearranged to calculate the cage's velocity.

The velocity is less than 25 m/s, so the cage won't break.

The Earth's gravity accelerates falling objects at a constant rate of 9.8 m/s².

there are no Dumb Questions

Q: I've seen values of 10 m/s^2 or 9.81 m/s^2 for acceleration due to gravity used in other books. These values are both close to 9.8 m/s^2 but...

A: The AP Physics B table of information gives the value of acceleration due to gravity as 9.8 m/s^2. That's what we're going to use in this book.

Q: But surely I should practice with the value I'll use in my exam?

A: Yes, that'll be fine. Your answers won't work out too different from ours, and the exact value you're supposed to use will become second nature to you.

Q: I'm worried about the AP B multiple choice exam though. Doing calculations involving 9.8 without a calculator is a bit time-consuming, to say the least!

A: That's right - in the AP B multiple choice exam, you need to do mental arithmetic because you're not allowed to use a calculator. But at the start of the multiple choice exam, it says 'Note: To simplify calculations, you may use g = 10 m/s^2 in all problems.'

Multiplying and dividing by 10 is much, much easier than dealing with 9.8 s, so there's no need to worry about that.

Q: Hmmm. Why do you use '/s^2' to indicate dividing by s^2 with units, but use 10^{-2} to indicate dividing by 10^2 in scientific notation?

A: These are the conventions that the AP physics course, table of information, and exam all use. It's also possible to write m/s^2 as ms^{-2} - using the same convention as you do for scientific notation. If this is what you're more used to, then do feel free to write your units like this instead.

Q: OK, so I got the value for the cage's acceleration - but what about its displacement after 2.0 seconds?!

A: You'll work that out in chapter 7 ...

Gravity accelerates everything at the same rate (if air resistance is minimal)

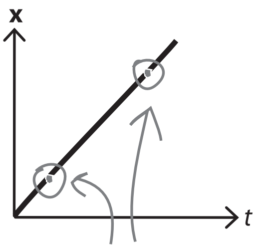
Although everyday experience might lead you to think otherwise, the earth's gravity accelerates everything at the same rate of 9.8 m/s^2 if you ignore the effect of air resistance.

The reason 'light' things like feathers fall more slowly than 'heavy' things like ball-bearings or cages is because they're falling through the air. The feather has a large surface area compared to its weight, so it's held up more by the air. If the air wasn't there, the feather and ball-bearing would land at the same time.

Most of the time in physics, you'll be dealing with things like cages and ball-bearings, which have small surface areas compared to their weight. Unless a question states otherwise, you can safely ignore air resistance.

Sometimes it's useful to make an assumption that simplifies a problem so you can solve the easier version.

While we're here, I've been wondering how it's possible to calculate an 'instantaneous' velocity at a single point on a displacement-time graph. Surely you need to use **two** points, so you can work out the change that happens **between** them?!



Whichever two points you choose, $\frac{\Delta x}{\Delta t}$ will be the same, as the velocity is constant.

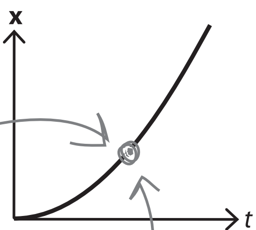
You do need two points - but they can be really really close together

The first time you calculated a velocity from a displacement-time graph, it was for a cyclist who traveled with a constant velocity. So the graph had a constant slope, and it didn't matter which two points you chose to use to calculate $\Delta \mathbf{x}$ and Δt since the value of the slope (and, therefore, the velocity) was always the same.

Now, you've had to deal with a falling thing for which the velocity is continually increasing. This produced a curved displacement-time graph where the slope was never the same from one moment to the next.

You can think of calculating the slope at a single **point** on this graph as calculating $\Delta \mathbf{x}$ and Δt over a very, very small interval, using two points infinitesimally close to the point you're interested in.

If you want to calculate the instantaneous velocity at a point, you need to choose two points that are really, really, really close together – practically on top of each other!



If you do this, it's more correct to write $\frac{dx}{dt}$ instead of $\frac{\Delta x}{\Delta t}$ to show that the changes are infinitesimally small.

A capital Δ implies that you mean a reasonable-sized change in a quantity.

This means "smaller than any possible measure".

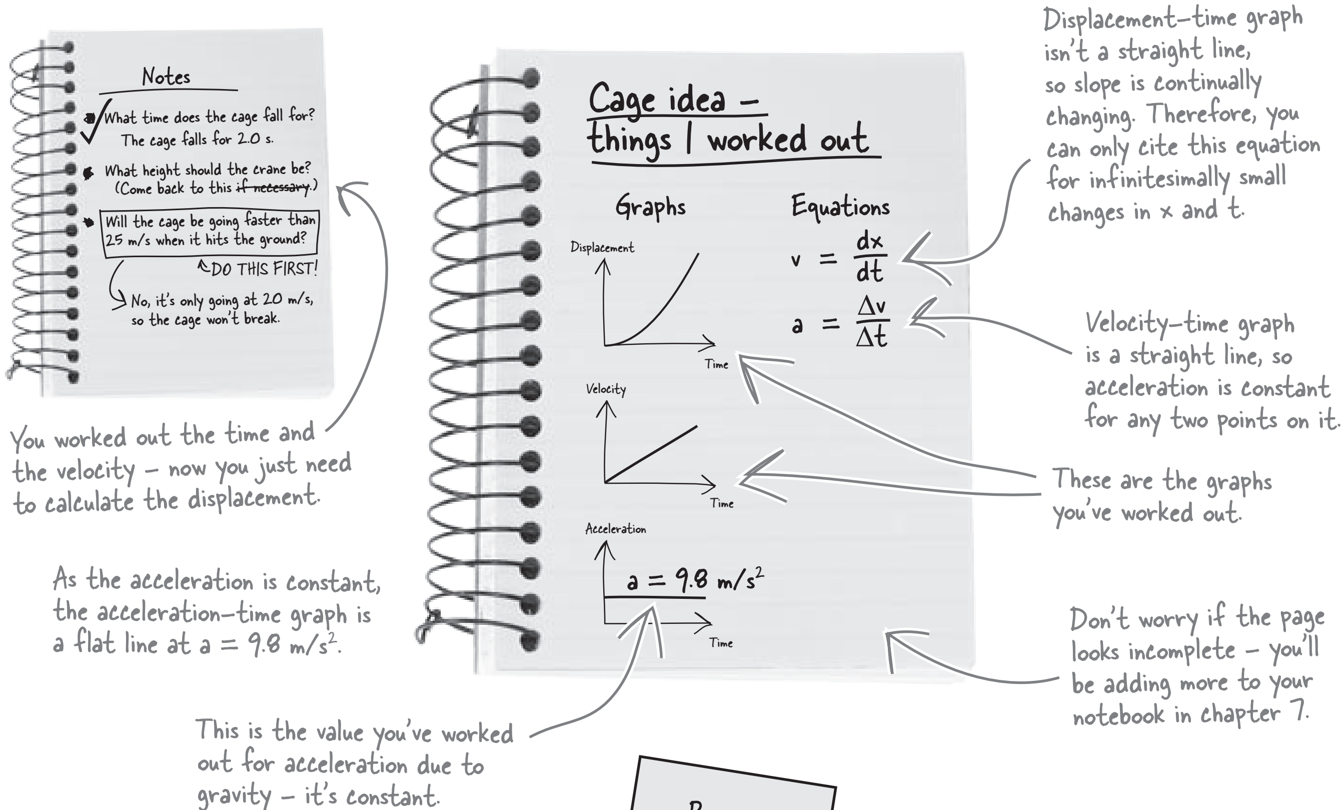
If you're talking about an **instantaneous** velocity at a single point like this, a more 'correct' way of writing your equation is to use a small letter 'd' to mean 'change in' instead of a capital 'Δ'. This means that the change is infinitesimally small.

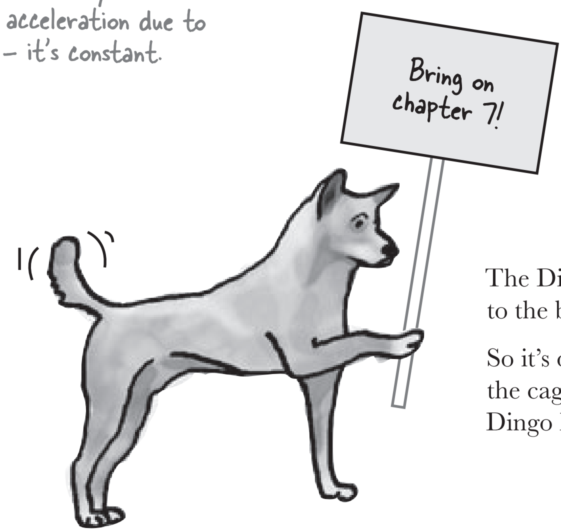
So you'd write the equation for instantaneous velocity as $\mathbf{v} = \frac{d\mathbf{x}}{dt}$ to make it clear that you mean an instantaneous velocity measured over a tiny, tiny change in **x** and t rather than the larger change that using $\Delta \mathbf{x}$ and Δt would imply.

Practically speaking, there are two ways of calculating $\frac{d\mathbf{x}}{dt}$: drawing a tangent (as you've already done) and using calculus (which isn't part of this book).

Now onto solve for the displacement!

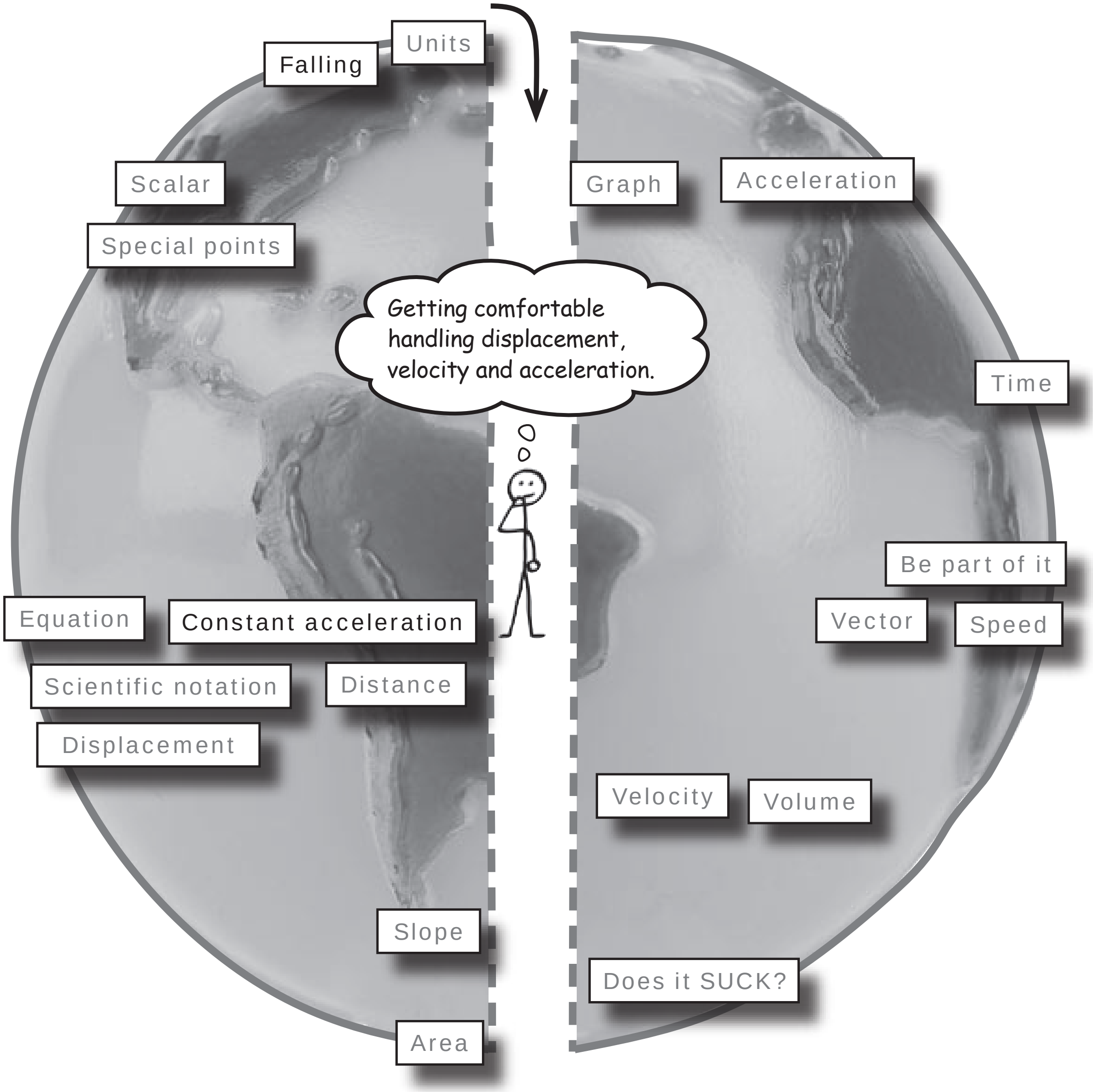
You know that the cage **won't** be going fast enough to break when it hits the ground 2.0 s after being dropped, so it's OK to go ahead with the plan. The notebook summarizes everything you've learned in this chapter about falling objects.



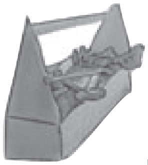


The Dingo's plan to stop the Emu and invite him to the birthday party is taking shape!

So it's on to chapter 7 - where you'll calculate the cage's **displacement** after 2.0 s. so that the Dingo knows how high to set the crane platform ...



☑	Constant acceleration	Something with constant acceleration has a straight line velocity-time graph described by the equation $a = \frac{\Delta v}{\Delta t}$
☑	Falling	Something falling close to the Earth is accelerated by gravity at a constant rate of 9.8 m/s^2.



Your Physics Toolbox

You've got Chapter 6 under your belt, and you've added some terminology and problem-solving skills to your toolbox.

Experiment -> graph -> equation

When you do an experiment, you'll usually use the results to draw a graph, then use the slope of the graph to work out an equation.

Slope of a graph

The slope of a displacement-time graph is equal to the velocity.

The slope of a velocity-time graph is equal to the acceleration.

Constant velocity

If something has constant velocity, its displacement increases at a steady rate.

This means that its displacement-time graph has a constant slope equal to its velocity.

Something with a constant velocity has an acceleration of zero, as its velocity isn't changing.

Constant acceleration

If something has constant velocity, its displacement increases more and more each second as time goes on.

Its velocity-time graph has a constant slope equal to its acceleration.

Falling object

A falling object is accelerated by gravity – so has a constant acceleration of 9.8 m/s^2 downwards close to the the Earth's surface..

Acceleration due to gravity

Acceleration due to gravity has a value of 9.8 m/s^2 downwards when you're close to the Earth's surface.

7 Equations of motion (part 1)

Playing With Equations

It's time to take things to another level.

So far, you've done experiments, drawn graphs of their results and worked out equations from them. But there's only so far you can go, since sometimes your graph isn't a straight line. In this chapter, you'll expand your math skills by making **substitutions** to work out a key **equation of motion** for a curved displacement - time graph of a falling object. And you'll also learn that **checking** your GUT reaction to an answer can be a good thing.

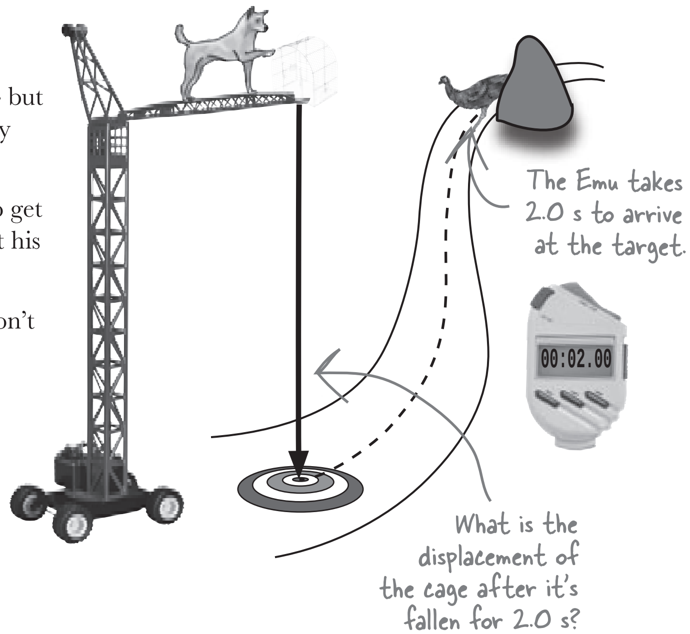
How high should the crane be?

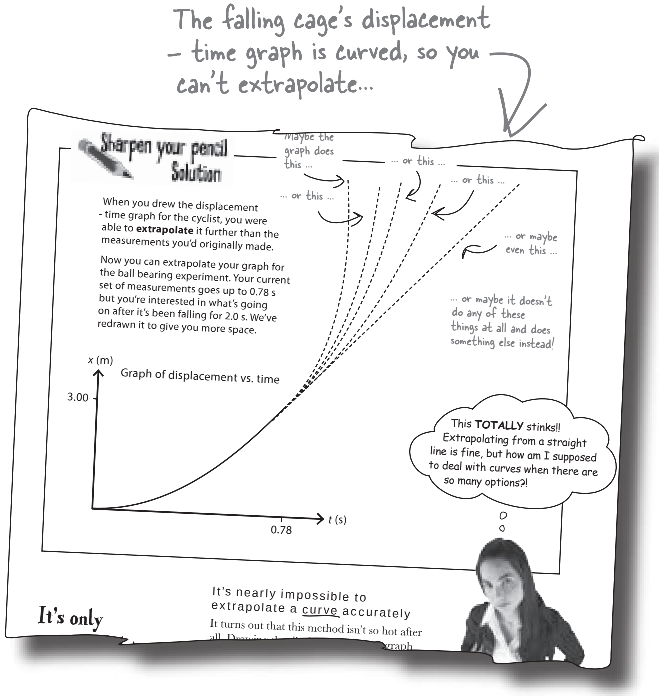
The Dingo wants to invite the Emu to his birthday party - but the only way he'll get him to stay still for long enough is by catching him in a cage!

In chapter 6, you figured out that it takes the Emu 2.0 s to get from the corner to the target on the road while running at his constant speed.

You also figured out that the cage's **velocity** after 2.0 s won't lead to it shattering on impact, by drawing its **velocity - time graph** and working out the **equation** $\mathbf{a} = \frac{\Delta \mathbf{v}}{\Delta t}$.

But the Dingo wants to know how high to set the crane. Which means that you now need to work out the cage's **displacement** after it's been falling for 2.0 s.





Sharpen your pencil Solution

When you drew the displacement - time graph for the cyclist, you were able to **extrapolate** it further than the measurements you'd originally made.

Now you can extrapolate your graph for the ball bearing experiment. Your current set of measurements goes up to 0.78 s but you're interested in what's going on after it's been falling for 2.0 s. We've redrawn it to give you more space.

Maybe the graph does this ...

... or this ...

... or this ...

... or this ...

... or maybe even this ...

... or maybe it doesn't do any of these things at all and does something else instead!

Graph of displacement vs. time

x (m)

3.00

0.78

t (s)

This **TOTALLY** stinks!! Extrapolating from a straight line is fine, but how am I supposed to deal with curves when there are so many options?!

It's nearly impossible to extrapolate a curve accurately

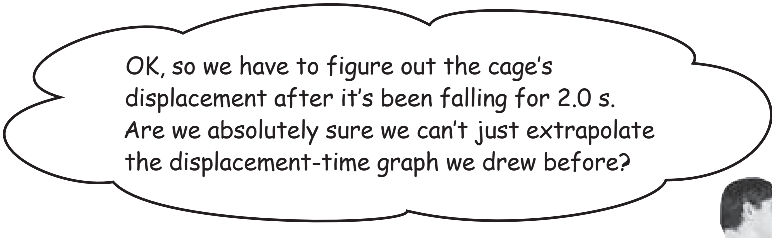
It turns out that this method isn't so hot after

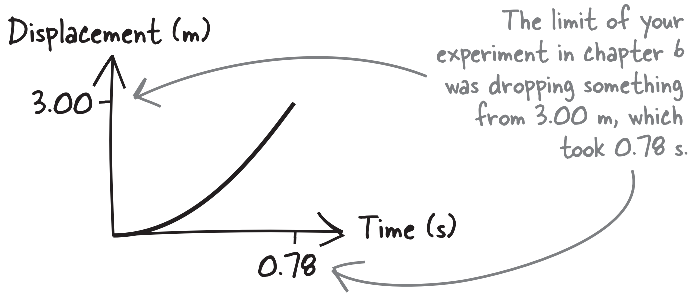
It's only

You already drew a **displacement - time graph** for a falling object, but you can't **extrapolate** it to read off the displacement after 2.0 s because the graph is curved.

BRAIN POWER

If you can't read the value for the displacement after 2.0 s off your graph, what can you do?



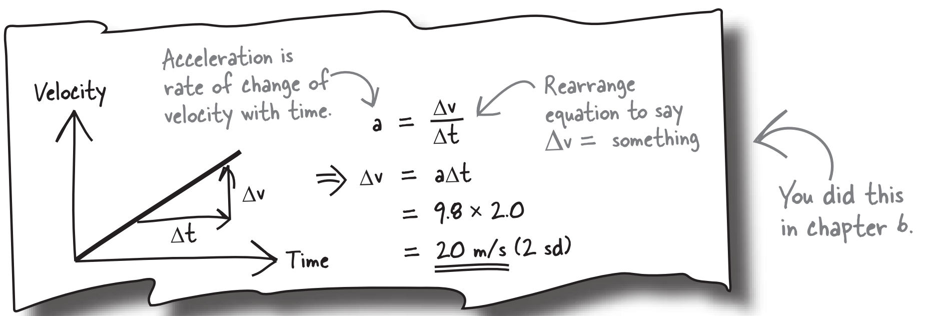


Jim: It's a **curve**, so we don't really know what it's going to do next. If the last point we'd plotted was close to 2.0 s we could probably make an educated guess, but not when we're so far away.

Frank: But 0.78 s is only a little bit less than 2.0 s. We'd only need to continue the graph for another 1.22 s - that's hardly any time at all!

Jim: It's a lot of time compared to what we already have. We've plotted less than half the graph between $t = 0.0$ s and $t = 2.0$ s.

Joe: Maybe we could try working out an **equation**, like we did before to get a value of the cage's velocity from its velocity - time graph?



Graphs and equations are both ways of representing reality.

Frank: But the velocity - time graph is a **straight** line. Our displacement - time graph is a **curved** line!

Jim: Yeah, I dunno if it's possible for a curved graph to be represented by an equation.

Joe:: I'm sure it must be possible, if graphs and equations are both ways of representing reality ...

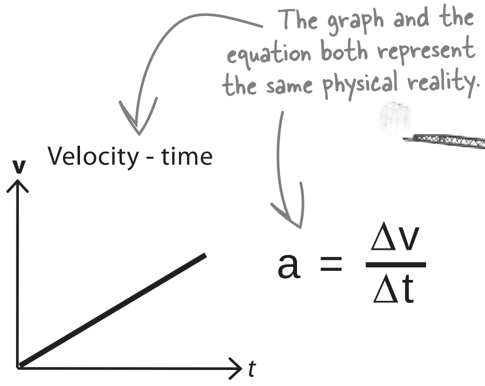
BRAIN POWER

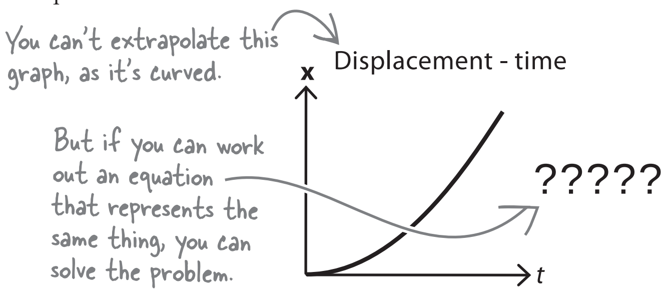
Do you think it's possible for a curved graph to be represented by an equation?

Graphs and equations both represent the real world

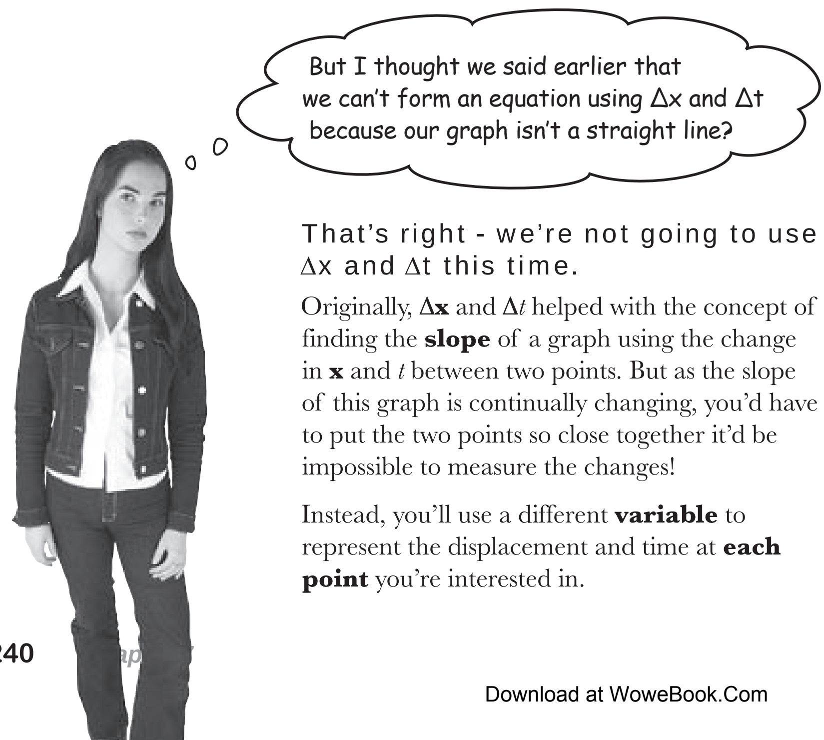
Graphs are a way of representing the real world visually. **Equations** are a way of representing the real world symbolically. They both allow you to **predict** what will happen to a quantity when other things that affect it change as well.

For example, $\mathbf{a} = \frac{\Delta \mathbf{v}}{\Delta t}$ is the equation for the cage's velocity-time graph. The equation and the graph both represent the same physical reality. The equation shows you symbolically how the velocity, acceleration and time interrelate when the acceleration is constant. If you know values for two of the quantities in the equation, you can use the equation to calculate the third by **rearranging** the equation.





So, if you can work out the **equation that represents your displacement - time graph**, you'll be able to use it to solve the problem of how high the cage needs to be.



That's right - we're not going to use Δx and Δt this time.

Originally, $\Delta \mathbf{x}$ and Δt helped with the concept of finding the **slope** of a graph using the change in **x** and *t* between two points. But as the slope of this graph is continually changing, you'd have to put the two points so close together it'd be impossible to measure the changes!

Instead, you'll use a different **variable** to represent the displacement and time at **each point** you're interested in.

Use a different variable to represent each of the values at the points you're interested in.

You're interested in the start and end points

We're only really interested in two points in the cage's motion - the **start** (when it's on the platform) and the **end** (when it hits the ground) - as we want to calculate the cage's displacement between these points.

In the equation you work out for your curved displacement - time graph we're going to use variables to represent every value we might be interested in at these start and end points:

$\mathbf{x}_0$ is the displacement at the start (when $t = 0$).
$\mathbf{v}_0$ is the velocity at the start (when $t = 0$).
$\mathbf{x}$ is the displacement at the end.
$\mathbf{v}$ is the velocity at the end.
$\mathbf{a}$ is the acceleration (which you already know is constant).

You call this "v nought" if you're speaking out loud.

You can tell that v and v_0 are both displacements because they use the letter v. But that they're different quantities because they have different subscripts.

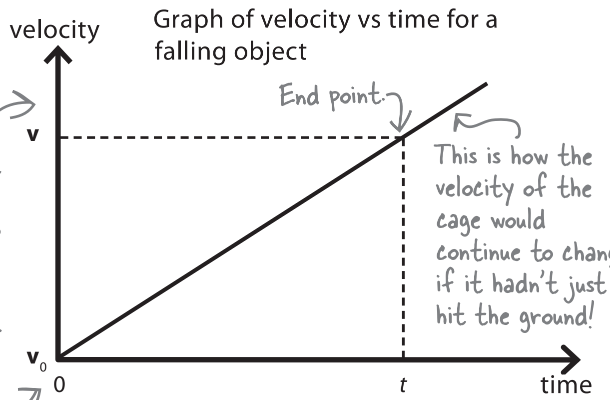
The little '0' is part of the variable name, and is called a subscript.

There's no point in having a_0 and a, as the value for the acceleration is always the same.

Use the same letter to represent the same type of thing, and subscripts to say which is which.

Sharpen your pencil

We've drawn in the interesting start and end points on your velocity - time graph.



You might find it helpful to draw or write on the graph.

Start point.

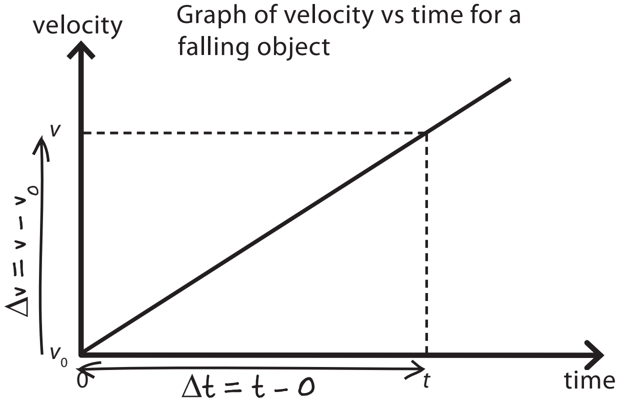
a. Write down an equation you already know that involves **a**, Δ**v** and Δt.

b. Use the values on the graph to rewrite this equation as an equation involving **a**, $\mathbf{v}_0$, **v** and t.

c. Rearrange your equation so that it says "**v** = something".

Sharpen your pencil Solution

We've drawn in the start and end points we're interested in on your velocity - time graph.



a. Write down an equation you already know that involves **a**, Δ**v** and Δ*t*.

Acceleration = Rate of change of velocity

$$a = \frac{\Delta v}{\Delta t}$$

b. Use the values on the graph to rewrite this as an equation involving **a**, $\mathbf{v}_0$, **v** and *t*.

$$a = \frac{\Delta v}{\Delta t} = \frac{v - v_0}{t - \cancel{0}}$$

$$a = \frac{v - v_0}{t}$$

c. Rearrange your equation so that it says "**v** = something".

$$a = \frac{v - v_0}{t}$$

Multiply both sides by t.

$$at = \frac{v - v_0}{\cancel{t}} \times \cancel{t}$$

Add v_0 to both sides.

$$v_0 + at = v \cancel{- v_0} \cancel{+ v_0}$$

Swap the sides over.

$$v = v_0 + at$$

Now you have v on its own.

Are we putting in letters, like 't' for time, instead of values, like 2.0 seconds, to make it more general?

You want your equation to be as general as possible so you can use it elsewhere.

At the moment, you're dealing with a falling cage. You could stick in the numbers you already know (t = 2.0 s, $\mathbf{a}$ = 9.8 m/s^2) but then you'd end up with an equation that you can only use once.

If you leave everything as letters for now and only put the numbers in at the end, you'll end up with a **general** equation you can use to deal with falling things, jet skis, racing cars ... anything that has a period of **constant acceleration**.

If your equation is general, you can reuse it in other problems .

there are no Dumb Questions

Q: Why did you choose particular variable names, like v (with no subscripts) for the final velocity and v_0 for the initial velocity?

A: It's a common convention to use x_0 and v_0 for the initial values of displacement and velocity and x and **v** for the final values. It's what's used in lots of textbooks, as well as the AP Physics B exam.

Q: But the convention isn't consistent! The initial velocity is called v_0, but the initial time doesn't even have a symbol - we just put in its value of 0.

A: The convention assumes that everything you're interested in starts at t=0. The '0' subscript in v_0 stands for 'at t=0'', so you can read v_0 as "the velocity at t=0". Similarly, t_0 would stand for "the time at t=0". So there's no need to bother with a t_0 symbol, as you already know that t=0 when t=0!

Q: Do I have to use these letters? Before, I've used s instead of x for displacement, and u instead of v_0 for initial velocity. I'm finding this confusing!

A: The main thing is that you understand the **physics concepts** that lie behind the equations. It doesn't matter which set of letters you use for that. It's fine to show your work using the letters you're more familiar with that already make sense for you!

Q: So what physics concepts are the most important here?

A: A graph and an equation can represent the same thing in real life. In this problem, they both describe what happens to the velocity of the falling cage as time passes.

Q: OK. But why have I used letters in the equation when I already worked out all the values of the things in the equation?! I know what v, v_0, a and t are for the falling cage!

A: One reason is that you can reuse a general equation again and again. If the crane is a different distance away from the corner, the cage would fall for a different time. Your general equation, $v = v_0 + at$, will give you the value of **v** for any time. All you need to do is put in the new numbers.

Q: And the other reason for not putting in the values yet?

A: If you keep the equation **general**, you'll be able to use it for anything with constant acceleration, even if it isn't 9.8 m/s^2. All you need to do is to put in the new numbers for your new problem.

Hey! We're supposed to be figuring out an equation for displacement, **x**. But the equation we just worked out doesn't have an **x** in it, so how's it gonna help?!?!

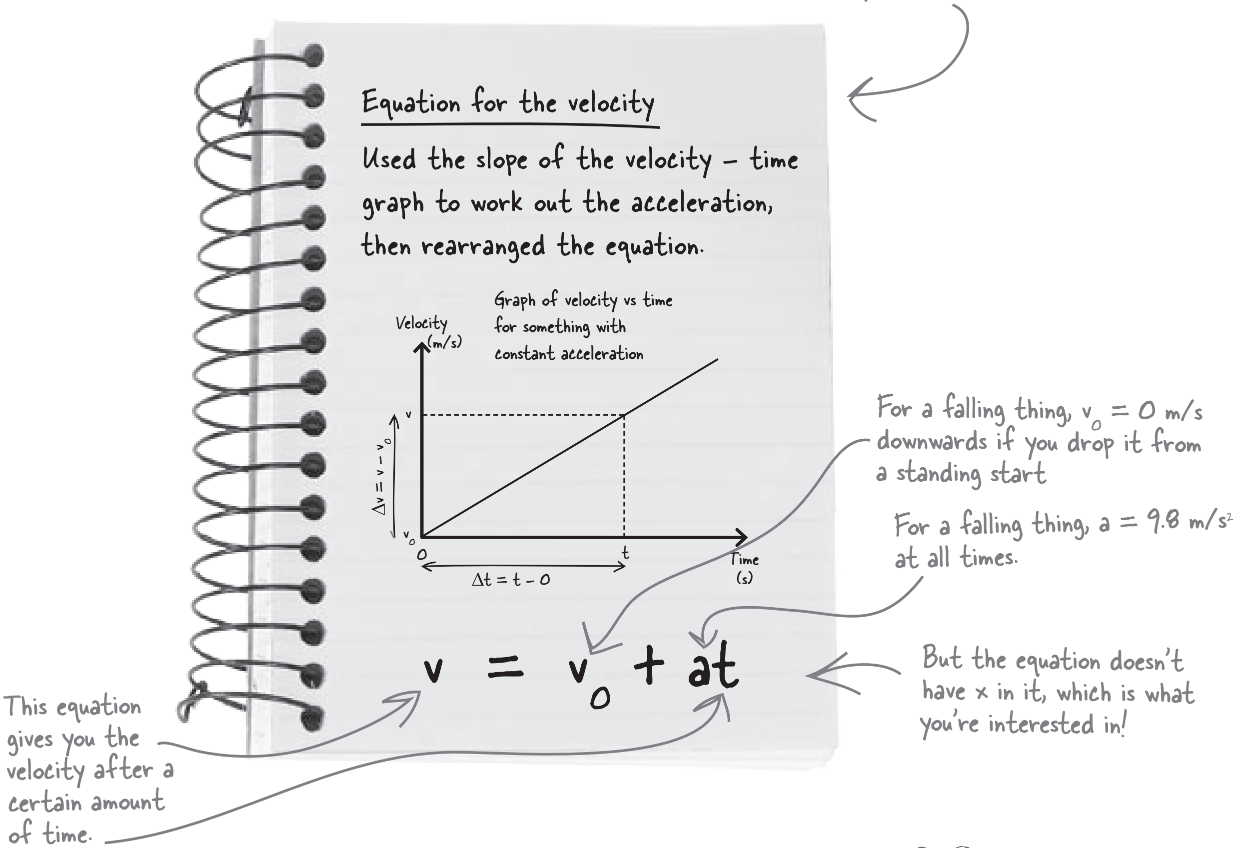
The equation shows how different variables depend on one another. You can use it as a stepping stone to get what you really want.

Your equation $\mathbf{v} = \mathbf{v}_0 + \mathbf{a}t$ shows you how the variables $\mathbf{v}$, $\mathbf{v}_0$, $\mathbf{a}$ and t depend on each other. But it doesn't have an $\mathbf{x}$ in it, so you can't use it to directly calculate a value for the displacement,

However, as displacement is rate of change of velocity, the displacement and the velocity must depend on each other. So although you can't use this equation directly, you'll be able to use it as a stepping stone towards calculating the displacement of the cage after 2.0 s.

You have an equation for the velocity - but what about the displacement?

You've worked out the equation $\mathbf{v} = \mathbf{v}_0 + \mathbf{a}t$, which comes from the slope of your velocity - time graph. It gives you an object's **velocity**, **v**, after a certain amount of time (if you know its initial velocity, $\mathbf{v}_0$, and its acceleration, **a**).

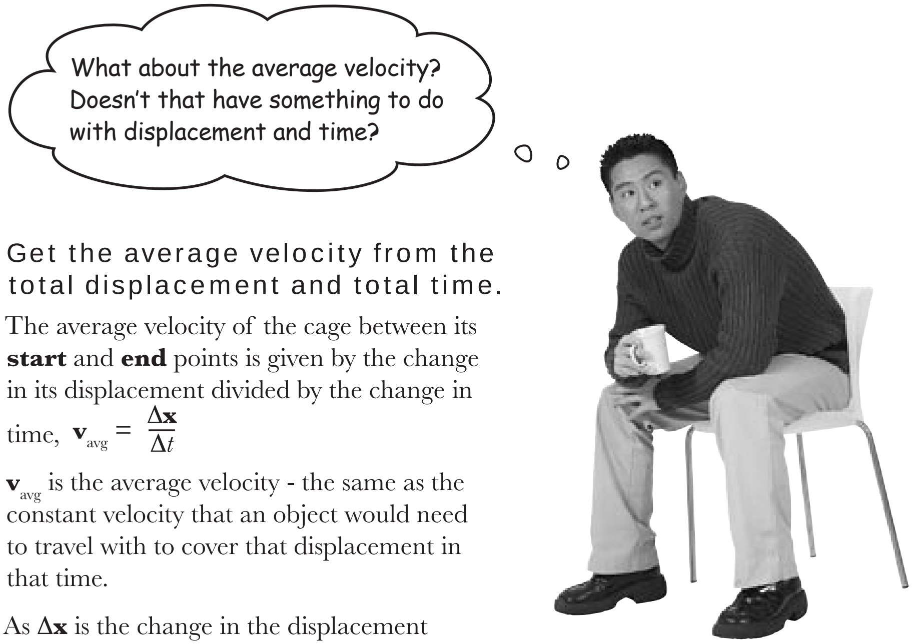


But what we're really interested in is the **displacement**, **x**, after a certain amount of time. If you have an equation for that, you can say how far the cage will fall in 2.0 s.

The velocity equation might be useful later on, as velocity and displacement must be related somehow. But right now, you really need an equation with an **x** in it to move forward ...

BRAIN POWER

How might you get an equation that involves the displacement?



The average velocity is the same as the constant velocity you could have gone at to cover the displacement between your start and end points in the same time

Get the average velocity from the total displacement and total time.

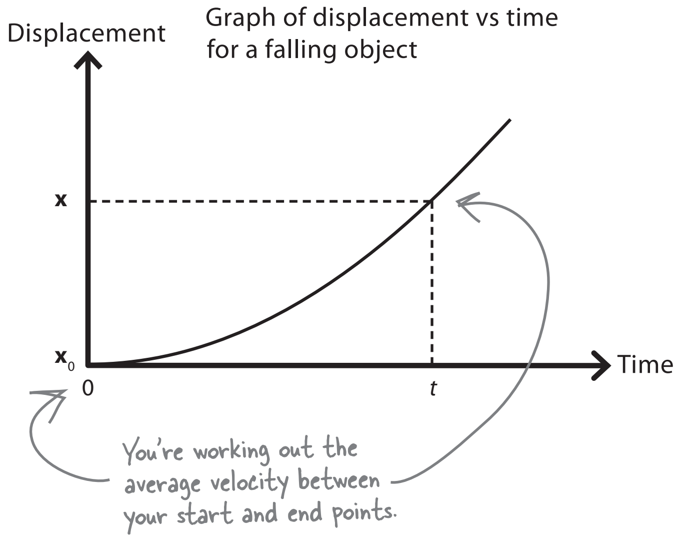
The average velocity of the cage between its **start** and **end** points is given by the change in its displacement divided by the change in time, $\mathbf{v}_{avg} = \frac{\Delta \mathbf{x}}{\Delta t}$

$\mathbf{v}_{avg}$ is the average velocity - the same as the constant velocity that an object would need to travel with to cover that displacement in that time.

As $\Delta\mathbf{x}$ is the change in the displacement between the start and end points, the equation for the average velocity will have an **x** in it - which is what you want to calculate!

x is the displacement at the end point.

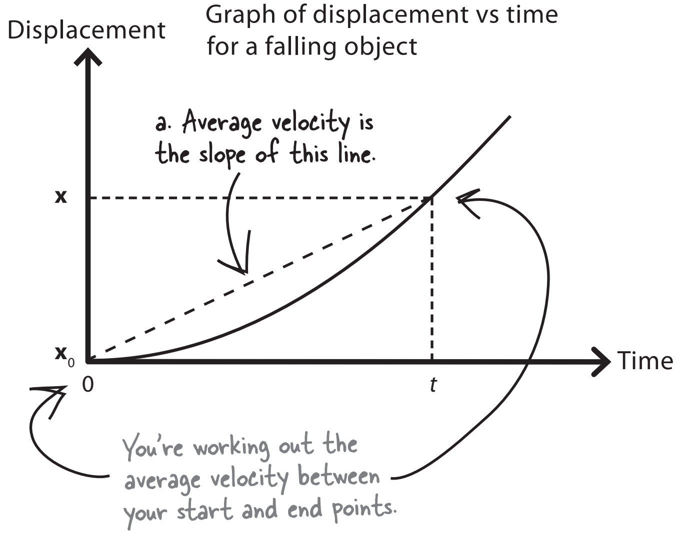
Sharpen your pencil



a. Draw a line on your displacement - time graph to represent the cage's average velocity between times 0 and *t*.

b. Use the graph to come up with an equation for the average velocity, $\mathbf{v}_{avg}$, in terms of $\mathbf{x}_0$, $\mathbf{x}$ and *t*.

Sharpen your pencil Solution



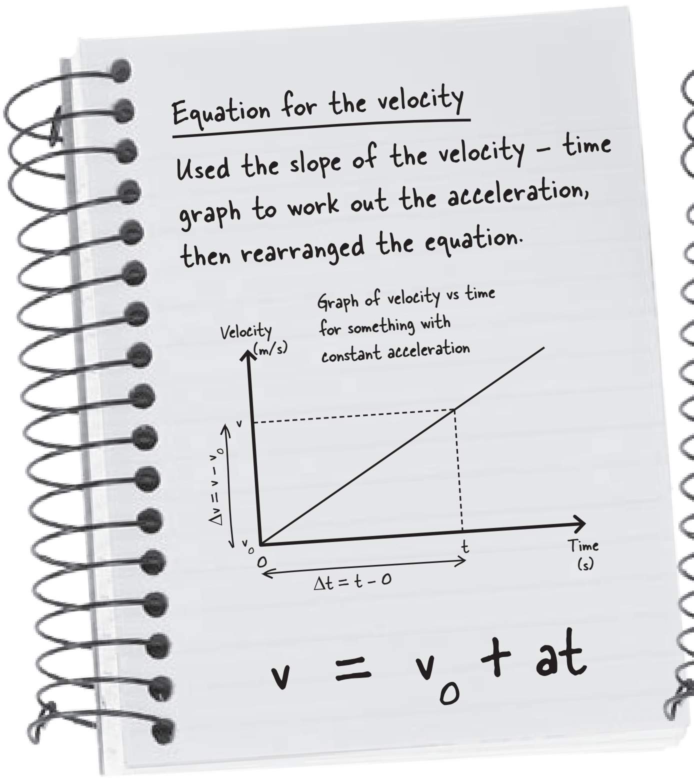
a. Draw a line on your displacement - time graph to represent the cage's average velocity between times 0 and *t*.

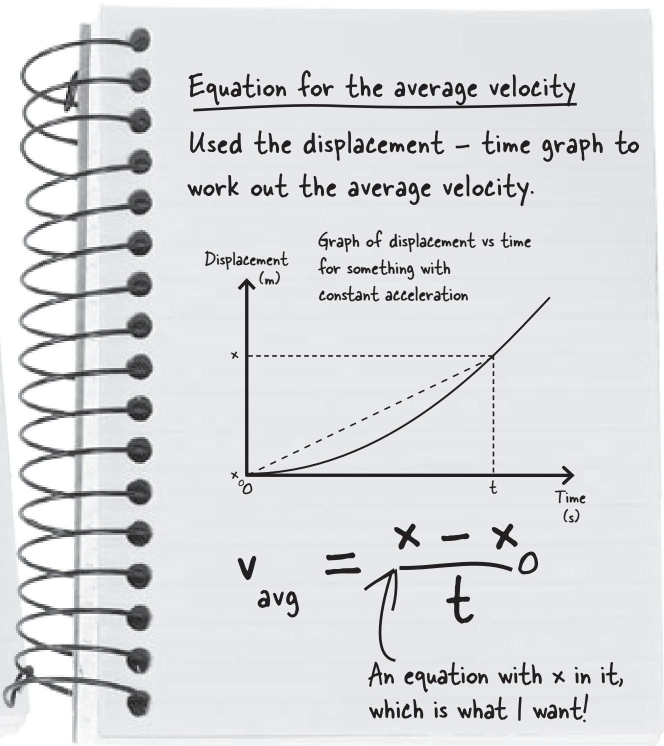
b. Use the graph to come up with an equation for the average velocity, $\mathbf{v}_{avg}$, in terms of $\mathbf{x}_0$, $\mathbf{x}$ and *t*.

$$\text{Average velocity} = \frac{\text{Total displacement}}{\text{Total time}}$$

$$v_{avg} = \frac{\Delta x}{\Delta t} = \frac{x - x_0}{t - 0}$$

$$v_{avg} = \frac{x - x_0}{t}$$





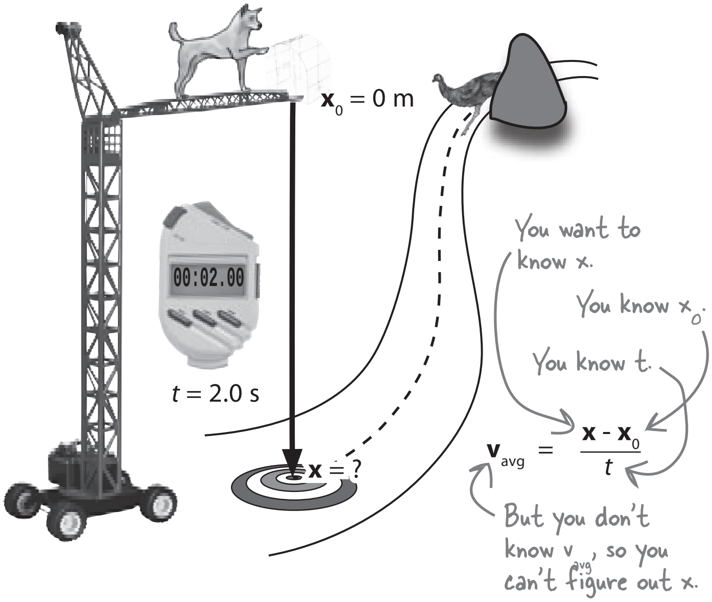
That's right - we don't know the value of the average velocity.

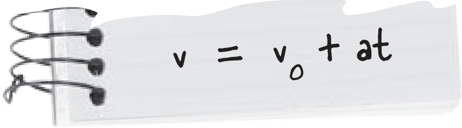
You've come a long way, and have two equations from the graphs you drew:

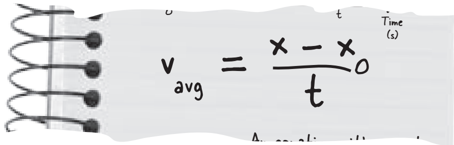
$\mathbf{v} = \mathbf{v}_0 + \mathbf{a}t$ and $\mathbf{v}_{avg} = \dfrac{\mathbf{x} - \mathbf{x}_0}{t}$.

The second of these equations has an **x** in it, which is what you want - but it also has $\mathbf{v}_{avg}$, the average velocity, in it.

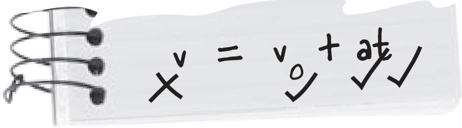
Since you don't know what the average velocity is, you can't use this equation to calculate **x** and tell the Dingo how high to put the crane platform right now. But **you're definitely making progress** ...

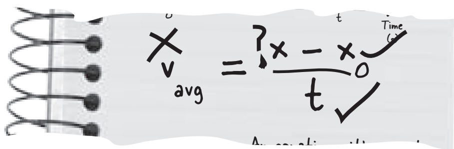






Jim: Let's just see how they help us. I'm gonna put a question mark by the x, because we want to work that out. Then I'll tick what we already know values for, and cross the variables we don't know ...





Jim: ... hmmm, neither equation helps us. The one on the left is for the velocity, **v**, which we're not interested in. The one on the right has the displacement, **x**, in it, which is what we want to work out ... but it also has the average velocity, $\mathbf{v}_{avg}$, in it. And we don't know what $\mathbf{v}_{avg}$ is.

Joe: Is there another equation we can use?

Frank: What do you mean?

Joe: Our problem is $\mathbf{v}_{avg}$, right? We can't just rearrange the equation to say "**x** = something" and put in the values for the other variables because we don't have a value for $\mathbf{v}_{avg}$. But what if there was another equation we could use to calculate the value of $\mathbf{v}_{avg}$?

Frank: I like your thinking. But we already worked out $\mathbf{v}_{avg}$ the only way we know how - from the slope of our **displacement - time graph**.

Jim: Hang on! What about our **velocity - time graph**? Maybe if we look at that, we can eyeball a second equation for $\mathbf{v}_{avg}$.

Joe: You might be on to something there ... let's try it!

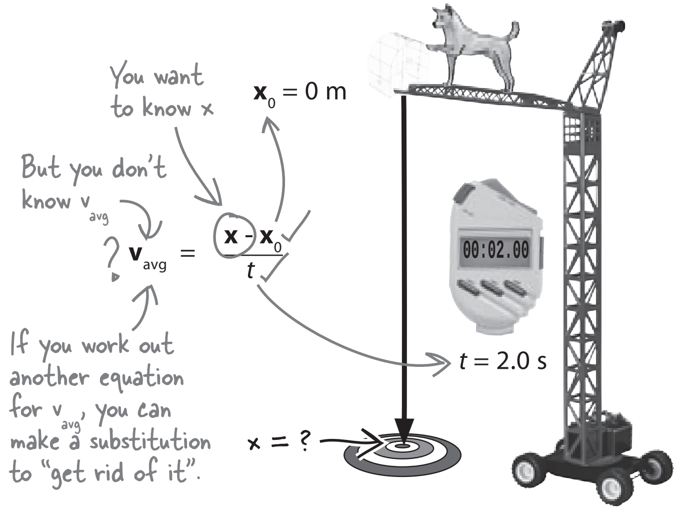
If you don't know the value for a variable in your equation, try to find another equation which includes that variable.

See the average velocity on your velocity-time graph

You've already worked out one equation for the falling cage's average velocity, $\mathbf{v}_{avg}$, from its **displacement - time graph**.

Now you can use your **intuition** to spot $\mathbf{v}_{avg}$ on the cage's **velocity - time graph** , which will lead you to a second equation for the cage's average velocity.

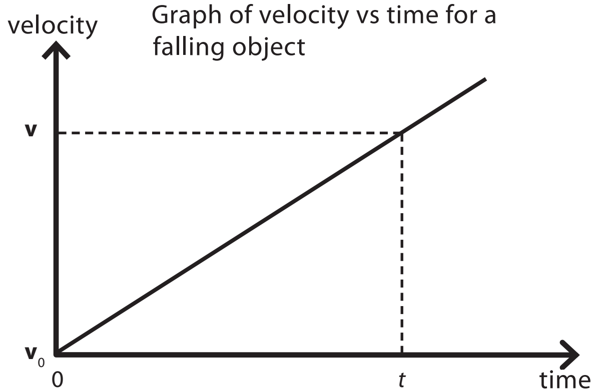
(And once you know the cage's average velocity, you can use that to get its displacement, which is what you really want to know!)



Sharpen your pencil

It'll be somewhere between v_0 and v.

a. On your graph, draw in where you think the cage's average velocity is between times 0 and *t*.



b. Explain how you worked that out visually.

c. Circle the equation from the choices on the right that matches most closely with your ideas.

$$v_{avg} = \frac{v - v_0}{2}$$

$$v_{avg} = \frac{v + v_0}{t}$$

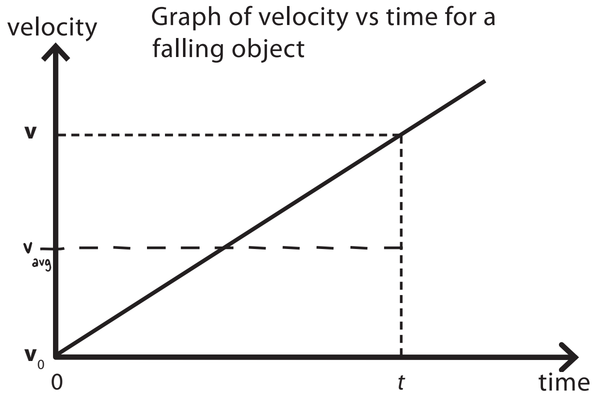
$$v_{avg} = \frac{v - v_0}{t}$$

$$v_{avg} = \frac{v + v_0}{2}$$

Sharpen your pencil Solution

It'll be somewhere between v_0 and v.

a. On your graph, draw in where you think the cage's average velocity is between times 0 and *t*.



$$v_{avg} = \frac{v - v_0}{2}$$

$$v_{avg} = \frac{v + v_0}{t}$$

$$v_{avg} = \frac{v - v_0}{t}$$

$$v_{avg} = \frac{v + v_0}{2}$$

b. Explain how you worked that out visually.

I think that v_{avg} will be halfway between v_0 and v.

In fact it'll be the average of v_0 and v!

The equation I've picked adds together v_0 and v then divides by 2, which is how you take an average of 2 numbers.

c. Circle the equation from the choices on the right that matches most closely with your idea.

I didn't do that. I tried putting in v_0 = 0 and t = 2 into the equations, since those are the values for Dingo's cage problem. But **all** the equations gave me the same answer!

The right equation will work for ALL values of v_0 and t.

If you tried putting in the Dingo's values for $\mathbf{v}_0$ and *t* into the equations, then **well done**. You're definitely thinking along the right lines.

If you're still not sure which equation is correct, the next thing to do is to try some different numbers for $\mathbf{v}_0$ and *t* and see what happens.

Even if you think you already know which equation is right, it's always a good idea to double-check it with some numbers.

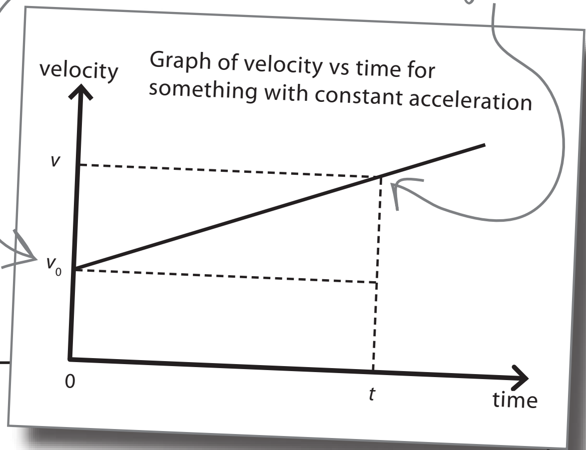
Test your equations by imagining them with different numbers

If you're not sure whether an equation's right and want to test it, you can **try some numbers in it** to see if the answers you get are plausible.

You have four different equations for $\mathbf{v}_{avg}$ to choose from. They can't all be right! So it's time to try some different values for $\mathbf{v}$, $\mathbf{v}_0$ and *t* in each equation to see which gives you consistently sensible answers that are the right **size**.

v_0 may not be 0 m/s. if the thing is already falling when you start the timer.

t could be anything at all, not just 2.0 s.



Sharpen your pencil

Fill in the table to show the value for $\mathbf{v}_{avg}$ given by each of these equations for various values of $\mathbf{v}_0$, $\mathbf{v}$ and *t*.

Is the average velocity the size you'd expect it to be?

Does the equation have the right units?

Possible equation for v_{avg}	v_0 = 0 m/s v = 10 m/s t = 5 s	v_0 = 0 m/s v = 10 m/s t = 100 s	v_0 = 9 m/s v = 10 m/s t = 5 s	Does this equation SUCK?
$\frac{v - v_0}{2}$	$\frac{10 - 0 \text{ m/s}}{2}$ = 5 m/s			
$\frac{v + v_0}{t}$				
$\frac{v - v_0}{t}$				
$\frac{v + v_0}{2}$				

Sharpen your pencil Solution

Fill in the table to show the value for v_{avg} given by each equation for various values of v_0, v and t.

Possible equation for v_{avg}	v_0 = 0 m/s, v = 10 m/s, t = 5 s	v_0 = 0 m/s, v = 10 m/s, t = 100 s	v_0 = 9 m/s, v = 10 m/s, t = 5 s	Is this equation plausible?
$\frac{v - v_0}{2}$	$\frac{10 - 0 \text{ m/s}}{2} = 5$ m/s	$\frac{10 + 0 \text{ m/s}}{2} = 5$ m/s	$\frac{10 - 9 \text{ m/s}}{2} = 0.5$ m/s	No. In the 3rd answer the average must be between 9 and 10. It isn't!
$\frac{v + v_0}{t}$	$\frac{10 + 0 \text{ m/s}}{5 \text{ s}} = 2 \text{ m/s}^2$	$\frac{10 + 0 \text{ m/s}}{100 \text{ s}} = 0.1 \text{ m/s}^2$	$\frac{10 + 9 \text{ m/s}}{5 \text{ s}} = 3.8 \text{ m/s}^2$	No. The units are wrong and the 2nd and 3rd answers are way too low.
$\frac{v - v_0}{t}$	$\frac{10 - 0 \text{ m/s}}{5 \text{ s}} = 2 \text{ m/s}^2$	$\frac{10 - 0 \text{ m/s}}{100 \text{ s}} = 0.1 \text{ m/s}^2$	$\frac{10 - 9 \text{ m/s}}{5 \text{ s}} = 3.8 \text{ m/s}^2$	No. The units are wrong and the 2nd and 3rd answers are way too low.
$\frac{v + v_0}{2}$ (circled)	$\frac{10 + 0 \text{ m/s}}{2} = 5$ m/s	$\frac{10 + 0 \text{ m/s}}{2} = 5$ m/s	$\frac{10 + 9 \text{ m/s}}{2} = 9.5$ m/s	Yes. All of the answers for v_{avg} are between v_0 and v, which is sensible.

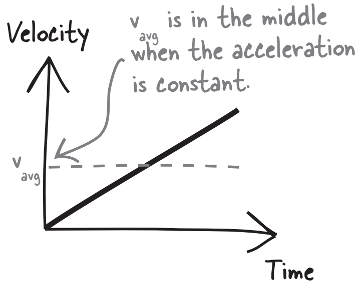
The right equation for v_{avg}

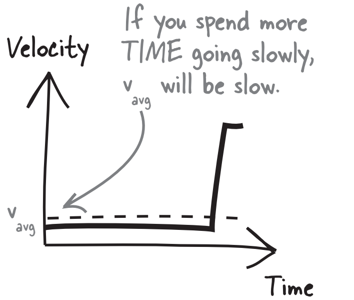
The equation only works when the acceleration's constant, right?

That's right - the average velocity will be different if the acceleration isn't constant.

This equation only works because your velocity - time graph is a straight line (your object, the cage, has constant acceleration).

If you didn't know what happened in between the initial and final velocities $\mathbf{v}_0$ and v, you couldn't work out the average velocity. If someone goes at 1 m/s for most of the time then at 5 m/s right at the end, they won't have an average velocity of 3 m/s as they spent most of their time travelling slowly.

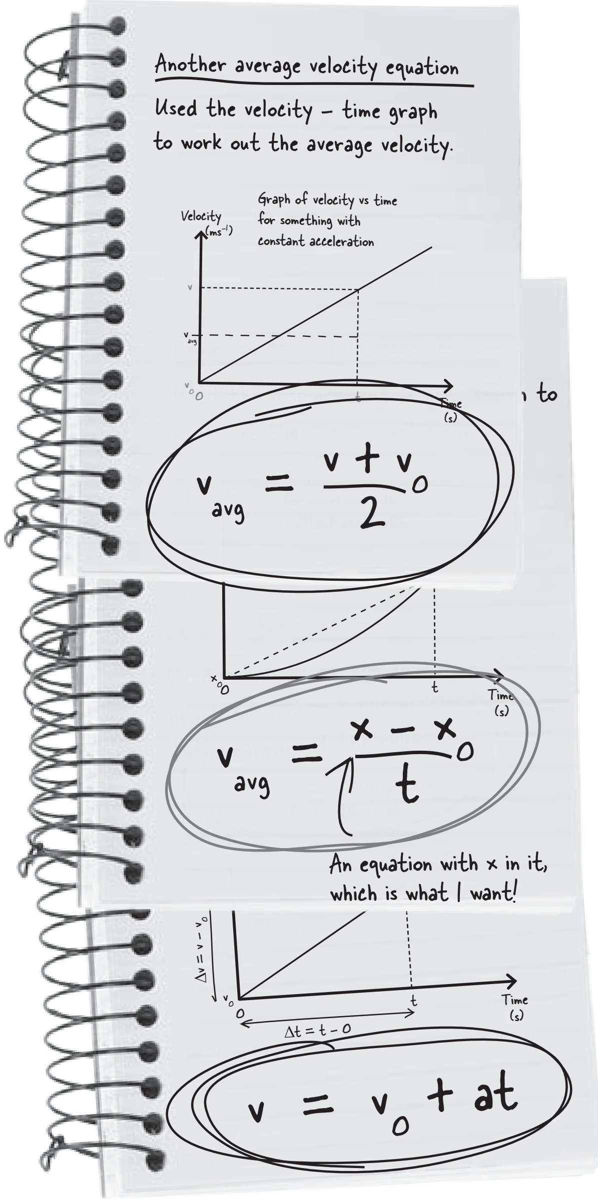




Calculate the cage's displacement!

You've used your displacement - time and velocity - time graphs to work out some equations that describe what the cage is doing as it falls towards the ground.

Now, you can use them to calculate a value for the cage's displacement after 2.0 s!



Sharpen your pencil

a. Draw a sketch of the cage and platform, with values and/or vector arrows representing what you already know about **x**, $\mathbf{x}_0$, $\mathbf{v}_0$, **v**, **a** and *t*.

This collates together all the information you already know in a visual format.

The cage has to fall for 2.0 seconds, and acceleration due to gravity is 9.8 m/s^2.

b. Use what you already know to calculate **v**.

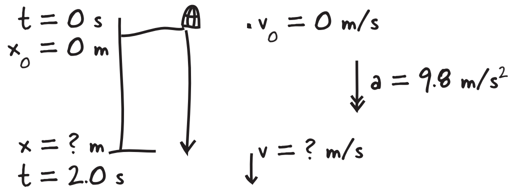
c. Use your answer from part b. to calculate $\mathbf{v}_{avg}$.

d. Use your answer from part c. to calculate **x**.

Sharpen your pencil

The cage has to fall for 2.0 seconds, and acceleration due to gravity is $9.8\ m/s^2$.

a. Draw a sketch of the cage and platform, with values and/or vector arrows representing what you already know about **x**, $\mathbf{x}_0$, $\mathbf{v}_0$, **v**, **a** and *t*.



This collates together all the information you already know in a visual format.

b. Use what you already know to calculate **v**.

It's also OK to write in the value you calculated in chapter 6.

$$v = v_0 + at$$

$$v = 0 + 9.8 \times 2.0$$

$$v = 19.6\ m/s = \underline{\underline{20\ m/s}}\ (2\ sd)$$

c. Use your answer from part b. to calculate $\mathbf{v}_{avg}$.

$$v_{avg} = \frac{v + v_0}{2} = \frac{20 + 0}{2}$$

$$v_{avg} = \underline{\underline{10\ m/s}}\ (2\ sd)$$

d. Use your answer from part c. to calculate **x**.

$$v_{avg} = \frac{x - x_0}{t}$$

$$x - x_0 = v_{avg} t$$

$$x = v_{avg} t + x_0$$

$$x = 10 \times 2.0 + 0$$

$$x = \underline{\underline{20\ m}}\ (2\ sd)$$

Each part of this problem involves using a different equation that you originally worked out from your graphs.

You know how high the crane should be!

You've calculated the cage's displacement after 2.0 s, so you know how high the crane should be - an awesome result!

At last, the Dingo will be able to make the Emu pause for long enough to invite him to his birthday party ...

Start every problem with a <u>sketch</u> to bring together everything you know in a visual way.

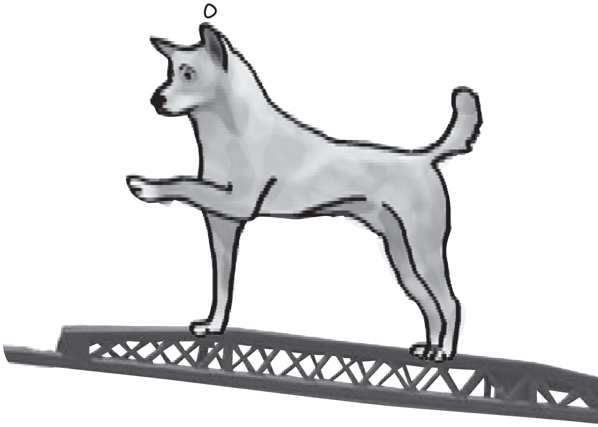
Then use what you already know to work out what you don't know.

But now the Dingo needs something more general

But the Dingo's soon back, with the sad news that his crane won't go high enough! What's more, he's not sure where he's going to find another spot for the crane, so has to stay flexible.

The Dingo really needs to be able to work out the cage's displacement for **any** time that the Emu might take to get to the target.

Can you help me work out the displacement for **any** time?



We can already do this, yeah?! You use the **time** and **acceleration** to calculate the **velocity**, and then you use the velocity to calculate the **average velocity**, and then you use the average velocity to calculate the **displacement**!

You don't want to do all of that every time when you just want the displacement!

You're right - you can do this by calculating intermediate values for **v** and $\mathbf{v}_{avg}$ every time you want to know a new value for **x**. But to go through all of that every time you want to calculate a displacement isn't very efficient, and will take a long time.

This is exactly what you did on the opposite page.

What you really want is an equation that says "**x** = something" where the right hand side only contains variables that you already know values for ($\mathbf{x}_0$, **a** and *t*). Somehow, we need to "get rid" of the intermediate variables **v** and $\mathbf{v}_{avg}$ from your equations to come up with a general equation for the displacement that you'll be able to use again and again.

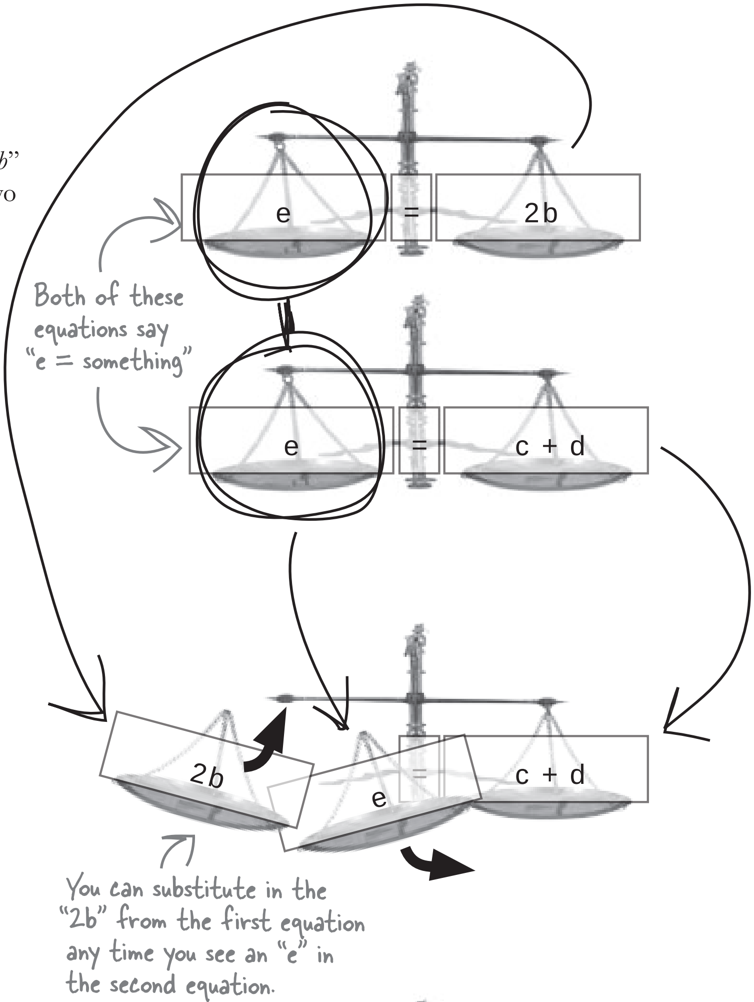
The time you spend getting your equations into this form will be made up by the time you save by using the new equation!

BRAIN POWER

Can you think of any way of using the equations you already have to "get rid of" **v** and $\mathbf{v}_{avg}$, the variables you don't know - and don't want?

A substitution will help

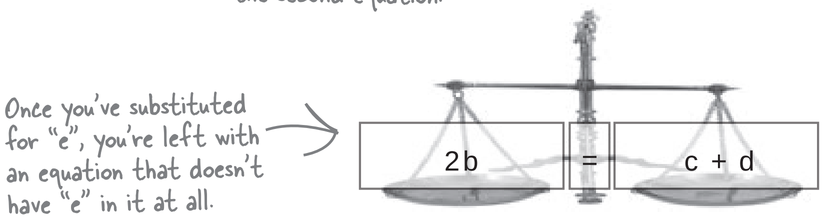
Suppose you have two equations, that say "$e = 2b$" and "$e = c + d$" respectively. By definition, the two sides of an equation are equal.



Both equations say "$e =$" on one side. So $2b$ is equal to e and $c + d$ is also equal to e.

Therefore, we can write down the new equation: $2b = c + d$. This works because if $2b$ **and** $c + d$ are both equal to e, they must also be equal to each other.

This is called making a **substitution** because the equation you're left with doesn't have any mention of e in it. It's like the e has been substituted and gone off the playing field, like substitutes do in sports.



You can get rid of a variable that isn't helpful to you by making a substitution.

Substitution can be useful if e is a quantity that's difficult to measure, but you're interested in the other variables in the equations. Instead of having two equations which both have an e in them, you can combine them to completely get rid of the e by doing a substitution.

Don't worry if you're not sure how a substitution will help you with the crane problem yet. We're almost there...

there are no Dumb Questions

Q: Can I do substitutions with any equations I like?

A: Only with equations where there's some form of overlap between the variables. In the same way that some dominos don't match, some equations don't have any variables in common.

So if your equations are $a = b$ and $c = d$, you can't make any substitutions because there's no common ground.

Q: What if I have two equations, but the letters mean different things. Like in one equation "*a*" means acceleration but in another "*a*" means altitude?

A: Then you can't make a substitution. The letters need to represent the same thing each time for it to be meaningful.

Q: If I'm given some equations to use in a test, how am I supposed to know what the letters in them represent?

A: Many exam boards provide an equation sheet. For example, the AP Equation Table gives you a list of what the letters stand for in the equations for each section of the syllabus. You're doing Newtonian Mechanics at the moment, so should look in that section to find out what the symbols mean.

Q: But I don't really want to look backwards and forwards to an equation table all the time to look up what the letters mean.

A: That's why we've mentioned what each symbol means as we've introduced it, and keep on slipping in hand-written reminders when they come up again. You'll pick a lot up as you go along, which is great if you're doing the AP course as you don't get an equation table at all in the multiple choice part!

Q: So if I get used to the equations by USING them, I won't really need the table at all and can go a lot faster?

A: You got it! It can be nice to have the equation table to double-check what you think you can remember, to check units or simply for inspiration if you're not sure where to go with a problem.

But learning the equations through understanding and using them is the most successful route.

Q: But the equations on the scales over there aren't physics equations!

A: OK, so it's time to get back to the Dingo's crane where your substitution skills are about to come in handy ...

You can only substitute one equation into another if they have at least one variable in common between them.

If you're working with more than one equation, make sure that the same letter represents the same thing in all of them!

You can rearrange one equation so the thing you want to get rid of is on its own

If the equations you're dealing with don't have $\mathbf{v}_{avg}$ on its own, you'll have to **rearrange** at least one of them before you can make a substitution.

But once you've rearranged one of your equations to say "$\mathbf{v}_{avg}$ = something", you can insert the "something" into your second equation every time you see $\mathbf{v}_{avg}$ mentioned.

You can make substitutions using any equation as long as you're able to rearrange it.

there are no Dumb Questions

Q: Why is it useful for me to make a substitution?

A: Sometimes the equation you want to use has a variable in it that you don't have a value for. If you do a substitution to get rid of that variable, you'll be able to use the new equation to get what you want.

Q: How do I get rid of a variable by making a substitution?

A: You also need a second equation that contains that variable. If you rearrange that equation so that the variable you want to get rid of is on its own on the left, you can then substitute in everything on the right hand side every time you see that variable in your original equation.

Q: What's wrong with just calculating a value for the variable and putting that in the original equation instead?

A: There's nothing inherently wrong with that - but it's a process you'll have to repeat again and again in the future if you want to do the same calculation using different numbers. You also might run into problems with rounding your intermediate values, or making a calculator typing mistake.

Q: So is making a substitution kind-of the same as calculating an intermediate value, except with letters not numbers?

A: Great spot! Instead of substituting in a value for the variable, you'll be substituting in some other variables that it's equal to.

Making a substitution instead of calculating intermediate values saves you time in the long run.

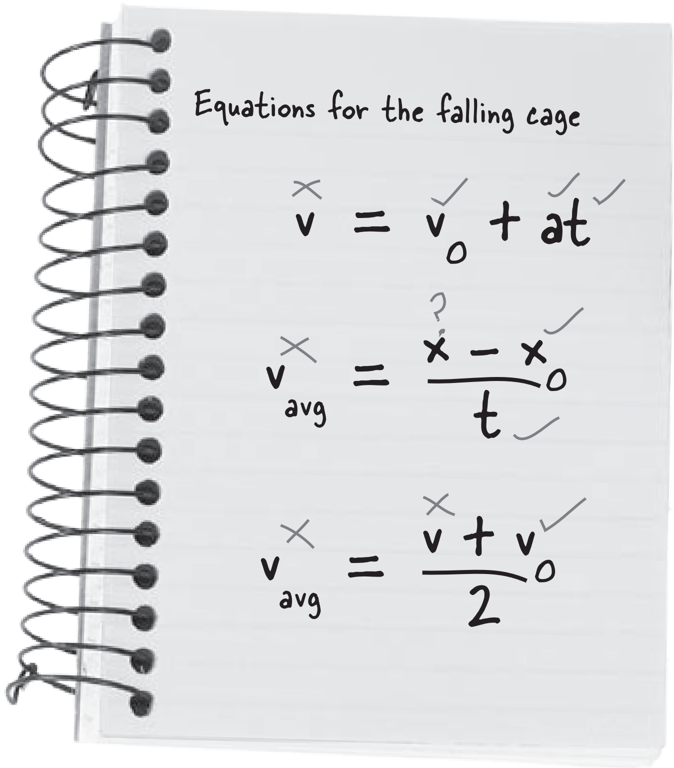
Get rid of the variables you don't want by making substitutions

You've managed to work out three different equations from your graphs.

But there are a couple of variables, **v** and $\mathbf{v}_{avg}$, which you didn't know at the start, and it would be good to get rid of them by making substitutions so that you don't have to spend time calculating them.

If you can do that, you'll be left with a **general equation** that gives you the displacement after any time, but doesn't include $\mathbf{v}_{avg}$ or **v**. You'll be able to use this equation to work out how far the cage falls in any time - and also for any problem where an object has constant acceleration.

So, it's time to get on with making some substitutions to get rid of the variables you don't want, **v** and $\mathbf{v}_{avg}$.



Sharpen your pencil

The equations you've worked out so far (with the variables you knew values for at the start checked off) are in the notebook above.

Decide which of $\mathbf{v}_{avg}$ and **v** is the easiest to get rid of first by making a substitution. Then go ahead and do the substitution!

Sharpen your pencil Solution

$$v = v_0 + at$$

$$v_{avg} = \frac{x - x_0}{t}$$

$$v_{avg} = \frac{v + v_0}{2}$$

The equations you've worked out so far (with the variables you knew values for at the start checked off) are in the notebook above.

Decide which of $\mathbf{v}_{avg}$ and $\mathbf{v}$ is the easiest to get rid of first by making a substitution. Then go ahead and do the substitution!

I already have two equations that say "v_{avg} = something", so make the substitution to get rid of v_{avg} first.

$$v_{avg} = \frac{v + v_0}{2} \quad \text{and also} \quad v_{avg} = \frac{x - x_0}{t}$$

$$\Rightarrow \frac{v + v_0}{2} = \frac{x - x_0}{t}$$

Try to spot which part of the math will be the easiest to do.

You already have two equations that both say "$\mathbf{v}_{avg}$ = something" so doing a substitution with them is the most straightforward thing to do.

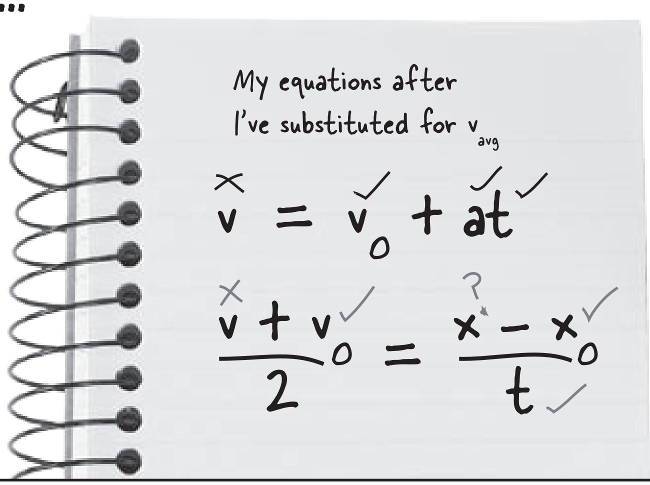
Doing the easier substitution first means that you're less likely to make a careless slip. If you start with the more difficult substitution, you're could mess up and get stuck when you don't have to.

If you can choose which order to do the math in, always try to spot the easier part and do it first.

Continue making substitutions ...

Now that you've made one substitution, your equation with **x** and *t* in it doesn't have $\mathbf{v}_{avg}$ in it anymore But in addition to $\mathbf{v}_0$ and *t* (which you've known values for from the start) the equation contains the variable **v**.

Since we're deriving a general equation for **x** that doesn't require you to know a value for **v**, you can use the other equation to make a substitution that 'gets rid' of the **v** as well.



Sharpen your pencil

a. Using your two equations, make a substitution to 'get rid' of the variable **v**.

b. Then rearrange your new equation into the form "**x** = something".

Sharpen your pencil Solution

$$\overset{\times}{v} = \overset{\checkmark}{v_0} + \overset{\checkmark}{a}\overset{\checkmark}{t} \quad (1)$$

$$\frac{\overset{\times}{v} + \overset{\checkmark}{v_0}}{2} = \frac{\overset{?}{x} - \overset{\checkmark}{x_0}}{t\checkmark} \quad (2)$$

It's a good idea to number your equations (1) and (2) so you can refer to them later on.

a. Using your two equations, make a substitution to 'get rid' of the variable **v**.

Equation (1) already says "v = something".

Remember to explain what you're doing and why.

$$v = v_0 + at$$

Rearrange equation (2) to say "v = something".

$$\frac{v + v_0}{2} = \frac{x - x_0}{t}$$

Multiply both sides by 2.

$$= \quad v + v_0 = \frac{2(x - x_0)}{t}$$

Subtract v_0 from both sides.

$$\Rightarrow \quad v = \frac{2(x - x_0)}{t} - v_0$$

Make a substitution:

If the x starts off on the left, it's easier to rearrange the equation to say "x = something".

$$\frac{2(x - x_0)}{t} - v_0 = v_0 + at$$

b. Then rearrange your new equation into the form "**x** = something".

$$\frac{2(x - x_0)}{t} - v_0 = v_0 + at$$

Add v_0 to both sides so that there's only the term that contains x on the left hand side.

$$\frac{2(x - x_0)}{t} = 2v_0 + at$$

Multiply both sides by t so that the $x - x_0$ bit isn't divided by anything.

$$2(x - x_0) = 2v_0t + at^2$$

Divide both sides by 2 so that the $x - x_0$ is left on its own.

$$x - x_0 = v_0t + \tfrac{1}{2}at^2$$

Add x_0 to both sides so that you have x on its own.

$$x = x_0 + v_0t + \tfrac{1}{2}at^2$$

Your final answer should be in the simplest form possible.

Can we save time by making substitutions without rearranging **both** equations to say "v = something"?

It's OK to do substitution in a different way as long as you understand what's going on.

If you want to rearrange just one of your equations to say "$\mathbf{v}$ = something" then substitute the "something" in every time you see $\mathbf{v}$ in your other equation, then that's fine.

Doing it this way would look like this:

$$v = v_0 + at \qquad (1)$$

Substitute $v_0 + at$ for v in equation (2):

$$\frac{v + v_0}{2} = \frac{x - x_0}{t} \qquad (2)$$

$$\frac{v_0 + at + v_0}{2} = \frac{x - x_0}{t}$$

Then you'd go on to rearrange the equation to say $\mathbf{x} = \mathbf{x}_0 + \mathbf{v}_0 t + \frac{1}{2}\mathbf{a}t^2$ as you did before.

Doing the math this way is fine. The main thing is that you always **understand** what you're doing and why.

Sharpen your pencil

This is the equation you've worked out. But is it right?

$$x = x_0 + v_0 t + \frac{1}{2}at^2$$

You've made a lot of substitutions, and come up with a nice-looking equation for x that only involves variables you already knew the values for at the start.

But does your equation SUCK?

Use the space on the right to jot down as many different ways you can think of to **check over** your equation, which is supposed to work OK for any values of the variables in it.

Don't worry if you're not sure how to do a particular check yet. Just say what you'd like to do if possible.

Sharpen your pencil Solution

This is the equation you've worked out. But is it right?

$$x = x_0 + v_0 t + \frac{1}{2}at^2$$

You've made a lot of substitutions, and come up with a nice-looking equation for x that only involves variables you already knew the values for at the start.

But does your equation SUCK?

Use the space on the right to jot down as many different ways you can think of to check over your equation, which is supposed to work OK for any values of the variables in it.

S – Size. I guess I could try the value $t = 2.0$ s in the equation to see if it gives the same answer as it did before. And maybe try some other values as well.

U – Units. I'd like to check the units, but I'm not sure how to do that.

C – Calculations. I think I did the substitutions OK.

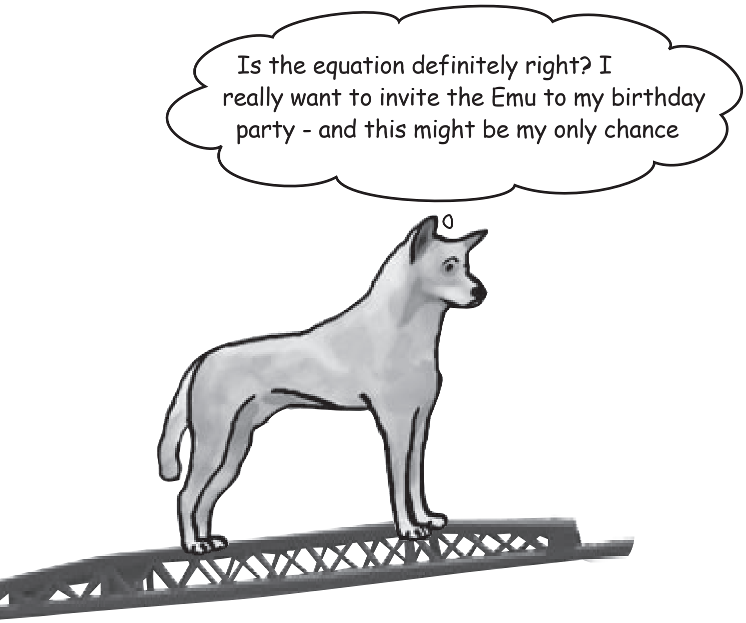
K – 'K'ontext. The equation says that x depends on v_0, a and t, which it probably should!

You did it - you derived a useful equation for the cage's displacement!

The equation you've worked out for the displacement after a certain amount of time is $\mathbf{x} = \mathbf{x}_0 + \mathbf{v}_0 t + \frac{1}{2}\mathbf{a}t^2$. With not a **v** or $\mathbf{v}_{avg}$ to be seen anywhere!

But **is your equation correct**? You don't want the Dingo's birthday plans to go wrong because of a calculation error.

When you work out an equation, you should check it over before you use it.



When you're checking over a numerical answer, asking yourself if it SUCKs is a good tool. But not all of the parts of SUCK are applicable to an equation.

So instead, we're going to think GUT.

G - Graph. Does your equation describe the graph?

U - Units. Does each term have the same units?

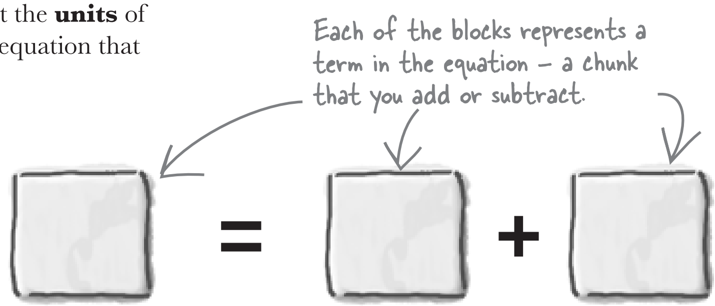
T - Try out extreme values (or values you already know the answer for) in your equation.

We're going to do the units part first, as it's the quickest check you can do once you've got used to how it works.

Check your equation using Units

A quick way of checking your equation is to think about the **units** of each of its **terms** (a term is one of the chunks in your equation that you **add** or **subtract**).

You're only allowed to add or subtract things that have the same units, as something like "2 seconds + 3 meters" is meaningless. So **every term in your equation must have the same units**.



Every term in your equation must have the same units, so you can add or subtract them.

A term can be anything from a single variable to a group of variables and numbers multiplied together. You need to keep track of the units of each variable to make sure that each term has the same units.

A term may be just a single variable.

A term may be a load of numbers and variables multiplied or divided by each other.

$$x = x_0 + v_0t + \frac{1}{2}at^2$$

A term can be anything from a single variable to a group of variables and numbers multiplied together.

Sharpen your pencil

Does your equation make sense - do all of its terms have the same units? Fill in the table to find out.

Some answers are already filled in for you.

Term	x	x_0	v_0t		$\frac{1}{2}at^2$		
Units of variable	$[x]$	$[x_0]$	$[v_0]$	$[t]$	$[\frac{1}{2}]$	$[a]$	$[t]$
			m/s	s			
Units of term	$[x]$	$[x_0]$	$[v_0t]$ = m/s × s		$[\frac{1}{2}at^2]$ = ...		
			$\frac{m}{\not{s}} \times \not{s} = m$				

Square brackets means 'units of'. So [x] = meters.

As these variables are multiplied together, you can work out the units of the term by multiplying the units of each variable together.

Sharpen your pencil Solution

Numbers are dimensionless and don't have units.

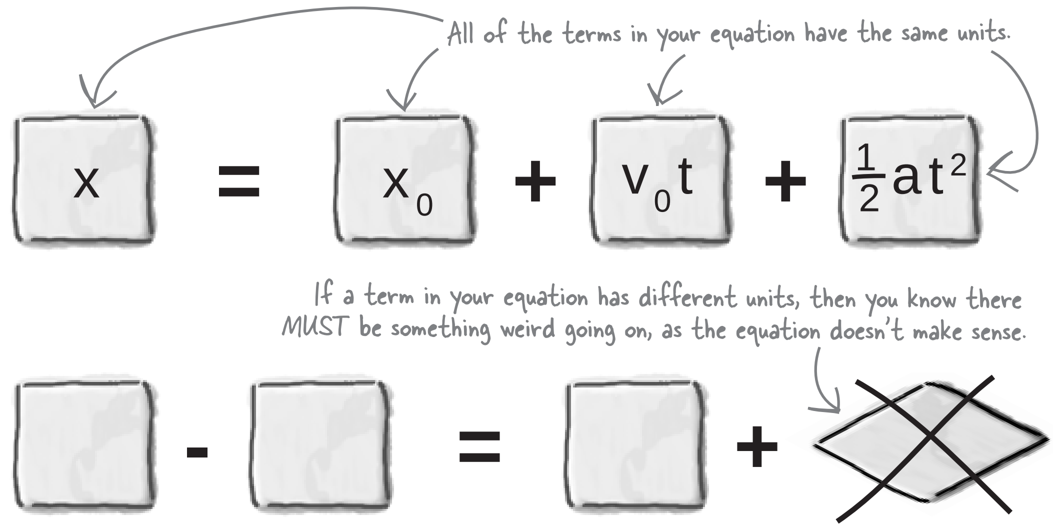
Does your equation make sense - do all of its terms have the same units? Fill in the table to find out.

Term	x	x_0	v_0t		$\frac{1}{2}at^2$		
Units of variable	$[x]$	$[x_0]$	$[v_0]$	$[t]$	$[\frac{1}{2}]$	$[a]$	$[t]$
	m	m	m/s	s	No units.	m/s²	s²
Units of term	$[x]$	$[x_0]$	$[v_0t]$ = m/s × s		$[\frac{1}{2}at^2]$ = m/s² × s²		
	m	m	$\frac{m}{\cancel{s}} \times \cancel{s}$ = m		$\frac{m}{\cancel{s^2}} \times \cancel{s^2}$ = m		

Square brackets means 'units of'. So [x] = meters.

All of the terms have units of m, so the equation's units check out OK.

All of the terms in your equation have the same units (meters) so your equation makes sense.



Anything you add or subtract must have the same UNITS.

Each TERM in the equation

there are no Dumb Questions

Q: Why is it important for all the terms in an equation to have the same units?

A: You can't add things together which have different units. The question "What is 2.0 seconds + 3 meters?" is meaningless, because you can't add meters to seconds.

Q: But you can multiply and divide things that have different units, right?

A: Yes - for example, when you divide a displacement by a time to work out something's velocity in meters per second.

Q: What would I do if it turned out that one of the terms in my equation had different units from the rest?

A: If one of the terms has different units from the others, you probably made a little slip with the math when you were rearranging an equation.

Q: What kinds of mathematical slips should I be on the lookout for?

A: If one of the terms sticks out when you compare the units, look back and see if you made a slip with that term when you were rearranging your equation.

For example, maybe you meant to multiply everything by *t* when you were rearranging your equation, but missed doing that to one of the terms.

Q: Does checking the units like this guarantee 100% that my equation's right?

A: No, not totally. Thinking about the units of each term of your equation will help to catch any mistakes that altered the units.

Q: What kinds of mistakes don't alter the units?

A: Perhaps you meant to multiply everything by 2 when you were rearranging your equation. Since 2 is just a number, it doesn't have any units, so this method wouldn't pick up on that.

The other thing that sometimes happens is missing off a subscript, like writing x instead of x_0. As x and x_0 both have the same units, this wouldn't get picked up.

Q: How would I find a mistake if it doesn't involve units?

A: You can compare your equation with your graph and try out some extreme values in it to see if it really does describe reality ...

If all of the terms in your equation that need to be added to or subtracted from each other have the same units, you're doing great!

Check your equation by trying out some extreme values

You've worked out an equation that describes the displacement - time graph for a falling object: $\mathbf{x} = \mathbf{x}_0 + \mathbf{v}_0 t + \frac{1}{2}\mathbf{a}t^2$. And you've just confirmed that each term in the equation has the same **units**.

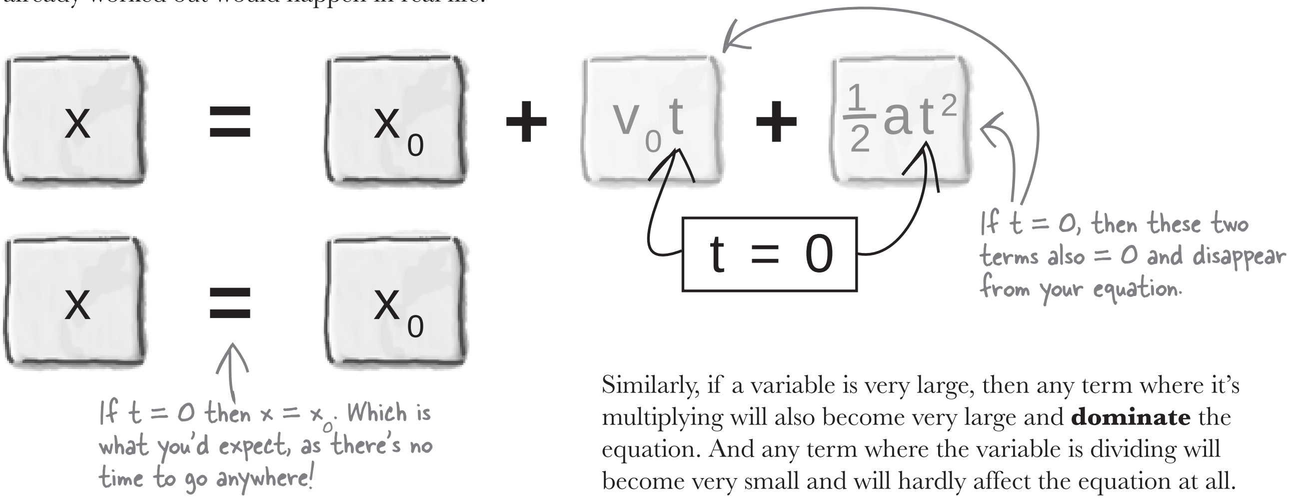
But that doesn't totally guarantee that the equation is right.

It's also a good idea to check your equation by trying out some **extreme numbers** in it, and comparing what your equation says to what would happen in real life. So ask yourself things like "what would happen if the time was **zero**?" or "what would happen if the initial velocity was **very large**?"

In real life, if the time was zero, then you wouldn't have time to go anywhere. Your displacement would be the same as $\mathbf{x}_0$, your initial displacement.

Your equation says $\mathbf{x} = \mathbf{x}_0 + \mathbf{v}_0 t + \frac{1}{2}\mathbf{a}t^2$. But if t = 0, then the term $\mathbf{v}_0 t$ is 0 because anything multiplied by zero is zero. Similarly, the term $\frac{1}{2}\mathbf{a}t^2$ is zero. So your equation becomes $\mathbf{x} = \mathbf{x}_0 + 0 + 0$, or just $\mathbf{x} = \mathbf{x}_0$. Which is what you already worked out would happen in real life!

If a variable is zero, then any term where it's multiplying must also be zero.



If a variable is very large, then any term where it's multiplying must also be very large. And any term where it's dividing must be very small.

Similarly, if a variable is very large, then any term where it's multiplying will also become very large and **dominate** the equation. And any term where the variable is dividing will become very small and will hardly affect the equation at all. This is because dividing by a very large number gives you a very small answer.

In real life, if the initial velocity was very large, you'd expect the displacement to be very large as well.

Your equation says $\mathbf{x} = \mathbf{x}_0 + \mathbf{v}_0 t + \frac{1}{2}\mathbf{a}t^2$. If $\mathbf{v}_0$ is very large, then the $\mathbf{v}_0 t$ term will also be very large and will dominate the equation. So your equation would become "$\mathbf{x}$ = something very large", which is what you know will happen in real life.

BE the equation

Your job is to imagine you're the equation. What's going to happen to you at various EXTREMES? When the acceleration is zero? When the acceleration is very large? When the time is zero? When the time is very large? How does v_0 affect you? And most importantly - do you describe reality?!

$$x = x_0 + v_0 t + \frac{1}{2}at^2$$

Extreme	What happens in real life?	What happens to your equation?	Does your equation describe what happens in real life?
$t = 0$	You've had no time to move, so you won't have gone anywhere.		
t is large		x is large because the v_0t and the $\frac{1}{2}at^2$ terms dominate.	
$a = 0$			
a is large			
v_0 is zero		The equation becomes $x = x_0 + \frac{1}{2}at^2$	
v_0 is large	Your displacement is large, as you're going really fast right from the start.		

BE the equation - SOLUTION

Your job is to imagine you're the equation. What's going to happen to you at various EXTREMES? When the acceleration is zero? When the acceleration is very large? When the time is zero? When the time is very large? How does v_0 affect you? And most importantly - do you describe reality?!

$$x = x_0 + v_0 t + \frac{1}{2}at^2$$

Extreme	What happens in real life?	What happens to your equation?	Does your equation describe what happens in real life?
$t = 0$	You've had no time to move, so you won't have gone anywhere.	$x = x_0$	Yes – it says that you stay where you started and don't go anywhere.
t is large	Your displacement is large because you travel for a long time.	x is large because the v_0t and the $\frac{1}{2}at^2$ terms dominate.	Yes – it predicts that your displacement is large.
$a = 0$	The velocity is constant.	The equation becomes $x = x_0 + v_0t$	Yes. This is similar to the equation distance = speed × time except with vectors and the initial displacement, x_0, in there.
a is large	The velocity will get faster and faster more quickly.	The $\frac{1}{2}at^2$ term dominates more and more as t^2 increases.	Yes – it says that you get faster more quickly.
v_0 is zero	x will depend only on the acceleration and x_0 as you have no velocity at the start.	The equation becomes $x = x_0 + \frac{1}{2}at^2$	Yes – displacement only depends on acceleration and x_0, and not on v_0.
v_0 is large	Your displacement is large, as you're going really fast right from the start.	x is large because the v_0t term dominates.	Yes – it predicts that you go a long way.

Graph and equation story magnets

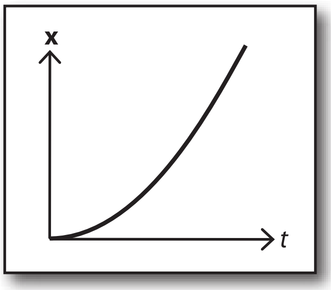
As graphs and equations both represent reality, a third (and final) way you can check your equation is to see if it **tells the same story as your graph**.

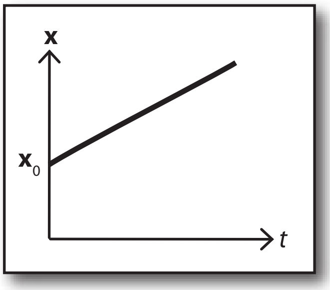
Your job is to match up each graph and an equation with a story. Each magnet will be used exactly once.

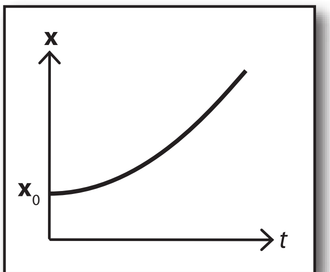
All of the equations are based on the equation you worked out, $x = x_0 + v_0t + ½at^2$ but with some of the variable equal to zero (like you've just been thinking about) which leads to some of the terms being missing.

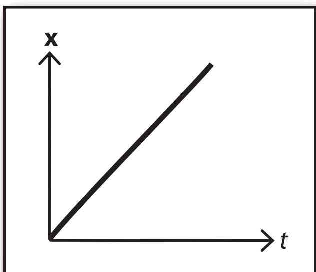
A couple of the story magnets have been left blank so that you can make up your own stories to describe the graphs! Think about using phrases like "constant velocity", "zero velocity", "constant acceleration" or "zero acceleration" in your stories.

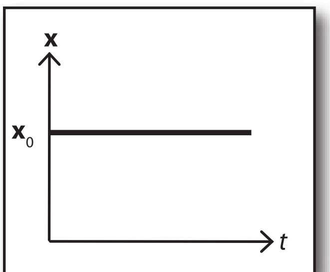
Write your own stories on the blank magnets.











A car sitting at traffic lights at displacement x_0 from home pulls away with constant acceleration.

A person in an office sits in the same position for a long time.

A cage is dropped from a crane from initial displacement $x_0 = 0$.

$x = x_0$

$x = x_0 + v_0t$

$x = v_0t$

$x = x_0 + ½at^2$

$x = ½at^2$

Graph and equation story magnets - SOLUTION

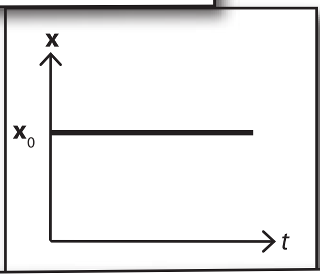
As graphs and equations both represent reality, a third (and final) way you can check your equation is to see if it **tells the same story as your graph**.

Your job is to match up each graph and an equation with a story. Each magnet will be used exactly once.

All of the equations are based on the equation you worked out, $x = x_0 + v_0t + ½at^2$ but with some of the variable equal to zero (like you've just been thinking about) which leads to some of the terms being missing.

A couple of the story magnets have been left blank so that you can make up your own stories to describe the graphs! Think about using phrases like "constant velocity", "zero velocity", "constant acceleration" or "zero acceleration" in your stories.

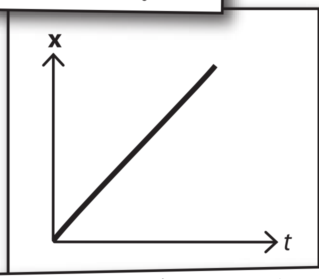
$x = x_0$



A person in an office sits in the same position for a long time.

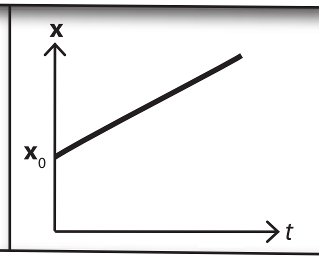
The equation doesn't have the variable t in it. If x doesn't depend on t, then x must be constant.

$x = v_0t$



A person walks from home (where x = 0) to work at a constant velocity (with zero acceleration).

$x = x_0 + v_0t$

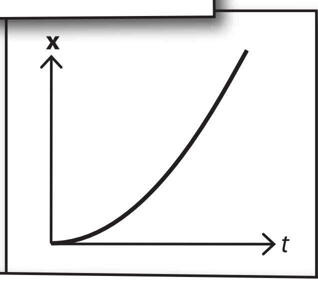


A car joins a long straight freeway at position $x = x_0$, and hits cruise control to travel at a constant velocity (zero acceleration)..

It doesn't matter what story you made up, as long as you realized that a constant slope on the x–t graph means a constant velocity.

Think of graphs and equations "telling stories" about what happens.

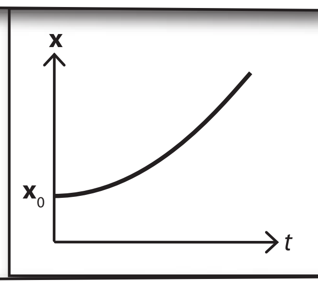
$x = ½at^2$



A cage is dropped from a crane from initial displacement $x_0 = 0$.

These graphs are curved because of the t^2 part of the equation. $1^2 = 1$, $2^2 = 4$, $3^2 = 9$, and so on. As t increases, the t^2 term dominates and makes the graph get steeper and steeper.

$x = x_0 + ½at^2$



A car sitting at traffic lights at displacement x_0 from home pulls away with constant acceleration.

Your equation checks out!

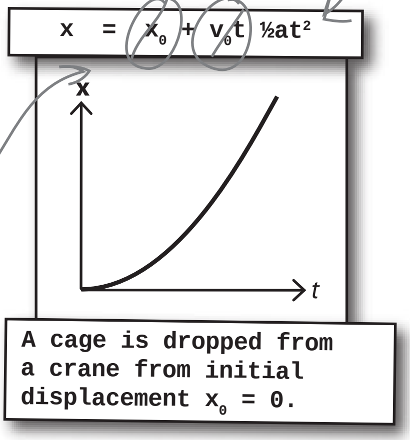
Your equation $\mathbf{x} = \mathbf{x}_0 + \mathbf{v}_0 t + \frac{1}{2}\mathbf{a}t^2$ passes all of the tests you've done on it.

To check your equation over, check your GUT.

G

The **graph** and the equation both tell the same story.

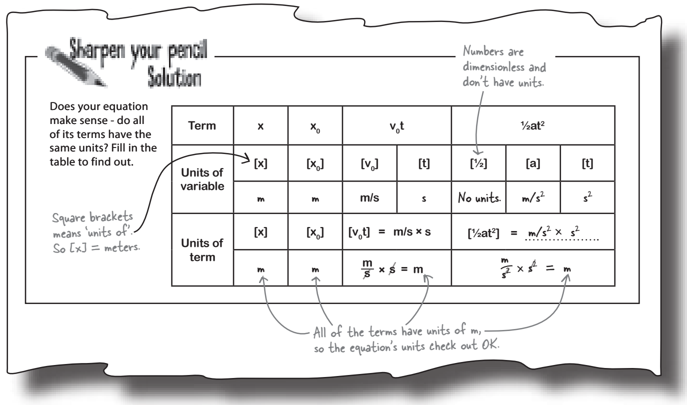
The t^2 part means that the x–t graph is curved.



On this occasion, $x_0 = 0$ (so the graph starts at $x = 0$) and $v_0 = 0$ (as you're dropping it from a standing start).

U

All of the terms in the equation have the same **units** (meters).



Sharpen your pencil Solution

Does your equation make sense - do all of its terms have the same units? Fill in the table to find out.

Term	x	x_0	v_0t		½at^2		
Units of variable	[x]	[x_0]	[v_0]	[t]	[½]	[a]	[t]
	m	m	m/s	s	No units.	m/s^2	s^2
Units of term	[x]	[x_0]	[v_0t] = m/s × s		[½at^2] = $m/s^2 \times s^2$		
	m	m	$\frac{m}{s} \times s = m$		$\frac{m}{s^2} \times s^2 = m$		

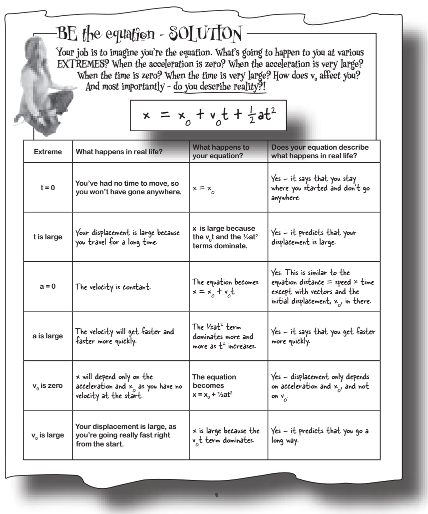
Square brackets means 'units of'. So [x] = meters.

Numbers are dimensionless and don't have units.

All of the terms have units of m, so the equation's units check out OK.

T

When you **try** out extreme values, the equation corresponds to reality.



BE the equation - SOLUTION

Your job is to imagine you're the equation. What's going to happen to you at various EXTREMES? When the acceleration is zero? When the acceleration is very large? When the time is zero? When the time is very large? How does v_0 affect you? And most importantly - do you describe reality?!

$$x = x_0 + v_0t + \frac{1}{2}at^2$$

Extreme	What happens in real life?	What happens to your equation?	Does your equation describe what happens in real life?
t = 0	You've had no time to move, so you won't have gone anywhere.	$x = x_0$	Yes – it says that you stay where you started and don't go anywhere.
t is large	Your displacement is large because you travel for a long time.	x is large because the v_0t and the ½at^2 terms dominate.	Yes – it predicts that your displacement is large.
a = 0	The velocity is constant.	The equation becomes $x = x_0 + v_0t$	Yes. This is similar to the equation distance = speed × time except with vectors and the initial displacement, x_0, in there.
a is large	The velocity will get faster and faster more quickly.	The ½at^2 term dominates more and more as t^2 increases.	Yes – it says that you get faster more quickly.
v_0 is zero	x will depend only on the acceleration and x_0 as you have no velocity at the start.	The equation becomes $x = x_0 + ½at^2$	Yes – displacement only depends on acceleration and x_0, and not on v_0.
v_0 is large	Your displacement is large, as you're going really fast right from the start.	x is large because the v_0t term dominates.	Yes – it predicts that you go a long way.

Sharpen your pencil

The Dingo now wants the cage to fall for 1.5 s. How high should he set the platform of the crane?

Sharpen your pencil Solution

The Dingo now wants the cage to fall for 1.50 s. How high should he set the platform of the crane?

$t = 0\ s$, $x_0 = 0\ m$; $v_0 = 0\ m/s$

$a = 9.8\ m/s^2$

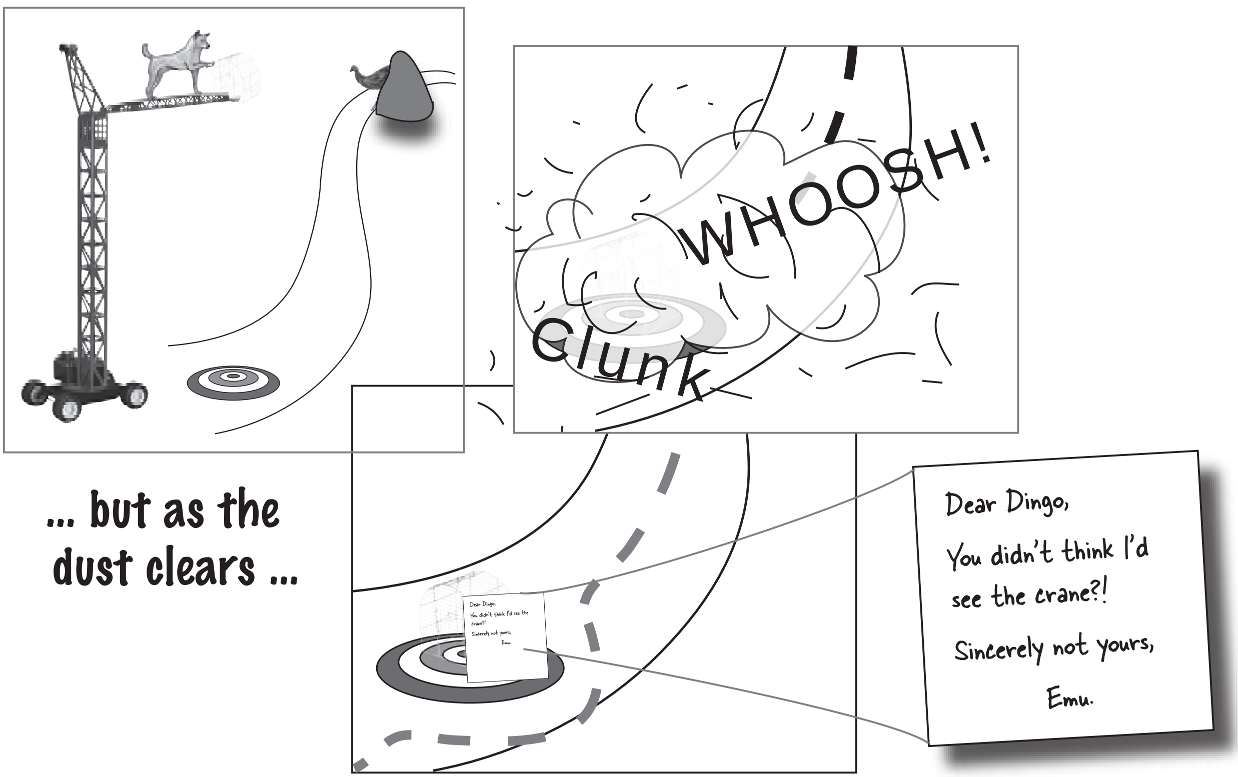
$x = ?\ m$, $t = 1.50\ s$; $v = ?\ m/s$

$$x = x_0 + v_0 t + \tfrac{1}{2}at^2$$

$$x = 0 + 0 + 0.5 \times 9.8 \times 1.50^2$$

$$x = \underline{\underline{11.0\ m}}\ (3\ sd)$$

So the Dingo drops the cage ...



... but as the dust clears ...

To be continued ...

Question Clinic: The “Substitution” Question

A question that asks you to find an “equation”, “expression” or “formula” for something often involves manipulating equations you already know or are given in the question. The main thing is not to panic, especially if you’re given equations you’ve not seen before, and to identify the variables that need to be eliminated from the final equation.

This indicates that you probably don’t have to work out any extra equations apart from the ones they give you.

If you’re aware of the units of your variables, you can test your final answer by making sure each term in your equation has the same units (see page 266).

7. Given the equations:

$$v = v_0 + at \qquad v_{avg} = \frac{x - x_0}{t} \qquad v_{avg} = \frac{v + v_0}{2}$$

a. Write down what each of these symbols conventionally means / represents in physics, and give their units.

b. Find an expression for x in terms of x_0, v_0, a and t.

Always remember that physics equations MEAN something and aren’t just meaningless letters to do algebra with.

This means that they want an equation that says “x = something”, i.e. the x is on its own on the left hand side.

This means that they want the right-hand side to only have x_0, v_0, a and t in it, and NO other variables.

Watch for the variables that you DON’T want in the final equation, and try to work out how to make substitutions that get rid of them.

These questions are mostly algebra and involve substitution to get rid of the variables you don’t want in the final equation. Don’t forget to check your equation, using GUT - Graphs, Units and Trying some extreme numbers - to make sure your equation represents reality!

Try to get a FEEL for what your equation is saying at the extremes of variables. Does it represent reality?

Question Clinic: The "Units" or "Dimensional analysis" Question

Sometimes you'll be asked to show that an equation makes sense by using "dimensional analysis" or by "considering the units". You do this by figuring out the units of each term in the equation. All terms must have the same units, as you're not allowed to add or subtract terms that have different units. The main thing you need to be able to do is to handle units written in scientific notation. This is basically the same as being able to handle numbers written in scientific notation - go back to Chapter 3 if you feel you need a review.

Each TERM in the equation must have the same units.

You're only given the units of t and x, not v_0 or x_0 or a. But you only need the units of one of the terms, and as x is a term you're fine.

8. In the equation $x = x_0 + v_0 t + ½at^2$, the units of t are seconds and the units of x are meters. What are the units of a?

9. In the equation $x = x_0 + v_0 t + ½at^2$, t has dimensions of time and x has dimensions of length. What are the dimensions of a?

Look carefully to see whether the question asks you to give your answer in terms of UNITS or DIMENSIONS.

A question that asks you about dimensions is exactly the same as one that asks you about units, except that you need to give a final answer of "length" instead of "meters", or "time" instead of "seconds" etc.

Some questions may ask you about DIMENSIONS (length, time) instead of units (meters, seconds). The easiest thing to do is to do all your work with units (which you've practiced with more), then give your final answer in dimensions, for example if your answer has units of m/s^2, its dimensions are $length/time^2$.

Honest Harry has a problem

At Honest Harry's Autos, they're always thinking of new ways to make their second-hand cars stand out from the competition. Detailed specs on their website, a one year guarantee, and fluffy dice all help them to stay ahead of the rest.

But Harry's latest idea has got him into trouble.

"Most dealers have the car's acceleration from 0-60 miles per hour in their specs," he explains. "But you can go at 80 mph on the freeway. So I decided to come up with specs for acceleration between 60-80 mph myself. No one else has that!"

But his figures got Harry into hot water when scores of angry motorists returned their cars threatening to sue him for misinformation, as the car's performance from 60-80 mph was far poorer than he'd claimed.

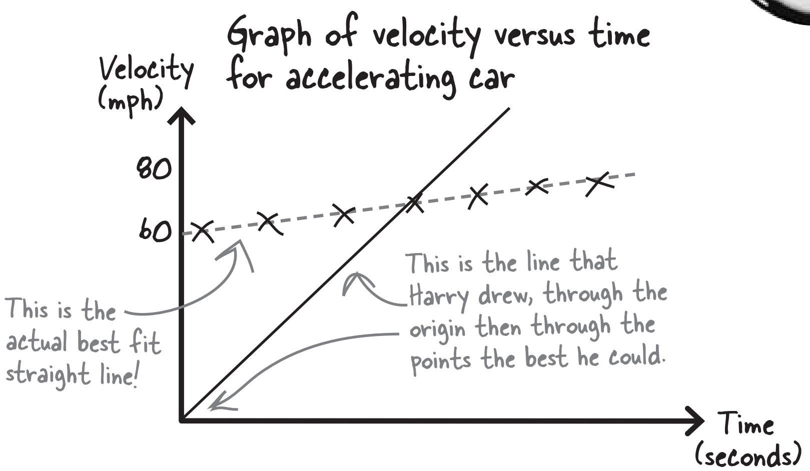
"I dunno what went wrong," Harry says mournfully. "I set things up to plot the speedometer reading on a graph as I accelerated from 60 to 80 mph. Then I drew a best fit straight line through my data points, making sure it went through the origin, and used the slope of that to work out the acceleration that went on the spec sheet."

Why did Harry calculate the wrong acceleration?

The origin of Harry's problem ... is the origin!

Why did Harry calculate the wrong acceleration?

Harry drew his straight line through the origin of his graph (where the velocity is 0 mph and the time is 0 s). But the car's speed at t = 0 was actually 60 mph, so the line he drew should have crossed the vertical axis at 60 mph, not at the origin.



This meant that the slope of his line was much steeper than it should have been - so his values for the acceleration of each car were far too high. No wonder Harry's customers came back to complain!

The equation for the graph is $v = v_0 + at$. where v_0 is the velocity at $t = 0$. Harry's right that he can work out the car's acceleration from the slope of a best fit straight line - but only if the line he draws passes through v_0!

A best fit straight line doesn't need to go through the origin.

Do I really need to **memorize** all of this chapter for my exam? Dude, that was tough!

You only need to memorize a couple of this chapter's equations for your exam

The two important equations from this chapter are $\mathbf{v} = \mathbf{v}_0 + \mathbf{a}t$ and $\mathbf{x} = \mathbf{x}_0 + \mathbf{v}_0 t + \frac{1}{2}\mathbf{a}t^2$. These are the two things you need to memorize for your exam - so that **you can solve all kinds of problems similar to this one**!

And don't worry - you'll pick the equations up soon enough by **using** them to do problems. As long as you practice, there'll be no need for you to do any rote memorization.

Think like a physicist!

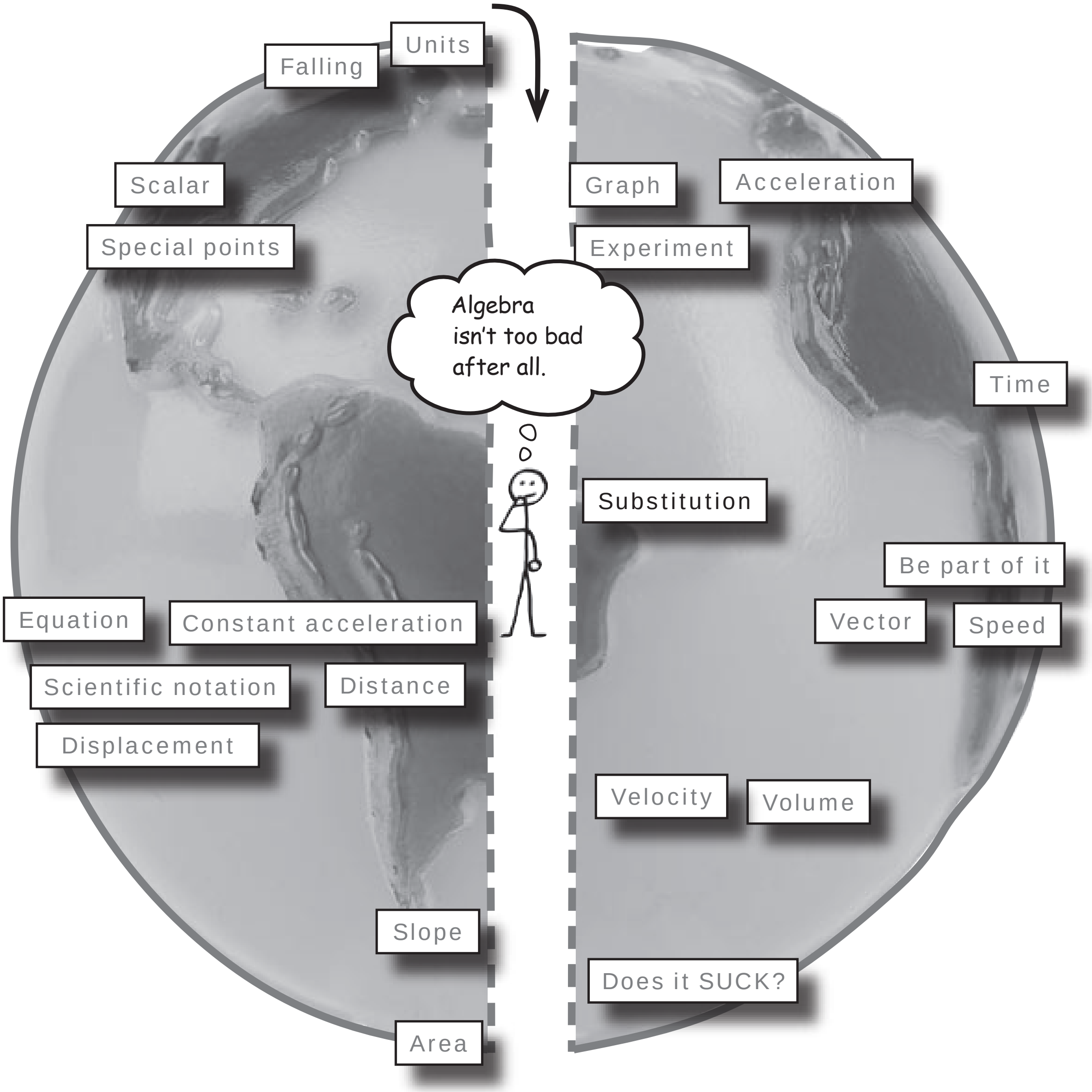
If you practice your tennis forehand, you can guarantee that your tennis results will improve even though you'll never play two identical forehands because each game situation is unique.

In a similar way to your forehand. you can also develop your equation-handling skills to enable you to solve all kinds of problems that are **like** this one, even if they're not exactly the same.

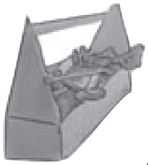
Grooving your physics skills means that you can solve all kinds of problems, not just ones you've memorized.

Throughout your physics career (and definitely in your exam) you'll be interpreting graphs and working out equations from them. If you want to understand what's going on and do well in physics, you need to be able to **rearrange equations**, make **substitutions** and **check your answers**. Which is what you've been practicing in this chapter.

Now you can solve all kinds of problems that are like this one.



☑ **Substitution** Making a substitution is 'getting rid' of a variable from an equation by replacing it with an expression it's equal to (usually from a second equation).



Your Physics Toolbox

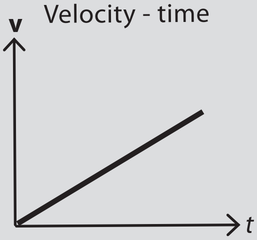
You've got Chapter 7 under your belt and added some problem-solving and answer-checking skills to your toolbox.

Equation of a graph

The equation of a graph has the form:

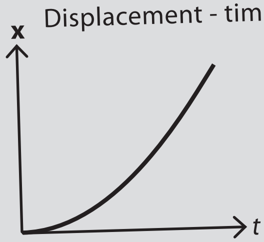
vertical = something to do with horizontal

A fundamental equation of motion



$$v = v_0 + at$$

Another fundamental equation of motion



$$x = x_0 + v_0 t + \frac{1}{2}at^2$$

GUT check

Graph – Do your equation and graph tell the same story?

Units – Do all the terms in your equation have the same units?

Try extreme values – Does your equation mirror reality when you make the variables zero or very large one at a time?

Substitution

Making a substitution is replacing a variable in an equation with an expression that's equal to it.

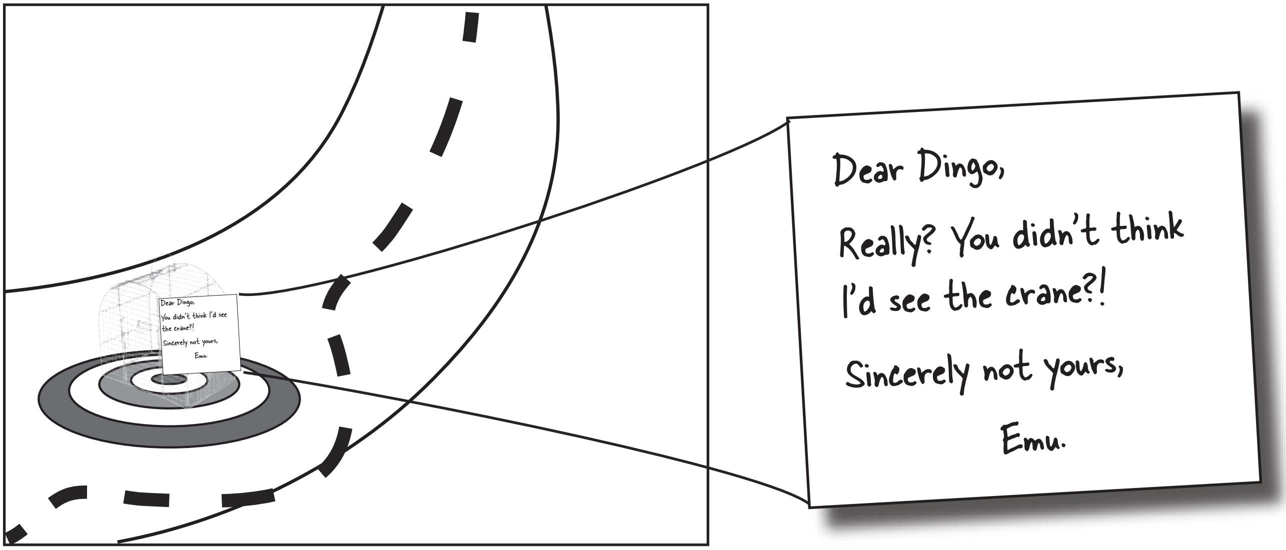
This can be useful if you have a variable in your equation that you don't know a value for.

8 equations of motion (part 2)

Up, up, and... back down

What goes up must come down. You already know how to deal with things that are falling **down**, which is great. But what about the other half of the bargain - when something's launched **up** into the air? In this chapter, you'll add a third key **equation of motion** to your armory which will enable you to **deal with** (just about) **anything**! You'll also learn how looking for a little **symmetry** can turn impossible tasks into manageable ones.

Previously ...

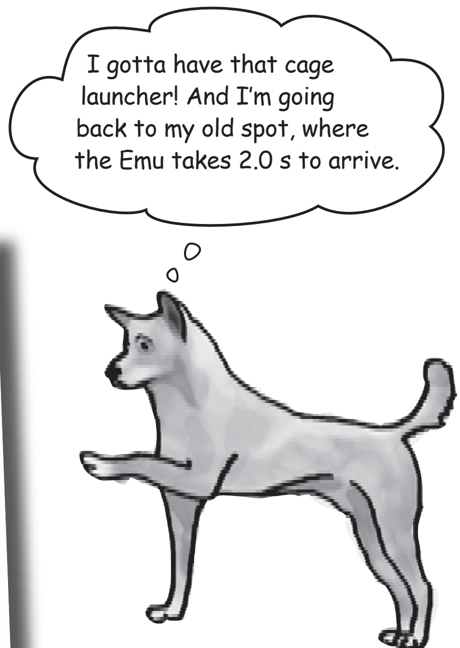


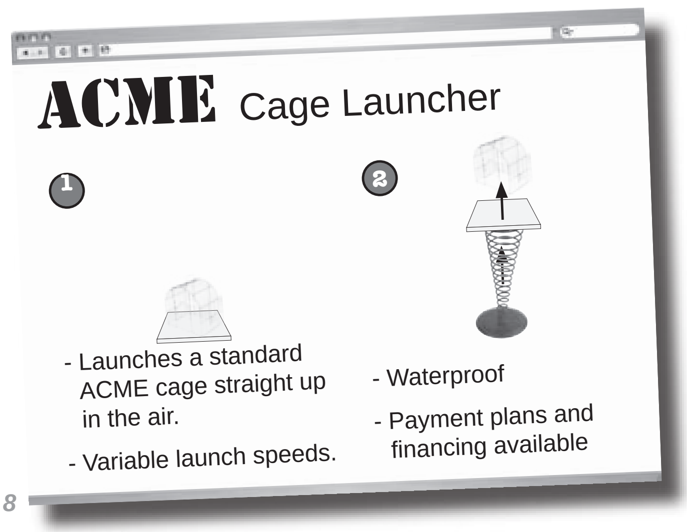
Now ACME has an amazing new cage launcher

But you can't keep the Dingo down for long - especially when ACME has an amazing new cage launcher! Once installed, it'll propel a standard ACME cage straight up in the air at a speed of your choice.

It's ideal for a more subtle approach - you can launch the cage from ground level, instead of having a big crane that the Emu will spot.

You just need to work out what velocity to launch it at so that it lands back on the target when the Emu arrives, exactly 2.0 s after you launched it.





Joe: Hmm, could we try using the equation we worked out in chapter 7? Y'know, $\mathbf{x} = \mathbf{x}_0 + \mathbf{v}_0 t + \frac{1}{2}\mathbf{a}t^2$.

Frank: But that was for a **falling** thing. This time, the cage is **going up**. It's not the same thing.

Joe: But once the cage gets to its maximum height, it falls back down again. So it's kinda the same. Or, well, at least the falling down part of it is!

Frank: But what about the first half, when the cage is going up?

Jim: Actually, the equation might work OK then too. It's supposed to work in any situation where the acceleration's constant, right? And I think that the acceleration due to gravity is constant, whatever direction you're moving in.

Frank: But how can the cage be accelerating downwards, when it's moving upwards?

Joe: Acceleration is rate of change of velocity, right? And the cage gets **slower** as it goes up. So the acceleration vector must be pointing downwards, or else it wouldn't get slower.

Frank: I think I'd find it easier to visualize with a sketch ...

Always start with a sketch!

Sharpen your pencil

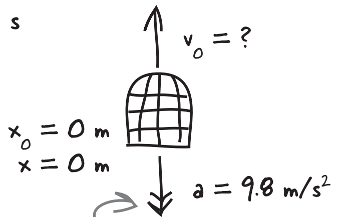
Draw a sketch of the cage just after it's been launched straight up in the air. Mark on the initial velocity vector $\mathbf{v}_0$, the acceleration vector $\mathbf{a}$, and any other information you know about the problem.

Do you think it's going to be OK to reuse the equation $\mathbf{x} = \mathbf{x}_0 + \mathbf{v}_0 t + \frac{1}{2}\mathbf{a}t^2$ in this new scenario? Why / why not?

Sharpen your pencil Solution

Draw a sketch of the cage just after it's been launched straight up in the air. Mark on the initial velocity vector $\mathbf{v}_0$, the acceleration vector $\mathbf{a}$, and any other information you know about the problem.

Cage takes t = 2.0 s to go up then down.



The cage starts and ends at the same point, so x_0 and x are both 0 m.

Give your hand-drawn acceleration vectors double-arrowheads to distinguish them from velocity vectors.

Do you think it's going to be OK to reuse the equation $\mathbf{x} = \mathbf{x}_0 + \mathbf{v}_0t + ½\mathbf{a}t^2$ in this new scenario? Why / why not?

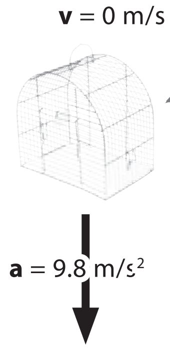
The acceleration vector points down regardless of the direction of the cage's velocity.

The acceleration will be constant at 9.8 m/s² downwards. The equation's supposed to be used for constant acceleration, so it'll probably work.

The acceleration due to gravity is constant

The **equation of motion** you worked out last time, $\mathbf{x} = \mathbf{x}_0 + \mathbf{v}_0t + ½\mathbf{a}t^2$, is supposed to be OK in any situation where the **acceleration is constant**.

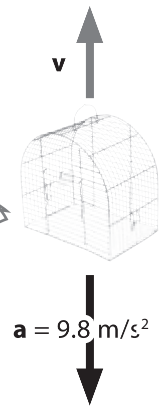
If you can reuse this equation, it'll be much much faster than going through the rigmarole of trying to design and carry out an experiment where you shoot things straight up into the air.



When the cage's velocity is zero, its acceleration due to gravity is still 9.8 m/s².

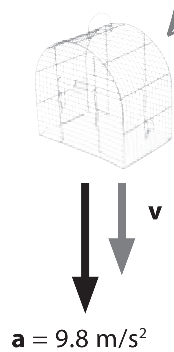
When the cage is going down, it gets faster. This is because it's being accelerated downwards at a rate of 9.8 m/s²

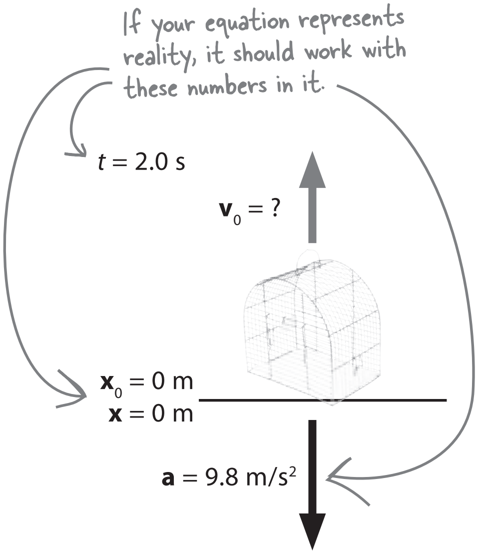
If an object is acted on ONLY BY GRAVITY, it has an acceleration of 9.8 m/s² downwards, whatever its velocity is.



When the cage is going up, it gets slower. This is because it's being accelerated downwards at a rate of 9.8 m/s²

Although you originally worked out this equation from a graph you plotted by dropping things down from a height, any object that's accelerated only by gravity has a **constant acceleration of 9.8 m/s² downwards.** It doesn't matter whether its velocity vector points up, down, sideways, or at an angle.





Good thinking - try to imagine your equation with some numbers in it.

The cage starts and finishes at ground level. This means that both **x** and $\mathbf{x}_0$ are zero, and your equation becomes $0 = 0 + \mathbf{v}_0 t + \frac{1}{2}\mathbf{a}t^2$ when you put these values in.

So for the equation to be true, the two terms on the right hand side, $\mathbf{v}_0 t$ and $\frac{1}{2}\mathbf{a}t^2$, must add up to zero. But how can two terms added together be zero? It's time to ...

BE the equation

Your job is to imagine you're the equation. Here, both x and x_0 are 0 because the cage starts and finishes in the same place. This means that the left hand side of the equation = 0, and on the right hand side, there are two non-zero terms ADDED together. Is there any way this is possible, or will we need to work out a different equation for the cage?

$$x = x_0 + v_0 t + \frac{1}{2}at^2$$

BE the equation - SOLUTION

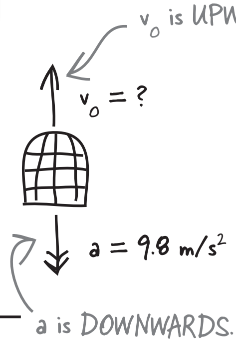
Your job is to imagine you're the equation. Here, both x and x_0 are 0 because the cage starts and finishes in the same place. This means that the left hand side of the equation = 0, and on the right hand side, there are two non-zero terms ADDED together? Is there any way this is possible, or will we need to work out a different equation for the cage?

$$x = x_0 + v_0 t + \frac{1}{2}at^2$$

Velocity and acceleration are vectors.

v_0 is upwards, and a is downwards – as they point in opposite directions, they'll have opposite signs.

It IS possible for a positive number and a negative number to equal zero when you add them together. So the equation could be OK.

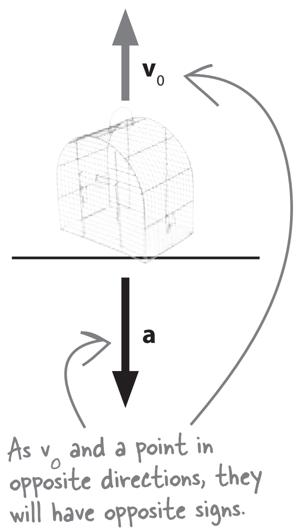


Velocity and acceleration are in opposite directions, so they have opposite signs

When **x** and $\mathbf{x}_0$ are both zero, your equation becomes $0 = 0 + \mathbf{v}_0 t + \frac{1}{2}\mathbf{a}t^2$. So the terms on the right hand side of your equation, $\mathbf{v}_0 t$ and $\frac{1}{2}\mathbf{a}t^2$, must add up to zero. You're interested in what's happening at $t = 2.0$ s, so both t and t^2 are positive. Therefore, the signs of the terms $\mathbf{v}_0 t$ and $\frac{1}{2}\mathbf{a}t^2$ are determined by the signs of $\mathbf{v}_0$ and **a**.

You're working with **vectors**! So as well as having a **size**, $\mathbf{v}_0$ and **a** have a **direction**. The acceleration due to gravity, **a**, always acts **downwards**. But the initial velocity, $\mathbf{v}_0$, is **upwards**. So $\mathbf{v}_0$ and **a** have opposite **signs**. One is positive and the other is negative.

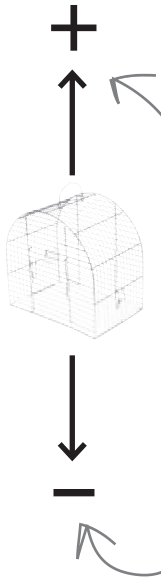
As $\mathbf{v}_0$ and **a** have opposite signs, it's perfectly reasonable to say that there's a certain value for $\mathbf{v}_0$ where $\mathbf{v}_0 t + \frac{1}{2}\mathbf{a}t^2 = 0$. And that's the value you want to work out, as it's the launch velocity for the cage!



Vectors have DIRECTION!

You use positive and negative signs to show the direction.

Do we get to decide which **direction** is positive and which is negative? So this is just another way to use vectors?



Yes. With vectors that point in opposite directions, you get to decide which way is positive and which is negative.

Vectors are vectors, so as long as you are consistent and dealing with **opposites**, you get to choose whether to make up or down the positive direction.

When something's moving through the air, it's usually easiest to make **up the positive direction** and **down the negative direction**.

It's conventional to make up the positive direction and down the negative direction.

But when we were **dropping** the cage, we said that down was positive. Why suddenly change now to make up positive - it's confusing!

Choose the direction that makes the math easier.

It's easy to 'lose' or forget about **minus signs** when you're doing calculations and end up with the wrong answer. When you dropped the cage, its displacement, velocity and acceleration all pointed in the **same direction** - downwards. So by making down the positive direction, you didn't have to deal with minus signs.

But now that the cage is going up then down, with $\mathbf{v}_0$ and **a** pointing in **opposite directions**, you've nothing to gain by making down positive. You're better off sticking with the convention of making up positive so that when you draw a displacement-time graph, up on the graph is the same direction as up in the real world.

You're less prone to making mistakes when doing math with only positive numbers than you are when there are negative numbers around.

there are no Dumb Questions

Q: I still don't get how you can add two numbers together and get zero as your answer.

A: Numbers can be negative as well as positive. If one of the numbers is negative, it can work out like that. For example, if v_0t = -2 and $½at^2$ = 2, then you have -2 + 2 = 0.

Q: Sorry ... I still don't quite see how two numbers added together can be zero?

A: Suppose you spend $10 entering a competition, then win a $10 prize in it. You've earned (-10) + 10 = 0 dollars. The sum of the (negative) entry fee and the (positive) prize win comes out to a zero balance.

Q: Why would I want to add a negative number to a positive number when I can just do a subtraction?

A: Because when you have an equation like $x = x_0 + v_0t + ½at^2$, you don't know in advance which variables are positive and which are negative. But when you put the numbers in, it'll work out as long as you get the minus signs right.

Q: So the variables x_0, x, v_0, v and a could all be negative because they're vectors?

A: Yes, vectors have both a size and a direction. When all of your vectors lie along one line (like in this case, they're either pointing up or down) then you can choose to make one direction positive and the other direction negative.

Q: And I get to choose whatever direction I want to be positive? Either up or down?

A: Yes, as long as you're **consistent** throughout. But it's usual to make up the positive direction so that when you plot a graph, up on the graph corresponds to up in real life.

Q: What would happen if I made down the positive direction instead?

A: The math would all still work out, as long as you make sure you're consistent. You just have to be careful to do the right thing when you're adding or subtracting a negative number.

Q: How would I figure out if I'd made a mistake with the minus signs?

A: You can see if your answer SUCKs. If a minus sign has gone astray, then the answer may end up a very different size from what you'd expect. So make sure you have a rough idea of what size your answer's going to be at the back of your mind, then compare it with the result of your calculation.

Q: OK, so I think I have the negative numbers, vectors and directions all figured out. Is there anything else I should do?

A: As you've never used this equation before to deal with something going up then down, it wouldn't hurt to sketch some graphs to confirm that it's going to be OK ...

Vectors in opposite directions have opposite signs.

Frank: I guess we need to do some graphs that **show** it's OK to use the equation to deal with the cage going straight up in the air and back down again.

Jim: But that's gonna be difficult. We usually draw graphs of **experimental** results, but I don't think we can do that this time. We only get one shot at launching the cage, and if we miniaturize the experiment, we can't really measure the launch speed.

Joe: Hmmm. Maybe we could **sketch graphs** of what we know happens to something that goes up and down. Then put some numbers into the equation and plot it to see if it comes out the same **shape** - like doing that GUT check thing - comparing the equation with the **g**raph by **t**rying out some values.

Frank: But how do we sketch a displacement-time graph for something that goes up then down? Is it curved? Is it straight? Does the shape depend on whether it's going up or down? I don't think we can do that straight off.

Joe: We could start off with the **acceleration-time graph**. We know that has a constant value of 9.8 m/s^2.

Jim: You mean **-**9.8 m/s^2 ... up is positive, so down is negative!

Joe: Yeah, well, acceleration is rate of change of velocity. So the value of the acceleration is the slope of the velocity-time graph. The value of the acceleration is constant: -9.8 m/s^2, so the **slope** of the **velocity-time graph** must also be constant at -9.8 m/s^2.

Frank: Err, a **negative slope**?! What would that look like?!

Jim: I guess it would go the other way, down from left to right instead of up?! Like going downhill instead of uphill?

Joe: That sounds about right. Then when we have the velocity-time graph, we can use the fact that velocity is rate of change of displacement. So the value of the velocity is the slope of the **displacement-time** graph.

Frank: Which lets us draw a displacement-time graph as well.

Jim: Great!

Equations represent reality. If you sketch a graph of what happens in real life, it should be the same shape as the graph for your equation.

Don't worry about numbers here – the important thing is the SHAPES of the graphs.

BRAIN POWER

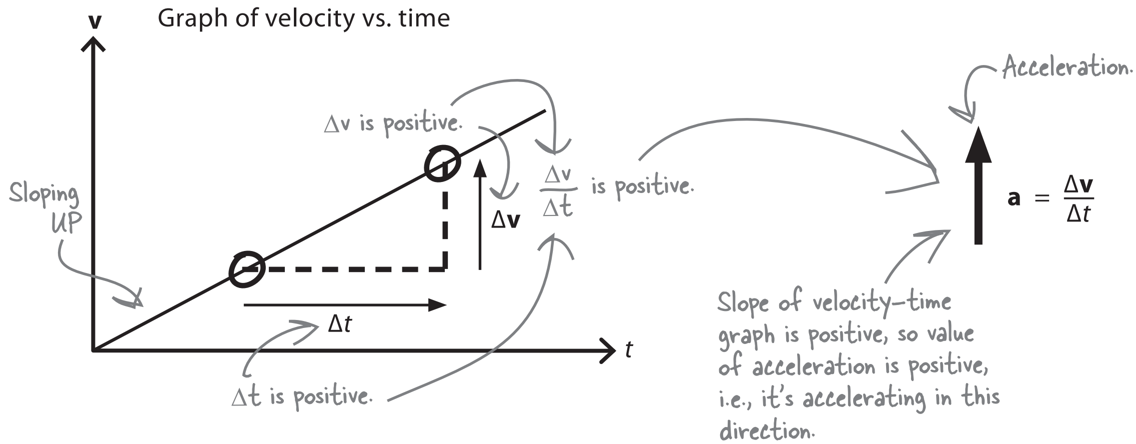
What might the velocity-time and displacement-time graphs look like?

Slope Up Close

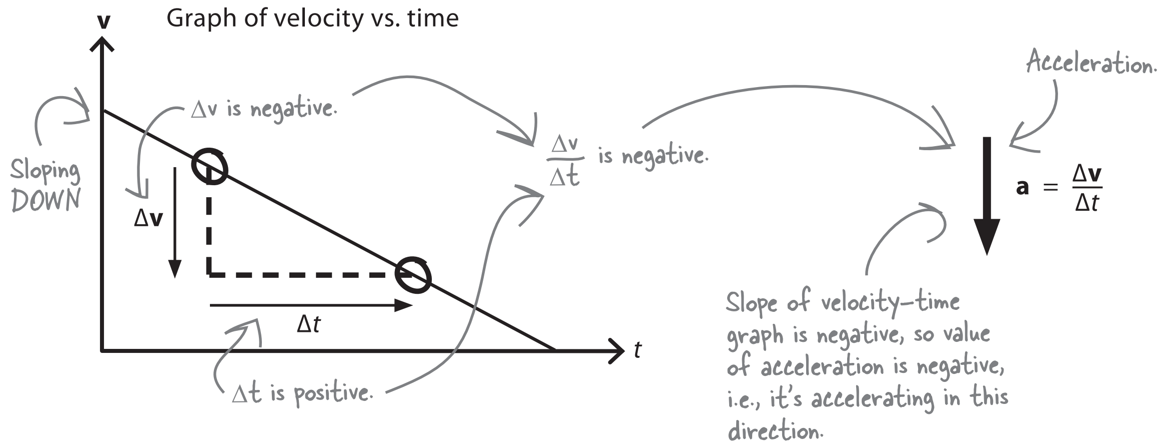
The same principle applies to the slope of a displacement-time graph and the value of the velocity.

Since velocity is a **vector**, it can be either positive or negative. The **slope** of a velocity-time graph shows you the **value** of the acceleration.

If the change in velocity is positive, then the graph slopes **up,** and the acceleration is **positive**. If the change in velocity is **negative**, then the graph slopes down, and the acceleration is **negative**.



If a graph goes up, its slope is positive.



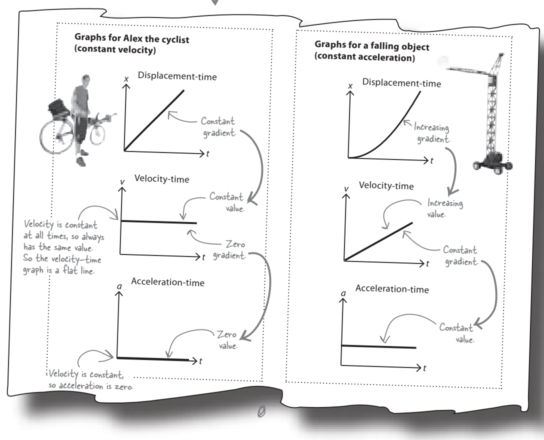
If a graph goes down, its slope is negative.

You already sketched graphs by thinking about values and slopes. What you're doing here isn't too different.

You can use one graph to work out the <u>shapes</u> of the others

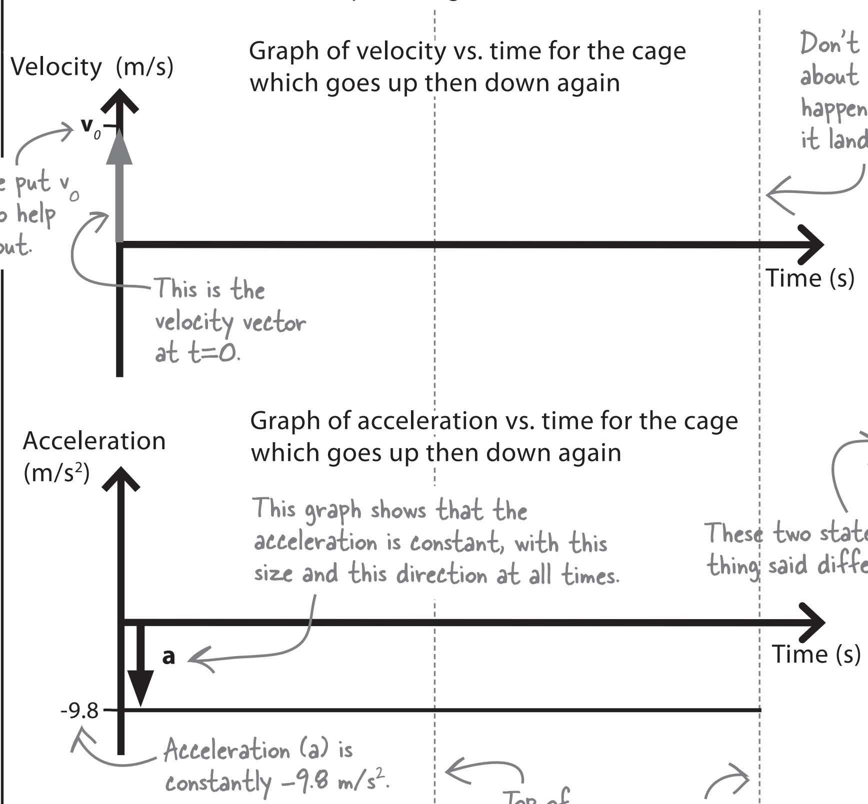
Acceleration, velocity, and displacement are all related to each other. If you have the graph of one of them and some initial values for the others, you can use these to sketch the shapes of the other graphs.

You know that the launched cage has a constant acceleration of -9.8 m/s². Since the **value** of the acceleration is constant and negative, the **slope** of the velocity-time graph must be constant and negative too.



Sharpen your pencil

Use the value of your acceleration-time graph to sketch the shape of the velocity-time graph. Try to imagine the velocity of the cage as it goes up then back down again. The first dotted line is the top of its flight, and the second one is when it lands again.



Don't worry about the displacement-time graph for the moment – you'll do that next.

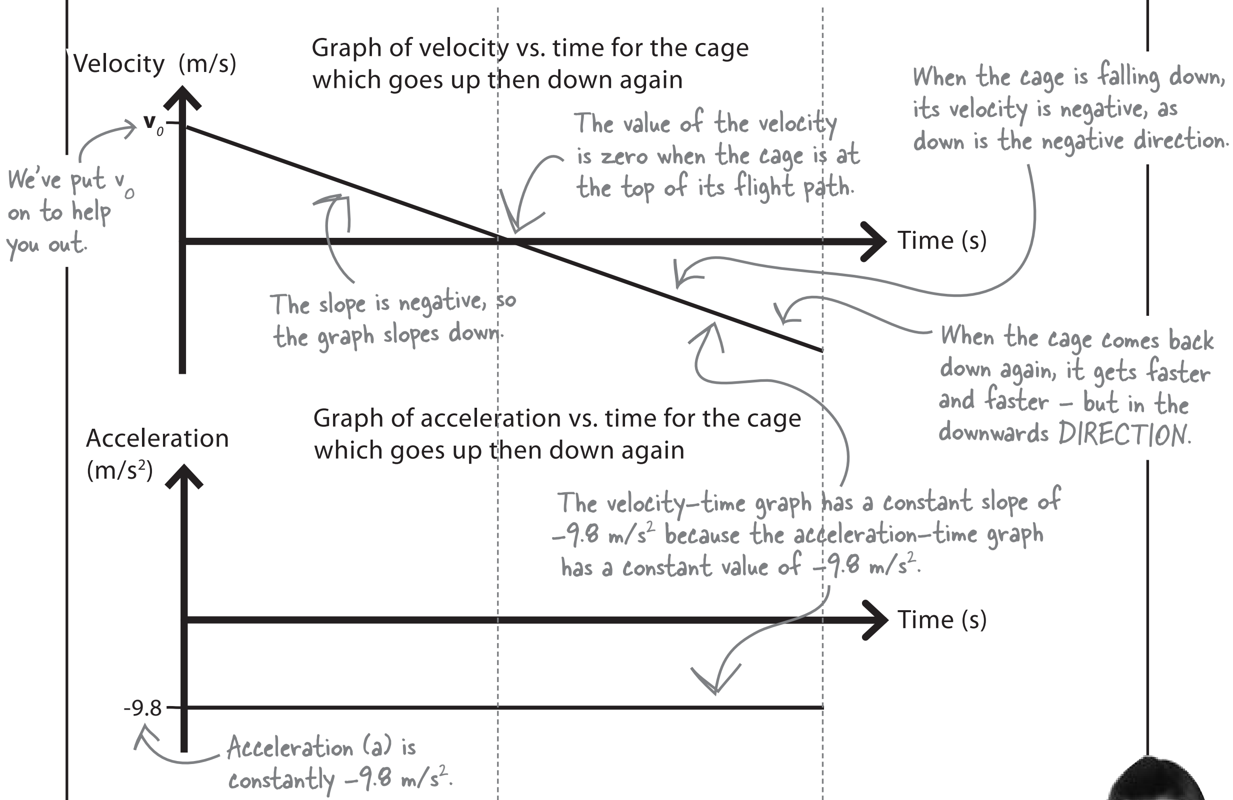
The <u>slope</u> of the velocity-time graph gives the <u>value</u> of the acceleration.

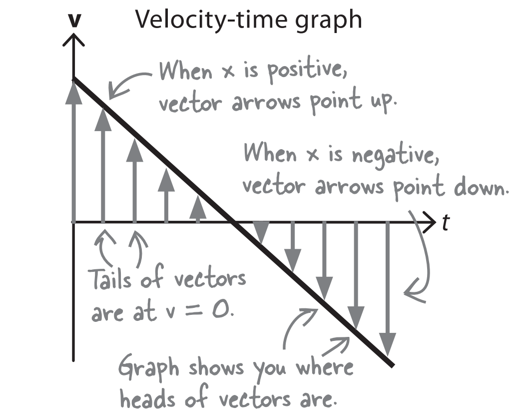
These two statements are the same thing said different ways around.

The <u>value</u> of the acceleration gives the <u>slope</u> of the velocity-time graph.

Sharpen your pencil Solution

Use the value of your acceleration-time graph to sketch the shape of the velocity-time graph. Try to imagine the velocity of the cage as it goes up then back down again. The first dotted line is the top of its flight, and the second one is when it lands again.



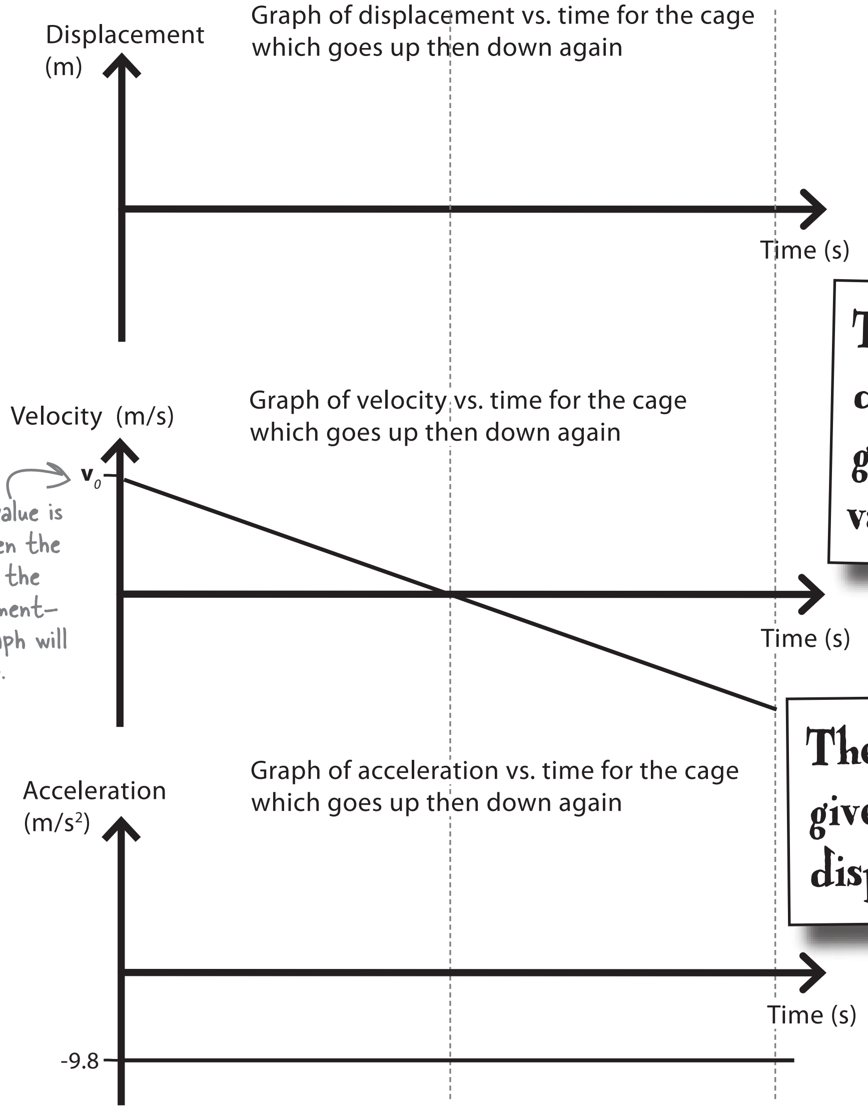


The graphs show the size and direction of a vector at any time

The line on a graph shows the **size** and **direction** of the quantity you're plotting at any **time**. If you imagine the tail of the arrow on the horizontal axis, the head will be just touching the line.

Sharpen your pencil

Now it's time for you to sketch the displacement-time graph for the cage, then annotate it to explain what you've done. The first dotted line is the top of its flight, and the second one is when it lands again.



Another way to do this (or to check over your answer) is to "be the cage" and think about what's going on as you go up then down again.

If this value is high, then the slope of the displacement-time graph will be steep.

The slope of the displacement-time graph gives the value of the velocity.

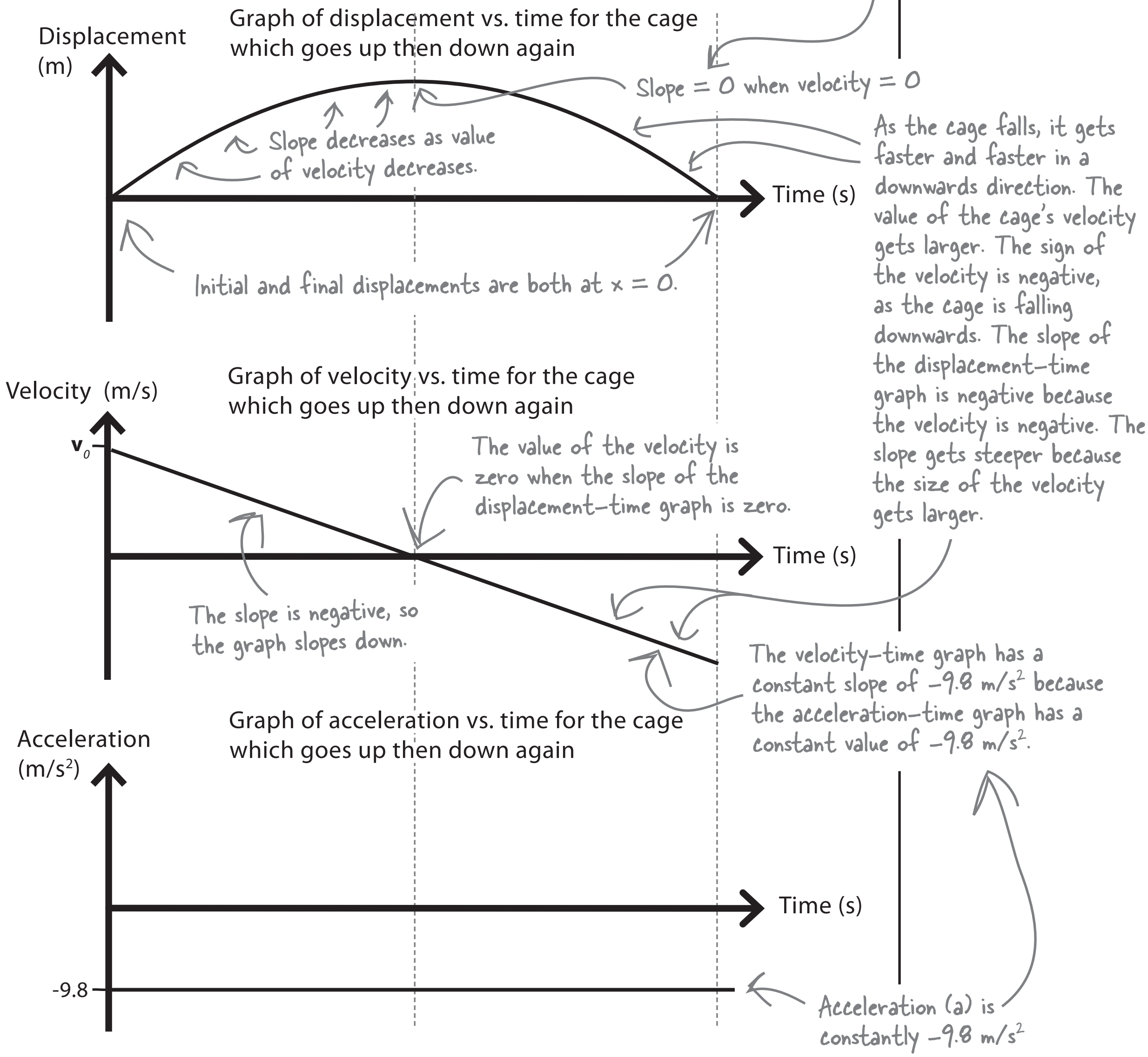
These two statements are the same thing, said different ways around.

The value of the velocity gives the slope of the displacement-time graph.

Sharpen your pencil Solution

Now it's time for you to sketch the displacement-time graph for the cage, then annotate it to explain what you've done. The first dotted line is the top of its flight, and the second one is when it lands again.

You might be used to using the word 'gradient' to indicate the steepness of a graph. In this book, we're using the word 'slope' to indicate the steepness of a graph.



The second half of each graph looks quite like when we dropped the cage from the crane.

If you launch an object straight up, the object's velocity = 0 at the top of its flight.

Yes. The second half is like dropping something from a stationary start.

What goes up must come down! At its highest point, the cage isn't going up any more and hasn't quite started coming down yet. The top of its flight is a 'special point' where its velocity is zero.

So for the second part of its motion, when it's coming back down, the cage behaves exactly as if it's been dropped from its maximum height.

The graph is the same shape as last time, though it's the other way up because this time up is positive.

there are no Dumb Questions

Q: I can't quite believe that the velocity-time graph is just a diagonal line like that. It seems too perfect.

A: The acceleration is constant and negative. So the slope of the velocity-time graph also has to be constant and negative. This is another way of saying that the velocity-time graph is a straight line graph that slopes downwards.

Q: How do you know where to start drawing the diagonal line? You could put it anywhere on the velocity-time graph!

A: Good point! If you didn't know that the initial velocity was v_0, then you wouldn't know where to start drawing the graph.

Q: How do you know what angle to slope the velocity-time graph at?

A: This is just a sketch graph, with no scales on the axes, so the angle doesn't matter much here. Just make sure the line goes through v= 0 when the object is at the top of its flight.

If you were plotting a velocity-time graph with values on your axes, then you'd make its slope the same as the value of the acceleration-time graph.

Q: You said that the line should go through v = 0 when the object is at the top of its flight. How can something's velocity be zero when it's up in the air?

A: There's a split second when the object reaches its maximum height. There, the object's just stopped going up and is just about to come down. At that point, the velocity is zero.

Q: So when the velocity of a launched object is zero, does that mean the displacement's zero as well?

A: No ... it means that the slope of the displacement-time graph is zero.

Q: So you're saying that we can work backwards from the acceleration-time graph to draw the other graphs?

A: Close but not quite - the cage started with velocity v_0 and at $x_0 = 0$. If you hadn't known these initial values, you wouldn't have known where to start sketching the graphs.

Q: But I knew some start values, so the sketches are OK. Can I get on with plotting the equation $x = x_0 + v_0t + ½at^2$ to see if it's the same shape as the sketch?

A: Well, alright then ...

Is a graph of your equation the same shape as the graph you sketched?

You can work out whether it's OK to use the equation $\mathbf{x} = \mathbf{x}_0 + \mathbf{v}_0 t + \frac{1}{2}\mathbf{a}t^2$ by **plotting** it on a graph. If the **shape** is the same as the shape of the graph you already sketched, then you're good to go!

You're already reasonably sure that it's OK to use the equation, as it's supposed to work when the acceleration is constant. But you want to make ABSOLUTELY sure!

As you're just doing a GUT check by drawing a **g**raph and **t**rying some values to see the shape of the graph, it's OK to just choose a value for $\mathbf{v}_0$. After all, you're expecting the cage to go up and eventually come back down again whatever $\mathbf{v}_0$ is (as long as it's positive!), so if the equation gives you that shape, it'll be fine to use it to give the Dingo an answer.

Back in chapter 7, we tried out extreme values for the 'T' of GUT to make sure that the equation we worked out there wasn't completely ridiculous. Here, we're trying some reasonable values to draw a graph. The principle is the same.

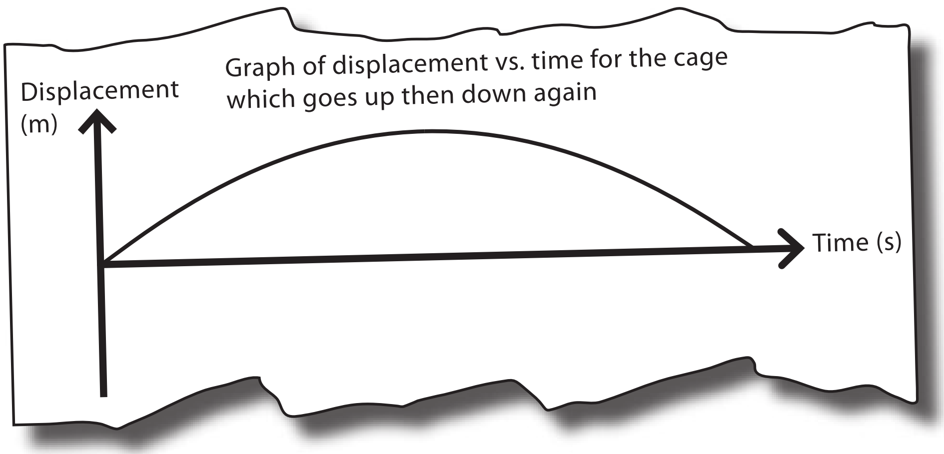
Sharpen your pencil

You want to plot a graph of your equation $\mathbf{x} = \mathbf{x}_0 + \mathbf{v}_0 t + \frac{1}{2}\mathbf{a}t^2$ to see if it's the same shape as the displacement an object has when it goes up then down in real life.

If the equation is correct, then it should produce the same **shape** of graph whatever value of $\mathbf{v}_0$ you choose. We're going to get you to plot the graph of the equation using $\mathbf{v_0} = \mathbf{15\ m/s}$.

a. Fill in the table of values.

b. Plot the graph. Is it the same shape as your sketch graph?



This is the sketch graph you already did a couple of pages ago.

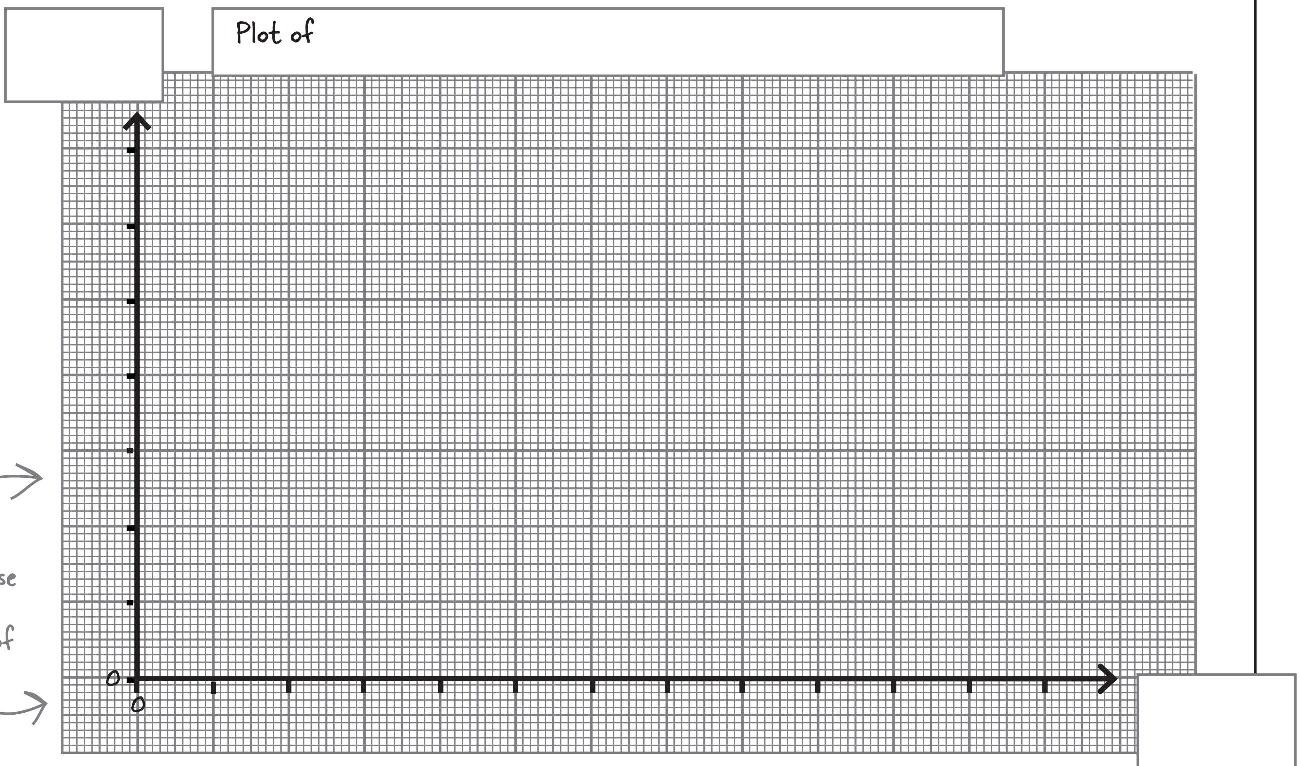
Horizontal axis → time (s)

We're trying $v_0 = 15$ m/s → v_0t

Remember $a = -9.8$ m/s^2 → $\frac{1}{2}at^2$

Vertical axis → $x = x_0 + v_0t + \frac{1}{2}at^2$

time (s)	v_0t	$\frac{1}{2}at^2$	$x = x_0 + v_0t + \frac{1}{2}at^2$
0.0	$15 \times 0 = 0$	$0.5 \times (-9.8) \times 0^2 = 0$	$0 + 0 = 0$
0.5			
1.0			
1.5			
2.0			
2.5			
3.0			



It's up to you to choose a scale for both axes of your graph.

Sharpen your pencil Solution

If you're adding or subtracting, quote your answers to the same number of decimal places as the least precise number.

You want to plot a graph of your equation $\mathbf{x} = \mathbf{x}_0 + \mathbf{v}_0 t + \frac{1}{2}\mathbf{a}t^2$ to see if it's the same shape as the displacement an object has when it goes up then down in real life.

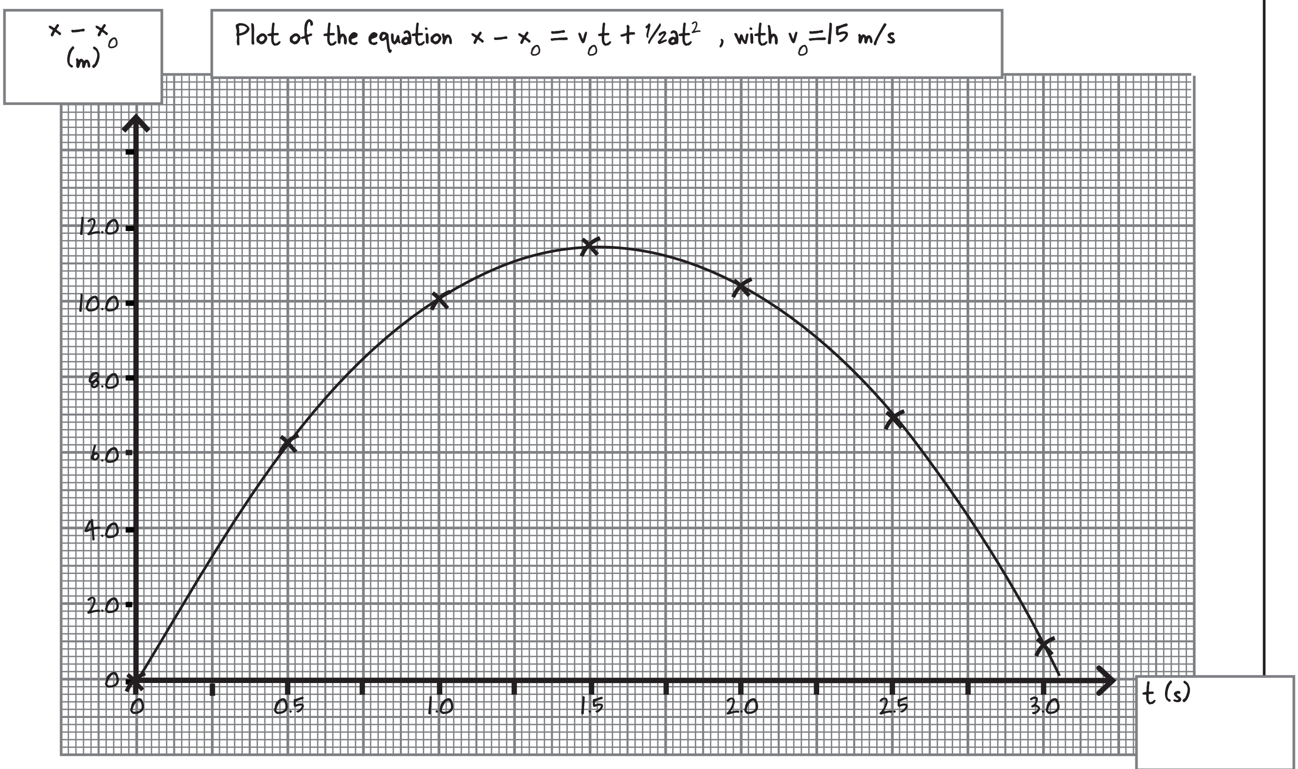
If the equation is correct, then it should produce the same **shape** of graph whatever value of $\mathbf{v}_0$ you choose. We're going to get you to plot the graph of the equation using $\mathbf{v}_0$ **= 15 m/s**.

a. Fill in the table of values.

b. Plot the graph. Is it the same shape as your sketch graph?

It's the same shape as the sketch graph, so it's OK to use the equation!

time (s)	v_0t	$\frac{1}{2}at^2$	$x = x_0 + v_0t + \frac{1}{2}at^2$
0.0	15 × 0 = 0	0.5 × (-9.8) × 0² = 0	0 + 0 = 0
0.5	15 × 0.5 = 7.5	0.5 × (−9.8) × 0.5² = −1.23	7.5 + (−1.23) = 6.3 (1 dp)
1.0	15 × 1.0 = 15.0	0.5 × (−9.8) × 1.0² = −4.90	15.0 + (−4.90) = 10.1 (1 dp)
1.5	15 × 1.5 = 22.5	0.5 × (−9.8) × 1.5² = −11.0	22.5 + (−11.0) = 11.5 (1 dp)
2.0	15 × 2.0 = 30.0	0.5 × (−9.8) × 2.0² = −19.6	30.0 + (−19.6) = 10.4 (1 dp)
2.5	15 × 2.5 = 37.5	0.5 × (−9.8) × 2.5² = −30.6	37.5 + (−30.6) = 6.9 (1 dp)
3.0	15 × 3.0 = 45.0	0.5 × (−9.8) × 3.0² = −44.1	45.0 + (−44.1) = 0.9 (1 dp)

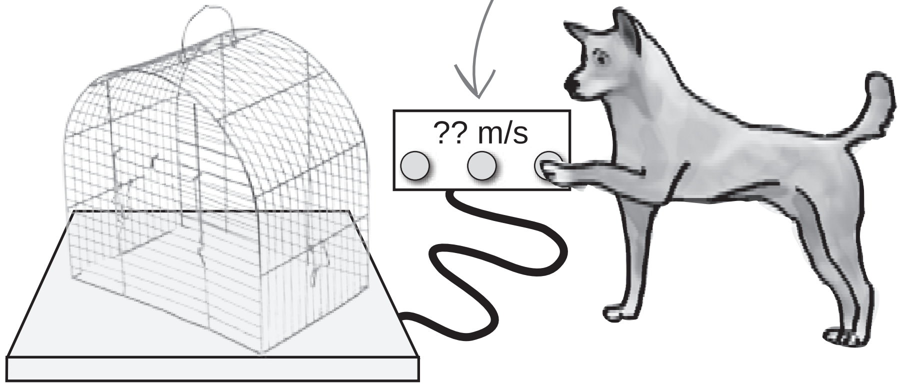


You can check equations by picking values and plotting a graph.

Ready to launch the cage!

The cage is ready for lift-off! You've sketched graphs and tried numbers to confirm that the equation of motion $\mathbf{x} = \mathbf{x}_0 + \mathbf{v}_0 t + \frac{1}{2}\mathbf{a}t^2$ works in any situation where the acceleration is constant - whether the cage starts off falling or going up.

So you're ready to rock and roll and work out a launch velocity for the Dingo!



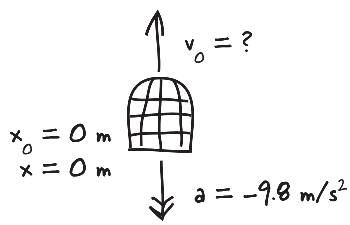
Sharpen your pencil

The Dingo wants to catch the Emu by launching a cage straight up in the air as the Emu rounds a bend. The cage needs to land 2.0 s later, when the Emu reaches the launch site.

What should the initial launch velocity be?

t = 0 s at start and
t = 2.0 s at end.

Up is positive direction.



This is a typical question. We've included the sketch you drew before so that you don't have to do that again.

Sharpen your pencil Solution

The Dingo wants to catch the Emu by launching a cage straight up in the air as the Emu rounds a bend. The cage needs to land 2.0 s later, when the Emu reaches the launch site.

What should the initial launch velocity be?

$t = 0$ s at start and $t = 2.0$ s at end.

Up is positive direction.

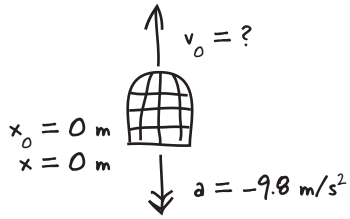
Need to know what v_0 is for x to be 0 after 2.0 seconds.

$$x = x_0 + v_0 t + \frac{1}{2}at^2$$

Rearrange equation to say "v_0 = something."

$$v_0 t = x - x_0 - \frac{1}{2}at^2$$

$$v_0 = \frac{x - x_0 - \frac{1}{2}at^2}{t}$$



Put in numbers.

$$v_0 = \frac{0 - 0 - 0.5 \times (-9.8) \times 2^2}{2}$$

Two negative numbers multiplied together is a positive number.

$$v_0 = 9.8 \text{ m/s (2 sd)}$$

The initial launch velocity should be 9.8 m/s (2 sd).

BULLET POINTS

- **Always start with a sketch.** Draw in all the **sizes** and **directions** of the things you already know plus the ones you want to find out.
- If your equations involve **vectors**, make sure you decide **which direction is positive** and stick to it!
- Be very careful when you're dealing with **negative numbers**!!
- If you already have any one of the displacement-time, velocity-time or acceleration-time **graphs**, you can work out the other two (though sometimes you may need initial values to know where to start drawing the other graphs).
- Before you 'reuse' an equation, think about the **context** you're trying to reuse it in. For instance, does it only work when the velocity is constant, and is the velocity constant in your situation?
- You can use your key **equations** $v = v_0 + at$ and $x = x_0 + v_0 t + \frac{1}{2}at^2$ in any situation where there's **constant acceleration.**

there are no Dumb Questions

Q: I did exactly the same as you did when I rearranged the equation to say "v_0 = something" but got a different answer when I put the numbers in. Why was that?

A: You might need to spend a bit more time practicing with your calculator.

Q: It doesn't seem fair that I do all that work then get marked wrong because I typed something in wrong.

A: Understanding the physics is the most important thing - you won't get marked wrong for the whole problem. In many mark schemes, including AP Physics B, you get most of your points for setting up the answer using the fact that you understand the physics.

Q: Is that why everyone's always going on about how I should show my work and not just write down an equation or an answer?

A: Partly. If you show your work, it's easier for you to spot any little slip you might have made on the way through that caused your answer to turn out wrong.

Q: But people who are good at doing physics and math don't make little slips like losing minus signs or typing in the numbers wrong ... do they?

A: You'd be surprised! That's why we've been practising checking over your answers a lot, using things like SUCK and GUT. If you get into thinking in this way, it's like having a second line of defense against little mathematical slips.

Q: So you're saying that people who've more experience of physics than me make this kind of mistake as well? That's a relief!

A: Oh yes!

Q: So the main thing is that if I show my work, then I can go back and try to fix it if I spot something's not quite right with my final answer?

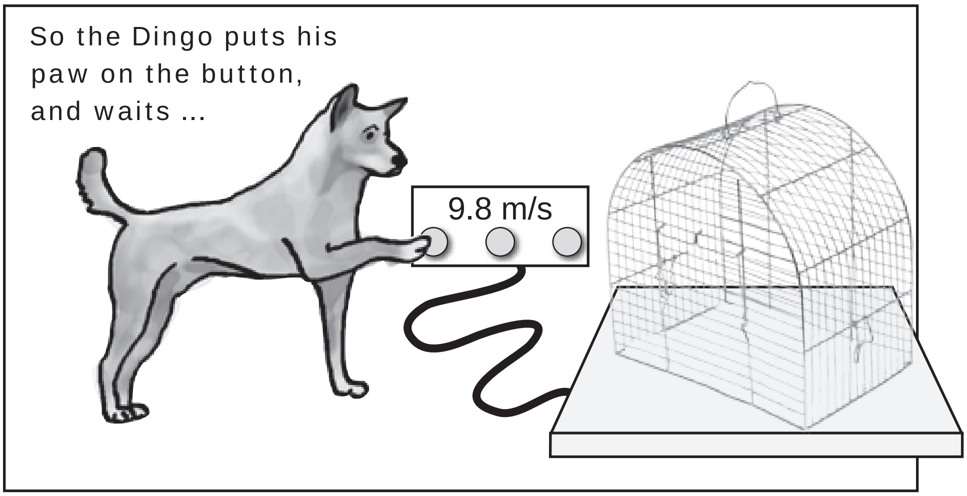
A: Exactly! It might be something as simple as inserting a minus sign and retyping the numbers into your calculator.

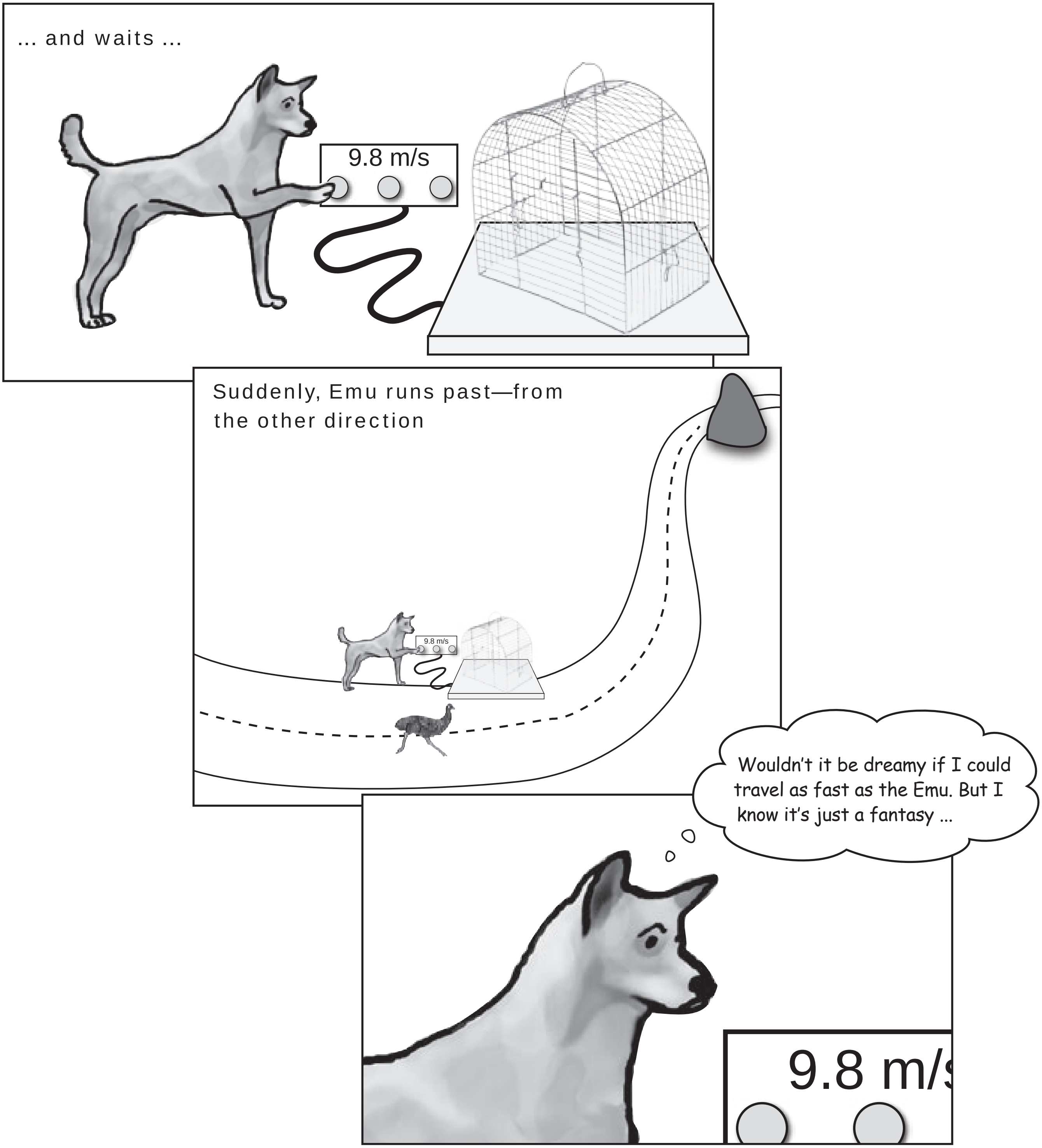
Q: Can we go see how the Dingo's getting on with the launcher?

A: Oh yes ...

If you show your work, it's easier to spot - and fix - little mathematical slips.

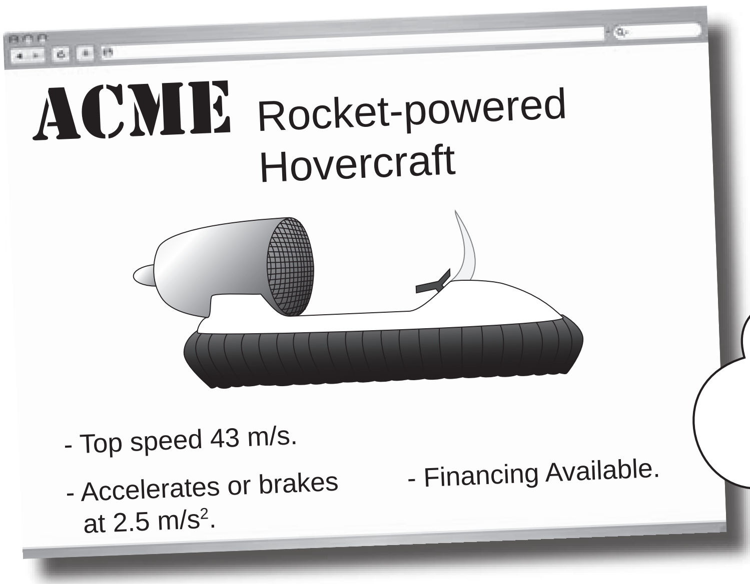
Your launch velocity of 5.0 m/s is definitely right!



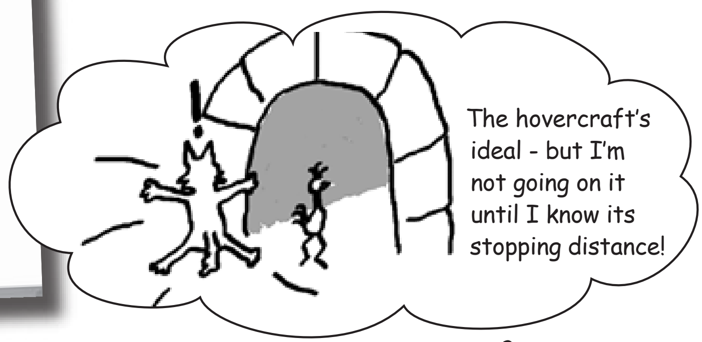

... and waits ...
9.8 m/s
Suddenly, Emu runs past—from the other direction
9.8 m/s
Wouldn't it be dreamy if I could travel as fast as the Emu. But I know it's just a fantasy ...
9.8 m/s

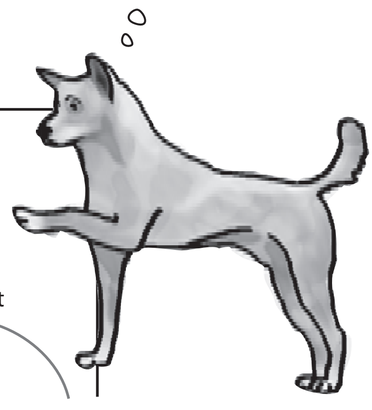
Fortunately, ACME has a rocket-powered hovercraft!

The Dingo goes back to ACME and discovers a rocket-powered hovercraft, which is tailor-made for his needs! Usually, he can't run as fast as the Emu can, but now he can set the hovercraft to any **speed** he likes and use it to catch up with the Emu. More importantly, he can exactly match his speed with the Emu by setting the controls to 15 m/s. Perfect for passing on a party invitation!.



However, the Dingo's a bit nervous, as he's tried this kind of thing before and it didn't go well. Before he goes anywhere near the hovercraft, he wants to know its **stopping distance** (the distance that it travels between applying the brake and the hovercraft actually stopping).





Sharpen your pencil

You need to figure out the stopping distance of the hovercraft. So start off by drawing a **sketch** of the hovercraft. Do one sketch showing the moment that it puts on the brakes while traveling at 15 m/s. Do another sketch showing the moment that the hovercraft comes to a halt (before you release the brakes). Put on all the values you already know, plus the ones you want to find out.

You're only going to do the sketch now. You'll do the calculation later.

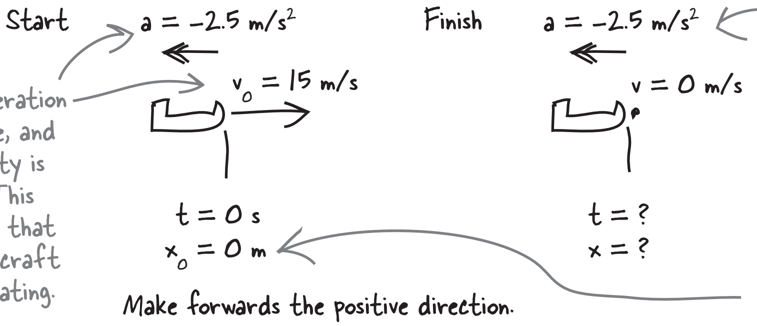
There are various values on the website above.

Hint: Draw two separate hovercraft drawings – one with all the values at the start next to it and the other with all the values at the end next to it.

Sharpen your pencil Solution

You didn't use all of the values on the hovercraft website – for example, the hovercraft's top speed. That's OK – sometimes you won't need to use all the values you're given.

You need to figure out the stopping distance of the hovercraft. So start off by drawing a **sketch** of the hovercraft. Do one sketch showing the moment that it puts on the brakes while traveling at 15 m/s. Do another sketch showing the moment that the hovercraft comes to a halt (before you release the brakes). Put on all the values you already know, plus the ones you want to find out.



The acceleration is negative, and the velocity is positive. This shows you that the hovercraft is decelerating.

The acceleration is backwards because putting on the brakes makes the hovercraft's velocity change in the opposite direction from the direction the hovercraft is currently traveling in.

It's nearly always best to define $x_0 = 0$ as your starting point.

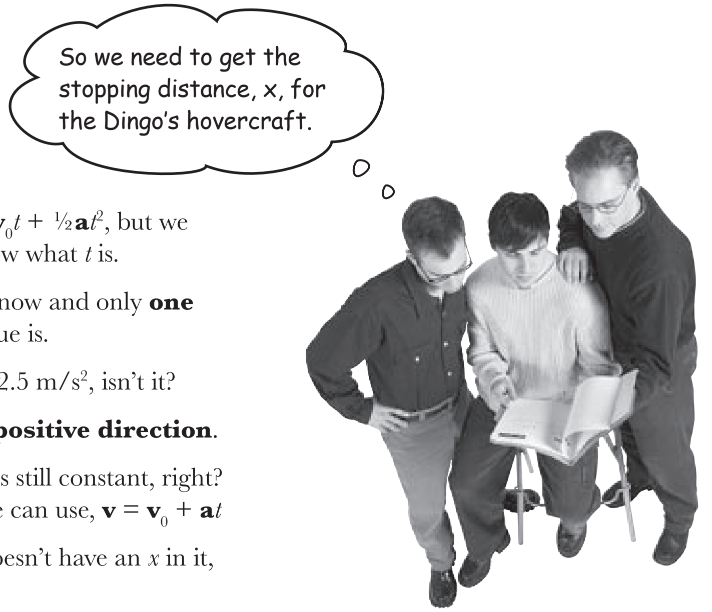
So we just plug these values into the same equation we used before, and that gives us the stopping distance, right?

You don't know the time it takes to stop!

The equation you used before, $\mathbf{x} = \mathbf{x}_0 + \mathbf{v}_0 t + \frac{1}{2}\mathbf{a}t^2$, has several variables in it: $\mathbf{x}$, $\mathbf{x}_0$, $\mathbf{v}_0$, t, and $\mathbf{a}$. You're trying to work out the stopping distance, $\mathbf{x}$, and you already know $\mathbf{v}_0$ and $\mathbf{a}$. So far, so good.

But you don't know t! If you have **one equation** with **two unknowns**, you can't use it (on its own) to work out either of the things you don't know.

Before you try to use an equation, make sure that the value you want to find out is the ONLY variable you don't already know.



Jim: The equation we used last time is $\mathbf{x} = \mathbf{x}_0 + \mathbf{v}_0 t + \frac{1}{2}\mathbf{a}t^2$, but we can't use that equation here because we don't know what t is.

Frank: Yeah, if there are **two** values you don't know and only **one** equation, then you can't work out what either value is.

Joe: The hovercraft's acceleration's **constant** at 2.5 m/s^2, isn't it?

Jim: MINUS 2.5 m/s^2 ! We made forwards the **positive direction**.

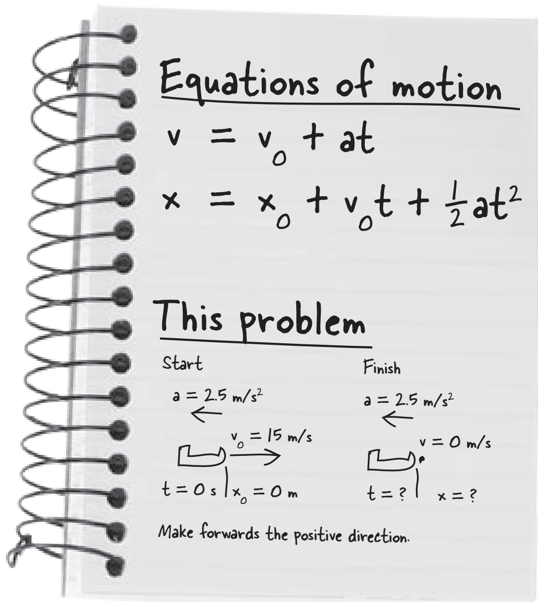
Joe: OK, the acceleration's -2.5 m/s^2 then, but it's still constant, right? So there's a second key **equation of motion** we can use, $\mathbf{v} = \mathbf{v}_0 + \mathbf{a}t$

Frank: I don't see how it helps - that equation doesn't have an x in it, which is what we want to work out!

Joe: But it does have t in it. We know $\mathbf{v}_0$, we know $\mathbf{v}$, and we know $\mathbf{a}$ - the only other variables in the equation. So we could use this equation to work out a value for t to use in the other equation.

Jim: That sounds good, but I don't want to have to work out an **intermediate value** for t every time I want to do something like this. Is there any way we could make a **substitution** to get a more **general equation** that we could use to work out something's displacement when we don't know the time interval? That would be really useful!

Joe: OK, I think you're right about trying to be general. It's more efficient, and we can use our new equation again and again to solve **similar problems**. But how are we going to wind up with an equation for $\mathbf{x}$ that doesn't involve t ?



BRAIN POWER

How might you use your two equations, $\mathbf{v} = \mathbf{v}_0 + \mathbf{a}t$ and $\mathbf{x} = \mathbf{x}_0 + \mathbf{v}_0 t + \frac{1}{2}\mathbf{a}t^2$, to come up with a general equation that lets you work out $\mathbf{x}$ when you don't know what t, the time interval is?

Working out a general equation is better than scribbling down a whole lot of intermediate values.

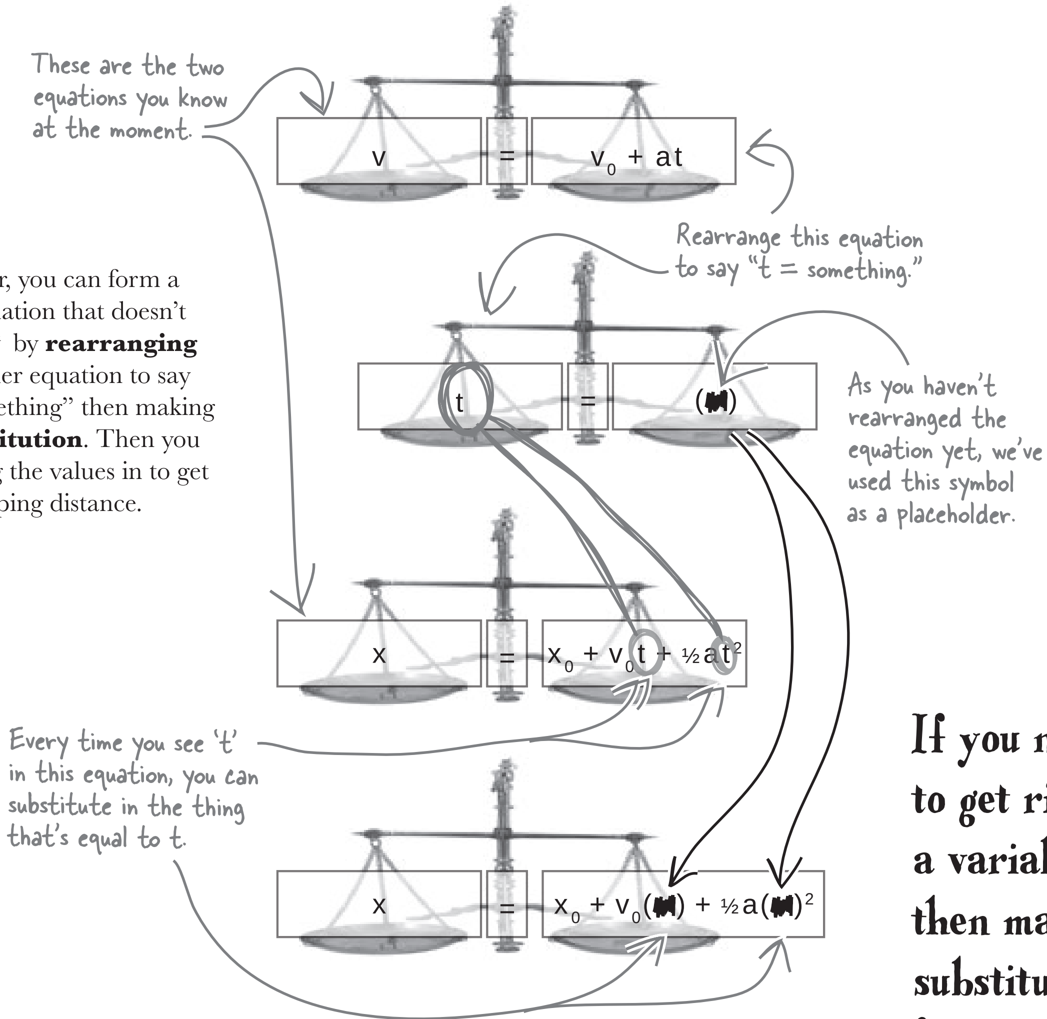
You can work out a new equation by making a substitution for *t*

So far, you know two key equations of motion:

$\mathbf{v} = \mathbf{v}_0 + \mathbf{a}t$ and $\mathbf{x} = \mathbf{x}_0 + \mathbf{v}_0 t + \frac{1}{2}\mathbf{a}t^2$

You want to work out *x*, the stopping distance for the Dingo's hovercraft. The hovercraft is traveling with a particular **velocity** and braking with a particular **acceleration**. However, you don't know the **time** the hovercraft takes to stop, so you can't use $\mathbf{x} = \mathbf{x}_0 + \mathbf{v}_0 t + \frac{1}{2}\mathbf{a}t^2$ directly.

However, you can form a new equation that doesn't involve *t* by **rearranging** your other equation to say "*t* = something" then making a **substitution**. Then you can plug the values in to get the stopping distance.



If you need to get rid of a variable, then make a substitution for it.

Sharpen your pencil

Rearrange the equation $\mathbf{v} = \mathbf{v}_0 + \mathbf{a}t$ so that you can substitute for t in the equation $\mathbf{x} = \mathbf{x}_0 + \mathbf{v}_0 t + ½\mathbf{a}t^2$ and end up with an equation for $\mathbf{x}$ in terms of $\mathbf{v}_0$, $\mathbf{v}$, and $\mathbf{a}$.

You may find it helpful if you use **parentheses** (also called brackets) when you make the substitution, to keep everything that's equal to t together.

We're only asking you to make the substitution at the moment. You don't need to simplify the equation once you've substituted for t – we'll do that next.

Sharpen your pencil Solution

Rearrange the equation $v = v_0 + at$ so that you can substitute for t in the equation $x = x_0 + v_0t + ½at^2$ and end up with an equation for x in terms of v_0, v, and a.

You may find it helpful if you use **parentheses** (also called brackets) when you make the substitution, to keep everything that's equal to t together.

I have two equations to rearrange, then substitute to get rid of t:

Remember to EXPLAIN what you're doing at each stage.

$$x = x_0 + v_0t + ½at^2 \quad (1)$$

$$v = v_0 + at \quad (2)$$

If you number your equations, it makes it easier for you to refer to them in your explanations.

Rearrange (2) to say "t = something" then substitute it into (1).

$$at = v - v_0$$

$$t = \frac{v - v_0}{a} \quad (2')$$

Number this equation 2' to show it's a rearranged version of 2, not an entirely new equation.

Put this in parentheses to make it clear that EVERYTHING in the parentheses is to be multiplied by the other part of the term.

Substitute (2') into (1).

$$x = x_0 + v_0\left(\frac{v - v_0}{a}\right) + ½a\left(\frac{v - v_0}{a}\right)^2$$

Remember to include the 2 part, as the t you replaced was squared.

That's a very complicated-looking equation. Can we try to **simplify** it a bit by **multiplying out the parentheses**?

If you're asked to work out an equation, always give it in its most simple form.

Often when you do a substitution, you're left with a complicated-looking equation with some **parentheses** in it. You can make life a lot easier for yourself by multiplying out the parentheses and seeing if some of the terms in the equation **cancel** by dividing out or adding to zero.

You're going to spend the next few pages making this ugly equation more clear and simple so that you're less likely to make mistakes when you use it in the future.

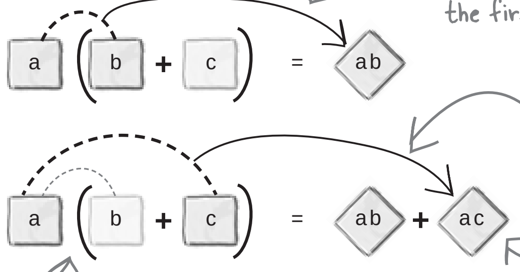
Clear, simple equations are nicer to work with than ugly, complicated equations.

Multiply out the parentheses in your equation

You need to know how to deal with an equation where you have stuff inside parentheses that you need to multiply by something that's outside the parentheses.

For example, $a(b + c) = ab + ac$

Doing this in steps makes it easier. First of all, multiply what's outside by the first thing in the parentheses.



Then multiply what's outside by the second thing in the parentheses (and so on if you have more than two things in there).

Remember – ac means a × c.

Remember – when you write two things next to each other, there's an implied × sign in between.

You can sort out one of the terms on the right hand side like this

The first term on the right hand side of your equation is:

$$\mathbf{v}_0\left(\frac{\mathbf{v} - \mathbf{v}_0}{\mathbf{a}}\right)$$

When you write two things next to each other like this, there's an implied × sign in between.

Everything **inside** the parentheses needs to be multiplied by $\mathbf{v}_0$, which is **outside** the parentheses.

Sharpen your pencil

Multiply out the parentheses for this term on the right hand side of your equation.

$$x = x_0 + v_0\left(\frac{v - v_0}{a}\right) + \tfrac{1}{2}a\left(\frac{v - v_0}{a}\right)^2$$

You can leave the terms we've greyed out as they are for the moment.

Sharpen your pencil Solution

Multiply out the parentheses for the first term on the right hand side of your equation.

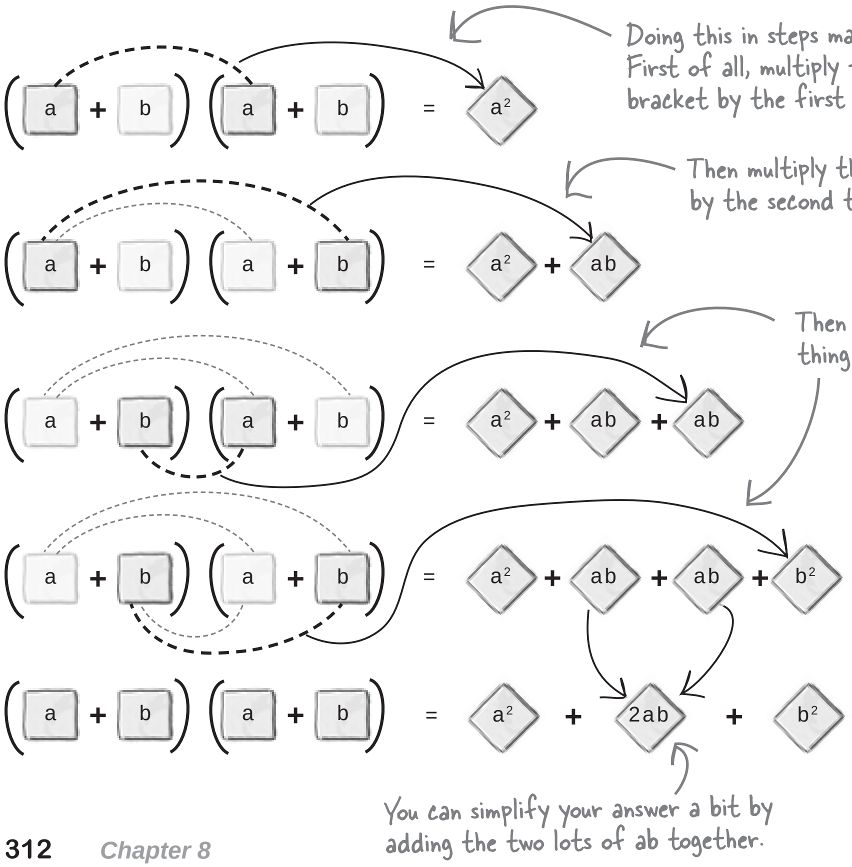
$$x = x_0 + v_0\left(\frac{v - v_0}{a}\right) + \tfrac{1}{2}a\left(\frac{v - v_0}{a}\right)^2 \Rightarrow x = x_0 + \frac{v_0 v - v_0^2}{a} + \tfrac{1}{2}a\left(\frac{v - v_0}{a}\right)^2$$

When you multiply a fraction by a number that's not a fraction, you only multiply the bit on the top of the fraction. So the bit on the bottom stays as 'a,' and doesn't become '$v_0 a$.'

You have two sets of parentheses multiplied together

The other nasty-looking term on the right hand side of your equation involves something inside parentheses squared. If you square something, it means you multiply it by itself.

For example, if you have $(a + b)^2$, it's the same as $(a + b)(a + b)$. You need to multiply everything in the first set of parentheses by everything in the second set of parentheses. This gives you $a^2 + ab + ab + b^2$, which simplifies to $a^2 + 2ab + b^2$ when you add the two lots of ab together.



Multiply every term in the second set of parentheses by every term in the first set of parentheses, one term at a time.

Then you can figure out your second term on the right hand side

The second term on the right hand side of your equation is:

$$\frac{1}{2}\mathbf{a}\left(\frac{\mathbf{v} - \mathbf{v}_0}{\mathbf{a}}\right)^2$$

Because the stuff in the parentheses is squared, this is the same as writing

$$\frac{1}{2}\mathbf{a}\left(\frac{\mathbf{v} - \mathbf{v}_0}{\mathbf{a}}\right)\left(\frac{\mathbf{v} - \mathbf{v}_0}{\mathbf{a}}\right)$$

Everything in the second set of parentheses needs to be multiplied by **everything in the first set of parentheses**, like in the example on the opposite page.

Do the top of the fractions first. Multiply everything on the top by everything on the top. Then do the bottom of the fractions. Multiply everything on the bottom by everything on the bottom.

Sharpen your pencil

Multiply out the parentheses for the second term on the right hand side of your equation.

It's probably easiest to do the squared part inside the parentheses first (similar to the example on the opposite page), then multiply everything through by ½**a**, which is outside the parentheses.

$$x = x_0 + \frac{v_0 v - v_0^2}{a} + \frac{1}{2}a\left(\frac{v - v_0}{a}\right)^2$$

It's easiest to write out the thing in the parentheses × itself first, so you can multiply out the parentheses more easily.

You can leave the terms we've greyed out as they are for the moment.

Sharpen your pencil Solution

Multiply out the parentheses for the second term on the right hand side of your equation.

It's probably easiest to do the squared part inside the parentheses first (similar to the example on the opposite page), then multiply everything through by ½*a*, which is outside the parentheses.

$$x = x_0 + \frac{v_0 v - v_0^2}{a} + \frac{1}{2}a\left(\frac{v - v_0}{a}\right)^2$$

It's easiest to write out the thing in the parentheses × itself like this, so you can multiply out the parentheses more easily.

$$x = x_0 + \frac{v_0 v - v_0^2}{a} + \frac{1}{2}a\left(\frac{v - v_0}{a}\right)\left(\frac{v - v_0}{a}\right)$$

negative × negative = positive

so $(-v_0) \times (-v_0) = v_0^2$

$$x = x_0 + \frac{v_0 v - v_0^2}{a} + \frac{1}{2}a\left(\frac{v^2 - 2vv_0 + v_0^2}{a^2}\right)$$

On the bottom of the fraction, $a \times a = a^2$

Now you have to multiply everything inside the brackets by ½a ...

$$x = x_0 + \frac{v_0 v - v_0^2}{a} + \frac{\frac{1}{2}v^2 - vv_0 + \frac{1}{2}v_0^2}{a}$$

... but the a on the top and the a^2 on the bottom cancel to leave everything divided by a.

You can still pass physics, but algebra is important too.

In your exam you get points for **understanding physics concepts** - but you also get points for being able to **explain** physics using **graphs** and **equations**.

For much of your course, you won't need to do that much algebra. But if you need to **rearrange** equations and make **substitutions** to get a solution, you won't get full credit if your algebra is a bit hazy - though you can probably score enough points to pass if you can explain **how** you would do the question.

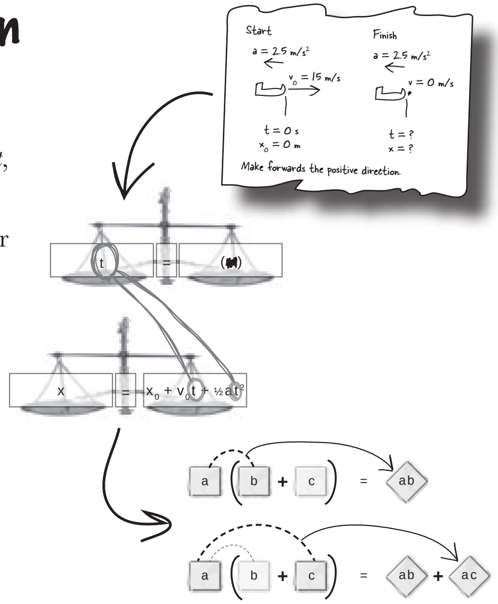
Your priority is definitely understanding the physics. There aren't that many patches of nasty algebra in the rest of this book, so hang in there!

Where you're at with your new equation

You're working out an **equation** that'll give you the stopping distance for the Dingo's new rocket-powered sled given $\mathbf{v}_0$, its initial velocity. However, the only equation you had for **x**, the displacement, included *t*, the time it would take to stop. But you don't know what *t* is.

You're already most of the way through working out a new equation for **x** that doesn't involve *t* by **rearranging** some equations you already knew and **substituting** in for the variable *t* to get rid of it.

You want the equation to be as clear and simple as possible so that you're less likely to make mistakes when you use it. You just multiplied out the parentheses to try to simplify the equation. But it doesn't look particularly simple at the moment!



You need to simplify your equation by grouping the terms

Now that you've multiplied out the parentheses, your equation has a lot of terms in it! If you **group** together all the terms that are the same letter (or letters multiplied together), you'll be able to **simplify** your equation.

For example, if you have the equation $a = b + c - b - 2c$, you have b and $-b$ on the right hand side. When you group them together, they become $b - b = 0$. You also have c and $-2c$. When you group them together, they become $c - 2c = -c$

Written out, the work looks like:

$$a = b + c - b - 2c$$

$$a = b - b + c - 2c$$

$$a = -c$$

If you do something similar with the equation for the hovercraft's stopping distance, it'll be a lot clearer to work with and less prone to error. We don't want the Dingo to get hurt when he only wants to invite the Emu to his birthday party.

Sharpen your pencil

Group together the similar terms on the right hand side of your equation, and simplify it down.

(We've already started by joining together the terms that were all being divided by a).

$$x = x_0 + \frac{v_0 v - v_0^2 + ½v^2 - v v_0 + ½v_0^2}{a}$$

Sharpen your pencil Solution

Group together the similar terms on the right hand side of your equation, and simplify it down.

(We've already started by joining together the terms that were all being divided by a).

We've spaced the lines of the answer out so that the annotations can fit!

$$x = x_0 + \frac{v_0 v - v_0^2 + \frac{1}{2}v^2 - v v_0 + \frac{1}{2}v_0^2}{a}$$

Did you spot that vv_0 and v_0v are just the same thing written in a different order?

On this line, the similar terms are grouped together.

$$x = x_0 + \frac{v_0 v - v v_0 + \frac{1}{2}v^2 - v_0^2 + \frac{1}{2}v_0^2}{a}$$

These two terms add up to zero and cancel each other out.

This is what happens when you group together the v_0^2 terms.

$$x = x_0 + \frac{\frac{1}{2}v^2 - \frac{1}{2}v_0^2}{a}$$

It's nicer to write $\frac{1}{2}v^2 - \frac{1}{2}v_0^2$ than it is to write $-\frac{1}{2}v_0^2 + \frac{1}{2}v^2$.

$$x = x_0 + \frac{v^2 - v_0^2}{2a}$$

This is just a neater way of writing the same thing that you have in the line above.

This equation is usually rearranged on equation tables to say:

$$v^2 = v_0^2 + 2a(x - x_0)$$

Relax

You won't need to do all that again!

We've been working out this key equation as an opportunity to practice the **kind of math** you'll come across while you're doing some parts of physics. Now that you know the equation, you can use it time and time again - either from memory (in your multiple choice paper) or by looking it up on your equation table (in the free response section).

You've been learning some important algebra as you go along.

Although we said that you don't need **lots** of algebra to pass physics, we'd rather you **understand** as much as possible. If you manage to get your head around this, you're setting yourself up to do **really well** if you take an exam.

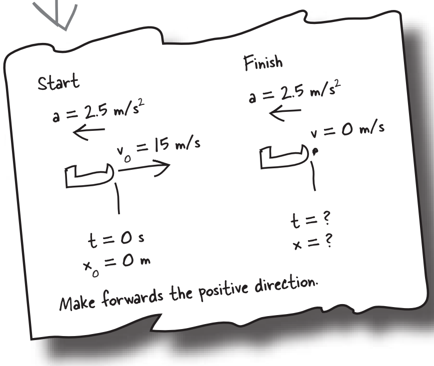
It's important to be able to do things like multiplying out parentheses now so that you won't feel lost or confused later on. It also means that you won't have to completely skip parts of exam questions where you understand the physics perfectly because they involve algebra at a similar level to this.

You can use your new equation to work out the stopping distance

You've worked out an equation you can use to calculate the stopping distance of the hovercraft for the Dingo.

$$x = x_0 + \frac{v^2 - v_0^2}{2a}$$

This is the sketch of the problem that you drew before.



Sharpen your pencil

A rocket-powered hovercraft is traveling at 15 m/s. When the brakes are applied, it decelerates at a rate of 2.5 m/s^2. What is its stopping distance?

Sharpen your pencil Solution

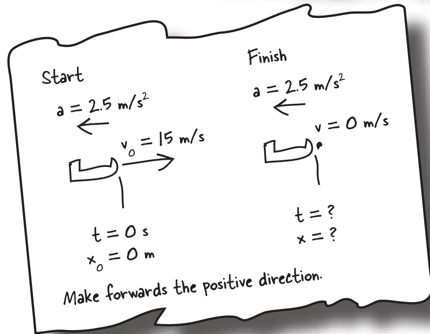
A rocket-powered hovercraft is traveling at 15 m/s. When the brakes are applied, it decelerates at a rate of 2.5 m/s². What is its stopping distance?

$$x = x_0 + \frac{v^2 - v_0^2}{2a} \Rightarrow x = 0 + \frac{0^2 - 15^2}{2 \times (-2.5)}$$

$$x = 45 \text{ m (2 sd)}$$

The stopping distance is <u>45 meters</u> (2 sd).

$$x = x_0 + \frac{v^2 - v_0^2}{2a}$$



There are THREE key equations you can use when there's constant acceleration

As well as a stopping distance to pass onto the Dingo, you now know the **three key equations** that will enable you to deal with **any** problem where the acceleration is constant.

Falling things, launched things, rocket hovercrafts, cars, boats - you name it.

Now **that's a real superpower!**

$$x = x_0 + \frac{v^2 - v_0^2}{2a}$$

This is the equation you just worked out. it's sometimes called the "no time" equation because the variable 't' doesn't appear in the equation.

This is exactly the same equation, but rearranged so that there are no fractions in it. We've mentioned it here because it's the version of the equation you'll find on equation sheets.

Equations of motion

The three key equations for something with constant acceleration.

$$v = v_0 + at$$

$$x = x_0 + v_0 t + \frac{1}{2}at^2$$

$$v^2 = v_0^2 + 2a(x - x_0)$$

So the bottom line with the math stuff is that if I can **use** these equations, I'll do **OK**. But if I can rearrange, substitute, multiply out parentheses, and simplify equations, I can do even better?

That's right - math is just another tool to help you get better at physics.

Most physics books assume you can already do algebra to use the equations they give to you ready-baked. This book is different. You've gradually been introduced to a lot of algebra **in the context of physics** so that you can use it as a tool to help you **understand** physics the best you can.

If you're not so sure of some of the math, you can still do OK, but it's up to you to **practice** anything you initially find difficult until you get better at it.

there are no Dumb Questions

Q: So am I supposed to memorize all of these equations? That's an awful lot! I thought this was supposed to be about understanding, not memorization!

A: The first equation, $v = v_0 + at$, says that your new velocity is the same as your old velocity, plus the effect of your acceleration. You don't care about x_0 or x.

The second equation, $x = x_0 + v_0t + ½at^2$, says that your new displacement depends on your old displacement, your initial velocity and your acceleration, as well as the length of time you've been going for. You don't care about v.

The third equation, $v^2 = v_0^2 + 2a(x-x_0)$, is the one you just worked out. It gives you your final velocity when you don't know t, the time you're traveling for.

Or don't care about t!

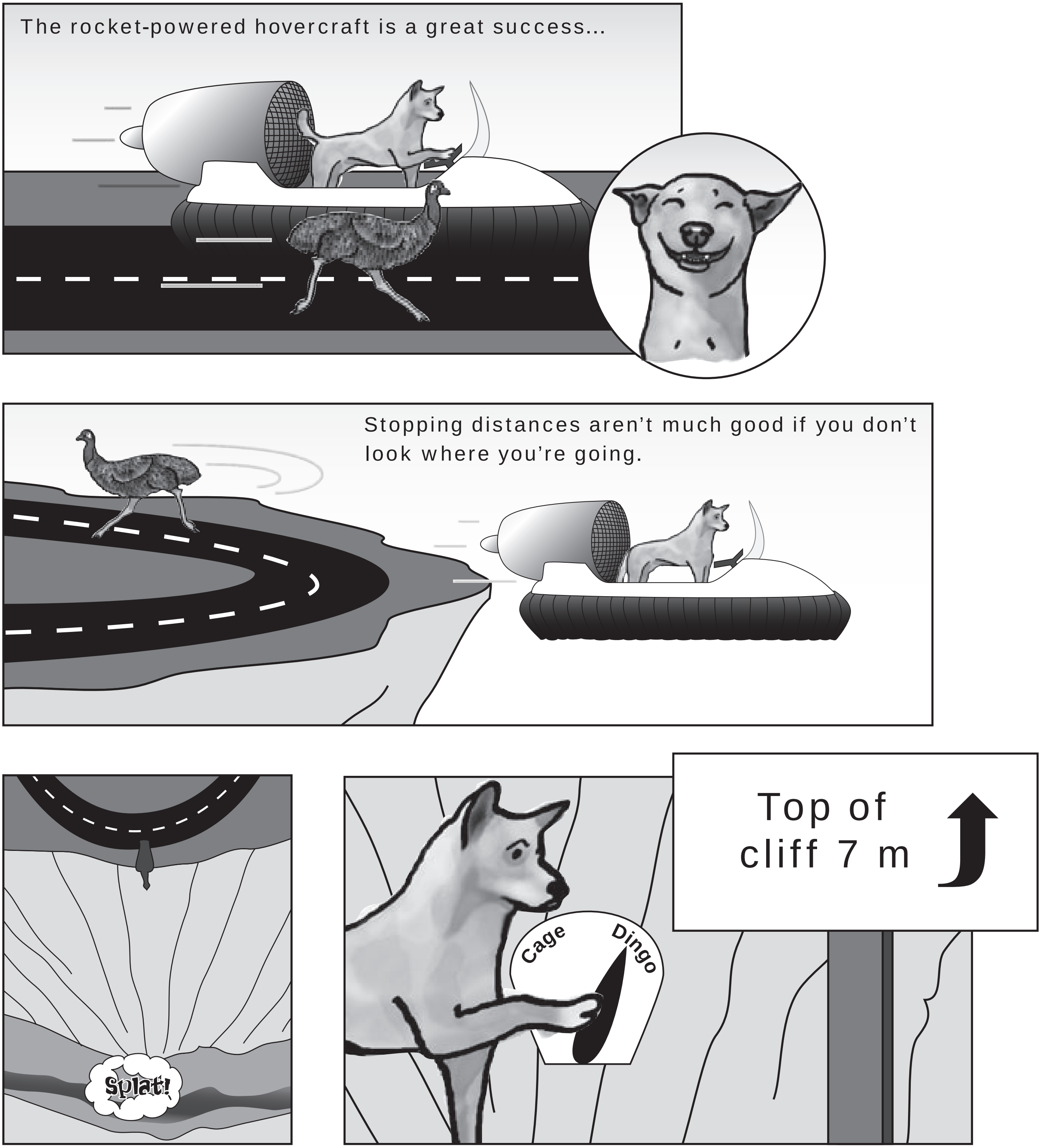
Q: So if the best way to learn equations is to do lots and lots of problems, how come Head First Physics doesn't have hundreds of examples at the end of each chapter, like most other physics books do?

A: One reason is that you're learning and doing problems throughout each chapter.

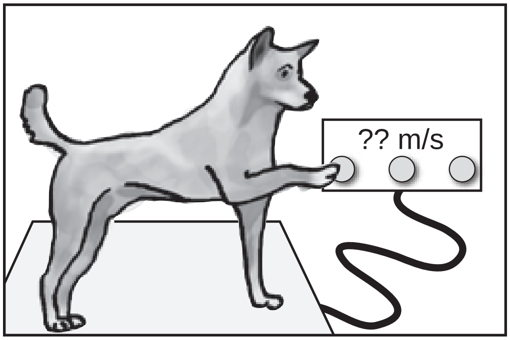
This book is about trying things out while you're learning. You use the concepts as you go along so that you really understand the physics, rather than the 'read along and nod' method that most textbooks use. There are plenty of resources with practice questions out there - do spend time working through lots of problems to reinforce what you're learning here.

Q: So if I keep a lookout on the Head First website, will there be something up there to help me practice?

A: Yes, we'll be producing some online resources to go with the book.


The rocket-powered hovercraft is a great success...
Stopping distances aren't much good if you don't look where you're going.
Splat!
Cage
Dingo
Top of cliff 7 m

You need to work out the launch velocity that gets the Dingo out of the Grand Canyon!



The Dingo's fallen over a cliff, but he had a soft splash landing. Fortunately, he still has his ACME launcher. And a new idea! If he launches himself into the air so that the top of his flight is exactly level with the top of the cliff, he should be able to pass the Emu the party invitation - but in a situation where the Emu doesn't feel scared.

The thing is that the Dingo doesn't know what what his launch **velocity** should be. If he sets the launch velocity too low, he won't make it up to the edge. If he sets the launch velocity too high, he'll still be going up when he reaches the edge and might not have time to pass on the invitation.

What should the Dingo set as the launch velocity this time?

Sharpen your pencil

Do as much of this as you can from memory, then turn back to page 314 to copy them down.

The Dingo is stuck at the bottom of the cliff with a launcher. If the cliff is 7.00 m high at this point, what launch speed will mean that the top of his flight is at the top of the cliff?

a. Start with a sketch! Draw everything you already know about.

b. Write down your three key equations for doing these kinds of problems. Next to each variable in each equation, put a ? if you want to find it out, a tick if you know it, and a cross if you don't know it.

This is an important problem-solving skill.

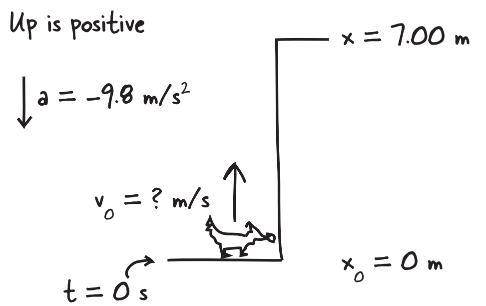
c. Do you think you can do this problem straight off, or do you need some extra information?

Sharpen your pencil Solution

Do as much of this as you can from memory, then turn back to page 314 to copy them down.

The Dingo is stuck at the bottom of the cliff with a launcher. If the cliff is 7.00 m high at this point, what launch speed will mean that the top of his flight is at the top of the cliff?

a. Start with a sketch! Draw everything you already know about.



b. Write down your three key equations for doing these kinds of problems. Next to each variable in each equation, put a ? if you want to find it out, a tick if you know it, and a cross if you don't know it.

This is an important problem-solving skill.

$$x = x_0 + v_0 t + \tfrac{1}{2}at^2$$

(x ✓, x_0 ✓, v_0 ?, t ✗, a ✓, t^2 ✗)

$$v = v_0 + at$$

(v ✗, v_0 ?, a ✓, t ✗)

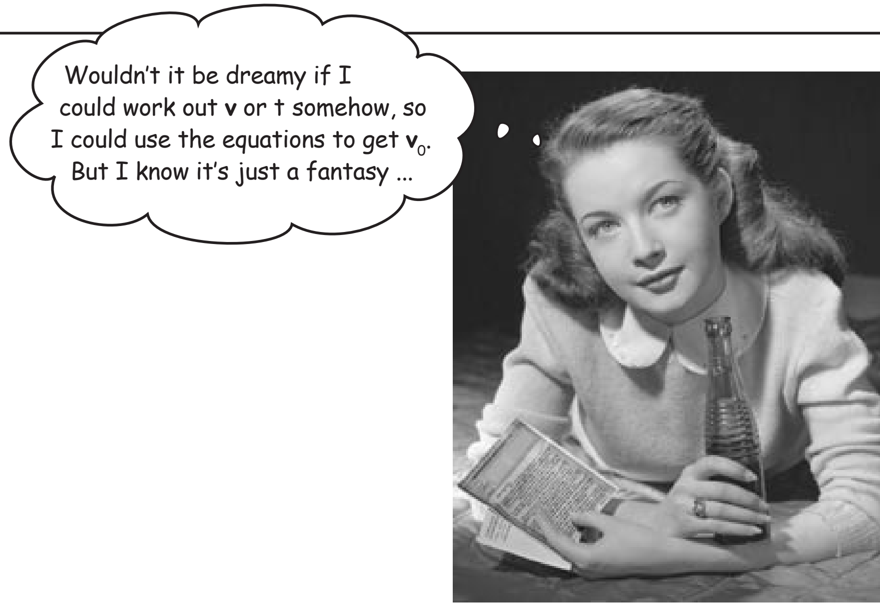
$$v^2 = v_0^2 + 2a(x - x_0)$$

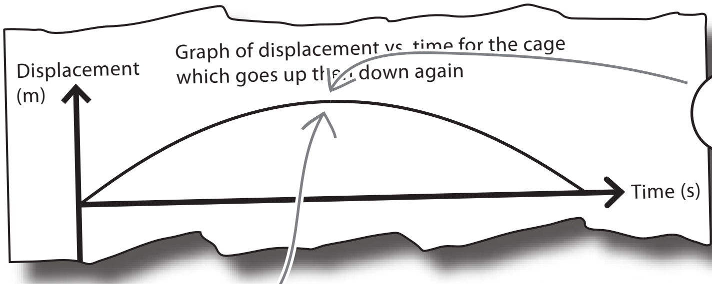
(v ✗, v_0 ?, a ✓, $(x - x_0)$ ✓)

If you don't know which equation to use, you should always have a go at doing this.

c. Do you think you can do this problem straight off, or do you need some extra information?

All of the equations have something else in them that I don't know (either v or t, or both!), as well as v_0 (which is what I want to find out). So I need some extra information to do the problem.





Hey ... didn't we say that velocity = 0 at the very top of something's flight?

Slope of displacement-time graph is zero, so velocity MUST be zero.

Sometimes you'll need to spot a 'special point.'

Don't settle just for the information you're given in a problem. Start with a sketch. Be the Dingo. **Spot the 'special point'**!

You've just realized that the top of something's flight is a special point where it's stopped going up but hasn't quite started going down again. So the velocity is zero there.

This is a value you can use in your equations to help you work out other things.

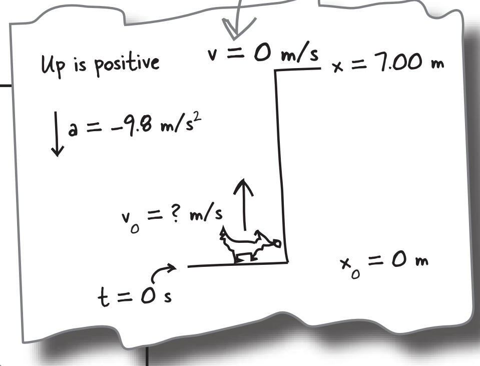
Sharpen your pencil

Now that you've spotted the special point, use your extra information that **v** = 0 at the top of the Dingo's flight together with what you did on the opposite page to calculate what his launch velocity should be.

You should add v = 0 to your sketch, and tick 'v' every time you see it in an equation.

Sharpen your pencil Solution

Now that you've spotted the special point, use your extra information that v = 0 at the top of the Dingo's flight together with what you did on the opposite page to launch him out of the Grand Canyon.



Extra information: v = 0 m/s at the top.

$$v^2 = v_0^2 + 2a(x - x_0)$$

v_0 is now the only unknown in this equation.

Rearrange to say "v_0 = something"

$$v_0^2 = v^2 - 2a(x - x_0)$$

This works out as negative × negative × positive = positive

$$v_0 = \sqrt{v^2 - 2a(x - x_0)} = \sqrt{0^2 - 2 \times (-9.8) \times (7.00 - 0)} = \sqrt{137.2}$$

$$v_0 = 11.7 \text{ m/s (3 sd)}$$

The Dingo needs to be launched at <u>11.7 m/s</u> (3 sd).

Extra information: you know what v is.

$$x = x_0 + v_0 t + \tfrac{1}{2}at^2$$

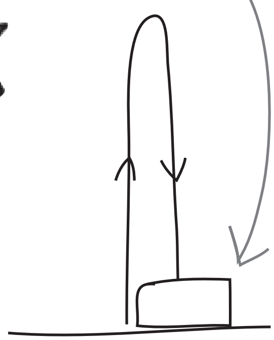
$$v = v_0 + at$$

$$v^2 = v_0^2 + 2a(x - x_0)$$

The launch velocity's right!

You just used the fact that **v** = 0 m/s at the top of the cliff to work out that the Dingo's launch velocity should be 11.7 m/s.

I wanna know the time it takes for me to go up then back down to the bottom of the cliff. Is it long enough for my safety thingy to inflate?



While he's in the air, a safe landing area inflates.

But the Dingo doesn't like getting wet, and wants a better plan for landing than splashing into the river again. He has an ACME inflatable landing area and wants to press its inflate button at the same time as launching himself up in the air.

The landing area takes 1.00 s to inflate. How long will the Dingo be in the air for? Will the landing area have enough **time** to inflate before he comes back down?

Jim: Yeah, we already know a lot of values! $\mathbf{x}_0$ and $\mathbf{x}$ will both be zero (as he's starting and finishing at the bottom of the canyon). And $\mathbf{v}_0$ = 11.7 m/s.

Joe: Not forgetting $\mathbf{a}$ = -9.8 m/s^2. I think we know more values right now than we've done for any other problem!

Frank: So which equation can we use? We don't know what v is, so we can't use either of the equations with v in them.

Jim: Well, that leaves $\mathbf{x} = \mathbf{x}_0 + \mathbf{v}_0 t + \frac{1}{2}\mathbf{a}t^2$, which should be cool. We know $\mathbf{x}$, $\mathbf{x}_0$, $\mathbf{v}_0$, and $\mathbf{a}$ ... that leaves only t, which is what we want to calculate!

Joe: One equation, one unknown - sounds ideal!

Frank: So I guess we rearrange the equation so that it says "t = something."

Jim: Yeah, let's get on with it!

Important problem-solving skill!

Write down your equations, and tick or cross the variables you know or don't know.

Sharpen your pencil

a. Try rearranging the equation $\mathbf{x} = \mathbf{x}_0 + \mathbf{v}_0 t + \frac{1}{2}\mathbf{a}t^2$ to say "t = something," so you can use it to work out the time it takes the Dingo to go up and back down again.

b. Write down your thoughts about whether this idea will work or not.

Sharpen your pencil Solution

a. Try rearranging the equation $\mathbf{x} = \mathbf{x}_0 + \mathbf{v}_0 t + \frac{1}{2}\mathbf{a}t^2$ to say "t = something," so you can use it to work out the time it takes the Dingo to go up and back down again.

$$x = x_0 + v_0 t + \tfrac{1}{2}at^2$$

Try to get this t on its own to say "t = something"

$$v_0 t = x - x_0 - \tfrac{1}{2}at^2$$

$$t = \frac{x - x_0 - \tfrac{1}{2}at^2}{v_0}$$

But now there's a t^2 over here, so that doesn't work.

b. Write down your thoughts about whether this is a good idea or not.

This isn't going to work. If you try to rearrange to say "t = something," there's a t^2 on the other side. And if you get the t^2 on its own to take a square root, there'll still be the t on the other side. You can't simplify the equation enough to say "t = something."

Strictly speaking, this is possible if you use something called the quadratic formula. If you already know how to do this, then feel free to use this method. But if you've never heard of the quadratic formula before, don't worry. You're about to learn a much less complicated way ...

You need to find another way of doing this problem

Your equation $\mathbf{x} = \mathbf{x}_0 + \mathbf{v}_0 t + \frac{1}{2}\mathbf{a}t^2$ contains **two terms with *t* in them**, $\mathbf{v}_0 t$ and $\frac{1}{2}\mathbf{a}t^2$. Because one of the terms has t in it, and the other has t^2 in it, there's no easy way to rearrange your equation to say "t = something" because the t and t^2 won't cancel by adding to zero.

However, if either $\mathbf{v}_0 = 0$ or $\mathbf{a} = 0$, then one of these terms would **disappear**. This means you'd be able to rearrange the equation to say "t = something" and use it to calculate the value of t.

If any variable that's multiplying a term is zero, then the whole term is zero and disappears.

$$x = x_0 + v_0 t + \frac{1}{2}at^2$$

If $v_0 = 0$, the equation becomes $x = x_0 + \frac{1}{2}at^2$

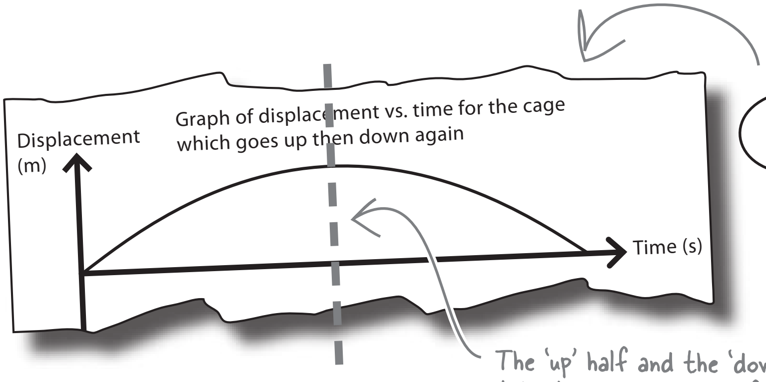
If $a = 0$, the equation becomes $x = x_0 + v_0 t$

What would happen to your equation if different variables were zero? Would this make the equation easier to solve?

If $\mathbf{a} = 0$, then the equation becomes $\mathbf{x} = \mathbf{x}_0 + \mathbf{v}_0 t$, which is the equation you worked out before for something that moves with a constant velocity without accelerating. But the acceleration isn't zero in this scenario.

But if $\mathbf{v_0} = \mathbf{0}$, then the equation would simplify to $\mathbf{x} = \mathbf{x}_0 + \frac{1}{2}\mathbf{a}t^2$, which you can rearrange to say "t = something." That's incredibly useful!

If only there was a way of reframing the problem so that $\mathbf{v}_0 = 0$, then all of this would be possible ...



The displacement-time graph's **symmetrical**. Symmetry's helped us before. I wonder if symmetry will help us again?

Sharpen your pencil

A dingo is launched with an initial velocity of 11.7 m/s upwards. The top of his flight path is 7.00 m from the level he started at.

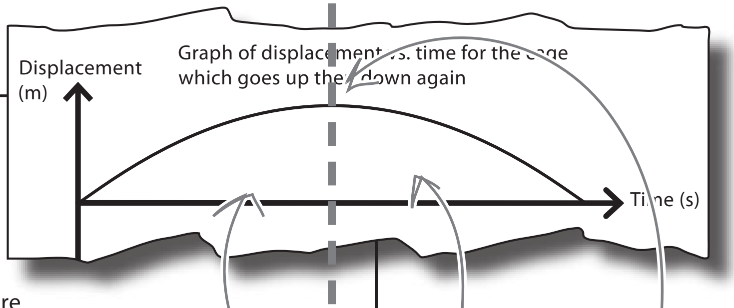
a. Use **symmetry** to make a new problem where $\mathbf{v}_0 = 0$. This will enable you to rearrange the equation $\mathbf{x} = \mathbf{x}_0 + \mathbf{v}_0 t + \frac{1}{2}\mathbf{a}t^2$ to say "t = something." Use this equation to calculate the time it takes for the Dingo to return to the bottom of the cliff.

You're creating a new problem, so start with a new sketch.

b. The Dingo's safety device takes 1.00 s to inflate. Will the safety device have inflated before the Dingo lands?

Look out for symmetry that might make a hard problem easier ...

Sharpen your pencil Solution



A dingo is launched with an initial velocity of 11.7 m/s upwards. The top of his flight path is 7.00 m from the level he started at.

a. Use **symmetry** to make a new problem where $\mathbf{v}_0 = 0$. This will enable you to rearrange the equation $\mathbf{x} = \mathbf{x}_0 + \mathbf{v}_0 t + \frac{1}{2}\mathbf{a}t^2$ to say "t = something." Use this equation to calculate the time it takes for the Dingo to return to the bottom of the cliff.

Work out the time it takes him to fall 7.00 m, then double it to get the time it takes him to go up then down.

You spend half the time going up ... ... and the other half going down.

So the TOTAL time it takes to go up and down is double the time it takes to fall from the same height.

Down is positive $v_0 = 0$ m/s

$x_0 = 0$ m

$t = 0$ s

$\downarrow a = 9.8$ m/s^2

Now that you're ONLY dealing with vectors pointing downwards, you can make down the positive direction so that the math is easier.

$t = ?$ s

$x = 7.00$ m

$$x = x_0 + v_0 t + \tfrac{1}{2}at^2$$ Rearrange this to say "t = something"

$v_0 = 0$ so this term is zero.

$$\tfrac{1}{2}at^2 = x - x_0$$

$$at^2 = 2(x - x_0)$$

Now there's only t^2 and no t, so this is an equation you can solve.

$$t = \sqrt{\frac{2(x - x_0)}{a}} = \sqrt{\frac{2(7.00 - 0)}{9.8}} = 1.20 \text{ s (3 sd)}$$

You get the same answer if you make "up" the positive direction, but there's more risk of making a mistake with minus signs.

b. The Dingo's safety device takes 1.00 s to inflate. Will the safety device have inflated before the Dingo lands?

Yes, the safety device inflates before the Dingo lands. Result!

The top of the flight path is exactly HALFWAY through the flight.

The time it takes to go up and down is DOUBLE the time it takes to fall from the top of the flight path.

If it's **symmetrical**, does that mean that you have the **same speed** when you hit the bottom as you did when you were launched, but in the opposite **direction**?

At a single height, a launched object has the same speed whether it's going up or down.

The Dingo left the ground with a speed of 11.7 m/s and a velocity of 11.7 m/s upwards. Since going up then down again is symmetrical, his speed at the bottom will also be 11.7 m/s, and his velocity will be 11.7 m/s downwards.

Also, at any displacement in his flight, his speed will be the same whether he's going up or down. This is another special **symmetry** thing that you can sometimes use to solve problems. If you start and end at the same height, then $\mathbf{v} = -\mathbf{v}_0$.

there are no Dumb Questions

Don't worry about what this is if you don't already know. You won't need to use it if you can spot symmetry in problems!

Q: So why was I trying to find a special point where $v_0 = 0$? I wasn't quite sure about that bit.

A: You want to use the equation $x = x_0 + v_0t + \frac{1}{2}at^2$ to calculate *t*. But because it has both *t* and t^2 in it, you can't easily rearrange it to say "*t* = something."

But the term with the *t* in it is actually v_0t. So if you can reframe the problem into one where v_0 is zero, you lose that term entirely and can solve the equation to find *t*.

Q: But couldn't I just use the quadratic formula to solve for *t* without having to do all of that symmetry stuff?

A: If you already know how to solve quadratic equations like this one, that's fine. Feel free to use any method you understand.

But spotting the symmetry and working out how long it takes to fall, then doubling it is actually easier mathematically and a very useful shortcut to know about. Symmetry makes hard problems easier.

You'll also learn another approach to doing a problem like this in chapter 14, which you can use instead if you want to.

Q: But what if I'm in a situation where I can't make $v_0 = 0$? How do I solve that kind of equation then?

A: In that case, it's usually most straightforward to work out a value for v, so you can use the simpler equation $v = v_0 + at$. You'd do this using the other one of the three key equations, $v^2 = v_0^2 + 2a(x - x_0)$.

Q: Ack. I have trouble remembering these equations.

A: Don't worry - it's on your equation table. And you'll naturally memorize it as you practice using it.

Symmetry makes hard problems easier.

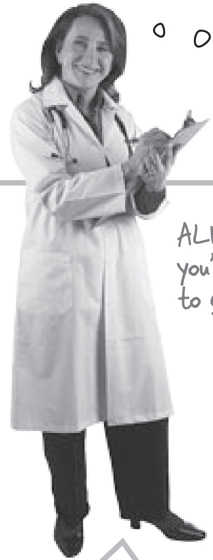
The start of a beautiful friendship

Thanks to you, the Dingo has managed to deliver his party invitation. And the party marks the start of a beautiful friendship.

TOGETHER AT LAST

A HEAD FIRST PRODUCTION

Question Clinic: The "Sketch a graph" or "Match a graph" Question



Sometimes you'll be asked to sketch a graph or match a sketched graph with an equation or story. The point is to show that you understand what's going on. Sketching a graph will usually be part of a free response question, and choosing graphs that match stories or scenarios is a standard style of multiple choice question.

ALWAYS START WITH A SKETCH!! Even if you're asked for a graph, sketch the situation to get your whole brain zoned in on the job.

Make a note of initial DIRECTIONS, and decide which direction to make positive.

2. A cage is propelled straight up from ground level by a launcher with initial velocity v_0. Sketch graphs of:

a. The displacement

b. The velocity

c. The acceleration

of the cage with respect to time, from the moment it's launched until it hits the ground again. State any assumptions you make.

Make a note of any initial VALUES, as you should include them on your graph.

You don't have to actually draw the graphs in this order. The easiest one is the acceleration, as it's a constant $-9.8\ m/s^2$. So start with that and work backwards.

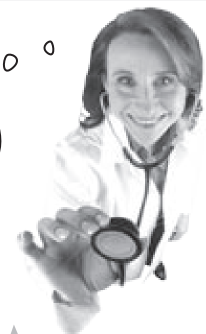
This tells you where your sketch should end.

This tells you what to put on the horizontal axis.

This means get the SHAPE right. Don't plot only put on values you know/have been given.

Don't forget to do this bit, or you'll lose points!

If you're asked to sketch more than one graph (like displacement, velocity, and acceleration), it's usually best to work out which graph is easiest to draw, and start with that one. You can then work out what the others look like by thinking about values and slopes.



Question Clinic: The "Symmetry" and "Special points" Questions

Some of the questions you'll be asked about moving things will have one simple step. But in many, you'll have to spot symmetry to take a shortcut - or to make the problem solvable at all. You may also need to spot 'special points' that give you extra information because of symmetry - like **v** = 0 at the top of something's flight, or the fact that something's speed will be identical at the same height whether it's going up or coming down (though the direction of the velocity is different).

ALWAYS START WITH A SKETCH!!

It starts and finishes at the same HEIGHT, so it will have the same SPEED both times, just in opposite directions.

3. A cage is propelled straight up from ground level by a launcher with initial velocity v_0 = 10 m/s. If it goes up then comes back down again to ground level,

a. What is the value of its velocity just before it hits the ground

b. How long does it remain in the air for?

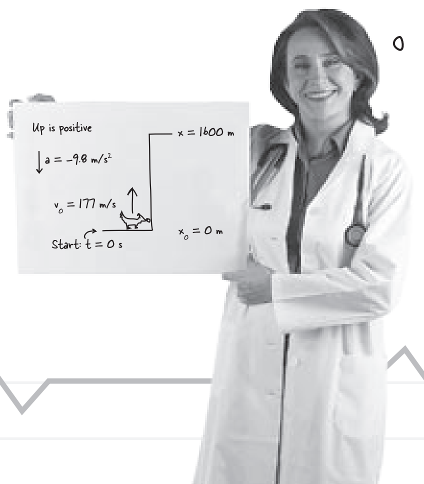
c. What is the maximum height it reaches?

This is asking for a time. If you already know the height, you could work out how long it takes to fall that far and double it.

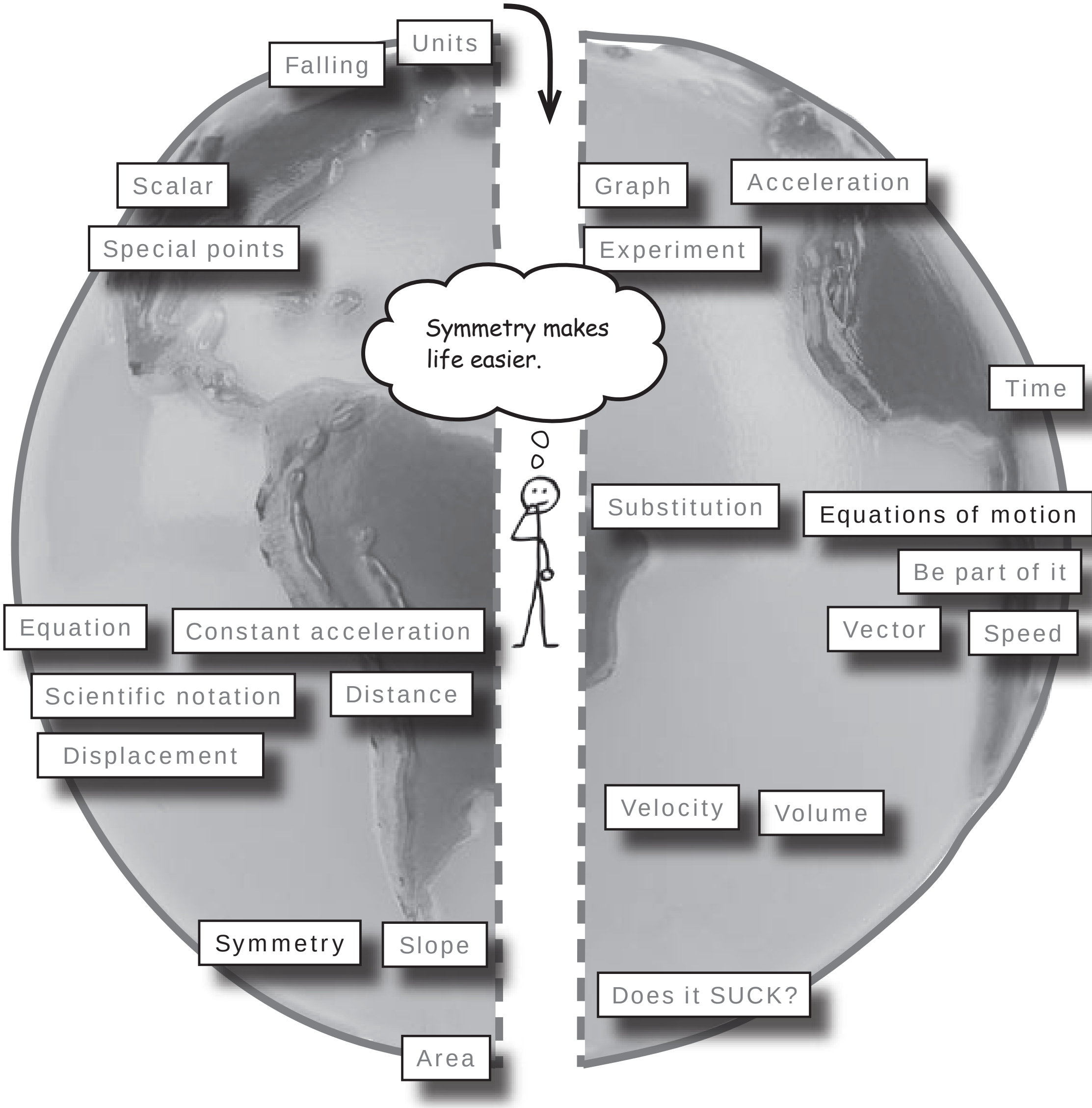
But here you know v and v_0 already (as $v_0 = -v$), so you can use $v = v_0 + at$.

You know that the maximum height is a 'special point' where the velocity is zero and that it gets there in half the time it takes to go up and down..

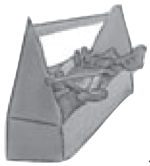
DO ANOTHER SKETCH!! As this part of the problem has a DIFFERENT END POINT (the top of the flight path instead of ground level), some of your variables will have different values.



The secret is to keep calm and **start with a sketch**. Write down the three key equations, add in 'special points' to your sketch, then play with the terms and the variables in your equations until you know which of them to use to get the answer you want.



☑	Equations of motion	Three equations that you can use to calculate the motion of an object that is moving with constant acceleration.
☑	Symmetry	In physics, the second half of something's motion sometimes mirrors the first. For example, going up into the air then back down again is symmetrical.



Your Physics Toolbox

You've got Chapter 8 under your belt and added some equations, math techniques and problem-solving skills to your tool box.

Equations of motion

The three key equations for something with constant acceleration.

$$v = v_0 + at$$

$$x = x_0 + v_0 t + \frac{1}{2}at^2$$

$$v^2 = v_0^2 + 2a(x - x_0)$$

Vectors: positive direction

When you're dealing with vectors that all lie along one line, you have to decide which direction is positive.

Usually, you'd make up positive so that your graphs come out the same way round as real life.

If all your vectors point down (e.g., if an object is falling), then making down positive helps to reduce errors with minus signs.

Parentheses

When you're multiplying out parentheses, you multiply every term inside by every term outside:

$$a(b + c) = ab + ac$$

$$(a + b)(a + b) = a^2 + 2ab + b^2$$

Special points

Special points help you to simplify and solve problems.

Vertical velocity = 0 at the top of an object's flight.

The object may also be moving horizontally. We'll talk about this in chapter 9 ...

Which equation of motion should I use?

If you don't know which equation of motion to use, write down all three of them. Tick the variable you know values for, cross the variables you don't know, and put a ? by the variable you want to find out. For example:

$$\overset{\checkmark}{x} = \overset{\checkmark}{x_0} + \overset{?\times}{v_0}t + \overset{\checkmark\times}{1/2at^2}$$

$$\checkmark\ v = \overset{?}{v_0} + \overset{\checkmark\times}{at}$$

$$\checkmark\ v^2 = \overset{?}{v_0^2} + 2\overset{\checkmark}{a}(\overset{\checkmark}{x - x_0})$$

Symmetry

When an object goes up then down, the up part and the down part of its motion both take the same TIME.

When something goes up then down, its VELOCITY has the same size at any one height regardless of whether the direction of the velocity is up or down.

9 triangles, trig and trajectories

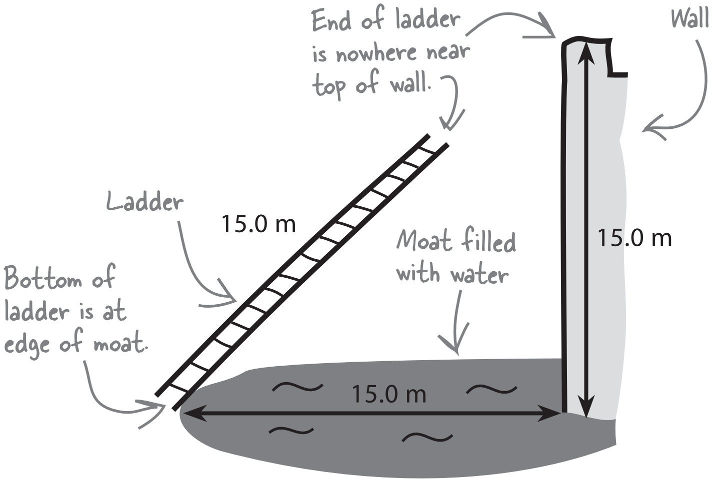
Going two-dimensional

So you can deal with one dimension. But what about real life?

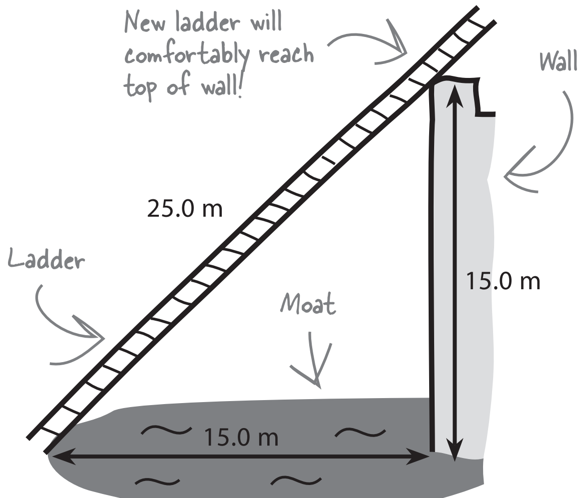
Real things don't just go up or down - they go sideways too! But never fear - you're about to gain a whole new bunch of **trigonometry** superpowers that'll see you spotting **right-angled triangles** wherever you go and using them to **reduce complicated-looking problems into simpler ones that you can already do**.

Camelot - we have a problem!

The Head First castle is in imminent danger! Back when it was built, the longest ladder available from Sieges-R-Us was 15.0 m long. So the castle was designed with a moat 15.0 m wide and a wall 15.0 m high, making it impossible for anyone to put a ladder from the edge of the moat to the top of the wall.



But the Sieges-R-Us website has just been updated with a new top of the range 25.0 m ladder. It's only a matter of time before someone comes to attack your castle armed with the new ladder, and your current defense system just isn't big enough...



If you don't act quickly, someone will turn up with the new ladder, and you'll be toast. It's time to **design a new castle defense system**!

Sharpen your pencil

Check the box next to the idea you think is best.

Here are some ideas for a new castle defense system. Unfortunately, you don't have many spare stones lying around, only some shovels and food.

Write down at least one advantage and disadvantage of each idea, and check the box next to the idea you think is the best.

☐ **Coat the top of the wall with something slippery.**

Advantage(s)	Disadvantage(s)
....	....
....	....

☐ **Make the moat wider.**

Advantage(s)	Disadvantage(s)
....	....
....	....

☐ **Run away! Run away!**

Advantage(s)	Disadvantage(s)
....	....
....	....

☐ **Make the wall higher.**

Advantage(s)	Disadvantage(s)
....	....
....	....

☐ **Put a health and safety rep on the top of the wall to recite the working height directive repeatedly to anyone who gets higher than 2 m. Also to ask when their ladders were last inspected and if they have been trained in the proper use of ladders. And insist that they all wear safety harnesses, hard hats, ear defenders, goggles, gloves and toe protectors.**

Advantage(s)	Disadvantage(s)
....	....
....	....
....	....

Sharpen your pencil Solution

Here are some ideas for a new castle defense system. Unfortunately, you don't have many spare stones lying around, only shovels and food.

Write down at least one advantage and disadvantage of each idea, and check the box next to the idea you think is the best.

☐ **Coat the top of the wall with something slippery.**

Advantage(s)	Disadvantage(s)
Ladders will slip, it might delay them a bit.	Washes off in the rain, attracts cats, not a long-term solution.

☑ **Make the moat wider.**

Advantage(s)	Disadvantage(s)
I have a shovel to dig a wider moat.	I'm not sure how wide to make the moat.

It's a lot easier to dig a wider moat than it is to build a higher wall.

☐ **Run away! Run away!**

Advantage(s)	Disadvantage(s)
I won't be there when they get into the castle.	They'll get into the castle.

☐ **Make the wall higher.**

Advantage(s)	Disadvantage(s)
This would definitely keep them out of the castle ...	... but there's no stone available to build a higher wall.

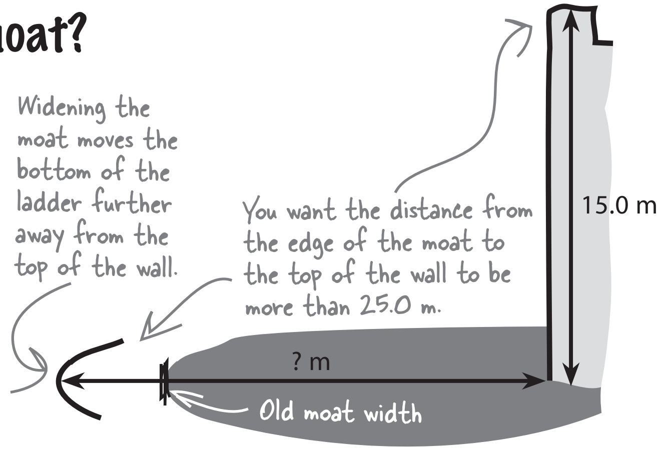
☐ **Put a health and safety rep on the top of the wall to recite the working height directive repeatedly to anyone who gets higher than 2 m. Also to ask when their ladders were last inspected and if they have been trained in the proper use of ladders. And to Insist that they all wear safety harnesses, hard hats, ear defenders, goggles, gloves and toe protectors.**

Advantage(s)	Disadvantage(s)
The attackers might get bored and attack a different castle.	By wearing ear defenders, they can ignore him and continue attacking the castle anyway.

How wide should you make the moat?

The best way of defending the castle against the 25.0 m ladder is to make the moat wider. The moat is already 15.0 m wide—so how much wider do you need to make it?

You could try making the moat the same size as the ladder —25.0 m— so that the distance from the edge of the moat to the bottom of the wall is the same length as the ladder. That would make sure that attackers couldn't simultaneously put one end of the ladder at the edge of the moat and the other end on the top of the wall.



But time is of the essence, and you don't want to start out digging a 25.0 m moat if a narrower moat will do the same job. The important thing is the **distance from the edge of the moat to the top of the wall.** If that's more than 25.0 m, there won't be anything to lean the ladder on. And you might be able to achieve that with a narrower moat... a **sketch** should help.

Sharpen your pencil

a. Draw a sketch of the 15.0 m castle wall, 25.0 m ladder, and extended moat, where the ladder is only just too short to reach the top of the wall from the side of the moat.

(This sketch is just a quick drawing to get the visual parts of your brain working—the lengths on it don't have to be accurate as long as everything's labelled correctly.)

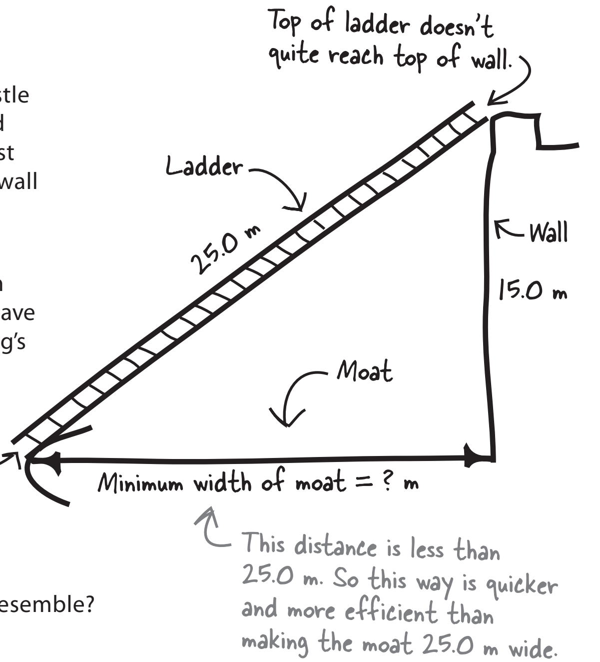
b. What shape does your sketch resemble?

....

Sharpen your pencil Solution

a. Draw a sketch of the 15.0 m castle wall, 25.0 m ladder, and extended moat, where the ladder is only just too short to reach the top of the wall from the side of the moat.

(This sketch is just a quick drawing to get the visual parts of your brain working—the lengths on it don't have to be accurate as long as everything's labelled correctly.)



b. What shape does your sketch resemble?

It looks like a triangle.

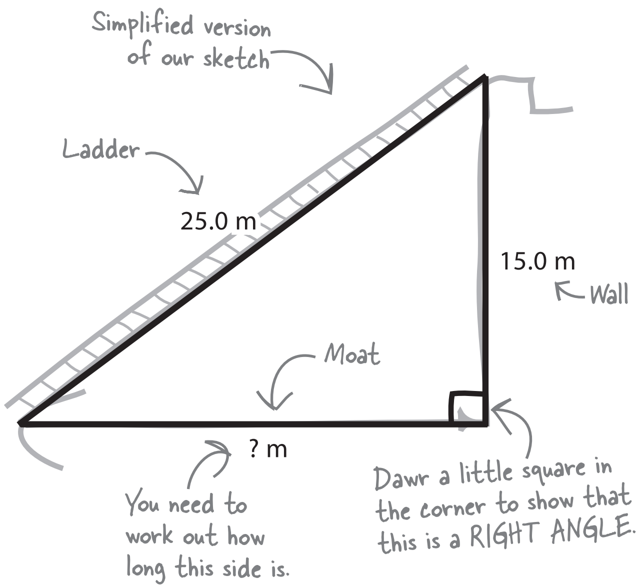
Looks like a triangle, yeah?

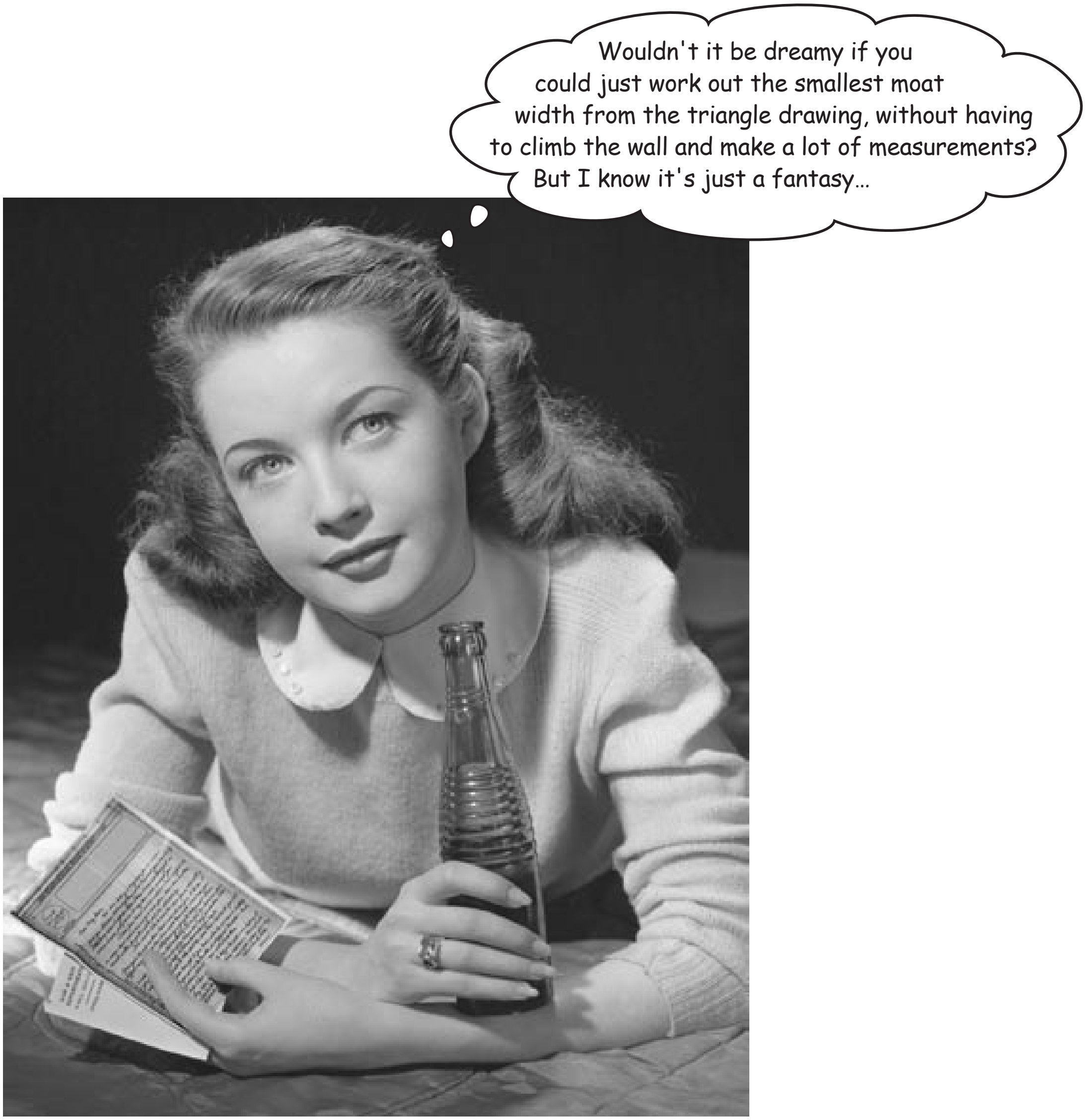
You can turn your complicated-looking castle, ladder, and moat sketch into a more simple picture of a **right-angled triangle**, with a 90° angle (a right angle) between the wall and the moat. You already know the lengths of two of the triangle's sides and want to find out how long the third side is.

You could figure this out by ordering a 25.0 m ladder, putting one end at the top of the wall, and seeing where the ladder touches the ground. But the attackers might arrive with their new ladder first!

If you don't want to wait, you can measure 25.0 m of rope, tie one end to the top of the wall, and see where it touches the ground when you pull it tight.

But that still involves a lot of steps and equipment.



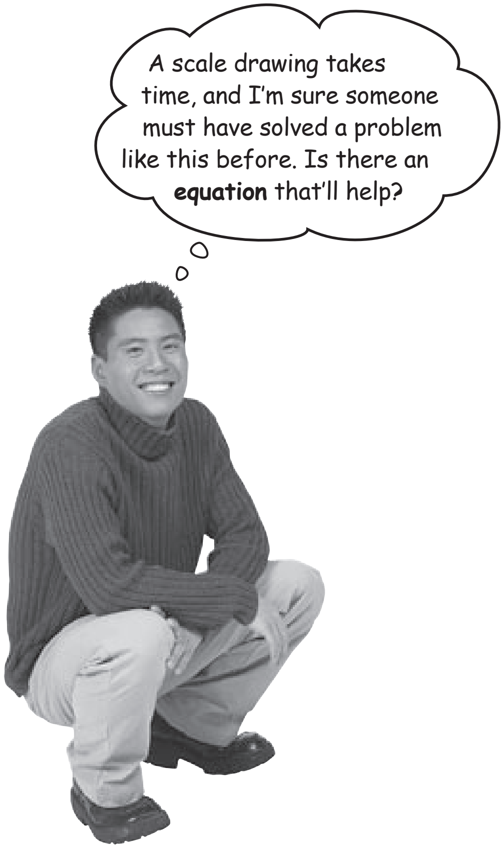

Wouldn't it be dreamy if you could just work out the smallest moat width from the triangle drawing, without having to climb the wall and make a lot of measurements? But I know it's just a fantasy...

A scale drawing can solve problems

This is one way of getting the best moat width from a triangle drawing. But it's not the best way.

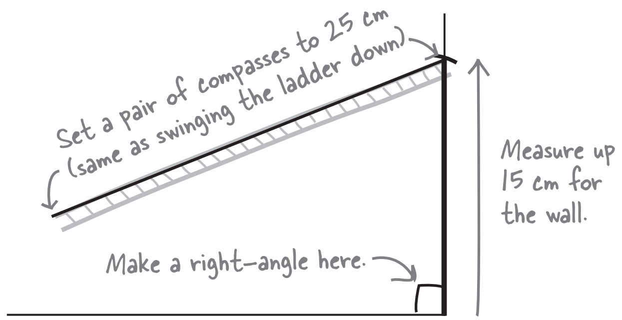
At the moment, your drawing is only a **sketch**—the triangle's side lengths aren't to **scale** (though the lengths you've written beside them are correct).

A **scale drawing** is one where you say something like "1 cm on the drawing = 1 m in real life." You can then do the same as you would with the 15.0 m castle wall and 25.0 m ladder, except with 15 cm and 25 cm!



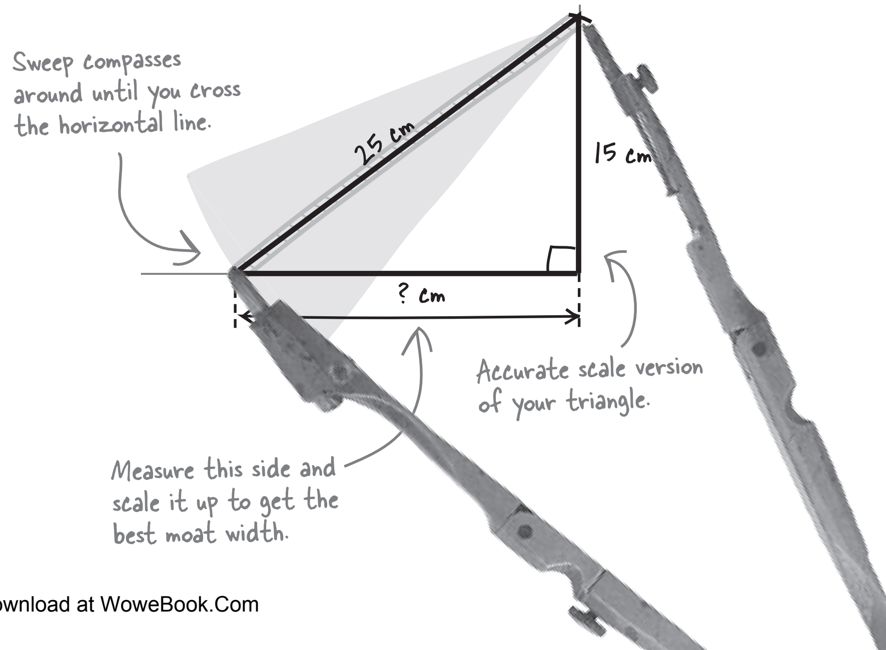
Start off by making a right-angle to represent where the wall meets the moat. Then measure up 15 cm to represent the wall.

Now, you want to swing the ladder down and see where it hits the ground. So set a pair of compasses to 25 cm...



... and swing down from the top of the wall to meet the ground line. Now you can measure the most economical moat width.

That's a lot of effort just to work out a simple length though ...

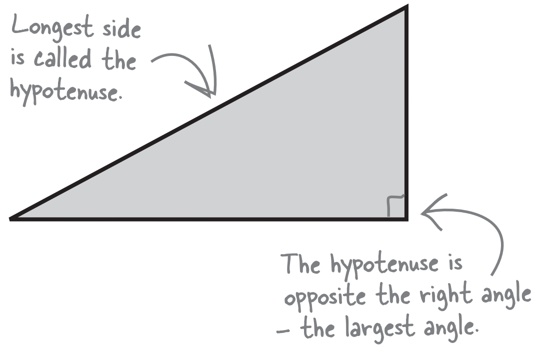


You can solve some problems with scale drawings, but it takes time and effort.

Pythagoras' Theorem lets you figure out the sides quickly

Pythagoras' theorem is an **equation** for solving this kind of problem without waving ladders around or making a super-accurate drawing. You only need to know the lengths of two sides, and the equation will tell you the third.

The longest side of the triangle is opposite the biggest angle (the right angle). This side has a special name and gets called the **hypotenuse**.

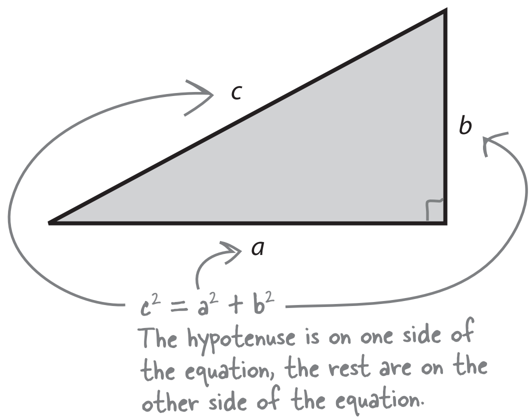


If you square the length of the hypotenuse, the answer is equal to the answer you get if you square the length of the other two sides individually, then add the squares together.

That's very wordy. So here's the equation - if you label the sides of your triangle *a*, *b* and *c* (where *c* is the hypotenuse) then Pythagoras' Theorem says:

$$c^2 = a^2 + b^2$$

It doesn't really matter which letters you use for the sides. We've chosen the same letters as the AP physics equation table.



Pythagoras' Theorem **only works for right-angled triangles**. You can't use it if your triangle doesn't have a right angle.

Pythagoras only works for right-angled triangles.

If you already know two sides of the right-angled triangle, Pythagoras gets you the third.

If the triangle is right-angled:

The square of the hypotenuse

is equal to the sum

of the other two sides squared:

$$c^2 = a^2 + b^2$$

there are no Dumb Questions

Q: You've called Pythagoras a theorem and an equation so far. And I've seen things like that called a formula too. So which is it?

A: Equation, formula, and theorem mean the same thing really. They all describe relationships where you write down "something = another thing."

Q: How do I try to remember Pythagoras? I mean, how do I remember which sides are *a*, *b* and *c*, then what order to put them in the equation?

A: If you can remember the form of the equation, you don't need to remember the letters. The hypotenuse is the longest side, whatever letter you use to name it. So hypotenuse goes on the left of the equation.

And on the right of the equation, you square each of the other sides, then add them together. You can think about the S in SUCK - size matters. So it's the square of the longest side that goes on its own.

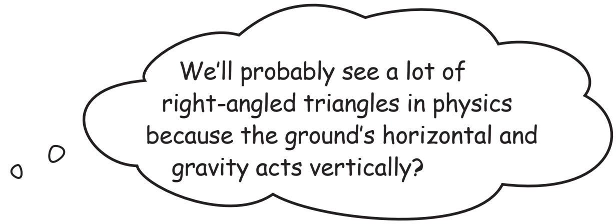
Q: What if I know the hypotenuse and want to calculate the length of one of the other sides?

A: You can rearrange the equation so that the side you don't know is on its own on the left.

Q: So where does Pythagoras' Theorem come from? Aren't we going to go through proving it?

A: It's only really worth going into understanding where an equation comes from if the understanding you gain helps you see how the world works, so you can solve physics problems (and other problems) better.

Being able to prove Pythagoras' Theorem doesn't help with this, so we've not gone into that here.



You'll spot many right angles between the horizontal and vertical

Right-angled triangles are going to be one of your most important physics tools. There are lots of right angles in physics, often between the horizontal ground and vertical walls or vertical acceleration vectors that exist as a result of gravity.

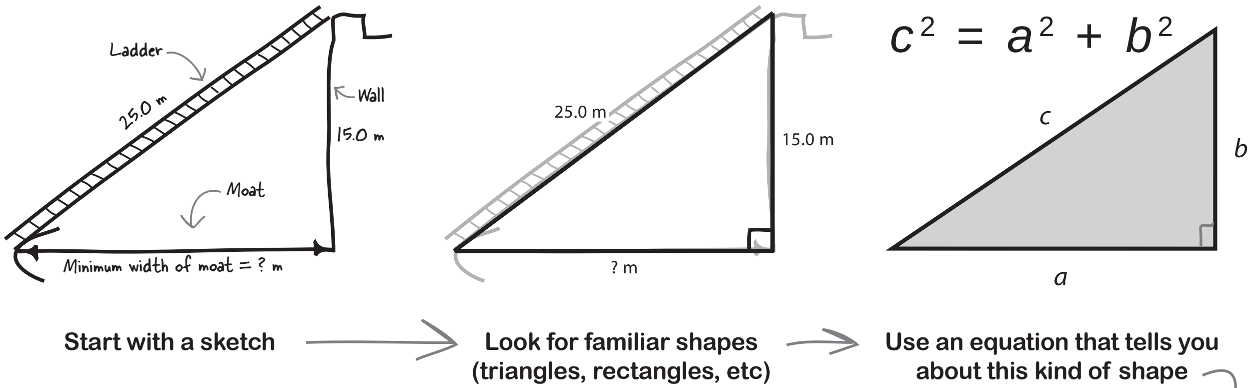
Keep your eyes open for them as this chapter progresses ...

Sketch + shape + equation = Problem solved!

Back to the castle and the new Sieges-R-Us ladder!

You started with a **sketch** and spotted a right-angled triangle **shape** in it. After toying with the idea of a scale drawing, Pythagoras popped up with an **equation**!

So now you can work out the best moat width—and save the castle!



Sharpen your pencil

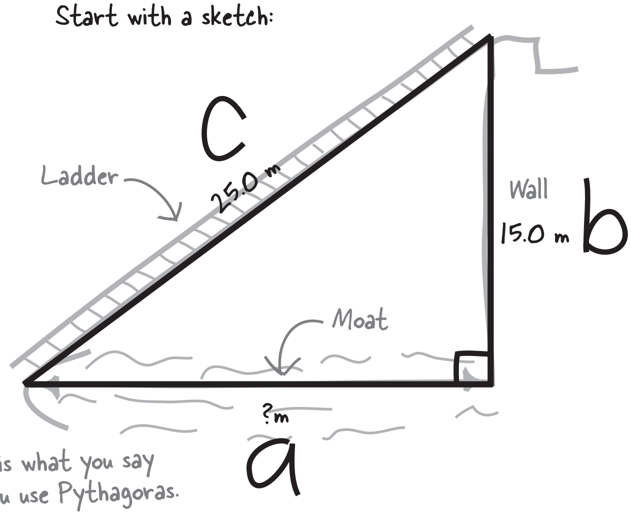
Solve your problem!

A castle is built on flat ground with 15.00 m walls. How wide must the moat be to ensure that a 25.00 m ladder only just touches the top of the wall? Assume that the base of the ladder is placed at the edge of the moat.

Sharpen your pencil Solution

A castle is built on flat ground with 15.00 m walls. How wide must the moat be to ensure that a 25.00 m ladder only just touches the top of the wall? Assume that the base of the ladder is placed at the edge of the moat.

Start with a sketch:



This is what you say if you use Pythagoras.

Want to know a, the width of the moat.

By Pythagoras, $c^2 = a^2 + b^2$

Rearrange equation for the side you want. → $a^2 = c^2 - b^2$

$a^2 = 25^2 - 15^2$

$a^2 = 400$

$a = \sqrt{400} = 20.0 \text{ m}$

So the best moat width is <u>20.0 m</u> (3 sd).

Values in your question were given to 3 sd, so your answer should have 3 sd too.

there are no Dumb Questions

Q: There were square roots in that solution, but it's been a while since I used these, and they're a bit hazy. Remind me how they work again?

A: As we saw in chapter 3, the square of a number is the number times itself.

or $3^2 = 3 \times 3 = 9$.

If you take the square root (√) of a number, the answer you get is the number you'd have to square to get the one you started off with. For example, $\sqrt{9} = 3$ because $3^2 = 9$.

Q: I noticed that I got nice round numbers – 15.0, 20.0 and 25.0 – for the side lengths. Does that always happen with right-angled triangles?

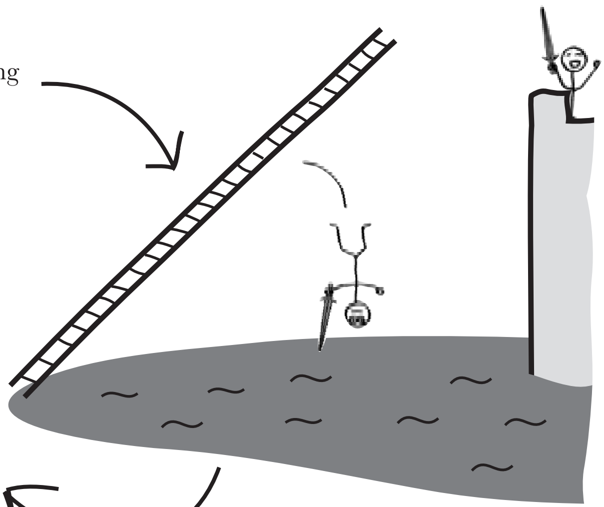
A: Here, the wall, ladder and moat formed a right-angled triangle which has nice side lengths in a 3:4:5 ratio. $3^2 + 4^2 = 5^2$, and a 15:20:25 ratio is just a 3:4:5 ratio multiplied by 5.

But usually that doesn't happen - your calculator will give you answers that you'll have to round to the same number of significant digits as the values you were initially given to work with.

You kept them out!

Under your instructions, the castle workmen start digging your 20.0 m wide moat immediately.

A couple of hours after they've finished, some attackers come along with the new 25.0 m ladder ... and have an unexpected bath!



But the attackers get smarter!

BRAIN POWER

Phoning for a pizza just isn't an option. How could you try to scare them away so you can get more supplies in?

BULLET POINTS

- The routine
  SKETCH
  SHAPE
  EQUATION
  is an excellent one!
- You'll often see **right-angled triangles**, as the ground is **horizontal** and walls, gravity and such operate **vertically**.
- The **hypotenuse** is the side opposite the right angle.
- Pythagoras' Theorem is an **equation** you can use to get the third side of a right-angled triangle if you already know the other two.
- Pythagoras' Theorem **only** works on right-angled triangles!
- If you spot a triangle with side lengths in a 3:4:5 ratio, then you know it must be right-angled (because Pythagoras' Theorem only works on right-angled triangles).
- If you forget which way around Pythagoras is, then think about the lengths of the sides and what makes sense. Or sketch out a 3:4:5 triangle and work out what Pythagoras must be from that.

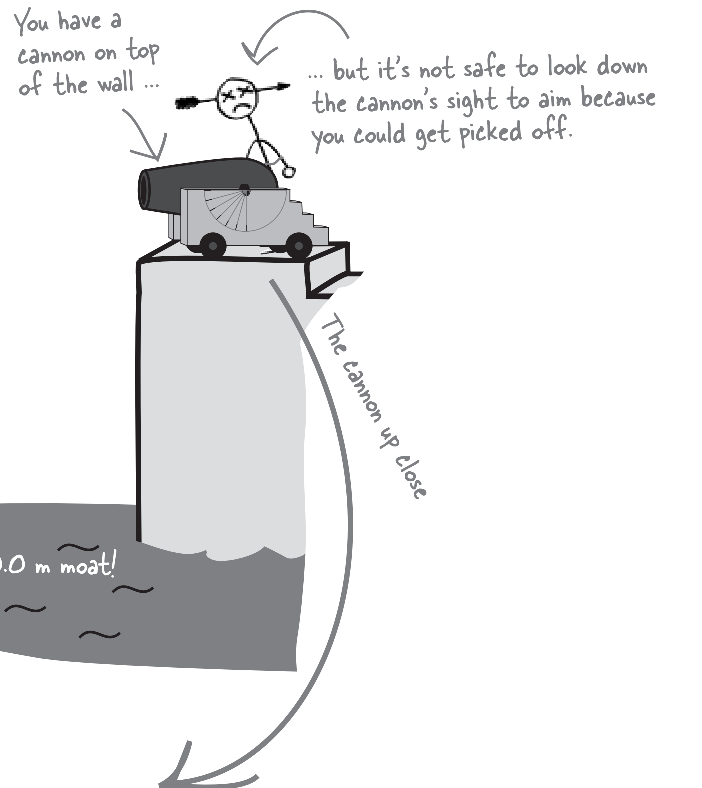
Camelot ... we have ANOTHER problem!

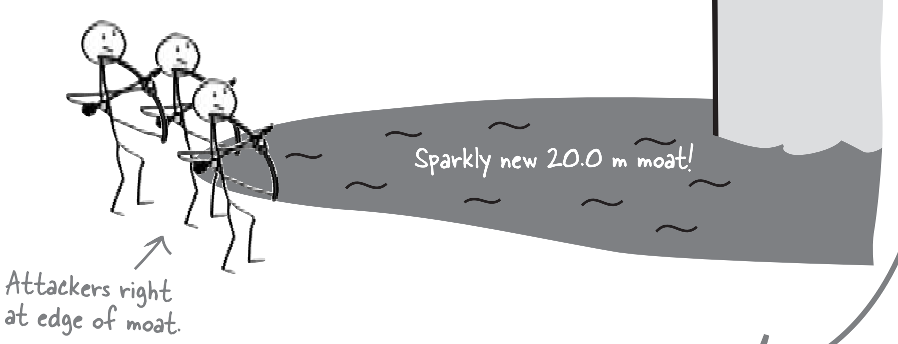
You dealt with the new ladders—and gave the attackers an early bath by widening the moat.

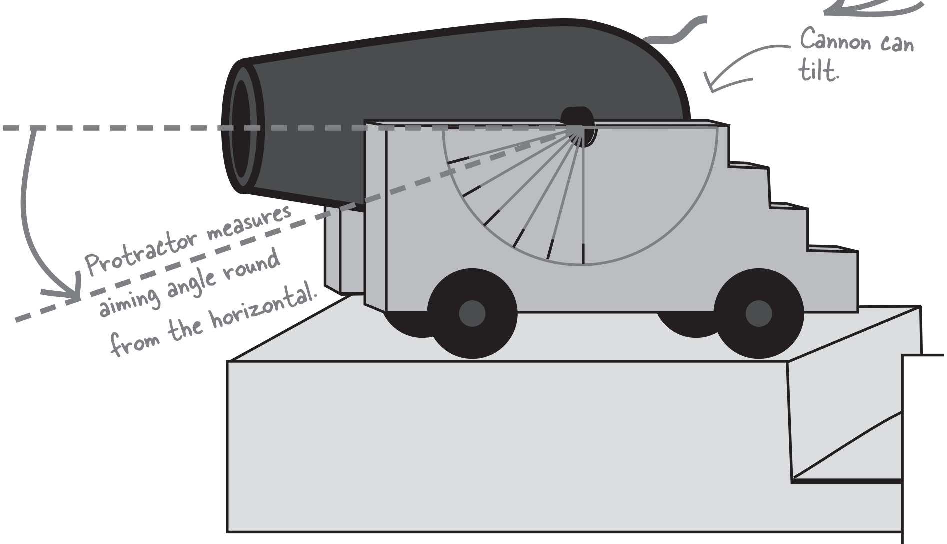
But now they're camped at the edge of the moat, waiting for you to run out of food and surrender. It would be someone else's problem ... if you weren't stuck in the castle too!

On the bright side—you have a cannon!

On the not-so-bright side, you need to aim before you fire. And you can't look over the top of the wall to aim the cannon, as they'll probably shoot at you before you can aim at them...

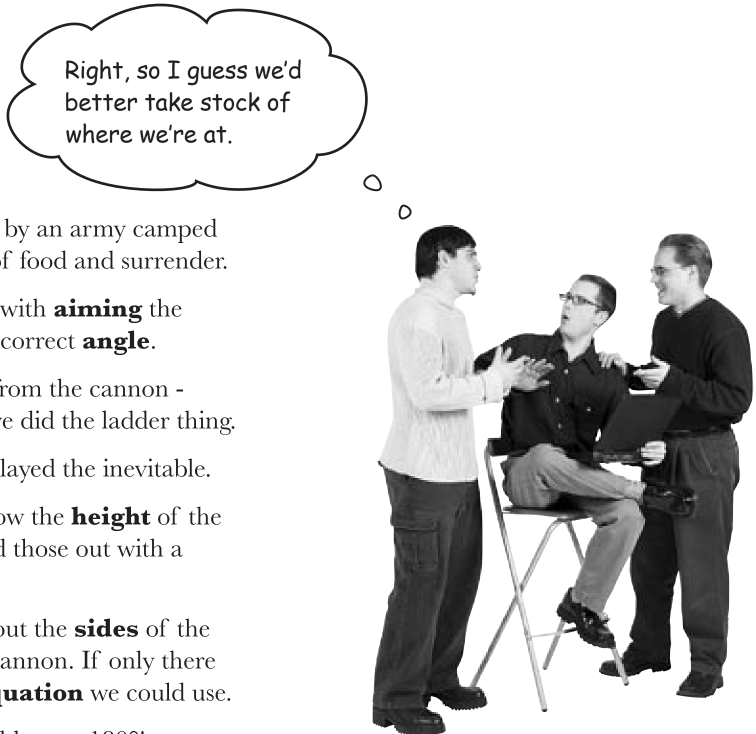






BRAIN POWER

Is it possible to aim the cannon at the attackers by the moat even though you don't have a direct line of sight?



Frank: That's easy! In a castle, surrounded by an army camped at the edge of our moat... until we run out of food and surrender.

Jim: Uhhhh. I kinda meant where we're at with **aiming** the cannon, when we can't see to point it at the correct **angle**.

Joe: Well, we know the **distance** they are from the cannon - 25.0 m! We already worked that out when we did the ladder thing.

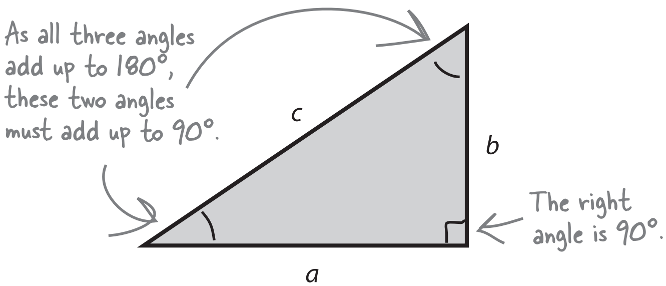
Frank: Oh yeah ... well looks like it only delayed the inevitable.

Joe: Think positive - we can do this! We know the **height** of the wall and the **width** of the moat. We worked those out with a sketch and Pythagoras' Theorem.

Jim: Yeah, we know all there is to know about the **sides** of the triangle. But we need an **angle** to aim the cannon. If only there was some kind of 'Pythagoras for angles' **equation** we could use.

Joe: But there is! The angles in a triangle add up to 180°!

Jim: And we already know that the right angle is 90°, so the other two angles must add up to 180° - 90° = 90°. That's progress!



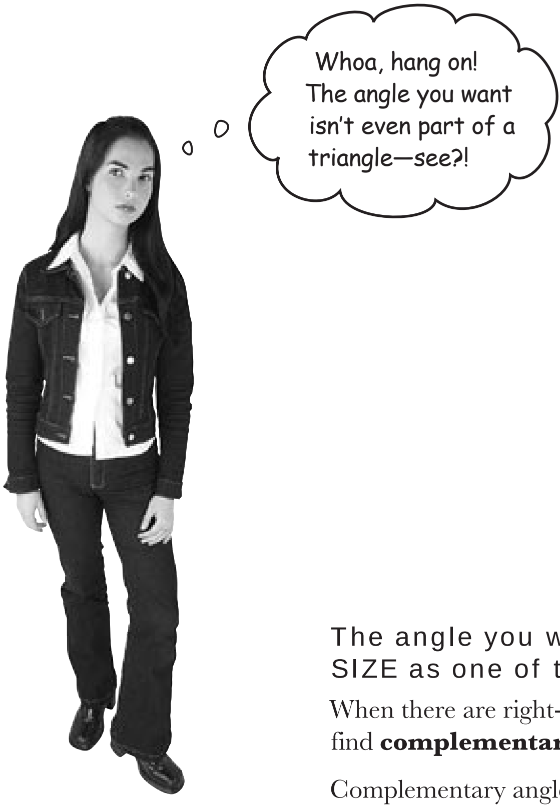
The three angles in a triangle add up to 180°.

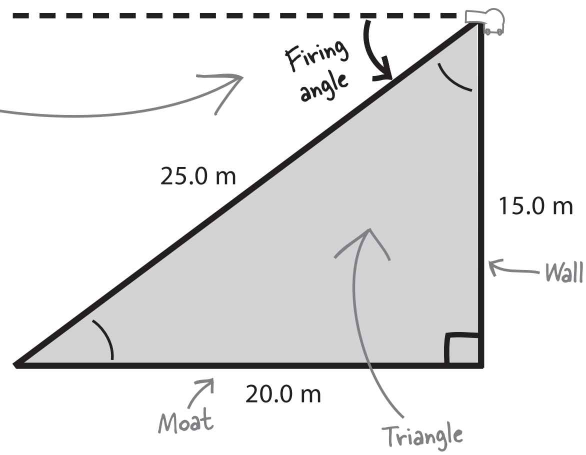
Frank: Not a lot of progress though. The other two angles could be 1° and 89°, or 45° and 45°, or 18.2° and 71.8°.

Jim: Oh yeah. You can't find out **two** things you don't know if you only have **one** equation to work with.

Joe: Well, since we only have one equation at the moment, maybe we can do an **experiment** with some different right-angled triangles and see if we can figure something out?

Frank: OK, that kinda thing's worked for us before ...





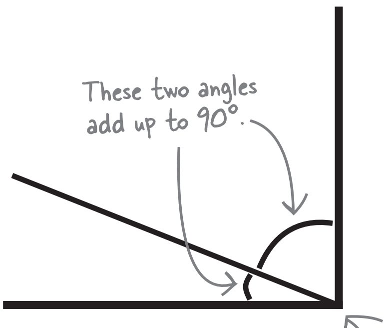
The angle you want might be the SAME SIZE as one of the angles in your triangle.

When there are right-angled triangles around, you'll often find **complementary** and/or **supplementary** angles.

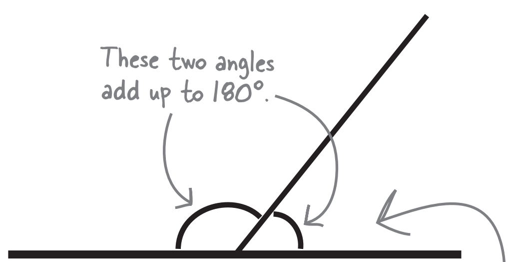
Complementary angles add up to 90°, and supplementary angles add up to 180°. They're useful because they help you work out the sizes of angles that aren't in your triangle.

You don't need to remember which is which to be able to actually USE them!

Try to spot angles that add up to 90° or 180° in and around your right-angled triangles.



Complementary angles add up to 90°.

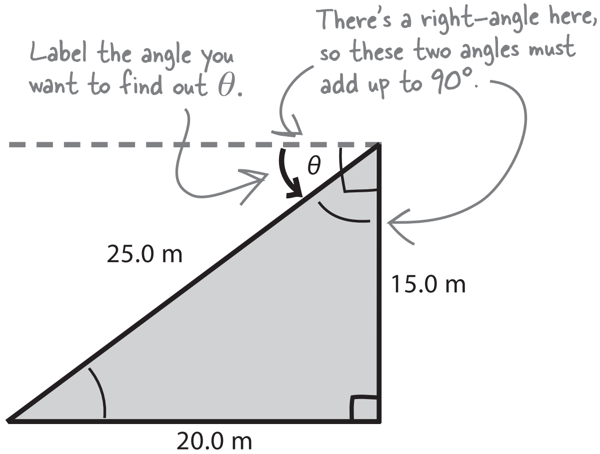


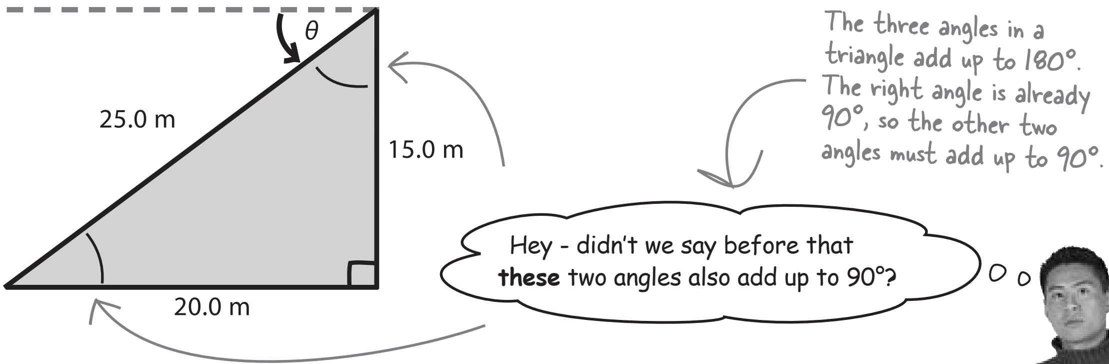
Supplementary angles add up to 180°.

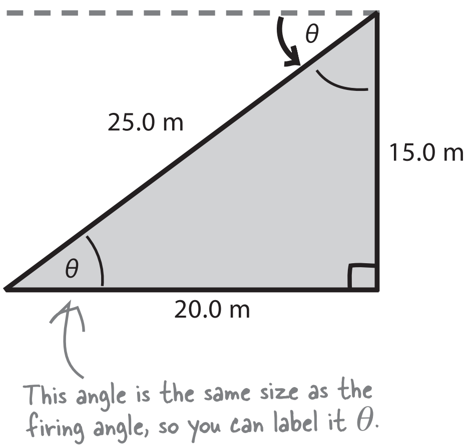
Relate your angle to an angle inside the triangle

We're going to give the firing angle the symbol θ. This is the Greek letter theta and is often used in physics to represent an angle that you're interested in.

You want to calculate the firing angle - but it isn't part of your triangle. However, when there are right-angled triangles around, there are often angles that add up to 90° or 180° that you can use to work out the angle you're interested in. Here, the firing angle and the angle at the top of your triangle add up to 90°.



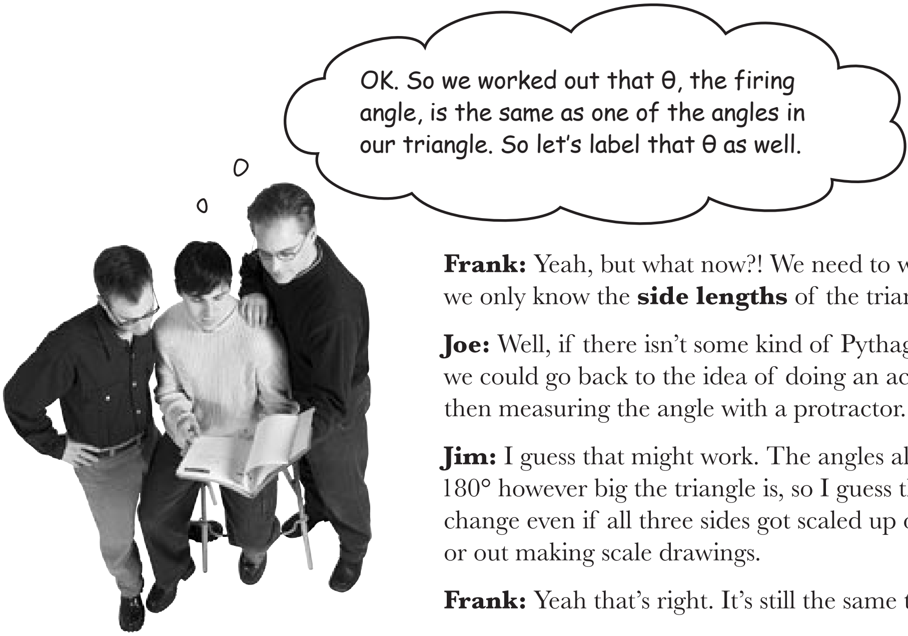




The firing angle is the same size as an angle in your triangle.

Now you have two sets of angles that add up to 90°. The first set is the firing angle, θ, and the angle at the top of the triangle. The second set is the angles at the bottom and top of the triangle.

This means that θ and the angle at the bottom of the triangle must both be the same size. So you can label the angle at the bottom of the triangle θ as well, and get on with trying to work out what size it should be to fire the cannonball ...



Frank: Yeah, but what now?! We need to work out an **angle** - but we only know the **side lengths** of the triangle.

Joe: Well, if there isn't some kind of Pythagoras for angles, maybe we could go back to the idea of doing an accurate scale drawing, then measuring the angle with a protractor.

Jim: I guess that might work. The angles always have to add up to 180° however big the triangle is, so I guess that the angles wouldn't change even if all three sides got scaled up or down as we zoom in or out making scale drawings.

Frank: Yeah that's right. It's still the same triangle!

Jim: It says here that triangles with **equal angles** (but different side lengths) are called **similar triangles**.

Joe: So the scale drawing would be of a similar triangle. And the angles would be the same as the original, big triangle. Cool!

Frank: But what if the attackers move? We'd have to do another scale drawing, and that's gonna take time.

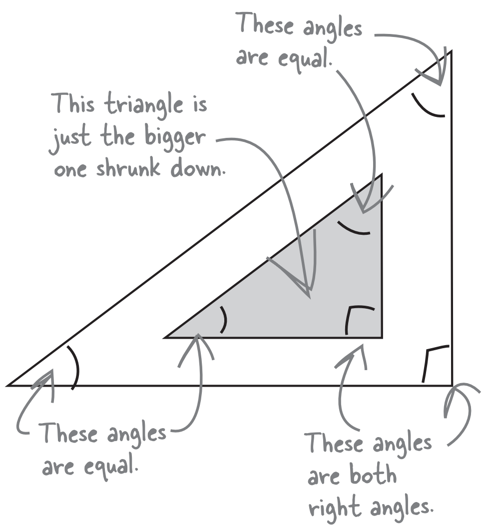
Jim: We can do that in advance. We can draw all the right-angled triangles that you could ever get and measure their angles.

Joe: Yeah, we can make the information into a **table** so that you can look up the angles of any right-angled triangle without having to measure them. And we could get a computer or a calculator to **look up** the angles when we tell it the sides—that bit would be really quick.

Frank: But if we were drawing all possible right-angled triangles, some of them would be really huge, like, miles long!

Jim: Not necessarily. We just worked out that **the angles of similar triangles are always the same**. So if we have a triangle in the table with side lengths 3 cm, 4 cm, and 5 cm, we don't also need one with 3 miles, 4 miles, 5 miles or 1500 miles, 2000 miles, 2500 miles, etc, as they're all just the same triangle, except zoomed in or out a bit.

Joe: Cool! Let's get cracking!



Triangles with equal angles are called similar triangles.

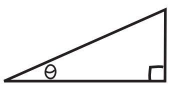
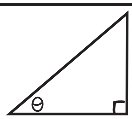
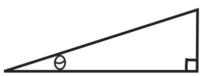
Not so fast! How are you going to **arrange** the table so that you can actually **find** the triangle you're trying to look up?

You can classify triangles by their shape - but need a way of looking them up later on.

Using **similar triangles** in your table is the best bet, so each type of triangle only needs to appear in the table once. For example, instead of the table including lots of different triangles with side lengths in a ratio of 3:4:5, it only needs to include one.

But the problem now is how to arrange or **index** the table. How are you actually going to **find** the triangle when you go to look it up?

The problem is that the triangle someone wants to look up could be any SIZE or zoomed in or out from the SHAPE of its entry in the table. So how on earth can you ever find the row with the right SHAPE in it?

Triangle shape	**Angle θ (°)**	Other **angle (°)**
	24.6	65.4
	40.6	49.4
	18.5	71.5

If you look up the shape of your triangle, the table will give you its angles.

The angles in this table have 3 significant digits because that's easier to write – but the angles in the completed table could have more significant digits.

You need some way to classify the SHAPE of a triangle that doesn't involve its angles (which are what you want to look up).

BRAIN POWER

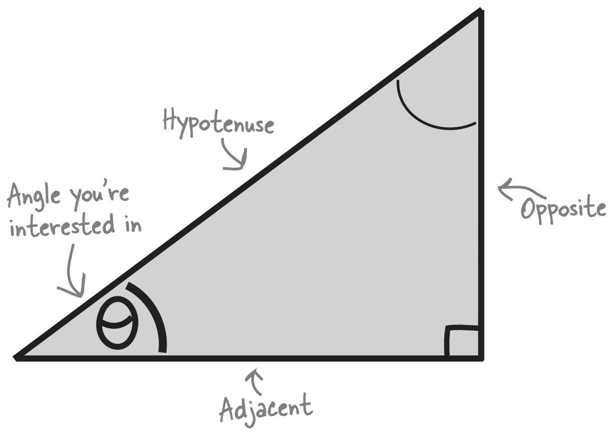
How might you classify the **shapes** of the triangles in your table?

Classify similar triangles by the ratios of their side lengths

If you take a triangle and magnify or reduce it (zoom in or out), you make a **similar triangle** which has the same angles as the triangle you started off with. For example, a scale drawing of a bigger triangle is a similar triangle.

Similar triangles don't have the same side lengths, but they do have the same **ratios** of side lengths. To work those out, you have to decide on a way of naming the sides so that everyone knows what you're talking about:

> The hypotenuse it labelled '**hypotenuse.**'
> The side opposite 'your' angle is labelled '**opposite.**'
> And you label the third side '**adjacent.**'



Label the sides the same way every time.

The **ratio** of two side lengths is simply **one side length divided by the other side length**. There are three sides and three ratios, which all have names:

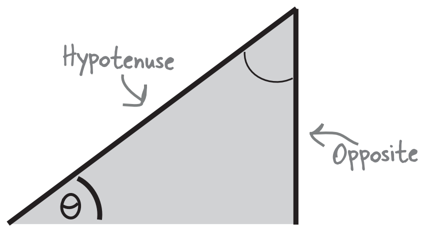
> **Sine** (pronounced "sign")
>
> **Cosine** (pronounced co-sign)
>
> **Tangent**

When you're writing these as part of an equation, sine is abbreviated to sin, cosine becomes cos and tangent becomes tan.

If you were saying this out loud, it would be "sine theta."

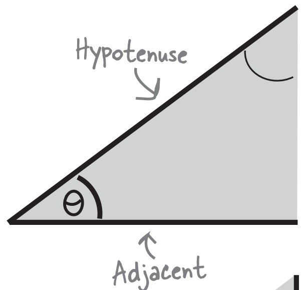
$$\sin(\theta) = \frac{\text{opposite}}{\text{hypotenuse}}$$

The RATIO of two side lengths is one side length divided by another side length.



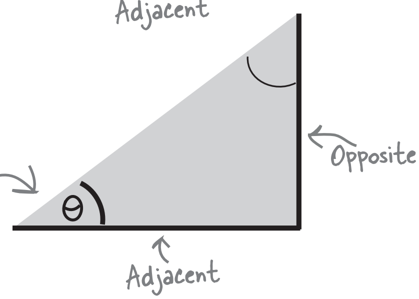
$$\cos(\theta) = \frac{\text{adjacent}}{\text{hypotenuse}}$$

Don't worry about remembering which name is which ratio at the moment.



$$\tan(\theta) = \frac{\text{opposite}}{\text{adjacent}}$$

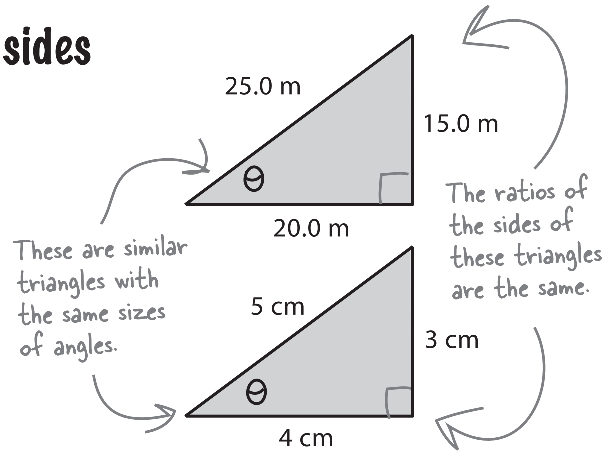
The angle you are interested in goes in brackets after the name of the ratio.



Similar triangles have the same RATIOS of side lengths. Sine, cosine, and tangent are RATIOS.

Tangent has the same name as the tangent to a curve – which can be confusing at first. But it's always abbreviated to 'tan' in equations.

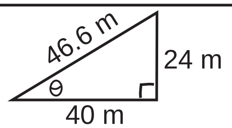
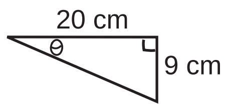
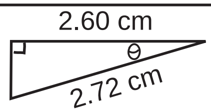
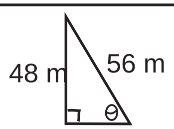
Sine, cosine and tangent connect the sides and angles of a right-angled triangle



The ratios sine, cosine and tangent are a way of classifying the similar triangles in your table. Suppose you have two **similar triangles**, one with side lengths 3 cm, 4 cm and 5 cm, and the other with side lengths 15.0 m, 20.0 m and 25.0 m. As they're similar triangles, you know that their **angles** must be equal.

And although their sides aren't equal, the **ratios** of their sides are. In the first triangle, the ratio of the two shortest sides is $\tan(\theta) = \frac{3}{4} = 0.75$; in the second triangle, $\tan(\theta) = \frac{15}{20} = 0.75$.

Label the sides of these right-angled triangles with 'h' (hypotenuse), 'o' (opposite), and 'a' (adjacent), then fill the blanks in the table below. If a side length is missing, then use Pythagoras to work it out in the space under the table. The angle θ is the one you're interested in.

	Triangle	sin(θ)	cos(θ)	tan(θ)
a		$\frac{o}{h} = \frac{24}{46.6} = 0.515$		
b				
c				
d				

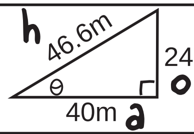
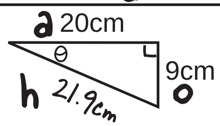
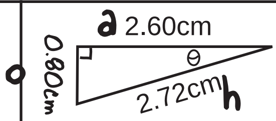
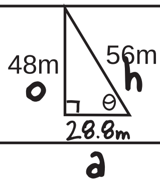
Use the definitions of sine, cosine and tangent on the opposite page to fill in these boxes.

Space for working out the missing side lengths using Pythagoras.

At the moment, this table just has ratios in it. The final table will have the angle that each ratio corresponds to, so you can look up an ANGLE if you have a RATIO, and vice-versa.

Exercise Solution

Label the sides of these right-angled triangles with 'h' (hypotenuse), 'o' (opposite), and 'a' (adjacent), then fill the blanks in the table below. If a side length is missing, then use Pythagoras to work it out in the space under the table. The angle θ is the one you're interested in.

	Triangle	sin(θ)	cos(θ)	tan(θ)
a		$\frac{o}{h} = \frac{24}{46.6} = 0.515$	$\frac{a}{h} = \frac{40}{46.6} = 0.858$	$\frac{o}{a} = \frac{24}{40} = 0.600$
b		$\frac{o}{h} = \frac{9}{21.9} = 0.411$	$\frac{a}{h} = \frac{20}{21.9} = 0.913$	$\frac{o}{a} = \frac{9}{20} = 0.45$
c		$\frac{o}{h} = \frac{0.80}{2.72} = 0.294$	$\frac{a}{h} = \frac{2.60}{2.72} = 0.956$	$\frac{o}{a} = \frac{0.80}{2.60} = 0.308$
d		$\frac{o}{h} = \frac{48}{56} = 0.857$	$\frac{a}{h} = \frac{28.8}{56} = 0.514$	$\frac{o}{a} = \frac{48}{28.8} = 1.67$

The ratios don't have units, as $\frac{\text{length}}{\text{length}}$ is dimensionless.

By Pythagoras, $h^2 = o^2 + a^2$

Triangle b:

$h^2 = 20^2 + 9^2$

$\Rightarrow h = \sqrt{481} = 21.9$ cm (3 sd)

Triangle c:

$o^2 = h^2 - a^2$

$o^2 = 2.72^2 - 2.60^2$

$\Rightarrow o = \sqrt{0.6384} = 0.80$ cm (3 sd)

Triangle d:

$a^2 = h^2 - o^2$

$a^2 = 56^2 - 48^2$

$\Rightarrow a = \sqrt{832} = 28.8$ m (3 sd)

The units are important, as some triangles are in m and some are in cm.

When you've labelled your triangle 'h', 'o' and 'a' (hypotenuse, opposite and adjacent) it's fine to use these letters instead of a, b and c in Pythagoras' Theorem.

The side length ratios - sine, cosine and tangent - are always the same for similar triangles.

How to remember which ratio is which??

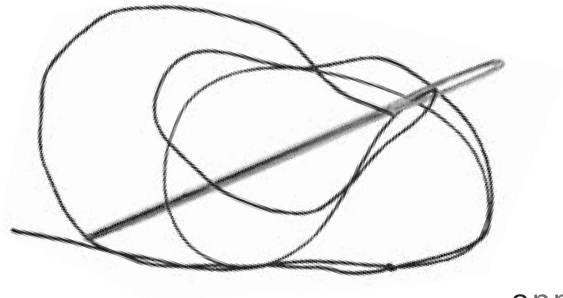
The mnemonic 'SOH CAH TOA' helps you to remember which ratio involves which sides.

SOH - Sine is **O**pposite divided by **H**ypotenuse.

CAH - Cosine is **A**djacent divided by **H**ypotenuse.

TOA - Tangent is **O**pposite divided by **A**djacent.

SOH



$$\sin(\theta) = \frac{\text{opposite}}{\text{hypotenuse}}$$

CAH

Like they say it in Boston.

$$\cos(\theta) = \frac{\text{adjacent}}{\text{hypotenuse}}$$

TOA

$$\tan(\theta) = \frac{\text{opposite}}{\text{adjacent}}$$

Say it a few times before you turn the page to help you remember.

"SOH CAH TOA"
"SOH CAH TOA"
"SOH CAH TOA"
"SOH CAH TOA"
"SOH CAH TOA"

If you forget which sides go with which ratio, write down this mnemonic, and go on from there.

Sine Exposed

This week's interview: sine is mighty fine!

Interviewer: So, sine, would you say that your bad reputation is justified?

sine: Sigh. The problem is that I'm sometimes get called 'sin.' But I'm still pronounced sin**e** - with an 'e' - like 'pine.' I'm not bad—I'm a **trigonometric function**!

Interviewer: Err ... a trigono-what-now?!

sine: Trigonometric - that means I help you with triangles. And I'm a function, so you give me a number, and I give you a different number back.

Interviewer: Hmmm. No, sorry, you've lost me. Why would I ever want to swap numbers with you?

sine: Well, the number you give me is an **angle**, from a right-angled triangle. I give you back the **ratio** of the side opposite the angle divided by the hypotenuse.

Interviewer: Riiiiight. I'm not sure why I'd ever care about that, but there you go.

sine: I'm the missing link! I'm what connects what you can know about the **lengths of a triangle's sides** to what you can know about the sizes of its **angles**.

Interviewer: Hmm. So what?

sine: Right angled triangles and angles are very important in physics. In fact, I'm one of the most important things in your entire physics toolbox!

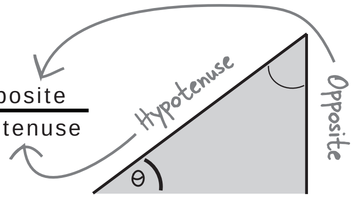
Interviewer: So, without you, people wouldn't be able to do most of the stuff in the rest of this book.

sine: Exactly!

Interviewer: You sound extremely important then. Can you just run past us again how you work?

sine: You give me an angle–probably one you found in a right-angled triangle. And I give you a number back, which is the ratio of the side opposite the angle divided by the hypotenuse. Like this:

$$\sin(\theta) = \frac{\text{opposite}}{\text{hypotenuse}}$$



Interviewer: But I don't really see how that helps.

sine: Well, if you already know the angle and the side opposite it, you can rearrange that equation to get the hypotenuse. And if you already know the angle and the hypotenuse, you can get the opposite side.

Interviewer: But what if I know the length of the adjacent side, plus one other side? Should I get Pythagoras to work out the missing side before calling on you to help?

sine: Not necessarily—you could call on my close relatives cosine or tangent.

Interviewer: And what do they do?

sine: Well, cosine is the ratio of the side adjacent to the angle divided by the hypotenuse. And tangent is the opposite divided by the adjacent.

Interviewer: Oh, so you guys cover all possible combinations of two sides of the triangle between you. So if I already know an angle and a side, I can get the length of any other side in one step. Cool. But what if I don't know any angles at all? Can I use you to work out the **angles** in a triangle too?

sine: Going the other way - from side lengths to angles?

Interviewer: That's right - some guys in a castle were trying to do that just before we went on air.

sine: You'll want my **inverse** to go the other way. He looks up the table of angles and ratios in the opposite direction. So you give my inverse a ratio, and he gives you the angle that the ratio corresponds to.

Interviewer: That sure sounds useful. Thank you, sine, you've been just swell.

there are no Dumb Questions

Q: Why choose to have three different ratios? Why not one, or two ... or five or six?

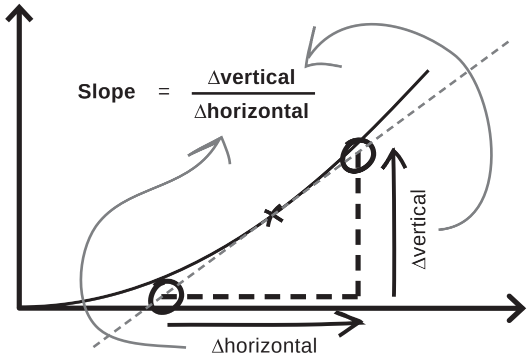
A: Suppose you only know two sides of your right-angled triangle, and want to find out one of the angles. If there were only one or two ratios in your calculator's table, you'd often have to work out the third side using Pythagoras before you could work out a ratio to get the angle. Having three ratios in the table covers all the different combinations of two sides.

Q: And why not five or six? You could have hypotenuse divided by adjacent, for example.

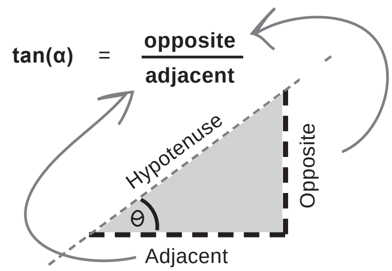
A: There are special names for these other ratios, but they're not important right now. You'll never need to use them on your physics course.

Q: OK. Now, I was wondering something else about 'tan' - or tangent, to give it its full name. I did tangents before, right? But that was something to do with working out the gradient of a curve, wasn't it?

A: Good point. Very good point! A tangent is a straight line that only touches a curve at one point - and you did use one to work out the slope of a line in chapter 7:



Your formula for tan(θ) is basically a rewrite of the "slope of a line" equation, which is where the name 'tangent' comes from.



The trigonometric functions sine, cosine and tangent connect what you know about the <u>sides</u> of a right-angled triangle to what you know about its <u>angles.</u>

If you put a value into a function, then it gives you a different value back. For example, if you give an angle to the sine function, it gives you back the ratio of the opposite and hypotenuse.

Q: Suppose I work out a ratio (sine, cosine or tangent) from two of the sides of my triangle. How do I use that to get an angle?

A: You need to calculate the firing angle for the cannon using the ratio of two sides of the moat-wall triangle. We're just getting on to that ...

Calculators have sin(θ), cos(θ) and tan(θ) tables built in

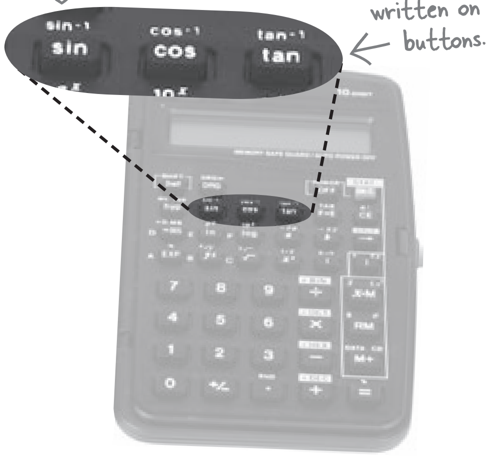
Your calculator already contains a table a bit like the one you've been filling in, where the angles of similar triangles are indexed using the ratios sine, cosine and tangent.

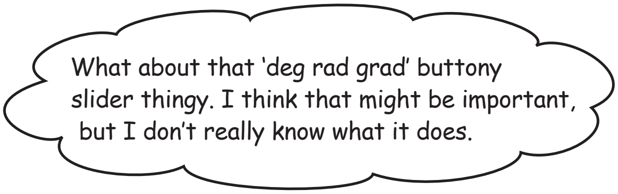
To get a ratio when you have an angle, use the sin(θ), cos (θ) and tan (θ) functions. They're usually printed on a calculator button.

To get an angle when you have a ratio, you need to use the **inverse** sine, cosine and tangent functions. They're usually written above the 'sin' 'cos' and 'tan' buttons. You use the inverse functions by pressing the 'shift' or '2nd fn' button first.

The inverse functions are usually called $\sin^{-1}$, $\cos^{-1}$ and $\tan^{-1}$. This is weird, as at first glance the '$^{-1}$' bit looks like scientific notation. But it's not - it's just a convention for indicating an inverse function that you'll unfortunately need to get used to.

Sometimes the inverse functions are called arcsine, arccosine and arctangent. These are abbreviated to asin, acos and atan on calculators. Make sure you know what they're called on your calculator!





Make sure your calculator's set to degrees!

If you and your calculator are speaking different languages, there's no way you're going to get the right answer. There's more than one way of measuring angles. Right now, make sure your calculator's set to work in degrees.

Usually, there'll be a switch or a button with a 'deg' or ° sign on it that you can use to sort this out.

It's also good to spend time playing with your calculator to see which **order** you need to press the buttons in. On some calculators, you type the number in, then press the button for the function you want. Other, more expensive, calculators let you type things in the same order you'd write them.

Play with your calculator, and make sure you know how it works before you go on to find the firing angle.

Know how your calculator works and inwhich order you should press the buttons!

Then the calculator will do what you want it to.

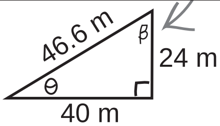
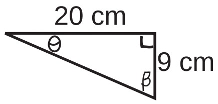
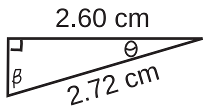
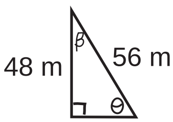
Exercise

Here's the table you already started filling in on pages 351-352. It's similar to the kind of table in your calculator, and it's time to practice moving smoothly between sides and angles before you do the critical mission of calculating the angle back at the castle.

The side lengths you were originally given are written in type. Your job is to skip the Pythagoras step and use the appropriate **ratio** — sin(θ), cos (θ) or tan (θ) — to get from the two given sides to find the angle θ. Show which ratio you'd use by circling it.

Once you've worked out θ, try to spot a quick way of calculating β for each triangle as well.

We've used another Greek letter, β (beta), to represent the other angle.

	Triangle	sin($\boldsymbol{\theta}$)	cos($\boldsymbol{\theta}$)	tan($\boldsymbol{\theta}$)	Angle $\boldsymbol{\theta}$ (°)	Angle $\boldsymbol{\beta}$ (°)
a		0.515	0.858	0.6	31.0	
b		0.411	0.913	0.45		
c		0.294	0.956	0.308		
d		0.857	0.514	1.67		

Triangle a – could use any two of three given sides.
Use tangent with opp and adj.

$$\theta = \tan^{-1}(0.6) = 31.0° \text{ (3 sd)}$$

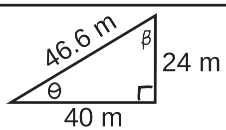
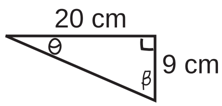
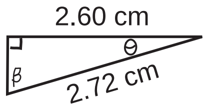
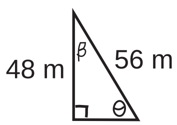
If you're not sure what to do with your calculator, play with the one we've already done, and see which buttons you need to press to get the same answer.

Exercise Solution

Here's the table you already started filling in on pages 351-352.

The side lengths you were originally given are written in type. Your job is to skip the Pythagoras step and use the **ratio** (sin, cos or tan) you can get from the two given sides to find the angle α. Show which ratio you'd use by circling it.

Once you've worked out θ,try to spot a quick way of getting β for each triangle as well.

	Triangle	sin(***θ***)	cos(***θ***)	tan(***θ***)	Angle ***θ*** **(°)**	Angle ***β*** **(°)**
a		0.515	0.858	0.6	31.0	59.0
b		0.411	0.913	0.45	24.2	65.8
c		0.294	0.956	0.308	17.1	72.9
d		0.857	0.514	1.67	59.0	31.0

Triangle a: could use any two of three given sides.
Use tangent with opp and adj.

$\theta = \tan^{-1}(0.6) = 31.0°$ (3 sd)

Triangle b: $\theta = \tan^{-1}(0.45) = 65.8°$ (3 sd)

Triangle c: $\theta = \cos^{-1}(0.956) = 17.1°$ (3 sd)

Triangle d: $\theta = \sin^{-1}(0.857) = 59.0°$ (3 sd)

Slow way of getting β: find sine, cosine or tangent of β.

Quick way of getting β: $\theta + \beta = 90°$

there are no Dumb Questions

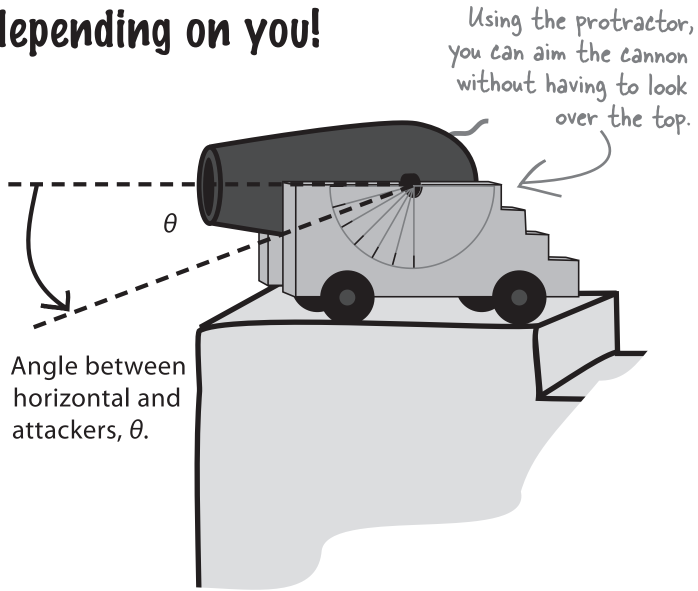
Q: I was playing with my calculator and typed in $\sin^{-1}$ (random number), and it gave me an error instead of an angle. Why was that?

A: It's great that you took the time to play with your calculator. Since the hypotenuse is the longest side, the length of any other side divided by the length of the hypotenuse must always be less than 1. So if you type in $\sin^{-1}$ of a random number bigger than 1, it's not in the table, and the calculator gives you an error.

Back at the castle, everyone's depending on you!

Back at the castle, things are starting to get slightly desperate. The supply of food has just run out, and morale is low.

We're running out of time, and we'll need to work out how to aim the cannon and make the attackers retreat.



The trigonometric functions sine, cosine and tangent let you work out angles from side lengths, and vice-versa.

This is how it might be worded in a physics exam.

Sharpen your pencil

A cannon sits on top of a 15.0 m castle wall. Outside the castle, at the edge of its 20.0 m moat, are some attackers. What angle should the cannon make with the horizontal if it is to be pointed directly at the attackers?

Sharpen your pencil Solution

A cannon sits on top of a 15.0 m castle wall. Outside the castle, at the edge of its 20.0 m moat, are some attackers. What angle should the cannon make with the horizontal if it is to be pointed directly at the attackers?

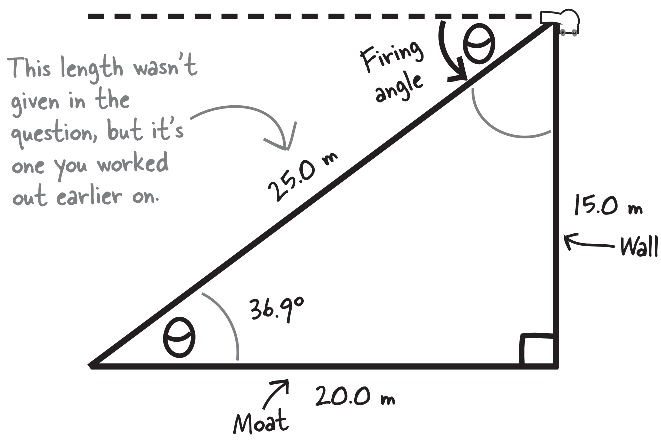
The firing angle is the same as the angle θ in the triangle.

$$\tan(\theta) = \frac{o}{a} = \frac{15.0}{20.0} = 0.75$$

Then use $\tan^{-1}$ button to look up table in calculator:

$$\theta = \tan^{-1}(0.75) = 36.9^\circ \text{ (3 sd)}$$

The cannon points at the attackers if it's aimed at 36.9° below horizontal.



Remember to give some MEANING to your answer – it's more than a number.

You can know everything! *

When you have a **right-angled triangle** and know the length of one **side**, plus **one other fact** (either another side length or an angle), you now have superpowers that enable you to work out **all** the other sides and angles.

You're going to see a lot of right-angled triangles through the rest of the book since the ground is horizontal and gravity accelerates things vertically at right-angles to the ground.

If you know ONE SIDE, plus ONE OTHER FACT (a side or an angle), you can work out EVERYTHING about a right-angled triangle using sine, cosine, tangent and Pythagoras.

*** Well, everything about a right-angled triangle at least!**

Does your answer SUCK?

Remember to check your answer once you've got it! Does the angle feel like it's the right kind of **size**? Did you remember the **units**? How about the **calculations**? And what about the '**kontext**' - stepping back and thinking about the big picture before moving on.

Well done if you already did this automatically when you worked it out!

Sharpen your pencil

Fill in the sections to see if your answer to the cannon question SUCKs. Remember to think about the 'k'ontext of what you're actually being asked to do!

S — **SIZE** ....

....

....

U — **UNITS** ....

....

....

C — **CALCULATIONS** ....

....

....

K — **"K'ONTEXT** ....

....

....

Sharpen your pencil Solution

This is an EXTREMELY useful way of checking if your angle is plausible.

Fill in the sections to see if your answer to the cannon question SUCKs. Remember to think about the 'k'ontext of what you're actually being asked to do!

S

SIZE I'm expecting an angle less than 45°, as it's opposite the smallest side of the triangle. So 36.9° is very plausible, as the smallest side isn't that much smaller than the other two, and my angle isn't that much less than 45°.

U

UNITS I've put them in – it's an angle, measured in degrees.

C

CALCULATIONS Well, they look OK. I rearranged the equation and used the inverse sine button OK (I didn't press the normal sine one by mistake).

K

"K'ONTEXT It's a cannon firing a cannonball from the top of a wall, so it goes along a straight line that's the hypotenuse of a right-angled tri ... hang on.
OH NO – WE FORGOT ABOUT GRAVITY!!

Before you launch in, think:
"Am I actually answering the question I was asked?"

It saves time to do this BEFORE starting on the math rather than afterwards.

Uh oh. Gravity...

Everyone forgot about **gravity**! The calculations assumed that the cannonball's going to follow the same **straight** path as the ladder. So although you got an answer which was the correct size with correct units and flawless calculations, it wasn't the answer to the problem we have to solve!

Gravity makes objects like baseballs and footballs travel along **curves** as they fly through the air. So the cannonball's going to curve too because gravity will **accelerate it downwards**.

Solving the wrong problem is a common mistake.

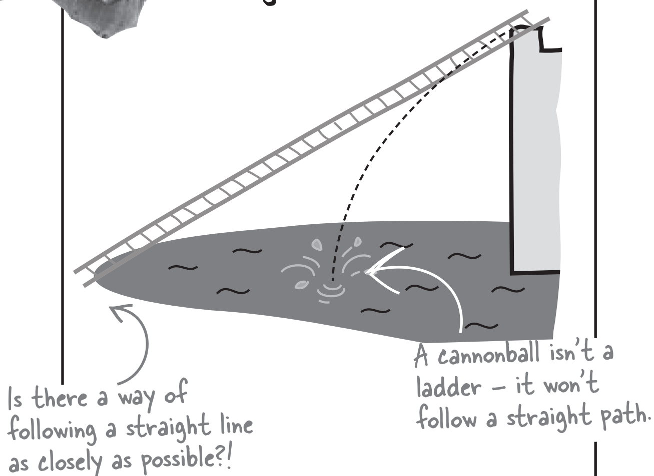
This happens more often than you'd think! Because the triangle thing worked for the ladder problem, the guys kept plowing ahead, without backing up first to make sure what they were doing was an appropriate way of solving the new problem.

Any time you get asked something new, sit back and work out what you're supposed to do before thinking about how you'll do it. **First what, then how**. Then you'll be fine.

So what affects how much the cannonball curves? A baseball and a bullet appear to curve by different amounts. So maybe the firing angle will be OK after all if the cannonball doesn't deviate from its part all that much. To work out if the angle's OK, it's time to **be** the cannonball!

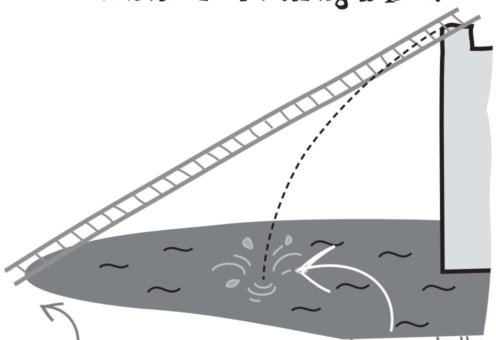
BE the cannonball

Your job is to imagine you're the cannonball. What makes you change direction as you go through the air.? And what affects how much you deviate from a straight line?



BE the cannonball - SOLUTION

Your job is to imagine you're the cannonball. What makes you change direction as you go through the air? And what affects how much you deviate from a straight line?



I'm being accelerated downwards by gravity.

If I was going slowly, I'd land very close to the wall.

If I was going quickly, I'd land further out.

If I was going really quickly, I'd almost go along a straight line.

My velocity affects how much I deviate from a straight line.

Jim: We might still be OK though. I just looked up the cannon on the Sieges-R-Us website, and its muzzle **velocity** is 90 m/s! That's high compared with the distance it's traveling, so maybe there won't be time for it to deviate from its original path too much.

Joe: We could use our **equations of motion**... except that we don't know the cannonball's total **displacement**! We know that the straight line distance from the cannon to the enemy is 25.0 m. But how do we get the length of its curved path?

Jim: Maybe that doesn't matter though. Back in the desert, we didn't need to know everything about the cage to be able to use equations of motion to work things out.

Joe: So ... what do we know? The cannonball's initial velocity ($\boldsymbol{v}_0$), its initial displacement ($\boldsymbol{x}_0$) and the acceleration due to gravity ($\boldsymbol{a}$).

Frank: Except - how are we going to put the numbers into the equations? Before, the acceleration and velocity vectors were always along the same straight line, either in the same direction or in opposite directions. So we defined one direction as positive and the other as negative. But with the cannon, the acceleration vector points down, and the velocity vector points at an angle. They're not opposites! How do we deal with that?

Joe: Ah ... I see what you mean. And it's even worse - the direction the cannonball's going in keeps changing. So both the size and the direction of the velocity vector are changing all the time!

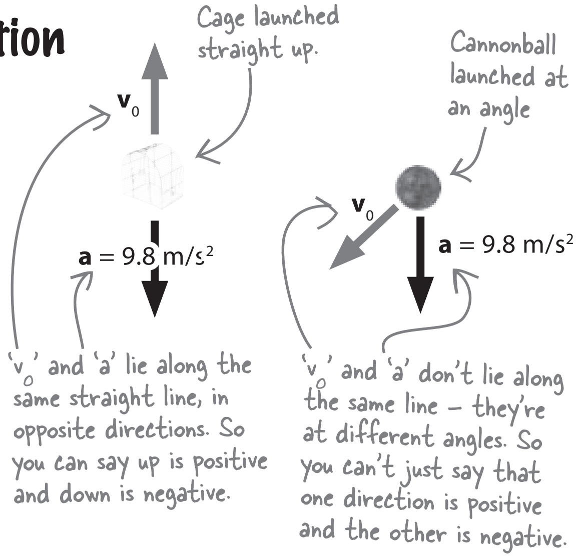
Jim: Mm-hm. How on earth are we gonna deal with that?

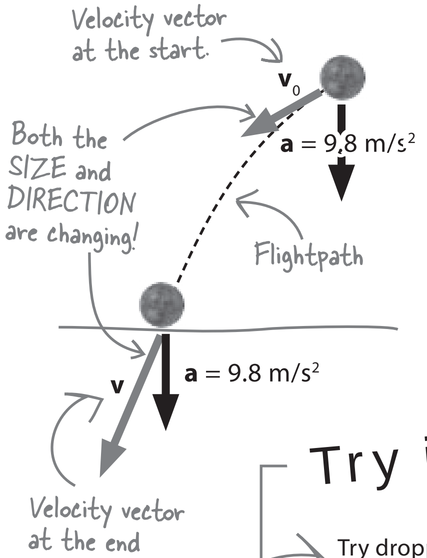
Joe: We could always go back to the experiment idea since we're doing something totally new now ...

The cannonball's velocity and acceleration vectors point in different directions

There's a big difference between the cannonball and an object launched vertically. The cannonball's velocity and acceleration vectors point in **different directions**. Not opposite directions - they're at completely different **angles**.

This creates a problem with the math. Before, we defined 'up' as the positive direction, and 'down' as the negative direction. This was because a launched object's velocity vector points up, and its acceleration vector points down. But the cannonball's velocity doesn't point up or down—it's at an angle—so you can't do that.





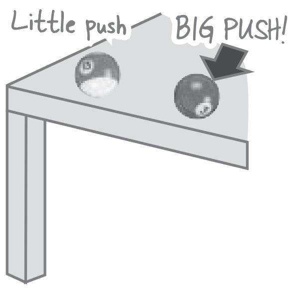
And even worse - as the cannonball accelerates, it **changes direction** and curves towards the ground. As time goes on, the direction of the velocity vector changes to point more and more towards the ground. That sounds difficult to deal with.

What IS possible is to **try** things out. You can see if there's any difference between how gravity acts on an object that is dropped versus an object that is already moving horizontally (like the cannonball is) before it falls.

Try it!

Try dropping two balls off the edge of a table at the same time, one with a little push and one with a big push. Keep the pushes horizontal for now so you're not 'helping' either of them downwards by pushing them towards the ground.

Look out for where they land - and when they land - and write down anything you notice.



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Or you could use two pens instead – the type of object doesn't matter.

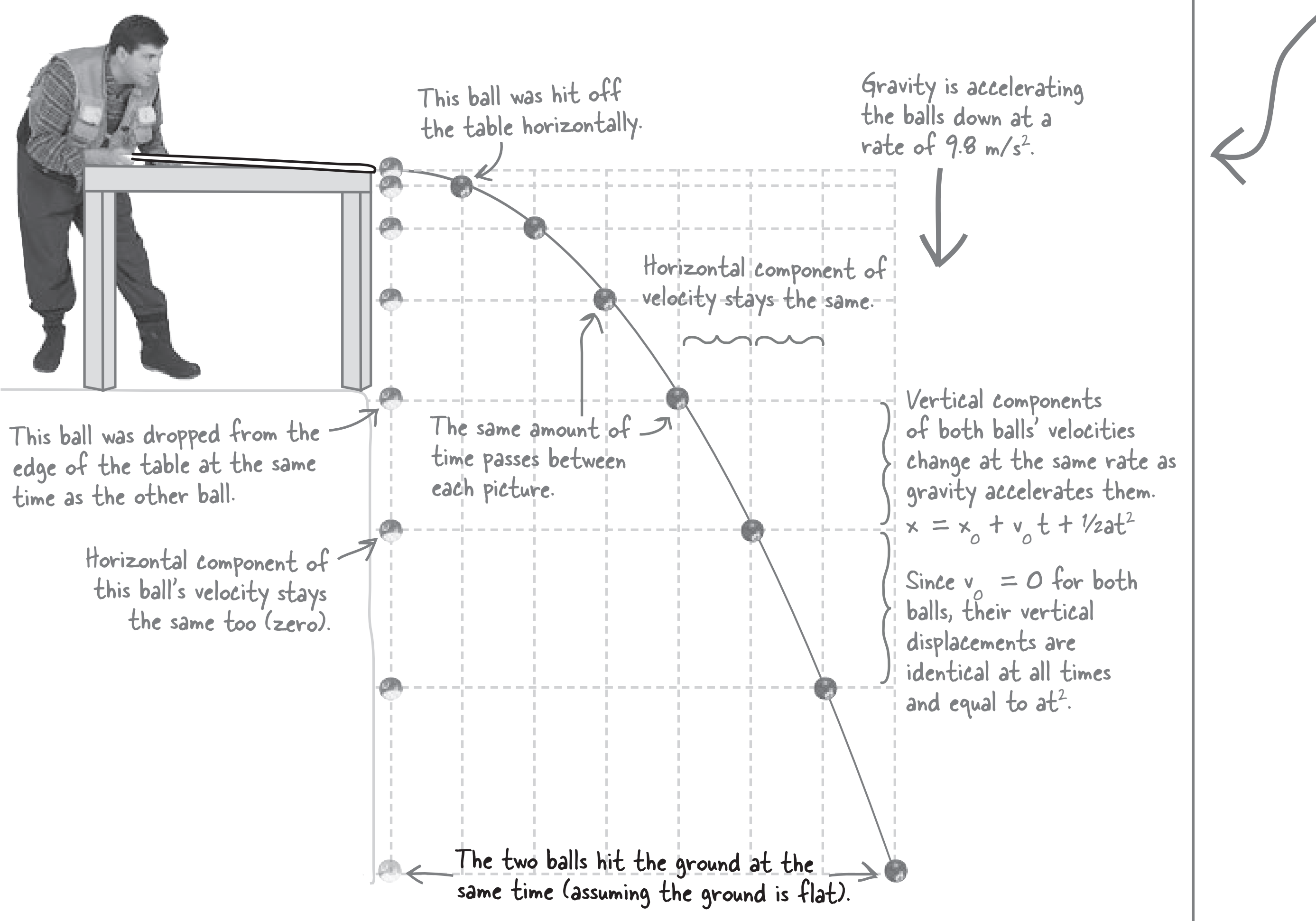
Gravity accelerates everything downwards at 9.8 m/s^2

Back when you were thinking about the Dingo dropping/launching his cage, you worked out that **gravity** always **accelerated** the cage **vertically** at 9.8 m/s^2, whether the cage was going up or coming back down again.

This is also the case if an object's velocity has a **horizontal** part (or component). Gravity accelerates **everything** downwards at the same rate of 9.8 m/s^2.

Tried it!

You've just been pushing two things off a table at the same time, but with different horizontal velocities, and seeing which hit the ground first.



The horizontal component of the velocity can't change once you've let go

Gravity accelerates things downwards at a rate of 9.8 m/s^2. When something moves through the air, gravity is the only thing affecting it (assuming that it doesn't have an engine like an airplane). The cannonball doesn't have an engine, so gravity changes the **vertical component** of its velocity at a rate of 9.8 m/s^2.

The technical term for 'something moving through the air' is a PROJECTILE.

Strictly speaking, there'll be air resistance, but for a cannonball, this will be tiny, and you can ignore it for now.

Once you've launched the cannonball, nothing affects the **horizontal component** of its velocity. It will keep on doing exactly what it was doing.

That's why the two objects you knocked off the table hit the ground at the same time, even though you gave one object a big horizontal push and let the other object drop straight down vertically.

The **vertical components** of both objects' velocities were zero at the start - they weren't falling before they left the table. Gravity accelerated both objects downwards at the same rate of 9.8 m/s^2, changing the horizontal components of each object's velocity by the same amount.

The **horizontal component** of both objects' velocities were unaffected by gravity. The dropped object landed directly under where it started, but the pushed object landed further away.

But the cannonball is launched at an **angle** - so its velocity initially has both horizontal and vertical components. What happens to something like that? Time to try it ...

Try it!

Get a ball, and throw it straight up in the air while standing still.

Now start to walk along a straight line, still throwing the ball in the same way you were before. Walking forwards gives the ball a **horizontal velocity component** which is the same as your walking velocity. Throwing the ball upwards gives it a **vertical velocity component**.

Write and sketch anything you notice.

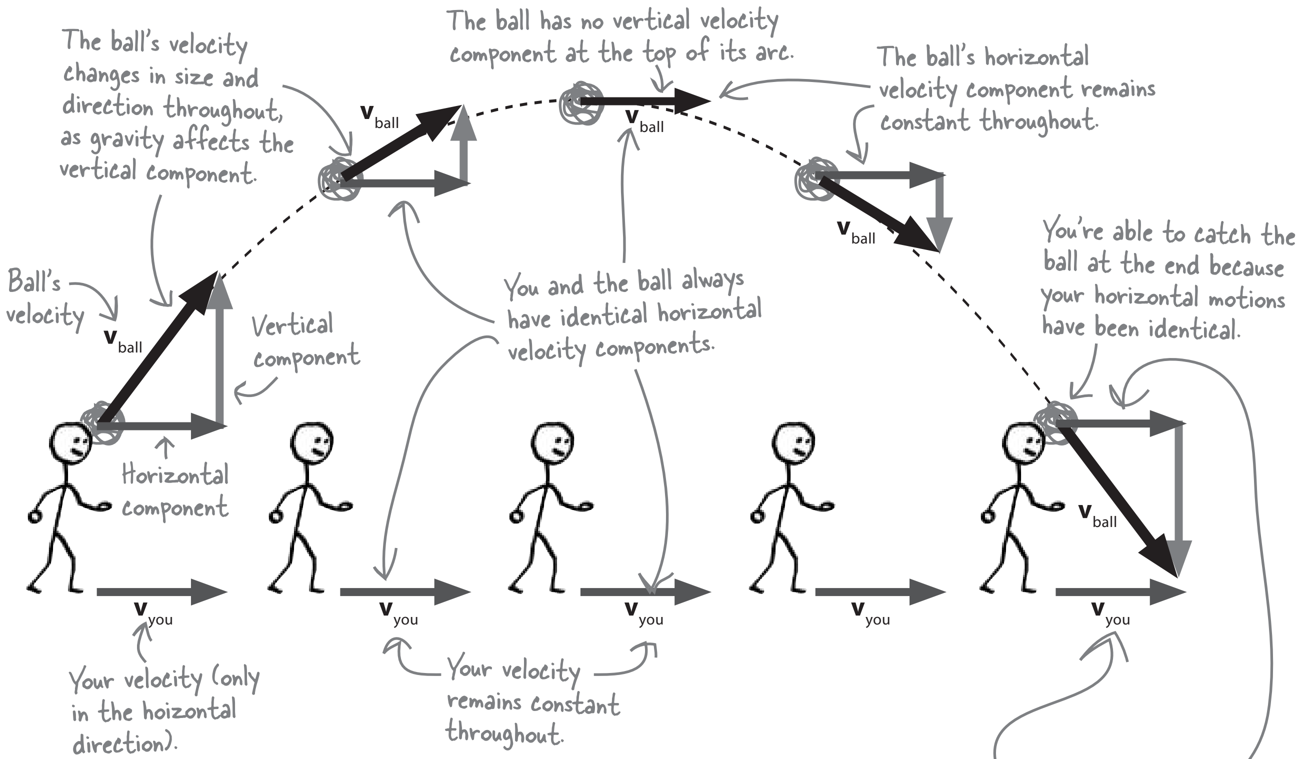
Gravity only affects the vertical component of the velocity.

The horizontal component of a projectile's velocity is constant

A ball you throw straight up comes straight back down into your hand for you to catch.

A ball you throw straight up while moving forwards also comes back down into your hand. Even though your hand stays the same distance away from your body as you throw the ball (so you throw it straight up), the ball's velocity also has a **horizontal component** . This is because you're moving forwards at the time that you throw the ball.

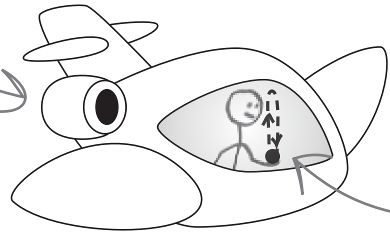
A "horizontal component" is sometimes called a "horizontal component vector."



Throughout the ball's flight, the **horizontal component** of your velocity and the horizontal component of the ball's velocity are exactly the same. The horizontal component of the ball's velocity doesn't change even though the ball's going up and down.

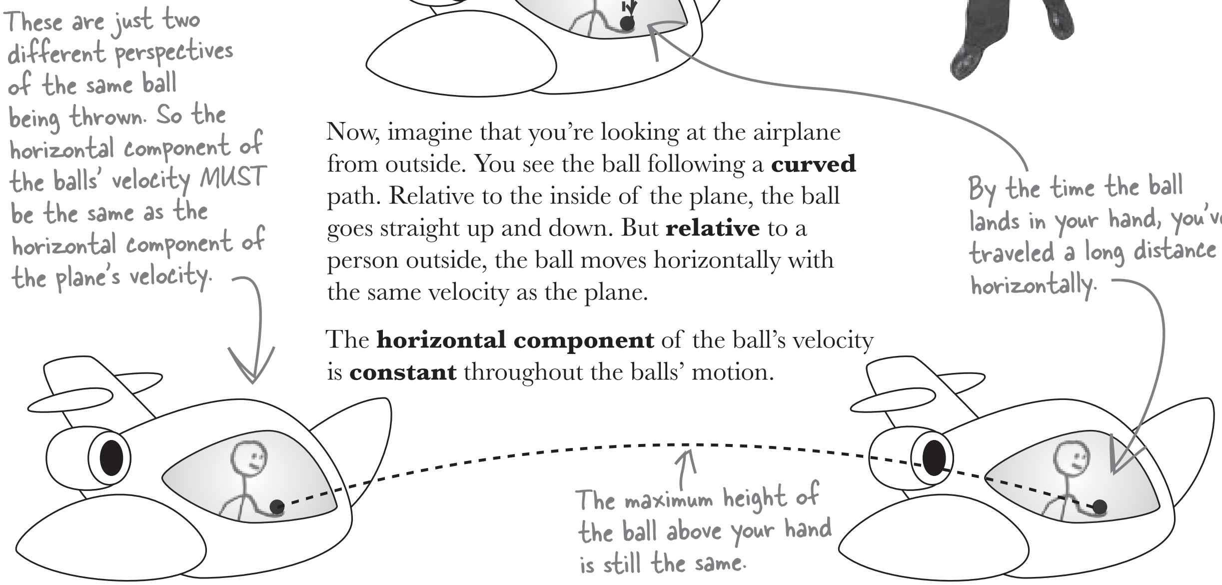
Would you, could you, on a plane?

OK, so imagine yourself on an airplane, traveling horizontally through the sky. If you throw a ball straight up in the air, it comes straight back down again.



Now, imagine that you're looking at the airplane from outside. You see the ball following a **curved** path. Relative to the inside of the plane, the ball goes straight up and down. But **relative** to a person outside, the ball moves horizontally with the same velocity as the plane.

The **horizontal component** of the ball's velocity is **constant** throughout the balls' motion.



The horizontal component of a projectile's velocity isn't affected by gravity, so is CONSTANT throughout the projectile's flight.

Yes. You can treat the vertical and horizontal parts of the problem separately.

Back in chapter 6, you solved an Emu-catching problem by treating the problem's horizontal and vertical parts separately. You were able to do this because the cage and the Emu are separate objects that are completely **independent** of each other.

You calculated the **time** it took the cage to fall using the cage's velocity and acceleration - which are both **vertical**.

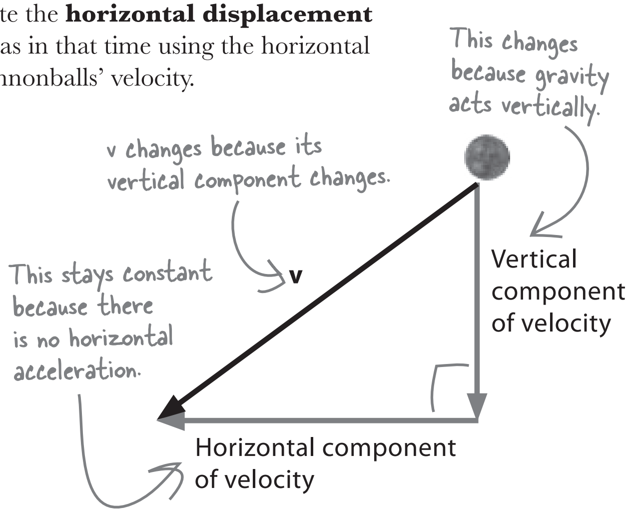
Then you calculated the **horizontal distance** that the Emu would cover in that time.

As the horizontal and vertical components of the cannonball's velocity are completely **independent**, you can do the cannonball problem the same way you did the Emu problem.

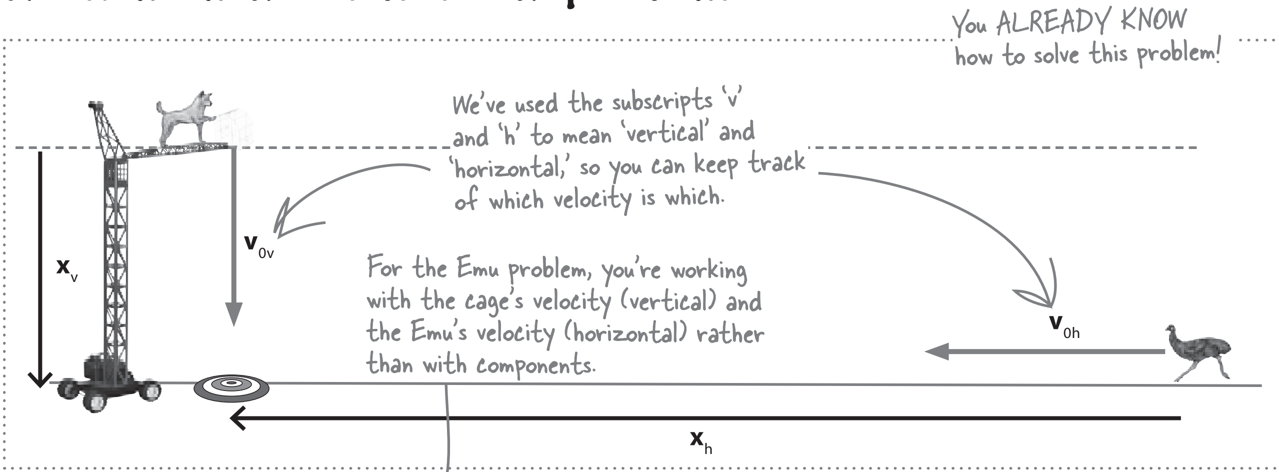
You can calculate the time it **takes** for the cannonball to land by thinking about the **vertical component** of the cannonball's velocity and the cannonball's acceleration, which is also vertical.

Then you can calculate the **horizontal displacement** that the cannonball has in that time using the horizontal component of the cannonballs' velocity.

The vertical component of a projectile's velocity behaves like an object launched straight up or down at that velocity.



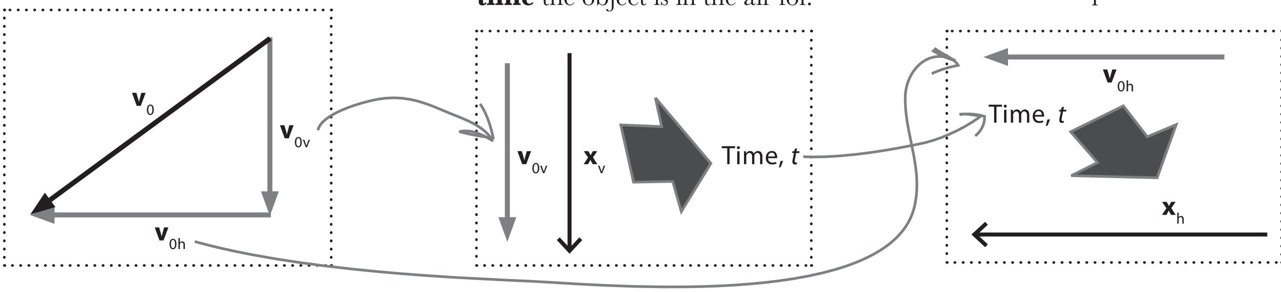
The same method solves both problems

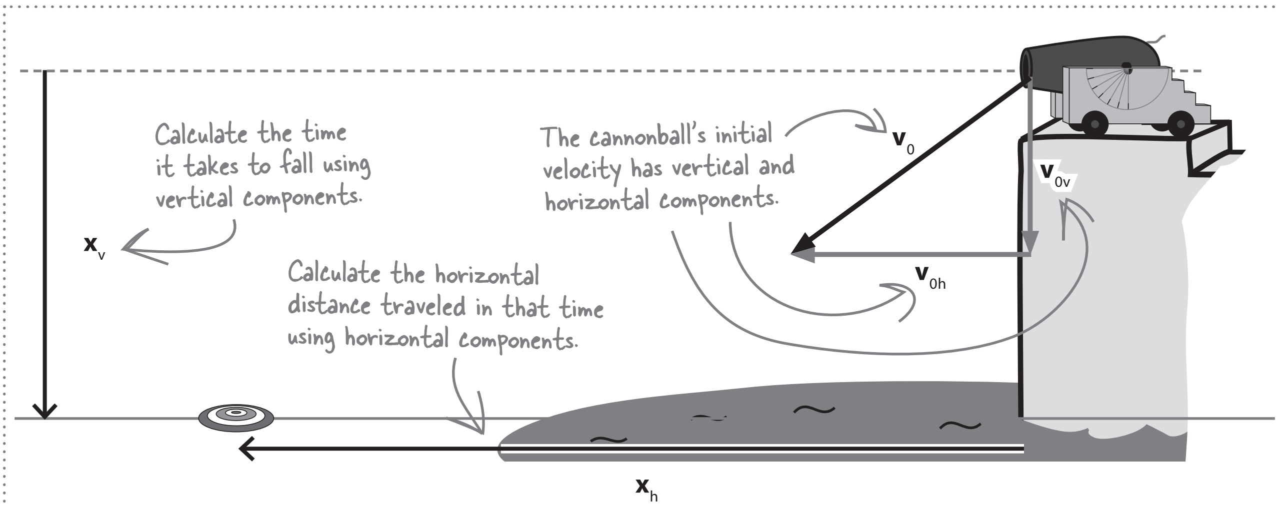


Step 1: Work out the vertical and horizontal **components** of the initial velocity, $\mathbf{v}_{0v}$ and $\mathbf{v}_{0h}$.

Step 2: Use the **vertical velocity component** and **vertical displacement** to work out the **time** the object is in the air for.

Step 3: Use the **horizontal velocity component** and the **time** to work out the object's horizontal displacement.





Question Clinic: The "Projectile" Question

Any question that involves an object projected through the air usually means you have to turn its velocity into horizontal and vertical components. Then you can use your right-angled triangle superpowers together with your equations of motion (from chapters 6, 7, and 8) to get your answers. The question presented here is a typical question.

This is just our problem worded differently.

Remember to start with a sketch, and to add information to the sketch as you work things out, so you have it all in one place.

You need to work out the vertical and horizontal velocity components first, before you do parts b and c.

5. You are in a castle where the wall is 15.0 m high. A cannon at the top of the wall is aimed directly at an enemy 20.0 m from the base of the castle wall. The cannon's muzzle velocity is 90.0 m/s.

a. What angle does the cannon make with the ground?

b. How long does it take for the cannonball to reach the ground?

c. How far from the attackers does the cannonball land?

You've already done part a of the question with the displacement vector triangle.

When you see the word 'angle,' start trying to spot right-angled triangles, and thinking about "sin, cos, tan & Pythagoras." Use the displacement triangle to calculate the angle.

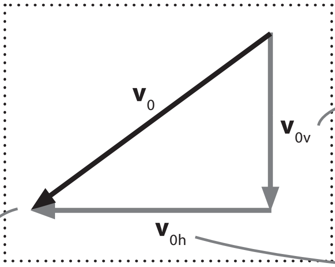
This involves using the VERTICAL component of the cannonball's velocity to see how long it takes to fall vertically.

This involves using the HORIZONTAL component of the cannonball's velocity to see how far it travels horizontally in that time.

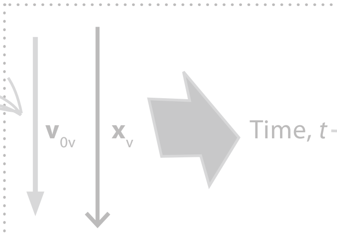
The word 'angle' also gives you the hint that component vectors might be important.

The projectile question almost always requires you to work out the **time** it takes for an object to fall (using the **vertical** velocity component), and then the **distance** it travels horizontally in that time (using the horizontal velocity component).

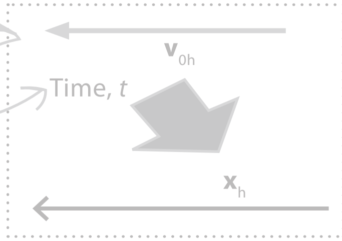
Step 1: Work out the vertical and horizontal **components** of the initial velocity, $\mathbf{v}_{0v}$ and $\mathbf{v}_{0h}$.



Step 2: Use the **vertical velocity component** and **vertical displacement** to work out the **time** the object is in the air for.



Step 3: Use the **horizontal velocity component** and the **time** to work out the object's horizontal displacement.



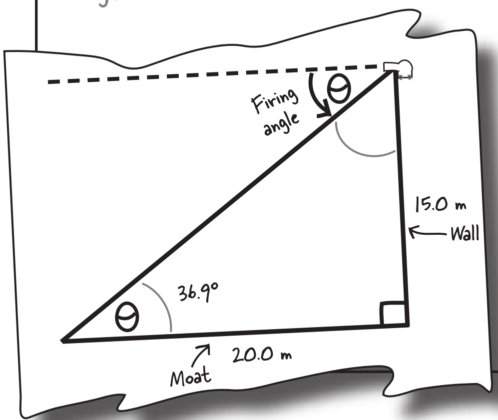
Sharpen your pencil

You are in a castle where the wall is 15.0 m high. A cannon at the top of the wall is aimed directly at an enemy 20.0 m from the base of the castle wall. The cannon's muzzle velocity is 90.0 m/s.

You've already worked out that the cannon makes an angle of 36.9° with the horizontal.

Now it's time to work out the **vertical and horizontal components of the cannonball's velocity.** Use subscripts in your symbols, $\mathbf{v}_v$ for the vertical component and $\mathbf{v}_h$ for the horizontal component.

This is a displacement vector triangle. You need to draw a velocity vector triangle and work out the lengths of its sides.



Sharpen your pencil Solution

You are in a castle where the wall is 15.0 m high. A cannon at the top of the wall is aimed directly at an enemy 20.0 m from the base of the castle wall. The cannon's muzzle velocity is 90.0 m/s.

You've already worked out that the cannon makes an angle of 36.9° with the horizontal.

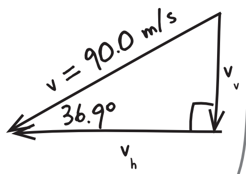
Now it's time to work out the **vertical and horizontal components of the cannonball's velocity.** Use subscripts in your symbols, $\mathbf{v}_v$ for the vertical component and $\mathbf{v}_h$ for the horizontal component.

There are TWO different ways of doing this question!

It doesn't matter which one you used – they both work!

Doing it using sine, cosine and tangent.

This way would be the quickest if you hadn't already been playing with the displacement triangle earlier on.



Start by writing out the equation.

Vertical component

$$\sin(36.9°) = \frac{opp}{hyp} = \frac{v_v}{90}$$

Then put the numbers in/rearrange it.

$$\Rightarrow v_v = 90\sin(36.9°) = \underline{\underline{54.0 \text{ m/s (3 sd)}}}$$

Make sure you use headings, so you and others know what you're trying to do.

Horizontal component

$$\cos(36.9°) = \frac{adj}{hyp} = \frac{v_h}{90}$$

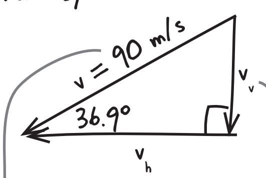
$$\Rightarrow v_h = 90\cos(36.9°) = \underline{\underline{72.0 \text{ m/s (3 sd)}}}$$

Doing it using similar triangles

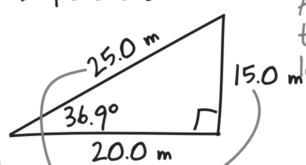
The position and velocity triangles are SIMILAR TRIANGLES.

Their angles are the same, as they both have a 90° angle and a 36.9° angle.

Velocity



Displacement



As they're similar triangles, the ratios of their side lengths are identical.

Vertical component: $\frac{v_v}{90} = \frac{15}{25} \Rightarrow v_v = \frac{15 \times 90}{25} = \underline{\underline{54 \text{ m/s}}}$

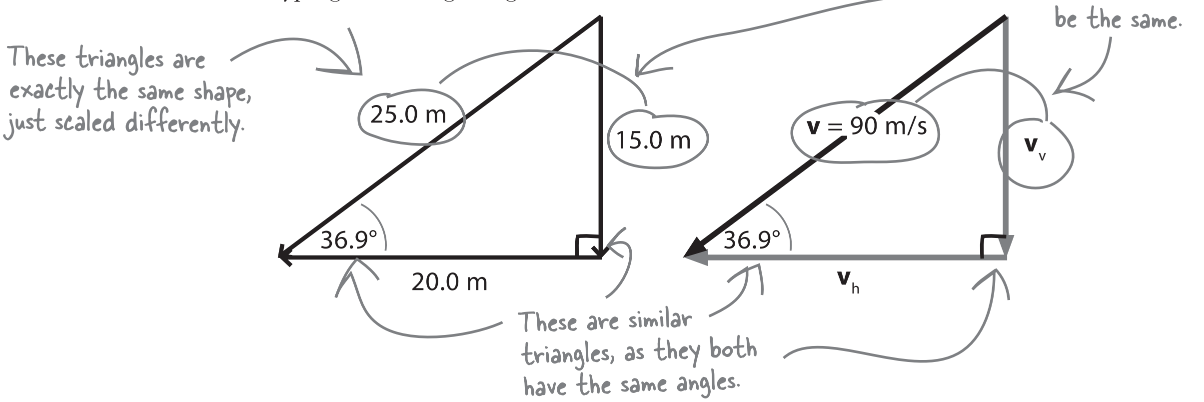
Horizontal component: $\frac{v_h}{90} = \frac{20}{25} \Rightarrow v_v = \frac{20 \times 90}{25} = \underline{\underline{72 \text{ m/s}}}$

Sometimes there's more than one way of doing the same calculation.

There often isn't one single "right way" to solve a problem in physics. As long as you reach the correct destination, it doesn't really matter how you got there, as long as what you did makes sense.

Here, the usual way to work out the horizontal and vertical components would be to use sine and cosine, along with the angle and side you'd been given for the velocity triangle. That's the method given in the first answer.

But because you already knew all three sides of the displacement triangle (from doing the ladder thing), it was possible to take a shortcut this time. Shortcuts are a good idea - they involve doing fewer calculations. Fewer calculations mean you type less into your calculator - and there's a smaller chance that you'll mess up by accidentally typing the wrong thing.



This particular shortcut works because the displacement and velocity triangles are similar triangles. You already know that the ratios of their side lengths will be the same. $\frac{15}{25}$ in the first triangle = $\frac{\mathbf{v}_v}{90}$ in the second triangle. You don't need to use the angle and sine, cosine or tangent to calculate the ratios of the side lengths.

5. You are in a castle where the wall is 15.0 m high. A cannon at the top of the wall is aimed directly at an enemy 20.0 m from the base of the castle wall. The cannon's muzzle velocity is 90 m/s.

a. What angle does the cannon make with the ground?
b. How long does it take for the cannonball to reach the ground?
c. How far from the attackers does the cannonball land?

Now it's time to get on with part b of the original problem!

Calculating the horizontal and vertical components of the cannonball's velocity wasn't part of the original question. Is there a way of spotting that we need to do that?

Sometimes you have to work out the intermediate steps yourself.

Many processes in life involve a series of **intermediate steps** to get from where you are to where you want to be. To get into your house, first of all you need to find your keys! Physics problems can be like this too.

In order to calculate the time it takes for the cannonball to fall, you need the vertical component of the cannonball's velocity. And to calculate the horizontal displacement, you need the horizontal component of the cannonball's velocity.

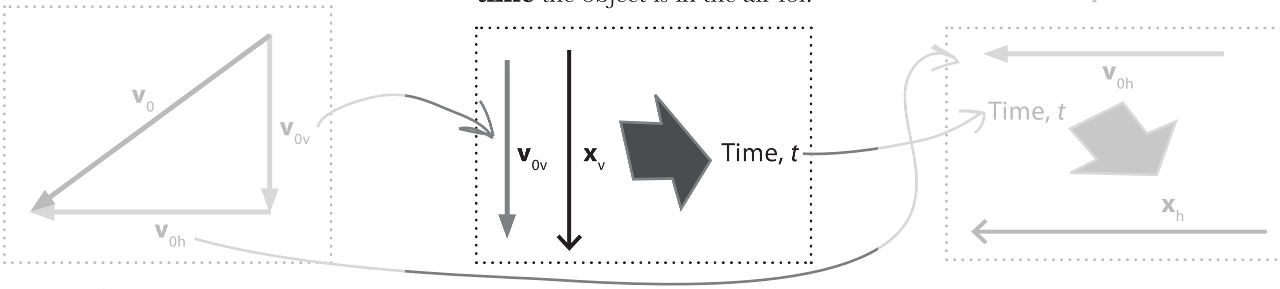
Spotting these intermediate steps comes with experience and practice with a variety of physics problems. You've been building up the ability to spot what you need to do to solve a problem as you've been learning to think like a physicist. But using what you've learned in this book to do practice problems and exam questions from elsewhere is important too.

In the Question Clinics throughout the book, we've taken several typical exam-style questions and broken them down to tell you what kinds of clues to look out for.

Step 1: Work out the vertical and horizontal **components** of the initial velocity, $\mathbf{v}_{0v}$ and $\mathbf{v}_{0h}$.

Step 2: Use the **vertical velocity component** and **vertical displacement** to work out the **time** the object is in the air for.

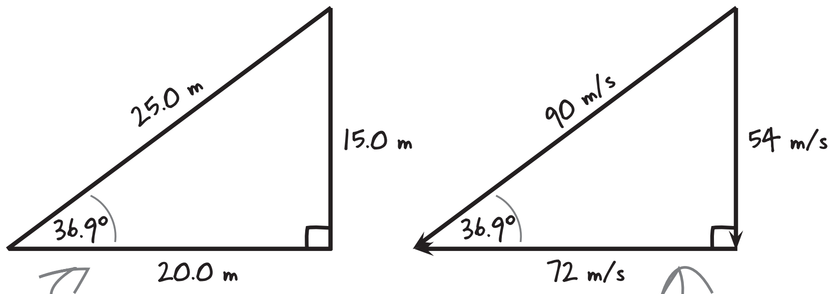
Step 3: Use the **horizontal velocity component** and the **time** to work out the object's horizontal displacement.



Sharpen your pencil

b. How long does it take for the cannonball to reach the ground?

Hint: Treat the cannonball like something launched directly downwards with the vertical component of the cannonball's velocity. Do a sketch that only deals with the VERTICAL direction and go on from there.

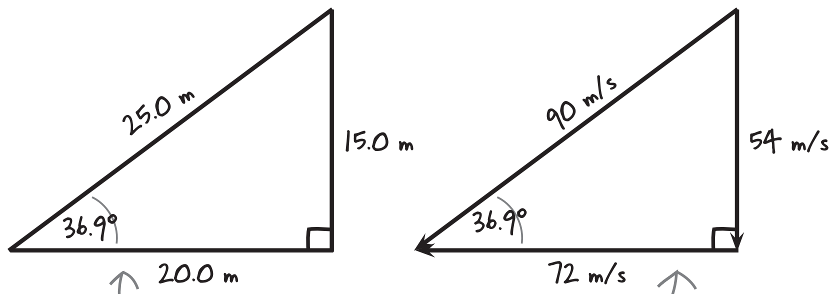


These are the triangles you've already drawn. You can still refer to them for this part of the question.

Hint: You may need to use more than one equation of motion to solve this problem.

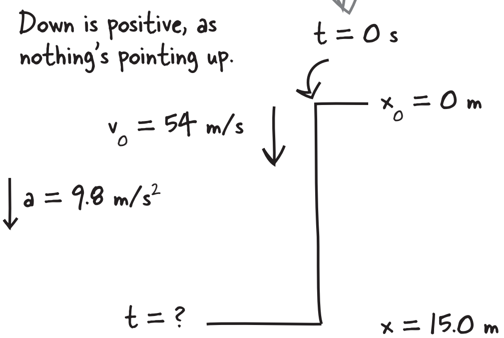
Sharpen your pencil Solution

b. How long does it take for the cannonball to reach the ground?



These are the triangles you'd already have drawn on your page during earlier parts of the question.

This is a sketch that only deals with the vertical direction. It's much clearer to do this than it is to try to work with parts of triangles.



$$x - x_0 = v_0 t + \tfrac{1}{2}at^2$$

$$v = v_0 + at$$

$$v^2 = v_0^2 + 2a(x - x_0)$$

Write down your three key equations, and make a note of what you do and don't know.

In the first equation, the only thing I don't know is t. But there's both t and t^2 in the equation, and I can't rearrange it to say "t = something."

So use third equation to work out v, then use that value in $v = v_0 + at$ to work out t.

$$v^2 = v_0^2 + 2a(x - x_0)$$

$$v = \sqrt{54^2 + (2 \times 9.8 \times 15)} = 56.7 \text{ m/s (3 sd)}$$

If you're only dealing with one component (in this case the vertical component) of the displacement, velocity and acceleration, you can omit the 'horizontal' and 'vertical' subscripts from your variables to reduce the clutter in your work.

Use this value in $v = v_0 + at$ to work out t.

$$v = v_0 + at$$

$$at = v - v_0$$

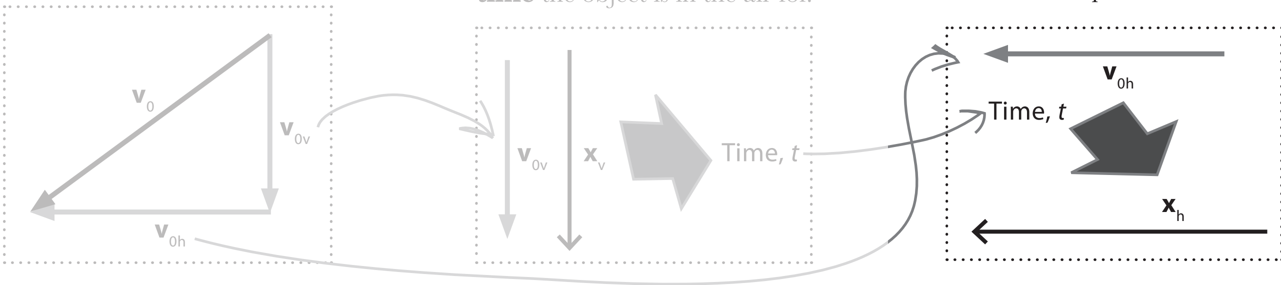
$$t = \frac{v - v_0}{a} = \frac{56.7 - 54}{9.8} = 0.276 \text{ s (3 sd)}$$

It takes 0.276 s (3 sd) for the cannonball to reach the ground.

Step 1: Work out the vertical and horizontal **components** of the initial velocity, $\mathbf{v}_{0v}$ and $\mathbf{v}_{0h}$.

Step 2: Use the **vertical velocity component** and **vertical displacement** to work out the **time** the object is in the air for.

Step 3: Use the **horizontal velocity component** and the **time** to work out the object's horizontal displacement.

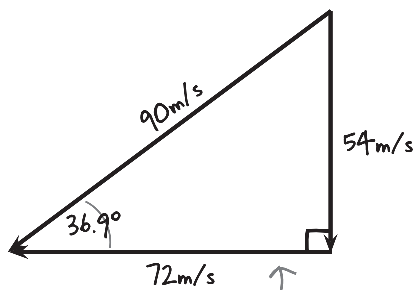


Sharpen your pencil

Now for part c (or step 3 in the illustration above). Hang in there; you're nearly done!

c. How far from the attackers does the cannonball land? (The attackers are at the edge of the moat, 20.0 m from the base of the wall.)

Hint: Treat the cannonball like something launched horizontally with the horizontal component of the cannonball's velocity. Do a sketch that only deals with the HORIZONTAL direction and go on from there.



You'll need to use the velocity triangle you already worked out and the time from part b (it takes 0.276 s for the cannonball to reach the ground).

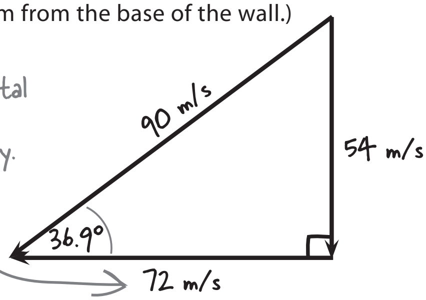
Sharpen your pencil Solution

c. How far from the attackers does the cannonball land?
(The attackers are at the edge of the moat, 20.0 m from the base of the wall.)

t = 0.276 s
v = 72 m/s

Distance = ?

This is the horizontal component of the cannonball's velocity.



You worked this out in part b.

Cannonball takes 0.276 s to reach the ground.

You worked this out in the 'missing step' between parts a and b.

Horizontal component of velocity = 72 m/s.

Horizontal distance traveled in this time = speed × time

= 72 × 0.276

= 19.9 m (3 sd)

This is the right method – once you know how long it takes to reach the ground VERTICALLY, work out how far the cannonball travels HORIZONTALLY in this time.

Attackers are 20.0 m away from the foot of the castle wall. This is another way of saying that they are 20.0 m horizontally from the cannon.

So the cannonball lands 20.0 – 19.9 = 0.1 m short of the attackers, but probably gets them running scared ...

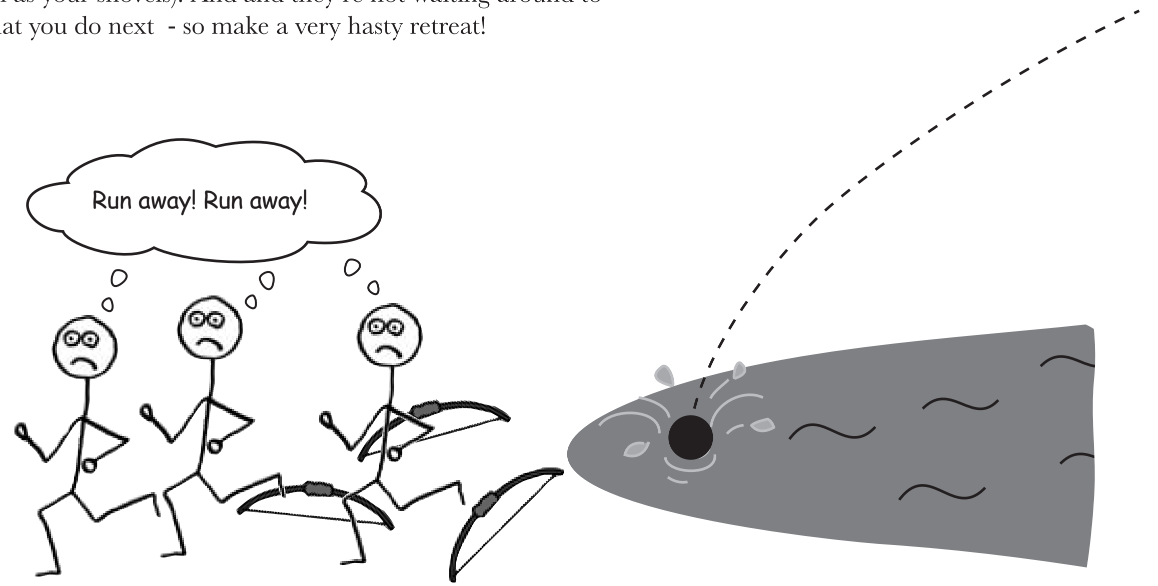
Did you answer the question you were ACTUALLY ASKED (distance from the attackers) or did you leave it as 19.9 m (distance from the wall)?

Check: "Have I actually answered the question I was asked?"

Make sure you don't forget to do something simple at the end, like convert units or subtract one length from another.

And so they ran away ...

The cannonball lands only 10 cm away from the attackers and soaks them! They didn't know you had a cannon up your sleeve (as well as your shovels). And and they're not waiting around to see what you do next - so make a very hasty retreat!



BULLET POINTS

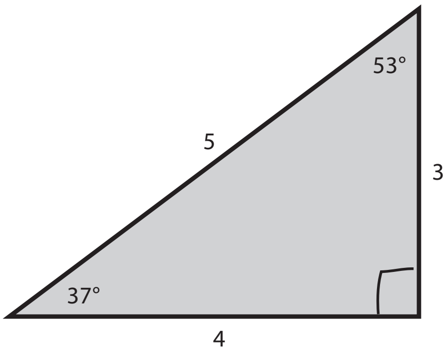
- With a right-angled triangle where you know the length of **one side plus one other fact** (a side length or an angle), you can work out all the other sides and angles using sine, cosine, tangent and Pythagoras.
- Similar triangles have the same sizes of angles.
- Similar triangles are useful because the **ratios** of the similar sides are the same. This means you can often find side lengths without having to work out an angle.
- If something's velocity is in a different direction from its acceleration, try breaking the velocity down into vector **components** parallel to and perpendicular to the acceleration.
- Vectors should always be added **nose to tail**.
- Once your displacement/velocity/acceleration vectors are broken down into components at 90° to each other, you can treat the two directions **independently**.
- You might want to work out the **time** it takes something to happen from one component, then use that in an equation involving the other component.
- Add together the component vectors at the end to find out what's happened to the original vector.

If you're doing the AP Physics exam, you're not allowed a calculator in the multiple choice section. So they give you a table – like the one you worked out earlier – of values for sine, cosine and tangent for certain 'common angles.'

Hey ... I've noticed that the AP table of information has sine, cosine and tangent for 'common angles.' What makes an angle 'common'?!

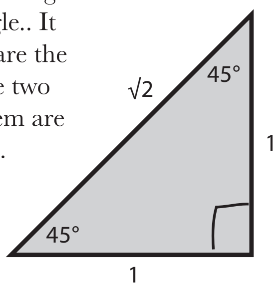
There are some standard triangles that you should look out for.

You've already met the 3:4:5 triangle in this chapter. Its angles (to 2 sd) are 37° and 53°. So if you see a question, especially on the multiple choice section (where you can't use a calculator) involving these side length ratios or angles, you know what kind of triangle it is.

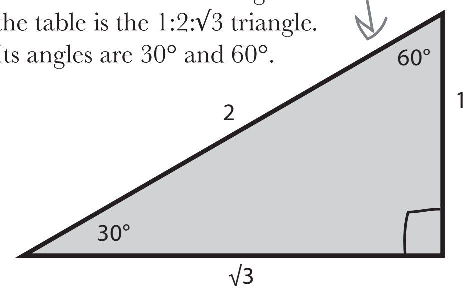


Note that the smallest side is always opposite the smallest angle, and the largest side is opposite the largest angle.

Another standard triangle is the 1:1:√2 triangle.. It has two sides that are the same length, so the two angles opposite them are the same size - 45°.



The third standard triangle in the table is the 1:2:√3 triangle. Its angles are 30° and 60°.



Question Clinic: The "Missing steps" Question

Often, you'll get a multi-part question, which doesn't directly ask you to carry out some of the steps you need to get from one bit of the question to the next. So there are **missing steps** that you need to figure out yourself. If you're familiar with the methods that are used in certain **types** of questions, you'll be fine with this.

The way the question is set up at the start, it could be a 'wheat from the chaff' one where these details are irrelevant and just in there to distract you.

So you start with a sketch and do the first bit with trigonometry (sine, cosine and tangent) which may make you think Pythagoras will be next.

5. You are in a castle where the wall is 15.0 m high. A cannon at the top of the wall is aimed directly at an enemy cannon at 20.0 m from the base of the castle wall. The cannon's muzzle velocity is 90 m/s.

   a. What angle does the cannon make with the ground?

   Work out the horizontal and vertical components of the cannonball's velocity.

   b. How long does it take for the cannonball to reach the ground?

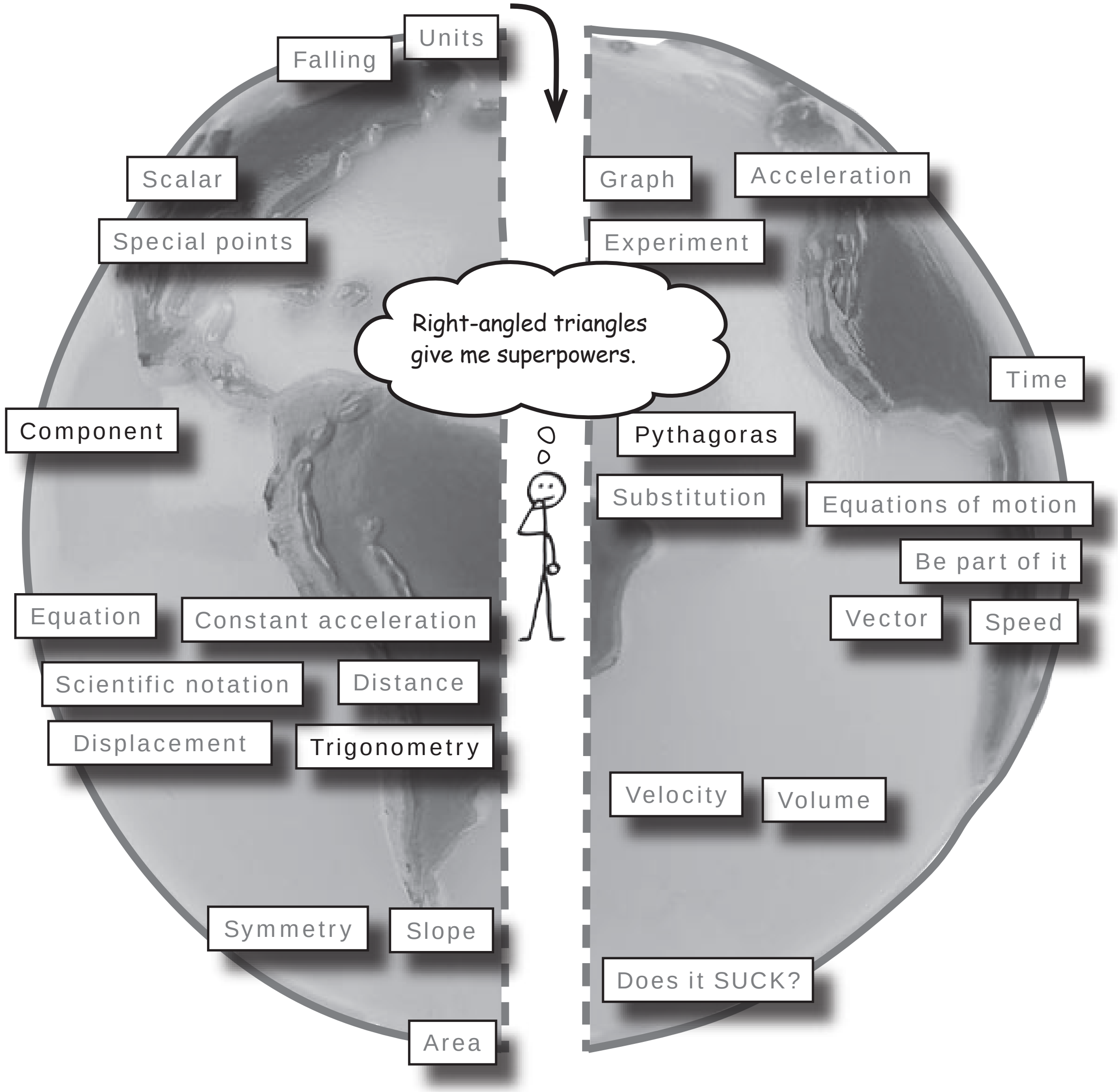
   c. How far from the attackers does the cannonball land?

This is the missing step and the KEY to being able to do the question.

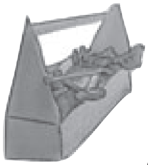
If you don't realize that this is REALLY important, you might wrongly try to do the question using Pythagoras and a straight flight path along the hypotenuse of a right-angled triangle, like we did earlier.

Parts b and c need you to use equations of motion – and you MUST carry out the missing step to be able to do them correctly.

This shows you how important it is to practice thinking about and doing certain **types** of questions, so that the in-between steps become something you'd do naturally, and you don't get stuck or waste time going down the wrong path.



☑	Pythagoras	An equation that you can use to find the third side of a right-angled triangle when you already know two sides.
☑	Trigonometry	Using the ratios sine, cosine and tangent to relate ratios of side lengths to angles.
☑	Component	'Part' of a vector. For example, you can turn a vector that's at an angle into horizontal and vertical conponent vectors.



Your Physics Toolbox

You've got Chapter 9 under your belt and added some terminology and answer-checking skills to your tool box.

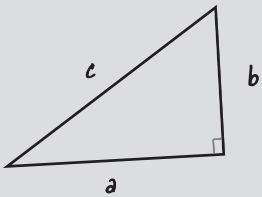
Spot the triangle

Anytime you do a sketch that involves two dimensions, look out for triangles.

Keep a special look out for right-angled triangles formed when there are things going on both horizontally and vertically.

Pythagoras' Theorem

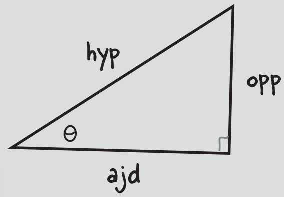
If you know the lengths of two sides of your right-angled triangle, you can calculate the length of the third side.



$$c^2 = b^2 + a^2$$

sine, cosine and tangent

sine, cosine and tangent are ratios of the sides of a right-angled triangle.



$$\sin(\theta) = \frac{opp}{hyp}$$

$$\cos(\theta) = \frac{adj}{hyp}$$

$$\tan(\theta) = \frac{opp}{ajd}$$

Right-angled triangle facts

If you know one side and one other fact (either a side or an angle), then you can work out EVERYTHING there is to know about a right-angled triangle using Pythagoras and sine, cosine and tangent.

Component vectors

If an object is moving at an angle, it can be useful to break down its velocity vector into horizontal and vertical component vectors.

The horizontal component remains constant.

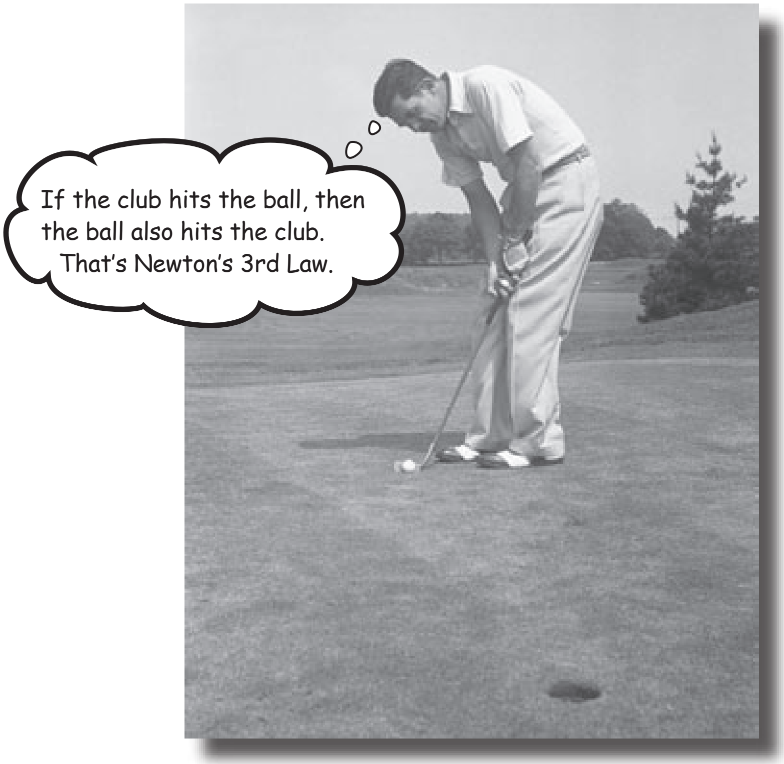
The vertical component is affected by gravity.

This lets you break down a complicated problem into two simpler problems that you already know how to do.

TOP TIP – Make sure you know how your calculator works! Do you have to press the 'sin,' 'cos' or 'tan' button before or after entering the angle? Is your calculator definitely in the degree mode of measuring angles? It's annoying to understand the physics only to get the problem wrong by making silly mistakes!

10 momentum conservation

What Newton Did



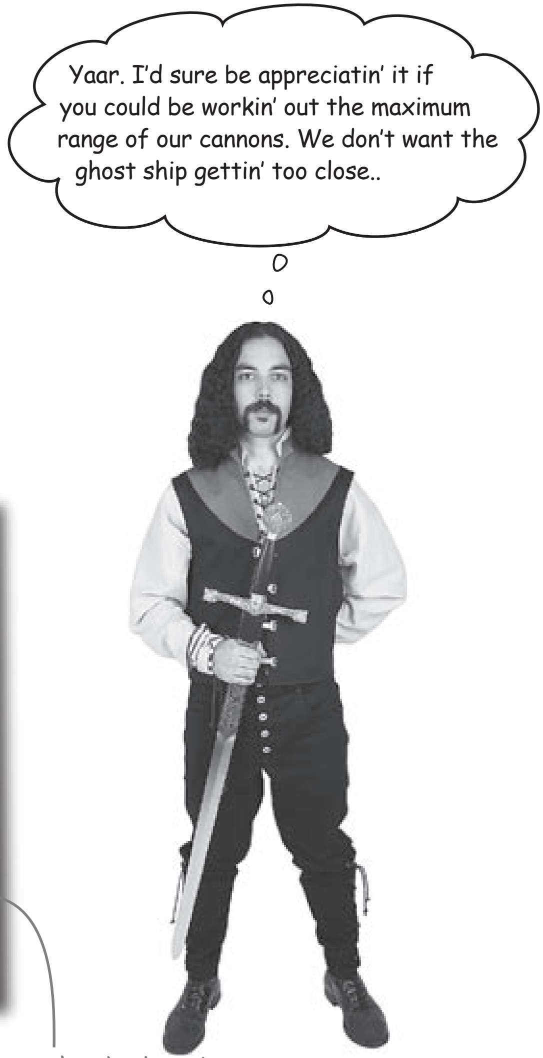
No one likes to be a pushover. So far, you've learned to deal with objects that are already moving. But what makes them go in the first place? You know that something will move if you push it - but *how* will it move? In this chapter, you'll overcome **inertia** as you get acquainted with some of **Newton's Laws**. You'll also learn about **momentum**, why it's **conserved**, and how you can use it to solve problems.

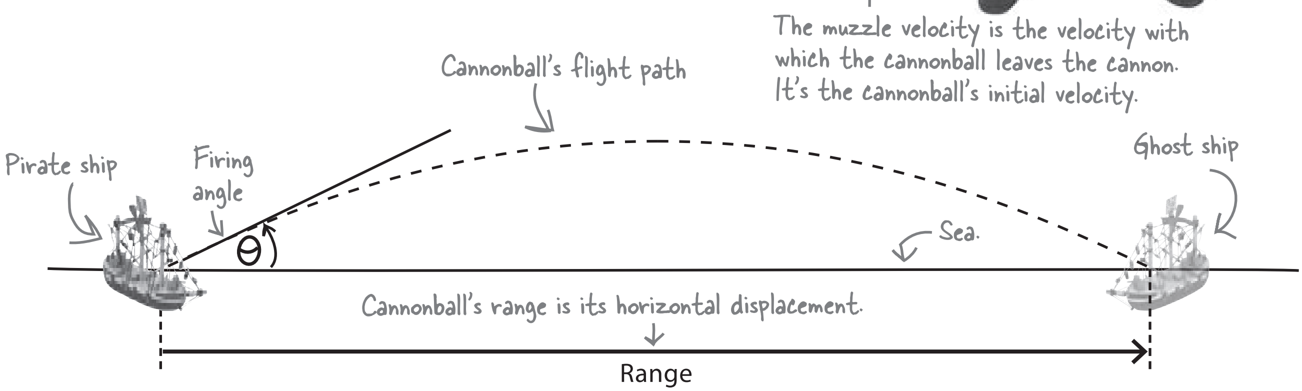
The pirates be havin' a spot o' bother with a ghost ship ...

The pirate captain is being chased across the seas by a ghost ship and needs to make sure it keeps its distance.

His ship's fitted with some Sieges-R-Us battle cannons. The captain wants to know the maximum **range** of his cannons - the maximum horizontal distance he can fire a cannonball, and you've been called in as the expert.

But the supply of cannonballs is limited at sea - so he won't actually be able to fire a cannon until you've got it all worked out.

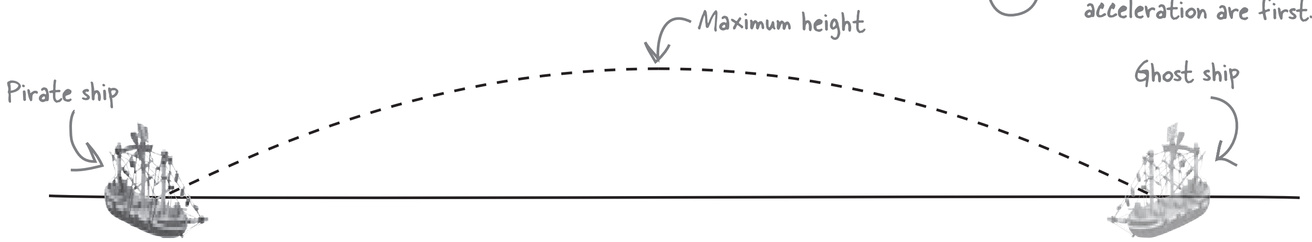


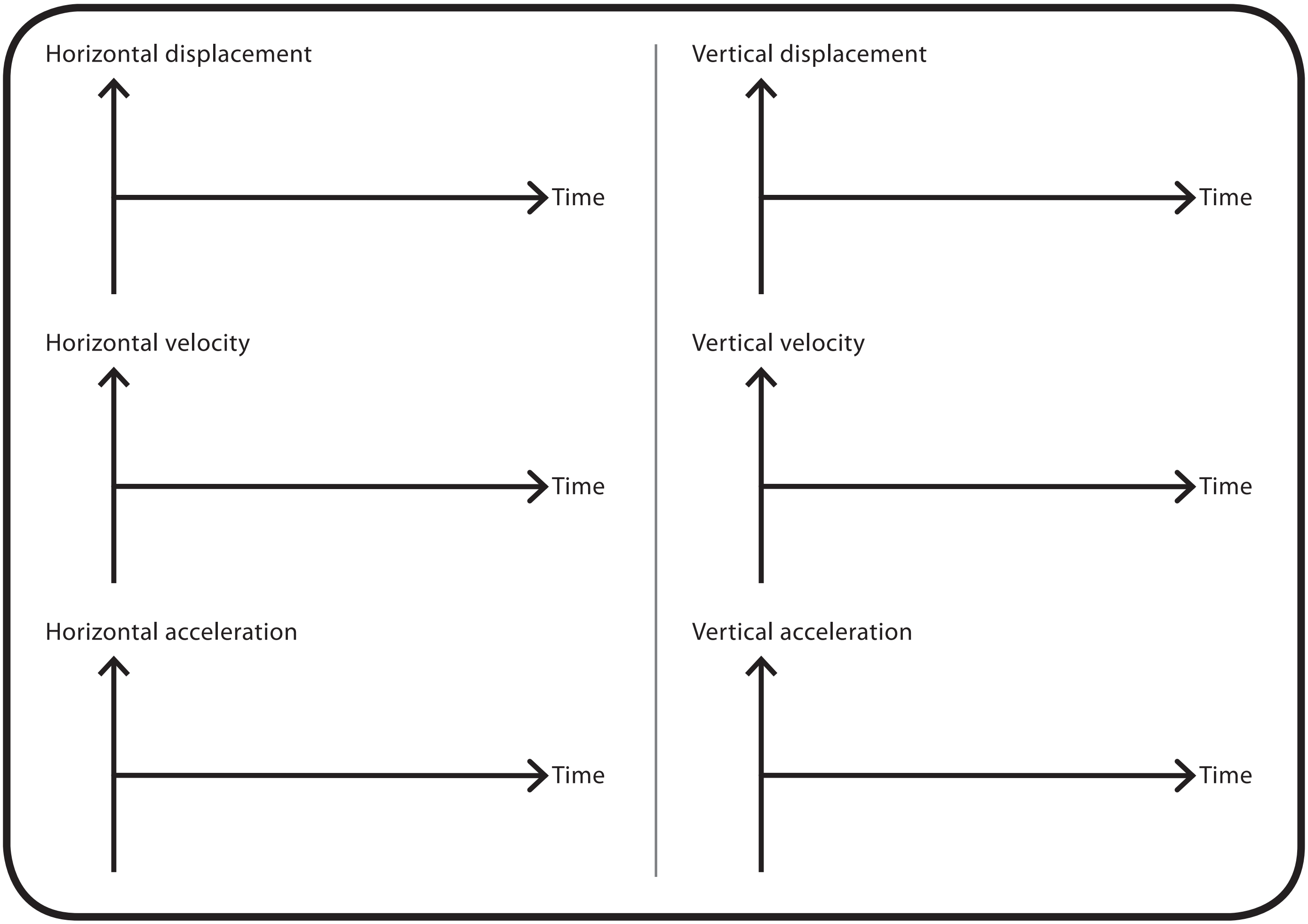


Whiteboard Wipeout - Cannonball

Time to get the hang of what the cannonball's doing. Your job is to sketch graphs that show how the horizontal and vertical components of the displacement, velocity and acceleration change with time. Think about it one component at a time, and do the easier graphs first!

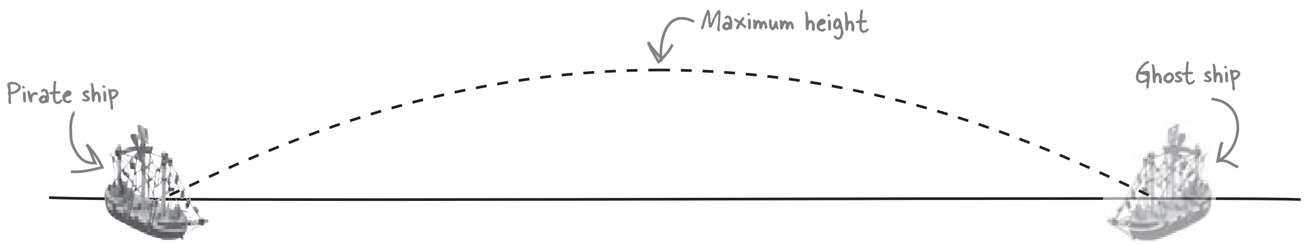
Hint: Start off by thinking about what the cannonball's horizontal acceleration and vertical acceleration are first.

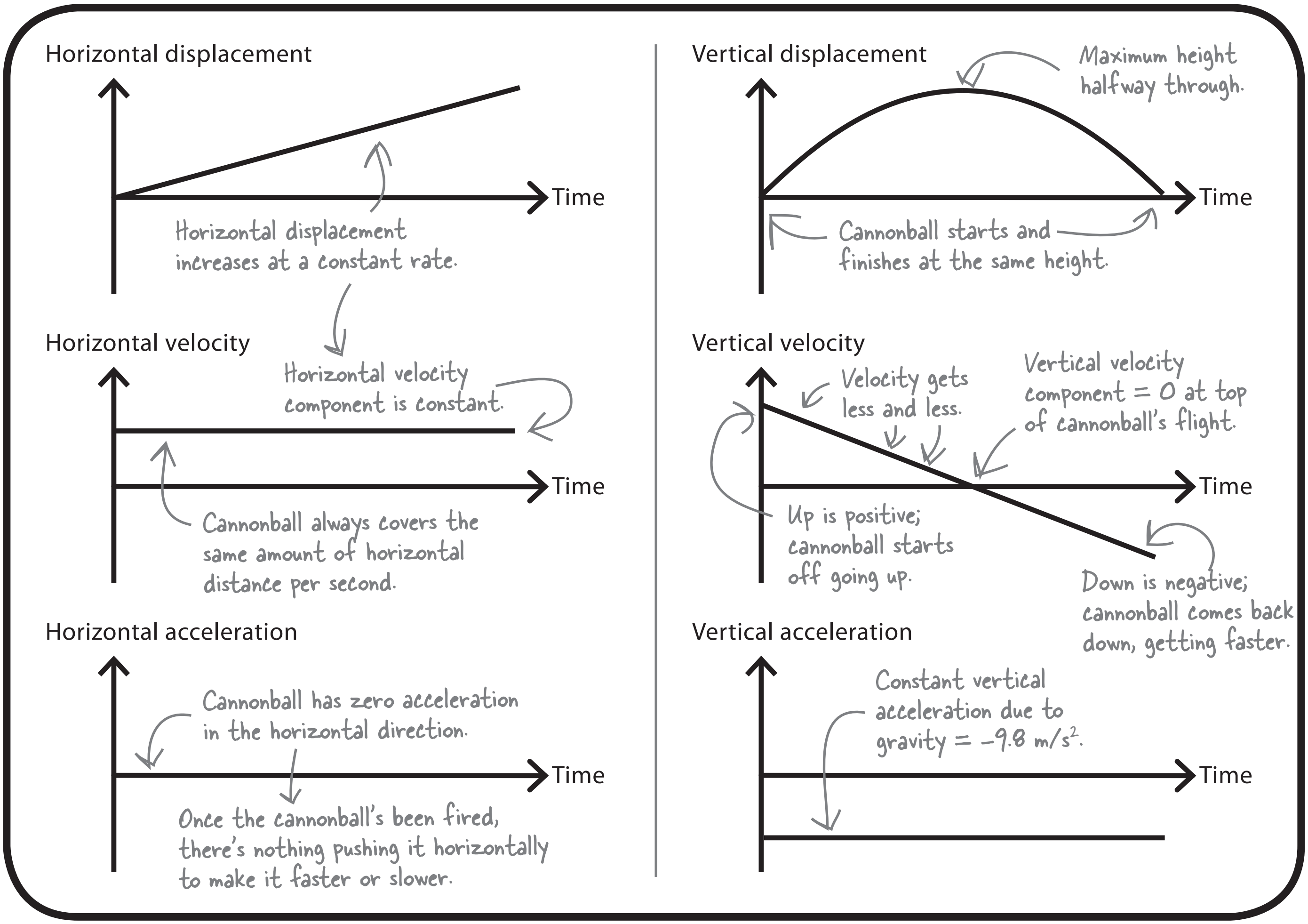




Whiteboard Wipeout - Cannonball SOLUTION

Time to get the hang of what the cannonball's doing. Your job is to sketch graphs that show how the horizontal and vertical components of the displacement, velocity and acceleration change with time. Think about it one component at a time, and do the easier graphs first!

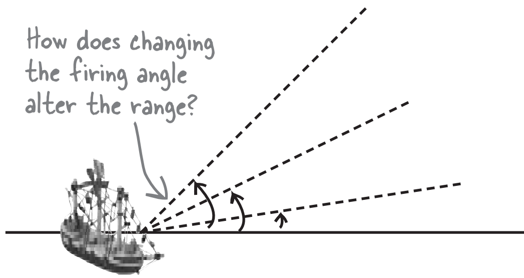




What does the maximum range depend on?

Now that you've sketched the graphs of its component vectors, you can think which **variables** may affect the maximum range of the cannonball.

For example, you already know from the last chapter that the firing **angle** will make a difference to the range. But what angle will be the best? Can you figure it out by thinking about what will happen at the **extremes** of the range of angles you could fire the cannonball at? And is there anything else you might need to take into account?



Sharpen your pencil

a. Write down all the things that could possibly affect the range of the cannonball.

Now think about some **extreme** angles to help you work out what's important.

b. Imagine - and draw - what will happen for **small firing angles** (close to 0° or horizontal) and describe this in terms of horizontal and vertical velocity components.

Space to draw.

c. Imagine - and draw - what will happen for **large firing angles** (close to 90° or vertical) and describe this in terms of horizontal and vertical velocity components.

Space to write in.

d. What do you think the **optimal firing angle** will be to give the **maximum range**? (You don't have to be right - just guess and trust your instincts).

Sharpen your pencil Solution

a. Write down all the things that could possibly affect the range of the cannonball.

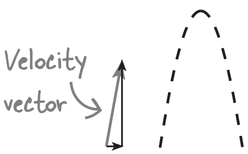
Firing angle
Velocity of cannonball
Wind speed and direction
Velocity of ship
A big wave could change the angle / velocity

Now think about some **extreme** angles to help you work out what's important.

b. Imagine - and draw - what will happen for **small firing angles** (close to 0° or horizontal) and describe this in terms of horizontal and vertical velocity components.

If the firing angle is small, then the horizontal component is large, and the vertical component is small. Although it's going fast horizontally, it isn't spending much time in the air because the vertical component is so low.

c. Imagine - and draw - what will happen for **large firing angles** (close to 90° or vertical) and describe this in terms of horizontal and vertical velocity components.



If the firing angle is large, then the vertical component is large, and the horizontal component is small. So the cannonball spends a long time off the ground – but doesn't travel very far horizontally in that time.

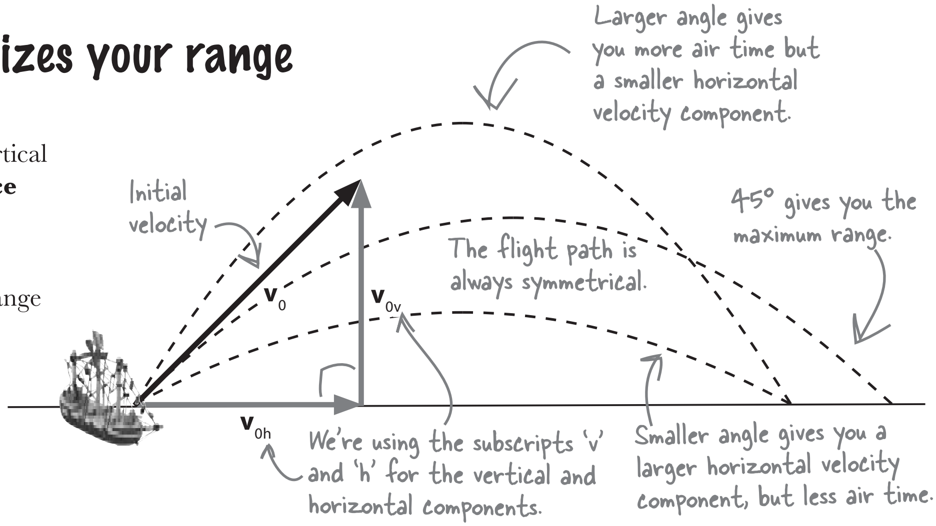
d. What do you think the **optimal firing angle** will be to give the **maximum range**? (You don't have to be right - just guess and trust your instincts.

The optimal firing angle will be between these two extremes – probably at around 45°.

Firing at 45° maximizes your range

An **angle of 45°** gives you the best balance between **time** in the air (vertical component of velocity) and **distance** covered horizontally in that time (horizontal component of velocity).

So you end up with the maximum range possible for that velocity.



You can't do everything that's theoretically possible - you need to be practical too

The pirate captain is delighted at your suggestion of achieving a maximum range using a 45° firing angle. Unfortunately, the pirates aren't able to aim properly for angles greater than 10°, so a 45° firing angle isn't **practical**. Sometimes what would be **theoretically** possible is restricted by what's physically possible.

But on the bright side, you know that if the pirates fire their cannon at 10°, the cannonball will go further than it would for any other possible angle, as 10° is the closest to 45° you can practically get. So you can **calculate the range** ...

Sharpen your pencil

Work out the maximum range of the cannon when it's fired at an angle of 10° with an initial velocity of 90.0 m/s. (Assume that the cannonball is fired from sea level to sea level.)

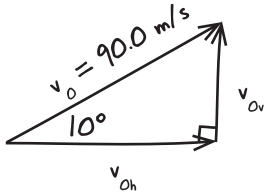
Remember to start with a sketch, and say which direction is positive. You might want to use subscripts to represent the vertical and horizontal components, like v_h and v_v.

Page 328 in chapter 8 and page 375 in chapter 9 should help you if you're not sure how to break this problem down into smaller parts.

Sharpen your pencil Solution

Work out the maximum range of the cannon when it's fired at an angle of 10° with an initial velocity of 90.0 m/s. (Assume that the cannonball is fired from sea level to sea level.)

Start off by working out horizontal and vertical velocity components.



$\sin(10°) = \frac{opp}{hyp} = \frac{v_{0v}}{v_0}$

$\Rightarrow v_{0v} = v_0 \sin(10°) = 15.6$ m/s (3 sd)

$\cos(10°) = \frac{adj}{hyp} = \frac{v_{0h}}{v_0}$

$\Rightarrow v_{0h} = v_0 \cos(10°) = 88.6$ m/s (3 sd)

Either use subscripts like this, or be very very careful when you treat the vertical and horizontal components separately!

Get time from vertical velocity component (working ONLY with vertical components):

$a_v = -9.8$ m/s^2 (down) Up is positive

$x_v - x_{0v} = 0$ m

$v_{0v} = 15.6$ m/s (up)

$t = ?$

$v_v = -15.6$ m/s (down)

$v_v = v_{0v} + at$

$\Rightarrow at = v_v - v_{0v}$

$\Rightarrow t = \frac{v_v - v_{0v}}{a_v} = \frac{(-15.6) - (15.6)}{-9.8} = 3.18$ s (3 sd)

Work out horizontal change in position during that time using horizontal velocity component

$v_h = 88.6$ m/s (O →)

No acceleration in horizontal direction.

$t = 3.18$ s

$x_{0h} = 0$ m

$x_h = ?$

$v_h = \frac{\Delta x_h}{\Delta t} = \frac{x_h - x_{0h}}{t - 0} = \frac{x_h}{t}$

$\Rightarrow x_h = v_h t = 88.6 \times 3.18 = 282$ m (3 sd)

The range of the cannonball fired at 10° is 282 m (3 sd)

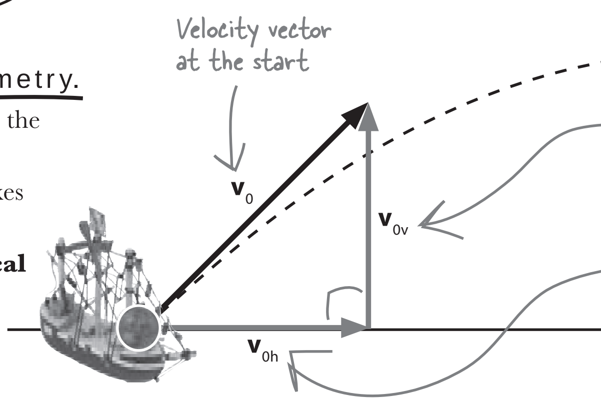
Spotting that the initial and final vertical velocity components are the same **size** (although in **opposite directions**) is pretty useful, right?

Look out for shortcuts involving symmetry.

Symmetry is often a useful shortcut - especially if the overall vertical displacement is zero.

Sometimes you can use the fact that a projectile takes **equal times to go up and down** as a shortcut.

And sometimes you can use the fact that the **vertical components of v_0 and v have the same size** (but opposite directions) at the same height.



there are no Dumb Questions

Q: Is it OK if I didn't spot the symmetry but still got most of the way through the problem?

A: Yes, it's absolutely fine. But it's always a good idea to keep on the lookout for symmetry, because it sometimes lets you solve problems more quickly.

Q: Can you still get the right answer to a problem like this even if you don't spot the symmetry?

A: Yes - you can use the equation $v^2 = v_0^2 + 2(x - x_0)$. Here, $x - x_0 = 0$ because the cannonball starts and finishes at the same height. So the equation simplifies to $v^2 = v_0^2$.

Q: If I have the equation $v^2 = v_0^2$, doesn't that mean that $v = v_0$?

A: Not necessarily. If you multiply two negative numbers together, then you get a positive number. So the solution to the equation $v^2 = v_0^2$ could either be $v = v_0$ or $v = -v_0$.

Q: If I have an equation with two possible solutions, how do I work out which solution is correct?

A: Look at the context (or 'k'ontext). Here, the vertical component of the cannonballs' velocity points up at the start and down at the end. So they're in opposite directions. Therefore, the solution must be the one where the vertical velocity components point in opposite directions: $v = -v_0$.

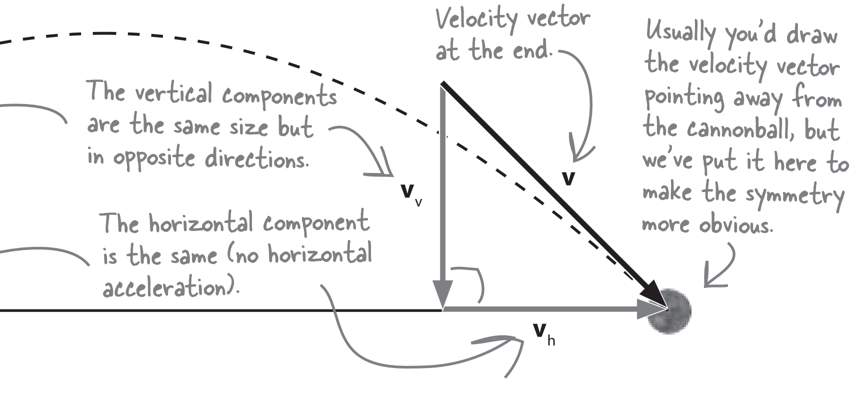
Q: I noticed that there were some subscripts used in the sharpen answer. Should I use subscripts too?

A: Subscripts sometimes help you keep track of things - and sometimes make things look messy! As long as you stay organized and write out what you're doing, you'll be OK.

Q: What if the ghost ship's more than 282 m away? I guess the pirates can't just get new cannons that have a higher muzzle velocity?

A: The pirates have to stick with the same cannons. But the website does say that the muzzle velocity is for a standard iron cannonball ...

If a projectile starts and finishes at the same height, the vertical component of its velocity has the same size, but the opposite direction, at the start and finish.



Sieges-R-Us has a new stone cannonball, which they claim will increase the range!

The Sieges-R-Us website has just been updated! As well as the standard iron cannonball, they now have a stone cannonball. Although both cannonballs are the same size, the stone cannonball is only a third of the **mass** of the iron cannonball because they're made from different materials.

An object's **mass** is a measure of the amount of stuff it's made from. Both cannonballs are the same size and have the same volume (so fit in the same cannon), but the iron one has more material in it.

The website claims that the stone cannonball has a longer range than the iron one if you fire it out of the same cannon. But the stone cannonball is so new that there are no tech specs - like the muzzle velocity.

So... **does the stone cannonball actually go further**, or is it all hype?

Even IF the stone cannonball **does** go further then the iron one, it might not be worth the money if it only goes a few meters further.

'Does it?' and 'How much?' are both questions you may need to answer.

Sometimes you need to answer a **qualitative** question, such as "**Does** a stone cannonball go further than an iron cannonball (which has a larger mass) if you shoot them from the same cannon?"

And sometimes you need to answer a **quantitative** question like, "**How much** further (if at all) does a stone cannonball go than an iron cannonball (which has a larger mass) if you shoot them both from the same cannon?"

Here, it's likely you'll need to answer both questions. If the stone cannonball does go further, then the pirates will want to know how much further, so they can keep the ghost ship as far away as possible.

I'm always very suspicious of anything that comes out of a marketing department.

Jim: Yeah, they don't even have any tech specs on there about the **velocity** the stone cannonball will travel at - only the fact that it's a third of the **mass** of the iron cannonball.

Frank: But the two cannonballs must have the same **size** if they both fit in the same cannon - so how can they have different masses?

Joe: 1 cm^3 of iron has a larger mass than 1 cm^3 of stone, doesn't it? If you have the same **volumes** of iron and stone, it takes more effort to lift the iron. So if the two cannonballs are the same size, then the iron cannonball must have a larger mass. The website's right about that!

Jim: But didn't we say before that all falling objects accelerate at the same rate no matter what their masses are? So there wouldn't be any difference for the stone and iron cannonballs.

Joe: Hmmm. But the cannonballs are coming out of the cannon before gravity takes over, aren't they? There'd be the same explosion to **push** them out of the cannon each time.

Frank: And I guess that's different from gravity - the explosion pushes the cannonball, but gravity doesn't have to make **contact** with the cannonball to accelerate it.

Jim: Maybe we could **be** the cannon. We could imagine pushing the stone and iron cannonballs to see what we think would happen.

Joe: But to us, both cannonballs are difficult to push. Maybe this is a good place to think in **extremes** - like pushing a large mass versus pushing a small mass ... pushing an elephant versus pushing a mouse ...

The cannon is different from gravity because it pushes the cannonball by making contact with it.

BE ... someone pushing something

Your job is to imagine pushing an elephant, then pushing a mouse with the same strength of push. You might want to think about each animal being on a skateboard so that you can actually see the effect of pushing them. How do their velocities vary?

BE ... someone pushing something - SOLUTION

Your job is to imagine pushing an elephant, then pushing a mouse with the same strength of push. You might want to think about each animal being on a skateboard so that you can actually see the effect of pushing them. How do their velocities vary?

If I push the mouse hard, it'll go really fast.

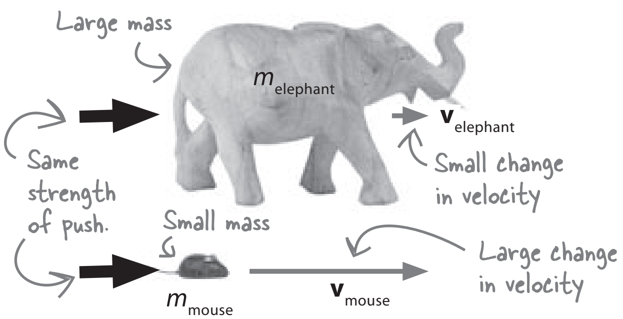
It's harder to get the elephant going because it has a larger mass. If I give it the same push I give the mouse, it'll hardly move at all.

The object with a lower mass has a higher velocity for the same push.

Massive things are more difficult to start off

If you stand an elephant and a mouse on a skateboard and give each the same size of push to make them move, then the mouse ends up traveling at a higher velocity than the elephant. Because the elephant has a larger mass, it's more difficult to change its velocity.

The elephant and the mouse both start with zero velocity.

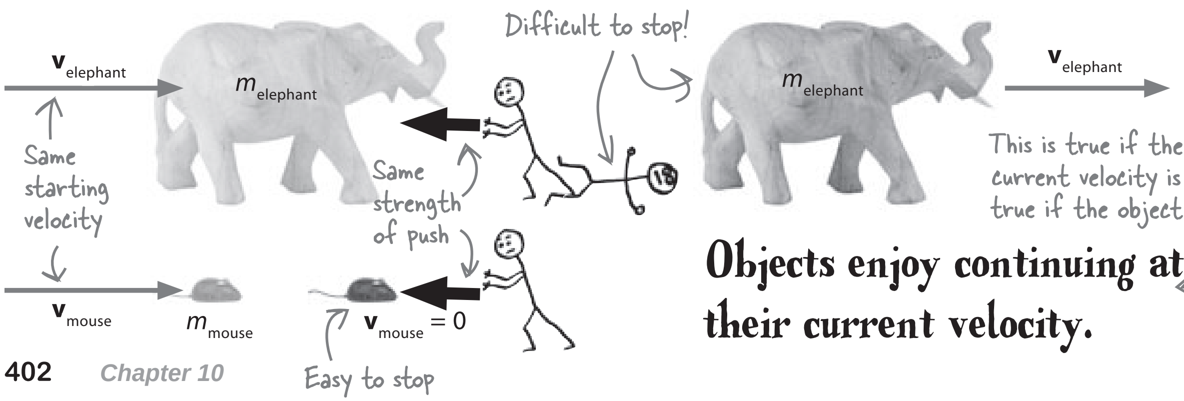


The larger an object's mass, the more difficult it is to change its velocity.

This is true if the object's initial velocity is zero. It's also true if the object is moving.

Massive things are more difficult to stop

If the elephant and the mouse are already traveling with the same velocity, the elephant is more difficult to stop. This is because its greater **mass** means it has a greater tendency to continue at its current velocity when you give it the same strength of push.



This is true if the object's current velocity is zero. It's also true if the object is moving.

Objects enjoy continuing at their current velocity.

Newton's First Law

You might have heard the saying "An object at rest tends to stay at rest; an object in motion tends to stay in motion." Newton's 1st Law takes this further.

Objects have **inertia**, which means that they will keep on moving with their **current velocity** unless you act on them with a **force** (for example, by giving them a push). A stationary object remains stationary unless something happens to make it move. And a moving object continues to move with its current velocity unless something happens to speed it up, slow it down, or change its direction of movement.

Another way of putting this is **Newton's First Law**, which says that an object will continue on with a **constant velocity** unless there's a **net force** acting on it.

'Net force' means total overall force.

We'll talk about this more later on.

Newton's First Law says that an object will carry on with the same velocity unless there's a net force acting on it.

The same SPEED in the same DIRECTION

there are no Dumb Questions

Q: If this is Newton's FIRST Law, does that mean there are others? How many are there?

A: You'll meet Newton's three Laws of Motion in this chapter and the next.

Q: I've heard something about Galileo's Law of Inertia, which sounds very similar to Newton's First Law. Do I need to know about that as well?

A: Galileo's Law of Inertia and Newton's First Law both say the same thing, there's no need to worry.

Q: OK, so does it matter what I call these laws? Do I have to learn their names or is it enough to understand the physics concepts?

A: Understanding the concepts is the most important thing, but in an exam, you may be asked to explain what's happening in terms of Newton's three laws. Then you'd have to remember which is which.

Q: If an object continues on with "constant velocity," it could either already be moving, or it could be completely still and have a velocity of zero, right?

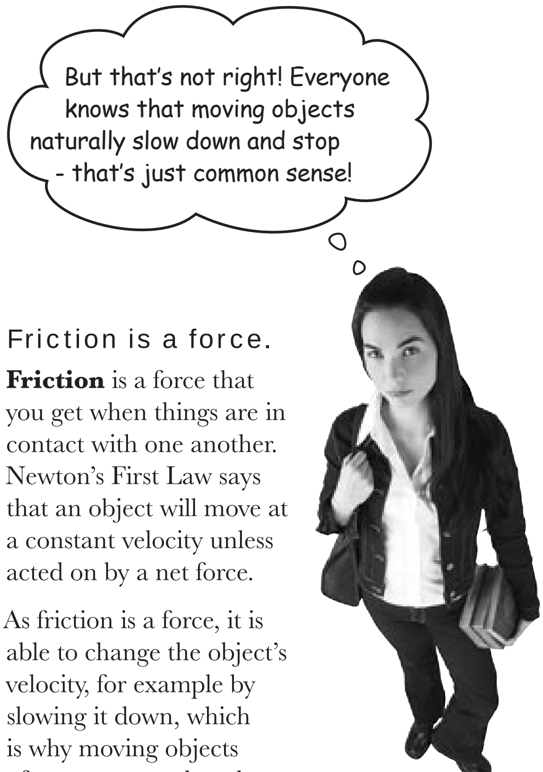
A: Yes, that's correct. Whether something's stationary or moving, you need a net force to change its velocity.

Q: If its velocity changes, that means it speeds up or slows down, right?

A: Velocity is a vector, so as well as speeding up or slowing down, a change in velocity could be a change in direction without a change in speed.

Q: What does 'net' force mean?

A: There might be more than one force acting on an object at the same time. The 'net' force is what you get when you add all the forces acting on an object together. Just like a company's net profit (or net loss) is when you add together all its incomings and (negative) outgoings.



Friction is a force.

Friction is a force that you get when things are in contact with one another. Newton's First Law says that an object will move at a constant velocity unless acted on by a net force.

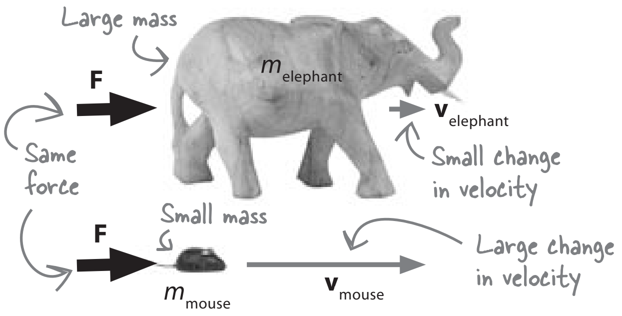
As friction is a force, it is able to change the object's velocity, for example by slowing it down, which is why moving objects often appear to slow down naturally when you don't interfere with them.

You'll learn more about friction in chapter 12.

Mass matters

If you push an elephant and a mouse with the same **force** each time, the mouse ends up with a larger **velocity** than the elephant because it has a smaller **mass**. And if you try to stop an elephant and a mouse that are already traveling with the same velocity, you'll get flattened by the elephant if you push it with the same force that you need to stop the mouse.

The more massive something is, the greater its **inertia**, or tendency to continue with its current velocity, and the larger the force you have to push it with to produce the same change in velocity.

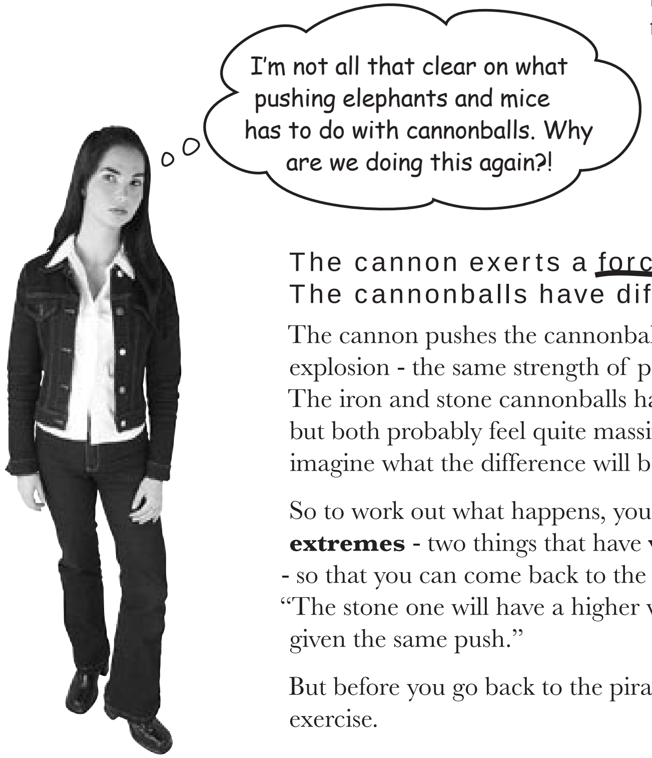


Q: Does this mean I have to do math with forces? How do I do that?

A: That's something for later on. Right now, you're not doing calculations with forces, just working out some general physics principles involving them.

Q: I've heard the word 'inertia' used to mean reluctance. Like, "I had to overcome a lot of inertia to get out of bed on a cold morning." Is this another meaning for the same word?

A: It's kind of similar actually. An object's inertia is its tendency to continue at its current velocity. In the example you mention, your inertia is your tendency to continue in your current sleeping place. So the usage is kind-of similar!



The cannon exerts a force.
The cannonballs have different masses.

The cannon pushes the cannonballs with the same explosion - the same strength of push, the same **force**. The iron and stone cannonballs have different masses, but both probably feel quite massive to you. It's hard to imagine what the difference will be.

So to work out what happens, you've been thinking about **extremes** - two things that have **very** different masses - so that you can come back to the cannonballs and say, "The stone one will have a higher velocity if they're both given the same push."

But before you go back to the pirate ship, here's a quick exercise.

Get used to thinking in extremes to work out what will happen in your situation.

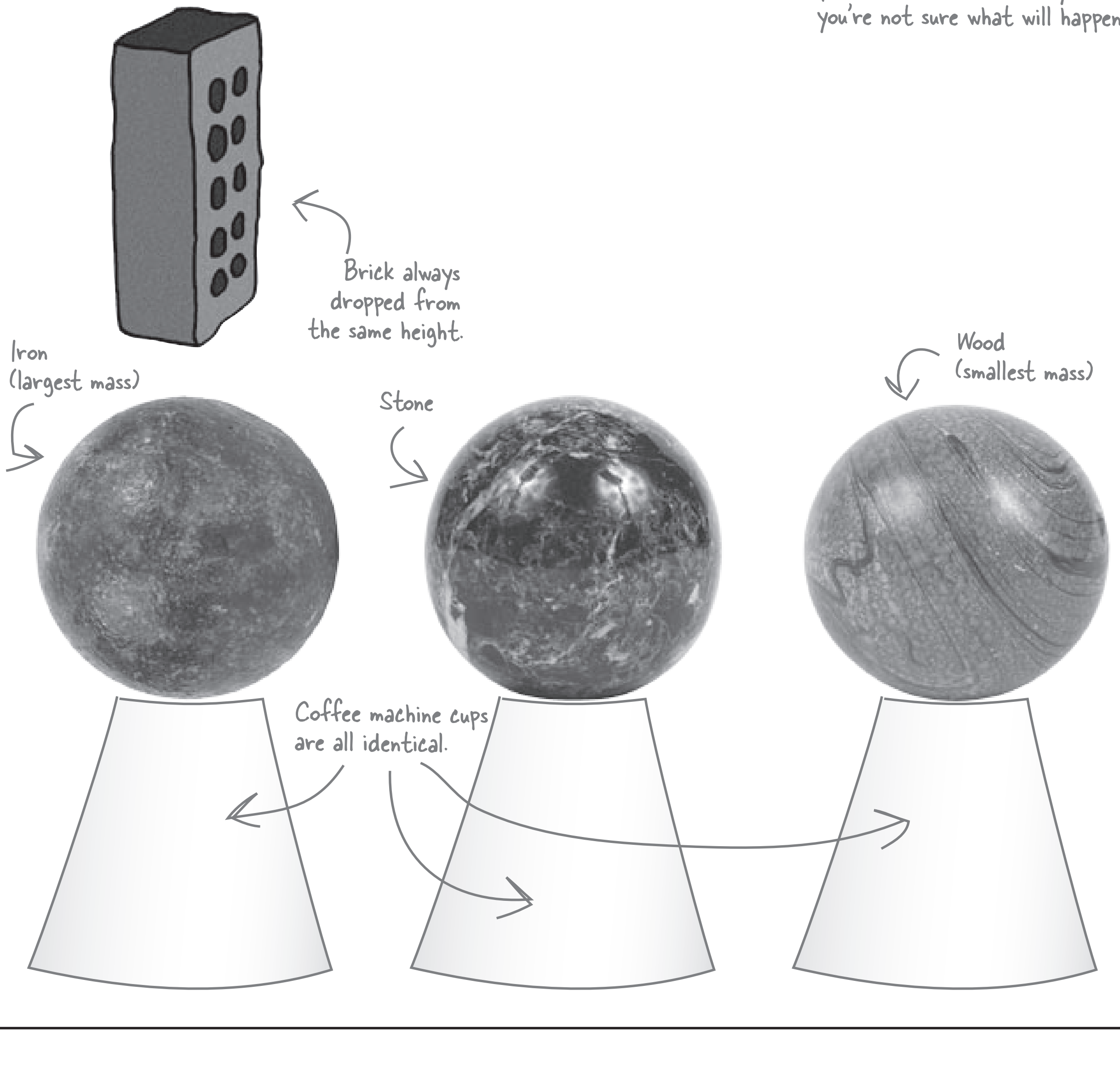
Exercise

In this exercise, you have three coffee machine cups. One has an iron cannonball balanced on top, the next has a less massive stone cannonball on top, and the third has an even less massive wood ball balanced on top.

The same brick is dropped directly onto each ball from the same height each time.

Which cup is damaged the least, and why?

Relax – we're looking for a prediction, so don't worry if you're not sure what will happen.



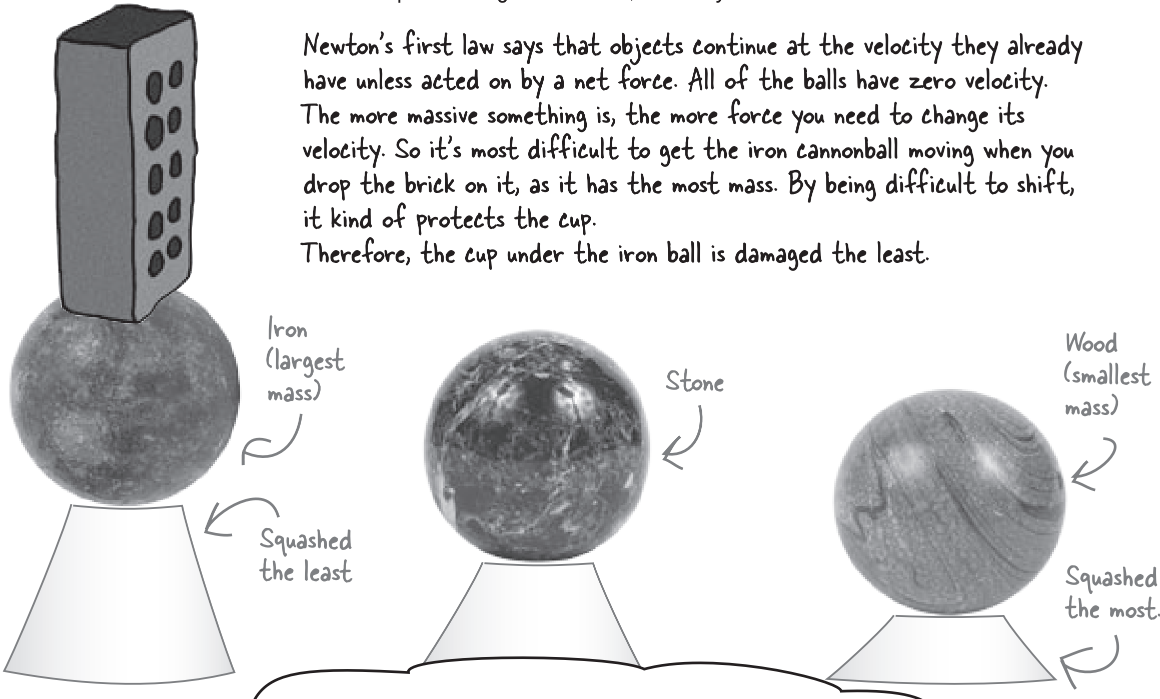
Exercise Solution

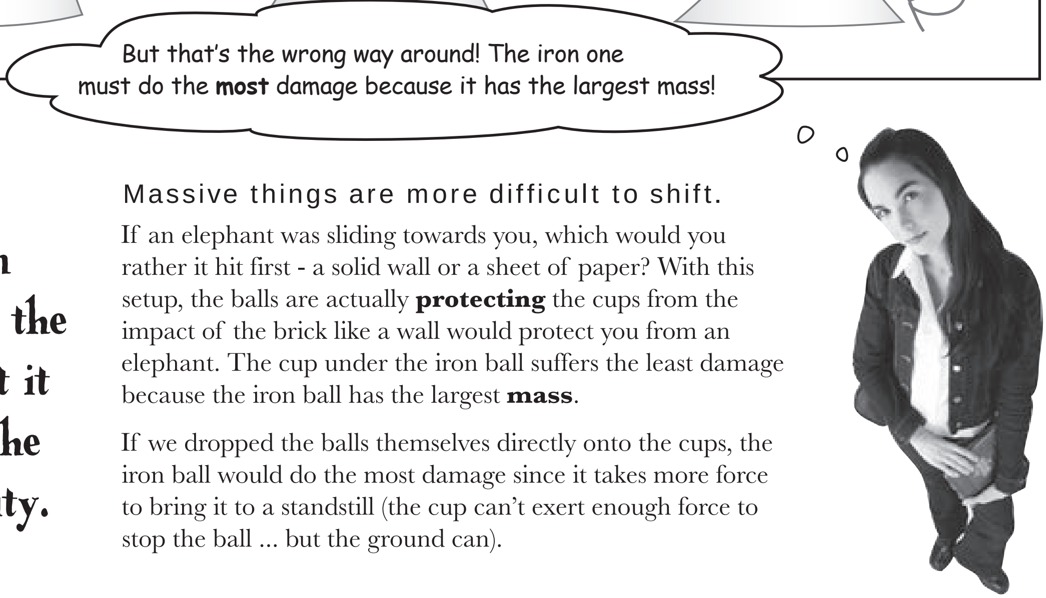
In this exercise, you have three coffee machine cups. One has an iron cannonball balanced on top, the next has a less massive stone cannonball on top, and the third has an even less massive wood ball balanced on top.

The same brick is dropped directly onto each ball from the same height each time.

Which cup is damaged the least, and why?

Newton's first law says that objects continue at the velocity they already have unless acted on by a net force. All of the balls have zero velocity. The more massive something is, the more force you need to change its velocity. So it's most difficult to get the iron cannonball moving when you drop the brick on it, as it has the most mass. By being difficult to shift, it kind of protects the cup.
Therefore, the cup under the iron ball is damaged the least.





The larger an object's mass, the more difficult it is to change the object's velocity.

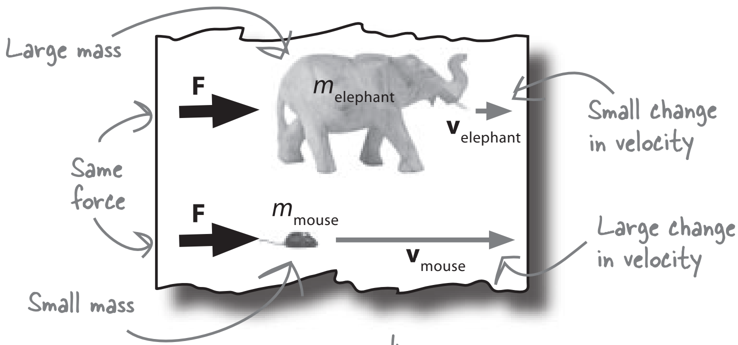
Massive things are more difficult to shift.

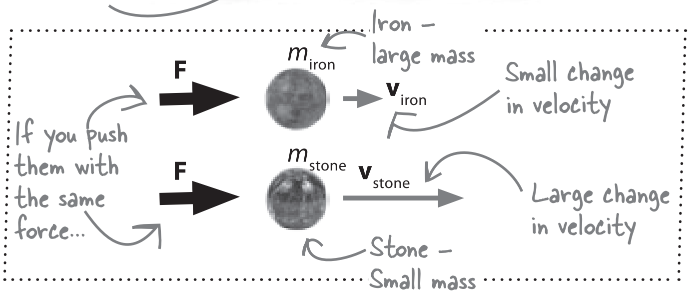
If an elephant was sliding towards you, which would you rather it hit first - a solid wall or a sheet of paper? With this setup, the balls are actually **protecting** the cups from the impact of the brick like a wall would protect you from an elephant. The cup under the iron ball suffers the least damage because the iron ball has the largest **mass**.

If we dropped the balls themselves directly onto the cups, the iron ball would do the most damage since it takes more force to bring it to a standstill (the cup can't exert enough force to stop the ball ... but the ground can).

A stone cannonball has a smaller mass - so it has a larger velocity. But how much larger?

As the stone cannonball has a smaller **mass** than the iron one, it will come out of the cannon with a greater **velocity**.





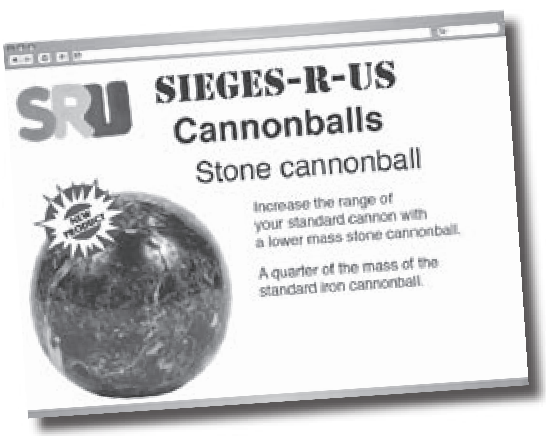
OK, so the new stone cannonballs have arrived. I don't mind doing a test firing - as long as it's **horizontal**. To stay under the radar - see?

However, the only thing you know about a stone cannonball is that it's a quarter of the **mass** of an iron cannonball. The pirate captain wants to know the range of a stone cannonball fired at 10°, but will only let you fire a stone cannonball horizontally.

Firing horizontally doesn't sound like it'll help, as you can't measure the **distance** the cannonball goes out to sea, and the cannonball's **velocity** will be far too high to measure directly.

You can't measure the distance when you're at sea!

If you know the velocity, you can calculate the range.



BRAIN POWER

Is there any way of working out the **range** or **velocity** for a stone cannonballs fired at 10° when you can only measure things that are actually on the ship?

So the stone cannonball will go further. Since the stone cannonball has less **mass** than the iron cannonball, the stone cannonball will come out of the cannon with a higher **velocity**.

Jim: Yeah, if the stone cannonball has a higher velocity, the vertical velocity **component** will be larger, so the cannonball will stay off the ground for longer. And the horizontal velocity component will also be larger, so the cannonball will go even further in that **time**.

Joe: So we've answered the "**Does it?**" question - it does go further. But now we need to answer the "**How much?**" question - how much further than the iron cannonball will the stone cannonball go?

Frank: But the stupid Sieges-R-Us website still hasn't been updated with proper tech specs. If we knew the muzzle **velocity** for the stone cannonball, it'd be easy. We might be able to do something if we knew the actual **masses** of the cannonballs. But all the website says is that the stone one is four times lighter than the iron one.

Joe: Maybe we could do some kind of **experiment**?

Frank: But the pirates won't let us fire their cannons at the ghost ship - they want the element of surprise...

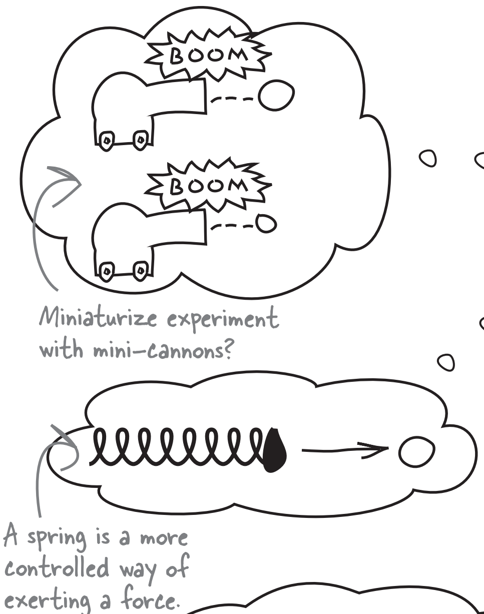
Joe: I mean, maybe we could miniaturize things to do a small experiment then scale it up, like we've done before.

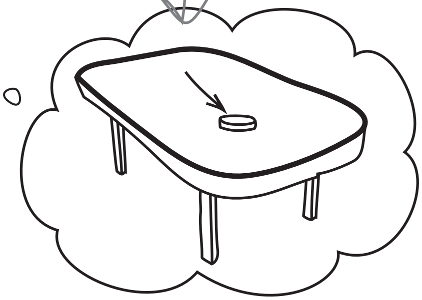
Jim: With a toy cannon, perhaps. We could make it fire two different objects, one four times the mass of the other.

Frank: I'm not sure how practical it is to calibrate a toy cannon. How about we use a **spring** to push an object horizontally? Then we're only thinking about one **dimension** - we can always extrapolate to two dimensions later on by using component vectors.

Joe: But if we push an object horizontally, they'll soon grind to a halt because of the **friction** between them and the table. There's relatively little friction when a cannonball goes through the air. It wouldn't be the same.

Jim: Hmm, air. Can we make some kind of hovercraft thingy to push with the spring? That floats on a cushion of air.





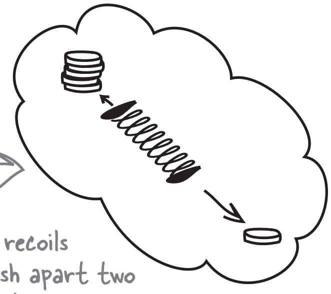
Frank: Or, no ... even better - can we use an air hockey table to reduce friction? We can push an object horizontally across an air hockey table, using a spring to make sure we use the same force each time.

Joe: I just thought of something else. Cannons **recoil**, right?

Jim: What do you mean?!

Joe: When you fire a cannon or a gun, the force of the explosion makes it kick back. That's why the pirates' cannon has wheels.

Frank: So how about putting the spring between **two** objects on the air hockey table. One has a large mass, like the cannon. The other has a smaller mass, like the cannonball. Then we get a **recoil** when we let the spring do its thing?

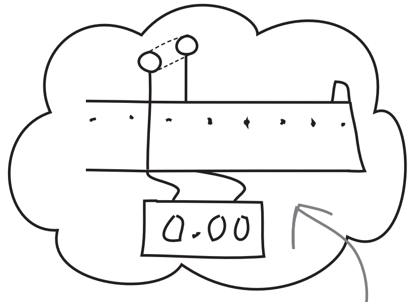


Joe: That sounds good, but I've also been thinking - how do we **measure velocity**? That's what we actually want to find out for the things with different masses - so we can scale it up for the two cannonballs and work out the range of the stone cannonball.

Jim: I guess we could mark out a **distance** we know, and **time** how long it takes for the thing we're pushing to cover it. We can work out a **velocity** from that.

Joe: But it's going to be difficult to do that **precisely**. It's not like we can use a regular stopwatch because the times here are going to be very short, maybe less than half a second and difficult to do by hand.

Frank: I've seen this thing in a physics lab before. It's a beam of light, and when something goes through and breaks the beam, a timer starts. Then when the beam's restored, the timer stops.



Jim: But there's only one beam, right? So how can we **time** something over a certain distance when the beam's always in the same place?

Frank: If we know the **length** of the object going through the beam, then we know that it's gone exactly that **distance** while breaking the beam. So we can work out the velocity.

Joe: And we could do that for both the small object (cannonball) and large object (cannon) so we get the **recoil velocity** as well.

Jim: Let's go see what equipment we've got in the lab ...

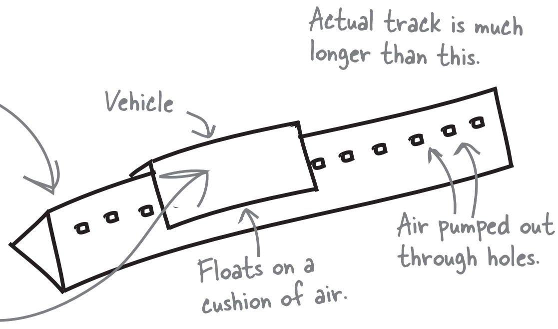
When you design an experiment, try to make it LIKE the situation you're modelling.

Here's your lab equipment

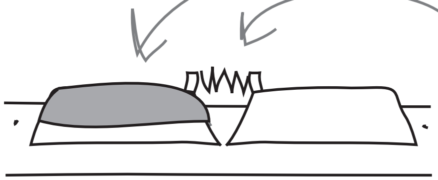
Here are the things you need for this experiment:

You need **a way of reducing friction**. You have an air track, which is like an air hockey table because it has small holes in it all the way along its length to create a 'cushion' of air and reduce friction as much as possible.

You need **objects to represent the cannon and cannonball**. The air track has specially-designed vehicles that sit on it.



An air track reduces friction, and the vehicles represent the cannon and cannonball.



You can add masses to the vehicles and push them apart with a spring

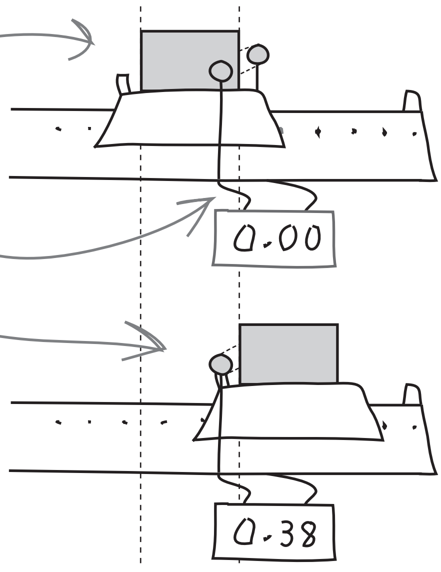
You need a way of **changing and measuring the vehicles masses**. The air track vehicles come with a set of masses that you can stack on top of them - both these and the vehicle are marked with their masses. So one vehicle can represent the cannonball and the other one the cannon.

You need something to **push the vehicles apart with the same force each time**. You can use a spring, which you push in to the same place every time. Then when you let go of the spring, it pushes the vehicles apart.

You need a way of **measuring velocity**. You can measure pieces of card and attach them along the length of each vehicle.

Then set up a light gate (sometimes called a photogate) for each vehicle to pass through. When the card breaks the beam, it starts a timer, and when the card has passed all the way through, the beam is restored and the timer stops. This gives you the **time** it took for the vehicle to go the same **distance** as the length of the card, which you can use to calculate the **velocity**.

Think about what you NEED to do. Then about what equipment you HAVE available to do it.



You can add a piece of card to a vehicle and time it with a light gate to work out its velocity.

How are force, mass and velocity related?

You now **have** all the equipment you **need** to **design** an experiment that will let you work out the effect that a cannonball's mass has on its muzzle velocity. But you need to put the equipment together in an **experimental setup**.

The other important thing at this stage is to work out what **variables** you can change during the course of your experiment. These are all the different things that might affect what you are measuring (in this case, the velocities of the vehicles).

Any time you design an experiment, think about what your VARIABLES are.

Sharpen your pencil

a. Design and draw an experimental setup with the equipment on the opposite page (plus anything else you'd like to use) to replicate a scenario where a force pushes apart a cannon and a cannonball, so that they both experience a change in velocity. Your aim is to see how the velocity of the cannonball varies with its mass (and whether this also affects the recoil velocity of the cannon).

b. Identify the things in your experimental setup that you can **vary** in order to produce a set of results. (At this stage you're not being asked how you would vary them.)

c. Explain what **measurements** you would make for one trial of your experiment, and how you would use them to fill in a table of masses and velocities.

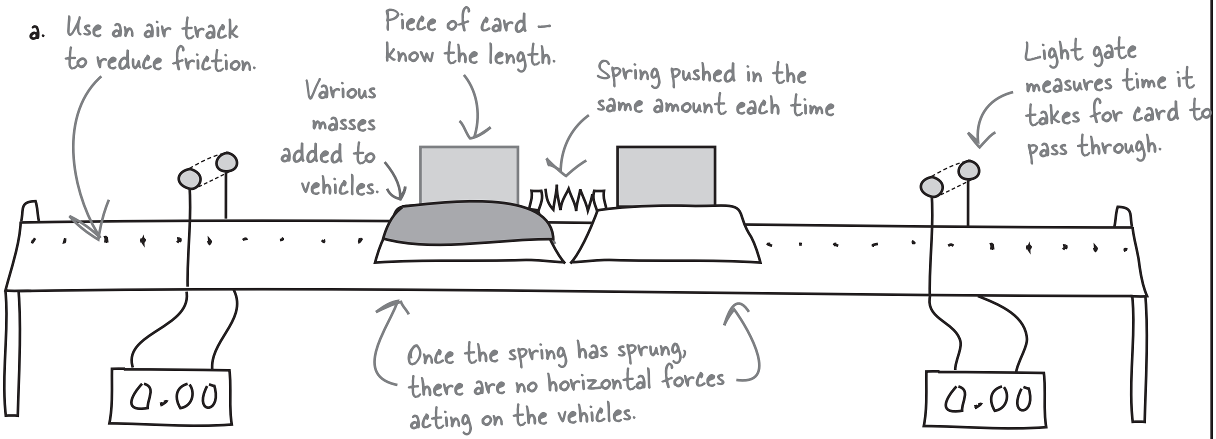
Sharpen your pencil Solution

a. Design and draw an experimental setup with the equipment on the opposite page (plus anything else you'd like to use) to replicate a scenario where a force pushes apart a cannon and a cannonball, so that they both experience a change in velocity. Your aim is to see how the velocity of the cannonball varies with its mass (and whether this also affects the recoil velocity of the cannon).

b. Identify the things in your experimental setup that you can **vary** in order to produce a set of results. (At this stage you're not being asked how you would vary them.)

c. Explain what measurements you would make for one trial of your experiment, and how you would use them to fill in a table of masses and velocities.

a.



b. I can vary the mass of each vehicle, and the strength of the spring.

c. For one trial, I would put masses on each vehicle. Then I'd use scales to find the mass of the vehicle plus the masses. I would use the light gates to measure the time it takes for each vehicle to pass through. Then I would use the length of the card and the time to work out each vehicle's velocity

there are no Dumb Questions

Q: Why am I using two vehicles again?

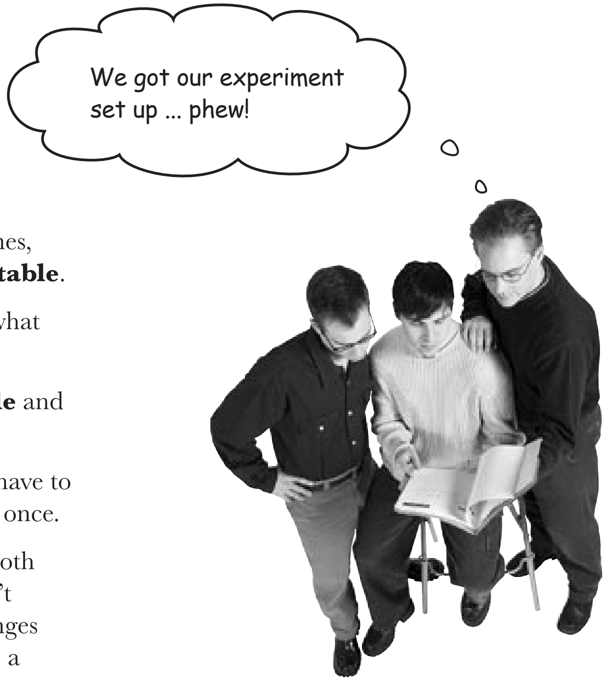
A: Cannons recoil. As well as the cannonball going forwards, the cannon goes backwards. So you're replicating that.

Q: Why am I measuring the 'cannon' vehicle's velocity when I'm only interested in the cannonball's velocity?

A: You never know what might be useful...

Q: But that's just extra work!

A: The spring pushes **both** vehicles, so knowing both velocities might help you come to a better conclusion than only knowing one.



Frank: Yeah, but what do we do now?

Jim: I guess we push the vehicles apart, measure some times, use them to work out velocities and write them down in a **table**.

Frank: But we gotta be more organized than that. Like, what will we **change** in between one trial and the next?

Joe: We can change the **mass of the "cannon" vehicle** and the **mass of the "cannonball" vehicle**.

Jim: So we just change both masses each time. We won't have to do so many trials that way, as we can change two things at once.

Joe: I'm not sure that's a good idea. Suppose we change both masses and get a different result for the velocities. We won't know why we got a different result. Which of the two changes was responsible for the difference - or did it come about as a result of both changes working together somehow?

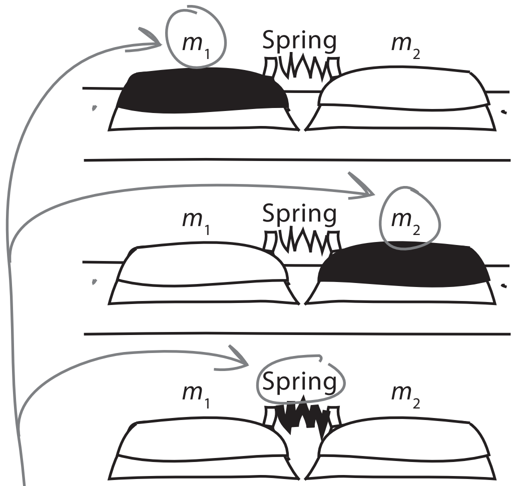
Frank: Yeah, that's a good point. In real life, the cannon's mass doesn't change. I think we should give one of the vehicles a really large mass - like the cannon has compared to the cannonball - then vary the mass of the other vehicle each time. we do a trial.

Jim: But what if we just happen to choose a mass for our 'cannon' vehicle that produces a special result. I think we need to change that mass as well, to make sure anything we work out isn't a special case.

Joe: So why don't we do one set of trials with a fixed 'cannon' mass and change the 'cannonball mass' each time, then do another set with a different fixed 'cannon' mass to see if that makes a difference?

Frank: And then do another lot with a different spring, to make sure that's not a special case too.

Jim: OK, I see what you mean. **Changing only one thing at a time** must be the best way of doing it. If something different happens, you know for sure that the thing you changed is what caused the difference.



There are THREE variables in your experiment.

You should only change one thing at a time in your experiment.

Vary only one thing at a time in your experiment

The point of doing an experiment is to find out what happens to one quantity if you **vary** another quantity. In this experiment, we want to find out what happens to the **velocities** of two objects that are pushed apart by a **force** when you vary the objects' **masses**. Then you can **extrapolate** your results to predict what will happen to the velocities of a cannon and cannonball when they're forced apart by an explosion.

If you vary both masses for each trial of your experiment, it'll be difficult to spot patterns in your results. You won't know which change had the most effect on your results - or if the changes somehow canceled each other out.

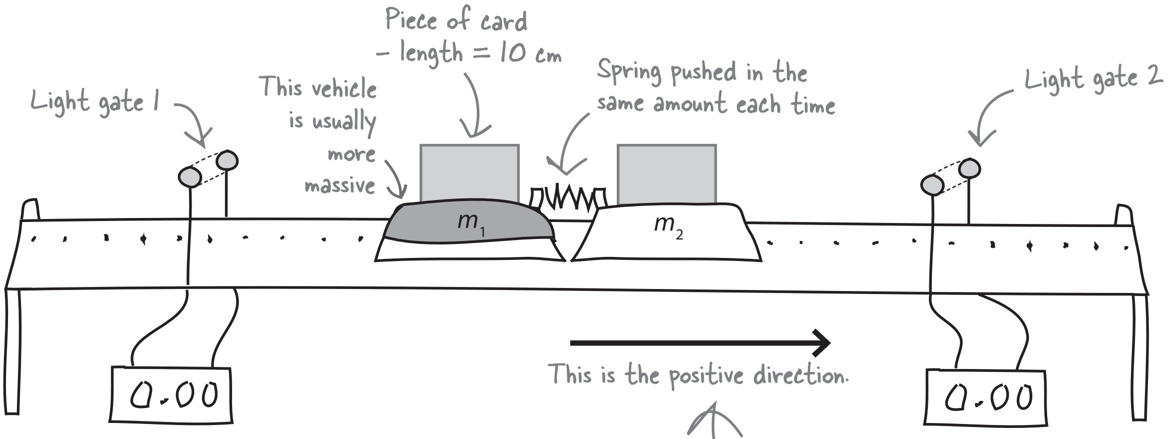
So by varying only **one** mass at a time then changing the other mass and doing it again will make sure that it'll be OK to extrapolate your findings to **any** two masses.

An experiment shows you what happens to one quantity when you vary another quantity.

Sharpen your pencil

Here are the results of your experiment. We've annotated the diagram with all of the relevant details like labelling which light gate is which, and the length of the pieces of card (10 cm). Be careful with the direction that each velocity is in!

Fill in the missing boxes in the table, and **use the space below to write down any patterns you see.**



Be careful with the minus signs that show the direction of each vehicle's displacement and velocity vectors.

Times measured to 2 sd so you should only quote your velocities to 2 sd.

Mass 1 (kg)	Mass 2 (kg)	LG 1 time (s)	LG 2 time (s)	Velocity 1 = $\frac{\Delta x}{\Delta t}$ (m/s)	Velocity 2 = $\frac{\Delta x}{\Delta t}$ (m/s)
0.150	0.150	0.19	0.20	$\frac{-0.10}{0.19}$ = -0.53 (2 sd)	$\frac{0.10}{0.20}$ = 0.50 (2 sd)
0.150	0.300	0.17	0.35		
0.150	0.450	0.16	0.48		
0.300	0.150	0.34	0.17		
0.300	0.300	0.27	0.26		
0.300	0.450	0.25	0.38		
0.450	0.150	0.47	0.16		
0.450	0.300	0.37	0.25		
0.450	0.450	0.34	0.33		

Repeat experiment with masses the other way round in case of systematic error/BIAS, e.g. the air track being on a slight slope.

Sharpen your pencil Solution

Here are the results of your experiment. We've annotated the diagram with all of the relevant details like labelling which light gate is which. Note: The pieces of card on the vehicles are 10 cm long.

Fill in the missing boxes in the table, and use the space below to write down any patterns you see.

Mass 1 (kg)	Mass 2 (kg)	LG 1 time (s)	LG 2 time (s)	Velocity 1 $= \frac{\Delta x}{\Delta t}$ (m/s)	Velocity 2 $= \frac{\Delta x}{\Delta t}$ (m/s)
0.150	0.150	0.19	0.20	$\frac{-0.10}{0.19}$ = -0.53 (2 sd)	$\frac{0.10}{0.20}$ = 0.50 (2 sd)
0.150	0.300	0.17	0.35	$\frac{-0.10}{0.17}$ = -0.59 (2 sd)	$\frac{0.10}{0.35}$ = 0.29 (2 sd)
0.150	0.450	0.16	0.48	$\frac{-0.10}{0.16}$ = -0.63 (2 sd)	$\frac{0.10}{0.48}$ = 0.21 (2 sd)
0.300	0.150	0.34	0.17	$\frac{-0.10}{0.34}$ = -0.29 (2 sd)	$\frac{0.10}{0.17}$ = 0.59 (2 sd)
0.300	0.300	0.27	0.26	$\frac{-0.10}{0.27}$ = -0.37 (2 sd)	$\frac{0.10}{0.26}$ = 0.38 (2 sd)
0.300	0.450	0.25	0.38	$\frac{-0.10}{0.25}$ = -0.40 (2 sd)	$\frac{0.10}{0.38}$ = 0.26 (2 sd)
0.450	0.150	0.47	0.16	$\frac{-0.10}{0.47}$ = -0.21 (2 sd)	$\frac{0.10}{0.16}$ = 0.63 (2 sd)
0.450	0.300	0.37	0.25	$\frac{-0.10}{0.37}$ = -0.27 (2 sd)	$\frac{0.10}{0.25}$ = 0.40 (2 sd)
0.450	0.450	0.34	0.33	$\frac{-0.10}{0.34}$ = -0.29 (2 sd)	$\frac{0.10}{0.33}$ = 0.30 (2 sd)

When the masses are the same, the velocities are roughly the same magnitude.

i.e. the same size – magnitude is another word for size.

When the masses are the same but both heavier than another trial, their velocities are smaller.

When the masses are different, the smaller one always has a higher magnitude of velocity.

The size of mass × velocity is the same for both vehicles, whether the masses are the same or not.

mass x velocity is always the same size for both vehicles in any one trial.

Don't worry if you didn't spot this last pattern. It's more difficult to see than the others.

there are no Dumb Questions

Q: I looked really hard for the pattern but didn't spot it. Is that OK?

A: If you spotted that when the mass of the vehicle is smaller its velocity is larger (as long as the other vehicle still has the same mass) then that's the main thing. Spotting that mass × velocity always has the same value was the bonus.

Q: So if I hadn't kept one vehicle's mass constant, I might not have spotted the pattern for the other vehicle so clearly?

A: That's right. Before you started actually doing your experiment, you worked out what things you are able to **vary**. Then you made sure that you varied only one thing at a time.

Keeping the mass of one of the vehicles constant enabled you to see the effect that varying the other vehicle's mass had. You were able to spot the pattern because you only changed one variable at a time.

Q: So I should turn all but one of the variables into constants for each trial - each set of measurements - that I make?

A: Precisely - just like you did in this experiment. Your first set of trials were for one mass of 'cannon' vehicle and your second set of trials for another.

Change only one variable at a time and keep the others constant when you do an experiment. This helps you reach better conclusions.

Q: And that was so I didn't inadvertently draw conclusions from just one set of trials, when the results might have looked different for a different 'cannon' vehicle mass?

A: You got it!

Q: Why are we using a 'cannon' vehicle and measuring its velocity in the first place? Why not just push the 'cannonball' vehicle off of a solid wall or something?

A: Because of the observation that cannons **recoil**. When the cannonball goes forward, the cannon rolls backwards - that's why they have wheels!

Q: So we're trying to replicate the 'big' scenario the best we can with our experiment?

A: Yeah, that's the idea.

Q: And now I get to use the results to come up with an equation that helps to work out the difference between firing the iron and stone cannonballs?

A: You got it!

Mass × velocity is the same **size** for both of the vehicles, but the two velocity vectors point in opposite **directions**. So we should be careful with negative signs, right?

Any time you're dealing with vectors, think about signs!

When you filled in your table of experimental results, you were talking about velocity, not speed. So each velocity should have a **sign** corresponding to the **direction** the vehicle is going in.

So far we've been saying that both vehicles have the same **size** of mass × velocity. But you're right, the **direction** of the vectors is about to become really important ...

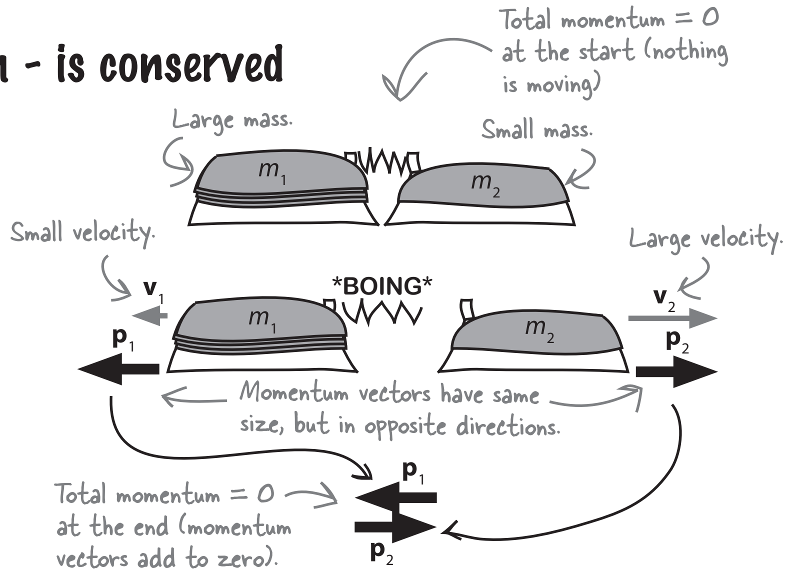
Mass x velocity - momentum - is conserved

From your experiment, you've worked out that when you push two objects apart, they end up with the same size of mass × velocity. The name given to mass × velocity is **momentum**. Momentum is given the symbol **p**.

An object's momentum is a **vector** that points in the same direction as its velocity vector. Your experiment has told you that **momentum is conserved** in this interaction when two things are pushed apart, as the total momentum is the same both before and after.

Mass is measured in kg and velocity in m/s. This means that momentum has units of kg·m/s The · between 'kg' and 'm' means 'multiplied by' The · helps to reduce confusion by keeping the 'kg' and the 'm' distinct

The total momentum is conserved in any interaction between different objects.



At the **start** of your experiment, the total momentum of the system is **zero**, as nothing's moving at all. Once the vehicles have been pushed apart, the total momentum of the system is still zero, as the two equally-sized momentum vectors point in **opposite directions** and cancel when you add them 'nose to tail'.

Momentum is always conserved in **any** interaction between two (or more) objects.

there are no Dumb Questions

Q: How can you say momentum is conserved? At the start, the vehicles don't have any momentum - they're sitting still. After they're pushed apart, they're moving, so they both have momentum.

A: If you just think about each vehicle individually, then you're right - at the start they have no momentum and at the end they do.

But when you add up the **total** momentum of all the individual objects involved in the experiment, then it's the same both before and after the interaction

Q: But I thought we said before that **mass × velocity is the same for both** vehicles once they're pushed apart. How can two things that are the same be zero when they're added together?

A: We said that the **size** of mass × velocity is the same for both vehicles. But as they're traveling away from each other in opposite **directions**, their velocity vectors - and momentum vectors - point in opposite directions.

Momentum is a vector!

Q: So when I add together two vectors that are the same size but point in opposite directions, the answer is zero?

A: Yes, that's right. Vectors add 'nose-to tail'. When you line up two vectors that are the same size and point in opposite directions, you end up where you started so the answer is zero.

Q: So now I've got my head around momentum conservation, do I get to play with equations to try and solve the stone cannonball problem?

A: Absolutely!

Hey, not so fast! Weren't we going to try some different springs as well? The results might work out differently.

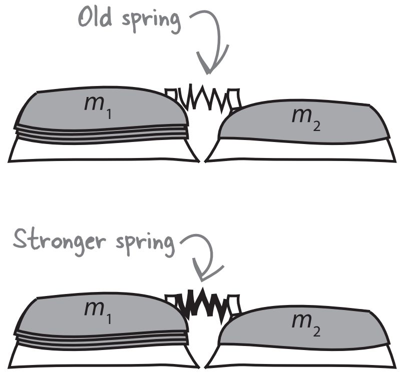
Good point - you need to work out what happens when the force is different

When you were designing the experiment, you realized that there are three things you can vary - the mass of vehicle 1, the mass of vehicle 2 and the strength of the spring that pushes them apart.

To make your experiment complete, you'd usually have to repeat it with a couple of different springs. But this time around, you're going to use your physics **intuition** to predict what would happen if you used a stronger spring ...

BE the experiment

Your job is to be the experiment you've just been doing, and imagine what will happen if you use an even stronger spring to push the two vehicles apart. How will the increased strength of spring affect mass x velocity for the vehicles?



BE the experiment - SOLUTION

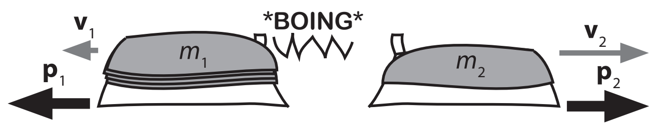
Your job is to be the experiment you've just been doing, and imagine what will happen if you use an even stronger spring to push the two vehicles apart. How will the increased strength of spring affect mass x velocity for the vehicles?

A stronger spring will push the two vehicles apart with a bigger force. So it'll have a bigger effect on the velocity of each vehicle. I think that momentum will still be conserved but the mass x velocity for each vehicle will be bigger, as they were pushed with a bigger force.

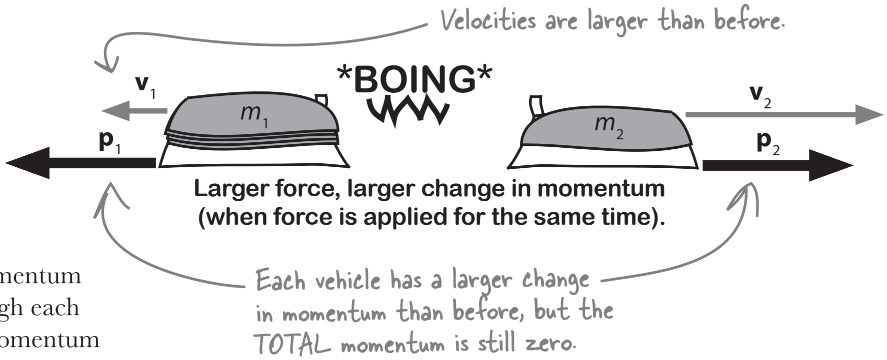
Pushing with a greater force for the same amount of time, leads to a greater change in momentum.

A greater force acting over the same amount of time gives a greater change in momentum

If you use a stronger spring, you increase the **force** that pushes the two vehicles apart. If you apply this larger force for the same time that you were applying the smaller force, it leads to a greater **change of momentum** for each vehicle. If both vehicles are still the same mass, their velocities will both be larger than before.



Small force, smaller change in momentum (when force is applied for the same time).



Larger force, larger change in momentum (when force is applied for the same time).

But because of **momentum conservation**, the **total** momentum is still equal to zero, even though each individual vehicle has more momentum than it did previously.

Write momentum conservation as an equation

The symbol for momentum is **p**, so you can write the equation $\mathbf{p} = m\mathbf{v}$ (momentum = mass × velocity).

When you're dealing with more than one object, you'll typically see **subscripts** to make it clear which object you're talking about. So vehicle 1 has mass m_1, velocity $\mathbf{v}_1$, and momentum $\mathbf{p}_1$. Vehicle 2 has mass m_2, velocity $\mathbf{v}_2$, and momentum $\mathbf{p}_2$.

The total momentum is always $\mathbf{p}_{total} = \mathbf{p}_1 + \mathbf{p}_2$ - this is what's **conserved**.

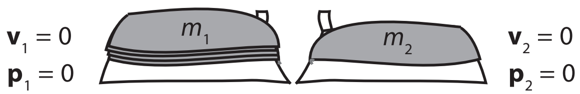
We've not drawn in the spring this time, to make what's going on with the vehicles a bit clearer.

At the start

$p_{total} = p_1 + p_2$

$p_{total} = 0$

At the start of your experiment, $\mathbf{v}_1$ and $\mathbf{v}_2$ are both 0, therefore $\mathbf{p}_1$ and $\mathbf{p}_2$ are both zero so $\mathbf{p}_{total} = 0$.

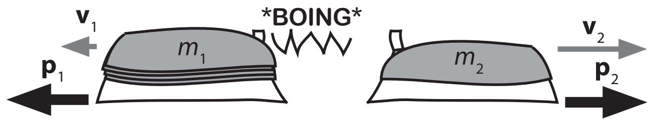


At the end

$p_{total} = p_1 + p_2$

$p_{total} = 0$

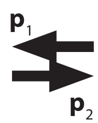
At the end of your experiment, the massive vehicle is moving slowly to the left, and the less massive vehicle is moving more quickly to the right. $\mathbf{p}_{total}$ is still 0, as momentum is conserved.



The symbol for momentum is p.

So you can write down the equation:

$$p_{total} = p_1 + p_2 = 0$$



$p_{total} = 0$, as the momentum vectors add to zero when you line them up nose to tail.

The total momentum is the same before and after.

You can also write the equation this way, which will be very useful for working with **masses** and **velocities**:

$$p_{total} = m_1v_1 + m_2v_2 = 0$$

$p_1 = m_1v_1$ $p_2 = m_2v_2$

This is zero because the total momentum was zero at the start. If the total momentum had a different value at the start, that value would be here instead.

If the total momentum is conserved in any interaction, does that mean that when two things interact, they both experience the same size of **force**?

There is a second law too – you'll meet that later.

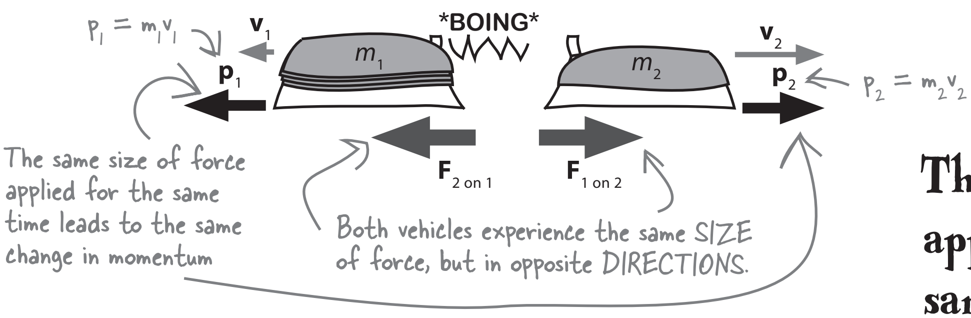
Momentum conservation and Newton's Third Law are equivalent

Newton's Third Law says that if you push something, it pushes back at you with an equal size of force in the opposite direction.

Imagine that instead of being separate, the spring is attached to the left hand vehicle, so the left hand vehicle 'pushes' the right hand vehicle. Then imagine that the spring is attached to the right hand vehicle, so that it 'pushes' the left-hand vehicle.

It actually doesn't make any difference which vehicle is "doing the pushing" - they still move apart in the same way. The size of each vehicle's **change in momentum** is the same, though their momentum vectors point in opposite directions. So both vehicles must have experienced the same size of **force** for the same amount of time, though in **opposite directions**. This is the case whichever vehicle you think of as "doing the pushing". Both vehicles experience the same size of force, but in opposite directions, as a result of their interaction.

In any interaction between two objects, they both experience the same size of force, but in opposite directions.



Newton's third law and momentum conservation mean that the cannon and cannonball exert the same **force** on each other when the cannon is fired. So the change in the cannonball's momentum is the **same size** as the change in the cannon's momentum. These changes in momentum happen in **opposite directions**, as the force that the cannon exerts on the cannonball is in the opposite direction from the force that the cannonball exerts on the cannon.

The same force applied for the same amount of time always produces the same change in momentum.

Not so fast! What if the cannon was attached to the ground? The cannonball would go forwards but the cannon wouldn't go backwards. So afterwards, the cannonball would have momentum but the cannon wouldn't, so momentum isn't conserved!

If the cannon's attached to the ground, you make the EARTH move backwards!

Momentum is **always** conserved in interactions between two or more objects. If the cannon exerts a force on the cannonball, then the cannonball exerts an equal-sized force on the cannon in the opposite direction.

This is Newton's Third Law.

If the cannon is on wheels, this force makes it recoil and roll backwards. But if the cannon is attached to the earth, then the whole earth experiences the force from the cannonball! Since momentum is conserved, the earth must recoil backwards - but because the earth is so massive, its recoil velocity is incredibly small. You can work out how small it is ...

Sharpen your pencil

Use momentum conservation to work out the approximate velocity that the earth would 'recoil' with if the cannon is firmly attached to it when it is fired.

The cannonball has a velocity of 90.0 m/s and a mass of around 1 kg.

The Sieges-R-Us website doesn't give its mass, but it feels like it's around 1 kg.

The earth's mass is 5.97×10^{24} kg.

If you need a refresher in scientific notation, turn to chapter 3.

Why don't you notice the earth recoiling like this?

Remember to start with a labelled sketch!

Sharpen your pencil Solution

Use momentum conservation to work out the approximate velocity that the earth would 'recoil' with if the cannon is firmly attached to it when it is fired.

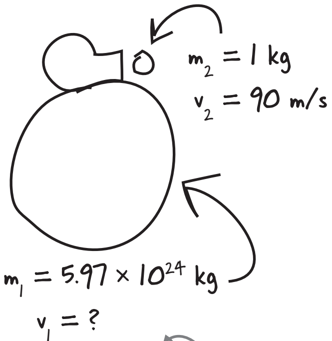
The cannonball has a velocity of 90.0 m/s and a mass of around 1 kg.

The earth's mass is 5.97×10^{24} kg.

Why don't you usually notice the earth recoiling like this?

Total momentum of earth and cannonball at start = 0

Total momentum of earth and cannonball after firing = 0 (momentum conservation)



Remember to draw a labelled sketch, so that it's easy for you (and your examiner) to work out what m_1, v_2 etc are.

$$p = m_1v_1 + m_2v_2 = 0$$

Want to know v_1, so rearrange equation.

$$m_1v_1 = -m_2v_2$$

$$v_1 = \frac{-m_2v_2}{m_1} = \frac{-(1 \times 90)}{5.97 \times 10^{24}}$$

$$v_1 = -1 \times 10^{-23} \text{ m/s (1 sd)}$$

The Earth's velocity is negative, as it moves in the opposite direction to the cannonball.

The earth's recoil velocity is around 1×10^{-23} m/s in the opposite direction to the cannonball's velocity.

This answer should only be quoted to 1 sd as the cannonball's mass is listed as 'around 1 kg,' so has 1 sd.

You don't notice this because it would take an incredibly long time to travel any noticeable distance at this velocity!

there are no Dumb Questions

Q: Momentum is always conserved, even when it doesn't look like it is?

A: That's right - it's easy to see a cannon recoil. But impossible to see the earth recoil!

Q: When am I allowed to use momentum conservation?

A: Any time two (or more) objects interact, momentum is conserved, so you can say $p_{before} = p_{after}$. As long as you keep track of which mass and velocity is which, you're fine.

Q: So I guess I can use the same type of calculation to work out what happens with the stone cannonball?

A: Yes - momentum conservation is the vital key to working out what happens there. Speaking of which ...

So, does the cannon exert the same force on the stone cannonball as it exerts on the iron cannonball?

Sometimes you can do this using an experiment (like here). Sometimes you'll need to think through assumptions using words, graphs or equations.

Test out any assumptions you have

If the cannon exerts the same force for the same amount of time on the stone cannonball as it does on the iron cannonball, the change in momentum would be the same for both cannonballs.

But right now this is an **assumption**, as the cannon may not always exert the same force for the same amount of time.

To investigate this further, you can revisit the experiment you did earlier...

Sharpen your pencil

The same size of **force** applied for the same amount of **time** always leads to the same **change in momentum**.

Therefore, if a spring pushing two vehicles apart always exerts the same force on both vehicles for the same amount of time, then each individual vehicle will experience the same change in momentum every time you do the experiment.

a. The table below is taken from your experiment a few pages ago, where you pushed two vehicles of varying masses apart with the same spring. Complete the table to show the change in momentum for each vehicle.

Mass 1 (kg)	Mass 2 (kg)	Velocity 1 (m/s)	Velocity 2 (m/s)	Change in momentum 1 $p_1 = m_1v_1$ (kg.m/s)	Change in momentum 2 $p_1 = m_1v_1$ (kg.m/s)
0.450	0.150	-0.21	0.63	0.450 × -0.21 = 0.094 (2 sd)	
0.450	0.300	-0.27	0.40		
0.450	0.450	-0.29	0.30		

b. Is the change in momentum of a single vehicle always the same in each trial of the experiment?

c. Try to think of reasons to explain the result you described in part b. Does this have implications for the iron and stone cannonballs, which are fired separately and have different masses?

Sharpen your pencil Solution

The same size of **force** applied for the same amount of **time** always leads to the same **change in momentum**.

Therefore, if a spring pushing two vehicles apart always exerts the same force on both vehicles for the same amount of time, then each individual vehicle will experience the same change in momentum every time you do the experiment.

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0.450	0.150	-0.21	0.63	0.450 × -0.21 = 0.094 (2 sd)	0.150 × 0.63 = 0.094 (2 sd)
0.450	0.300	-0.27	0.40	0.450 × -0.27 = 0.12 (2 sd)	0.300 × 0.40 = 0.12 (2 sd)
0.450	0.450	-0.29	0.30	0.450 × -0.29 = 0.13 (2 sd)	0.450 × 0.30 = 0.14 (2 sd)

b. Is the change in momentum of a single vehicle always the same in each trial of the experiment?

No, the change in momentum isn't the same. It seems to get larger as the masses of the vehicles get larger.

c. Try to think of reasons to explain the result you described in part b. Does this have implications for the iron and stone cannonballs, which are fired separately and have different masses?

The change in momentum is only the same on different occasions if the same force is applied for the same time.

As the masses of the vehicles get larger, the change in momentum increases. The larger a vehicle's mass, the more difficult it is to get the vehicle going. So maybe the vehicles with larger masses spend more time in contact with the spring, so the force isn't applied over the same time.

The iron cannonball has a larger mass than the stone cannonball. This means that it's more difficult to get it going. So it may spend a longer time inside the cannon, and end up with a larger momentum.

If the force is different every time, how are we supposed to solve the problem?!

You can use momentum conservation

You're allowed to do horizontal test firings. Although we didn't see any point in this before, as we can't measure the cannonball's velocity or displacement, we can measure things ON the ship.

This means we can measure the cannon's velocity for firing the iron cannonball, and the cannon's velocity for firing the stone cannonball.

Then we can use what we've learned about momentum conseervation to put the pieces together ...

Yaar, we can test-fire the cannon - as long as we aim it horizontally and away from the ghost ship!

Sharpen your pencil

This is the value given for this particular cannon and cannonball on the website.

Here are the results of two horizontal test firings, where the cannon's recoil velocity was measured.

Iron cannonball: Cannon velocity = 0.126 m/s, Cannonball velocity = 90.0 m/s

Stone cannonball: Cannon velocity = 0.063 m/s

The only other fact you know is that the mass of the iron cannonball is four times the mass of the stone cannonball. If you knew the mass of the cannon you'd be fine, but it's far too heavy to measure using scales. But there's another way to measure the mass of the cannon...

a. Use momentum conservation to calculate m_c the mass of the cannon, in terms of m_i the mass of the iron cannonball.

You won't be able to give your answer as just a number. Your answer will look something like $m_c = 400m_i$ which would tell you that the cannon has 400 times the mass of the iron cannonball.

By the way, this isn't the correct answer, it's just an example of what an answer would look like!

b. Use momentum conservation and your answer from part a. to calculate the velocity of the stone cannonball. The stone cannonball's mass is a quarter of the mass of the iron cannonball.

Don't worry about calculating the range yet – you'll do that next.

Sharpen your pencil Solution

Iron cannonball: Cannon velocity = 0.126 m/s, Cannonball velocity = 90.0 m/s

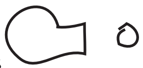
Stone cannonball: Cannon velocity = 0.063 m/s.

a. Use momentum conservation to calculate m_c the mass of the cannon, in terms of m_i the mass of the iron cannonball.

Iron cannonball

$m_c = ?$

$v_c = -0.126$ m/s



$m_i = ?$

$v_i = 90.0$ m/s

$$p = m_c v_c + m_i v_i = 0$$

$$m_c v_c = -m_i v_i$$

$$m_c = \frac{-m_i v_i}{v_c}$$

$$m_c = \frac{-m_i \times 90v_i}{-0.126}$$

$$m_c = 714 m_i \text{ (3 sd)}$$

Sometimes you'll be asked to do a problem where your final answer isn't a number. That's OK.

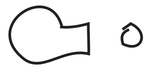
Don't put units on an answer like this. The units of m_c will depend on the units that m_i is measured in.

b. Use momentum conservation and your answer from part a. to calculate the velocity of the stone cannonball. The stone cannonball's mass is a quarter of the mass of the iron cannonball.

Stone cannonball

$m_c = 714 m_i$

$v_c = -0.063$ m/s



$m_s = \frac{m_i}{4} = 0.25 m_i$

$v_s = ?$ m/s

$$p = m_c v_c + m_s v_s = 0$$

You don't know actual values for any of the masses – but the terms in the momentum conservation equation all mention masses.

$$m_s v_s = -m_c v_c$$

$$v_s = \frac{-m_c v_c}{m_s}$$

If you know what all the masses are relative to each other, for example in terms of m_i, all mention of mass in your equation will divide out and cancel.

Make substitutions for m_c and m_s in terms of m_i.

$$v_s = \frac{-714 m_i v_c}{0.25 m_i}$$

$$v_s = \frac{-714 \times (-0.063)}{0.25}$$

$$v_s = 180 \text{ m/s (2 sd)}$$

Even though we **didn't know** the masses of the cannonballs, we could still do the question because the masses we didn't know divided out and cancelled.

Try the variables from your sketch in equations to see what happens

Often, you won't know how a problem's going to work out until you start writing down equations and playing with them.

So don't get stressed if you don't know many values at the start. Do your sketch, write down everything you know, and jot down **how** you'd do the problem and which equation(s) you'd use.

Then play with the equations! You'll often find that variables cancel and the equation simplifies down to a calculation you're able to do.

If you don't know values for the variables you're working with, play with the equations. Some of the variables may divide out and cancel.

You've calculated the stone cannonball's velocity...

Although the cannon doesn't always exert the same force on a cannonball, **momentum** is always conserved on every occasion that the cannon is fired.

You've just used **momentum conservation** twice. The first time was for firing the iron cannonball, which allowed you to calculate the **mass** of the cannon. The second time was for the stone cannonball, which allowed you to calculate the **velocity** of the stone cannonball: 180 m/s.

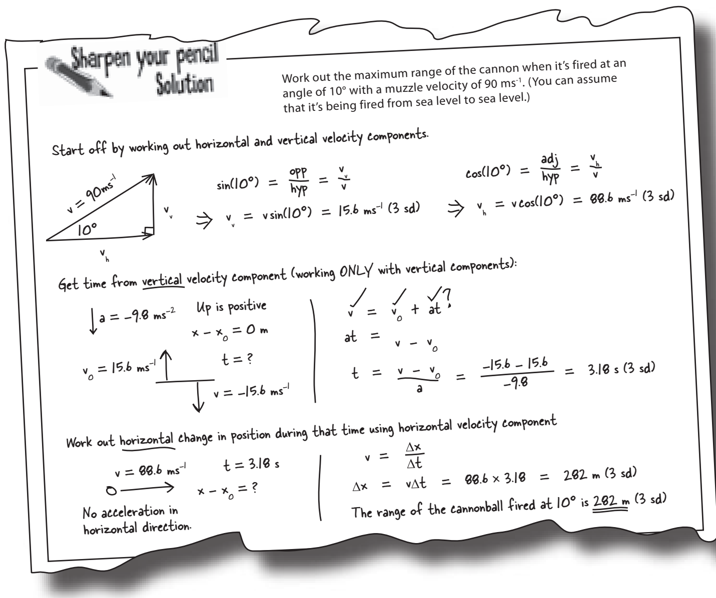
Momentum conservation allows you to calculate masses or velocities that you don't already know.

That's great and all, but we want to know the **range** of the stone cannonball.

... but you want the new range!

You already know that you can work out the cannonballs' **range** if you know its **velocity** - you already did that for the iron cannonball.

But that was a long, involved calculation which took you quite a while.



Sharpen your pencil Solution

Work out the maximum range of the cannon when it's fired at an angle of 10° with a muzzle velocity of 90 ms^{-1}. (You can assume that it's being fired from sea level to sea level.)

Start off by working out horizontal and vertical velocity components.

$v = 90 ms^{-1}$, $10°$, v_v, v_h

$\sin(10°) = \frac{opp}{hyp} = \frac{v_v}{v}$

$\Rightarrow v_v = v\sin(10°) = 15.6\ ms^{-1}$ (3 sd)

$\cos(10°) = \frac{adj}{hyp} = \frac{v_h}{v}$

$\Rightarrow v_h = v\cos(10°) = 88.6\ ms^{-1}$ (3 sd)

Get time from vertical velocity component (working ONLY with vertical components):

$a = -9.8\ ms^{-2}$ Up is positive

$x - x_0 = 0\ m$

$v_0 = 15.6\ ms^{-1}$

$t = ?$

$v = -15.6\ ms^{-1}$

$v = v_0 + at$

$at = v - v_0$

$t = \frac{v - v_0}{a} = \frac{-15.6 - 15.6}{-9.8} = 3.18$ s (3 sd)

Work out horizontal change in position during that time using horizontal velocity component

$v = 88.6\ ms^{-1}$

$t = 3.18$ s

$x - x_0 = ?$

No acceleration in horizontal direction.

$v = \frac{\Delta x}{\Delta t}$

$\Delta x = v\Delta t = 88.6 \times 3.18 = 282$ m (3 sd)

The range of the cannonball fired at 10° is 282 m (3 sd)

This is what you did to work out the iron cannonball's range.

Wouldn't it be dreamy if you could work out the new range without having to redo the whole calculation all over again with different numbers. But I know it's just a fantasy...

You used subscripts to keep track of which component is which.

Use proportion to work out the new range

Your earlier calculation to work out the range of the iron cannonball had three parts:

Part a: Work out the vertical and horizontal **components** of the initial velocity, $\mathbf{v}_{0v}$ and $\mathbf{v}_{0h}$.

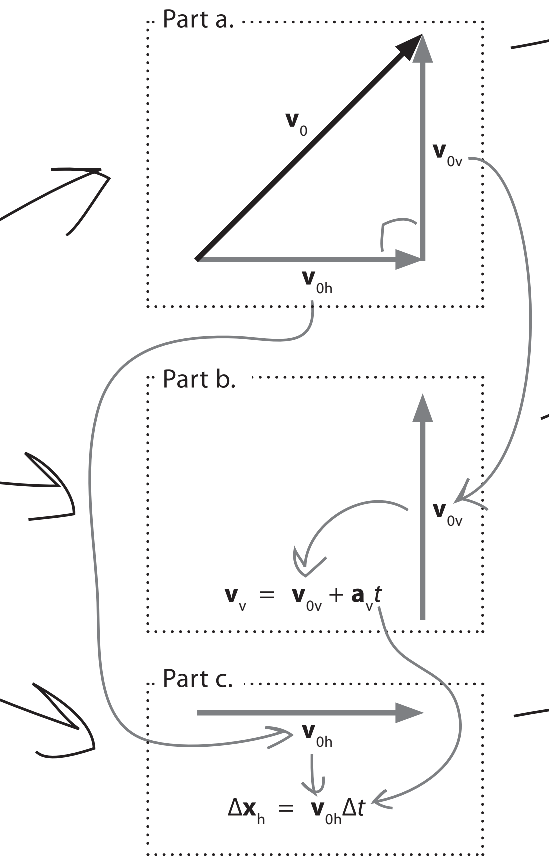
Part b: Use the **vertical component** and the equation $\mathbf{v}_v = \mathbf{v}_{0v} + \mathbf{a}_v t$ to work out the **time** the cannonball is in the air for.

Part c: Use the **horizontal component** and the equation $\Delta\mathbf{x}_h = \mathbf{v}_{0h}\Delta t$ to work out the cannonball's horizontal displacement in that time.

You've calculated that the stone cannonball's initial velocity $\mathbf{v}_0$ is two times greater than the iron cannonballs' velocity.

So rather than doing the entire calculation again with a different value for $\mathbf{v}_0$, you can use **proportion** to work out the new range ...

Sharpen your pencil



a. If the initial velocity is 2 times greater than before, how many times greater are its horizontal and vertical components?

b. The vertical component of the velocity is used in the equation $\mathbf{v} = \mathbf{v}_0 + \mathbf{a}t$ to work out the time in the air. How many times greater is the time in the air with the new cannonball?

c. The horizontal component of the velocity and the time in the air are used in the equation $\Delta \mathbf{x}_h = \mathbf{v}_h \Delta t$ to work out the range. How many times greater is the range with the new cannonball?

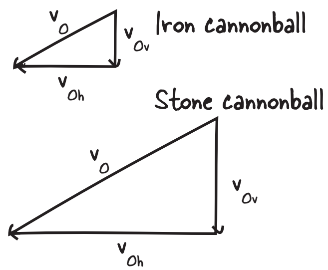
The range is the same as the horizontal component of the displacement.

d. The range of the iron cannonball fired at 10° is 282 m. What is the range of the stone cannonball when it's fired at 10°?

Sharpen your pencil Solution

a. If the initial velocity is 2 times greater than before, how many times greater are its horizontal and vertical components?

The angle is still the same, so they're similar triangles. If one side is two times longer, the other sides are too.



Both components are 2 times greater than before.

b. The vertical component of the velocity is used in the equation $\mathbf{v}_v = \mathbf{v}_{0v} + \mathbf{a}_v t$ to work out the time in the air. How many times greater is the time in the air with the new cannonball?

Both v_v and v_{0v} are 2 times larger. When you rearrange the equation to say "t = something" it becomes $t = \frac{v_v - v_{0v}}{a_v}$

When you're adding or subtracting (which you are before you divide by a_v) and the numbers you're dealing with become two times larger, your answer also becomes two times larger.

c. The horizontal component of the velocity and the time in the air are used in the equation $\Delta \mathbf{x}_h = \mathbf{v}_h \Delta t$ to work out the range. How many times greater is the range with the new cannonball?

Both v_h and t are 2 times greater then before.

As they're multiplied together to give Δx_h, this means that Δx_h is 4 times greater than before, as $2 \times 2 = 4$.

d. The range of the iron cannonball fired at 10° is 282 m. What is the range of the stone cannonball when it's fired at 10°?

New range is 4 times greater than old range.

New range $= 4 \times 282 =$ <u>1130 m</u> (3 sd)

there are no Dumb Questions

Q: What's the 'similar triangles' thing again? It rings a bell ...

A: If two triangles have identical angles, then the ratios of their sides are equal. You used that before to work out sin, cos, and tan.

Here, if you make the velocity two times larger, then the whole triangle also becomes two times larger - as do the components.

Q: Oh yeah. But what's this bit about adding rather than multiplying v_v and v_{0v}?

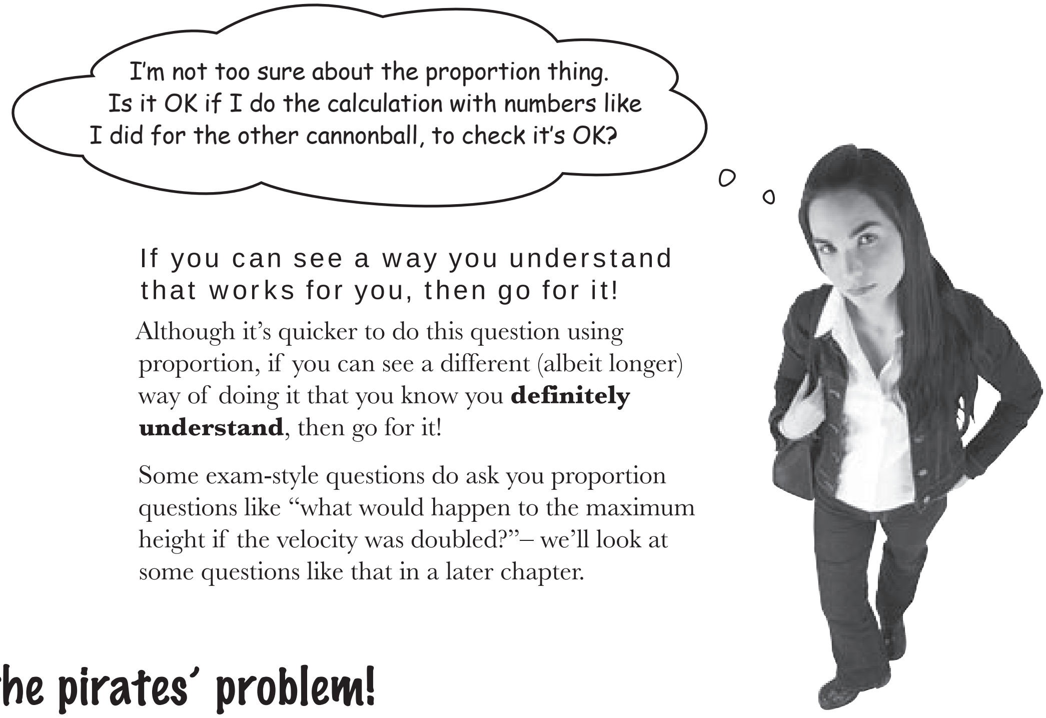
A: The rearranged equation is $t = \frac{v_v - v_{0v}}{a_v}$

Since $v_{0v} = -v_v$ (they have the same size but are in opposite directions), the bit on top of the fraction is 2 times greater than it was before. And since a is constant, *t* is also 2 times greater than before.

Q: What about part c?

A: That equation is $\Delta x_h = v_h \Delta t$. You're now dealing with the horizontal component of the velocity, v_h, which is two times greater than before. And the time, *t*, is also two times greater than before. So the term $v\Delta t$ is four times greater, as 2×2 = 4.

Using <u>proportion</u> to work out the answer to a question that's similar to one you already did can be a good shortcut.



If you can see a way you understand that works for you, then go for it!

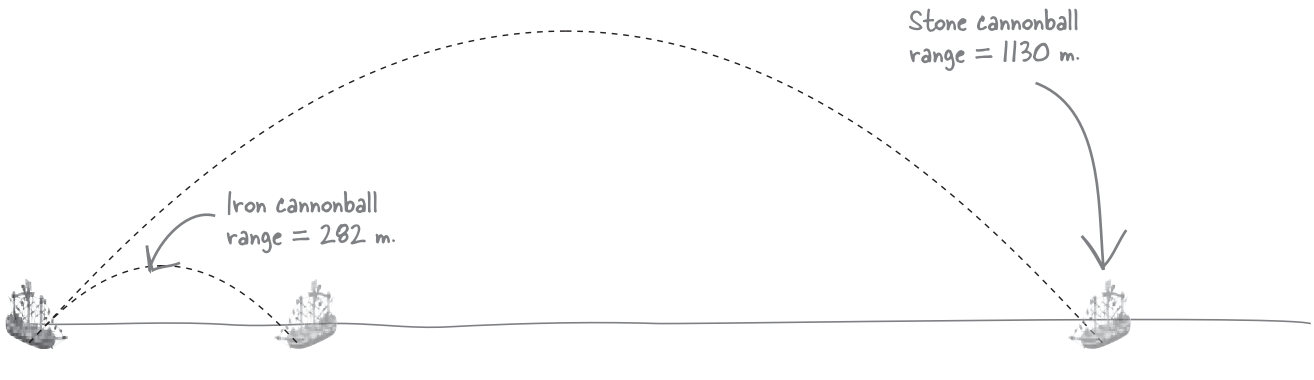
Although it's quicker to do this question using proportion, if you can see a different (albeit longer) way of doing it that you know you **definitely understand**, then go for it!

Some exam-style questions do ask you proportion questions like "what would happen to the maximum height if the velocity was doubled?"– we'll look at some questions like that in a later chapter.

You solved the pirates' problem!

The Sieges-R-Us hype is justified! The **vertical component** of the stone cannonball's velocity is **two** times greater than for the iron cannonball, so the **time** in the air is **two** times greater as well. If the horizontal component of the velocity was the same for both cannonballs, then the stone cannonball would go two times as far (as it's in the air for two times as long).

But the **horizontal component** of the stone cannonball's velocity is also **two** times greater than the horizontal component of the iron cannonballs' velocity. So the stone cannonball's **range** is a massive **four** times further than the iron cannonball's!

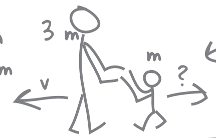


So the ghost ship has to stay 4 × 282 m = 1130 m away - where it doesn't cause problems any more. **Another job well done!**

Question Clinic: The "Proportion" Question (often multiple choice)

You will sometimes come across questions, especially in a multiple choice exam, which don't give you numerical values. Instead they may say that something is 'three times the mass' or has 'double the momentum' of something else. These are designed to test your understanding of the physics, rather than your ability to press buttons on a calculator.

Start with a sketch! Then you'll spot that this question is about momentum conservation.



Write what you know on your sketch.

This tells you that you can ignore the effect of friction.

2. An adult and a child are on an ice rink. The child has mass m and the adult has mass 3 m. The child pushes the adult, and as a result the adult slides backwards with velocity v. What is the size of the child's velocity?

a. v

b. $\frac{v}{3}$

c. 3v

d. $\frac{v}{2}$

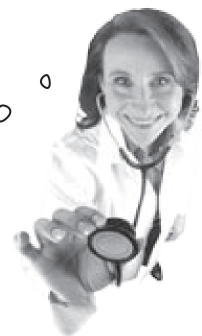
e. 9v

This tells you that it's a proportion question, so don't worry about not knowing any values.

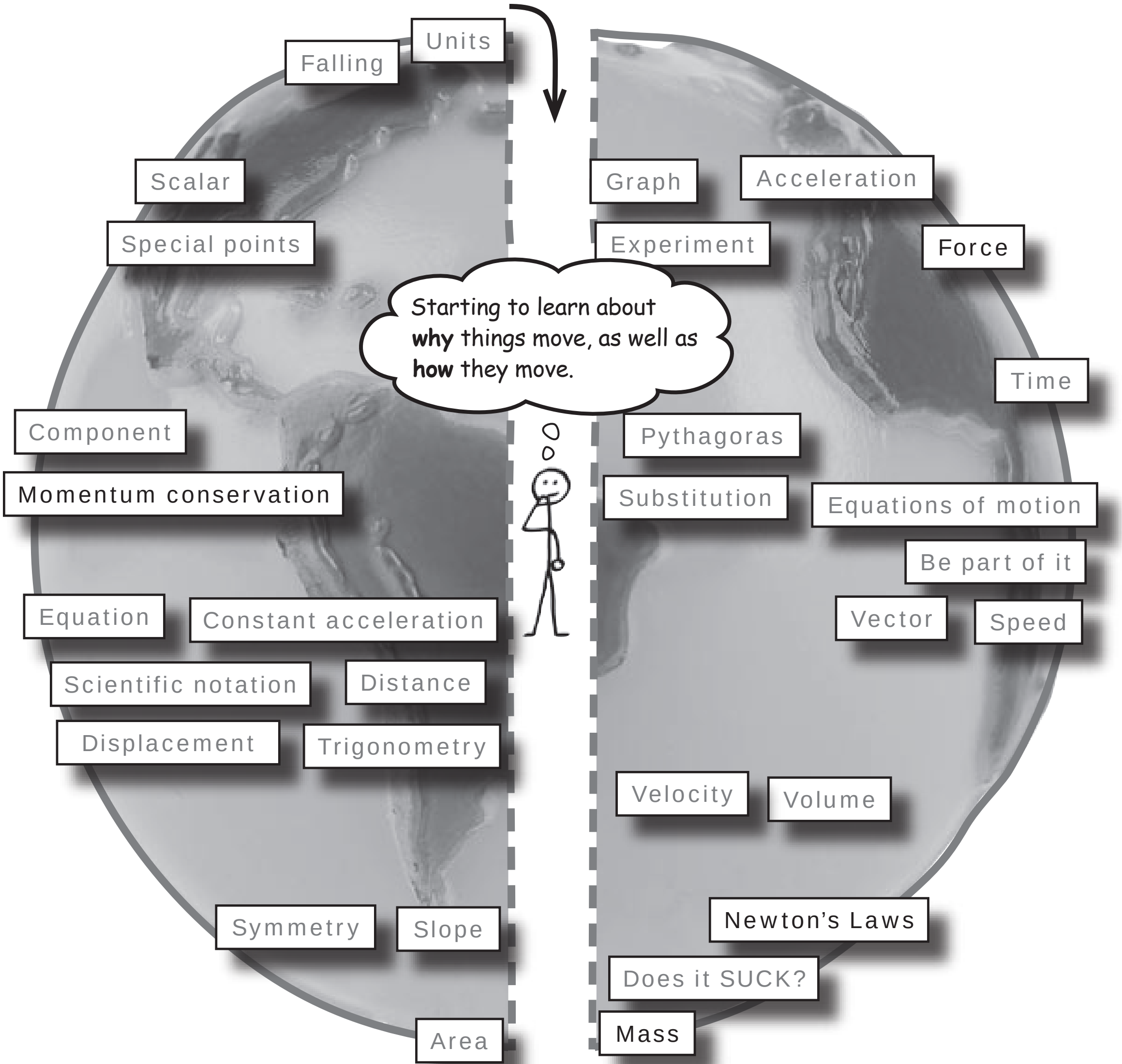
The child has a lower mass, so it will go faster than the adult when they push with the same force. So you can already eliminate any answer where the child is going slower than the adult, or at the same speed.

Velocity is a vector, but this question only wants the size, not the direction.

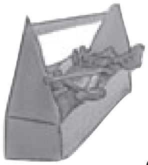
The adult and child experience the same force for the same amount of time (Newton's 3rd Law). So mass × velocity should have the same size for the adult and the child. As the child is a third of the mass of the adult, the child's velocity needs to be three times greater than the adult's velocity.



The 'proportion' question may look like it's from a 'weird' part of physics - for instance a question about an atomic nucleus that splits into two parts. The key is to see past the story - **start with a sketch** - what is it **like**? When you do this, you'll realize that it's the same as two things being pushed apart and you can use momentum conservation to solve the problem.



Mass	The amount of 'stuff' that something is made from.
Momentum conservation	Momentum is mass × velocity. When two (or more) objects interact, the total momentum is the same both before and after.
Force	Something that causes a change in an object's momentum, for example a push or a pull (though there are also other types of forces).
Newton's Laws	Three laws that tell you how objects move in the presence (or absence) of forces.



Your Physics Toolbox

You've got Chapter 10 under your belt and added some problem-solving skills to your ever-expanding toolbox.

Vary one thing at a time

If you're doing an experiment, you should only vary one thing at a time.

This means that you can know for certain what has caused any change to your results that you observe.

Newton's 1st law

An object continues with its current velocity unless it's acted on by a force.

In everyday life, things appear to slow down as time passes, but that's because of the force of friction.

Proportion

If you have an equation, you can work out what happens to the answer you get if you change one of the variables by a certain amount (e.g., doubling, tripling or halving it).

If you've already used the equation, this can be faster than redoing the same calculation with different values.

Momentum conservation

Momentum is mass × velocity, $p = mv$

Momentum is a vector that has the same direction as an object's velocity vector.

The same force always causes the same change in momentum

If two (or more) objects interact, their total momentum is the same both before and after the interaction. You can work out the total momentum by adding the momentum vectors 'nose-to-tail.'

Newton's 3rd law

When two objects interact, each experiences an equal force, but in opposite directions.

This is a direct consequence of momentum conservation. As the same force always causes the same change in momentum, and momentum is conserved, then the two objects must experience equal (but opposite) forces so that the total momentum is the same before and after.

Newton's 2nd law is in chapter 11!

11 weight and the normal force

Forces for courses

Sometimes you have to make a statement forcefully.

In this chapter, you'll work out **Newton's 2nd Law** from what you already know about momentum conservation to wind up with the key equation, $\mathbf{F}_{net} = m\mathbf{a}$. Once you combine this with spotting **Newton's 3rd Law force pairs**, and drawing **free body diagrams**, you'll be able to deal with (just about) anything. You'll also learn about why mass and **weight** aren't the same thing, and get used to using the **normal force** to support your arguments.

WeightBotchers are at it again!

WeightBotchers claim that their new product guarantees instant weight loss. It's been difficult to miss their brash advertisements since the campaign launched last week.

But the TV show FakeBusters doesn't buy those claims, and want you to investigate the details. If you can prove that the WeightBotchers machine is a hoax, your work will be featured in a special episode in their next series.

Memo

From: FakeBusters

Re: WeightBotchers

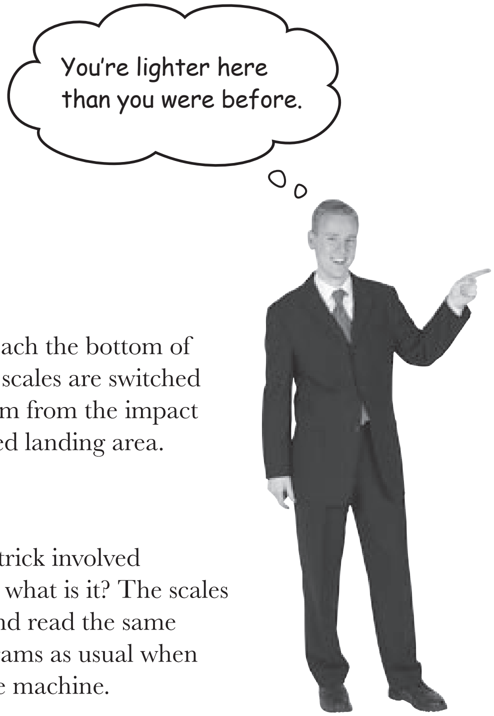
We're planning to include a 10 minute slot in our next series looking into the claims made by WeightBotchers about their latest product.

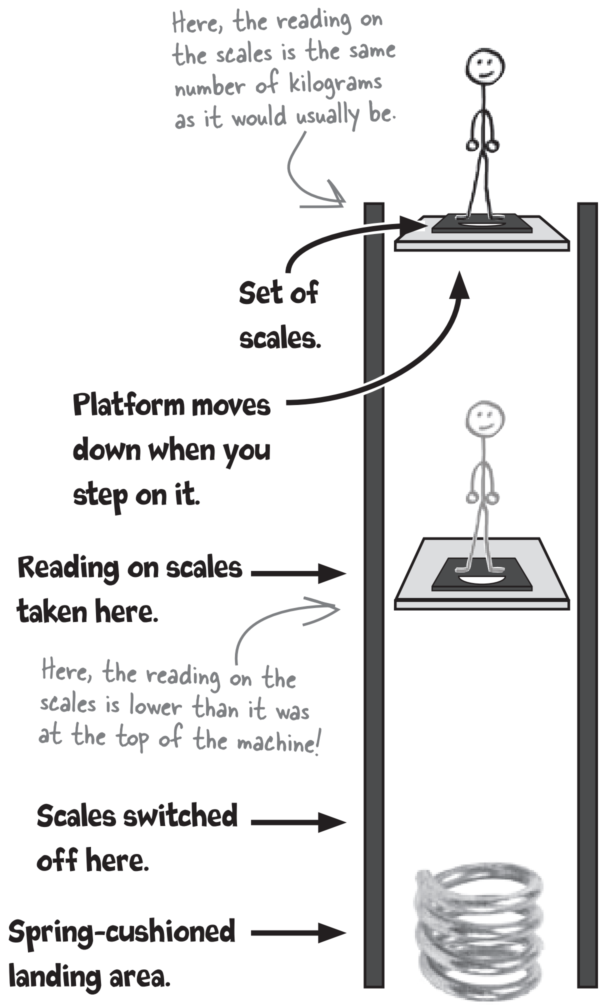
If you can bust the fake with physics, we'd love to have you on our show.

Is it really possible to lose weight instantly?!

Here's the deal. The machine has a platform at the top with some scales on it. When you stand on the scales, they read the same number of kilograms as they usually would in your bathroom. No surprises there.

But then the platform you're standing on suddenly moves downwards - and the reading on the scales becomes lower. Numbers don't lie - so if the reading's gone down then you must have lost weight. Right?





Just before you reach the bottom of the machine, the scales are switched off to protect them from the impact with the cushioned landing area.

There must be a trick involved somewhere... but what is it? The scales don't look fake and read the same number of kilograms as usual when they're not on the machine.

So maybe it's something to do with what the machine does and **how the scales produce a measurement.**

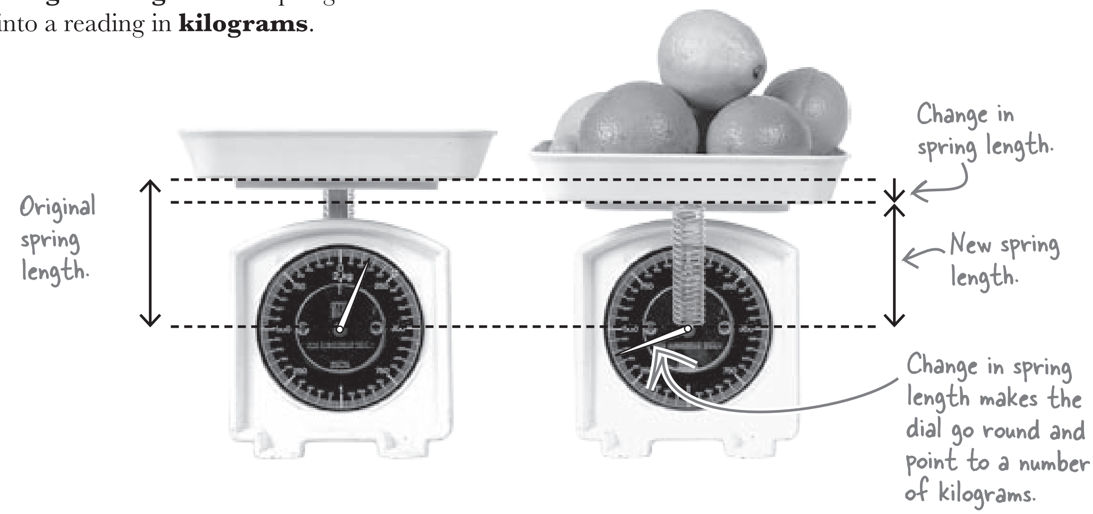
BRAIN POWER

How do you think the machine works?

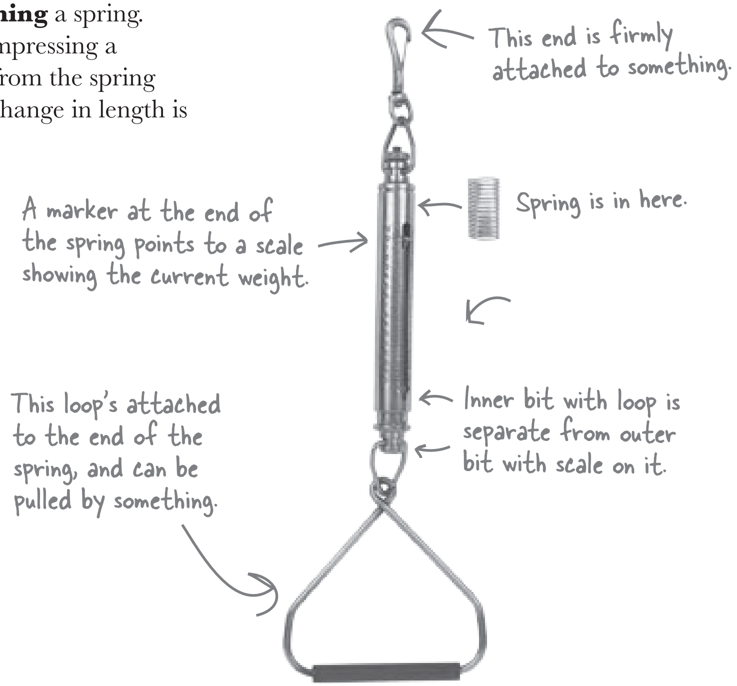
(How do scales actually produce a **measurement**?)

Scales work by compressing or stretching a spring

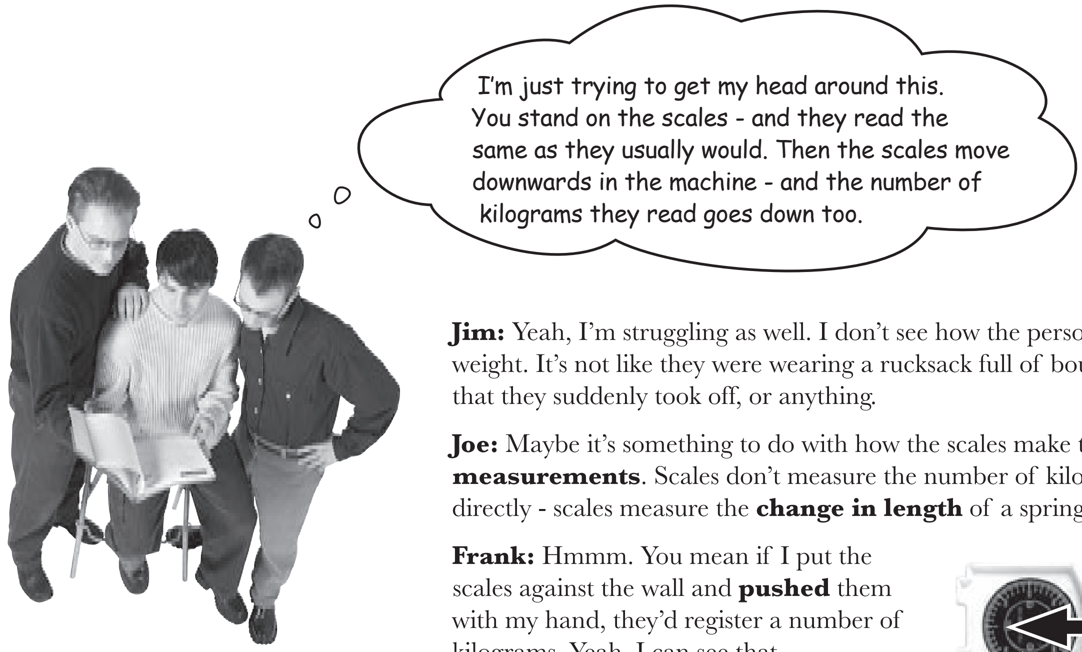
Some scales work by **compressing** a spring. If you put pieces of fruit on top that are all more or less the same size, the spring will compress by the same amount each time you add another piece. The **change in length** of the spring is **converted** into a reading in **kilograms**.



Another type of scales works by **stretching** a spring. This is exactly the same principle as compressing a spring, except that you hang an object from the spring rather than putting it on top. Again, a change in length is converted into a reading in kilograms.



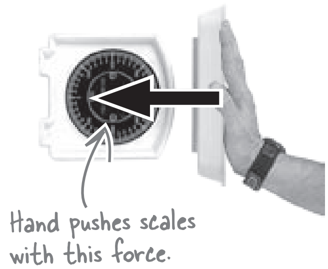
A spring will always compress/stretch by the same amount for the same load, to give a consistent reading.



Jim: Yeah, I'm struggling as well. I don't see how the person's lost weight. It's not like they were wearing a rucksack full of boulders that they suddenly took off, or anything.

Joe: Maybe it's something to do with how the scales make their **measurements**. Scales don't measure the number of kilograms directly - scales measure the **change in length** of a spring.

Frank: Hmmm. You mean if I put the scales against the wall and **pushed** them with my hand, they'd register a number of kilograms. Yeah, I can see that.



If you know HOW your measuring devices work, you can trouble-shoot your experiments when unexpected things happen.

Jim: That's weird. **Kilograms** are units of **mass**, right? Mass is the amount of 'stuff' something's made from. But if you push the scales sideways like that, the reading depends on the **force** that you push with, not on the amount of stuff your hand's made from.

Joe: I guess that's because the scales don't really measure kilograms directly - they measure the **change in the length** of the spring. And that must depend on the **force** that the spring's pushed with.

Frank: If I'm standing on the scales, I'm kinda pushing down on the spring inside them because of gravity. I guess that because of gravity, a certain number of kilograms must produce a certain force - and a certain change in length of the spring. So the scales always assume that you're standing on them when making a measurement.

Joe: So if you use the scales differently from how the manufacturer intended - by pushing them against the wall, or perhaps by making them move down like the WeightBotchers machine does - then you get a flakey reading.

Frank: That sounds plausible - but personally I'd like to get my head around how force (which seems to do with pushing) and mass (which is to do with the 'stuff' you're made of) are connected ...

Mass is a measurement of "stuff"

Mass is an indication of how much 'stuff' something is made from, and is measured in **kilograms**. Mass is a **scalar**, as 'stuff' can't have a direction - it's just what's there.

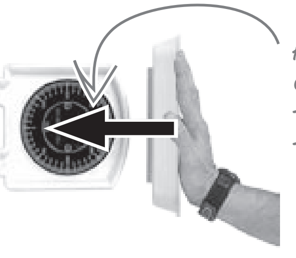
Even though the scales indicate otherwise, the person on the WeightBotchers machine always has the same mass - it's not like they took off a rucksack or had a haircut halfway down and lost a whole lot of matter.

MASS is how much "stuff" something's made of. It's a scalar, because 'stuff' doesn't have a direction.

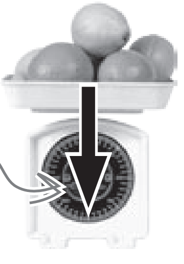
Weight is a force

If you put the scales against the wall, you can exert a horizontal **force** on them by pushing them with your hand and compressing the spring.

Force is a **vector** because it has direction - the direction that you're pushing the spring in.



Applying a force in this direction compresses the spring and makes the dial go round.



The force vector of the fruit's weight points in this direction.

Although the scales give a reading in kilograms, they actually make measurements based on the **change in length** of the spring. So if you put fruit on the scales and the spring's length changes, there must be a force involved.

The change in length comes about because the spring has to counteract the fruit's **weight**, which is there because the fruit is in the earth's gravitational field. The fruit's weight is the **force** exerted on it by the earth's gravitational pull. You can draw the fruit's weight as a force vector arrow pointing down, towards the center of the earth.

WEIGHT is the FORCE you experience as a result of being in a gravitational field.

As weight is a force, this is a force vector.

On Earth, your weight vector points down, towards the center of the earth.

But people say things like "I weigh 60 kilograms" all the time. How can you say that mass and weight are different?

Mass and weight are different!

In everyday speech, people use the words "mass" and "weight" like they're the same thing. But in physics we need use these words more carefully.

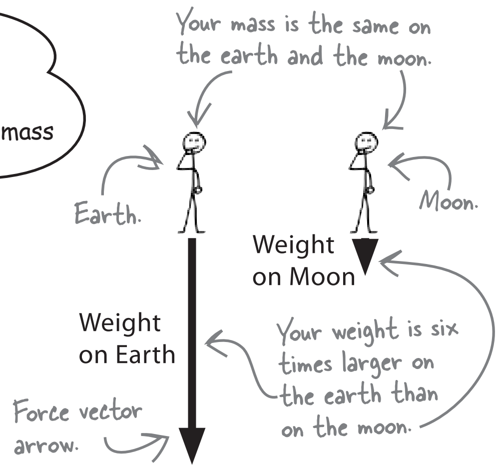
Mass. Stuff. Scalar.

If you go to the moon, your **mass** is the same number of kilograms as it is on earth, as you're still made from the same amount of 'stuff'.

Weight. Force. Vector.

But **weight** is the force you experience as a result of being in a **gravitational field**. And as the moon's gravitational field is smaller than the earth's, your weight is less on the moon than it is on Earth even though your mass is still the same.

So the scales measure the force it takes to compress a spring, then **convert** the force that causes a certain change in spring length into kilograms? It sounds like the **relationship** between mass and weight is really important here.



The way that the scales convert a force into a reading in kg is crucial.

If you stand on the scales on the moon, the scales will read the wrong number of kilograms - even though your mass hasn't changed. This is because the scales assume you're on earth when they **convert** the change in length (as a result of an applied force) into a reading in kilograms.

If you can work out the **relationship between force and mass** that the scales use to do this conversion, you'll be able to debunk the WeightBotchers machine.

BRAIN POWER

Have you seen an equation that involves both **force** and **mass** somewhere before?

The relationship between force and mass involves momentum

In chapter 10, you figured out that when you apply the same **force** for the same amount of **time** to any object, you always give it the same **change in momentum**. As long as there are no other forces acting on the object, you can write this as an equation:

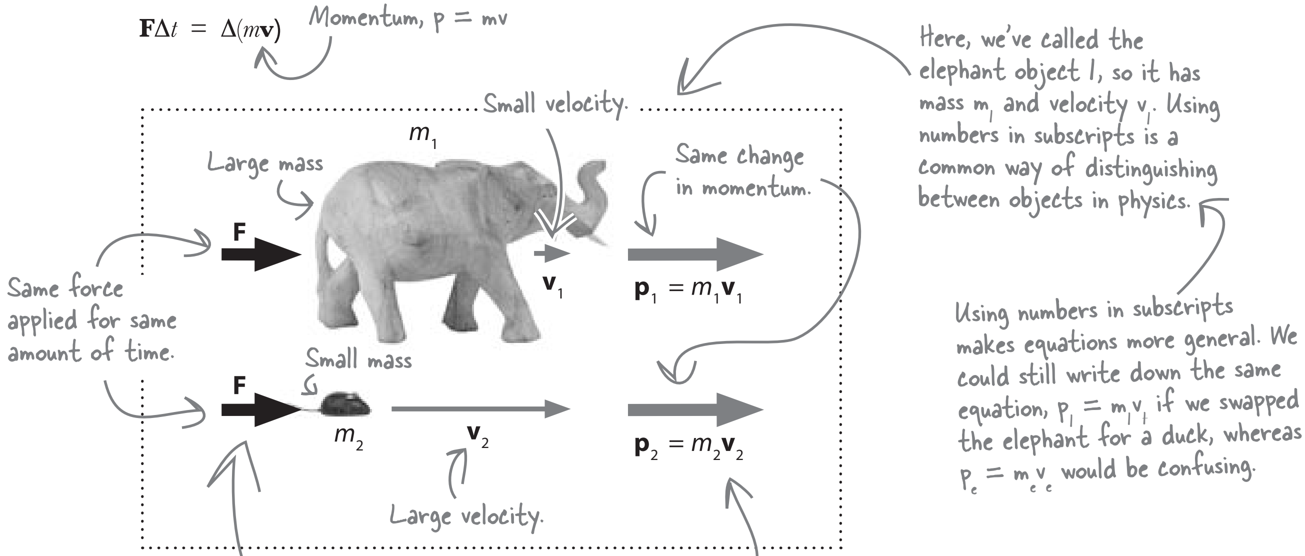
$$\mathbf{F}\Delta t = \Delta\mathbf{p}$$

Force applied ...
... for a period of time
... gives a change in momentum.

But momentum is mass × velocity. So you can substitute in $m\mathbf{v}$ every time you see a **p** and rewrite this equation as:

$$\mathbf{F}\Delta t = \Delta(m\mathbf{v})$$

Momentum, p = mv



$$F\Delta t = \Delta(mv)$$

This equation works if F is the only force acting on the object.

Momentum, p = mv

This equation works for ANY object. We don't have a specific object in mind here, so there are no subscripts on the 'm' or 'v'.

This is **Newton's Second Law**. It shows that objects with more mass have more **inertia**, or more resistance to changing how they're currently moving. If you apply the same force for the same time to push two different objects, the object with the larger mass is more 'resistant to change' and has a smaller change in velocity at the end.

The equation $\mathbf{F}\Delta t = \Delta(m\mathbf{v})$ gives you a **relationship between force and mass** that you can use to work out what's going on with the WeightBotchers machine.

But sometimes you push something with a force and it stays still. Where's the change in momentum there?

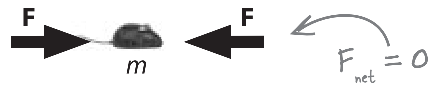
Newton's Second Law:
If you apply a NET force to any object for a period of time, the change in the object's momentum always has the same value.

$$F_{net}\Delta t = \Delta(mv)$$

This equation works for any number of forces acting on the object added together to make the net force, F_{net}.

It's the net force that matters.

Two people pushing the mouse with equal forces in opposite directions looks like this:



When you add together these force vectors by lining them up 'nose to tail', the overall, or **net** force you end up with is zero. And the mouse doesn't go anywhere, so its momentum doesn't change.

But if the left-to-right force became larger, it would start to 'overpower' the right-to-left force, and there'd be a net force to the right. So the mouse would start moving to the right - its momentum would change in the direction of the net force.

Sharpen your pencil

a. After introducing a subscript to make it clear that it is the **net force** that causes the change in momentum, the equation on the opposite page, $\mathbf{F}_{net}\Delta t = \Delta(m\mathbf{v})$ can be rearranged to say $\mathbf{F}_{net} = \frac{\Delta(m\mathbf{v})}{\Delta t}$ Use this equation to work out the units of force.

b. Your equation contains the term $\frac{\Delta(m\mathbf{v})}{\Delta t}$. Do both m and $\mathbf{v}$ change with time while a force is applied? (Assume that the situation is one where an elephant or mouse has been pushed with a net force.)

c. Does your answer to part b give you any ideas about how you might simplify your equation $\mathbf{F}_{net} = \frac{\Delta(m\mathbf{v})}{\Delta t}$

Hint: What other equations do you know where a variable changes with time?

Sharpen your pencil Solution

a. After introducing a subscript to make it clear that it is the **net force** that causes the change in momentum, the equation on the opposite page, $\mathbf{F}_{net}\Delta t = \Delta(m\mathbf{v})$ can be rearranged to say $\mathbf{F}_{net} = \frac{\Delta(m\mathbf{v})}{\Delta t}$
Use this equation to work out the units of force.

$[m] = kg \quad [t] = s \quad [F] = \frac{kg.m/s}{s} = kg.m/s^2$

$[v] = m/s$

If you say this out loud, it's: "kilogram-meters per second squared"

b. Your equation contains the term $\frac{\Delta(m\mathbf{v})}{\Delta t}$. Do both *m* and **v** change with time while a force is applied? (Assume that the situation is one where an elephant or mouse has been pushed with a net force.)

The velocity changes but the mass doesn't change.

c. Does your answer to part b. give you any ideas about how you might simplify your equation $\mathbf{F}_{net} = \frac{\Delta(m\mathbf{v})}{\Delta t}$

You could turn it into $F = m\frac{\Delta v}{\Delta t}$ as the mass is constant.

And $\frac{\Delta v}{\Delta t}$ is the acceleration. So it could become $F = ma$.

Don't worry if you didn't spot this.

If the object's mass is constant, $F_{net} = ma$

Newton's Second Law says that if you apply a net force to an object for a period of time, then its momentum changes. So **force is the rate of change of the momentum** of an object:

$$\mathbf{F}_{net} = \frac{\Delta(m\mathbf{v})}{\Delta t}$$

Rate of change of momentum

Typically, the **mass** of an object doesn't change during the time that the force is applied. This means that *m* is constant and only **v** changes with time. And you already know that $\frac{\Delta \mathbf{v}}{\Delta t}$ is the rate of change of velocity - in other words, the **acceleration**.

So you can rewrite Newton's Second Law as:

$$\mathbf{F}_{net} = m\mathbf{a}$$

This shows you that the units of force are $kg.m/s^2$. However, as this is a rather unwieldy unit to write out, physicists have come up with a new unit, the **Newton** (N) where $1\ N = 1\ kg{\cdot}m/s^2$.

So if you do a calculation to work out a force where the mass is in kg and the acceleration is in m/s^2, you'd write your answer as 10 N instead of 10 $kg.m/s^2$.

The form of Newton's Second Law that you'll use the most is:

$$\mathbf{F}_{net} = \mathbf{ma}$$

Net force

Mass

Acceleration

there are no Dumb Questions

Q: So why not just say "$F_{net} = ma$" from the start? Why all this stuff about momentum first?

A: This book is about understanding physics. Rather than nodding and accepting "$F_{net} = ma$" with no reason for it, we went back to what you discovered about momentum in chapter 10, when you used a force to change the momentum of various objects. You've just used what you already knew about momentum to work out this form of Newton's Second Law for yourself.

Q: Won't the mass of an object always be constant? So you can always use $F_{net} = ma$?

A: Sometimes, both the mass and velocity of an object can change. For example, a rocket going into space carries a large mass of fuel, which it continually burns. As time goes on, its velocity gets larger, but its mass gets smaller as the fuel gets used up. So both the mass and velocity change with time, which means that you'd need to treat the $\Delta(mv)$ part of the equation $F_{net}\Delta t = \Delta(mv)$ differently.

But you don't need to worry about this too much, since it's not the part of the physics that we'll cover in this book.

Q: If an object's mass stays the same, you can say $F_{net} = ma$. But if its mass changes, you have to say $F_{net}\Delta t = \Delta(mv)$?

A: Yes. The equation $F_{net}\Delta t = \Delta(mv)$ works for **any** object, whether its mass is constant or not.

The equation $F_{net} = ma$ only works for an object whose mass is constant.

Q: But how do I know which equation to use?

A: If you're interested in the object's **velocity** or **momentum** rather than its acceleration, $F_{net}\Delta t = \Delta(mv)$ is the most useful form of Newton's Second Law.

If you're interested in the object's **acceleration**, then $F_{net} = ma$ its the most useful form of Newton's Second Law (as long as the mass of the object is constant).

g is the gravitational field strength.

On earth, $g = 9.8\ m/s^2$

Different physics courses use slightly different values for g. AP Physics uses $9.8\ m/s^2$

Weight = mg

Weight is the force that causes an object to accelerate when it falls.

If you drop an apple, it **accelerates** at a rate of $9.8\ m/s^2$. This is because the earth's gravitational field strength is $9.8\ m/s^2$. You now know that for something to accelerate, a **net force** must act on it.

The only force acting on the falling apple is its **weight**. You can think of this as a **gravitational force** which results from the stuff that the earth's made of and the stuff that the apple's made of attracting each other.

Even when the apple isn't falling, it's still subject to the same gravitational force, so it still has the same weight - its mass × the gravitational field strength, or $m\mathbf{g}$ (we use the letter **g** to represent the gravitational field strength).

Jim: Right - and my **weight** is due to the "stuff" I'm made of and the "stuff" the earth's made of attracting each other. So we can think of weight as being a **gravitational force**.

Joe: Yeah, your weight is the reason you **accelerate** towards the ground at 9.8 m/s^2 when there's nothing to support you. And $\mathbf{F}_{net} = m\mathbf{a}$, so if I have a mass of 80.0 kg, my weight must be $80 \times 9.8 = 784$ N as that's the gravitational force on me.

Practical point: Different physics courses use slightly different values for g. AP Physics uses 9.8 m/s^2 – but generally expects you to quote answers to 3 significant digits even though this value for g given in the AP table of information only has 2 significant digits.

Frank: Yeah, and if you're not accelerating, that force of your weight's still there, and is still 784 N, as **weight** = $m\mathbf{g}$. I guess that means that if my mass is constant, then the force of my weight is constant whatever's going on - my weight is still $m\mathbf{g}$.

Jim: But the force that the WeightBotchers machine measures goes down when the scales move downwards!

Jim: Yeah, that's a puzzle. The scales can't be measuring weight directly, or else they would always have the same reading. So if the scales don't measure weight, what force do the scales measure?!

Joe: I think the key thing might be that the scales on the WeightBotchers machine are accelerating towards the ground when the reading changes.

Frank: But why would that change the reading?

Joe: I guess that the scales aren't **supporting** you as much as they were before they started to move.

Jim: Yeah ... when you stand on the scales, the spring inside the scales compresses until it provides enough force to support you - to stop you moving down any further. And it's the compression of the spring that the scales measure.

Joe: Yeah, the scales measure the support force!

Frank: So if the scales aren't totally supporting your weight, the reading would be less?

Joe: I think that's probably right.

If the scales (or the earth) didn't provide a support force, you'd just keep on falling!

The scales measure the support force

Your weight is always *m***g**. If you're falling, then the force of your weight causes you to accelerate. But if you're stationary, the thing you're standing on must be exerting a **support force** on you in the opposite direction from your weight - otherwise you'd be falling!

When you stand on scales that are sitting on the ground, you compress the spring and continue to move down slightly until the spring is compressed enough to exert a support force on you that's equal to your weight. Then the net force on you is zero.

If you are stationary, the net force acting on you must be zero.

Sharpen your pencil

a. What is the weight of an 80 kg person?

b. An 80 kg person stands stationary on some scales that are sitting on the ground. What support force do the scales need to exert on the person to prevent them from breaking or falling through the scales?

c. What is the net force on the person?

d. Draw a sketch of the person showing all of the forces acting on them. Use labeled vector arrows pointing away from the person to represent the forces.

ONLY draw the person - don't draw the scales or the earth or anything else.

e. Now, imagine that the person on the scales grabs onto a couple of rings hanging from the ceiling, and pulls down on them with a force of 200 N. Draw a sketch showing all the forces acting on the person, including both the support force provided by the rings, and the support force provided by the scales.

f. What reading (in kg) do the scales show when the person is partially supported by the rings, as described in part e?

Sharpen your pencil Solution

a. What is the weight of an 80 kg person?

weight = mass × gravitational field strength

= 80.0 × 9.8

weight = 784 N

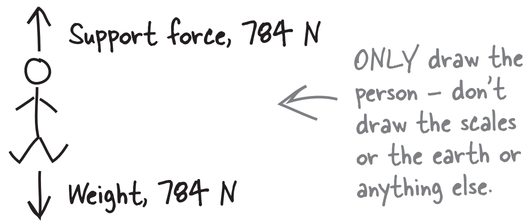
b. An 80 kg person stands stationary on some scales that are sitting on the ground. What support force do the scales need to exert on the person to prevent them from breaking or falling through the scales?

The person's weight is 784 N. So to stop them breaking the scales, the support force needs to be 784 N.

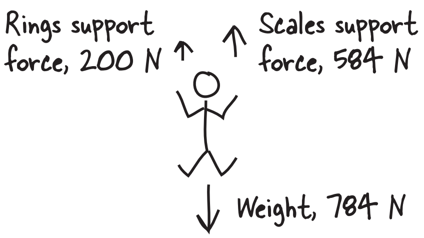
c. What is the net force on the person?

The net force is 0 N, as the weight acts downwards, the support force acts upwards, and the weight and support force are equal sizes.

d. Draw a sketch of the person showing all of the forces acting on them. Use labeled vector arrows pointing away from the person to represent the forces.



e. Now, imagine that the person on the scales grabs onto a couple of rings hanging from the ceiling, and pulls down on them with a force of 200 N. Draw a sketch showing all the forces acting on the person, including both the support force provided by the rings, and the support force provided by the scales.



f. What reading (in kg) do the scales show when the person is partially supported by the rings, as described in part e?

Scales provide a support force of 584 N and assume that F = mg

$$\Rightarrow m = \frac{F}{g} = \frac{584}{9.8}$$

$$m = 59.6 \text{ kg (3 sd)}$$

Scales measure the **support force** provided by the spring, then convert this into kilograms using **F** = *m***g** to tell you what your mass is.

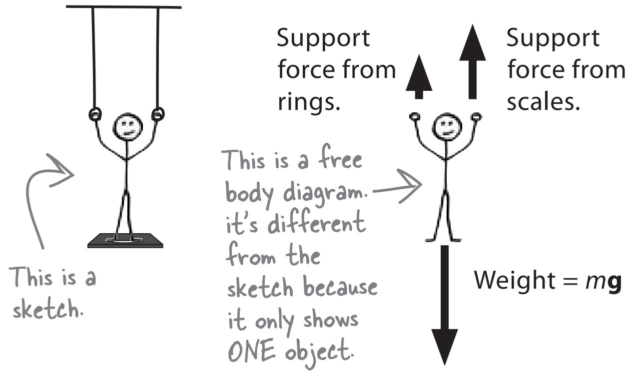
If, for some reason, the support force isn't equal to your weight, then the scales will still convert the support force into kilograms. However, this reading in kilograms will not be equal to your mass, as the support force wasn't the same size as your weight.

Scales measure the support force.

Now you can debunk the machine!

You've worked out that the scales measure the support force - and that if the support force is less than your weight, the reading on the scales will be less than your mass.

One of the tools you've used to do this is a **free body diagram**. This is just another name for a sketch of an object showing all of the forces acting on it, and it's a very useful way of analyzing forces.



A free body diagram shows an object plus all of the forces acting on it, and nothing else.

Speaking of analysis, the FakeBusters team think they've made a breakthrough! They've looked at the advertisement and worked out that the machine is accelerating downwards at a rate of 2.0 m/s^2 while the low reading is being taken.

You can use this fact to demonstrate why the WeightBotchers machine gives false readings.

A free body diagram only shows one object. So only draw the person – not the scales or the machine.

Sharpen your pencil

a. Draw a free body diagram for the person, mass *m*, on the WeightBotchers machine, inside the box to the right.

(There's no need to include the values of forces you've not calculated yet, just words to describe what they are.)

b. Newton's Second Law, $\mathbf{F}_{net} = m\mathbf{a}$, says that an object accelerates if all of the forces acting on it **add up** to a non-zero net force. The person on the machine accelerates downwards at a rate of 2.0 m/s^2. Use these facts to derive an equation for the support force from the scales.

Hint: The scales assume that they are measuring the force mg and convert that to kilograms.

c. The scales will give a reading in kg. What will this reading be?

Your answer won't be a pure number. It will still have m, the mass of the person, in it. For example 0.5 m kg means that the scales will give a reading that's half the person's mass.

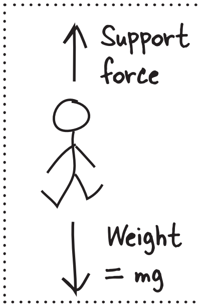
d. Explain why the reading on the scales is lower than it would be if the scales were stationary.

Be careful not to get the variable used to represent mass and the unit of meters mixed up. They are both represented by the letter 'm' (but in different contexts).

Sharpen your pencil Solution

a. Draw a free body diagram for the person, mass *m*, on the WeightBotchers machine, inside the box to the right.

(There's no need to include the values of forces you've not calculated yet, just words to describe what they are.)



b. Newton's Second Law, $\mathbf{F}_{net} = m\mathbf{a}$, says that an object accelerates if all of the forces acting on it **add up** to a non-zero net force. The person on the machine accelerates downwards at a rate of 2.0 m/s². Use these facts to derive an equation for the support force from the scales.

Make down the positive direction. ← Force is a vector, so you need to get the signs right.

$$F_{net} = ma$$

$$\Rightarrow mg - F_s = 2.0\ m$$

$$F_s = mg - 2.0\ m$$

The net force is the weight and the support force added together.

Start with Newton's 2nd Law, then substitute in.

We chose this symbol for support force.

c. The scales will give a reading in kg. What will this reading be?

Scales assume that measured F= mg, so divide support force by g to get mass in kg.

$$\text{Reading} = \frac{mg - 2.0\ m}{g}$$

$$\text{Reading} = m - \frac{2.0\ m}{g}$$

$$\text{Reading} = m - \frac{2.0\ m}{9.8} = m - 0.204\ m$$

$$\text{Reading} = 0.80\ m\ \text{kg}\ \ (2\ \text{sd})$$

This is like (1 – 0.204) m

d. Explain why the reading on the scales is lower than it would be if the scales were stationary.

As the person is accelerating downwards, there must be a net downwards force. This means that the support force (what the scales measure) is less than the person's weight.

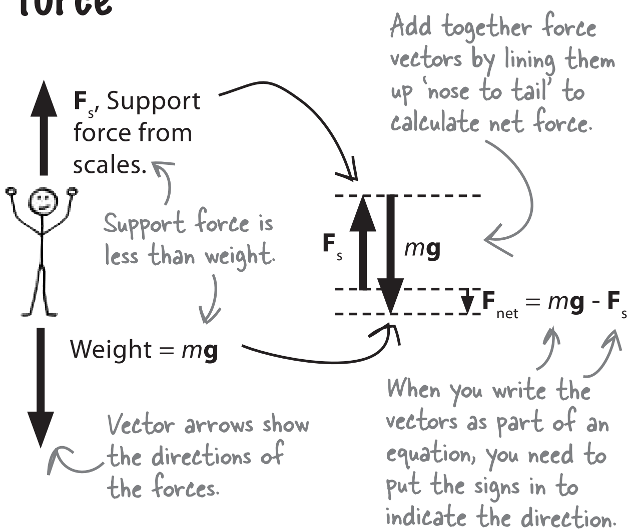
The scales are only partially supporting the person.

The machine reduces the support force

The WeightBotchers machine 'works' because scales measure the support force that the scales exert on the person that stands on them. If the support force is less than your weight, then the reading on the scales is also less then your weight.

The machine **accelerates** you downwards. This means that there must be a **net force** acting on you to produce the acceleration, as $\mathbf{F}_{net} = m\mathbf{a}$. The only force acting downwards on you is your weight, and the only force acting upwards is the support force.

So if you're accelerating downwards, the support force must be less than your weight and the scales will in turn have a lower reading - that's how WeightBotchers are doing it.



there are no Dumb Questions

Q: So the scales don't measure my mass ... and they don't even measure my weight?!

A: The scales measure the force that the spring is exerting on you, which we've called the support force here. They calculate the support force from the change in the length of the spring.

Q: So how is it possible for the scales to measure a support force that's less than my weight?

A: If the scales are only partially supporting you (for example, if you're partially pulling yourself up using rings, or you have one foot on the scales and one foot on the floor) then they will provide only part of the supporting force.

Q: So if I'm being supported by more than one thing, all of the supporting forces will add up to my weight?

A: If you're not being **accelerated** then yes, the supporting forces will add up to your weight because the net force on you is zero.

Q: Why does being accelerated change the supporting force?

A: Good question! But don't think of an acceleration 'causing' or 'changing' a force. It's the other way around - a non-zero net force causes an acceleration.

If you're accelerating, there must be a non-zero net force on you, because $F_{net} = ma$. If you're accelerating downwards, your weight (which points down) must be greater than the supporting force (which points up) to have a net downwards force that causes your acceleration.

Q: What would happen if the upwards force was larger than my weight? Like if I sat on a rocket or something?

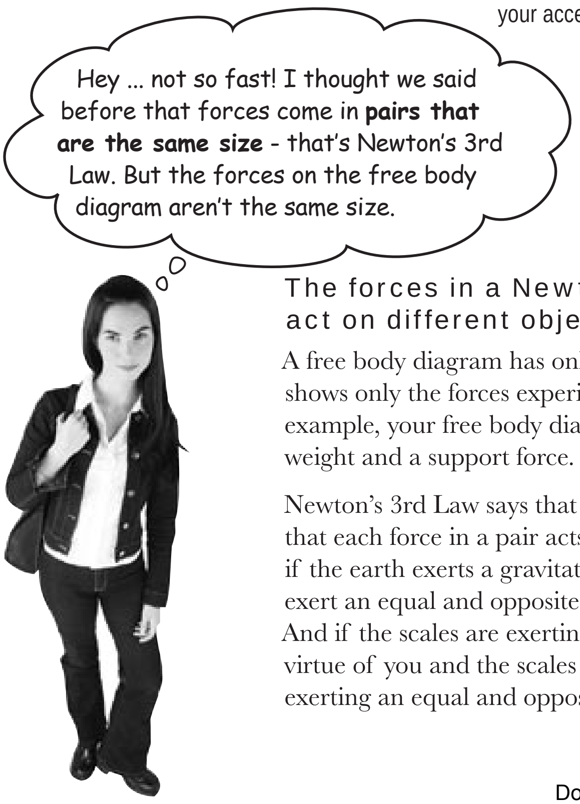
A: Then there'd be a net force upwards, so you would accelerate upwards.

Q: I've a question about free body diagrams. Why do the force vector arrows always point away from the object, even when some of them act from underneath (like the support force from scales does)?

A: If you're drawing a free body diagram with several different forces on it, this convention helps you to see at a glance which directions forces are operating in. An arrow above the object must be a force acting upwards, and so on.

Q: Now I've debunked the WeightBotchers machine, do I get to go on television?

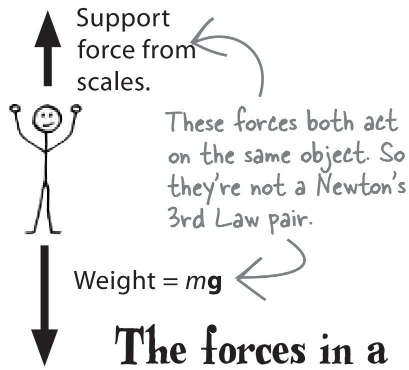
A: Well ...



The forces in a Newton's 3rd Law pair act on different objects.

A free body diagram has only **one** object in it, and shows only the forces experienced by that object. For example, your free body diagram might show your weight and a support force.

Newton's 3rd Law says that forces come in **pairs** - and that each force in a pair acts on **different** objects. So if the earth exerts a gravitational force on you, you exert an equal and opposite gravitational force on it. And if the scales are exerting a support force on you (by virtue of you and the scales being in contact), you're exerting an equal and opposite **contact force** on them.



The forces in a Newton's 3rd Law pair act on different objects.

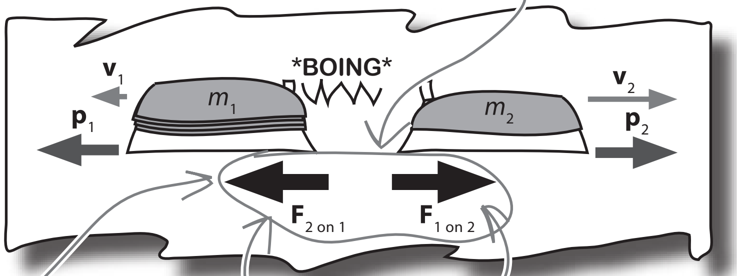
Force pairs help you check your work

You need to make sure you worked out the forces involved in the WeightBotchers machine correctly before you present your findings to FakeBusters. And the best way to do that is to make sure that each force you've drawn on your free body diagram is part of a **Newton's 3rd Law pair of forces**.

You originally met Newton's 3rd Law back in chapter 10 when you were dealing with pushing two objects apart. In order for momentum to be conserved (an experimental result that you worked out), each object must experience the **same size of force** but in **opposite directions** when they interact.

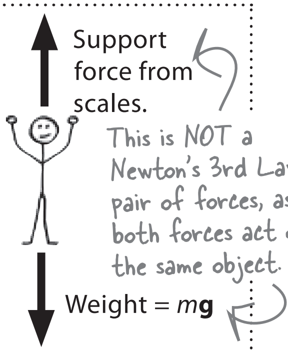
If the objects are "1" and "2" the pair of forces is:

The force that object 1 exerts on object 2.
The force that object 2 exerts on object 1.



If you draw a **free body diagram** of a person standing on scales, there are two forces acting on the person - their weight and the support force.

Although these forces are equal in size and in opposite directions, they are not a Newton's 3rd Law pair of forces, as both forces act on the same object.

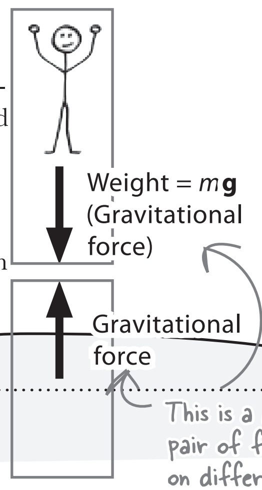


You can check your work by seeing if each of the forces you've drawn on the free body diagram is part of a Newton's 3rd Law pair.

To do this, you need to work out what the other object in each pair would be:

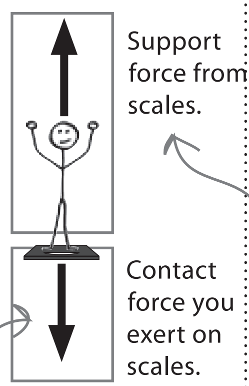
The force that pairs with your weight is the gravitational force that you exert on the earth - as the stuff you're made from and the stuff that the earth's made from mutually attract each other.

This pair of forces will also exist at a distance, even if you're not in contact with the earth.



The force that pairs with the support force from the scales is the **contact force** that you exert on the scales.

Neither the support force that the scales exert on you nor the contact force that you exert on the scales would exist if there was no contact.



But that doesn't make sense! There are two arrows pointing downwards, your weight and this 'contact force', so when you add them together, doesn't that mean you're pushing on the ground with double your weight?!

It's only meaningful to add together forces that act on the same object.

Your weight is a force that acts on you.

Your weight is a force that acts on you. It must act on you or else you wouldn't fall through the air as a result of it. It points downwards. If it helps, think of it as "gravitational attraction."

The contact force between you and the scales acts on the scales. It also points downwards - but it acts on the scales, not on you.

You can only add together forces if they act on the same object. As these two downwards forces act on different objects, you can't add them together.

It seems that the support force only exists when you're in **contact** with the scales?

There's a distinction between contact forces and non-contact forces

Gravitational forces are **non-contact** forces, which can act at a distance. The earth exerts a gravitational force on you whether you're in contact with it or not.

Contact forces only exist if there is contact between two objects. If you stand on the ground you're not accelerating. The net force on you is zero, so your weight and the support force that the ground exerts on you must add to zero.

Newton's 3rd Law says that if the ground is exerting a contact force to support you, then you must be exerting a contact force on it. This makes sense - you can only squash a bug by stepping on it, which involves contact. This can't be done by the force of your weight - as this is a force that acts on you, not on the bug. But it can be done by the contact force that pairs with the support force that the ground exerts on you.

The forces in a Newton's 3rd Law pair must both be of the same type (for example, both contact or both non-contact).

there are no Dumb Questions

Q: Why is it useful to think about Newton's 3rd Law pairs of forces?

A: When you draw a free body diagram, you're drawing only the forces that act on a single object. Each of these forces must be one of a pair - as when two objects interact, each of them experiences an equal-sized force in opposite directions.

Q: So when I draw a free body diagram, I should think about what the other force in the pair is?

A: That's right - and you should also think about which two objects are involved in the interaction. This prevents you from accidentally drawing two forces on your free body diagram that act on different objects.

Q: Where does this idea of a 'contact force' with the ground come from? I thought that the Newton's 3rd Law force pair was my weight and the support force.

A: Both your weight and the support force that the scales exert are forces that act on you. They can't be a Newton's 3rd Law force pair, as they don't act on different objects.

Q: Is there any other way for me to work out what forces might be in a pair?

A: The forces in a Newton's 3rd Law force pair must be of the same type. So either they both have to be contact forces, or they both have to be non-contact forces.

Q: Why doesn't the contact force I exert on the earth (via the scales, which are attached to the earth) lead to the earth accelerating? Don't I exert a net force on the earth?

A: As well as the contact force you exert on the earth, you also exert an attractive gravitational force on it, in the opposite direction. So the net force on the earth as a result of you being there is zero, and the earth doesn't accelerate.

Q: What if I wasn't standing on the earth? Then the gravitational force that the earth experiences as a result of me being there must accelerate the earth towards me, right?

A: That's completely spot on, and a great observation. When you're not standing on the earth, there's a gravitational force pair. The gravitational force on you (your weight) accelerates you towards the earth. And the gravitational force on the earth accelerates it towards you.

Q: So why don't I notice the earth accelerating towards me?

A: Newton's 2nd Law says that F = ma. Therefore, $a = \frac{F}{m}$. As the earth has a much much larger mass than you, its acceleration is very small compared to yours. So it's not something you'd notice.

Thinking about Newton's 3rd Law pairs of forces helps you make sure your free body diagram is correct.

You debunked WeightBotchers!

Your free body diagram of the person on the scales is correct. The support force exerted by the scales on the person is less than the person's weight.

This means that the net force on the person is not zero. A non-zero net force produces an acceleration: $\mathbf{F}_{net} = m\mathbf{a}$. The net force leads to the person accelerating downwards, as observed on the advert.

The scales measure the support force, and convert it to a mass by assuming that **F** = **weight** = m**g**. If the support force is less than the person's weight, then the scales read a smaller number of kilograms than they would if there was no acceleration.

FakeBusters are soon back in touch to congratulate you, and arrange for your segment of the show to be filmed.

But WeightBotchers are back!

We spoke too soon. WeightBotchers are back with a new machine - though only the magazine ads for it have appeared so far.

This time, the person on the scales is sliding down a hill. And miraculously (or so it seems), the scales read lower than they did when the person was standing on flat ground.

FakeBusters wants your help busting the new scam with science once again.

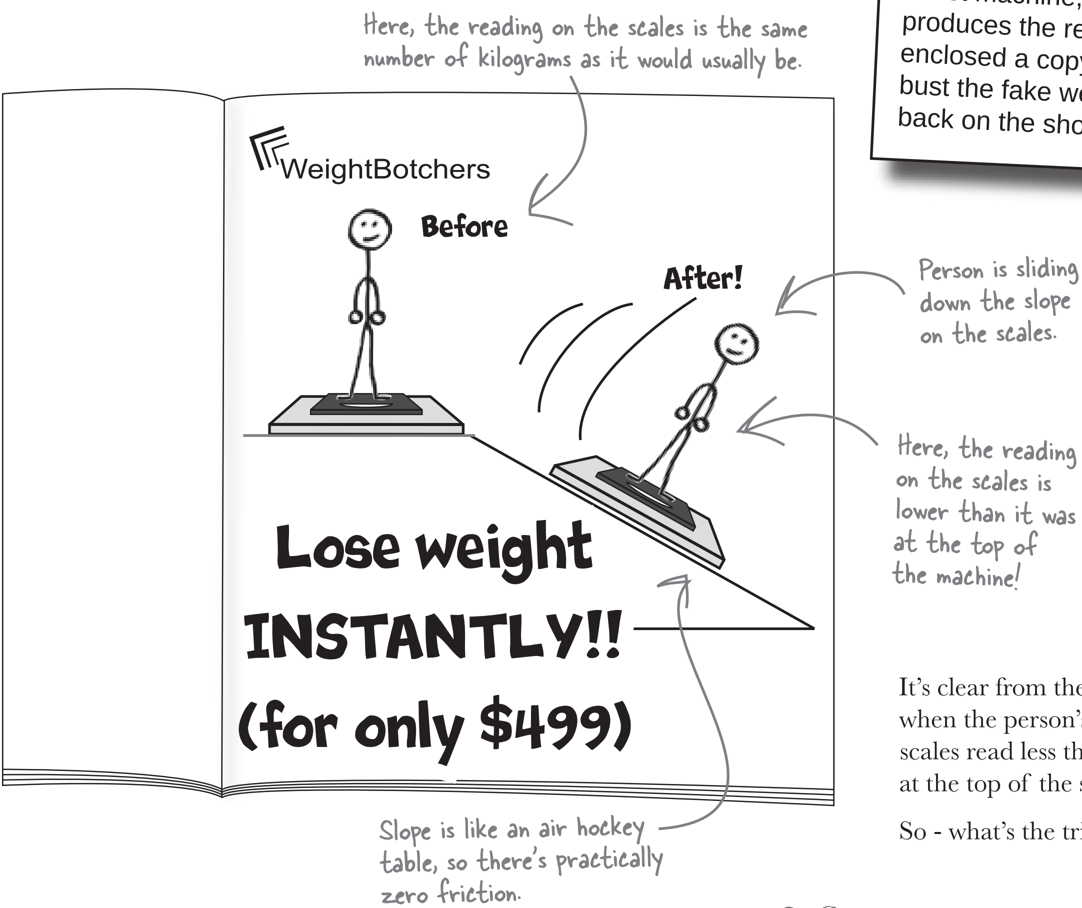
Memo

From: FakeBusters

Re: WeightBotchers

Great work busting WeightBotchers... but they're back.

Could you investigate WeightBotchers' latest machine, and work out how it produces the results it does? We've enclosed a copy of their ad. If you can bust the fake we'd love to have you back on the show.



It's clear from the WeightBotchers ad that when the person's going down the slope, the scales read less than they did on the flat ground at the top of the slope.

So - what's the trick this time?!

BRAIN POWER

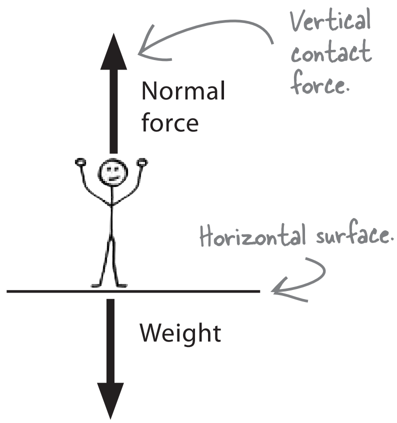
How do you think the new machine works?

A surface can only exert a force perpendicular (or normal) to it

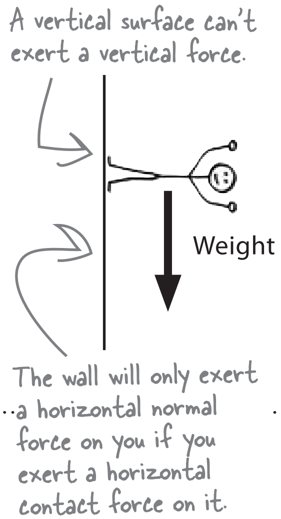
If you're standing on a **horizontal** surface, you don't accelerate. This is because the surface exerts a vertical **contact force** on you, **perpendicular** to the surface.

Up until now, we've been calling a perpendicular contact force exerted by a surface a "support force". But a more accurate name for the perpendicular contact force exerted by a surface is the **normal force**.

This is because whatever angle a surface is at, the surface can only exert a contact force perpendicular, or **normal**, to itself. Depending on the situation, the normal force may not be supporting an object's weight.

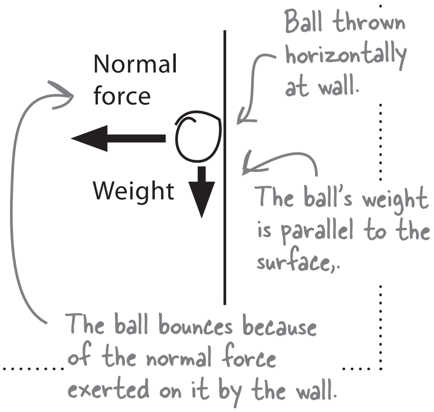


If you try to "stand" on a **vertical** surface (like a wall), you just fall straight down, as the vertical surface can only produce a horizontal normal force. The vertical surface is unable to provide any support for your weight.



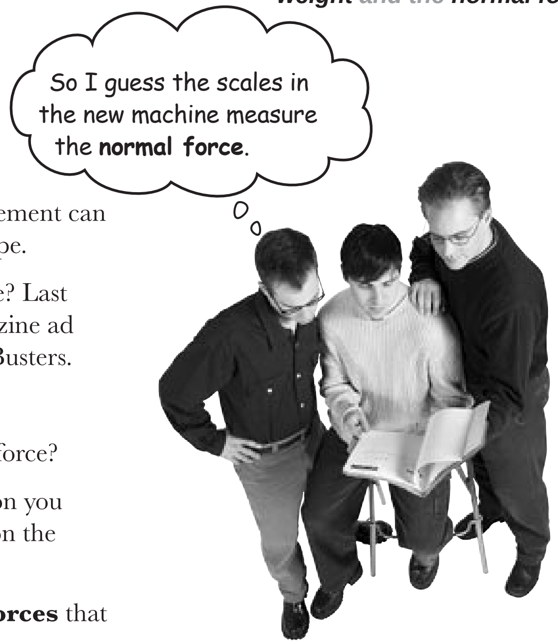
If you throw a ball horizontally at a vertical wall, it experiences a horizontal **normal force** from the wall that makes it bounce.

The ball also experiences the force of its weight, but as the ball's weight is parallel to the wall, it doesn't affect the size of the normal force.



The normal force that a surface exerts on you is the same size as the perpendicular force that you exert on the surface, but in the opposite direction.

The normal force that a surface exerts on you is the same size as the **perpendicular** force you are exert on it, but in the opposite direction. If you are stationary on a horizontal surface, the normal force is equal to your weight. If you push a wall horizontally with a force of 50 N, the normal force is 50 N.



Jim: Yeah, the spring in the scales that makes the measurement can only be compressed perpendicular - or normal - to the slope.

Joe: But how do we calculate the **normal force** this time? Last time we got data from the TV ad, but there's only a magazine ad this time. We need to do this before the deadline for FakeBusters.

Frank: I guess we can measure the **angle** of the slope...

Jim: How does the angle help us to calculate the normal force?

Joe: It says here: "the normal force that a surface exerts on you is the same size as the perpendicular force that you exert on the surface, but in the opposite direction."

Frank: I think that means we need to think in terms of **forces** that are **parallel** and **perpendicular** to the slope.

Jim: Yeah, I'm sure the **net force** and the equation $\mathbf{F}_{net} = m\mathbf{a}$ will come in to it somewhere. Though I'm not really sure where yet...

Sharpen your pencil

Hint: An object's acceleration must be in the same direction as the net force acting on the object, since $F_{net} = ma$ is a vector equation.

a. If a car accelerates horizontally, parallel to the ground, does its acceleration have a component perpendicular to the surface?

b. If a person on a sloped surface accelerates down the slope, parallel to it, does their acceleration have a component perpendicular to the surface?

c. If an object has zero acceleration, what can you say about the net force acting on it?

d. If one component of an object's acceleration is zero, what can you say about the net force acting in the direction of that component?

e. Do your answers to parts a-d give you any ideas about how to deal with the person on the WeightBotchers machine accelerating down the slope?

Sharpen your pencil Solution

Hint: An object's acceleration must be in the same direction as the net force acting on the object, since $F_{net} = ma$ is a vector equation.

a. If a car accelerates horizontally, parallel to the ground, does its acceleration have a perpendicular component?

No, it only has a parallel (horizontal) component as it is accelerating parallel to the surface.

b. If a person on a sloped surface accelerates down the slope, parallel to it, does their acceleration have a component perpendicular to the surface?

No, they only have a parallel component, as they are accelerating parallel to the surface.

c. If an object has zero acceleration, what can you say about the net force acting on it?

$F = ma$ so if the acceleration is zero then the net force is zero (even though there may be several forces acting on the object).

d. If one component of an object's acceleration is zero, what can you say about the net force acting in the direction of that component?

If the component of the acceleration in that direction is zero, then the net force in that direction must be zero.

e. Do your answers to parts a-d give you any ideas about how to deal with the person on the WeightBotchers machine accelerating down the slope?

The person is accelerating parallel to the slope but not perpendicular to the slope. If we think about acceleration and force components parallel and perpendicular to the slope, it might work out well, because the perpendicular components will all be zero.

there are no Dumb Questions

Q: I just want to get some terminology straight. Are the normal force and a support force the same thing or not?

A: The normal force is the contact force that a surface exerts on you. The normal force always acts perpendicular to the surface, and depends on the force you are exerting perpendicular to the surface

What we were calling the support force earlier on was a specific example of a normal force. We called it a "support force" because it was supporting your weight, which acts perpendicular to a horizontal surface. "Support force" was a good mental image to have when you were getting to grips with how scales work.

Q: Why are we now using the term 'normal force' instead of 'support force'?

A: Now, the surface isn't horizontal any more - it's at an angle. The normal force always acts perpendicular to the surface. So the normal force doesn't point in the opposite direction from your weight any more - it's at an angle because the surface is at an angle.

You can't think of the normal force as "supporting" your weight in quite the same way, as the weight vector and normal force vector are not parallel to each other. So it's best to use the term "normal force" rather than "support force" to avoid getting confused.

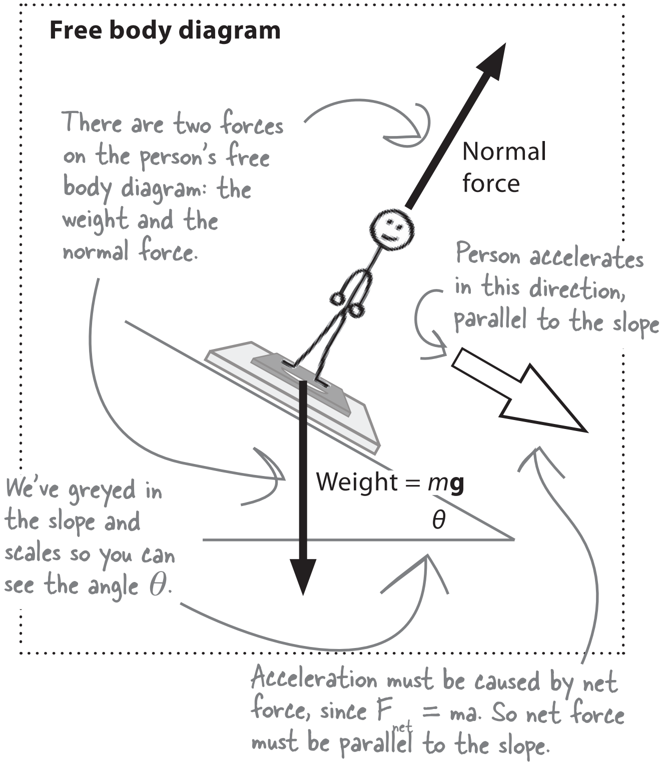
Q: We've always talked about horizontal and vertical vector components in earlier chapters. Why are we talking about parallel and perpendicular vector components now?

A: When you're dealing with projectiles, the net force that causes the projectile to accelerate at 9.8 m/s^2 is the gravitational force. The net force acts vertically, so the projectile accelerates vertically. There's zero net force in the horizontal direction.

When you're dealing with an object accelerating down a slope, the net force acts down the slope, so the object accelerates down the slope. The net force acts parallel to the slope. There's zero net force in the perpendicular direction.

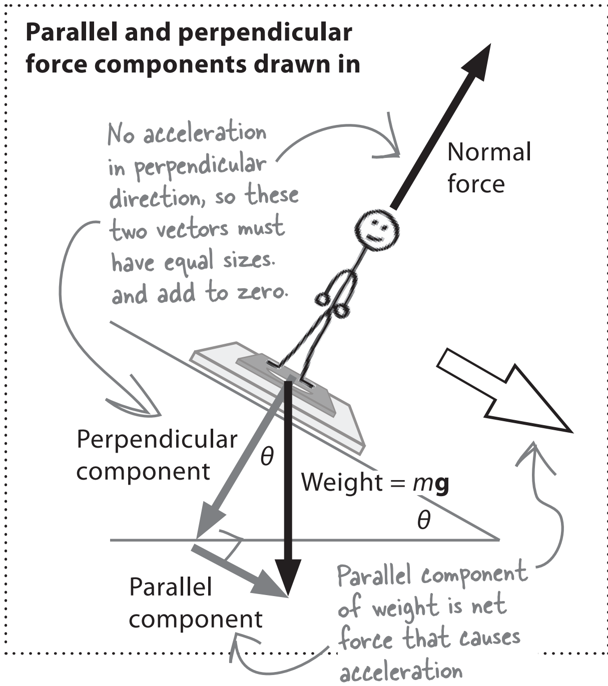
When you slide downhill, there's zero perpendicular acceleration

The person on the slope has two forces acting on them: their **weight** and the **normal force** from the slope.



The person slides down the slope, accelerating parallel to the slope. This means that the net force must act parallel to the slope, because Fnet = ma.

This means that the components of the forces perpendicular to the slope must add up to zero.



The NORMAL force, F_N, is always perpendicular to a surface.

The NET force, F_{net}, always points in the same direction as the acceleration it causes.

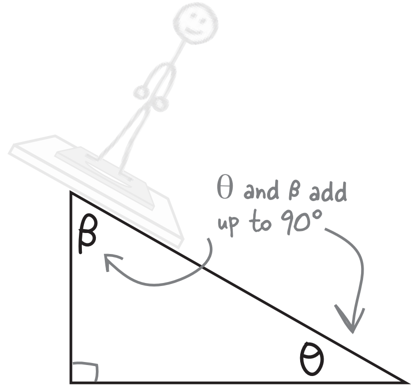
The only two forces with components perpendicular to the slope are the **weight** and the **normal force**. Therefore, the perpendicular component of the weight and the normal force must add up to zero, so that **the net force is parallel** to the slope.

I guess we can work out the angles of the force vectors from the other angles nearby, like we did before?

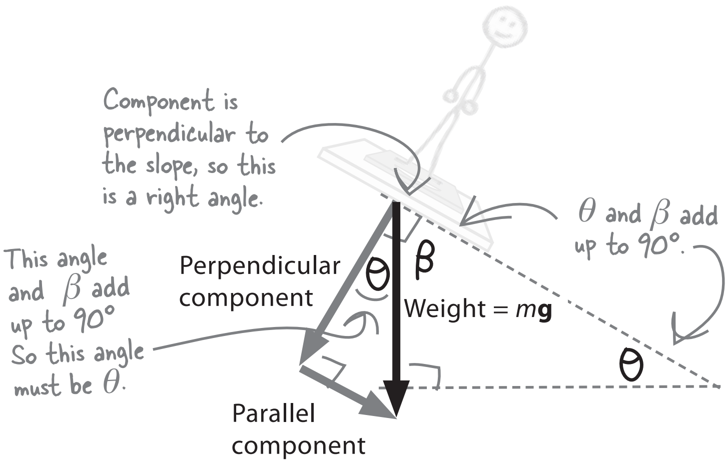
You did this back in chapter 9, with right-angled triangles

The force vector angles are related to the angle of your slope.

Your **slope** is like a **right-angled triangle**. You can label its angles by calling the angle it makes with the ground θ and its other angle β. You know that the angles in a triangle add up to 180°, and as there's already a right angle in there (90°), θ and β must add up to 90°.



Your **weight vector components** also form a right-angled triangle. Because two of the sides of the weight vector triangle are parallel and perpendicular to the slope, the slope triangle and weight vector triangle are **similar triangles**. So the angle *θ* from your slope will also appear in your weight vector component triangle.



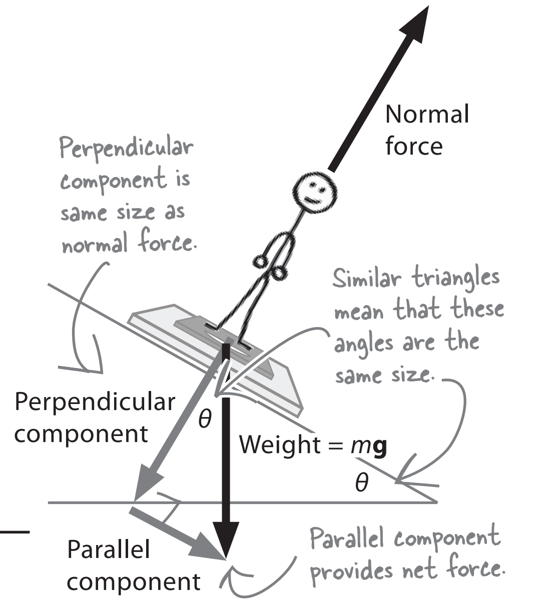
If a problem involves parallel components, perpendicular components and angles, look for similar triangles.

Use parallel and perpendicular force components to deal with a slope

The reading on the scales is the **normal force**, which is the same size as the **perpendicular component** of your weight. This is because the normal force always acts perpendicular to a surface, and always exerts the same size of force on you as you exert on the surface.

The **parallel component** of your weight is is the **net force** that leads to you accelerating down the slope.

Time to get on TV ...



Sharpen your pencil

The WeightBotchers ad shows an "instant 5% reduction" when the person goes down the slope. You're going to calculate the angle the slope would need to be to produce this reduction. Then the Fakebusters team will compare your theoretical calculation with the angle of the slope in the ad.

a. Draw a big vector triangle showing the weight and its components of the weight parallel and perpendicular to the slope, so you can write things on it.

b. If the WeightBotchers scales show a 5% reduction in someone's mass, what's the angle of the slope? (Assume that the person has mass *m*, and that the gravitational field strength is **g**.)

Hint: a 5% reduction means that the normal force is 95% of the person's actual weight.

c. What is the person's acceleration down the slope?

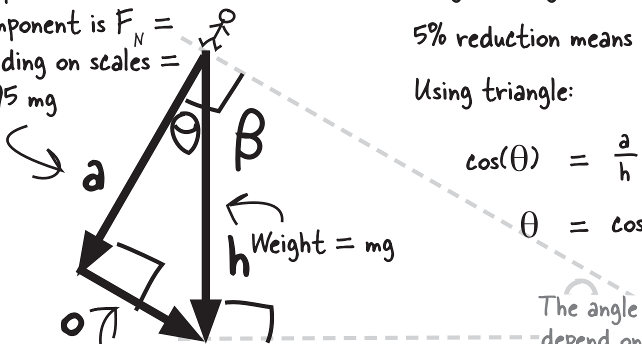
Hint: Calculate the net force down the slope, and use $F_{net} = ma$

Sharpen your pencil

The WeightBotchers ad shows an "instant 5% reduction" when the person goes down the slope. You're going to calculate the angle the slope would need to be to produce this reduction. Then the Fakebusters team will compare your theoretical calculation with the angle of the slope in the ad.

a. Draw a big vector triangle showing the weight and its components of the weight parallel and perpendicular to the slope, so you can write things on it.

Perpendicular component is F_N = reading on scales = 0.95 mg



Parallel component is F_{net} that causes acceleration down hill.

b. If the WeightBotchers scales show a 5% reduction in someone's mass, what's the angle of the slope? (Assume that the person has mass *m*, and that the gravitational field strength is **g**.)

Hint: a 5% reduction means that the normal force is 95% of the person's actual weight.

Weight = mg.

5% reduction means that normal force = 0.95 mg

Using triangle:

$$\cos(\theta) = \frac{a}{h} = \frac{0.95\,\cancel{mg}}{\cancel{mg}} = 0.95$$

$$\theta = \cos^{-1}(0.95) = \underline{\underline{18.2°}}\ (3\ sd)$$

'mg' is on the top and bottom of the fraction so it divides out and cancels.

The angle doesn't depend on the person's mass.

c. What is the person's acceleration down the slope?

Calculate net force then use $F_{net} = ma$ to get acceleration

Using triangle to work out net force (which is opp side)

$$\sin(\theta) = \frac{o}{h} = \frac{F_{net}}{mg}$$

$$F_{net} = mg\sin(18.2)$$

$$F_{net} = 0.312\ mg$$

$$\cancel{m}a = 0.312\ \cancel{m}g$$

'm' is multiplying both sides of the equation, so it divides out and cancels.

$$a = 0.312 \times 9.8 = \underline{\underline{3.06\ m/s^2}}\ (3\ sd)$$

The acceleration doesn't depend on the person's mass.

Another fake busted!

There are two forces acting on you when you stand on the second WeightBotchers machine - your **weight** and the **normal force** from the surface.

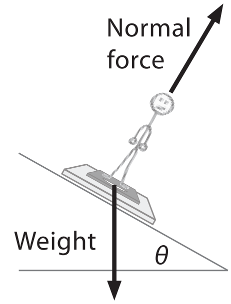
Your weight vector points straight down, but the normal force points **perpendicular** to the surface.

You accelerate down the slope, parallel to the surface. This means that the **net force** acting on you must be **parallel** to the surface, because $\mathbf{F}_{net} = m\mathbf{a}$. And the **perpendicular** components of the forces must add to zero.

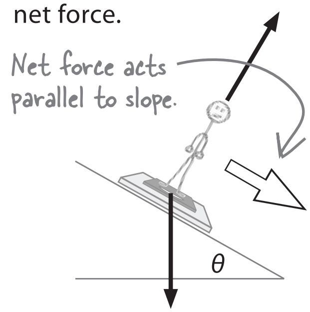
When you draw in the parallel and perpendicular components of your weight vector, you see that the parallel component of your weight provides the parallel net force.

The net perpendicular force on you is zero. Therefore, the normal force and the perpendicular component of your weight must add to zero. As the scales measure the normal force, and the normal force is only a component of your weight, the scales don't register your full weight. The reading on the scales is lower than it would be on a horizontal surface.

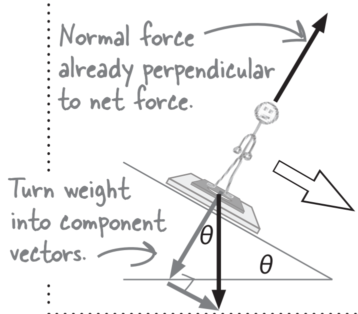
1. Start with a free body diagram



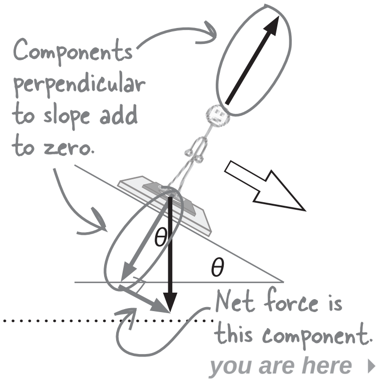
2. Work out the direction of the net force.

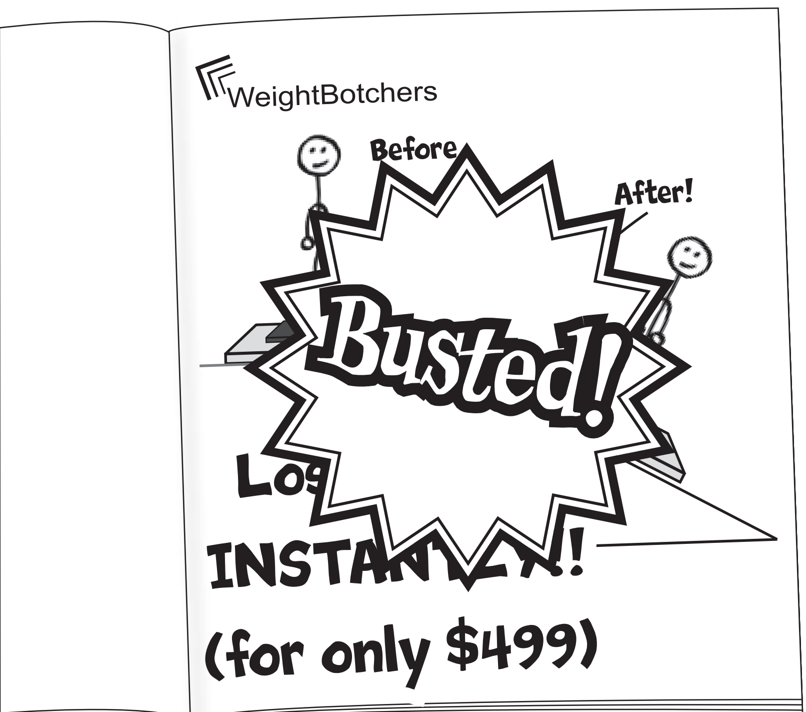


3. Draw in components parallel and perpendicular to the net force.



4. The components perpendicular to the net force must add to zero.





Question Clinic: The "Free body diagram" Question

Any time you have a problem that involves forces, always, always, ALWAYS start with a free body diagram! To do this, think of all the Newton's 3rd Law pairs of forces that the object is involved in. Then draw in all the forces from the pairs that act ON the object (but not the forces that the object exerts on other things).

If you know the mass, you might be able to use it to work out the net force ($F = ma$) or the momentum ($p = mv$) later on.

This should immediately get you thinking about horizontal and vertical components.

> 5. A hot air balloon, mass 3500 kg, travels horizontally at a constant speed of 2.0 m/s.
>
> a. Draw a free body diagram for the balloon, clearly labelling all the forces acting on it.
>
> b. If sandbags with a mass of 200 kg are dropped from the basket, draw the new free body diagram for the balloon.
>
> c. What is the acceleration of the balloon after the sandbags are removed?

This means that you should write down what your force vector arrows represent next to them.

This changes the weight of the balloon.

This means that there is no net force acting on the balloon, causing it to accelerate.

If there's no net force, then the weight and the buoyant force must be equal.

There's now a net force, as the buoyant force that holds up the balloon is greater than the new weight. So you can use $F_{net} = ma$ to work out the acceleration.

Remember to use the new mass without the ballast!

This will be the same as your old one, but with the new weight.

Remember - if the object is stationary or moving with a constant velocity, there's no net force acting on it - it's not accelerating. So all the forces you draw must add to zero when you line them up 'nose-to-tail'. This will help you not to forget forces that you should include on your free body diagram.

Question Clinic: The "Thing on a slope" Question

Sometimes, you'll be asked a question about an object on a slope. You should always **start with a sketch** - draw the slope, and write on all the information you know. If the question is about **forces** and **acceleration**, then make sure you draw a **free body diagram** as well. Look carefully to see if the wording of the question implies that there's a net force

5. A person, mass m, stands on scales that accelerate down a slope. The slope makes an angle **θ** with the ground.

   a. Draw a free body diagram for the person, clearly labelling all the forces acting on them.

   b. Calculate the normal force in terms of *m*, *θ* and **g**, the gravitational field strength.

   c. What mass do the scales say the person is?

   d. If the person accelerates down the slope at 3 m/s², what is θ?

   e. If there was no slope and the person was just falling, what would the reading on the scales be?

These are the values you know at the start of the problem.

This means that there is a net force acting on the person.

This gives you the direction of the net force – parallel to the slope.

A simple free body diagram shouldn't contain components – just the actual forces.

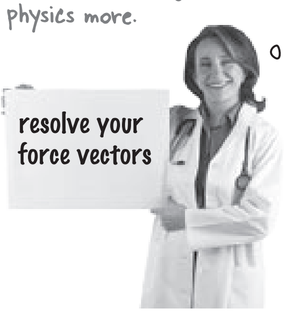
Work with components perpendicular to the slope.

This means that you're using the variables m, θ and g rather than numerical values, which tests your understanding of the physics more.

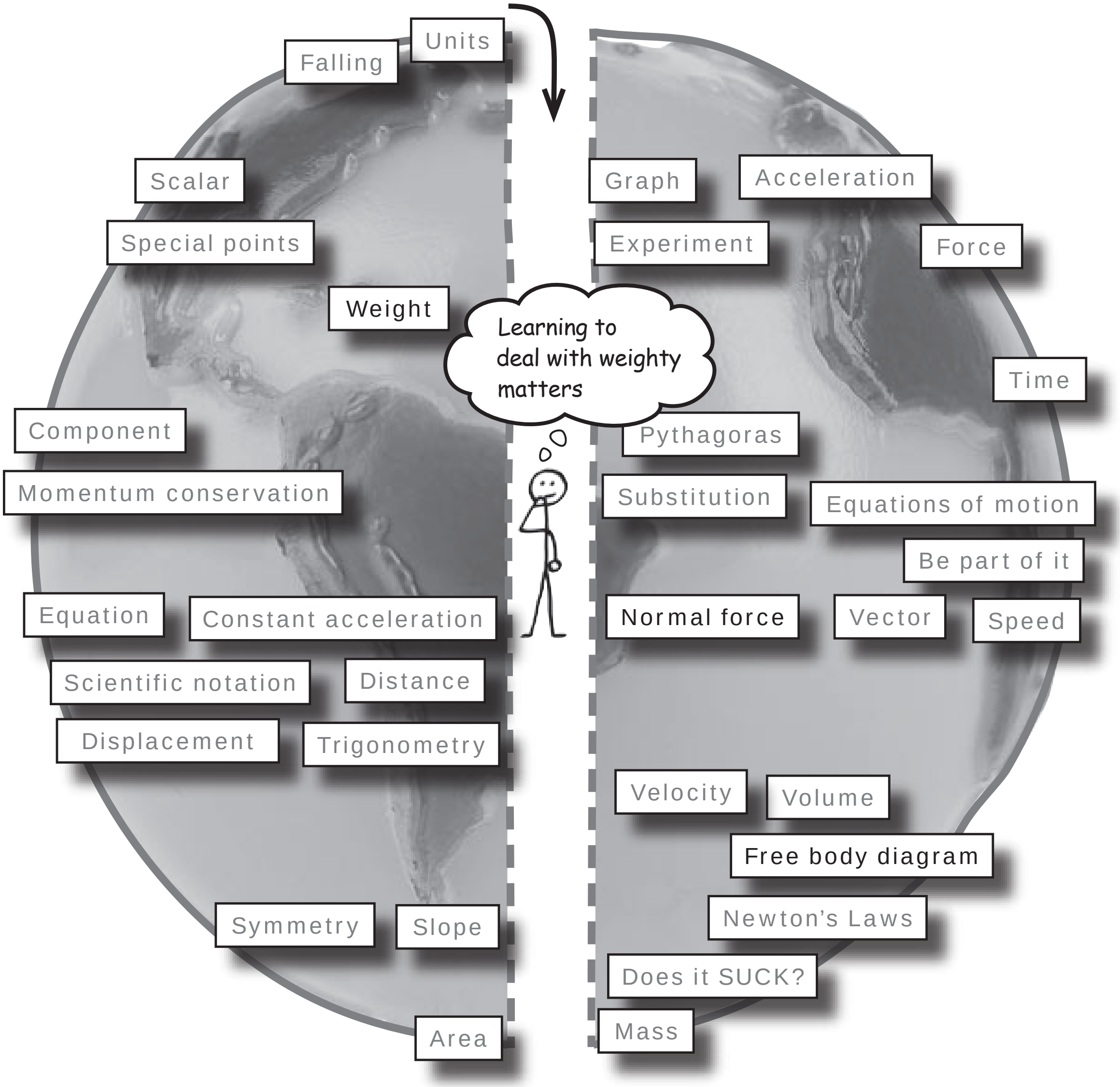
Note when you're asked for a mass rather than a weight.

Draw a new free body diagram. There's no normal force, so the scales will read zero.

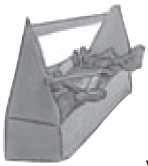
Use F = ma and components.



It's very important to resolve your force vectors into **components** parallel and perpendicular to the slope - especially the object's **weight** vector. This is because an object moving down a slope experiences no net force perpendicular to the slope. So drawing in the components of your forces helps you calculate the correct value for the normal force, and provides a way of working out the net force as well.



☑	Weight	The gravitational force exerted on an object by a much larger object, such as the earth. Weight = mg, where m is the mass and g is the gravitational field strength.
☑	Free body diagram	A diagram showing only one object, and all of the forces acting on it.
☑	Normal force	The contact force exerted by a surface on an object. This force is always perpendicular (or normal) to the surface



Your Physics Toolbox

You've got Chapter 11 under your belt and added some problem-solving concepts to your toolbox.

Free body diagram

For any problem that involves forces, you should always draw a free body diagram, giving the size and direction of every force acting on a single body.

Draw your force vector arrows pointing away from the object.

Net force

You can work out the net force on an object by adding together all the force vectors from its free body diagram by lining them up "nose to tail."

If the net force is zero, the object isn't accelerating.

If there is a non-zero net force, the object accelerates in the direction of the net force.

Newton's 2nd law

A net force applied for a time always leads to the same change in momentum:

$$F_{net}\Delta t = \Delta(mv)$$

If the mass of an object is constant, this can simplify to:

$$F_{net} = ma$$

Newton's 3rd Law pairs of forces

Newton's 3rd Law pairs of forces exist where two objects interact.

Both forces in the pair must be of the same type (contact or non-contact).

Each force in the pair must act on a different object.

Each force in the pair will have the same size, but the forces will be in opposite directions.

Object on a slope

If you have an object on a slope, the normal force and the perpendicular component of the weight are equal sizes.

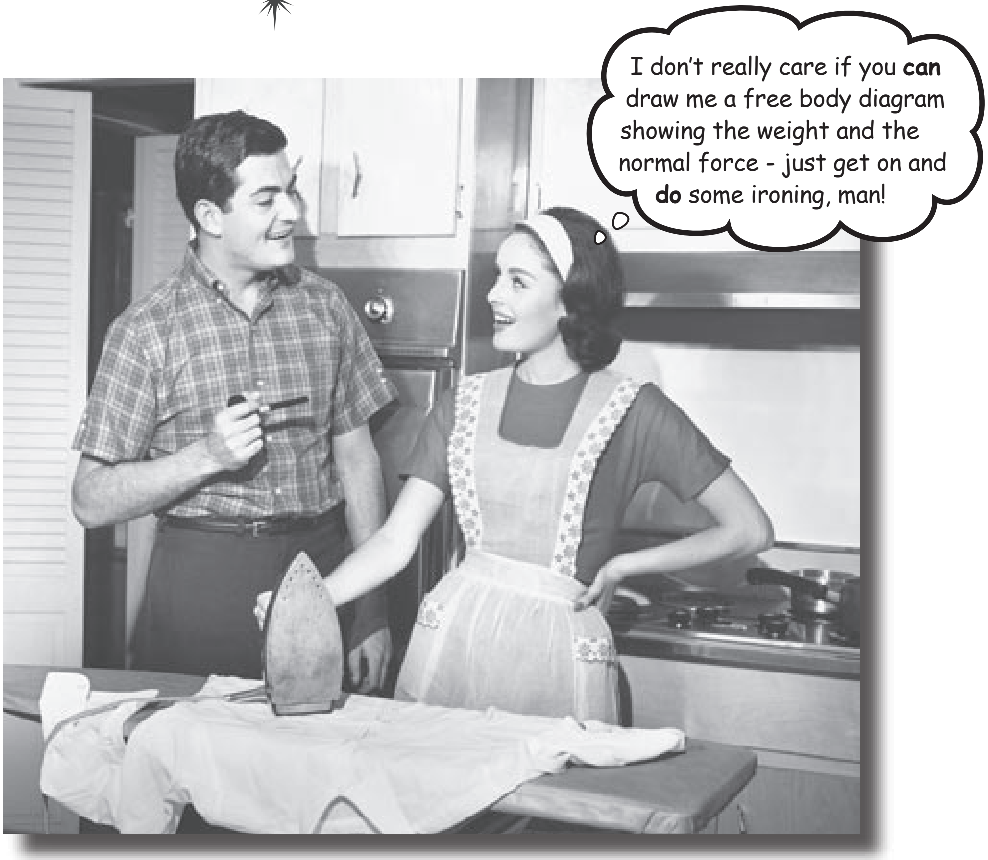
The net force and the parallel component of the weight are equal sizes.

Choosing component directions

If you have a problem where the net force is zero in one particular direction (e.g. perpendicular to a slope) then choose component vectors parallel and perpendicular to this direction to make your calculations easier.

12 using forces, momentum, friction and impulse

Getting on with it



It's no good memorizing lots of theory if you can't apply it.

You already know about equations of motion, component vectors, momentum conservation, free body diagrams and Newton's Laws. In this chapter, you'll learn how to fit all of these things together and apply them to solve a much **wider range** of physics problems. Often, you'll spot when a problem is **like** something you've seen before. You'll also add more realism by learning to deal with **friction** - and will see why it's sometimes appropriate to act on **impulse**.

It's ... SimFootball!

You've been contacted by the SimFootball team, who need your help with some of the physics in their video game. If you can help them figure out why the characters in the game aren't behaving like they would in real life - you'll get an all expenses paid trip to the X-Force Games.!

Memo

From: SimFootball

Re: Physics in our new game

We saw you on FakeBusters the other night, and thought you might like to be a consultant on our latest game.

We already have the graphics in place, but need advice on the **physics** engine for many of the components of the game - passing, tackling, tire drag (in training mode) and kicking. You will work closely with one of our programming team.

If you can help us get this all together in time, we'll send you to the X-Force Games...all expenses paid.

Sharpen your pencil

The SimFootball programming team have come up with a list of things they need physics advice on for their game. Your first job is to outline the physics you think you'll need to use.

So **start with a sketch** of each item to reduce it to its 'bare bones' and see if it's **like** something you already know how to do. Label things like velocity, acceleration, force etc where appropriate. And give a brief **outline** of the kind of physics you might use to solve each problem.

a. **Passing** - Working out the path of a ball that has been thrown through the air at a known angle with a known initial velocity.

b. **Tackling** - Players with known masses each running with a certain velocity collide with each other and grab on.

c. **Tire drag** - In training mode, a player with a rope around his waist runs, dragging a tire along the ground.

d. **Kicking** - Moving foot kicks stationary ball with a force, and is in contact for a known period of time.

Relax

Don't worry if you don't know much about football.

Each of the game elements are explained in the 'Sharpen your pencil'. You're only going to be working with the physics, so it doesn't matter if you don't know much about the rules.

This is American football, not soccer. But whatever you call it, don't worry!

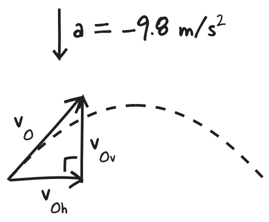
Sharpen your pencil Solution

The SimFootball programming team have come up with a list of things they need physics advice on for their game. Your first job is to outline the physics you think you'll need to use.

So **start with a sketch** of each item to reduce it to its 'bare bones' and see if it's **like** something you already know how to do. Label things like velocity, acceleration, force etc where appropriate. And give a brief **outline** of the kind of physics you might use to solve each problem.

a. **Passing** - Working out the path of a ball that has been thrown through the air at a known angle with a known initial velocity.

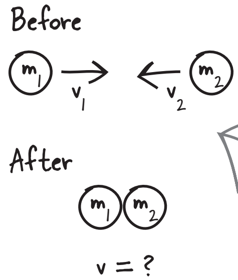
This looks like a projectile fired through the air at an angle. Use equations of motion and treat horizontal and vertical components separately.



b. **Tackling** - Players with known masses each running with a certain velocity collide with each other and grab on.

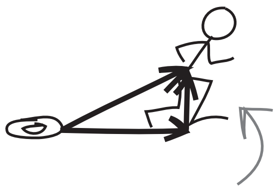
Players both have mass and velocity, so both have momentum before collision.

Momentum is conserved so it must be the same before and after.



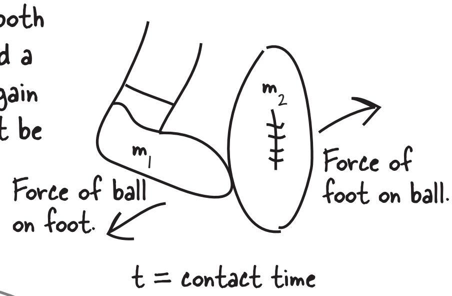
c. **Tire drag** - In training mode, a player with a rope around his waist runs, dragging a tire along the ground with a constant velocity.

The tire is being pulled at an angle, so you can maybe make a right-angled triangle and use component vectors of forces to work this out.



d. **Kicking** - Moving foot kicks stationary ball with a force, and is in contact for a known period of time.

Foot and ball both have a mass and a velocity, and again momentum must be conserved.



You don't know exactly how to do some of these problems yet, but don't worry – you've already got off to a great start!

If two objects interact, look out for being able to use momentum conservation or a form of Newton's 2nd Law (either $F = ma$ or $F\Delta t = \Delta p$) as both objects experience the same size of force.

If you're given a story, start with a sketch to work out what physics the story involves. What's it LIKE?

Jim: Yeah, but what about tackling? The players usually hit head on and grab on to each other. In the game we know their **masses** and **velocities** before the tackle. Ow!!! What are you doing?!

Joe: Just being a part of it! Looks like if I'm running faster when I tackle you, we move faster afterwards than when I run slowly.

Frank: And if your mass was larger, Jim would have gone flying!

Joe: The total **momentum**, mass × velocity, will be the same before and after - right?

Jim: I'm glad we're back to math now! Yeah, the game would need to move the players with the correct **velocity** after the tackle. We know the **mass** and velocity of each player before the tackle, so using **momentum conservation** sounds about right.

Sharpen your pencil

Two football players hit each other head on. One has a mass of 95.0 kg and is running from left to right at 8.50 m/s. The other has a mass of 120.0 kg and is running from right to left at 3.80 m/s

If the players lock together in the tackle, what velocity do they move with in the split second after the tackle?

Hint: If the players lock together, they move as one mass after the tackle

Sharpen your pencil Solution

Two football players hit each other head on. One has a mass of 95.0 kg and is running from left to right at 8.50 m/s. The other has a mass of 120.0 kg and is running from right to left at 3.80 m/s

If the players lock together in the tackle, what velocity do they move with in the split second after the tackle?

Before: $m_1 = 95.0$ kg $m_2 = 120.0$ kg

$v_1 = 8.5$ m/s $v_2 = -3.8$ m/s

After: $m_3 = 95.0 + 120.0 = 215.0$ kg

$v_3 = ?$

Left to right is positive.

Momentum is a VECTOR so you need to choose which DIRECTION to define as positive.

Use momentum conservation to work out v_3:

total momentum before = total momentum after.

$$m_1v_1 + m_2v_2 = m_3v_3$$

$$v_3 = \frac{m_1v_1 + m_2v_2}{m_3}$$

$$v_3 = \frac{95.0 \times 8.50 - 120.0 \times 3.80}{215.0}$$

$$v_3 = 1.63 \text{ m/s (3 sd)}$$

It's safest to rearrange your equation before you put the values in.

They go from left to right at 1.63 m/s (3 sd).

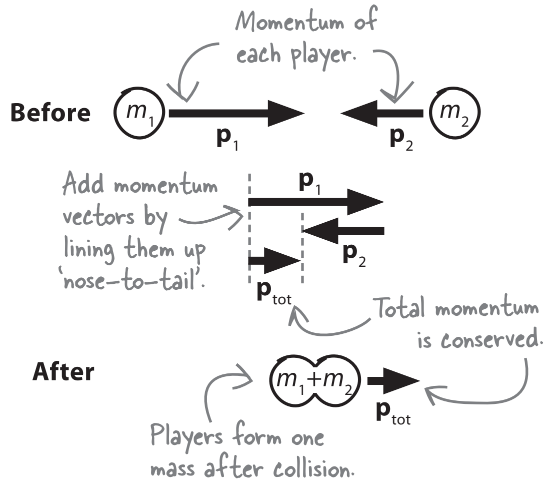
Momentum is conserved in a collision

Momentum is always conserved in an interaction between two or more objects. So when the two players **collide** in the tackle, the total momentum must be the same afterwards as it was before the collision.

This happens because each player experiences the same size of **force** when they collide, but in opposite directions - a **Newton's Third Law pair of forces**. The same size of force always causes the same change in momentum.

So the first object has its momentum changed in the direction of the force acting on it - and the second object has its momentum changed in the direction of the force acting on it.

But the forces are equal sizes and in opposite directions. So the changes in momentum are equal sizes and in opposite directions. This means that the total momentum is the same both before and after the collision. The changes in momentum make no difference to the total when you add them together.



When two objects collide, think about what happens. Do they become one object?

there are no Dumb Questions

Q: How do you know that the two masses that exist before the collision have turned into one mass afterwards?

A: You'll often do problems where two masses stick together after colliding. This means that they no longer move as two separate masses, but as one mass with a single velocity. Read the question carefully!

Q: Are there any buzzwords that indicate that the objects stick together?

A: Sometimes the term "inelastic" is used to indicate a situation where two objects collide without bouncing (in an "elastic" way).

Q: Is momentum always conserved? Or does that only happen when the objects stick together?

A: Momentum is always conserved in any interaction between two objects, whether they stick together or bounce off of each other. This happens because each object experiences an equal-sized forces in opposite directions as a result of the collision. The same size of force always leads to the same change in momentum.

So if one object's momentum changes by +10 kg.m/s and the other object's by -10 kg.m/s, the total momentum is still the same. The +10 and -10 add to zero when you add the "after" momentums together.

Q: So that happens because of a Newton's Third Law pair of forces?

A: Spot on! Newton's 3rd Law and momentum conservation are two sides of the same coin.

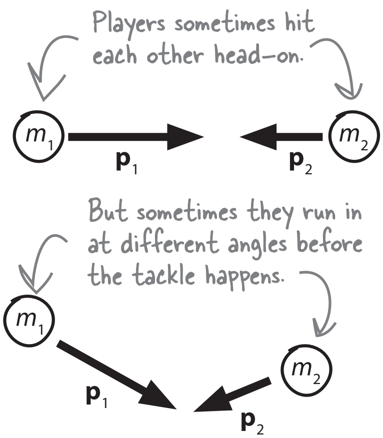
Q: What if the football player had a collision with an advertising billboard that stopped him completely? Where's the momentum conservation there?

A: The advertising billboard is attached to the Earth, which has a huge mass compared to the player. As momentum is mass × velocity, the Earth's huge mass means that the change in its velocity is far too small for you to notice.

But the collision might be at an angle

The SimFootball team are really happy with what you told them about tackling, and write it into the game!

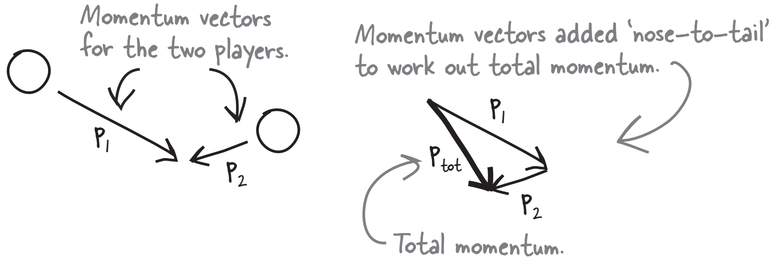
But they soon realize that the problem's more involved than they first thought. The players don't always collide head on - sometimes they hit each other at an **angle**. And they don't know how to deal with that.



How are we gonna to figure out what happens if they hit each other at an **angle** instead of head on like before?

Jim: Well, isn't **momentum** still conserved? We can figure out the total momentum before the collision just like we did before. This'll be the the same as the size and direction of the players' total momentum after the tackle, when they stick together.

Joe: We can do that in principle ... but in practice it's going to be difficult dealing with the momentum vectors if we add them together to work out the total momentum at the start. Look:



When you draw a sketch, make sure you think about angles.

You can't use what you know of Pythagoras, sine, cosine or tangent unless your triangle is right-angled.

Frank: But what's the big deal? The vectors make a **triangle** - and we can deal with triangles!

Jim: Correction ... we can deal with **right-angled triangles**. But that triangle sure ain't right-angled.

Frank: Oh yeah. When the players hit head on, we didn't need to think about angles, because all the action was taking place along a straight line that ran from left to right.

Jim: But can't we just use Pythagoras etc?

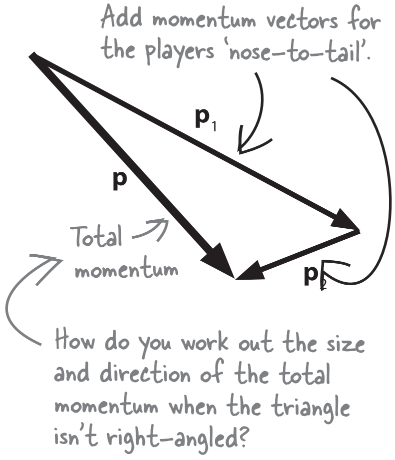
Joe: Pythagoras only works for right-angled triangles. And what we know about sine, cosine and tangent only works for right-angled triangles. I guess we could try to work out something that works for other triangles, but that sounds waaay hard.

Frank: Hmmm, a triangle with no right-angles like the one we're stuck with sure is awkward.

Jim: I wonder if we could somehow flip things around so that there are some right-angled triangles ..

A triangle with no right angles is awkward

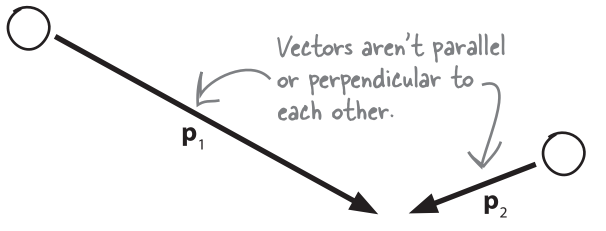
The main problem with this collision is that the players are running in at different **angles**. You can add together the players' **momentum** vectors to get the total momentum before the collision by lining them up nose-to-tail, like we've done here.



But the triangle formed by the players' momentum vectors isn't right-angled. This makes it difficult for you to calculate the total momentum. Pythagoras, sine, cosine and tangent only work with a right-angled triangle. A triangle with no right angles is awkward!

Use component vectors to create some right-angled triangles

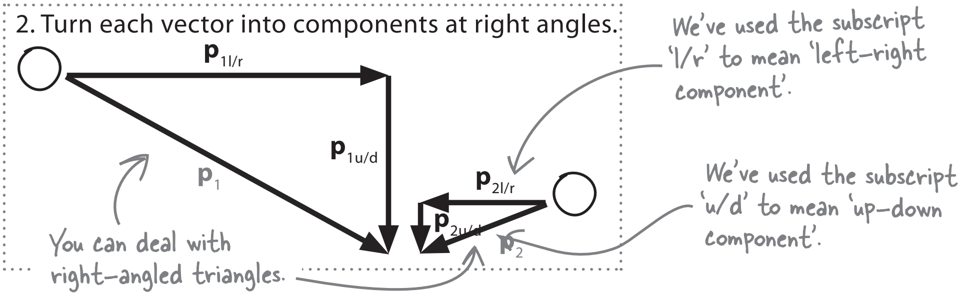
1. You need to add together vectors at an angle.



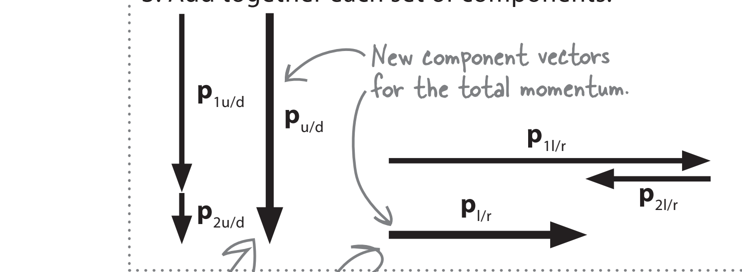
If your problem has two dimensions, think component vectors.

You can redraw any vector as two **component vectors** at **right-angles** to each other. This is especially useful if you have to **add two vectors together** that aren't parallel or perpendicular to each other.

2. Turn each vector into components at right angles.



3. Add together each set of components.

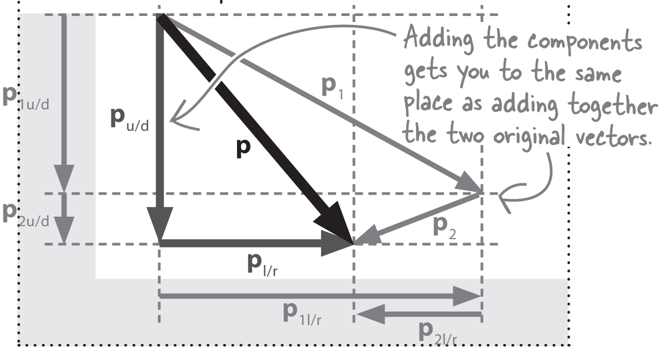


Now work with the components! Use right-angled triangles to add together the up/down and left/right components of each momentum vector separately.

This gives you the up/down and left/right components of the total momentum vector.

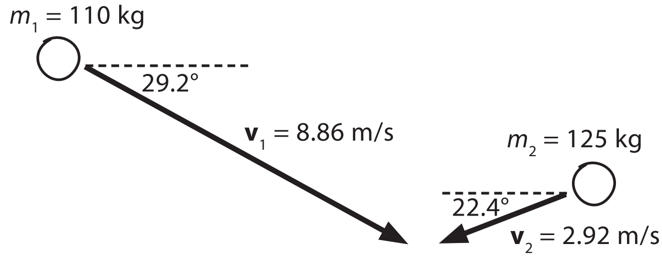
Finally, you can make a new right-angled triangle out of the up/down and left/right components of the total momentum, and use it to calculate the **total momentum** (which will be the same before and after the collision)

4. Add new components for total momentum.



Sharpen your pencil

Two players in the "SimFootball" game collide in a tackle and grab on to each other. Their masses and velocity vectors are shown here:



a. Calculate the size of the momentum vector for each player.

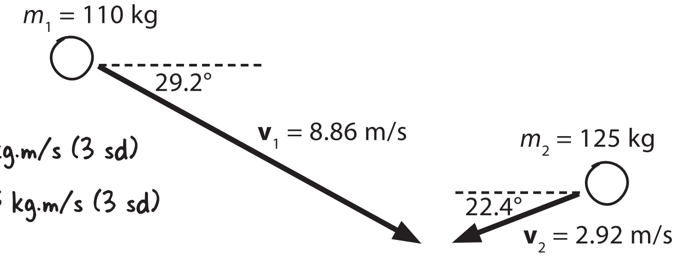
b. Draw a sketch to show the left/right and up/down components of each player's momentum, and calculate the sizes of these components.

c. Calculate the size and direction of the total momentum vector using your results from part b.

d. What velocity do the players move with after the tackle?

Sharpen your pencil Solution

Two players in the "SimFootball" game collide in a tackle and grab on to each other. Their masses and velocity vectors are shown here:

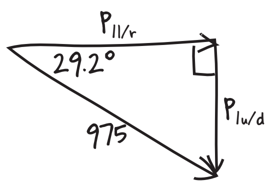


a. Calculate the size of the momentum vector for each player.

$$p_1 = m_1 v_1 = 110 \times 8.86 = 975 \text{ kg.m/s (3 sd)}$$

$$p_2 = m_2 v_2 = 125 \times 2.92 = 365 \text{ kg.m/s (3 sd)}$$

b. Draw a sketch to show the left/right and up/down components of each player's momentum, and calculate the sizes of these components.



$$\cos(29.2) = \frac{a}{h} = \frac{p_{1l/r}}{975}$$

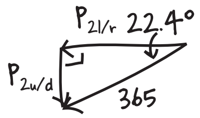
$$p_{1l/r} = 975\cos(29.2)$$

$$p_{1l/r} = 851 \text{ kg.m/s (3 sd) right}$$

$$\sin(29.2) = \frac{o}{h} = \frac{p_{1u/d}}{975}$$

$$p_{1u/d} = 975\sin(29.2)$$

$$p_{1u/d} = 476 \text{ kg.m/s (3 sd) down}$$



$$\cos(22.4) = \frac{a}{h} = \frac{p_{2u/d}}{365}$$

$$p_{2l/r} = 365\cos(22.4)$$

$$p_{2l/r} = 337 \text{ kg.m/s (3 sd) left}$$

$$\sin(22.4) = \frac{o}{h} = \frac{p_{2u/d}}{365}$$

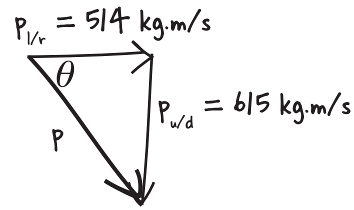
$$p_{2u/d} = 365\sin(22.4)$$

$$p_{2u/d} = 139 \text{ kg.m/s (3 sd) down}$$

c. Calculate the size and direction of the total momentum vector using your results from part b.

Left/ right components: $851 - 337 = 514$ kg.m/s right

Up / down components: $476 + 139 = 615$ kg.m/s down



Size: By Pythagoras, $p^2 = p_{l/r}^{\,2} + p_{u/d}^{\,2} = 514^2 + 615^2$

$$p = \sqrt{514^2 + 615^2} = \underline{\underline{802 \text{ kg.m/s}}} \text{ (3 sd)}$$

Direction: Given angles all measured from the horizontal, so do this too.

$$\tan(\theta) = \frac{o}{a} \Rightarrow \theta = \tan^{-1}\left(\frac{615}{514}\right) = \underline{\underline{50.1° \text{ (3 sd) from the horizontal, left to right.}}}$$

d. What velocity do the players move with after the tackle?

m = total mass $= 110 + 125 = 235$ kg

$$p = mv \Rightarrow v = \frac{p}{m} = \frac{802}{235} = \underline{\underline{3.41 \text{ m/s (3 sd) at } 50.1° \text{ (3 sd) from the horizontal, left to right.}}}$$

there are no Dumb Questions

Q: That was a lot of math to go through with all the component vectors!!

A: It wasn't any more difficult than what you've done previously. It's just that you had to calculate a number of sides and angles! But as long as you manage to organize your work so that you don't get mixed up, you're fine. Now you can handle component vectors and right-angled triangles, you have superpowers that let you deal with two-dimensional situations.

Q: How often will I need to do a problem involving momentum conservation like this one?

A: You may not come across many problems exactly like this. But the **general skill** of being able to turn vectors at awkward angles into component vectors so you can add them together is one you can use again and again with any vectors.

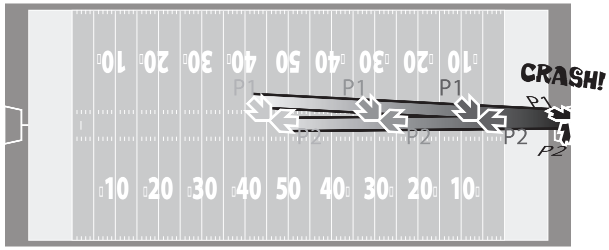
Q: I was just thinking ... what happens if the players bounce off each other after the tackle? Then I'd have two momentum vectors to deal with after the collision!

A: Great spot! You're right - this would be a more difficult problem, and you'll need to learn about energy before you can solve it. You'll come back to a similar scenario in a later chapter, so don't worry about it for now.

To add together two vectors at different angles, resolve the vectors into <u>components</u> at <u>right-angles</u>, then add together the components.

The programmer includes 2D momentum conservation ...

The SimFootball programmer gets to work, and quickly codes up what you've learned about 2D collisions using momentum conservation. Now the players move realistically for the split second after the tackle ...



... but the players keep on sliding for ever!

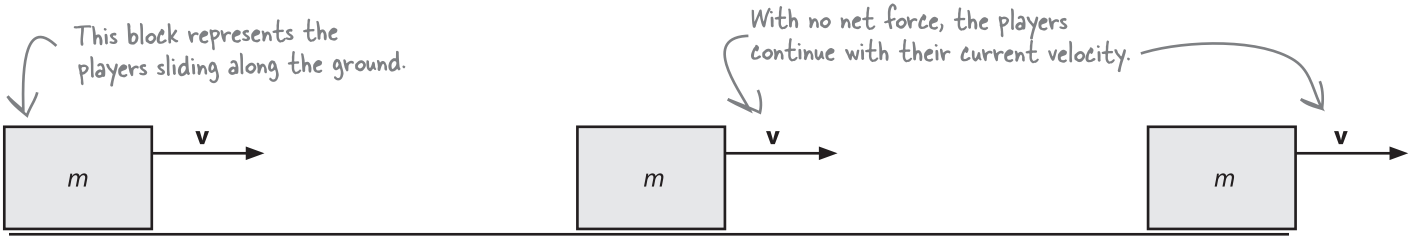
BRAIN POWER

What needs to be included in the game to stop the players just sliding on for ever?

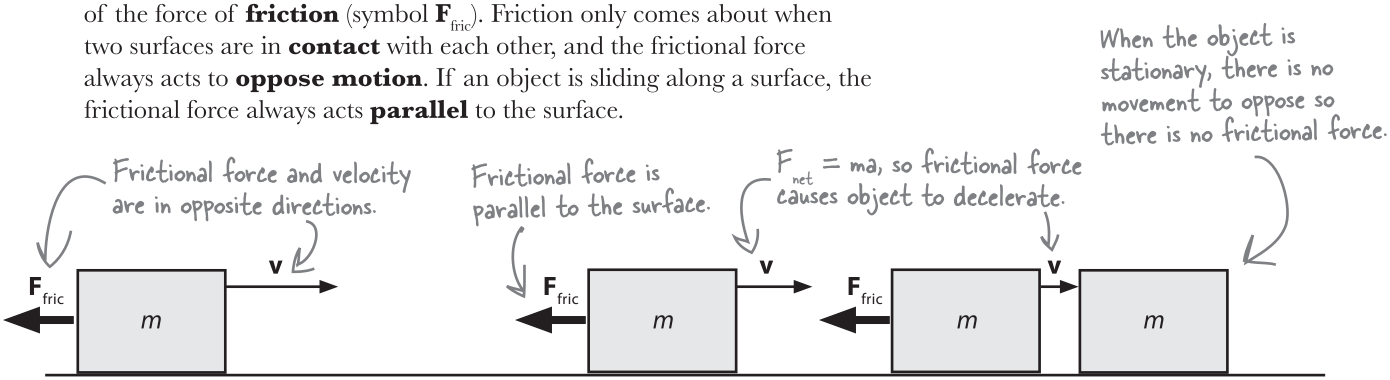
In real life, the force of friction is present

Newton's First Law says that an object will continue on at the same velocity unless acted on by a net force. At the moment, the SimFootball game isn't finished, and the only way sliding players can be stopped is if they crash into the advertising billboards or a goalpost.

So at the moment, the players just continue along at the same velocity after the tackle until they hit something.



In the real world, **moving** objects slow down and eventually stop because of the force of **friction** (symbol $\mathbf{F}_{fric}$). Friction only comes about when two surfaces are in **contact** with each other, and the frictional force always acts to **oppose motion**. If an object is sliding along a surface, the frictional force always acts **parallel** to the surface.

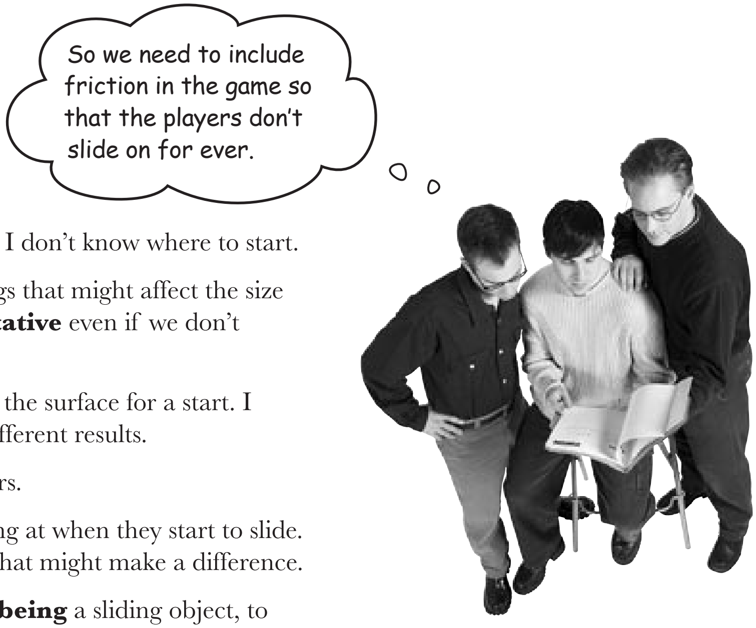


Friction always acts to OPPOSE motion. If an object is sliding along a surface, the frictional force on it acts PARALLEL to the surface.

If the object is **stationary**, its velocity is constant (zero) therefore there is no net horizontal force. If no other horizontal forces are present, and therefore no friction if there are no other horizontal forces.

If you start to gently push a stationary object, it won't move, as friction always opposes motion. But if you keep on pushing harder, you'll eventually manage to exert a larger force than the frictional force, and the object will move.

No-one knows for sure exactly how friction works. It's definitely a contact-dependent force, which you see when you try to slide one surface over another. Interactions between the surfaces act to oppose the motion until, eventually, the two surfaces end up at rest. But the exact nature of these interactions hasn't yet been pinned down.



Jim: How are we gonna calculate that?! I don't know where to start.

Joe: How about we think of all the things that might affect the size of the frictional force. We can be **qualitative** even if we don't know how to be **quantitative** yet.

Frank: Well, there's the **roughness** of the surface for a start. I think grass and astroturf will produce different results.

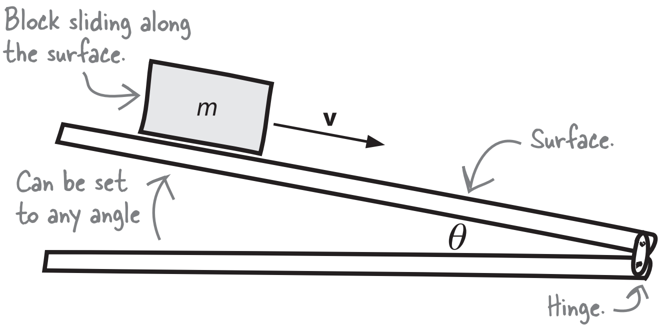
Jim: How about the **mass** of the players.

Joe: Yeah, and the **velocity** they're going at when they start to slide. Plus some football fields **slope** a little - that might make a difference.

Frank: I guess we'd better have a go at **being** a sliding object, to see how each of these variables might affect the frictional force.

BE the sliding object

Your job is to be a sliding object. We've drawn an experiment here that you can do yourself - or you can put yourself in the place of the sliding object / player. Write down the effect you think each factor will have on the size of the frictional force and give reasons for your answers.



a. Materials block and surface are made from.

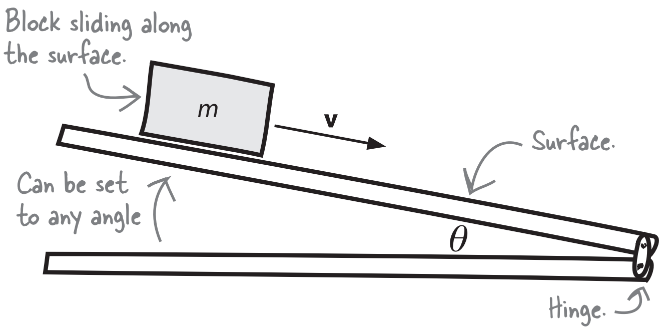
b. Mass / weight of block.

c. Angle / slope of surface.

d. Velocity of block.

BE the sliding object - SOLUTION

Your job is to be a sliding object. We've drawn an experiment here that you can do yourself - or you can put yourself in the place of the sliding object / player. Write down the effect you think each factor will have on the size of the frictional force and give reasons for your answers.



a. Materials block and surface are made from.

Some surfaces will have more friction than others. Players slide further on grass than on astroturf so frictional force is smaller.

b. Mass / weight of block.

The heavier something is, the harder it is to slide, as it's "pushed into" the surface. So a larger mass leads to a larger frictional force.

c. Angle / slope of surface.

The larger the angle, the smaller the frictional force, because the block isn't "pushed into" the surface so much.

d. Velocity of block.

It's difficult to tell what effect the velocity has on the friction without making more accurate measurements.

there are no Dumb Questions

Q: So the amount of friction depends on how rough the surfaces are, right? Like, sandpaper on sandpaper creates a lot of friction, but there's less friction with smooth surfaces?

A: Not quite. As anyone who's ever worn through their bicycle brakes may have spotted, there's actually more friction with two perfectly smooth surfaces (steel on steel) than with sandpaper, or rubber brake blocks on steel! Friction depends on the type of surface, but not necessarily on roughness.

Q: But everyone knows that when you oil something (i.e. make its surface more smooth) then there's less friction.

A: Friction occurs when two surfaces are in **contact** with each other. Oiling introduces a **layer** in between the two surfaces, which increases the **distance** between them. Oiling doesn't change the smoothness of the surfaces themselves.

Friction is a contact-dependent force.

Q: What about something moving through the air? It's not close to any surfaces, but it still slows down.

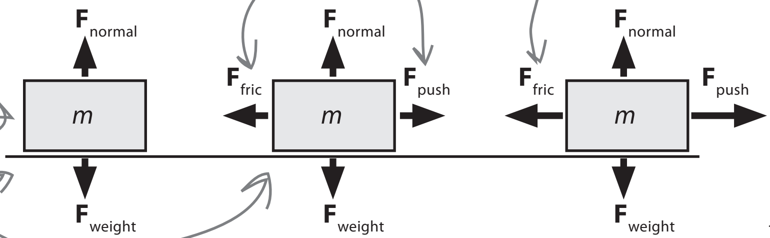
A: That's mainly air resistance, which isn't quite the same as friction. The force of friction experienced by an object sliding across a surface actually doesn't depend on its velocity. But the resistive force that something moving through the air experiences increases when its velocity increases, as the faster it goes, the more air molecules it has to 'push aside' every second to make progress.

It seems harder to **start** something moving than it is to **keep** it moving. Is that because friction's different when you have a standing start?

Static friction and kinetic friction have different values for the same object.

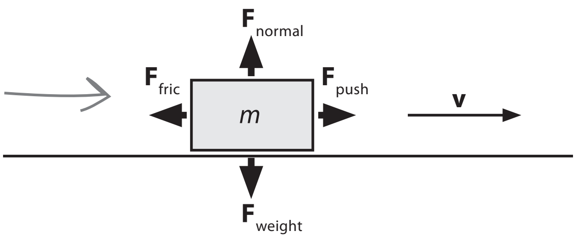
Static friction is the force you need to overcome to get an object to **start to slide** across a surface.

If you try to push a stationary object along a surface, the static friction force opposes what you're trying to do until you push with a greater force than the maximum amount of static friction which can exist between that surface and that object.



Kinetic friction is the force you need to overcome to keep an object moving. If an object is already moving and you push with a force equal to the kinetic friction, the object will move with a constant velocity, since there is no net force. The amount of friction doesn't depend on the object's velocity.

Newton's 1st Law says that an object will move with a constant velocity unless there is a net force acting on it.



Static friction is greater than kinetic friction between the same two surfaces. So you need a larger force to start something moving than you need to keep it moving.

If the parallel component of the pushing force and the kinetic friction are equal in size, your object will move with a constant velocity, since there is no net force.

So there are grass and astro fields in SimFootball. We need to be able to **calculate** the friction for each so that the game knows how the players should move.

Friction depends on the types of surfaces that are interacting

The frictional force that a moving object (such as two football players sliding after a tackle) experiences depends on the nature of the two surfaces that are interacting. In physics, this is expressed by the **coefficient of friction**, μ. The greater the value of μ, the greater the amount of friction. Values of μ can range from around 0.05 for two teflon surfaces to around 1.7 for a rubber tire on a road.

μ is the Greek letter 'mu' and is pronounced 'mew'.

The larger the coefficient of friction, , the larger the frictional force.

For the football pitch surfaces, $\mu = 0.8$ for astro and $\mu = 0.5$ for grass. So you can see that the frictional force as the players slide along astroturf will be higher than the frictional force from grass.

Friction depends on the normal force

The frictional force that a moving object experiences depends on how much the object is "pushed into" the surface. This is another way of saying that friction depends on the **normal force** that a surface exerts on the object. **The greater the normal force, the greater the amount of friction.**

The **equation** for the size of the frictional force experienced by an object is:

$$F_{fric} = \mu F_N$$

Frictional force → F_{fric}; Coefficient of friction → μ; Normal force → F_N

The larger the normal force, the larger the frictional force.

This is just an equation for the **size** of the frictional force. $\mathbf{F}_{fric}$ will always oppose the direction of motion (or attempted motion) parallel to the surface. So to get the **direction** of $\mathbf{F}_{fric}$, you need to look at the velocity of the object (for kinetic friction) or the parallel component of a pushing force (for static friction).

When you know the direction of $\mathbf{F}_{fric}$, you can use $\mathbf{F}_{net} = m\mathbf{a}$ to calculate the players' **acceleration**. You can then use the acceleration in equations of motion to see how the players move.

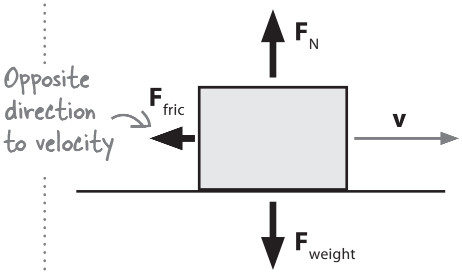
This applies to size. The normal force is perpendicular and the frictional force is parallel.

Be careful when you calculate the normal force

The normal force is always **perpendicular** to a surface. It should be the last force you add to a free body diagram, as it is the force exerted by the surface on an object so as to make the **net perpendicular force equal to zero**.

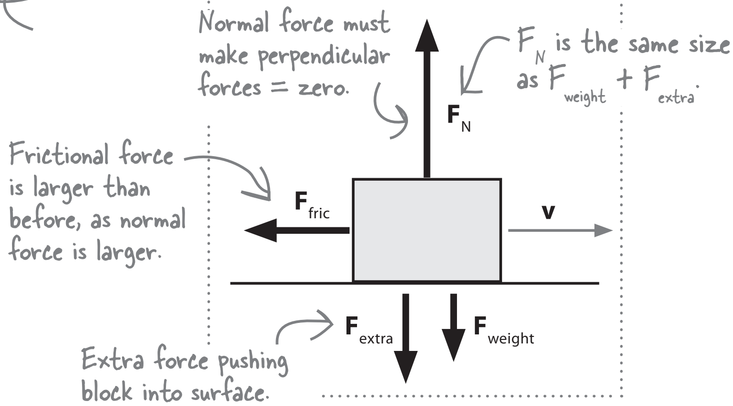
If the net perpendicular force wasn't zero, the object would either crash through the surface, burrow into the surface or bounce off the surface.

If surface is horizontal and there are no other forces acting on the object then the normal force is exactly the same as the object's weight.



If the surface is at an angle, the normal force is also at an angle, as it is always perpendicular.

If there are other forces acting on the object (for example, a player may be pushed into the ground by another player), then you need to ensure that you draw on the normal force so as to make the net perpendicular force equal to zero.

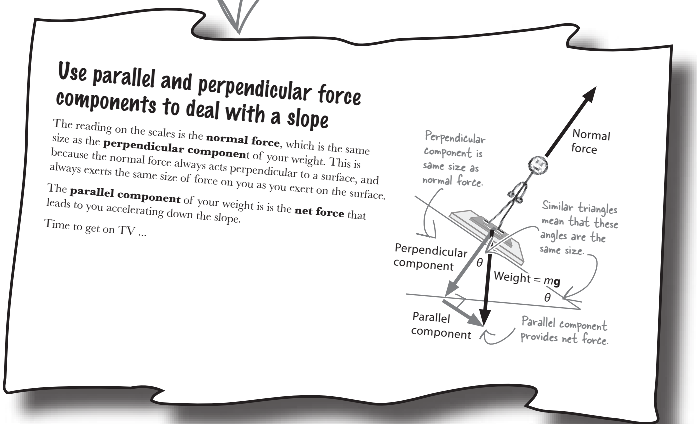


The normal force ensures that the net force perpendicular to the surface is zero.

If the object is on a **slope** then you'll have to work out the **perpendicular components** of all the forces acting on the object. Usually, this will just involve the object's weight, but it can sometimes involve extra forces if something else is pushing or pulling the object.

If there are other forces with perpendicular components in addition to the weight, you'll need to calculate the size of the normal force that makes the net perpendicular force equal to zero.

You already know how to calculate the normal force when an object is on a slope.


Use parallel and perpendicular force components to deal with a slope

The reading on the scales is the **normal force**, which is the same size as the **perpendicular component** of your weight. This is because the normal force always acts perpendicular to a surface, and always exerts the same size of force on you as you exert on the surface.

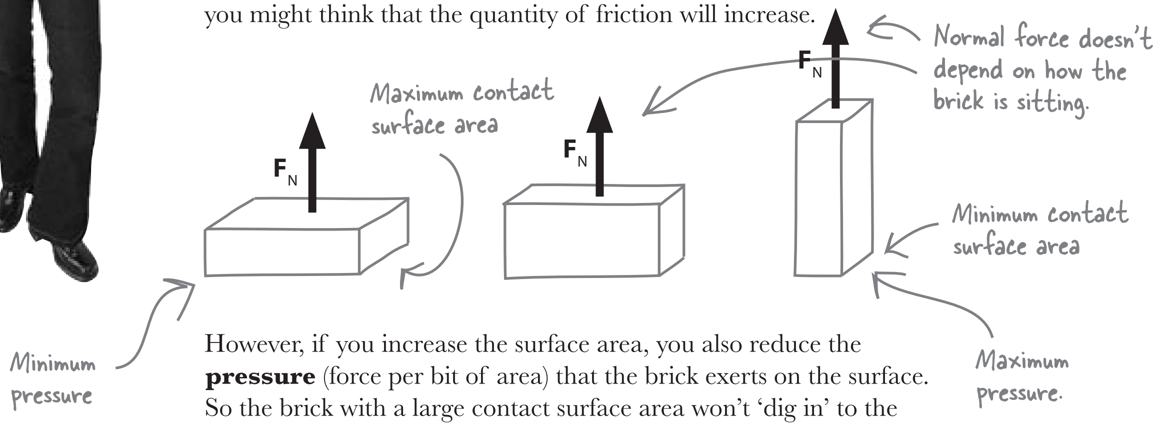
The **parallel component** of your weight is is the **net force** that leads to you accelerating down the slope.

Time to get on TV ...

Frictional force doesn't depend on surface area!

Since friction is a force that depends on **contact** between two surfaces, it seems logical to assume that you will have a greater value of friction when the surface area is greater.

But think of it this way instead. If you have a flat-sided brick, there are three different ways you can put it on a flat surface. The normal force is always the same each time, since the weight of the brick doesn't change. So if you maximize the contact surface area, you might think that the quantity of friction will increase.



However, if you increase the surface area, you also reduce the **pressure** (force per bit of area) that the brick exerts on the surface. So the brick with a large contact surface area won't 'dig in' to the surface as much as one on its end - in the same way as flat-soled shoes don't 'dig in' to a surface as much as high heels do.

It turns out that these two effects - increasing the surface area and reducing the pressure - exactly cancel each other out when it comes to their effect on the frictional force experienced by the brick. The frictional force depends **only** on the coefficient of friction and the normal force, i.e. $\mathbf{F}_{\text{fric}} = \mu \mathbf{F}_N$

The frictional force ONLY depends on the coefficient of friction and the normal force.

You're ready to use friction in the game!

You should start any problem that involves forces with a **free body diagram**. This is even more important if **friction** is involved. Remember that friction always acts to oppose motion, so you need to think about the object's **velocity** to get the direction of the frictional force.

Think about which direction the object is **accelerating** in (if any). If it isn't accelerating perpendicular to the surface, then all of the perpendicular force **components** will cancel each other out so that there is no net force in that direction. The value of the normal force will be whatever makes the net perpendicular force equal to zero.

Look at the pictures on page 489 if this sounds mind-boggling.

If a problem involves forces, start by drawing a free body diagram.

Sharpen your pencil

After a tackle, two players with a combined mass of 215 kg slide horizontally along the ground with an initial velocity of 3.70 m/s.

How long does it take for them to come to a complete stop on a. Astroturf ($\mu = 0.80$) and b. grass ($\mu = 0.50$)?

Hint: Use the normal force to calculate the frictional force. Work out the acceleration that the frictional force produces using $F_{net} = ma$. Then use the acceleration and equations of motion to calculate the stopping time.

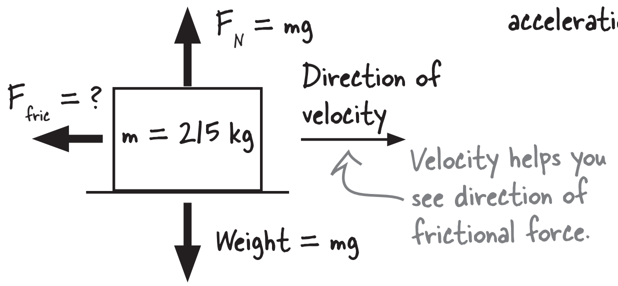
Hint: Draw a free body diagram to get the forces right. Draw a separate sketch to use with equations of motion.

Sharpen your pencil Solution

After a tackle, two players with a combined mass of 215 kg slide horizontally along the ground with an initial velocity of 3.70 m/s.

How long does it take for them to come to a complete stop on a. Astroturf ($\mu = 0.80$) and b. grass ($\mu = 0.50$)?

Free body diagram



Want to work out time. Use $F_{net} = ma$ to work out acceleration then equations of motion to get t.

$$F_{net} = ma$$

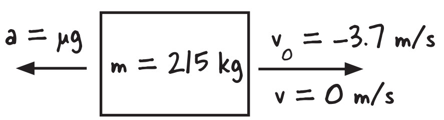
$$F_{net} = F_{fric} = \mu F_N$$

The net force is the frictional force.

$$\mu F_N = ma$$

$$a = \frac{F_{net}}{m} = \frac{\mu F_N}{m} = \frac{\mu \cancel{m} g}{\cancel{m}} = \mu g$$

Equations of motion sketch



Right to left is positive. ←

Choose a positive direction and stick with it.

$$v = v_0 + at \Rightarrow t = \frac{v - v_0}{a} = \frac{v - v_0}{\mu g}$$

a. For astro: $t = \frac{v - v_0}{\mu g} = \frac{0 - (-3.70)}{0.80 \times 9.8} = \underline{\underline{0.47\text{ s}}}$ (2 sd)

b. For grass: $t = \frac{v - v_0}{\mu g} = \frac{0 - (-3.70)}{0.50 \times 9.8} = \underline{\underline{0.76\text{ s}}}$ (2 sd)

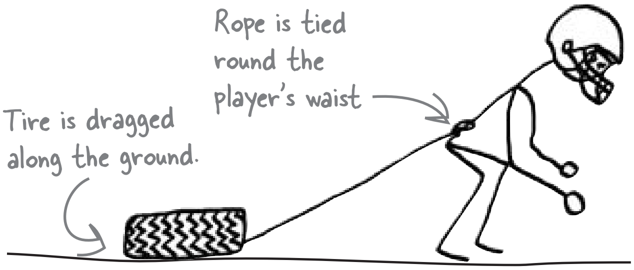
Including friction stops the players from sliding forever!

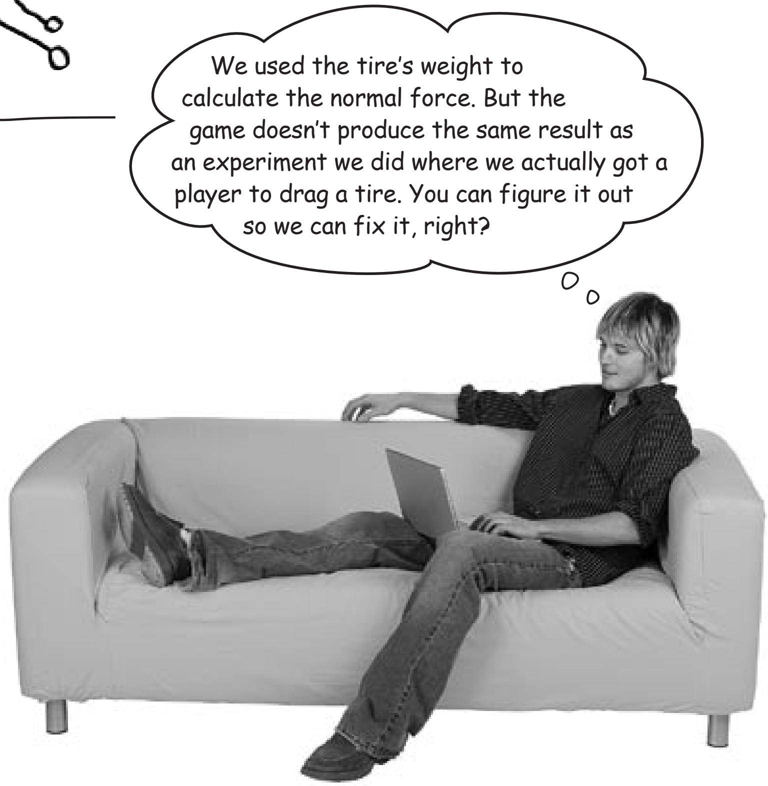
You explain to the programmer that the players sliding along a surface will experience a frictional force that opposes their current velocity, with size $\mathbf{F}_{fric} = \mu \mathbf{F}_N$.

When he includes this in the game, the players stop sliding endlessly, and come to a stop like you'd expect them to in real life. Which is great!

The sliding players are fine - but the tire drag is causing problems

Soon, the SimFootball team have another problem that involves **friction**. In training mode, the players can run dragging a tire behind them.



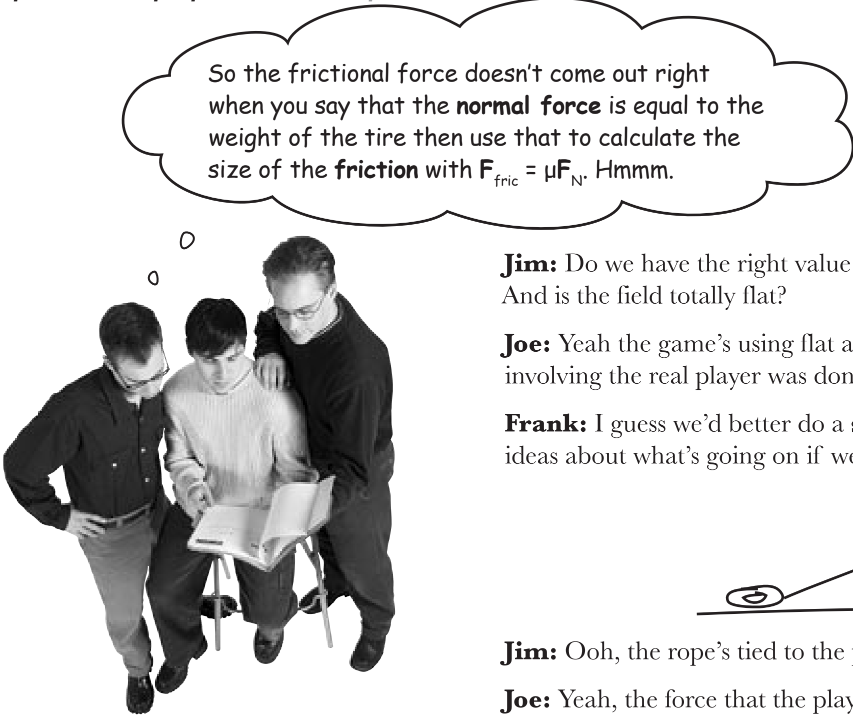


The programmer's tried working out the weight of the tire and making the normal force the same size as the tire's weight to calculate the friction. But the computer-generated players aren't behaving the way the programmer expects them to.

The frictional force that the game calculates appears to be larger than the frictional force actually is is in real life.

BRAIN POWER

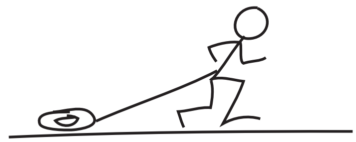
What could be behind the game calculating too high a value for the friction?



Jim: Do we have the right value for μ, the coefficient of friction? And is the field totally flat?

Joe: Yeah the game's using flat astroturf, and the experiment involving the real player was done on astroturf too, with no slope.

Frank: I guess we'd better do a **sketch** - we might have some more ideas about what's going on if we can actually see this.



Jim: Ooh, the rope's tied to the player's waist, isn't it?

Joe: Yeah, the force that the player exerts on the tire via the rope acts at an **angle**.

Frank: Will that make the normal force different somehow?

Jim: I think so - the player's kind-of pulling the tire up as well as along. Look at the **components** of the pulling force:

Joe: The rope's **supporting** the tire vertically as well as pulling it horizontally. The normal force will be smaller than the tire's weight.

Frank: So how do we work with that?

Jim: Well, the tire's weight vector points downwards. The vertical component of the force from the rope points upwards. And the normal force points upwards. The tire isn't rising or burrowing down - there's no net perpendicular force. So the tire's weight, the vertical force from the rope and the normal force must add to zero.

Joe: And the tire's moving horizontally with a **constant velocity,** so the frictional force and the horizontal component of the force from the rope must add to zero, so that there's **zero net horizontal force**. Or else the tire would accelerate.

If forces act at angles, think about resolving them into components that are parallel and perpendicular to a surface.

Sharpen your pencil

A football player has one end of a rope tied around his waist; the other end is attached to a tire. The programmer has done a brief experiment involving a real player and tire, and wants to calculate the force that the player exerts on the tire via the rope so he can use it in the game.

a. Draw a free body diagram of the tire when it is dragged along the ground with a constant velocity. Use $\mathbf{F}_r$ to represent the force that the player exerts on the tire via the rope.

Don't put on any values yet, just draw the force vector arrows and say what they represent.

b. Draw a new sketch showing the horizontal and vertical components of all the forces acting on the tire.

c. Use the vertical components to work out an equation for the normal force, $\mathbf{F}_N$.

d. Use the fact that the frictional force, $\mathbf{F}_{fric} = \mu \mathbf{F}_N$ to arrive at an equation that only involves the components of $\mathbf{F}_{fric}$, the mass of the tire, the coefficient of friction and $\mathbf{g}$, the gravitational field strength.

e. The tire has a mass of 10.0 kg, the rope is 2.00 m long and the player's belt is 120 cm above the ground, which is astro with $\mu = 0.80$. Use your equation from part d to work out $\mathbf{F}_r$, the force that the player exerts on the tire via the rope.

Sharpen your pencil Solution

A football player has one end of a rope tied around his waist; the other end is attached to a tire. the programmer has done a brief experiment involving a real player and tire, and wants to calculate the force that the player exerts on the tire via the rope so he can use it in the game.

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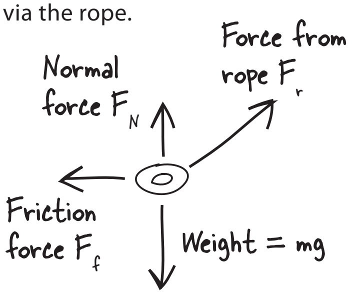
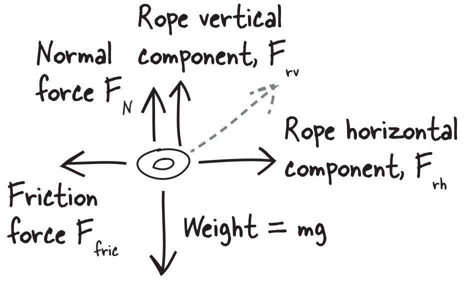


b. Draw a new sketch showing the horizontal and vertical components of all the forces acting on the tire.



c. Use the vertical components to work out an equation for the normal force, $\mathbf{F}_N$.

Up is the positive direction.

No net force so $F_N + F_{rv} - mg = 0$

$$F_N = mg - F_{rv}$$

d. Use the fact that the frictional force, $\mathbf{F}_{fric} = \mu\mathbf{F}_N$ to arrive at an equation that only involves the components of $\mathbf{F}_{fric}$, the mass of the tire, the coefficient of friction and $\mathbf{g}$, the gravitational field strength.

Right is the positive direction.

Constant velocity so $F_{rh} + (-F_{fric}) = 0$ (There's zero net horizontal force.)

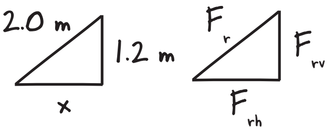
$$F_{fric} = F_{rh}$$

But also $F_{fric} = \mu F_N = \mu(mg - F_{rv})$

$$\Rightarrow \mu(mg - F_{rv}) = F_{rh}$$

e. The tire has a mass of 10.0 kg, the rope is 2.0 m long and the player's belt is 120 cm above the ground, which is astro with $\mu = 0.80$. Use your equation from part d to work out $\mathbf{F}_r$, the force that the player exerts on the tire via the rope.

Rope distance triangle and force triangle are similar triangles.



By Pythagoras, $x^2 + 1.2^2 = 2.0^2 \Rightarrow x = 1.6$ m

Similar triangles: $\dfrac{F_{rv}}{F_r} = \dfrac{1.2}{2.0} \Rightarrow F_{rv} = 0.60F_r$

$$\frac{F_{rh}}{F_r} = \frac{1.6}{2.0} \Rightarrow F_{rh} = 0.80F_r$$

Equation: $\mu(mg - F_{rv}) = F_{rh}$

$$\Rightarrow \mu(mg - 0.60F_r) = 0.80F_r$$

$$\Rightarrow \mu mg - 0.60\mu F_r = 0.80F_r$$

$$\Rightarrow \mu mg = F_r(0.60\mu + 0.80)$$

$$\Rightarrow F_r = \frac{\mu mg}{0.60\mu + 0.80} = \frac{0.80 \times 10 \times 9.8}{(0.60 \times 0.80) + 0.80}$$

$$F_r = 61 \text{ N (2 sd)}$$

Using components for the tire drag works!

Now that the programmer knows how to calculate the normal force, there's no stopping him, and the parts of the game that are affected by friction are soon in place.

And as well as sliding players and dragging tires, the game can even deal with dragging tackles!

there are no Dumb Questions

Q: What do I need to know in order to calculate the force of friction?

A: The equation for the size of the frictional force is $F_{fric} = \mu F_N$. So you need to know the coefficient of friction for the surfaces you have, and the normal force.

Q: How do I find out what the coefficient of friction is?

A: You can look it up in a book or on the web. And if you're taking a test, μ will either be something you're given or something you're asked to work out from the values of various forces.

Q: How do you get the normal force?

A: The object isn't accelerating into the surface, so the perpendicular components of the forces acting on it must add to zero. The normal force will have the value that makes this possible

Q: I've noticed that an object travelling at a constant velocity has come up more than once. Is there a reason for that, and what's the best way of dealing with it?

A: There are many situations where you'd want something to travel with a constant velocity. A constant velocity means that there's no net force on an object (Newton's 1st Law) - you'll be fine if you remember this.

Q: What if there is a frictional force acting on an object? How do you get a situation where there is no net force?

A: Either by pushing or pulling the object with a force equal to the frictional force, or by tipping the surface to a greater angle, so that the normal force (and therefore the frictional force) is smaller, and the component of the object's weight accelerating it down the slope is greater.

Q: Does the tire experience the same frictional force when it rolls?

A: No. It experiences a relatively small amount of rolling friction, due to the part of the tire in contact with the surface deforming.

Friction Exposed

This week's interview:
Getting to grips with friction.

HeadFirst: So, friction, you're a bit of an enigma, aren't you? Like, no-one really knows where you come from. What's your take on that?

Friction: Yeah, it's true that people don't really know why I'm around. But the important thing is that I'm here!

HeadFirst: But you're a bit of a stick-in-the-mud, aren't you? You always oppose everything!

Friction: It's true that I always **oppose motion**, but it's not something you should take personally.

HeadFirst: And you're a little without direction, aren't you? I mean, you always depend on what everyone else is doing!

Friction: OK, well I guess that's true. Because I'm a force that always opposes motion, I don't actually appear until something actually moves, or tries to move. But as long as you're watching closely, that shouldn't be a problem.

HeadFirst: So are you in surfaces all the time, just hiding and waiting to come out?

Friction: Not at all. I'm just not there unless something's moving or trying to move.

HeadFirst: You've used that phrase "moving or trying to move" a couple of times now. What do you mean by it?

Friction: Well, I come in a couple of different varieties. If you're already moving, the force that opposes this motion is called **kinetic friction**.

HeadFirst: Why is it called that?

Friction: Kinetic means "moving"!

HeadFirst: And what if an object's stationary then someone comes along and tries to move it?

Friction: Then there's **static friction**. I guess the two surfaces have longer to interact with each other because they're stationary, and so the frictional force you have to overcome is greater.

HeadFirst: But kinetic friction doesn't depend on velocity, right?

Friction: Right. That's why having a mental picture of "bonds forming and breaking" or something like that can be misleading

HeadFirst: So what's this about you and the normal force?

Friction: I was wondering when that would come up! I can only oppose motion if two surfaces are actually touching. And the more they're "pushed together" the larger the frictional force between them. The **normal force** is a measure of how hard the two surfaces are being "pushed together."

HeadFirst: And how might the normal force vary?

Friction: If the surface is at an **angle**, and there are no other forces present, then the normal force will be less than the weight of the object.

HeadFirst: Is it only the angle of the surface that affects the normal force?

Friction: No - if there are extra forces acting on the object as well as its weight and the normal force, then the perpendicular components all have to add to zero.

HeadFirst: Why is that?

Friction: If the object isn't burrowing into or bouncing off the surface, there's no net perpendicular force. So the perpendicular components of the forces acting on the object have to add to zero. The normal force is whatever it needs to be for that to be true.

Question Clinic: The "Friction" Question

Sometimes, you will be presented with a problem where friction plays a vital role. The main thing in this type of problem is to work out the normal force (which you can then multiply by μ to get the frictional force) - so always start with a free body diagram. NB: This example question has these 'intermediate steps' filled in for you; in other questions you may need to carry them out unprompted as you home in on your final answer.

If the overall velocity is constant (or zero), this means that there is no net force acting on the object.

Friction always opposes motion, so the frictional force will be in the opposite direction from the velocity.

2. A tire attached to a rope is dragged along the ground by a player with a constant velocity. The tire has a mass of 10 kg, the rope is 2.0 m long and the player's belt is 120 cm above the ground, which **is Astroturf with μ = 0.8**

a. Draw a free body diagram for the tire.

b. Draw in the components parallel and perpendicular to the ground for any forces that aren't entirely parallel or perpendicular.

c. Use the perpendicular components to work out the normal force and hence the frictional force in terms of F, the force the player exerts on the tire via the rope.

d. Use the parallel components to work out a value for F.

The normal force acts perpendicular to the surface – so you'll need to turn any vector that's not already perpendicular or parallel to the surface into components.

As the normal force depends on all of the other force components perpendicular to the surface (as it needs to balance them all so that the net force in the direction is zero), it should be the last thing you calculate.

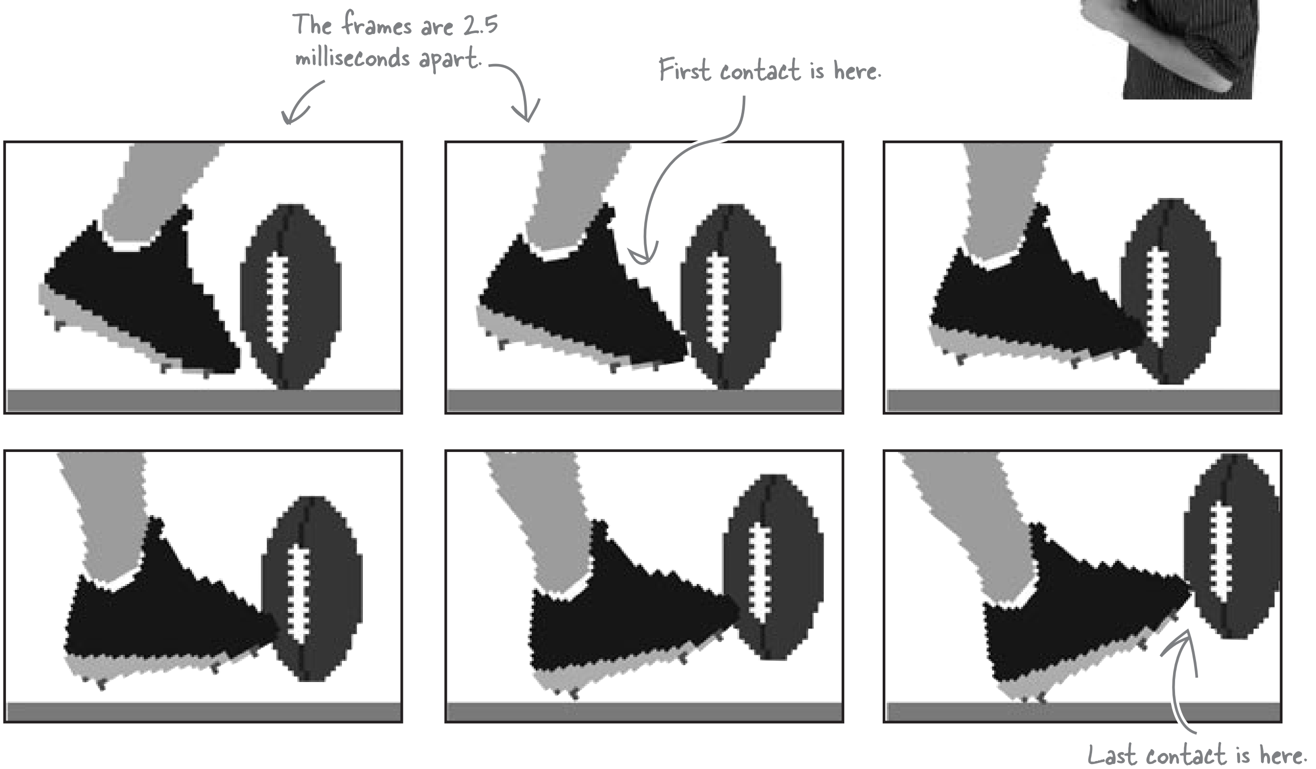
If the velocity in a particular direction is constant (or zero) then there is no net force in that direction, i.e. all the components must add up to zero.

An important thing to remember when doing problems that involve forces is that 'constant velocity' is shorthand for 'no net force'. Usually, this means that the frictional force will have the same size as the component of the force that's causing the object to move.

How does kicking a football work?

So the game's nearly complete ... but the SimFootball team want to make kicking the football as realistic as possible. They've got their hands on some freeze frame footage - and have worked out the ball's **velocity** as it heads for goal. But they need you to work out the **average force** of the kick so they can program it in

But how are you going to do that when you don't know the ball's **acceleration**, can't use $\mathbf{F}_{net} = m\mathbf{a}$?



BRAIN POWER

What might you be able to do with the images to work out the average force that the ball experiences when the player kicks it?

Jim: But we don't know the football's **acceleration** - we only know the **velocity** that it takes off with.

Frank: Hmm, good point.

Jim: Can we somehow use the freeze frame footage to work out the acceleration?

Joe: I think that's gonna be difficult. The ball deforms when it's kicked, then expands to its normal shape again. Which part of the ball would we use to work out the acceleration, when different parts are moving in different ways?!

Frank: But we **can** use the freeze frames to work out the **time** that the foot's in contact with the ball for. It looks like it's around 10 milliseconds... if we can use the time, it might help us somehow.

Joe: Oh ... hang on! Newton's second Law isn't actually $\mathbf{F}_{net} = m\mathbf{a}$, is it? In its purest form, it actually says that when you apply a force for a period of time, then it causes a **change in momentum**.

Jim: So you're saying that we can use $\mathbf{F}_{net}\Delta t = \Delta\mathbf{p}$ (where $\mathbf{p} = m\mathbf{v}$)? That's cool: momentum is mass × velocity, and we know what both of these are for the football!.

Frank: And we can get the time of contact from the freeze frame! That's the time that the force acts for, isn't it? 10 milliseconds?

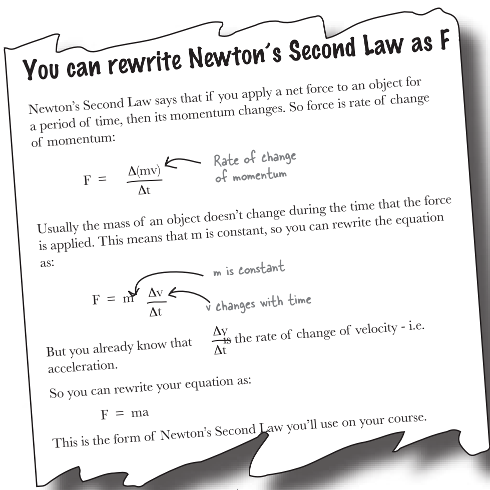
Jim: Yeah, that sounds good. Though the force doesn't look like it'll be the same all the time. I'm sure the middle of the kick exerts more force than the start or end of it ...

Joe: But we've been asked to find the **average force**. When we were finding average speeds it was only the overall change in position that counted. So with the football, we can use the overall change in momentum to work out the average force.

Frank: So we are using Newton's Second Law like I suggested all along - but just a **different form** of it.

Jim: Yeah. Come on - let's do it!

Earlier on, you rewrote Newton's Second Law, having originally worked it out from momentum conservation.


You can rewrite Newton's Second Law as F

Newton's Second Law says that if you apply a net force to an object for a period of time, then its momentum changes. So force is rate of change of momentum:

$$F = \frac{\Delta(mv)}{\Delta t}$$ ← Rate of change of momentum

Usually the mass of an object doesn't change during the time that the force is applied. This means that m is constant, so you can rewrite the equation as:

$$F = m\frac{\Delta v}{\Delta t}$$ (m is constant; v changes with time)

But you already know that $\frac{\Delta v}{\Delta t}$ is the rate of change of velocity - i.e. acceleration.

So you can rewrite your equation as:

$$F = ma$$

This is the form of Newton's Second Law you'll use on your course.

Newton's Second Law in its purest form is:

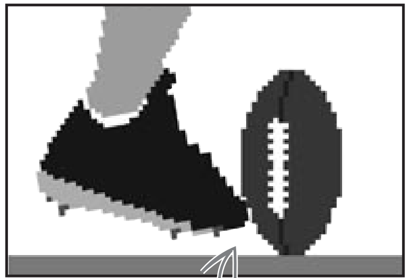
$$F_{net} = \frac{\Delta p}{\Delta t}$$

$F\Delta t$ is called impulse

You can work out the force of the kick using a slightly different form of Newton's Second Law, $\mathbf{F}\Delta t = \Delta\mathbf{p}$. The quantity $\mathbf{F}\Delta t$ is also called **impulse**, and the equation says that impulse is equal to the change in something's **momentum**.

If you have a problem where your first instinct is to use $\mathbf{F}_{net} = m\mathbf{a}$ but you don't know the acceleration, look to see if you know the **mass** and **velocity** at the start and at the end.

If you do, you can work out the change in momentum, which is equal to the impulse, and then get the force from that.



Foot is in contact with ball for time Δt during which it exerts force F on it.

The total momentum of everything taking part in an interaction is conserved.

Momentum = mass x velocity

Symbol: p

Symbol: a

The acceleration is the rate of change of an object's velocity.

These are related by the equation $F_{net} = ma$

Symbol: F

The force is the rate of change of an object's momentum.

If you apply a force for a short time, you get a smaller change in momentum than if you apply it for a long time.

The impulse is the actual change in an object's momentum.

Δp also happens to be called impulse. And you can write down the equation $F\Delta t = \Delta p$

Some people give impulse the symbol J, many others don't bother with a symbol.

Sharpen your pencil

a. The programmer wants to know the average force of the kick. The football has a mass of 400 grams and from the freeze frames, you can tell that the foot and ball are in contact for 10 ms (milliseconds). If the ball leaves the boot at an angle of 45° and travels 60 m, work out the force of the kick.

Hint: 45° is when the horizontal and vertical components of the ball's velocity are equal.

Hint: Use equations of motion to calculate the initial velocity. Look back at pages $$-$$ of chapter 10 if you get stuck, as the problem there is very similar.

Hint: Once you know the initial velocity, you can use $\mathbf{F}\Delta t = \Delta\mathbf{p}$. (Remember that $\mathbf{p} = mv$)

b. Experiencing a large net contact force hurts! Explain, using impulse, why football players wear padding.

Sharpen your pencil Solution

a. The programmer wants to know the average force of the kick. The football has a mass of 400 grams and from the freeze frames, you can tell that the foot and ball are in contact for 10 ms (milliseconds). If the ball leaves the boot at an angle of 45° and travels 60 m, work out the force of the kick.

Hint: 45° is when the horizontal and vertical components of the ball's velocity are equal.

Hint: Use equations of motion to calculate the initial velocity. Look back at pages $$-$$ of chapter 10 if you get stuck, as the problem there is very similar.

Hint: Once you know the initial velocity, you can use **F**Δ*t* = Δ**p**. (Remember that **p** = *m*v)

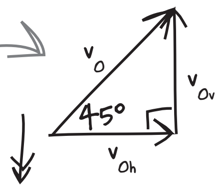
Use $F\Delta t = \Delta mv$ to work out the force. So work out the initial velocity, v_0, from 60 m range.

45° angle, so $v_{0h} = v_{0v}$

Pythagoras: $v_0^2 = v_{0h}^2 + v_{0v}^2$

$\Rightarrow v_0^2 = 2v_{0v}^2$

$v_0 = \sqrt{2}\, v_{0v}$



$a = -9.81\ m/s^2$

$x_0 = 0\ m$

$x = 60\ m$

UP is positive

RIGHT is positive

Symmetry: Replace v_v with $-v_{0v}$ in every equation you use

Get time from vertical component, then distance in that time from horizontal.

Vertically: $v_v = -v_{0v}$ because of symmetry. Use this in equation of motion:

$v_v = v_{0v} + at$

$t = \frac{-v_{0v} - v_{0v}}{a} = \frac{-2v_{0v}}{-9.8} = 0.204 v_{0v}$

Negative divided by negative is positive.

Use this value for t with horizontal component of velocity.

Horizontally: $v_{0h} = v_{0v} = \frac{x - x_0}{t}$

v_{0h} and v_{0v} are the same size. A right-angled triangle with two equal 45° angles has two equal sides.

$v_{0v} = \frac{60 - 0}{0.204 v_{0v}}$

There's a v_{0v} on both sides, so you can solve for v_{0v}, and then for v.

Multiply both sides by v_{0v} → $v_{0v}^2 = 294 \Rightarrow v_{0v} = 17.1$ m/s (3 sd)

From 45° triangle, $v_0 = \sqrt{2}\, v_{0v} \Rightarrow v_0 = \sqrt{2} \times 17.1 = 24.2$ m/s (3 sd)

This is contact time for foot and ball, not flight time!

Rearrange $F\Delta t = \Delta mv$ to get F: $F = \frac{\Delta mv}{\Delta t} = \frac{0.4 \times 24.2}{10 \times 10^{-3}} = 968\ N$

Force is measured in Newtons.

The average force of the kick is 970 N (2 sd).

b. Experiencing a large net contact force hurts! Explain, using impulse, why football players wear padding.

If you have velocity v and are tackled so that your final velocity = 0, your momentum has changed from mv to 0. And Newton's Second Law / impulse says that $F\Delta t = mv$

If you're not wearing padding, then this happens over a short time. So F is high and it hurts.

If you're wearing padding, then this happens over a longer period of time, as the padding deforms. So F is lower than if you weren't wearing any padding at all and it hurts less.

If a collision takes more <u>time</u>, the average <u>force</u> is lower - and it hurts less!

there are no Dumb Questions

Q: So why does this thing have the special name "impulse"? Why can't I just call it "change in momentum" like we've been doing all along?

A: Because "impulse" is what it's called! If you understand how it works that's great - but you need to be able to communicate with other people who call it impulse.

Q: But if I explain what I mean, won't they get it?

A: If your exam question asks you to explain something using impulse (like the question about padding did) then you need to know what impulse is.

Q: Yeah, the question about padding. Surely padding works because it absorbs some of the hit so you don't feel it as much by the time it gets to you? What does that have to do with impulse?

A: You just said "the time it gets to you". If the interaction takes place over a longer time, the average force is lower.

Q: What does that have to do with it?

A: Big forces hurt! If you were wearing a suit of armor instead of padding, it wouldn't deform. The collision would take the same time as it did before, and it would hurt just as much.

The game's great - but there's just been a spec change!

You and the SimFootball team have come up with a realistic game that's also fun to play! Big win! But before you all collect your VIP passes, the CEO takes a look at the game - and decides he wants to have a mode where you can play football on the moon!

The strength of the moon's gravitational field is lower then the Earth's

The moon is smaller and less massive than the Earth, so the gravitational force it exerts on objects is less, which means its gravitational field strength is less. You need to work out how this will affect the physics of the game

The players will be in a pressurized dome where there's plenty of air, so you don't need to worry about anything medical!

BRAIN POWER

How is being on the moon going to affect the game (if at all)?

Sharpen your pencil

Which aspects of the game will change as a result of being on the moon and which will stay the same? The SimFootball programming team have already had a go at guessing what will happen - and you need to decide whether each of these statements is correct or not.

If you think a statement and the reasoning is correct, please explain why, using physics.

If a statement is incorrect, or an incorrect reason is given for a correct statement, then please explain why using physics - to debunk the myth! Use relevant equations wherever you can.

a. The ball will go further when passed horizontally because it weighs less so is easier to throw.

....

....

b. The ball will go further when passed horizontally because it weighs less so spends longer in the air.

....

....

c. The ball will go further when passed horizontally because the gravitational field strength is less.

....

....

d. There will be less friction in the game so the players will slide further.

....

....

e. Tackles will involve less force because the players weight less.

....

....

f. The optimal angle where punts go furthest won't be 45° any more because the ball weighs less.

....

....

g. The ball will have a higher velocity when it leaves a player's boot because it weighs less.

....

....

h. If a player runs into and collides with a goalpost, it will hurt less because they weigh less.

....

....

Sharpen your pencil Solution

Which aspects of the game will change as a result of being on the moon and which will stay the same? The SimFootball programming team have already had a go at guessing what will happen - and you need to decide whether each of these statements is correct or not.

If you think a statement and the reasoning is correct, please explain why, using physics.

If a statement is incorrect, or an incorrect reason is given for a correct statement, then please explain why using physics - to debunk the myth! Use relevant equations wherever you can.

a. The ball will go further when passed horizontally because it weighs less so is easier to throw.

No – wrong reason! The ball still has the same mass. Throwing force F_{net} = ma so the acceleration (and velocity) depend on the ball's mass, not its weight, if it's thrown horizontally.

b. The ball will go further when passed horizontally because it weighs less so spends longer in the air.

Yes. The ball accelerates vertically because of its weight. On the moon, the ball weighs less. Therefore it will go further, as it'll have a longer time to travel horizontally.

c. The ball will go further when passed horizontally because the gravitational field strength is less.

Yes. This is just another way of wording the statement in b.

d. There will be less friction in the game so the players will slide further.

Yes. If the gravitational field strength is less, the players weigh less, and the normal force is less. Therefore, there will be less friction in the game and the players will slide further.

e. Tackles will involve less force because the players weight less.

No. Newton's 2nd Law is F_{net} = ma. Tackling is horizontal. So their weights have nothing to do with it (weight would only have an effect if they were tackling vertically!).

f. The optimal angle where punts go furthest won't be 45° any more because the ball weighs less.

No. The optimal angle is always 45° whatever planet you're on!

g. The ball will have a higher velocity when it leaves a player's boot because it weighs less.

No. $F\Delta t = \Delta(mv)$. Force, time and mass are still the same, so the velocity is still the same.

h. If a player runs into and collides with a goalpost, it will hurt less because they weigh less.

No. Same reason as a, e and g. Any time the change is in the horizontal direction but not the vertical direction, the important thing is the mass, not the weight.

For added realism, sometimes the players should slip

After successfully adding the 'moon mode', the SimFootball team have decided that the game needs one more element. At the moment, the players are able to make impossibly tight turns and **change direction** more or less instantly. But if they tried that in real life, they would **slip**.

But what makes someone slip? Or rather - what makes someone able to change direction in the first place? A change of direction means a change in velocity. Newton's First Law says that for a velocity to change, there must be a net force. But where does the force that enables a player to change direction come from?

Sharpen your pencil

a. If a player changes direction, they change velocity, so there must be a net force acting on them in the same direction as the change in velocity.

Explain where this force comes from, and why a player might slip in real life.

b. Describe in words how you'd go about working out whether a player will slip when they change direction.

Sharpen your pencil Solution

a. If a player changes direction, they change velocity, so there must be a net force acting on them in the same direction as the change in velocity.

Explain where this force comes from, and why a player might slip in real life.

The player can change direction because of friction between their foot and the ground.

If the player exerts a force on the ground with their foot, the ground exerts an equally-sized force on the player in the opposite direction - a Newton's Third Law pair of forces.

If the force required for the change in direction is smaller than the force that can be provided by friction, then the player will slip.

b. Describe in words how you'd go about working out whether a player will slip when they change direction.

Work out what the player's change in momentum is when they change direction.

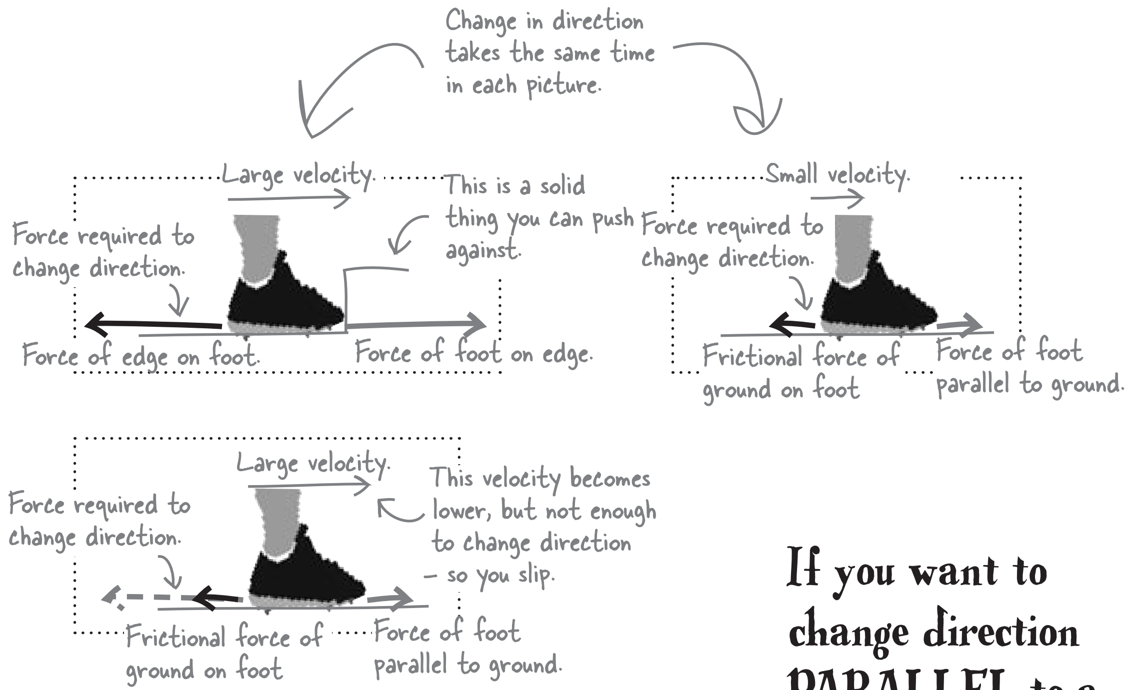
Work out what the maximum friction force is given the player's weight, the normal force and the surface he's playing on.

$F\Delta t = \Delta p$ Use what you worked out for F and Δp to work out how long the player's foot needs to be in contact with the ground to provide this change in momentum.

Estimate whether this is reasonable or not.

You can change only direction horizontally on a flat surface because of friction

If you're trying to change direction horizontally on a flat surface, friction is the only thing that can provide the force you require to change your momentum. Otherwise, you would slip (unless there's a convenient wall or curb you can push against instead).



If you want to change direction PARALLEL to a surface, friction is the only thing that can provide the force you require.

The game is brilliant, and going to X-Force rocks!

SimFootball is a success! Using physics, you were able to turn the real game into a computer game. Everything acts just like it should - on Earth and on the moon!

Newton's Laws give you awesome powers

You can use momentum conservation, Newton's Laws and free body diagrams to work out problems that involve forces.

The fundamental thing you can work out from experiments.

Total momentum is conserved.

Constant velocity (or zero velocity) means no net force.

Newton One: Things carry on.

This comes from momentum conservation

Newton Two: $F_{net} = ma$

Draw a FREE BODY DIAGRAM to work out the net force.

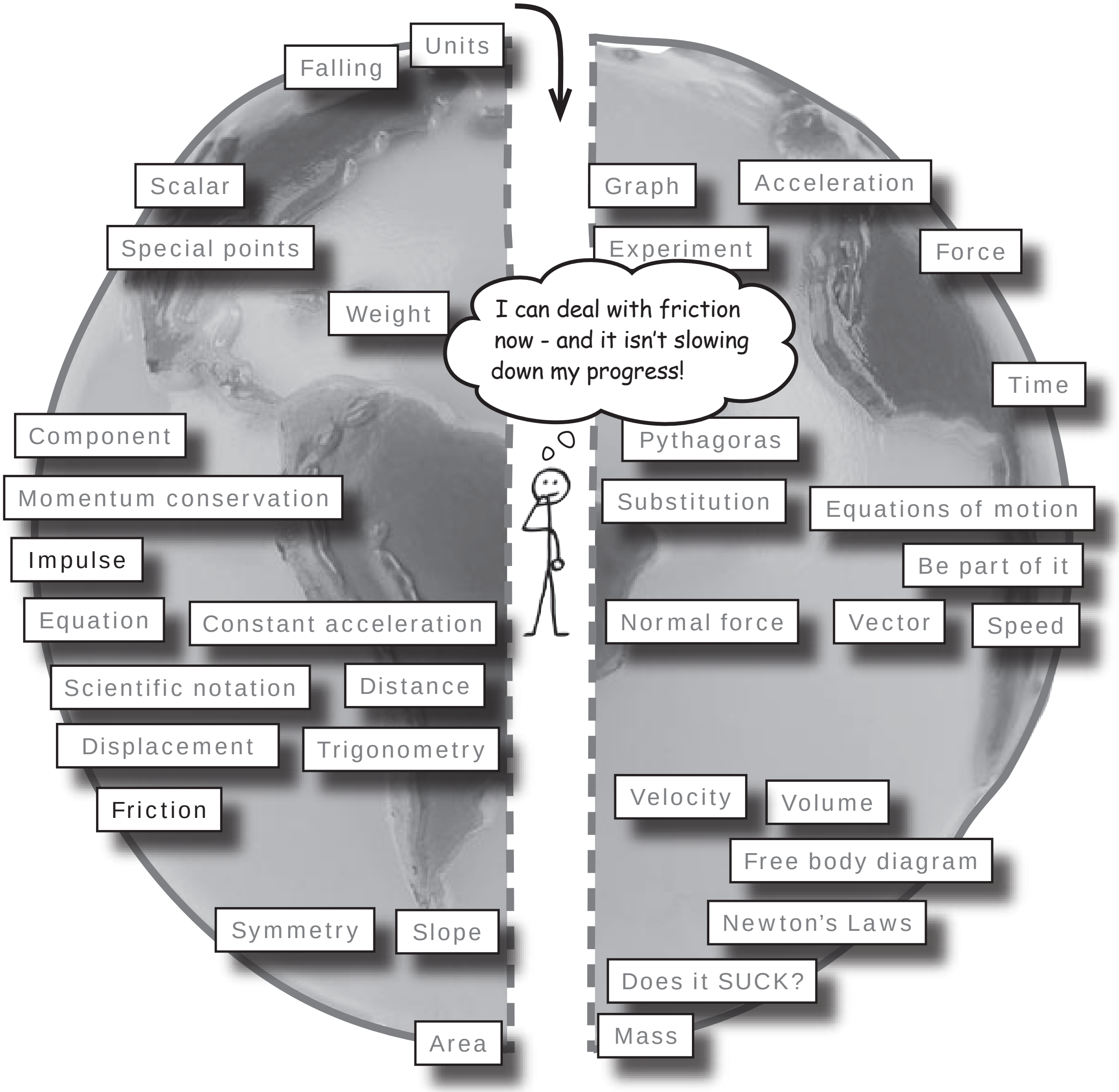
This comes from momentum conservation.

Newton Three: Push you, push me.

Forces come in pairs that are the same size in opposite directions and act on different objects.

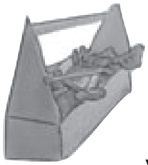
BULLET POINTS

- Always start with a free body diagram of all the forces acting on an object.
- Mark on all the forces.
- Is there a net force?
- Work out forces you don't know.
- Use $F_{net} = ma$ to determine how the object moves.
- Remember the impulse form of Newton's Second Law: $F_{net}\Delta t = \Delta p$
- Total momentum is conserved!



☑	Friction	A contact-dependent force that opposes motion.
☑	Impulse	Impulse is equal to the change in momentum, $F_{net}\Delta t$. Impulse is sometimes given the symbol J.

CHAPTER 12



Your Physics Toolbox

You've got Chapter 12 under your belt and added some problem-solving concepts to your toolbox.

How many objects?

Before you start a problem, think about how many objects there are interacting in it.

If there is only one object, you can probably use equations of motion to work out what happens

But if there are two or more objects, or if there are forces involved, then look to use Newton's Laws, momentum conservation or impulse (or a combination of these).

Working with forces and equations of motion

A common way of making progress on a problem is to use Newton's 2nd Law (usually in the form $F_{net} = ma$, though occasionally in the form $F_{net}\Delta t = \Delta p$) to calculate the acceleration (or velocity) an object experiences.

Then you can use this value in your equations of motion to find out how the object moves as a result of the force.

Calculating friction

The friction experienced by an object on a surface depends on the normal force and the coefficient of friction, μ, for that object and that surface.

$$F_{fric} = \mu F_N$$

The normal force

Be careful when calculating the normal force as part of a friction problem (or indeed any problem).

The normal force is perpendicular to a surface. As long as the object in contact with the surface isn't accelerating in the perpendicular direction, the normal force has the right size to make the net perpendicular force equal to zero.

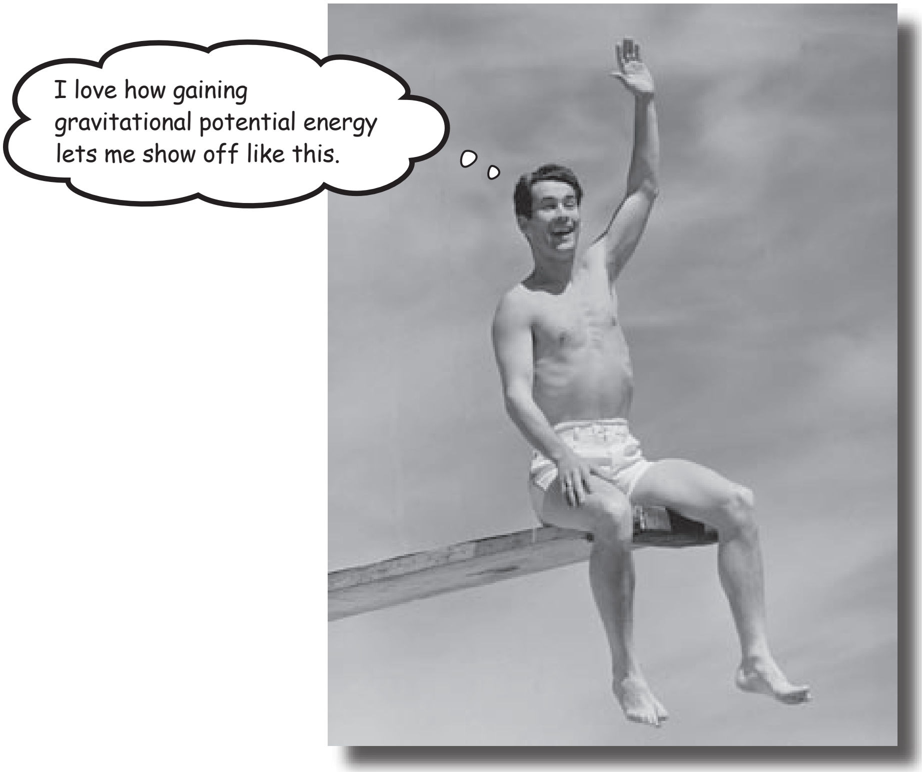
"Constant velocity"

If an object moves with "constant velocity", it means that there is no net force on an object

To solve a problem where an object moves with constant velocity, you should draw a free body diagram and start equating forces and/or components of forces.

13 torque and work

Getting a lift

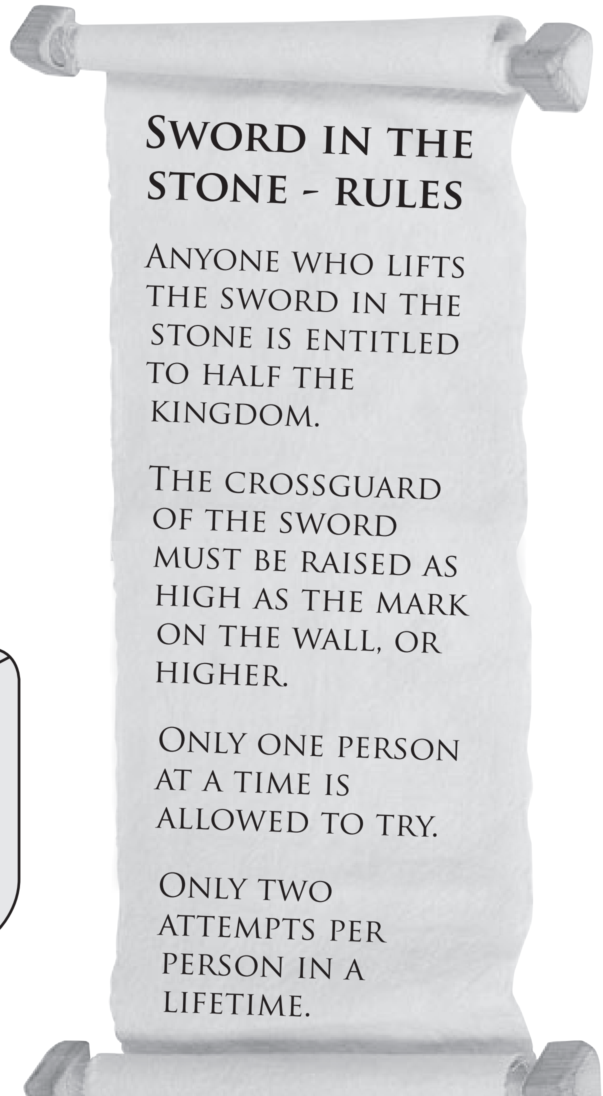


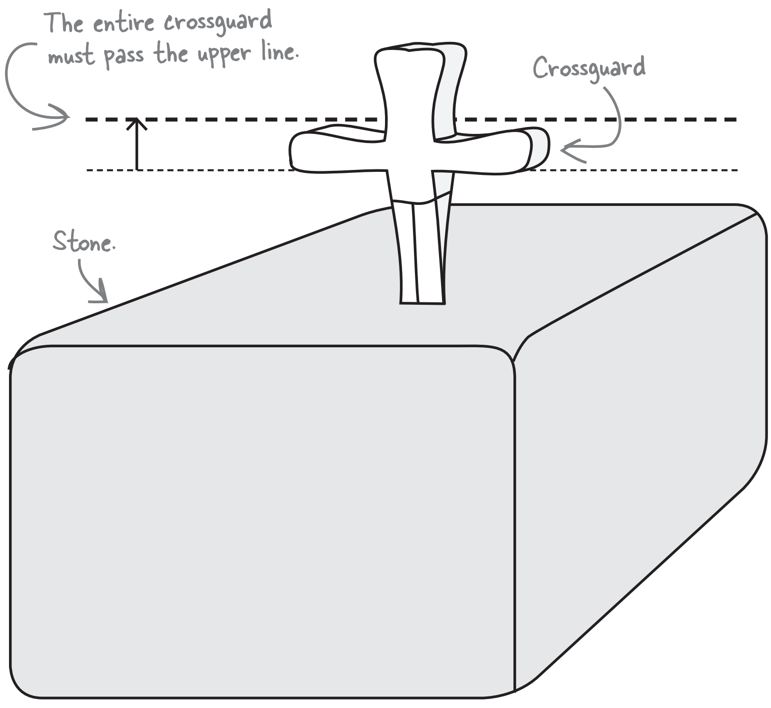
You can use your physics knowledge to do superhuman feats. In this chapter, you'll learn how to harness torque to perform amazing displays of strength, by using a lever to exert a much larger force than you could on your own. However, you can't get something for nothing - **energy** is always **conserved** and the amount of **work** you do to give something **gravitational potential energy** by lifting it doesn't change.

Half the kingdom to anyone who can lift the sword in the stone ...

The sword in the stone has acquired near-legendary status. But now, in a shock move, anyone can attempt to lift it.

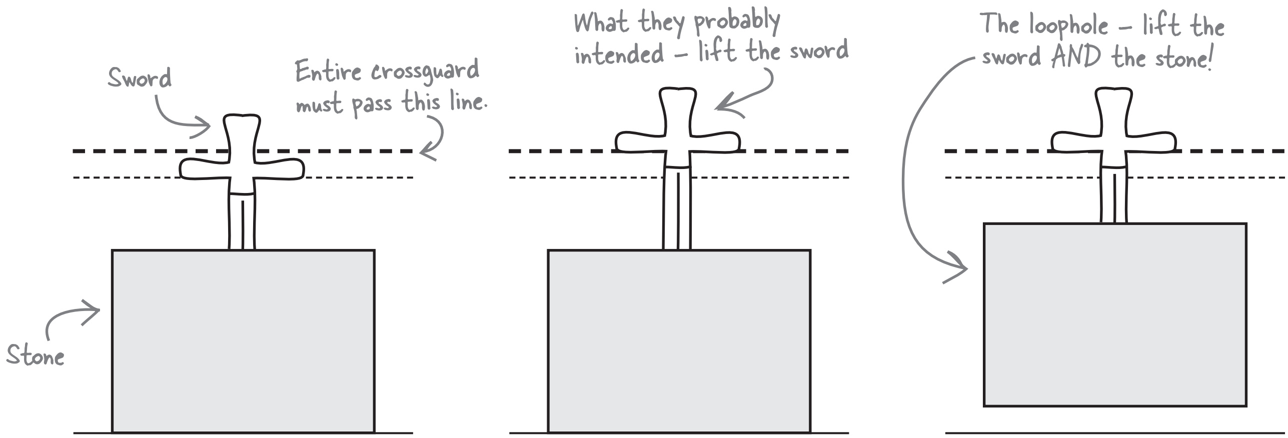
There are rules of course - but the promise of half the kingdom for anyone who succeeds is completely genuine.



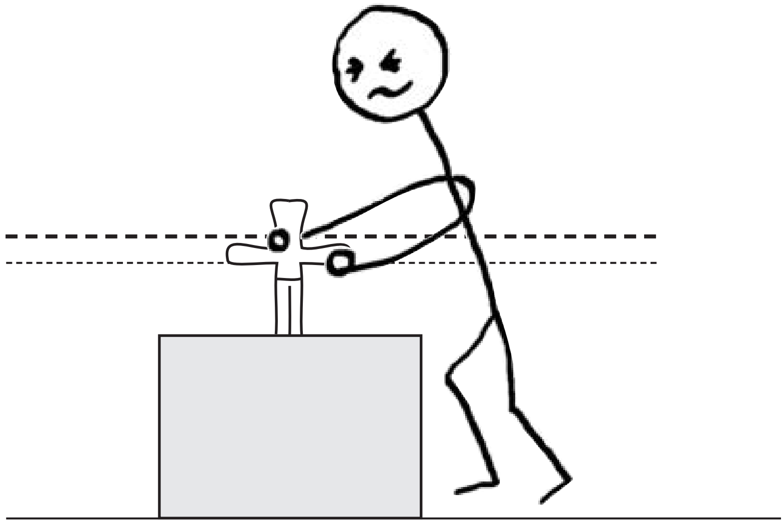


Can physics help you to lift a heavy object?

The rules say that the crossguard of the sword must go up by at least 10.0 cm to reach the line. But they don't say anything about whether the sword needs to be detached from the stone at the time!

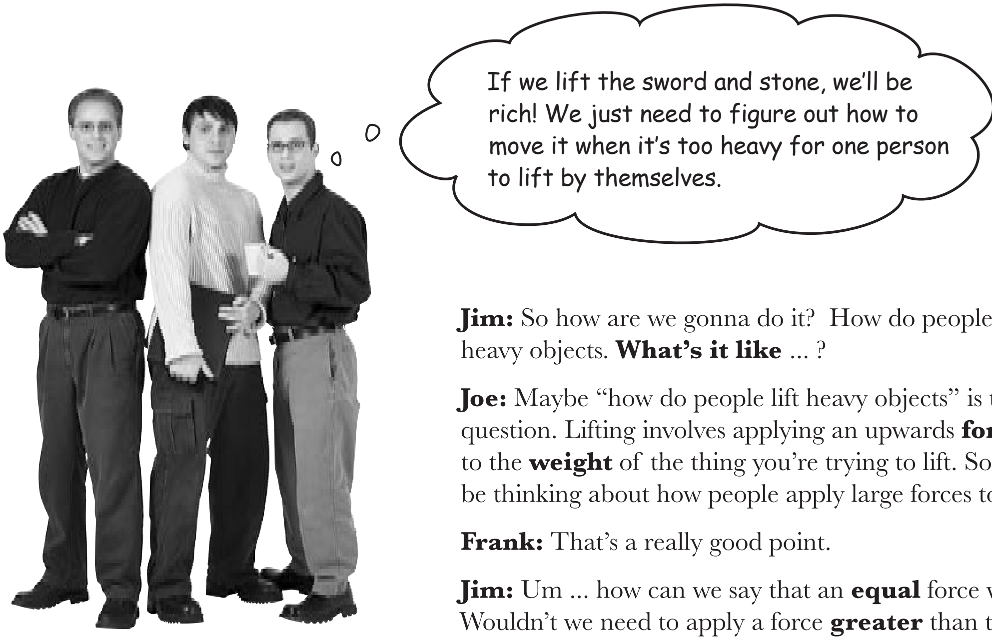


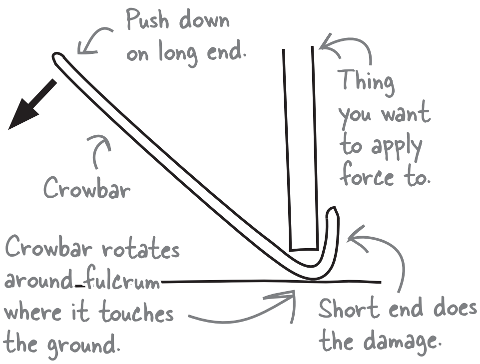
If you can use **physics** to lift both the sword **and** the stone 10.0 cm off the ground, you'll win. The only thing is the stone is far too **heavy** for one person to lift on their own, and it's not like you can take it to the moon to reduce its weight or use something like a crane that hasn't been invented yet...



BRAIN POWER

Think about the physics you've learned so far. How might you be able to lift the sword and the stone?





If you're asking "what's it like?" try to generalize. Ask "How can I apply a large force?" instead of "How can I lift a heavy thing?"

Jim: So how are we gonna do it? How do people use physics to lift heavy objects. **What's it like** ... ?

Joe: Maybe "how do people lift heavy objects" is the wrong question. Lifting involves applying an upwards **force** at least equal to the **weight** of the thing you're trying to lift. So maybe we should be thinking about how people apply large forces to objects.

Frank: That's a really good point.

Jim: Um ... how can we say that an **equal** force will lift it? Wouldn't we need to apply a force **greater** than the sword and stone's weight to get it to move upwards?

Joe: Once you've got the sword and stone going (with a force slightly larger than its weight) the most efficient way to lift it is to use a force equal to its weight. If there's no net force, it'll go up with a constant velocity.

Jim: Ah - I forgot about that Newton stuff. So if we can somehow apply a force equal to the sword and stone's weight, we'll be OK.

Frank: So how do people apply large forces? What circumstances might I want to use a large force in? **What's it like?**

Frank: Well, I guess that if you want to get through a locked door without a key, you could pry it open. You'd use a **crowbar** for that - to apply a large enough force to break either the door or the lock.

Joe: So how does that work?! I guess it has a long handle and a short claw ... you use it like a **lever**. You push down on the long end, and the short end does a lot of damage!

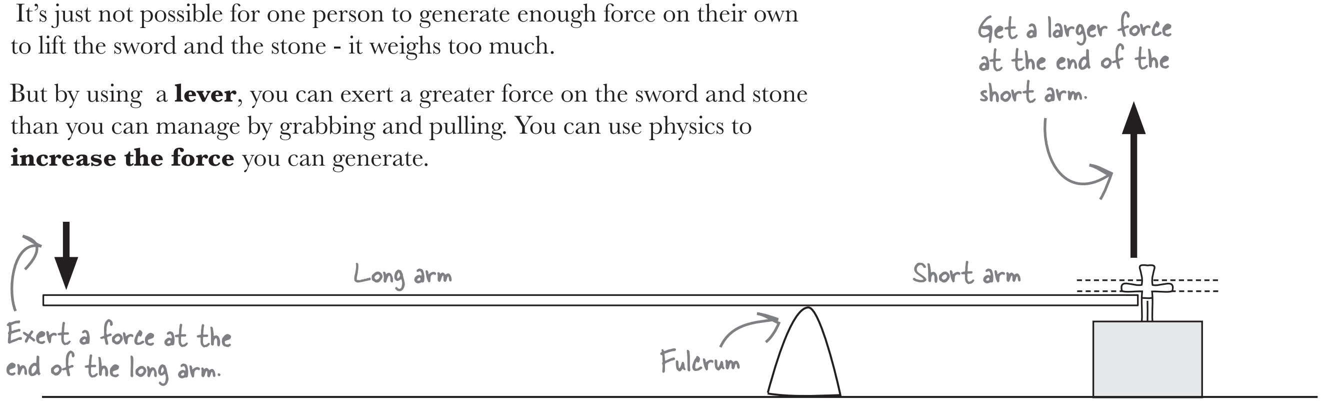
Jim: Yeah - far more damage than your pushing force would do if you just pushed on the door directly. Somehow, the force that the lever exerts on the door is greater than the force you exert on it.

Frank: So maybe we can rig up a lever with a long end and a short end to exert a larger force on the sword and stone than we can manage directly. I think we're on to something here ...

Use a lever to turn a small force into a larger force

It's just not possible for one person to generate enough force on their own to lift the sword and the stone - it weighs too much.

But by using a **lever**, you can exert a greater force on the sword and stone than you can manage by grabbing and pulling. You can use physics to **increase the force** you can generate.



A lever is a bit like a seesaw - a rigid bar that can **rotate** about a **fulcrum** (or pivot point). If you push down on one end, the other end goes up.

In physics, the two sides of the lever are called the **arms**. If the lever arms are different **lengths**, you can use the lever to exert a large force at the end of the short arm by pushing down on the long arm.

But what size of force do you need to generate to lift the sword and stone?

Use a lever to exert a larger force than you could on your own.

A lever's arms aren't like your arms – they always move together and can't move independently.

Sharpen your pencil

The stone is granite. We looked it up, and 1.00 cm^3 of this granite has a mass of 2.680 grams.

a. The stone is 1.0000 m by 0.8100 m by 0.6900 m. What is the mass of the stone?

Hint: Be careful with the units!

b. The sword's mass is 2.2 kg. What is the minimum force required to lift the sword and stone?

Sharpen your pencil Solution

The stone is granite. We looked it up, and 1.00 cm^3 of this granite has a mass of 2.680 grams.

a. The stone is 1.0000 m by 0.8100 m by 0.6900 m. What is the mass of the stone?

Work out volume of block in cm^3 then multiply that by 2.680 grams to get mass.

Volume = $100.0 \times 81.0 \times 69.0 = 558900\ cm^3$

Mass = $560000 \times 2.68 = 1500000\ g$ = **1498 kg** (4 sd)

It's generally best to give your answers in SI units - in this case, kg rather than grams.

b. The sword's mass is 2.2 kg. What is the minimum force required to lift the sword and stone?

Total mass = 1498 + 2.2 = 1500 kg (4 sd)

Minimum force will be the same size as the sword and stone's weight.

Weight = mg = 1500×9.8 = **14700 N** (3 sd)

Zero net force (weight + lifting force) means that you can lift the sword and stone with a constant velocity - Newton's 1st Law.

Be careful not to get 'g' (grams) and 'g' (gravitational field strength) mixed up!

there are no Dumb Questions

Q: We've assumed that the minimum force required to lift an object is equal to its weight. But surely you need to use a larger force?

A: Newton's 1st Law says that if the net force is zero, an object will move at a constant velocity. So the most efficient way to lift something is to exert a force on it that's a tiny bit larger than its weight for a short time. This gives it a small upwards velocity. Then you can continue with a force **equal** to its weight, so the object continues to move upwards with this velocity.

Q: So you DO need a force larger than the object's weight!

A: Yes, but only slightly larger and for a very short period of time to get it started. You can approximate this to a force equal to the object's weight (with an extra initial 'nudge').

Q: We've called the two ends of the lever "arms". But doesn't that imply that they can move independently (like my own two arms)?

A: Talking about the "arms" of a lever is physics terminology. Each side of the lever is an arm - but they're connected together and can't move independently.

Q: The whole setup looks like a seesaw, with two arms and a fulcrum. But how can you increase the force at the other end? Everyone knows that to balance a seesaw, you need the same weight - the same force - at each end.

A: If you have an adult and a child on a seesaw, you can balance them by moving the adult closer to the center. The force of the child's weight is smaller than the force of the adult's weight, but they can still balance.

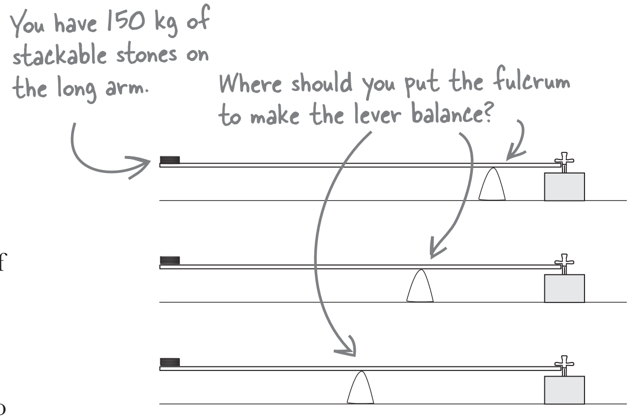
Q: OK, but the two sides of the seesaw are still the same length, right?

A: Yes, but the distance between the adult and the fulcrum has changed. So the child's small weight is able to provide enough force to lift the adult's larger weight (by balancing then doing a small initial 'nudge' to get going) - just like you'd like to do with the sword and stone.

Q: So you mean that if you get the adult and child to balance, then saw off the 'extra' bit of seesaw behind the adult, I get a lever where the two arms are different lengths, like we were already talking about?

A: You got it! Though you may have to reposition the adult slightly, to compensate for the missing bit of seesaw.

Do an experiment to determine where to position the fulcrum



If we use a **lever** to lift the sword and stone, a small force applied to the long arm will be able to exert a large force at the short arm. A quick look on the Sieges-R-Us website reveals that they have ten 15 kg stackable stones in stock - giving us a total of 10 ×15 = 150 kg we can place on the long arm.

But where should we put the fulcrum? The rules say only one attempt per lifetime! We need to make sure we have the fulcrum in the right spot before actually trying to use the lever to lift the sword and stone.

Time to design an experiment!

A small force on the long arm can balance out a large force on the short arm.

Sharpen your pencil

Don't worry if you're not sure what some of these items are. Just do your best!

Design an experiment that will allow you to determine the relationship between the two forces required to balance a lever and the distance from the fulcrum to the point each force is applied at.

a. Underline the items of equipment you will use to obtain the data: Stopwatch, Metal ruler, Scales, Protractor, Double-sided tape, Pipette, Triangular prism, Air track, Set of identical masses

b. On the tabletop below, sketch the setup you will use to obtain the data, labelling the fulcrum and any relevant forces and distances.

Table top.

c. Explain how you will carry out the experiment.

Sharpen your pencil Solution

Design an experiment that will allow you to determine the relationship between the two forces required to balance a lever and the distance from the fulcrum to the point each force is applied at.

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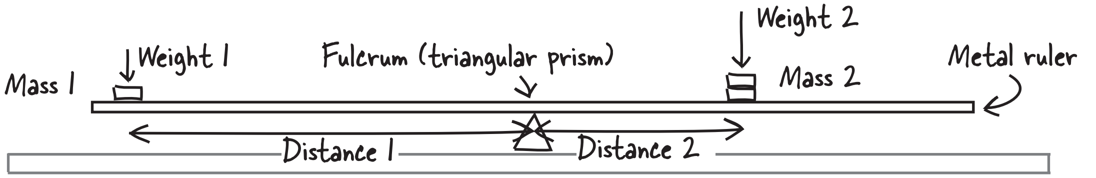
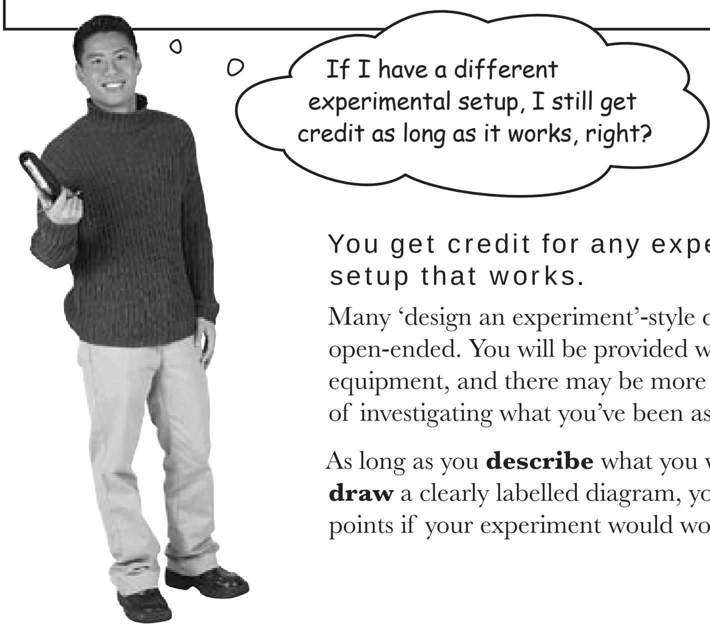


c. Explain how you will carry out the experiment.

Use the prism as the fulcrum and the ruler as the lever. Have the fulcrum in the middle of the ruler (so that the two halves of ruler balance), and put the masses at different distances from the fulcrum. Pile the masses on top of each other so that they press down on the same point, and use small amounts of blutac to avoid them slipping.

Start with one mass at the far end, and make sure one mass an equal distance away balances it. Then try moving two masses up and down to find the balance point. Repeat with three and so on.

Draw a table of results (mass 1, distance 1, mass 2, distance 2) and look for a pattern.



You get credit for any experimental setup that works.

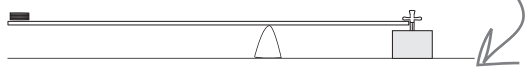
Many 'design an experiment'-style questions are open-ended. You will be provided with a range of equipment, and there may be more than one way of investigating what you've been asked about.

As long as you **describe** what you want to do and **draw** a clearly labelled diagram, you'll get the points if your experiment would work.

When designing an experiment, think about what you can DO with each piece of equipment.

Try it!

We've put the larger mass on the right in the experiment, because the larger mass (sword and stone) is on the right in the other picture.



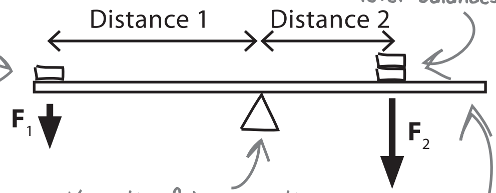
Now you can get on with doing this experiment! Your job is to find the balance point of the ruler when different weights are applied to each arm.

Find five large coins that all have the same value - you'll use these as your weights. You don't need to know the force exerted by a single coin in SI units, as you can use your own unit, the "coin-weight"!

Stick a round pen to a tabletop to use as your fulcrum. Keep it in the center of the ruler, and the smaller weight at one end, then slide the larger pile of coins up and down until the two sides balance. Use the measurements on the ruler to read off the distances between the center of each stack of coins and the fulcrum, and fill in the table below.

Keep this at the end of the ruler

Slide this mass to and fro until the lever balances.



Keep the fulcrum in the center of the ruler

Use a 30 cm (or longer) ruler as your lever.

You can use "coin-weights" as a unit because all of the coins have the same weight.

Force 1 (coin-weights)	Force 2 (coin-weights)	Distance 1 (cm)	Distance 2 (cm)
1	1	15	
1	2	15	
1	3	15	
1	4	15	

We chose 15 cm for Distance 1 because it's halfway along a 30 cm ruler. If your ruler is a different length, then change the value in this column to halfway along your ruler.

Do you see a pattern? Write down anything you notice about the forces and their distances from the fulcrum if the ruler is to balance.

Tried it!

Now you can get on with doing this experiment! Your job is to find the balance point of the ruler when different weights are applied to each arm.

Find five large coins that all have the same value - you'll use these as your weights. You don't need to know the force exerted by a single coin in SI units, as you can use your own unit, the "coin-weight"!

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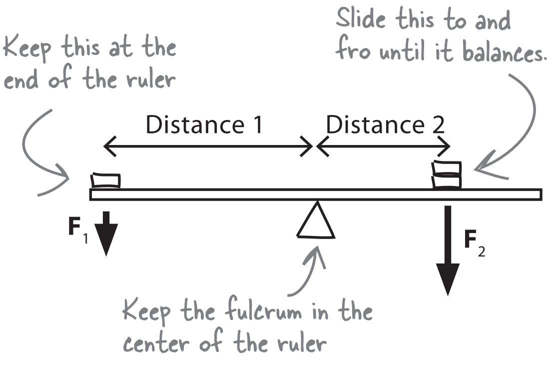


Force 1 (coin-weights)	Force 2 (coin-weights)	Distance 1 (cm)	Distance 2 (cm)
1	1	15	15
1	2	15	7.5
1	3	15	5.0
1	4	15	3.8

These are our results – it's OK if yours are slightly different.

Do you see a pattern? Write down anything you notice about the forces and their distances from the fulcrum if the ruler is to balance.

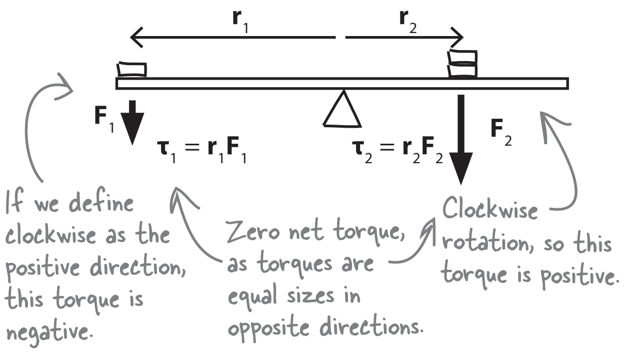
If I double the force (e.g. by using two coins instead of one) then I need to half the distance between it and the fulcrum to keep the ruler balanced.

I also noticed that number of coins × distance from fulcrum is the same for both sides when the ruler is balanced.

Zero net torque causes the lever to balance

A **torque** is like a ‘turning force.’ The greater the torque, the greater the effect it has on the **rotational** motion of the object that the torque is applied to. (A torque can also be referred to as a turning moment.)

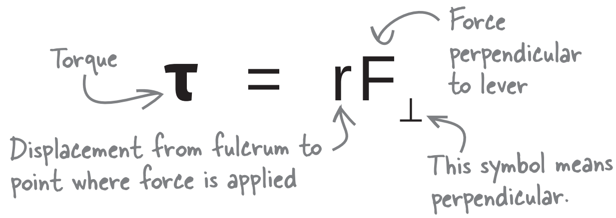
You might not have thought of the lever rotating – but that’s exactly what it would do if the ground wasn’t there, but the lever was still supported by its fulcrum.



Torque is a **vector** - you define clockwise as positive and counter-clockwise as negative. The direction of rotation depends on the direction of the force and the direction of the displacement. If two torques are the same size but would cause rotation in opposite **directions**, there’s zero net torque on the lever, and the lever balances.

The experiment you just did shows that the **size of a torque** is proportional to both the **size of the force** and the **distance from the fulcrum**. So if you double the force, the torque doubles. Or if you double the distance from the fulcrum, the torque doubles.

In physics, the Greek letter **τ** (pronounced ‘tau’) is used to represent a torque. When you have a fulcrum, torque is defined as the **displacement** from the fulcrum a force is applied at × the component of the **force** perpendicular to the lever. You can write this as an equation: $\boldsymbol{\tau} = \mathbf{r}\mathbf{F}_\perp$



Exercise

Use **torque** to explain:

a. Why you can lift the sword and stone using a force smaller than their weight.

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b. Why door handles are positioned far away from the hinges.

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c. Why a wrench (used to undo nuts and bolts) has a long handle.

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Exercise Solution

Use **torque** to explain:

a. Why you can lift the sword and stone using a force smaller than their weight.

If you use a lever whose arms are different lengths, a small force acting on the long end can produce the same torque as a large force acting on the short end, as torque = distance from fulcrum × force. So you can use a small force to lift a large weight.

b. Why door handles are positioned far away from the hinges.

To open a door, you need to produce a torque, so that the door rotates round its hinges (fulcrum). If the door handle is far away from the hinges, you need a smaller force to produce the same torque, as the same force applied a greater distance from the fulcrum produces a greater torque.

c. Why a wrench (used to undo nuts and bolts) has a long handle.

You use the handle of the wrench to apply a torque to a nut (the fulcrum is the center of the nut). The longer the handle, the smaller the force you need to apply to produce the same torque.

there are no Dumb Questions

Q: Why are we calling a torque a "turning force" when the seesaw isn't turning - it's just swinging up and down?

A: Swinging is just turning, but not full circle! A seesaw swings around a fulcrum, and if the ground wasn't there it would be able to rotate all the way round.

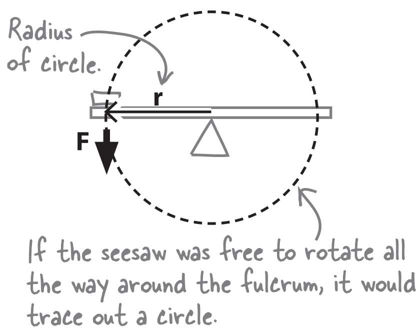
Q: What's the difference between a force and a torque?

A: To exert a torque on an object, there needs to be a fulcrum that it can rotate about. A torque is a force applied to the object at a distance from the fulcrum, with a component that's perpendicular to the axis of the lever.

There's a picture of this on the opposite page.

Q: In the equation $\tau = rF_{\perp}$, why is the letter r used to represent a displacement instead of x, the letter we usually use?

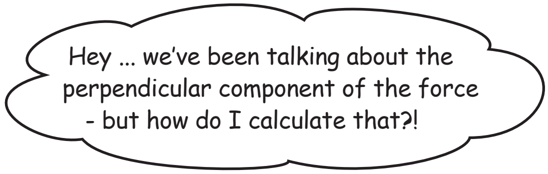
A: A torque produces a rotation. If you imagine the seesaw rotating all the way around, it would trace out a circle with the fulcrum at the center. The 'r' stands for 'radius', as the radius of a circle is the distance from its centre to its edge.



Q: It's confusing to use different letters to represent displacements. Why are we doing that?

A: Using 'r' in this equation is a physics convention that's followed any time circular motion is involved. It makes you think about the fact that the displacement is a radius, and that rotation is involved.

A torque causes an object to rotate about a fulcrum.

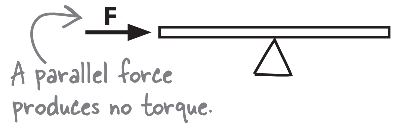


It's the perpendicular component of a force that produces a torque.

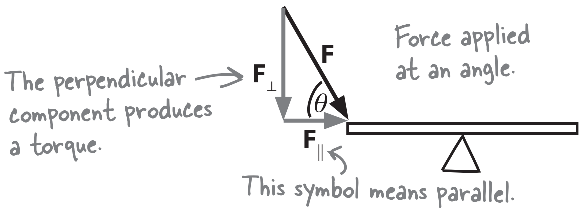
Only the force component perpendicular to the lever produces a torque.

In your experiment, the vertical forces you applied always acted perpendicular to the horizontal lever. But sometimes a force will be applied to a lever at an **angle**.

If a force is applied **parallel** to the lever, it won't rotate at all, and the torque will be zero.



If a force, **F**, is applied at an angle, θ, to the lever, only the **perpendicular component** of **F** will produce a torque.



Sharpen your pencil

A force, **F**, is applied to a horizontal lever at a point displacement **r** from the fulcrum. The force is applied in such a way as to make the angle θ with the horizontal.

a. Draw a large sketch showing the lever and the relevant components of **F**.

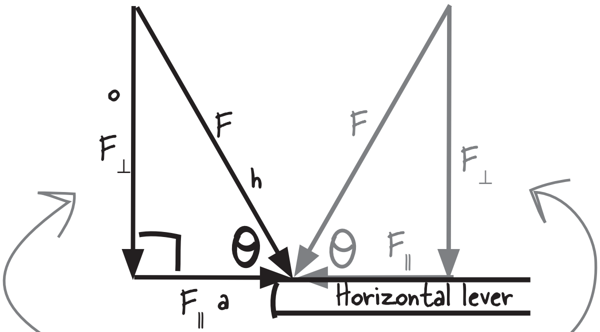
The sketch will be a large version of this one.

b. Use your sketch to derive an equation for the torque, $\boldsymbol{\tau} = \mathbf{r}\mathbf{F}_{\perp}$, in terms of **r**, **F** and θ.

Sharpen your pencil Solution

A force, **F**, is applied to a horizontal lever at a point displacement **r** from the fulcrum. The force is applied in such a way as to make the angle θ with the horizontal.

a. Draw a large sketch showing the lever and the relevant components of **F**.



It doesn't matter if the force is pointing "outwards" or "inwards" – the perpendicular component still has the same size and direction.

b. Use your sketch to derive an equation for the torque, $\tau = rF_\perp$, in terms of **r**, **F** and θ.

Force is applied at angle θ.

Perpendicular component is opposite side.

$$\sin(\theta) = \frac{o}{h} = \frac{F_\perp}{F}$$

$$\Rightarrow \quad F_\perp = F\sin(\theta)$$

$$\tau = rF_\perp$$

$$\Rightarrow \quad \tau = rF\sin(\theta)$$

This is the equation for torque given on many equation sheets. However, we'd recommend working it out for yourself with triangles, as the equation doesn't make it crystal clear how the angle θ is measured.

Newton's 1st law says that if there's no net force, an object continues at the same velocity. Is there an equivalent for torques?

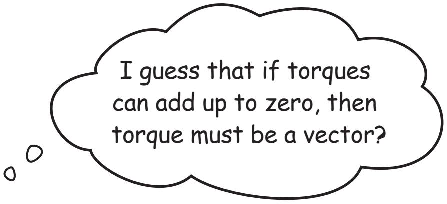
Zero net force = static equilibrium

Newton's 1st law says that if an object has zero net force exerted on it, then it will continue at its current velocity, in other words, it won't accelerate. This is also known as **static equilibrium.**

Zero net torque = rotational equilibrium

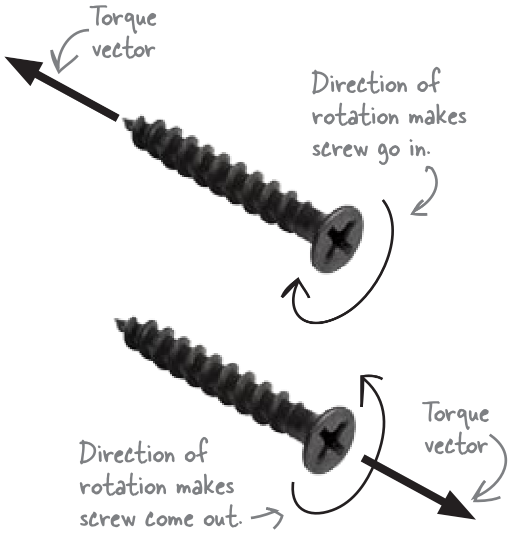
If the net torque on a lever (or another object) is zero, then its speed of rotation doesn't change, in other words its rotation won't get faster or slower. This is also known as **rotational equilibrium**.

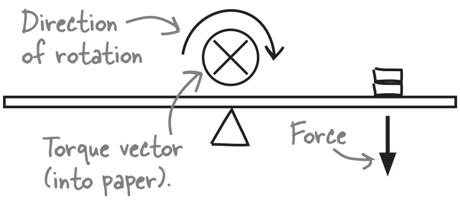
If a lever isn't rotating (or is spinning at a constant rate) the net torque must be zero.

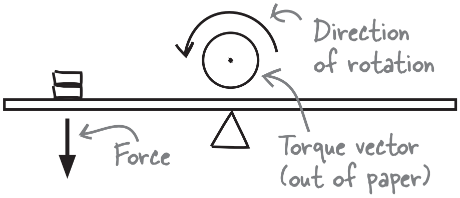


Torque is a vector.

Torque has a **direction** as well as a **size**, since you can turn clockwise or counterclockwise round the fulcrum. A **vector** arrow points in the direction in which a screw would move if you applied the torque to it. So clockwise = into the paper and counterclockwise = out of the paper.



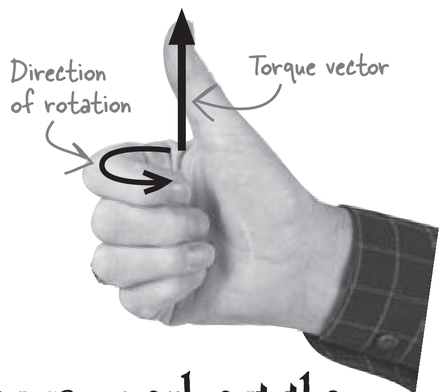




This sounds difficult to draw in only two dimensions, but there's a standard convention that helps.

The symbol for a vector going into the paper is ⊗ (like the head-on view of a screw) and for one coming out of the paper is ⊙, (like the view of the screwpoint).

If you can't imagine which way a screw would move if a torque was applied, then do a 'thumbs up' with your **right hand.** If you curl your fingers in the direction that the object's turning in, then your thumb points in the direction of the torque vector.



You can work out the direction of the torque vector by doing a 'thumbs up' with your right hand.

there are no Dumb Questions

Q: Can I just memorise the equation $\tau = rF\sin(\theta)$ or look it up on my equation sheet rather than working it out with triangles each time?

A: You can if you like ... but what if you come across a problem where you're given the angle that the force makes with the vertical, rather than the angle it makes with a horizontal lever. If you're used to starting with triangles you can work that out, but if you just assume the equation will be the same, you'll come undone.

Q: What is equilibrium?

A: Equilibrium is another word for balance. Static equilibrium is when forces are balanced - in other words, when the net force is zero. Rotational equilibrium is when the torques are balanced - in other words the net torque is zero.

Equilibrium is another word for balance.

Q: Why is there a distinction between static and rotational equilibrium? Equilibrium's just equilibrium, right?!

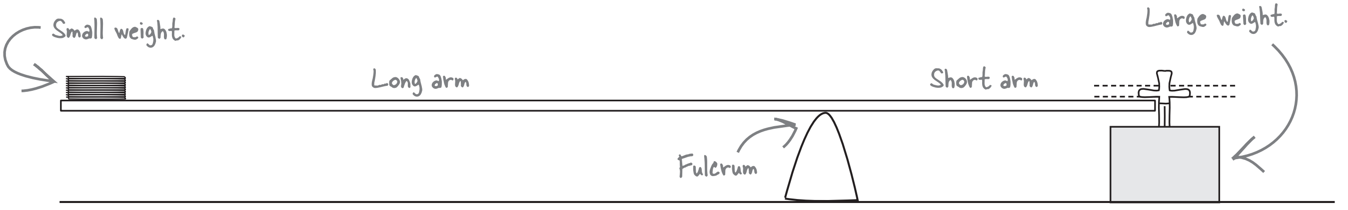
A: A rocket firework that's just been started off is in rotational equilibrium, as there's no torque on it. But it isn't in static equilibrium as it's accelerating.

A Catherine-wheel firework that's just been started off isn't in rotational equilibrium, as it's spinning faster and faster due to a non-zero net torque. But it is in static equilibrium - as it isn't going anywhere, the net force on the firework must be zero.

Use torque to lift the sword and the stone!

The force you need to lift the sword and the stone is equal to its weight, which is very, very large! But you can use physics to make it easier by designing a **lever**.

A lever consists of two arms of different lengths which can rotate around a **fulcrum**. Applying a force produces a **torque**, which can cause the lever to rotate around the fulcrum.



If an object will rotate around a fixed point, see if you can use torque to solve problems.

The equation for torque is $\tau = rF_{\perp}$.

So if you make a lever and put the sword and stone (which exerts a large force due to its weight) at the end of a short arm then apply an equal torque on a long arm using stackable stones, you'll be able to arrange a setup where there is **zero net torque** on the lever, which is therefore in **rotational equilibrium**. From there, a tiny nudge will be enough to lift the sword and stone. Fame and fortune beckon ... you just need to work out where to put the fulcrum.

Sharpen your pencil

A lever, length *L*, has two masses on it. At one end is a sword and stone, mass m_1 and displacement $\mathbf{r}_1$ from the fulcrum. At the other end of the lever is a stack of stones, mass m_2 and displacement $\mathbf{r}_2$ from the fulcrum.

a. Write down an equation for $\mathbf{L}$, the total length of the lever, in terms of $\mathbf{r}_1$ and $\mathbf{r}_2$. (Make left-to-right the positive direction, and make $\mathbf{L}$ a positive vector. $\mathbf{r}_1$ and $\mathbf{r}_2$ are also vectors - do a sketch and be VERY careful with signs!)

b. Write down the condition for the rotational equilibrium of the lever (from where a small nudge will allow you to lift the sword and stone).

c. Use these two equations to work out an equation for $\mathbf{r}_1$ in terms of m_1, m_2 and $\mathbf{L}$.

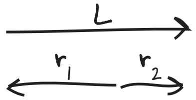
d. If the lever is 10.00 m long, the sword and stone have a mass of 1500 kg and the stackable stones a mass of 150 kg, how far from the sword and stone end should the fulcrum be placed?

Sharpen your pencil Solution

A lever has two masses on it. At one end is a sword and stone, mass m_1 and displacement $\mathbf{r}_1$ from the fulcrum. At the other end of the lever is a stack of stones, mass m_2 and displacement $\mathbf{r}_2$ from the fulcrum.

a. Write down an equation for **L**, the total length of the lever, in terms of $\mathbf{r}_1$ and $\mathbf{r}_2$. (Make left-to-right the positive direction, and make **L** a positive vector. $\mathbf{r}_1$ and $\mathbf{r}_2$ are also vectors - do a sketch and be VERY careful with signs!)

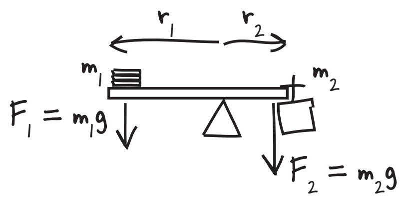
Make left to right the positive direction, and make L a vector going from left to right.



$$L = r_2 - r_1$$

If you chose a different way of defining the positive direction, your answer will work out the same in the end, but some of the minus signs in the algebra may be different.

b. Write down the condition for the rotational equilibrium of the lever (from where a small nudge will allow you to lift the sword and stone).



The net torque must be zero.

$$r_1F_1 + r_2F_2 = 0$$

$$\Rightarrow r_1m_1g + r_2m_2g = 0$$

$$\Rightarrow r_1m_1 + r_2m_2 = 0$$

Both terms are multiplied by g, so g divides out and cancels.

c. Use these two equations to work out an equation for $\mathbf{r}_1$ in terms of m_1, m_2 and **L**.

Need to make a substitution to get rid of r_2.
Rearrange equation from part a:

$$r_2 = L + r_1$$

Substitute this into equation from part b:

$$r_1m_1 + (L + r_1)m_2 = 0$$

$$r_1m_1 + Lm_2 + r_1m_2 = 0$$

$$r_1m_1 + r_1m_2 = -Lm_2$$

$$r_1(m_1 + m_2) = -Lm_2$$

$$r_1 = \frac{-Lm_2}{(m_1 + m_2)}$$

Put brackets in, so there's only one occurrence of r_1 on the left hand side.

d. If the lever is 10.00 m long, the sword and stone have a mass of 1500 kg and the stackable stones a mass of 150 kg, how far from the sword and stone end should the fulcrum be placed?

Use equation from part c. with values
(m_1 is stackable stones and m_2 is sword and stone)

$$r_1 = \frac{-Lm_2}{(m_1 + m_2)}$$

$$r_1 = \frac{-10 \times 1500}{(150 + 1500)} = -9.09 \text{ m (3 sd)}$$

This is negative as we made left to right the positive direction.

r_1 is the displacement from the stackable stones end - need to work out r_2.

Displacement from sword and stone end will be:

$$r_2 = L + r_1$$

$$r_2 = 10 - 9.09 = \underline{0.91 \text{ m}} \text{ (3 sd)}$$

The question asks for the displacement of the sword and stone from the fulcrum.

Question Clinic: The "Two equations, two unknowns" Question

If you have one equation, you can use it to work out the value of one unknown variable as long as you know values for the others. If you have two equations, you can use them to work out two unknowns (as long as it's the same two unknowns in both equations!)

Always, always, always start with a sketch!

The buzzwords 'lever' and 'fulcrum' tell you that torque is involved.

2. A lever has two masses on it. At one end is a sword and stone, mass m_1 and displacement r_1 from the fulcrum. At the other end of the lever is a stack of stones, mass m_2 and displacement r_2 from the fulcrum.

a. Write down an equation for **L** in terms of r_1 and r_2.

b. Write down the condition for the rotational equilibrium of the lever.

c. Use these two equations to work out an equation for r_1 in terms of m_1, m_2 and **L**.

If the question gives you variable names, make sure you use the same ones in your answer.

You may be expected to spot this is useful from your sketch, rather than being asked explicitly to do it.

'Condition' probably means equation in this context, though you should write it in words as well as you get points for it!

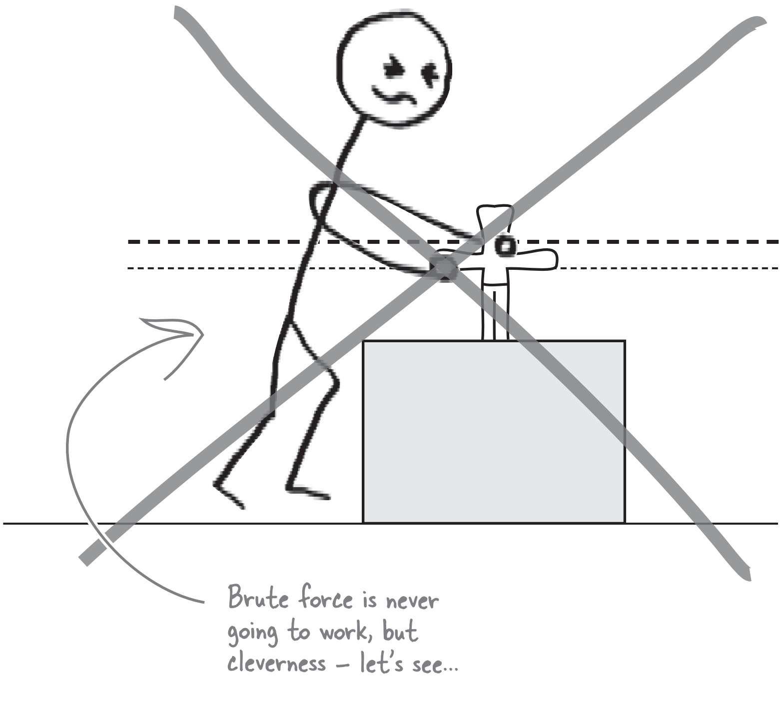
Work out what variable(s) you need to get rid of, and make sure you do what they ask you to!

In this book we've concentrated on making substitutions, but if there's another way that makes sense for you, then go for it!

You can solve for two unknowns either by using a **substitution** or by setting up **simultaneous equations**. These methods are effectively the same, and it's up to you which to choose. If one of your unknowns is 'buried' deep inside a term, it's probably easiest to do a substitution. If both unknowns are terms in their own right (or are multiplied by just a number) then simultaneous equations will be quicker - but a substitution will still work.

BULLET POINTS

- The **size** of a **torque** is equal to the component of the force perpendicular to the lever × the displacement from the fulcrum.
- You can work out the perpendicular force component using trigonometry.
- If a problem involves something turning, you need to work out where the fulcrum is.
- Torque is a vector - the **direction** you're turning in matters.
- If you curl the fingers of your right hand in the direction you're turning in, your thumb points in the direction of the torque vector.
- If an object is in rotational equilibrium, it means that vector sum of all the torques on the object is zero.

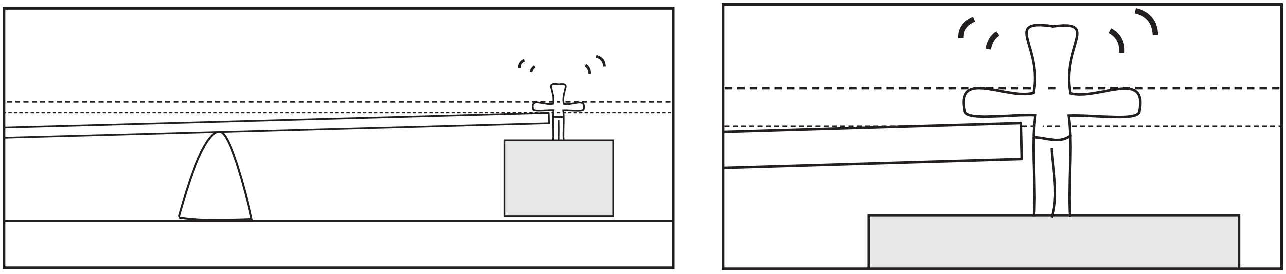


Time to try using a lever to lift the sword and stone. Hopefully, by using physics, you'll be more successful than than the person who tried before you ...

So you lift the sword and stone with the lever ...

You've used what you know about **torque** to predict that you can lift a 1500 kg sword and stone by using only 150 kg of stackable stones to generate a force much larger than their weight with a lever.

So you set everything up, and as you lift the final stone into place, the sword and stone begin to move...



... but they don't go high enough!

Although the sword and the stone get off ground, they don't go high enough. The crossguard of the sword needs to be raised by 10 cm for the lift to count, but it only went up by around 1 cm.

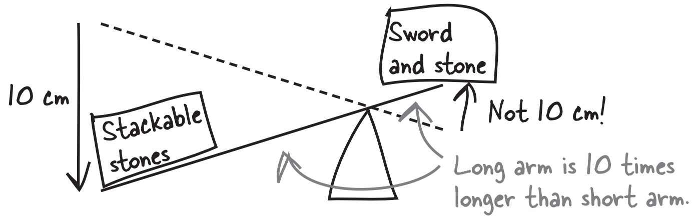
The stackable stones have definitely gone down by 10 cm at the other end of the lever - but the sword and stone haven't gone up by the same amount. What's going on?!

BRAIN POWER

The stackable stones have gone down by 10 cm. So why haven't the sword and the stone gone up by 10 cm?

Jim: But it didn't reach the line!. The stone end went down by 10 cm - why didn't the sword and stone end go up by 10 cm?

Joe: Hang on while I do a quick sketch ...



Frank: Ooh, triangles!

Joe: Yeah, **similar triangles**! Look - the stackable stones lever arm is ten times longer than the sword and stone lever arm. So if the stackable stones side goes down 10 cm, the sword and stone end only goes up a tenth of that distance - 1 cm.

Jim: If the stackable stones going down by 10 cm causes the sword and stone to rise by 1 cm, then I guess the stackable stones need to go down by 100 cm to make the sword and stone rise by 10 cm.

Frank: That sounds like an awful lot of **work** to lift the stackable stones that high in the first place! Maybe it's easier to make the two arms of the lever equal lengths. Then we only need to lift the stackable stones 10 cm - a much smaller **distance**.

Joe: But we'd need 1500 kg of stackable stones to lift the sword and stone, instead of 150 kg. That's ten times as many stackable stones! Even though you only have to lift them a tenth of the **distance**, you're lifting ten times more **weight** than you were before.

Jim: I'm kinda starting to think that you can't get something for nothing. We either lift a tenth of the weight ten times the distance, or the same weight the same distance.

Joe: Yeah, it's hard work either way around!

Always look out for triangles - and especially similar triangles!

Similar triangles don't have to be right-angled. They just need to have the same angles as each other.

BRAIN POWER

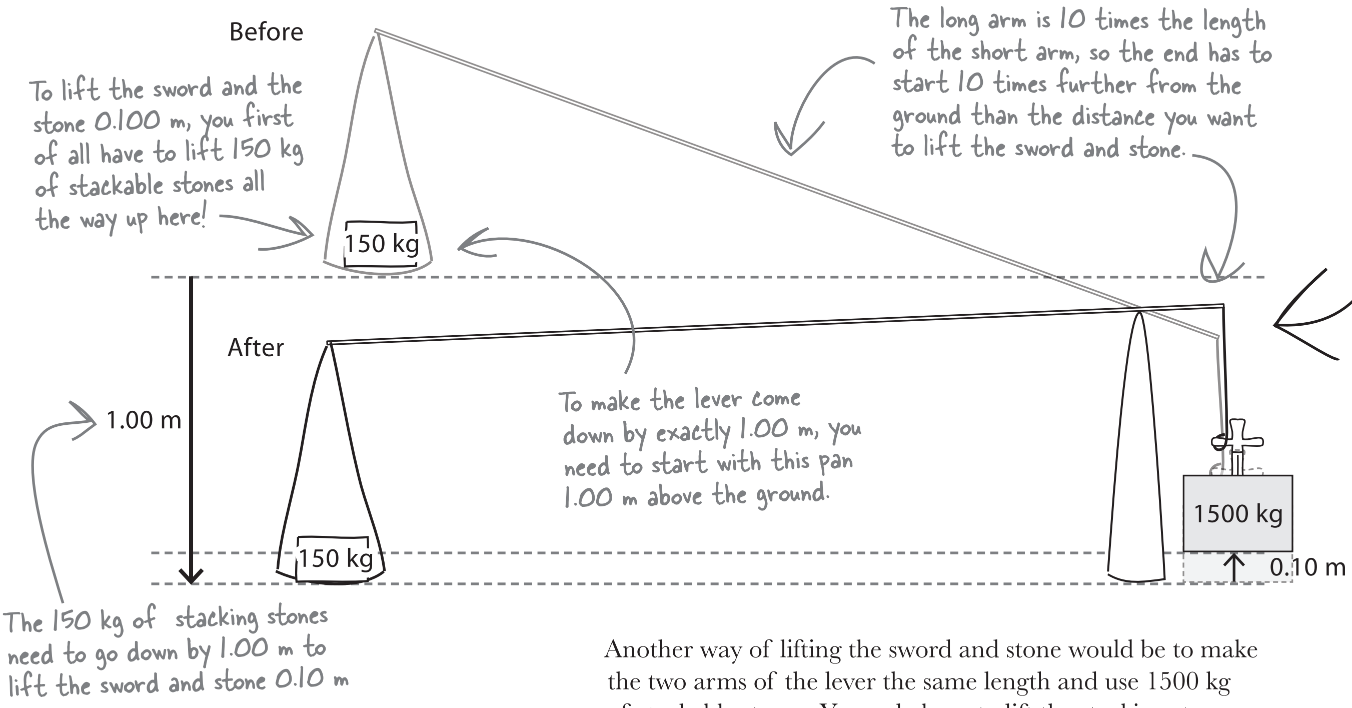
Which do you think involves more work - lifting 10 stackable stones 1 m each, or 100 of them 10 cm each?

You can't get something for nothing

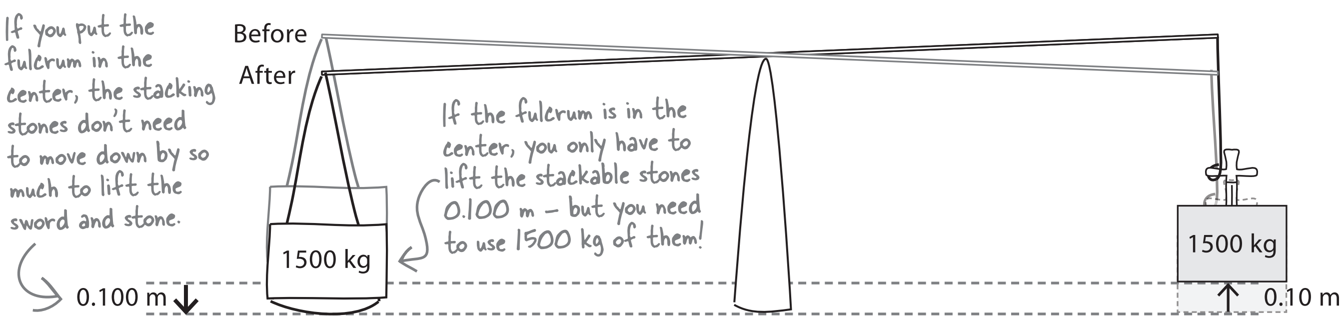
You can't get something for nothing. If you use 150 kg of stackable stones to lift a 1500 kg sword and stone, the long lever arm needs to be 10 times the length of the short one. But that means the sword and stone only get lifted a tenth of the distance that the stackable stones move through.

So to lift the 1500 kg sword and stone 0.10 m (10 cm) off the ground, you need to lift the 150 kg of stackable stones 1.00 m off the ground to start off with - 10 times as high. That's a lot of work for you!

We've now redesigned the lever as a pan balance. This is so that you only have to lift the stackable stones the minimum height necessary (onto the pan) and not all the way up to the top of the lever.



Another way of lifting the sword and stone would be to make the two arms of the lever the same length and use 1500 kg of stackable stones. You only have to lift the stacking stones 0.100 m instead of 1.00 m - but you have to lift 10 times as much weight to take advantage of the smaller displacement!



When you move an object against a force, you're doing work

You have a job to do - lift the 1500 kg sword and stone 0.100 m in the air by overcoming the gravitational force on it. In physics, if you **displace** an object in the **opposite** direction from a **force** that's acting on the object, you're said to be **doing work** on the object against the opposing force.

So when you lift a stackable stone, you **do work on** it against the force of gravity. And if a pile of stackable stones lifts the sword and stone (using a lever) then the stackable stones **do work on** the sword and stone against the force of gravity.

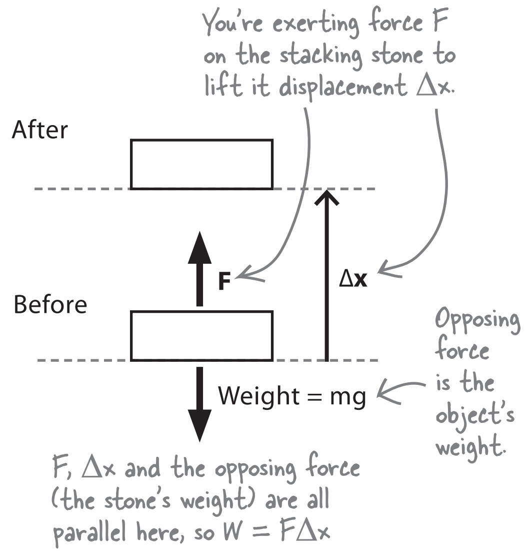
To do WORK on an object, you need to use a FORCE to DISPLACE the object in the opposite direction from another force that's acting on the object.

The work you need to do a job = force x displacement

In physics, the word **work** has a very specific meaning. When you do work on something against a force (e.g. by lifting it against the force of gravity), the amount of work you do depends on two things:

The component of the **force** you exert on the object that's **parallel** to other force you're working against. When you're lifting something against the force of gravity, this is the vertical component of the lifting force.

The **displacement** of the object in the same direction. When you're lifting something against the force of gravity, this is the vertical component of the displacement.



Work done on object

Component of the force you exert on it that's parallel to the force opposing it.

$$W = F_{||}\Delta x$$

Be careful – work begins with the same letter as weight!

This symbol means 'parallel'.

Displacement in same direction as $F_{||}$

The **work** you do on the object is defined as the he parallel component of the **force** you use to move the object × its **displacement**. This can be written as:

$$W = \mathbf{F}_{||}\mathbf{\Delta x}$$

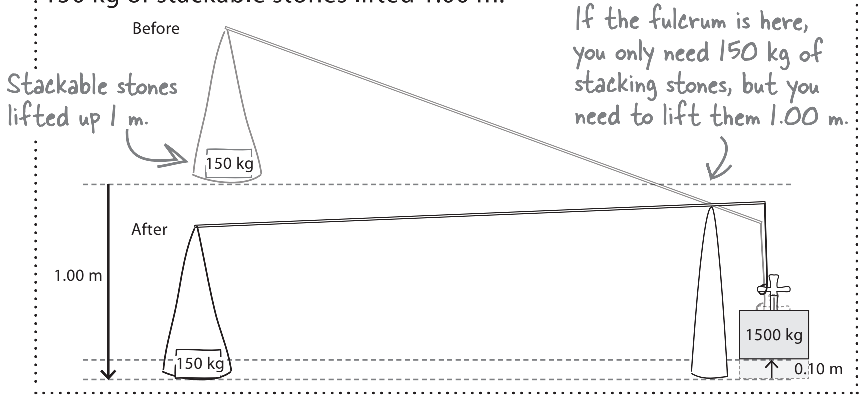
where W is the work, $\mathbf{\Delta x}$ is the displacement and $\mathbf{F}_{||}$ is the component of the force you're exerting on the object parallel to the object's displacement.

Which method involves the least amount of work?

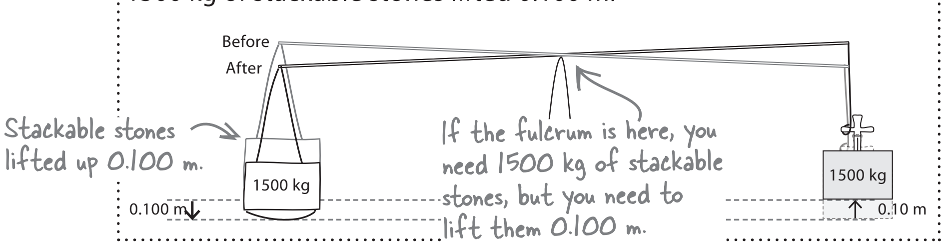
You've come up with two different ways of lifting the sword and stone using a lever and some stackable stones. You can either lift 150 kg of stones 1.00 m, or 1500 kg of stones 0.100 m (depending on where you put the fulcrum) to enable the stacking stones to lift the sword and stone.

When you lift an object, you do work on it by displacing it upwards with an upwards force that counters the downwards gravitational force.

150 kg of stackable stones lifted 1.00 m.



1500 kg of stackable stones lifted 0.100 m.



It would make sense to choose the easier method - the one that involves you having to do less work on the stackable stones in order to get them from the ground into a position where they can lift the sword and stone.

Sharpen your pencil

You need to do work on the stackable stones before they can do work on the sword and stone. But which method involves doing more work? Calculate the amount of work you need to do to:

a. Lift 150 kg of stackable stones 1.00 m.

b. Lift 1500 kg of stackable stones 0.100 m.

c. Comment on the sizes and the units of your answers.

Hint: Get the units of work from the units of the variable on the right hand side of the equation you use to calculate the work.

Sharpen your pencil

You need to do work on the stackable stones before they can do work on the sword and stone. But which method involves doing more work? Calculate the amount of work you need to do to:

a. Lift 150 kg of stackable stones 1.00 m.

Lifting force same size as weight, so $F = mg$

$W = F\Delta x = 150 \times 9.8 \times 1.00$

$W = 1470$ Nm (3 sd)

b. Lift 1500 kg of stackable stones 0.100 m.

Lifting force same size as weight, so $F = mg$

$W = F\Delta x = 1500 \times 9.8 \times 0.100$

$W = 1470$ Nm (3 sd)

c. Comment on the sizes and the units of your answers.

Both ideas involve doing the same amount of work on the stackable stones. The units of work are force × displacement. Force is measured in Newtons and displacement in meters, so the units of work are Nm.

You say "Newton meters" if you're saying this unit out loud.

Hey - torque and work have the same units, force x distance - Nm. Is that significant?!

Work and torque are different because they involve different displacements.

Even though they share the same units (Nm), torque and work are very different things.

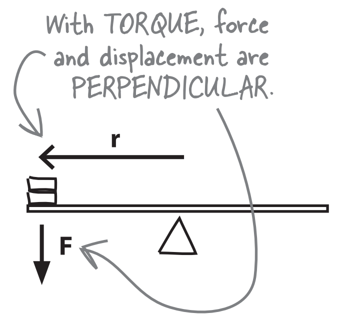
Torque, $\boldsymbol{\tau} = \mathbf{rF}_{\perp}$, tells you how good a force is at **turning** something. The **displacement** in the equation is from the fulcrum to where the force makes contact, which is multiplied by the component of the force **perpendicular** to this displacement.

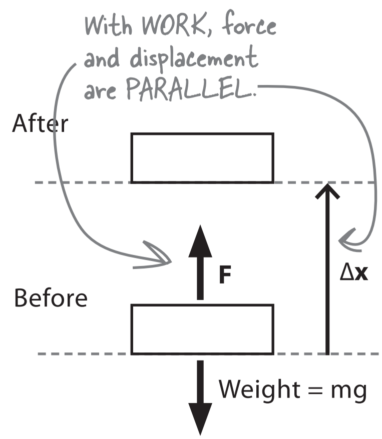
Torque is a **vector**, as the turning can be clockwise or counterclockwise along any axis in three dimensions.

Work, $W = \mathbf{F}_{||}\Delta \mathbf{x}$, is a measure of how much **energy** you need to move an object using a force. The **displacement** in the equation is the displacement of the moved object, which is multiplied by the component of the force **parallel** to this displacement.

Work is also a **scalar**, as the same amount of work will be done by the same size of force moving something the same distance, regardless of direction.

Torque is a vector, work is a scalar.





Work has units of Joules

To avoid getting confused because torque and work have the same units, scientists measure work in something called **Joules** (J) where 1 J = 1 Nm. If you're answering a question about work, you should always give your answer in Joules.

> But if I lift a stone quickly, it makes me more tired than if I lift it the same distance but more slowly. How can you say I'm doing the same amount of work both times?!

Work is measured in Joules.

Power is the rate at which you do work.

Power's measured in Joules per second, J/s

If you lift something in a shorter time, your power output is higher.

Work is measured in Joules. You do the same amount of work to lift the stone the same distance each time, regardless of the time it takes.

The difference is your **power** output. Power is the **rate** at which you do work and is measured in Joules per second (J/s).

Because of how your body functions, you get more tired if you do the same work in a shorter time with a higher power output. So how **tired** you feel won't always correlate with the amount of work you've done.

there are no Dumb Questions

Q: I don't like how physics steals words like 'work' and 'power' and makes them mean different things from usual.

A: Many words can be used in a variety of ways in everyday speech. You're right that in physics, words like work, power, and force have very **specific** meanings, but they have to so that people know exactly what you're talking about.

Q: I can make myself tired without doing any work at all if I hold my arm out at shoulder height with an object in my hand. The weight's not moving, so I'm not doing work on it. What's that all about?!

A: The fibers in your muscles are continually expanding and contracting (moving as a result of a force) to enable you to do that. You **are** doing work, but on your muscle fibers, not the object! A table can support the object quite happily without doing any work on it!

Q: You said over there that "work is a measure of how much energy you need to move something with a force." Is 'energy' another of these physics words with a very specific meaning?

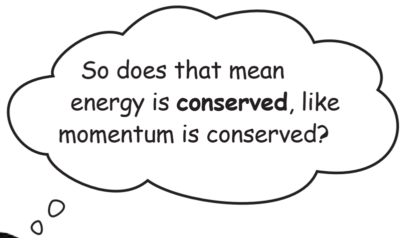
A: It sure is - and you're going to be doing a lot with energy in the next few chapters, starting now...

Energy is the capacity that something has to do work

If something has the **capacity** to do work, it means that it is able to exert a force to displace an object. But how much work might it be able to do?

Energy is the capacity that something has to do work. By lifting 150 kg of stackable stones 1.00 m, you are giving them the capacity to do $\mathbf{F}_{||}\Delta\mathbf{x} = 150 \times 9.8 \times 1.00 = 1470\,\text{J}$ of work.

Another way of putting this is that you have given the stackable stones 1470 J of **gravitational potential energy**.



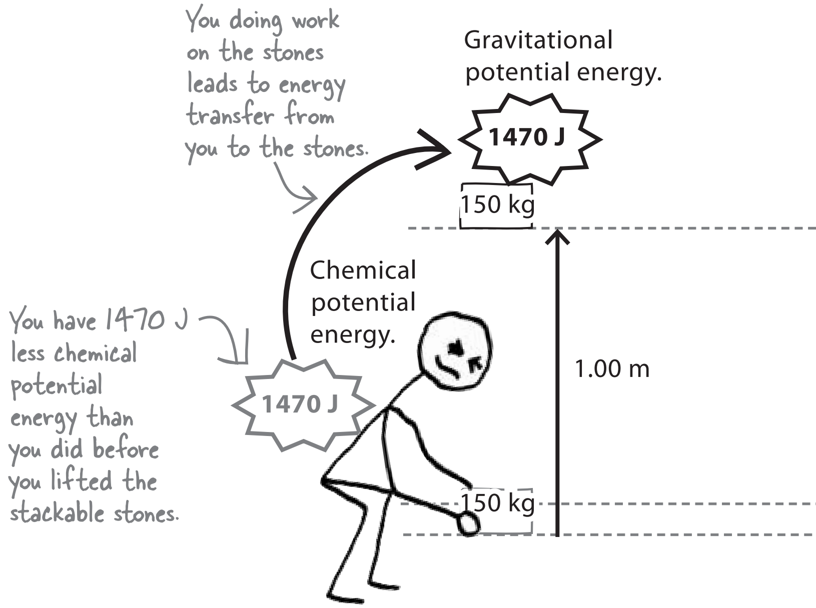
Energy is conserved

The total energy of everything in the entire universe is always constant. Closer to home, the total energy of an isolated system (for instance you, stackable stones, lever, sword and stone, plus surroundings) is always the same, or in other words energy is **conserved**.

Energy is <u>conserved</u>.

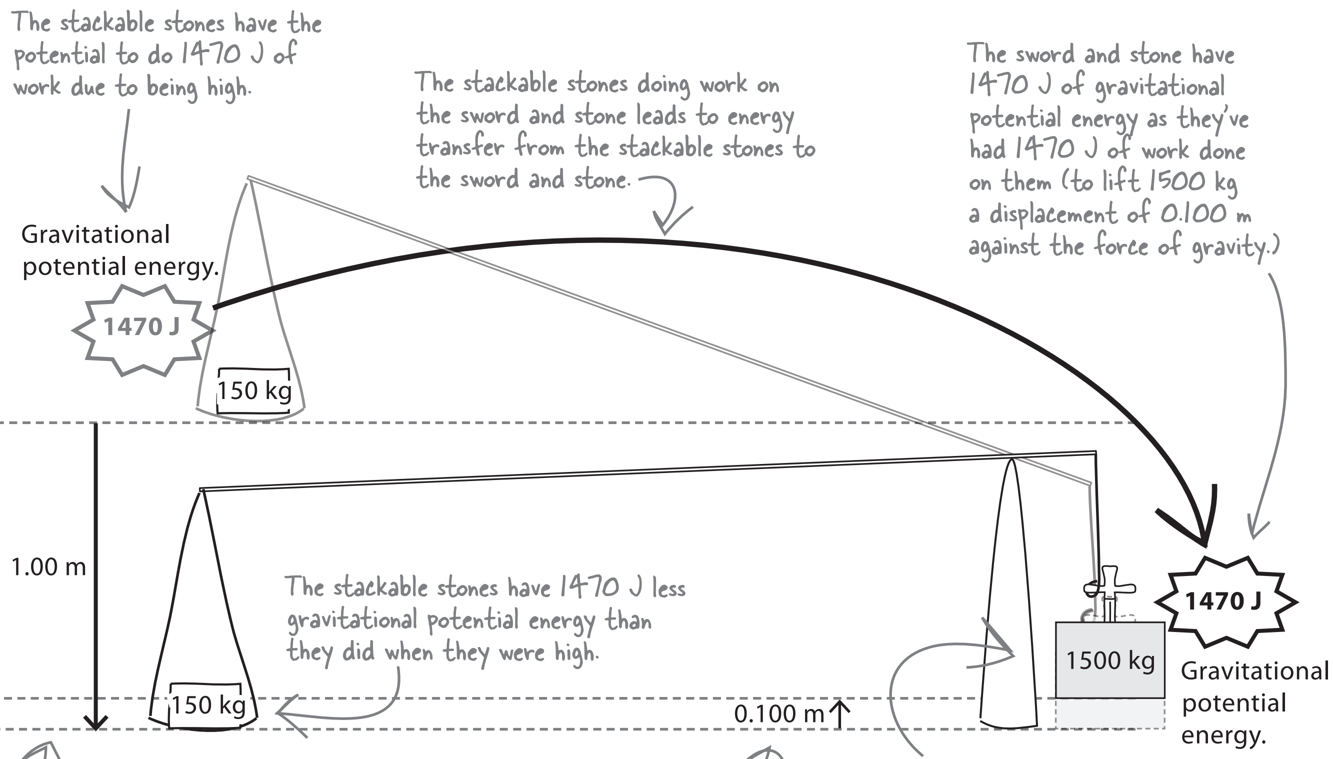
Lifting stones is like <u>transferring</u> energy from one store to another

The gravitational potential energy that the stackable stones gain when you lift them doesn't just 'appear' from nowhere. You can think of the food you eat as a store of **chemical potential energy**, which your body can tap into to do work.



If you do 1470 J of work on the stackable stones, giving them 1470 J of gravitational potential energy, you can think of the energy being **transferred** from one store to another, as your body has 1470 J less chemical potential energy than it had before, and the stackable stones have 1470 J more gravitational potential energy than they did before, so have the capacity to do 1470 J of work.

Doing work is a way of <u>transferring</u> energy.

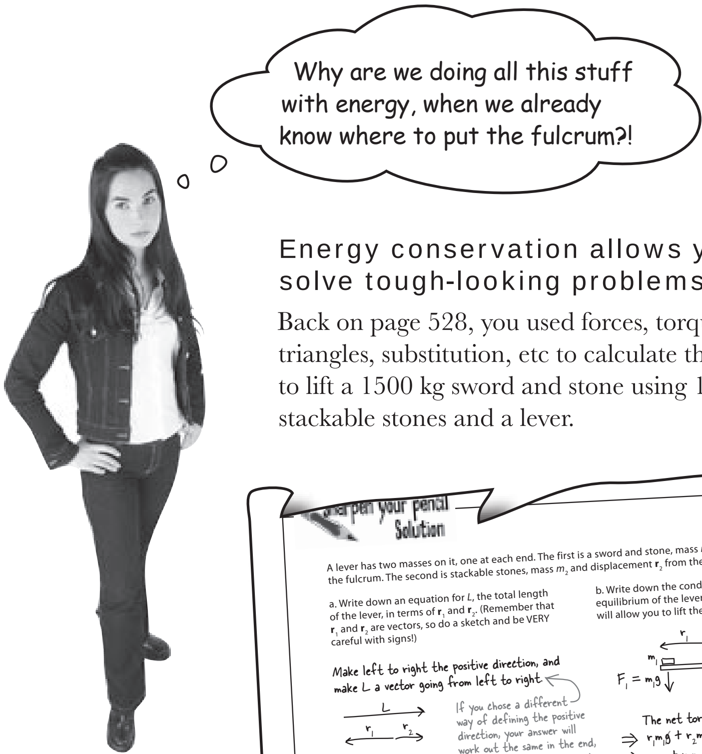


If the stackable stones then do 1470 J of work on the sword and stone (via the lever), you can also think of that as 1470 J of energy being **transferred** from one store to another.

The 1470 J that started out as chemical potential energy in your body has ended up being transferred to gravitational potential energy in the sword and stone (via the stackable stones).

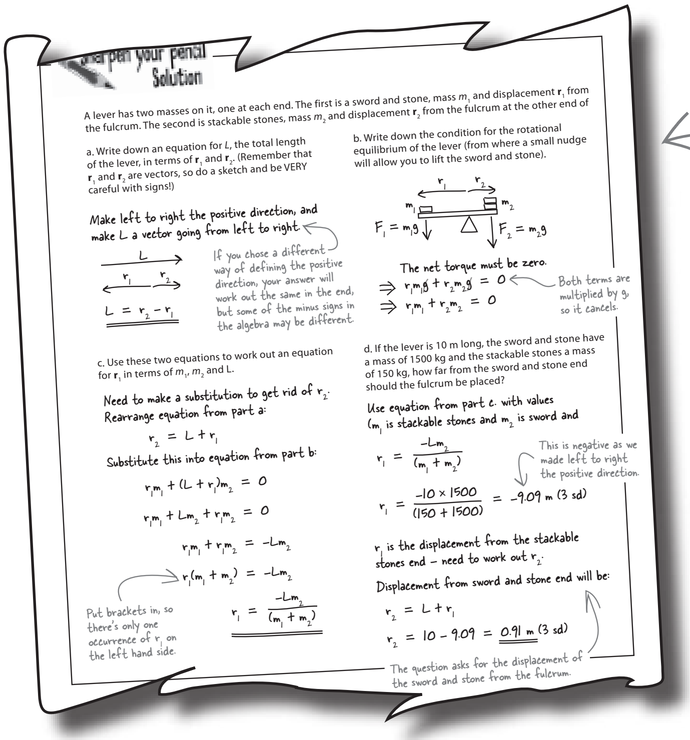
Here, we've used ground level as a reference point to measure all the displacements from.

You can think about any physical process that involves change in terms of energy transfer.



Energy conservation allows you to solve tough-looking problems quickly.

Back on page 528, you used forces, torques, similar triangles, substitution, etc to calculate the best way to lift a 1500 kg sword and stone using 150 kg of stackable stones and a lever.



arpen your pencil
Solution

A lever has two masses on it, one at each end. The first is a sword and stone, mass m_1 and displacement $\mathbf{r}_1$ from the fulcrum. The second is stackable stones, mass m_2 and displacement $\mathbf{r}_2$ from the fulcrum at the other end of the lever.

a. Write down an equation for L, the total length of the lever, in terms of $\mathbf{r}_1$ and $\mathbf{r}_2$. (Remember that $\mathbf{r}_1$ and $\mathbf{r}_2$ are vectors, so do a sketch and be VERY careful with signs!)

Make left to right the positive direction, and make L a vector going from left to right.

If you chose a different way of defining the positive direction, your answer will work out the same in the end, but some of the minus signs in the algebra may be different.

$$L = r_2 - r_1$$

b. Write down the condition for the rotational equilibrium of the lever (from where a small nudge will allow you to lift the sword and stone).

$F_1 = m_1 g$ $\quad$ $F_2 = m_2 g$

The net torque must be zero.

$$\Rightarrow r_1 m_1 g + r_2 m_2 g = 0$$

Both terms are multiplied by g, so it cancels.

$$\Rightarrow r_1 m_1 + r_2 m_2 = 0$$

c. Use these two equations to work out an equation for $\mathbf{r}_1$ in terms of m_1, m_2 and L.

Need to make a substitution to get rid of r_2. Rearrange equation from part a:

$$r_2 = L + r_1$$

Substitute this into equation from part b:

$$r_1 m_1 + (L + r_1)m_2 = 0$$

$$r_1 m_1 + Lm_2 + r_1 m_2 = 0$$

$$r_1 m_1 + r_1 m_2 = -Lm_2$$

$$r_1(m_1 + m_2) = -Lm_2$$

Put brackets in, so there's only one occurrence of r_1 on the left hand side.

$$r_1 = \frac{-Lm_2}{(m_1 + m_2)}$$

d. If the lever is 10 m long, the sword and stone have a mass of 1500 kg and the stackable stones a mass of 150 kg, how far from the sword and stone end should the fulcrum be placed?

Use equation from part c. with values (m_1 is stackable stones and m_2 is sword and

$$r_1 = \frac{-Lm_2}{(m_1 + m_2)}$$

This is negative as we made left to right the positive direction.

$$r_1 = \frac{-10 \times 1500}{(150 + 1500)} = -9.09 \text{ m (3 sd)}$$

r_1 is the displacement from the stackable stones end – need to work out r_2.

Displacement from sword and stone end will be:

$$r_2 = L + r_1$$

$$r_2 = 10 - 9.09 = 0.91 \text{ m (3 sd)}$$

The question asks for the displacement of the sword and stone from the fulcrum.

You don't want to have to go through all of this (and still not quite get the correct answer) when there's an easier way ...

But that was a complicated calculation where you had to be very careful with minus signs! And what's more - it wasn't obvious what height the stackable stones should start off at to lift the sword and stone the correct height (0.100 m off the ground) even when you'd worked out where to put the fulcrum.

Energy conservation helps you to solve problems with differences in height

The stackable stones are able to lift the sword and stone because of the **difference in height** between them.

With the lever in place, this difference in height causes a **change in height** for both the stackable stones and the sword and stone. This process involves **energy transfer**, from the stackable stones to the sword and stone.

You give an object gravitational potential energy by doing work on it, $W = \mathbf{F}_{||}\Delta\mathbf{x}$. As the force you're working against is the object's weight, $m\mathbf{g}$, and you're lifting it to a height, $\mathbf{h}$, the gravitational potential energy you give the object is $U_g = m\mathbf{gh}$.

Differences drive changes that lead to energy transfer.

Potential energy has the symbol U.

mg is the object's weight – the force you're doing work against.

$$U_g = mgh$$

The 'g' subscript shows you it's gravitational potential energy.

Here, the displacement is a height, so gets symbol 'h' to remind you of this.

One of our stackable stones is missing ...

Unfortunately, in between your first attempt at lifting the sword and stone (where it lifted but didn't go high enough) and this one, a stackable stone has gone missing. Now you only have 9 stones each with a mass of 15 kg instead of 10 ...

Sharpen your pencil

You are using a lever to lift the sword and stone (mass 1500 kg) a height of 0.100 m off the ground.

a. How much gravitational potential energy are you giving the sword and stone?

b. You have 9 stackable stones available to put on the other end of the lever, each with a mass of 15.0 kg. How much gravitational potential energy do you need to give the stackable stones in order for them to be able to lift the sword and stone to the required height?

c. What height do you need to lift the stackable stones to?

Sharpen your pencil Solution

You are using a lever to lift the sword and stone (mass 1500 kg) a height of 0.10 m off the ground.

a. How much gravitational potential energy are you giving the sword and stone?

$U_g = mgh$

$U_g = 1500 \times 9.8 \times 0.10$

$U_g = \underline{\underline{1470\ J}}$ (3 sd)

You need to give the sword and stone 1470 J of gravitational potential energy.

b. On the other side of the lever, you have 9 stackable stones, each with a mass of 15 kg. How much gravitational potential energy do you need to give them in order for them to be able to lift the sword and stone the required distance?

The stackable stones need to be able to do 1470 J of work on the sword and stone. Therefore you need to give them 1470 J of gravitational potential energy.

c. What height do you need to lift the stackable stones to?

Mass of stacking stones $= 9 \times 15 = 135$ kg.

$U_g = mgh$

$\Rightarrow h = \frac{U_g}{mg} = \frac{1470}{135 \times 9.8} = \underline{\underline{1.11\ m}}$ (3 sd)

This makes sense, as it's more than 1.00 m (the height you'd have needed to lift 10 stones).

there are no Dumb Questions

Q: Why are energy and work useful?

A: Using energy conservation and work to do problems is often more straightforward than using forces.

Q: Can you give an example of where using energy conservation to do a problem is easier than using forces?

A: If the problem involves **differences** in heights, for example if you need to lift an object, then using energy conservation is faster than using forces.

Any time there are differences that drive change, think about using energy conservation to solve problems.

Q: So what IS energy? It has the same units as work - does that mean they're both the same thing?

A: Not quite. Energy is the **capacity** that something has to do work.

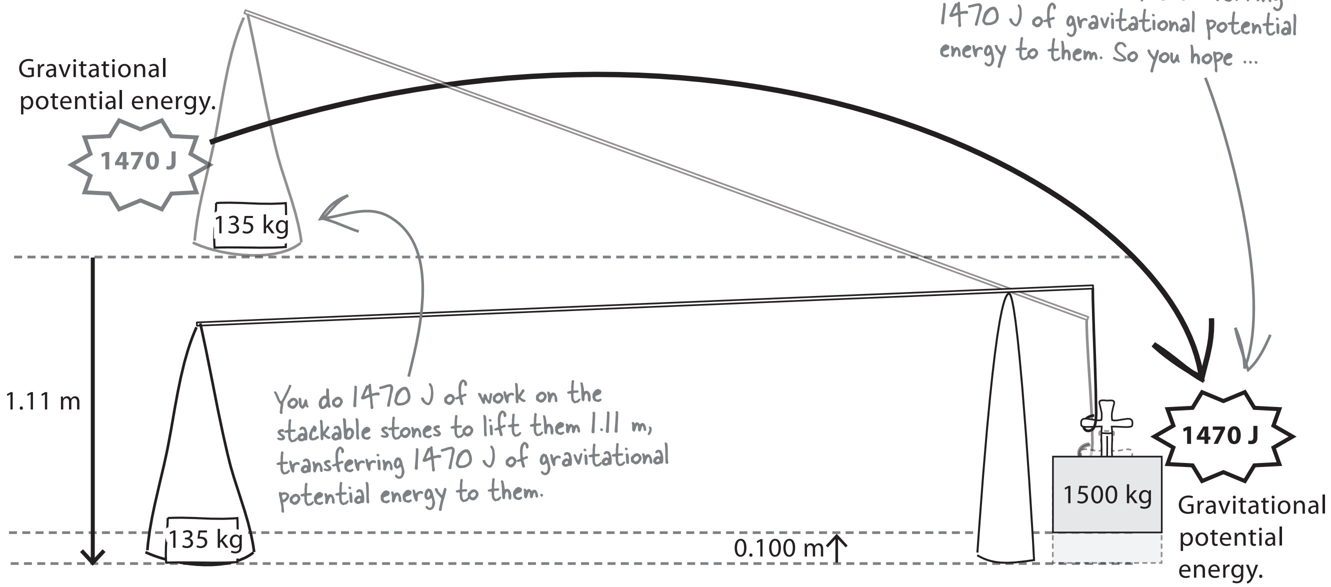
Q: But I thought that energy was something to do with electricity? Like when you use a food mixer or boil a kettle, you're using energy, aren't you?!

A: Energy can be stored in a number of different ways - we've already mentioned gravitational potential and chemical potential energy. Electrical potential energy is another way - but we won't be dealing with that in this chapter.

Think - what was the difference at the start, what change did it cause, and where has energy transfer taken place?

Will energy conservation save the day?

You've just calculated that lifting 135 kg of stackable stones 1.11 m gives them 1470 J of gravitational potential energy. This allows the stackable stones to do the 1470 J of work required to lift 1500 kg of sword and stone 0.100 m.



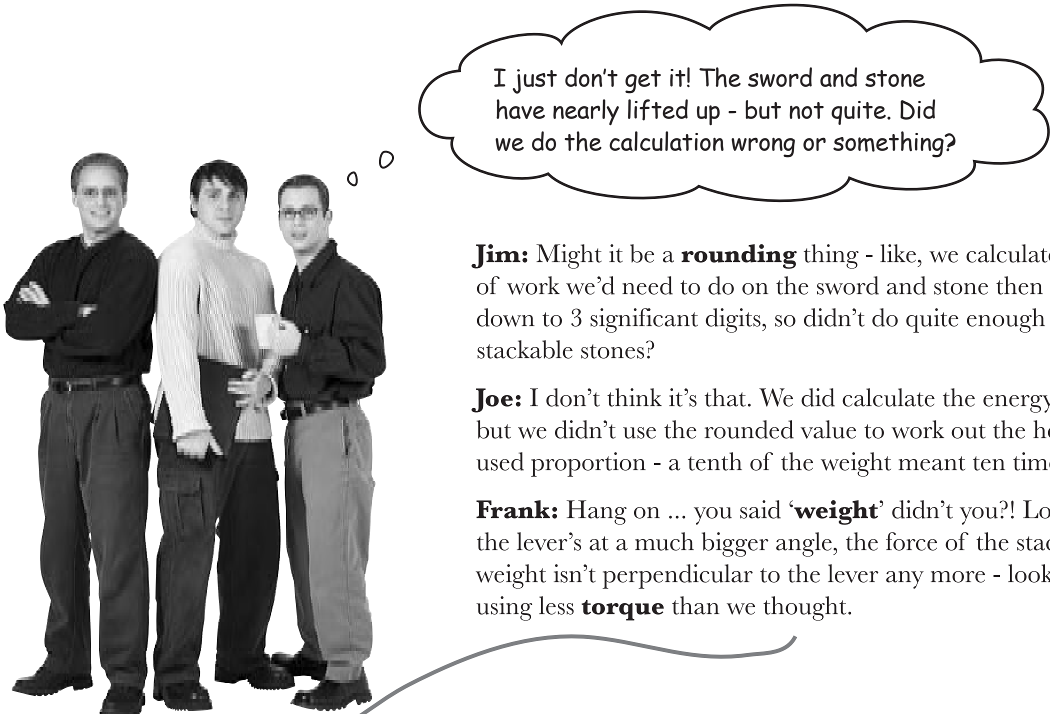
The rules say that you're allowed to have two attempts per lifetime, and this is your second attempt. Here goes ...

Oh. The sword and stone wobble a little, but **don't go up in the air**. They're very close to leaving the ground, but haven't quite made it.

If you can think quickly and work out what's gone wrong and how much extra you need to add to the stone side of the lever, there's still time to rescue the situation and claim your half of the kingdom!

BRAIN POWER

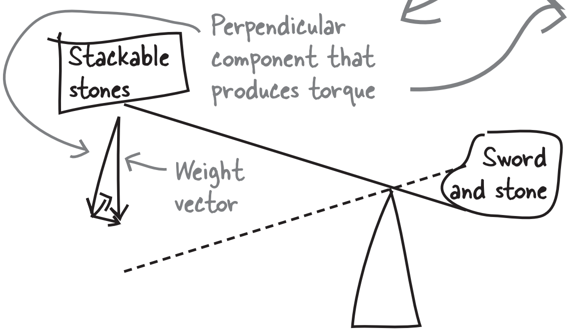
You've done the same amount of work on the stackable stones as you want them to do on the sword and stone. So why haven't you managed to lift the sword and stone?



Jim: Might it be a **rounding** thing - like, we calculated the amount of work we'd need to do on the sword and stone then rounded down to 3 significant digits, so didn't do quite enough work on the stackable stones?

Joe: I don't think it's that. We did calculate the energy at one point, but we didn't use the rounded value to work out the height. We just used proportion - a tenth of the weight meant ten times the height.

Frank: Hang on ... you said '**weight**' didn't you?! Look! Now that the lever's at a much bigger angle, the force of the stackable stones' weight isn't perpendicular to the lever any more - look! So we're using less **torque** than we thought.



Jim: But the weight vector of the sword and stone isn't perpendicular to the lever either. So its torque is less too. And as both force vectors make the **same angle** with the lever, the torques they produce will still be the same whatever angle the lever's at.

Frank: Hmmm. I think you're right. There must be another reason that we've not managed to do enough work on the sword and stone to lift it properly.

Joe: Wait...this isn't an ideal situation...what about friction?

Frank: How do you mean?

Joe: Well, if there's some **friction** in the fulcrum, we'd need to use more **force** to overcome it than if it's perfectly frictionless.

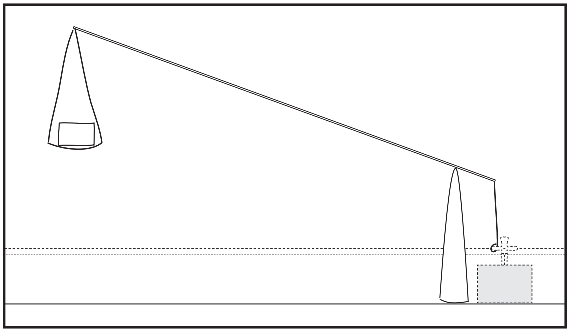
Frank: Ahh ... we'd be doing **work** against the force of friction as well as doing work against the force of gravity!

Jim: But how can you tell? It's not like the fulcrum gets lifted up.

Joe: Right...but before, we were expecting to use 100% of the stackable stones' gravitational potential energy to do work against gravity to lift the sword and stone. But we also need to do work against friction in the fulcrum. So not all of the stackable stones' energy is available to lift the sword and stone. Quick - find something else we can put on the lever ...

You need to do work against friction as well as against gravity

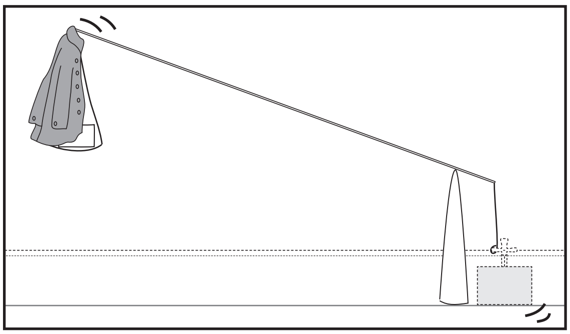
The sword and stone are teetering on the brink because it's impossible to be 100% **efficient**.



Your calculation assumed that all of the gravitational potential energy in the stackable stones would be available to do **work** on the sword and stone, so that all the stored energy you started with would be **transferred** in this way.

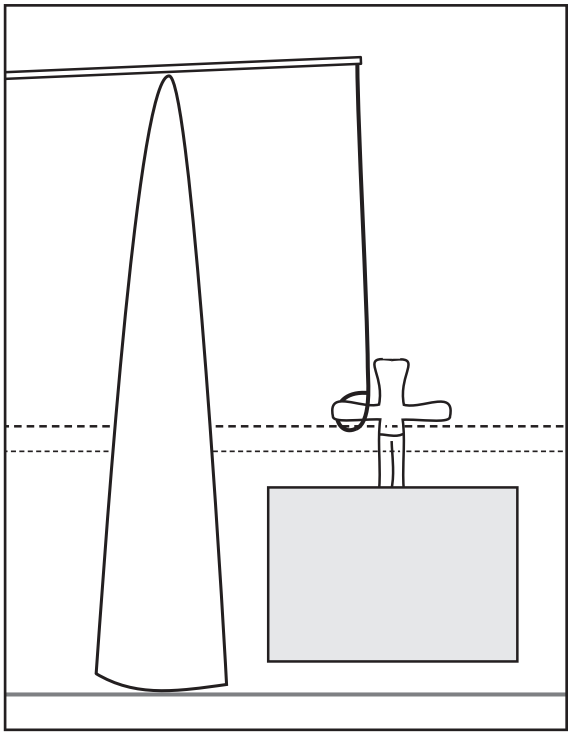
But there will be **friction** in the fulcrum of the lever, and some of the stackable stones' gravitational potential **energy** will go towards increasing the fulcrum's **internal energy** as you do work against the force of friction.

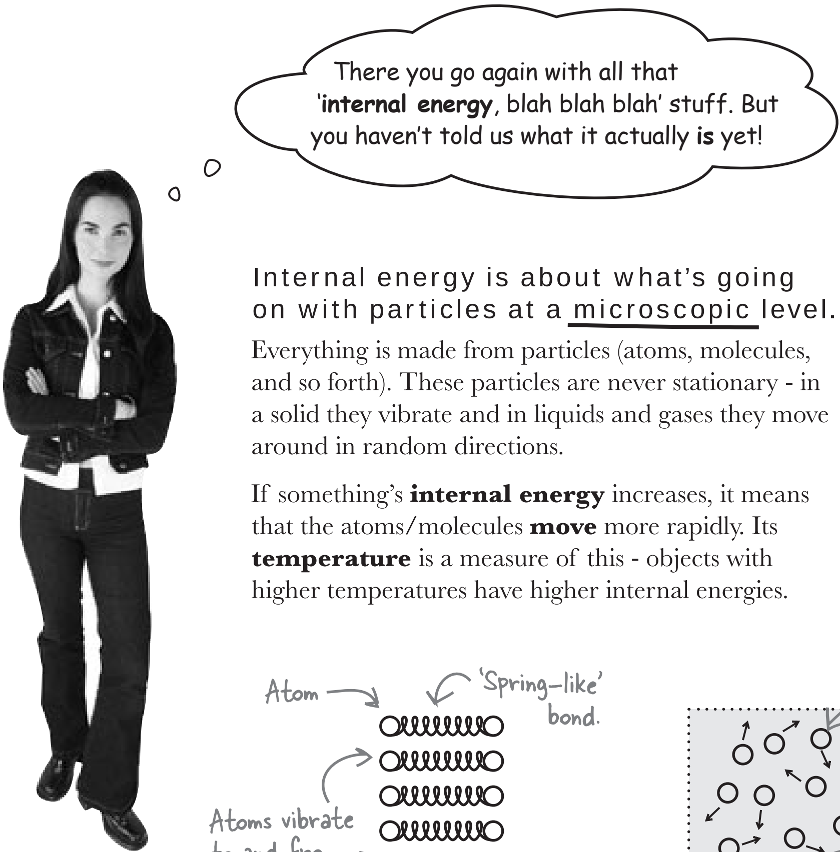
But as long as there isn't too much friction, it shouldn't be too difficult to tip the balance ...



Fortunately, the fulcrum's well-oiled, and the simple act of hanging your coat on the stackable stones is enough to tip the lever, lift the sword and stone, and gain you half the kingdom!

Result!

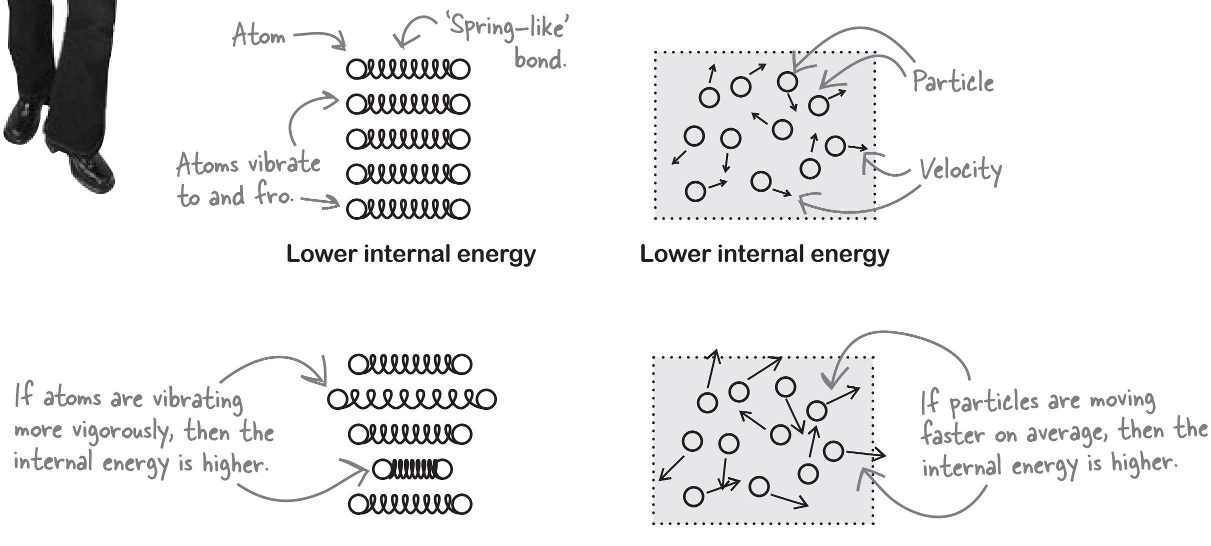




Internal energy is about what's going on with particles at a microscopic level.

Everything is made from particles (atoms, molecules, and so forth). These particles are never stationary - in a solid they vibrate and in liquids and gases they move around in random directions.

If something's **internal energy** increases, it means that the atoms/molecules **move** more rapidly. Its **temperature** is a measure of this - objects with higher temperatures have higher internal energies.



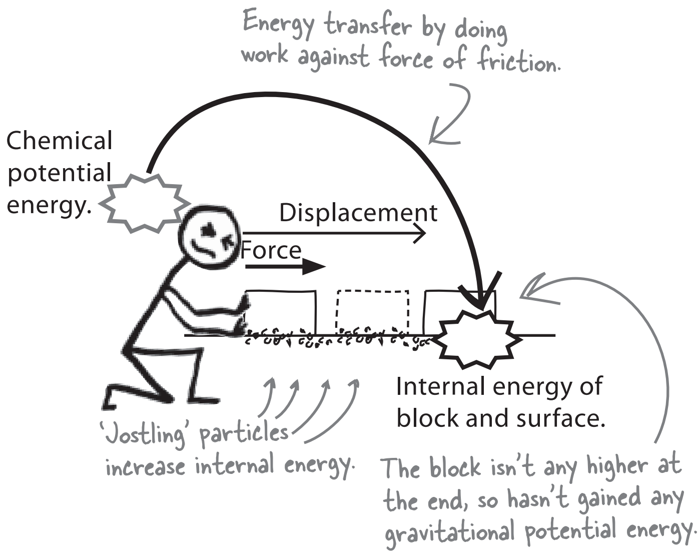
The higher something's temperature, the higher its internal energy.

Temperature is a measure of internal energy.

Doing work against friction increases internal energy

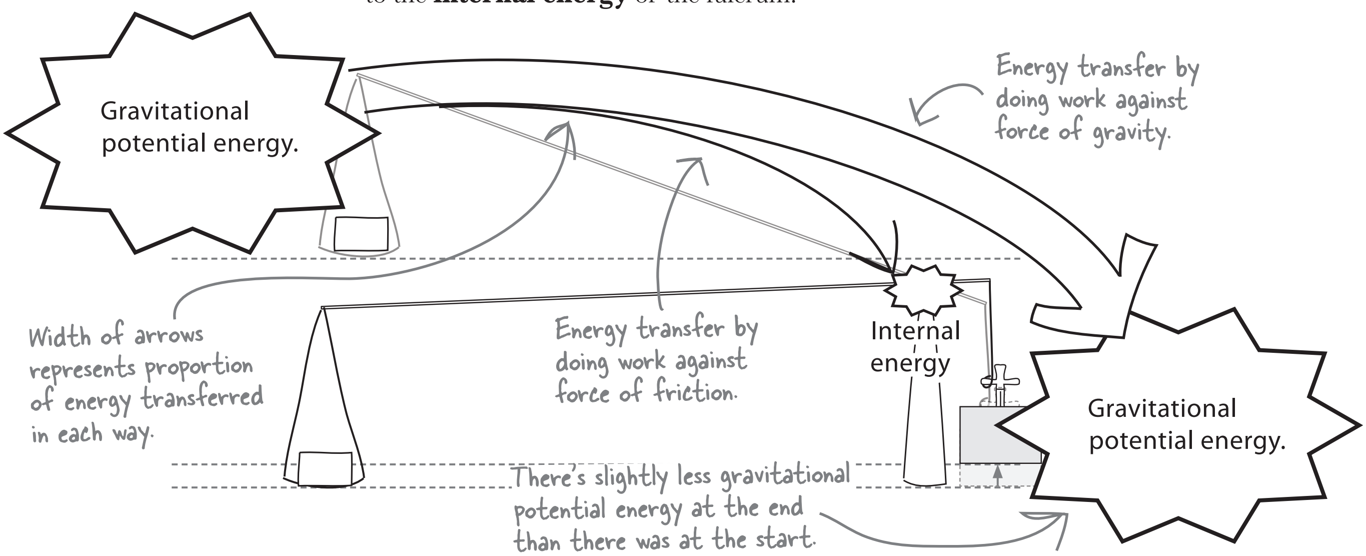
It's possible to increase something's internal energy by doing **work** on it against the force of **friction**. The most obvious way of seeing this is by pushing something along the ground. If you push it long enough, its temperature will increase (and so will the temperature of the ground).

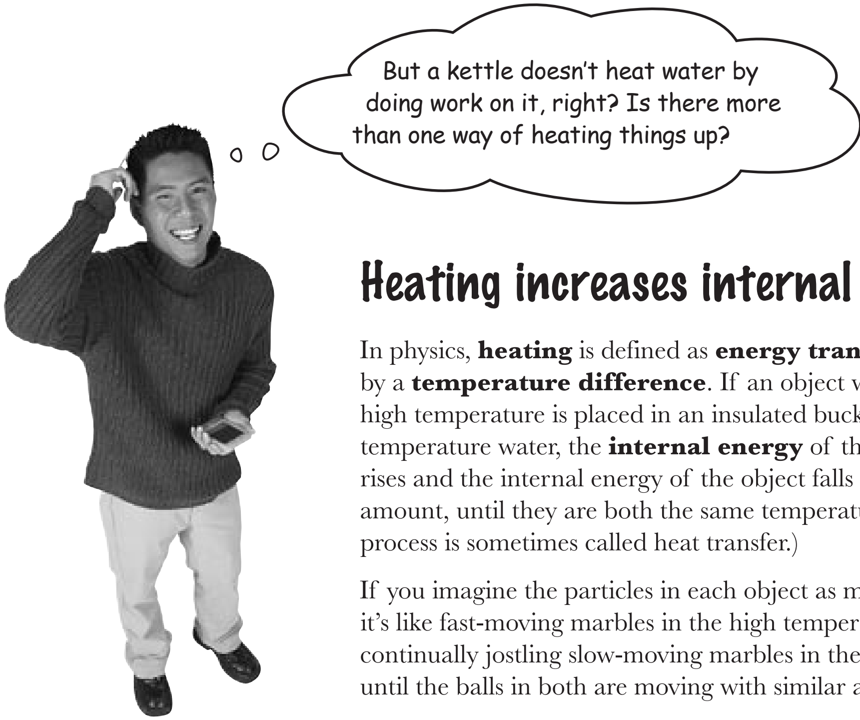
If you think about the particles as marbles, the two surfaces moving over each other is a bit like like jiggling and jostling a tray of marbles around so that the marbles end up with a greater average speed, and therefore a greater **internal energy**.



Something similar happens in the fulcrum of the lever, where the moving parts rub together. This time, most of the gravitational potential energy in the stackable stones does work against gravity and is transferred to gravitational potential energy in the sword and stone.

But work is also done against the **friction** that comes from the moving parts of the fulcrum rubbing together. This leads to some of the stackable stones' gravitational potential energy being transferred to the **internal energy** of the fulcrum.

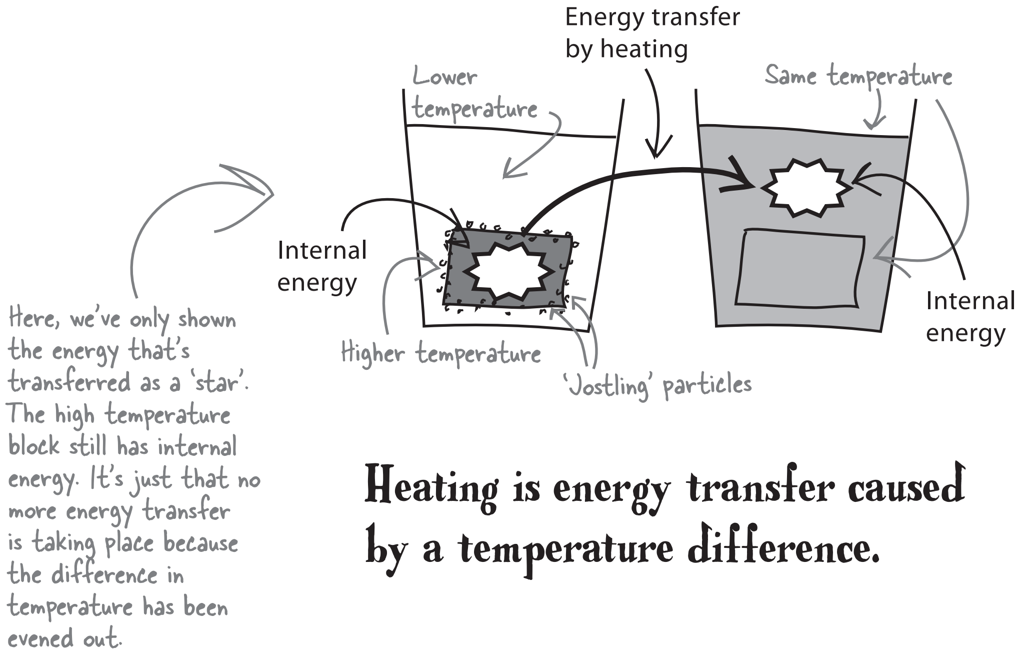




Heating increases internal energy

In physics, **heating** is defined as **energy transfer** caused by a **temperature difference**. If an object with a high temperature is placed in an insulated bucket of low temperature water, the **internal energy** of the water rises and the internal energy of the object falls by the same amount, until they are both the same temperature. (This process is sometimes called heat transfer.)

If you imagine the particles in each object as marbles, then it's like fast-moving marbles in the high temperature object continually jostling slow-moving marbles in the cold water, until the balls in both are moving with similar average speeds.



Heating is energy transfer caused by a temperature difference.

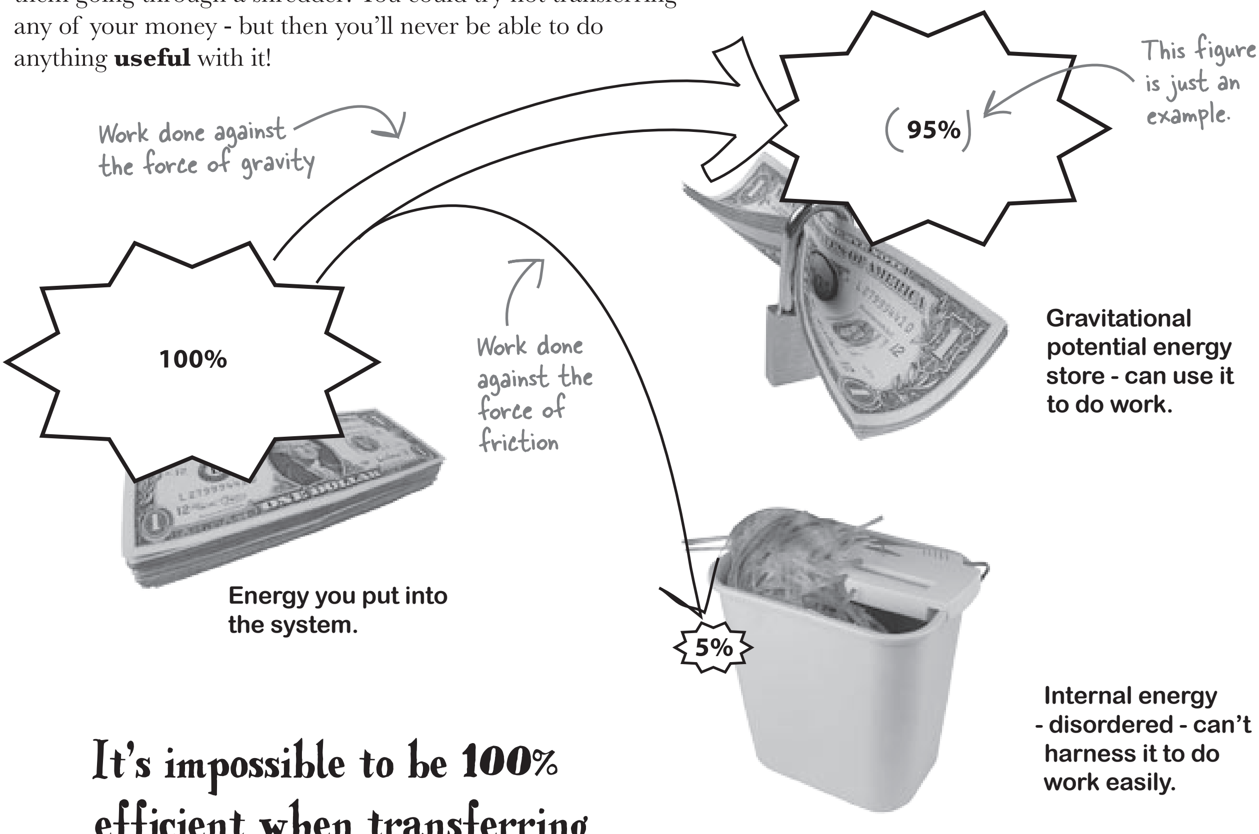
It's impossible to be 100% efficient

Back with the sword and stone, you've realized that not all of the gravitational potential energy in the stackable stones will be available to do work against the force of gravity by lifting the sword and stone. You also need to do work against the force of friction in the fulcrum - which you've successfully done by hanging your coat on the stackable stones!

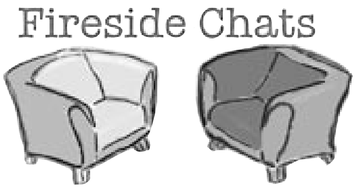
The **efficiency** of your lever is the **fraction** of energy that it can **usefully** transfer to the sword and stone. Increasing the internal energy of the fulcrum isn't useful to you! The total energy is always conserved - some is transferred to the sword and stone and some to the fulcrum.

It's like having a banking system where you're not able to transfer dollar bills from one place to another without some of them going through a shredder! You could try not transferring any of your money - but then you'll never be able to do anything **useful** with it!

Efficiency is the fraction of energy you can usefully transfer.



It's impossible to be 100% efficient when transferring energy from one store to another.



Tonight's talk: **Energy and Work go head to head.**

Energy: Oh hi work, what are you up to today?

Work: Whatever it is ... it'll be more useful than what you're up to, I'm sure!

Energy: Hey - there's no need to get so hot under the collar! What's up?

Work: Well, to tell you the truth, I'm feeling a little left out. It was great back at the start of the chapter when I was the star of the show. I loved the attention. But now you've come along, people are thinking almost exclusively about you instead.

Energy: Oh yes, I have rather hogged the limelight recently!

Work: Yeah, and it's me who's the useful one! If you want to get something moved by applying a force, I'm right in there!

Energy: Well, to be fair ... I'm in there too!

Work: But only as a noun - not a verb. If work is done on something, the energy is transferred. See?

Energy: But if it wasn't for me, you wouldn't be able to happen at all!

Work: Err ... I don't think so. How can you say that I depend on you?

Energy: Well, energy is the capacity that something has to do work. If something doesn't have a capacity to do work, then it can't do work. No me - no you!

Work: There you go - playing with words as usual and not letting yourself be pinned down. At least I know what I am - a displacement × the component of a force in the same direction. But what are you?

Energy: I'm the capacity that something has to do work!

Work: That's not what you are though - that's just words!

Energy: OK then - I'm always conserved, and I can't be created or destroyed. How about that then?!

Work: Well, it sounds a bit metaphysical - but is it physics?! And it still doesn't tell me what you actually are!

Energy: I guess that's because you can't pin me down really. You can't measure me directly. You can only measure changes as I'm transferred from one thing to another.

Work: Ha! Now there's where you can't get on with your life without me!! You need me!

Energy: Meaning ...

Work: Well, when work is done, energy is transferred. If it wasn't for me, you'd be totally stuck in one place all the time. No holidays, no day trips - just stuck.

Energy: That's not entirely true. Yes, doing work is one way of transferring energy, but I can also be transferred through heating, if there's a temperature difference between two things. The hot one gets cooler (so its internal energy gets lower) and the cool thing gets hotter (so its internal energy gets higher, until they both have the same temperature

Work: But that's all the same, isn't it? Where's the excitement?! At least when I act on something, you usually get transferred in a different way - like into something's potential energy. I give your life variety!

Energy: Yeah, but sometimes you do transfer me to raise something's internal energy as well - when there's friction involved.

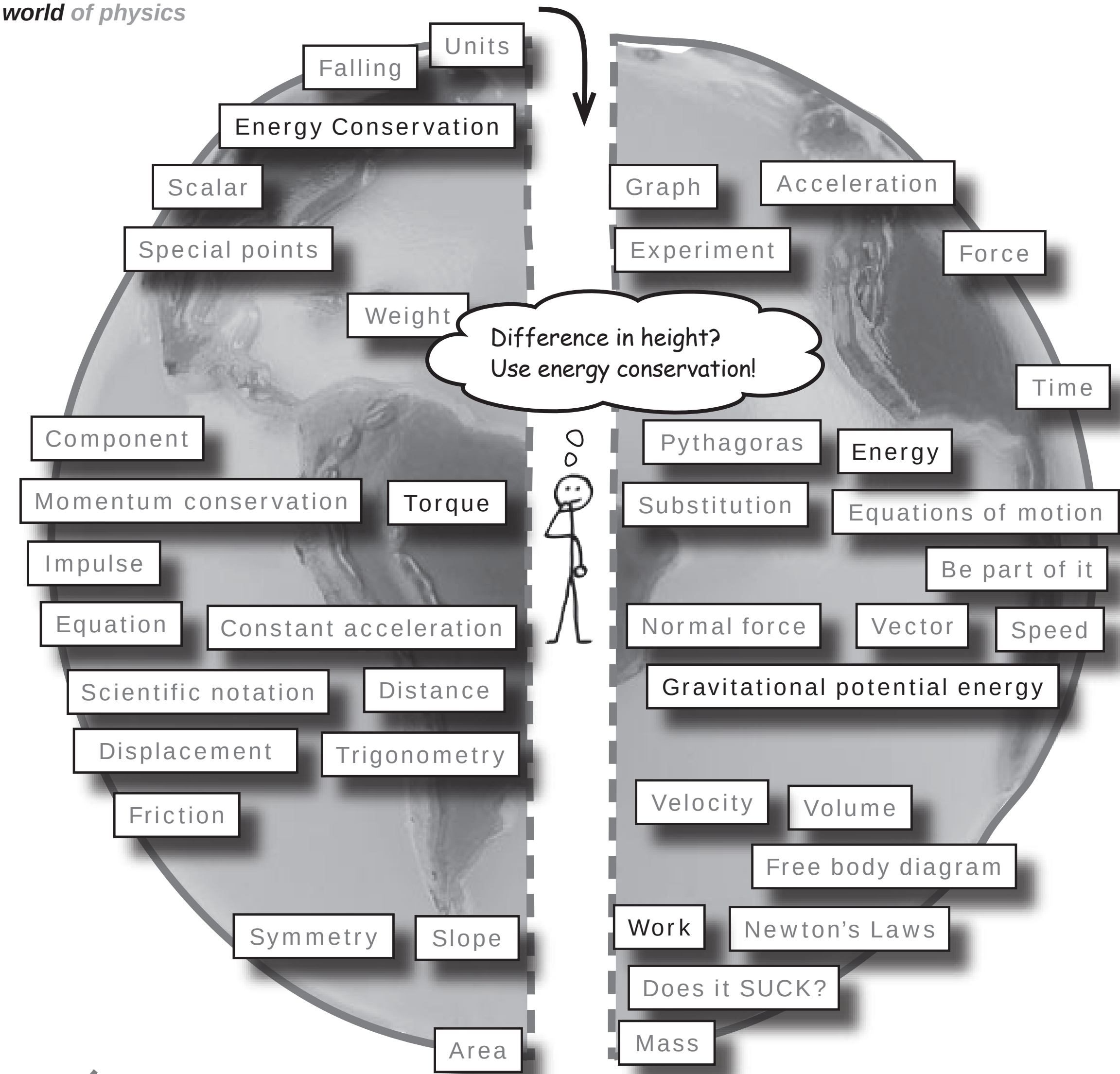
Work: Ah yes, friction. I hate him. Y'know, the reason I didn't get a bonus this year was because I wasn't efficient enough. And guess whose fault that was!

Energy: Yeah ... I guess that's something we agree on. I don't get to go to such interesting places if there's too much friction.

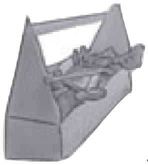
Work: But I'm still more useful than you!

Energy: And you still couldn't do anything without me - so there!

Work: Whaaatever.



☑	Torque	A "turning force" equal to component of force perpendicular to arm × distance from the fulcrum
☑	Work	When moving an object in the opposite direction from a force, work done = force × displacement parallel to force
☑	Energy	The capacity that something has to do work.
☑	Potential energy	The capacity something has to do work due to its increased height.
☑	Energy conservation	The total energy of a system must be the same before and after a change to the system, as energy is conserved.



Your Physics Toolbox

You've got Chapter 13 under your belt and added some problem-solving tools to your toolbox.

Lifting an object

If you have to lift an object in a physics problem, you should assume that the minimum force you require is equal to the object's weight (unless there is friction involved).

This is because Newton's 1st Law says that with zero net force, an object continues with a constant velocity. So once you've got it going upwards with an extra little 'nudge', you only need to exert a force equal to its weight to maintain its velocity.

"Zero net torque"

If you have a problem where something could potentially rotate and you have to work out the force you need to apply to make sure it doesn't, then you are being asked what force is required for there to be zero net torque.

When you're working with torques, take care to make one direction of rotation positive and stick with it.

Doing work

Doing work is a way of transferring energy from one store to another.

If you do work against a gravitational force by lifting an object, you increase its gravitational potential energy.

If you do work against a frictional force by pushing an object, you increase the internal energy of the object and of the surface it's in contact with.

"Spot the difference"

Differences drive changes that lead to energy transfer.

If there's a difference in the height of an object between the start and finish of a problem, look and see if you can use energy conservation instead of forces to solve it.

14 energy conservation

Making your life easier

Why do things the hard way when there's an easier way?

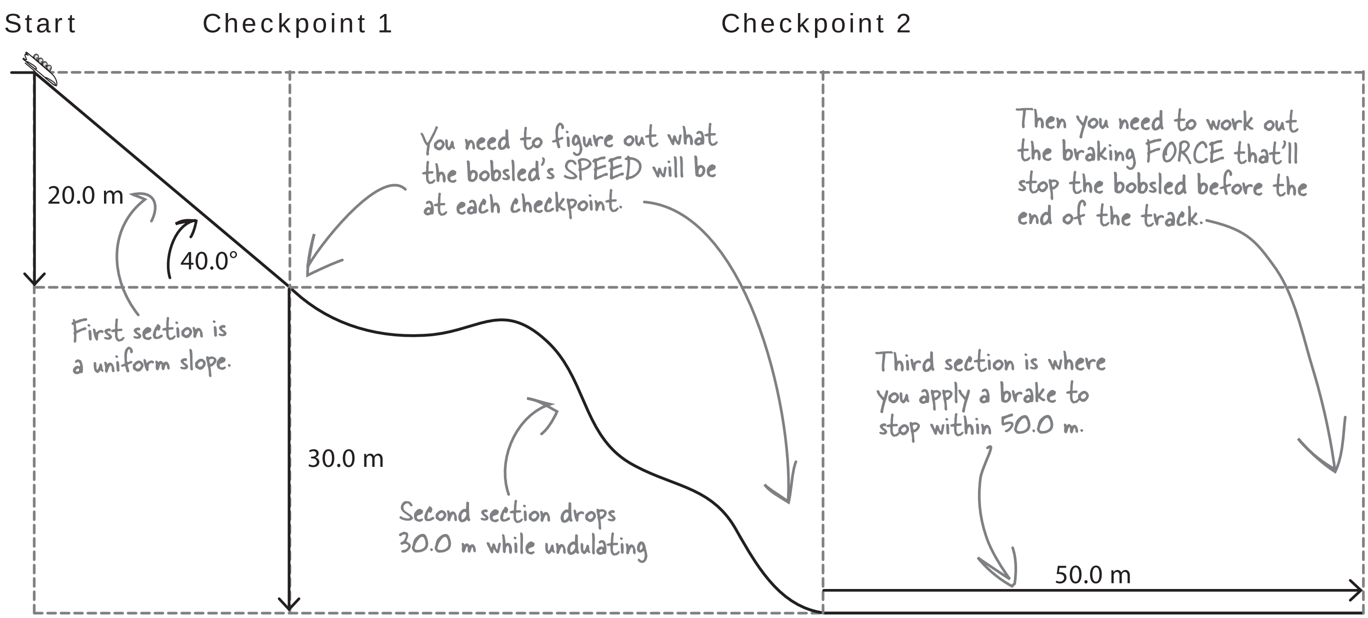
So far, you've been solving problems using equations of motion, forces and component vectors. And that's great - except that it sometimes takes a while to crunch through the math. In this chapter, you'll learn to spot where you can use **energy conservation** as a shortcut that lets you solve complicated-looking problems with relative ease.

The ultimate bobsled experience

The fairground has designed a unique, new, state-of-the-art bobsled track that's due to open soon. But before anyone can use it, it has to be designated as safe to ride. That's your job. Even though you're not a bobsledder, you can still use physics to figure out whether or not any adjustments have to be made.

From the start to checkpoint 1, the track has a **uniform slope**. Between checkpoints 1 and 2, the track **drops 30.0 m**, but the track **undulates** - the bobsled's even going uphill for a bit! Then the third part of the track is totally flat - where the bobsled is **stopped** by applying its brake to the ice.

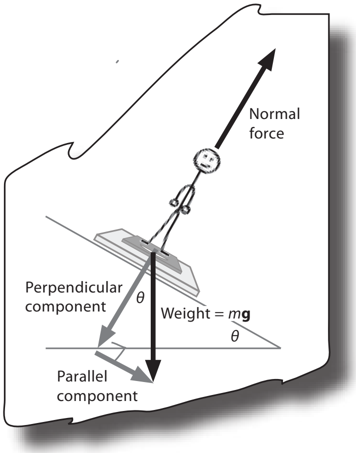
Your job is to work out what the bobsled's **speed** will be at each checkpoint, and how hard you need to brake at the end.



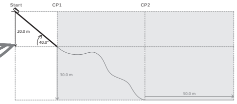
Start any problem with a sketch and by asking yourself "What's it LIKE?"

BRAIN POWER

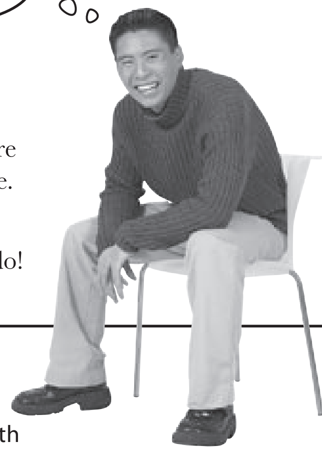
Can you see any parts of the bobsled track that you **already know** how to deal with?



Hey ... haven't we seem something going down a uniform slope before?!



You are here.



You already know how to do this!

The **first** section of track looks like the WeightBotchers machine from chapter 11 where the person on a set of scales slides down a slope.

Even if the later parts of the track look tough, you can still start with what you can **already** do!

Sharpen your pencil

You can already handle the first part of the track.

A bobsled, mass m, travels down a uniform slope that makes an angle of 40.0° with the horizontal and drops 20.0 m in height between the start and the first checkpoint.

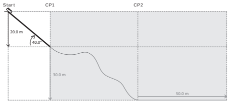
a. What distance does the bobsled travel between the start and the first checkpoint?

b. What is the component of the bobsled's weight parallel to the slope?

Hint: You don't know a value for the bobsled's mass, so this answer won't be purely numerical and will have 'm' in it

c. What is the bobsled's speed at the first checkpoint?

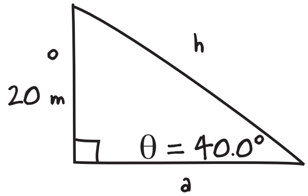
Hint: Use Newton's 2nd Law to move from net force to acceleration, then equations of motion to calculate the speed.



Sharpen your pencil Solution

A bobsled, mass *m*, travels down a uniform slope that makes an angle of 40.0° with the horizontal and drops 20.0 m in height between the start and the first checkpoint.

a. What distance does the bobsled travel between the start and the first checkpoint?

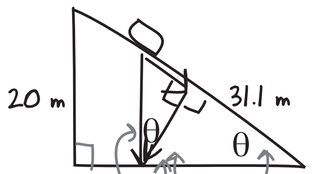


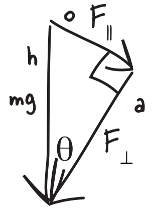
$$\sin(\theta) = \frac{o}{h}$$

$$h = \frac{20.0}{\sin(40.0)}$$

$$h = \underline{\underline{31.1\text{ m}}}\ (3\text{ sd})$$

b. What is the component of the bobsled's weight parallel to the slope?





Bobsled's weight = mg

$F_{\parallel}$ component down the slope is the side opposite θ.

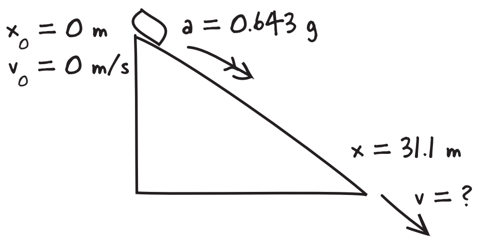
Use similar triangles to calculate side:

$$\frac{F_{\parallel}}{mg} = \frac{20.0}{31.1}$$

$$F_{\parallel} = \underline{\underline{0.643\ mg}}\ (3\text{ sd})$$

If your answer contains variables, such as 'm' and 'g', you don't need to put in units at the end, as the variables already have units. But you still need units for a purely numerical answer.

c. What is the bobsled's speed at the first checkpoint?



$$F_{net} = ma$$

$$0.643\ \cancel{m}g = \cancel{m}a$$

$$a = 0.643\ g$$

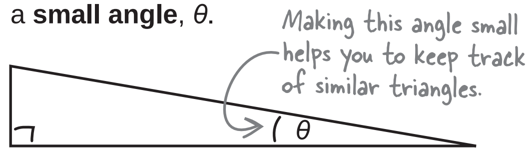
The acceleration is 0.643 g, so remember to put in the 9.8

$$v^2 = v_0^2 + 2a(x - x_0)$$

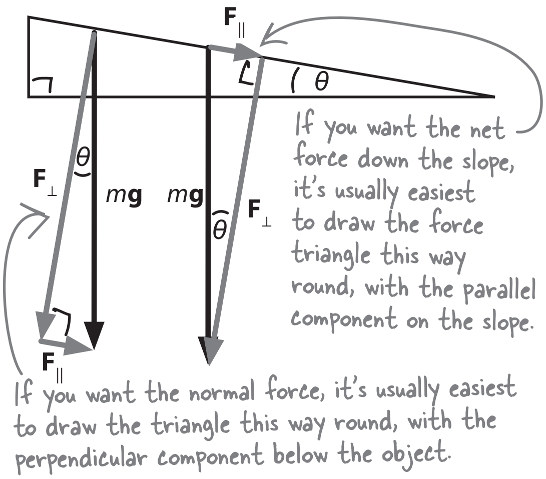
$$v = \sqrt{0 + 2 \times 0.643 \times 9.8 \times 31.1} = \underline{\underline{19.8\text{ m/s}}}\ (3\text{ sd})$$

Triangle Tip: sketch extreme angles

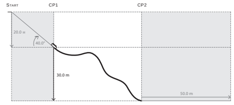
If you're not sure which angle in your force vector triangle corresponds to the angle of your slope, sketch a slope with a **small angle**, *θ*.

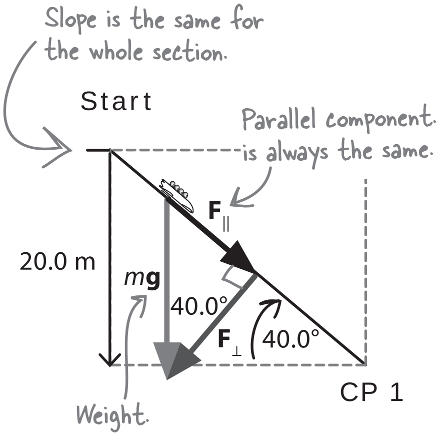


Now draw the force triangle. Draw on the weight pointing straight down. Then draw in the parallel and perpendicular components. It doesn't matter which way round you draw the components, as the triangle's sides will still be the same length.



θ is the small angle in the slope triangle - so *θ* will also be the small angle in the force triangle.





Forces and component vectors solve the first part ...

The first part of the track has a **uniform slope**, that always makes the same angle with the horizontal. This means that the bobsled always experiences the same **net force** down the slope, as the component of the bobsled's weight **parallel** to the slope is constant.

And you've just worked out that the bobsled will have reached a speed of 19.8 m/s when it reaches the first checkpoint. So far, so good ...

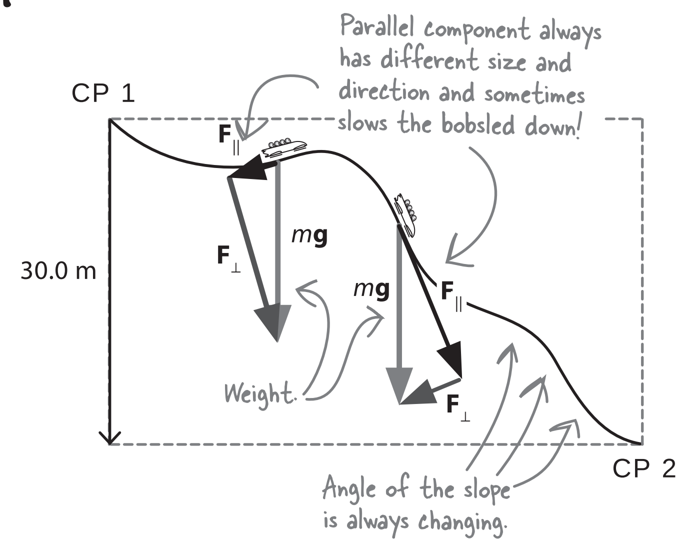
You want to know the bobsled's speed at each checkpoint.

...but the second part doesn't have a uniform slope

However, the second part of the bobsled run isn't quite as straightforward. Although the track drops a further 30.0 meters, the slope definitely isn't uniform - it has peaks and dips along the way. Sometimes the bobsled's even going uphill!

Any time the **angle** of the slope changes, the component of its weight parallel to the slope also changes. This means that the net force on the bobsled changes, so the size and direction of its **acceleration changes**.

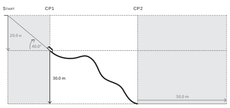
This is a problem, because the equations of motion only work when an object's acceleration is constant. So you can't use the same method as you did for the uniform slope.

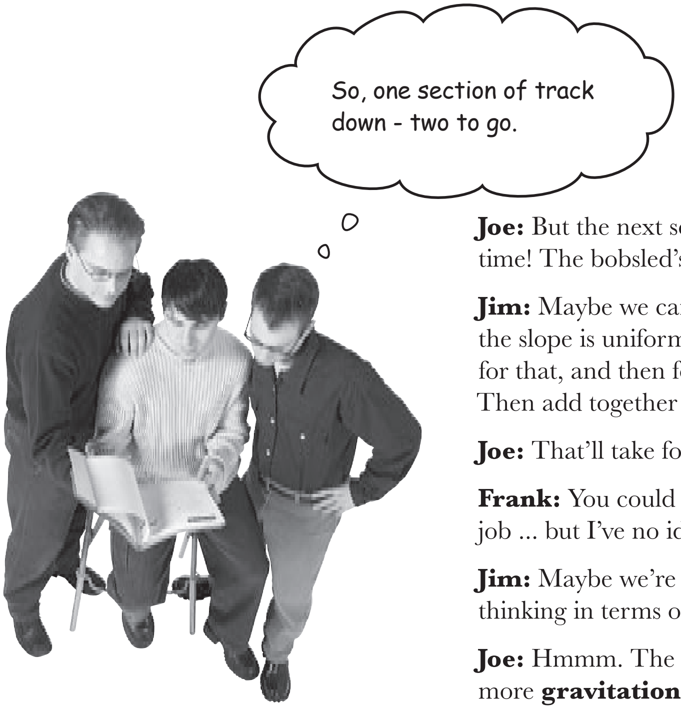


An object moving down a slope is accelerated by the component of its weight <u>parallel</u> to the slope.

BRAIN POWER

How can you deal with an **undulating** slope when all you know is that the bobsled drops 30.0 meters between the the first and second checkpoints?





Joe: But the next section's more difficult - the slope changes all the time! The bobsled's acceleration won't be constant.

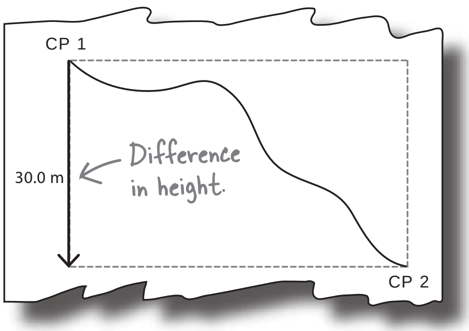
Jim: Maybe we can split the section up into lots of tiny sections? If the slope is uniform for a couple of meters, we could do a calculation for that, and then for the next bit, and then for the next bit and so on. Then add together all the changes in speed to get our final answer.

Joe: That'll take for ever! I don't think you could do it by hand.

Frank: You could probably program a computer to do that kinda job ... but I've no idea how to do that.

Jim: Maybe we're going about this the wrong way. We've been thinking in terms of **forces** so far, but what about **energy**?

Joe: Hmmm. The bobsled loses 30.0 m of height. So the bobsled has more **gravitational potential energy** at the start of the section than it does at the end because of the **difference in height**.



Any time you see a change in height, think about using energy conservation.

Frank: But how do we do a calculation with that? I'm not sure where that energy's been transferred to!

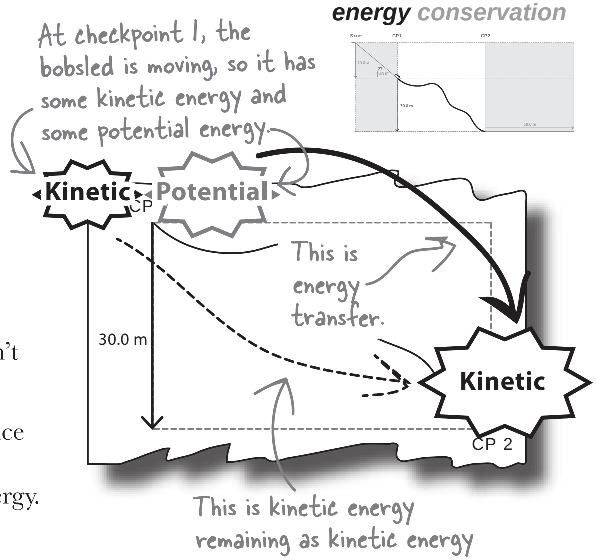
Jim: Yeah - but **energy's conserved**, right?! So the energy must be transferred in some way when the bobsled goes down the slope!

Jim: I wonder if we can say that the bobsled has energy because it's **moving**?! Energy's the capacity something has to do work, right?

Frank: Yeah ... if a moving object hits another object, the first object will exert a **force** that **displaces** the second object. So work gets done. A hammer! A moving hammer exerts a force to displace a nail into wood. That's doing work, isn't it?

Joe: I think we'd better take this further!

A moving object has kinetic energy



At the top of the slope the bobsled has more **gravitational potential energy** than it does part-way down or at the bottom because of the difference in **height**.

Moving objects are said to have **kinetic energy**. They're able to do work by exerting a force that displaces something - that's how hammering in a nail works. If the hammer was stationary and didn't have a **velocity**, you couldn't use it to do work on the nail!!

Differences drive changes that involve energy transfer. The difference in **height** leads to a change in the **velocity** of the bobsled. This means that the bobsled gains kinetic energy as it loses potential energy. If the bobsled's potential energy goes down by 1000 J due to the difference in height, its kinetic energy increases by 1000 J.

If you can work out an **equation** for the kinetic energy, you'll be able to use **energy conservation** to calculate the bobsled's velocity at checkpoint 2 due to its change in height.

The change in potential energy is the same size as the change in kinetic energy because energy is conserved.

This is assuming there's no friction – and there won't be that much on ice.

Sharpen your pencil

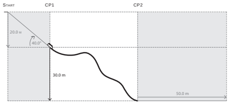
This is doing work: Work = $F\Delta x$.

a. Imagine a **moving** hammer doing work on a nail by exerting a force on it to move it a displacement into a piece of wood. The more **kinetic energy** the hammer has, the more work it can do. With this in mind, what variables do you think the hammer's kinetic energy might depend on, and why?

Now you can collect useful equations that will help you work out an equation for kinetic energy. In this "Sharpen your pencil", you're only writing the equations down - you'll do the rearranging and substituting on the next page. If you can't remember the equations, it's OK to look them up in appendix $.

b. Write down an equation for the gravitational potential energy of the bobsled, mass *m*, at its highest point, height **h**.

c. Write down an equation of motion for a falling object that involves **x**, $\mathbf{x_0}$, $\mathbf{v_0}$, **v** and **a**.



Sharpen your pencil Solution

a. Imagine a **moving** hammer doing work on a nail by exerting a force on it to move it a displacement into a piece of wood. The more **kinetic energy** the hammer has, the more work it can do. With this in mind, what variables do you think the hammer's kinetic energy might depend on, and why?

The hammer will do more work if it has a high speed.

The hammer will do more work if it has a large mass.

So kinetic energy must depend on both mass and velocity.

The more work the hammer does with each hit, the larger the displacement of the nail with each hit.

Now you can collect useful equations that will help you work out an equation for kinetic energy. In this "Sharpen your pencil", you're only writing the equations down - you'll do the rearranging and substituting on the next page. If you can't remember the equations, it's OK to look them up in appendix $.

b. Write down an equation for the gravitational potential energy of the bobsled, mass *m*, at its highest point, height **h**.

$$U_g = F\Delta x = mgh$$

This gives you an expression for the potential energy that will be transferred to kinetic energy.

c. Write down an equation of motion for a falling object that involves **x**, $\mathbf{x_0}$, $\mathbf{v_0}$, **v** and **a**.

$$v^2 = v_0^2 + 2a(x - x_0)$$

This will help you to connect the difference in height with the speed.

there are no Dumb Questions

Q: If I know the difference in height between the start and end of a uniform slope, I can work out an object's velocity at the end, right?

A: Yes. If a slope is uniform and always at the same angle (like the first part of the track was), you can use forces and component vectors to calculate the net force on an object, and therefore its acceleration and final velocity.

Q: What about an undulating slope? Can I do the same to solve that problem?

A: You could in theory ... though not easily. You'd need to work out the net force on the object for every tiny part of the slope. This is practically impossible unless you program a computer do to it for you.

Q: Does a problem with an undulating slope always need a computer to solve it?

A: No - you can use energy conservation. The **potential** energy that the object has at the top of the slope must be transferred to **kinetic** energy as the object goes down the slope.

Q: What's kinetic energy?

A: The capacity that something has to do work due to its velocity. Moving things have kinetic energy.

Q: I know that the potential energy is mgh, but what's the equation for kinetic energy?

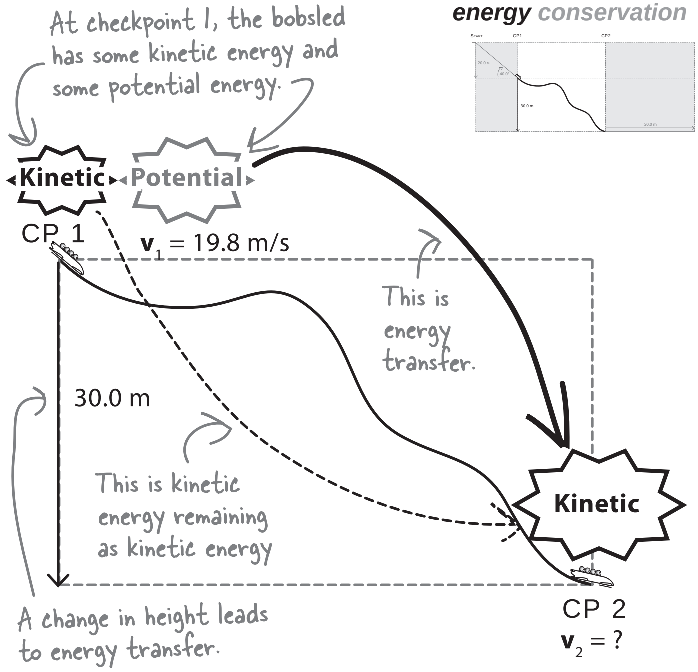
A: That's what you're in the process of working out. So far, you've realized that an object's kinetic energy must depend on its mass and velocity. And you're about to do some substitutions to see exactly how ...

The kinetic energy is related to the velocity

Moving objects have **kinetic energy**.

As the bobsled gets lower down the track, some of the **potential energy** it had at the start will be **transferred** to kinetic energy. The kinetic energy the bobsled gains must be **equal** to the potential energy that it loses.

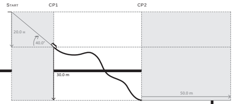
The kinetic energy must depend somehow on the **mass** and **velocity** of the bobsled. If you can work out an equation for the change in kinetic energy, you can use it to calculate the bobsled's change in velocity between the first and second checkpoints - and therefore its velocity at the second checkpoint.



Sharpen your pencil

a. The two equations you wrote down on the previous page, $U_g = m\mathbf{gh}$ and $\mathbf{v}^2 = \mathbf{v}_0^2 + 2\mathbf{a}(\mathbf{x} - \mathbf{x}_0)$, are both written down in the form that you'll find them on equation sheets - but they use different letters to represent the same quantities. If you're dealing with an object dropped straight down and accelerated by gravity, write down which variables in the equations are equivalent to each other.

b. By making a substitution for the change in height, show that the kinetic energy (K) of an object dropped straight down from a stationary start at height **h** which started with potential energy, $U_g = m\mathbf{gh}$ is $K = ½m\mathbf{v}^2$.



Sharpen your pencil Solution

a. The two equations you wrote down on the previous page, $U_g = m\mathbf{gh}$ and $\mathbf{v}^2 = \mathbf{v}_0^2 + 2\mathbf{a}(\mathbf{x} - \mathbf{x}_0)$, are both written down in the form that you'll find them on equation sheets - but they use different letters to represent the same quantities. If you're dealing with an object dropped straight down and accelerated by gravity, write down which variables in the equations are equivalent to each other.

$U = mgh \qquad v^2 = v_0^2 + 2a(x - x_0)$

In the first equation, the displacement is h; in the second it's $x - x_0$.

In the first equation, the acceleration is g; in the second it's a.

b. By making a substitution for the change in height, show that the kinetic energy (K) of an object dropped straight down from a stationary start at height **h** which started with potential energy, $U_g = m\mathbf{gh}$ is $K = ½m\mathbf{v}^2$.

Stationary start, so $v_0 = 0$.

Energy is conserved, so $U_{start} = K_{end}$

The change in height, h, is $x - x_0$ so rearrange and substitute.

Substitute 'g' for 'a'. → $v^2 = 2g(x - x_0)$

$(x - x_0) = \frac{v^2}{2g}$

Substitute 'h' for '$x - x_0$' → $h = \frac{v^2}{2g}$ ← Substitute for 'h'

$K_{end} = U_{start} = mgh = m\cancel{g}\frac{v^2}{2\cancel{g}}$

$K_{end} = ½mv^2$

It's sometimes a good idea to drop the 'g' subscript from U_g, the gravitational potential energy.

This is because the subscript is only to distinguish gravitational potential energy from U_s, the elastic potential energy of a spring.

In a situation where there are no springs, and there could be confusion with 'g', the gravitational field strength, it's best just to use the symbol 'U' with no subscript for gravitational potential energy. This also allows you to write U_{start} and U_{end} more easily.

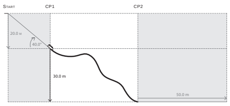
But that equation's for something that falls straight down, right? The bobsled doesn't do that!

The same change in height always leads to the same change in potential energy, regardless of path.

The amount of energy transferred only depends on the change in height.

The bobsled has a certain store of **potential energy** as a result of being at a certain height. Its potential energy store doesn't depend on what path it took to get up there in the first place.

The same change in height always leads to the same change in potential energy when the bobsled's going down as well as when it's going up. So that change in potential energy must be transferred to **kinetic energy**, whatever path the bobsled takes (assuming there's no friction). The bobsled will always have the same change in kinetic energy for the same change in height.



Calculate the velocity using energy conservation and the change in height

The kinetic energy of a moving object is given by the equation:

$$K = \tfrac{1}{2}m\mathbf{v}^2$$

This means that if you know the bobsled's **mass** and **kinetic energy**, you can use them to calculate the bobsled's **velocity**.

Kinetic energy → K; Mass → m; Velocity → v

$$K = \tfrac{1}{2}mv^2$$

You can calculate the bobsled's kinetic energy using energy conservation. The difference in its speed between checkpoint 1 and checkpoint 2 is because of the difference in height between the two checkpoints. So the change in **potential** energy and the change in **kinetic** energy must be the same size.

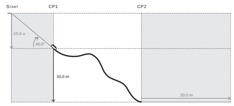
You're ready to work out the velocity at the second checkpoint? Cool!

Sharpen your pencil

A bobsled travelling down a track has just reached its first checkpoint (CP1). The second checkpoint (CP2) is at the bottom of the slope, 30.0 m lower than the first.

If the bobsled has a speed of 19.8 m/s at CP1, what is its speed at CP2? (The bobsled's potential + kinetic energy at the start will be equal to its kinetic energy at the end.)

Hint: It's a good idea to use subscripts to represent energy at each checkpoint, for example K_1, U_1, K_2, U_2, etc.



Sharpen your pencil Solution

A bobsled travelling down a track has just reached its first checkpoint (CP1). The second checkpoint (CP2) is at the bottom of the slope, 30.0 m lower than the first.

If the bobsled has a speed of 19.8 m/s at CP1, what is its speed at CP2? (The bobsled's potential + kinetic energy at the start will be equal to its kinetic energy at the end.)

CP1: $K_1 + U_1 = mgh + \frac{1}{2}mv_1^2$

CP2: $K_2 = \frac{1}{2}mv_2^2$

$v_1 = 19.8$ m/s

h = 30 m

$v_2 = ?$

$\frac{1}{2}\cancel{m}v_2^2 = \cancel{m}gh + \frac{1}{2}\cancel{m}v_1^2$

Each term is multiplied by 'm'. So 'm' divides out and cancels.

$v_2^2 = 2gh + v_1^2$

$v_2 = \sqrt{2gh + v_1^2}$

$v_2 = \sqrt{(2 \times 9.8 \times 30) + 19.8^2}$

$v_2 =$ **31.3 m/s** (3 sd)

If you can use energy conservation to do a problem, it's less complicated than using forces.

We said before that work is a scalar, right? Does that mean that kinetic energy is also a scalar?

Like work, energy is a scalar quantity.

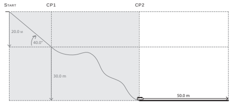
Work is a scalar, as the same amount of work will be done by the same size of force moving an object the same distance, regardless of direction.

Kinetic energy is also a scalar, as an object with velocity, **v**, will have the same quantity of kinetic energy, $K = \frac{1}{2}m\mathbf{v}^2$, regardless of the direction of its velocity.

When you have a vector, its direction is shown by its sign. But when you square a number, the result is always positive. Therefore, v^2 is a scalar, because the information about the direction of v (its sign) has been lost. This means that kinetic energy must also be a scalar, as $K = \frac{1}{2}mv^2$

A change in kinetic energy can be positive or negative. The sign doesn't signify a direction, just a change in the **amount** of kinetic energy. In the same way, a change in mass (another scalar quantity) signifies the change in the amount of 'stuff', and can be positive or negative.

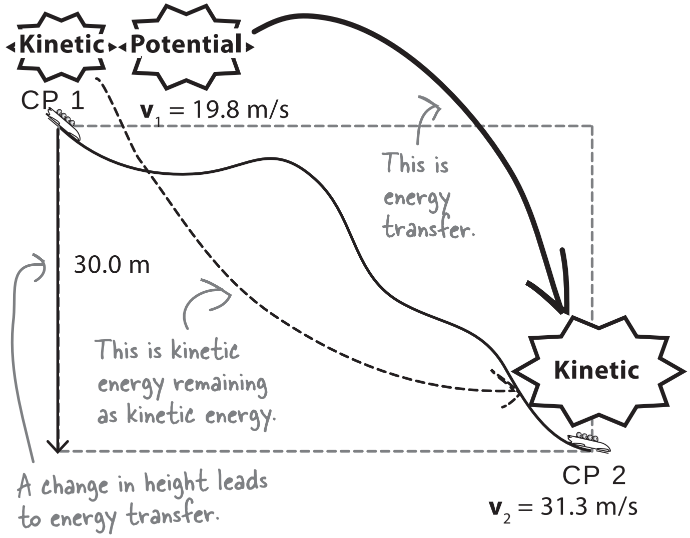
Same velocity means same kinetic energy, regardless of direction.



You've used energy conservation to solve the second part

You've used energy conservation to work out that the bobsled has a speed of 31.3 m/s when it hits the second checkpoint, 30.0 m below the first.

The same **change in height** will always leads to the same **change in potential energy** - and to the same **change in kinetic energy** if there's no friction.

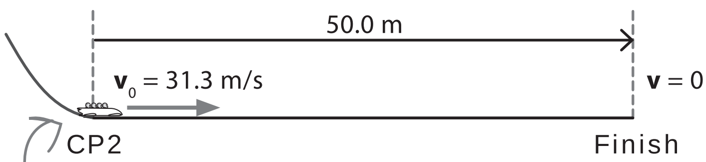


In the third part, you have to apply a force to stop a moving object

The third part of the track is flat, and is the stretch where the bobsled must be brought to a halt by applying a brake to the ice. The track design states that the 630 kg bobsled must be stationary by the time it's covered 50.0 m.

How should you go about calculating the **force** you need to apply with the brake?

630 kg is the maximum mass the bobsled is allowed to have when there are people in it.

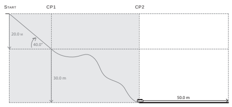


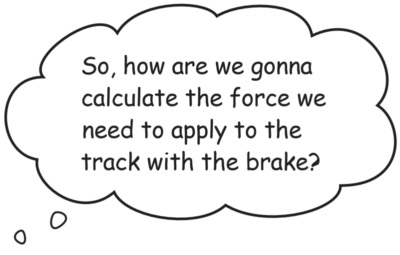
You need to work out the braking FORCE that'll stop the bobsled by the end of the track.

So if we can work out the braking force from that velocity, I can get my first run in!

BRAIN POWER

How could you calculate the **force** you need to apply with the brake to stop the bobsled?





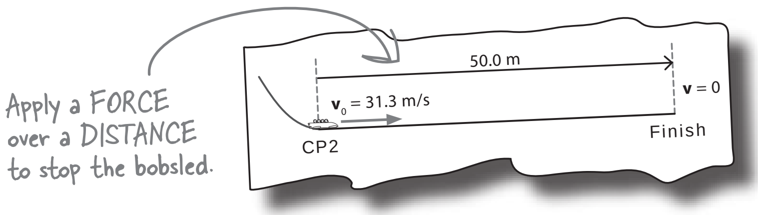
Jim: Well, there's no change in height, so we can't do anything with energy. I guess we can use **equations of motion** to work out the **acceleration**, and then Newton's Second Law, $\mathbf{F}_{net} = m\mathbf{a}$, to calculate the **force**. That would work!

Joe: But it would be a lot of calculations. Are you **sure** we can't do something with **energy conservation**? That was easier than using equations of motion for the previous part of the track

Jim: Well, fairly sure. There's no change in height, so no change in potential energy. So how would we use energy conservation?

Joe: I was thinking - we need to stop the bobsled within a certain **distance** by applying a **force**. That sounds like doing **work** to me!

Frank: Yeah, you do work on something by applying a force. But I thought you actually had to displace something with the force - not just shove a brake down as you slide along some ice!

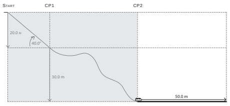


Any time you apply a force over a distance, think work and energy!

Jim: Wait, he might be right. When you catch a baseball, you exert a force on it with your hand. And your hand moves backwards as you catch it, so you apply the force over a distance. And that changes the kinetic energy of the ball.

Joe: So can we use the same kind of principle with calculating the force we need to stop the bobsled? We know its mass and velocity, so we can calculate its **kinetic energy** - AND we also know the distance that we need to apply the force over - 50.0 m.

Frank: But where does the kinetic energy of the bobsled - or the baseball for that matter - go when you stop it?! It's not like it gets transferred to potential energy, is it?



Putting on the brake does work on the track

You need to stop the bobsled by applying the brake before it reaches the end of the course. You've 50.0 m to stop it, but you need to know what **force** to apply with the brake.

One way of doing this problem would be to calculate the acceleration that the bobsled experiences as is slows down, then use $\mathbf{F}_{net} = m\mathbf{a}$ to calculate the force required. This method is fine, and will give you the correct answer.

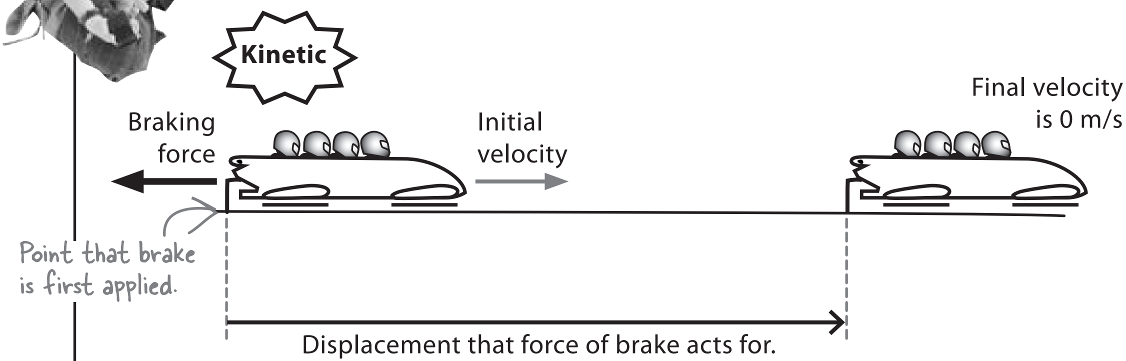
Do problems using energy conservation if possible.

However, for the earlier parts of this problem, you've discovered that using **energy conservation** to do a calculation can be a lot quicker and easier than using forces. Work = $\mathbf{F}\Delta\mathbf{x}$, so applying a force with the brake over the 50.0 m distance will do work.

But how is **energy transferred** from the moment the brake is applied to the moment the bobsled stops moving?

BE the brake

Your job is to imagine that you're the brake. What happens in terms of energy transfer from the moment that the brake is applied to the track to the moment that the bobsled comes to a complete stop? Write notes on the picture and give a written explanation below.



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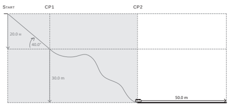
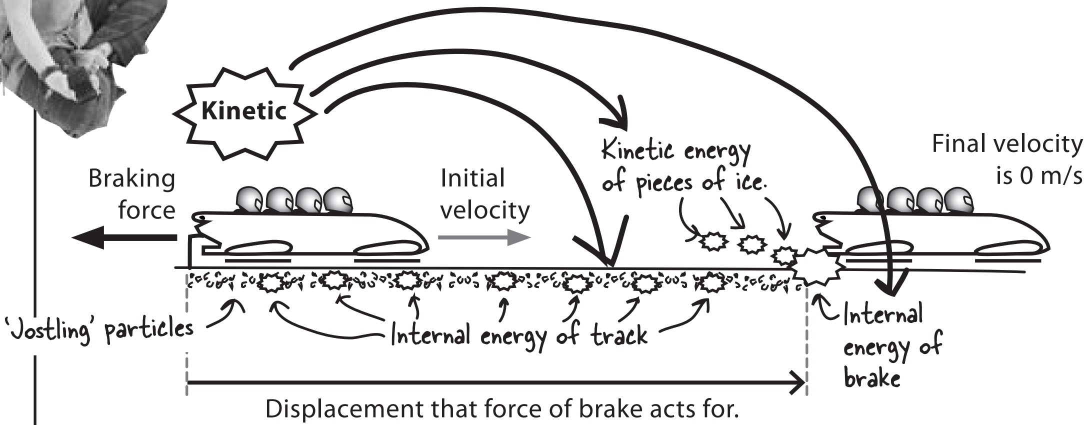


BE the brake - SOLUTION

Your job is to imagine that you're the brake. What happens in terms of energy transfer from the moment that the brake is applied to the track to the moment that the bobsled comes to a complete stop? Write notes on the picture and give a written explanation below.



At the start the bobsled has kinetic energy; at the end its kinetic energy is zero. The kinetic energy is transferred to the internal energy of the brake and the track, as the brake does work against friction. The movement of the brake over the track kind of 'jostles' the particles and makes them move around more vigorously inside the brake and the track.

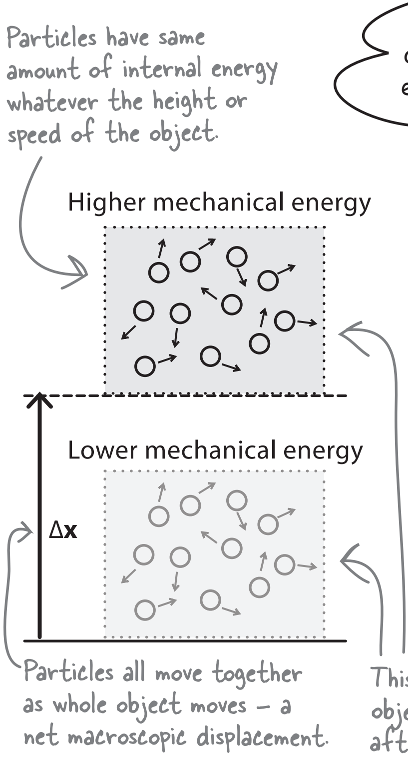
Doing work against friction increases the internal energy

When you put on the brake, you **transfer** some of the bobsled's **kinetic** energy to the **internal** energy of the brake and the track by doing work against friction. This reduces the kinetic energy of the bobsled - and therefore its speed, as K = ½mv2

Energy transfer also happens when small pieces of ice spray up from the track. Each individual piece of ice has a small amount of **kinetic** energy - not much compared with the bobsled's kinetic energy - but over 50.0 m this will have an effect.

When you do work against friction, you increase the INTERNAL ENERGY of the surfaces that are moving over each other.

What's the difference between increasing an object's internal energy (which increases the kinetic energy of its individual particles), and increasing the kinetic energy of the whole object?



Increasing mechanical energy moves particles together in an ordered way.

If an object's energy store can easily be used to do work, then it's said to have **mechanical energy**. **Kinetic** energy, **gravitational potential** energy and **elastic potential** energy (which is like a compressed or stretched spring) are all examples of mechanical energy.

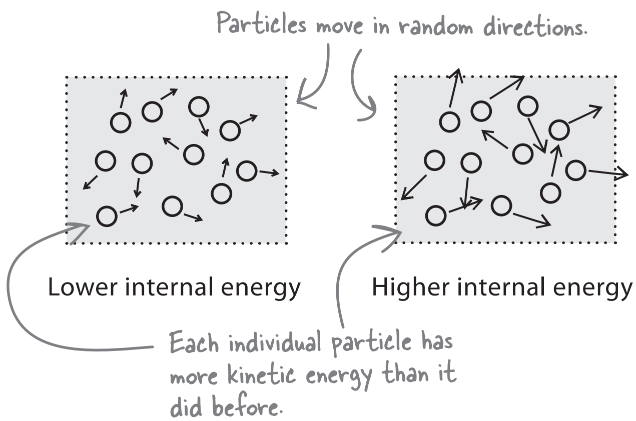
Giving something mechanical energy involves moving all of its particles **together**, for example by lifting it, compressing it or giving it a velocity. Although the individual particles inside the object will continue to move around at random, there's a **net displacement** on a **macroscopic scale** (large scale).

With potential or kinetic energy, the whole object moves in an ordered way.

Increasing internal energy makes particles move more vigorously in a disordered way.

If you increase something's internal energy, you make the **random** motion of its particles more pronounced.

If it's a liquid or a gas, you can think of this as increasing the kinetic energy of each individual particle as they move around.



If it's a solid, you can think of this as increasing the intensity with which atoms vibrate. This is like giving the atoms kinetic energy and their bonds potential energy (like springs).

Increasing internal energy involves many tiny increases of kinetic or potential energy on a **microscopic scale**. But as these changes occur in **random directions**, they don't lead to a change in mechanical energy on a macroscopic scale, because there's **no overall net displacement**.

there are no Dumb Questions

Q: So what's the difference between mechanical energy and kinetic energy? The names sound kinda similar.

A: Mechanical energy is a catch-all term for kinetic energy and potential energy - the kinds of energy stores that can be easily harnessed to do work.

Q: How do I calculate the mechanical energy of a system?

A: The mechanical energy is the total potential energy plus the total kinetic energy.

Q: Why is mechanical energy useful?

A: If there's no friction, the total mechanical energy is conserved. For example, you can work out the speed of an object that's at a lower height than it started off at by working out the change in its potential energy. This must be the same size as its change in kinetic energy, which you can use to calculate its speed.

Q: What is internal energy?

A: Every object or substance has internal energy due to the movement of the particles it's made from. Particles in a solid vibrate; particles in a liquid or gas move around.

Q: Why is internal energy different from mechanical energy?

A: An object's mechanical energy changes when all of its particles experience the same movement on a macroscopic (large) scale.

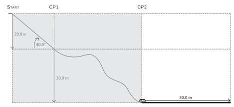
For example, if an object is lifted to a new height, its particles all move together in an ordered way and its potential energy increases. And if an object gets faster, its particles all move together in an ordered way and its kinetic energy increases.

Q: But internal energy also involves the movement of particles - which themselves have individual stores of kinetic or potential energy. Why is it different from the movement of particles in mechanical energy?

A: The particles move or vibrate in a random, disordered way. When you increase the internal energy of an object, its particles don't all suddenly spontaneously relocate all together. They do move with greater individual energies on a microscopic scale. But on a macroscopic scale, the object as a whole stays where it is!

Mechanical energy is about changes on a macroscopic scale.

Internal energy is about changes on a microscopic scale.



Sharpen your pencil

A 630 kg bobsled going at 31.3 m/s is to be brought to a halt by a brake applied to the ice track.

a. Describe this process in terms of energy transfer.

b. If the bobsled is to be stopped in 50.0 m, what force needs to be used?

c. Another way of helping to stop the bobsled is for the final part of the track to be slightly uphill. If the track goes up by 10.0 m during the final 50.0 m, what force would be required from the brake then?

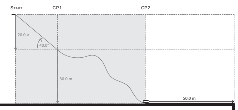
d. If the bobsled is to be returned to the top of the track (50.0 m above the bottom) what is the minimum time it would take a 10.0 kW engine time to lift it the requisite height (assuming 90% efficiency)?

Watts (W) are a measure of power. 1 Watt = 1 Joule per second.

This means that only 90% of the engine's energy is 'usefully' used. The rest goes towards increasing the internal energy of various moving parts.

Hint: Assume that there's no friction, so the engine only has to do work against gravity.

Hint: Calculate the total energy required, then see how many seconds it takes the engine to produce that amount of energy.



Sharpen your pencil Solution

A 630 kg bobsled going at 31.3 m/s is to be brought to a halt by a brake applied to the ice track.

a. Describe this process in terms of energy transfer.

The moving bobsled has kinetic energy. As it slows down, energy is transferred and the internal energy of the brake and the track increase. When the bobsled is stationary, the brake and the track will be hotter than they were before due to their increase in internal energy.

b. If the bobsled is to be stopped in 50.0 m, what force needs to be used?

$$\text{Work done} = F\Delta x \qquad K = \tfrac{1}{2}mv^2$$

Need to 'get rid of' all the bobsled's kinetic energy by doing work against friction.

$$F\Delta x = \tfrac{1}{2}mv^2$$

$$F = \frac{\tfrac{1}{2}mv^2}{\Delta x} = \frac{0.5 \times 630 \times 31.3^2}{50.0} = \underline{\underline{6170\ \text{N}}}\ (3\ \text{sd})$$

c. Another way of helping to stop the bobsled is for the final part of the track to be slightly uphill. If the track goes up by 10.0 m during the final 50.0 m, what force would be required from the brake then?

Some kinetic energy transferred to potential energy. Transfer the rest by doing work against friction.

$$F\Delta x + mgh = \tfrac{1}{2}mv^2$$

$$F = \frac{\tfrac{1}{2}mv^2 - mgh}{\Delta x} = \frac{(0.5 \times 630 \times 31.3^2) - (630 \times 9.8 \times 10.0)}{50.0}$$

$$F = \underline{\underline{4940\ \text{N}}}\ (3\ \text{sd})$$

d. If the bobsled is to be returned to the top of the track (50 m above the bottom) what is the minimum time it would take a 10.0 kW engine time to lift it the requisite height (assuming 90% efficiency)?

$$\text{Work required to lift bobsled} = mgh = 630 \times 9.8 \times 50.0 = 309000\ \text{J}\ (3\ \text{sd})$$

Engine produces 10000 Joules per second; 9000 Joules per second are useful.

$$\text{Time taken} = \frac{309000}{9000} = 34.3\ \text{s}\ (3\ \text{sd})$$

It doesn't matter how far the bobsled travels horizontally – as long as it ends 50.0 m higher than it starts, this is the energy required to get it up there.

there are no Dumb Questions

Q: If energy conservation lets me do complicated questions this easily, what's the point of equations of motion and force?

A: To understand energy conservation, you needed to build on top of a base of the other things you mention. And you can't solve every problem using energy conservation! It's another tool in your toolkit, but not one you'll be able to use exclusively.

Q: But I will be using it a lot, right?

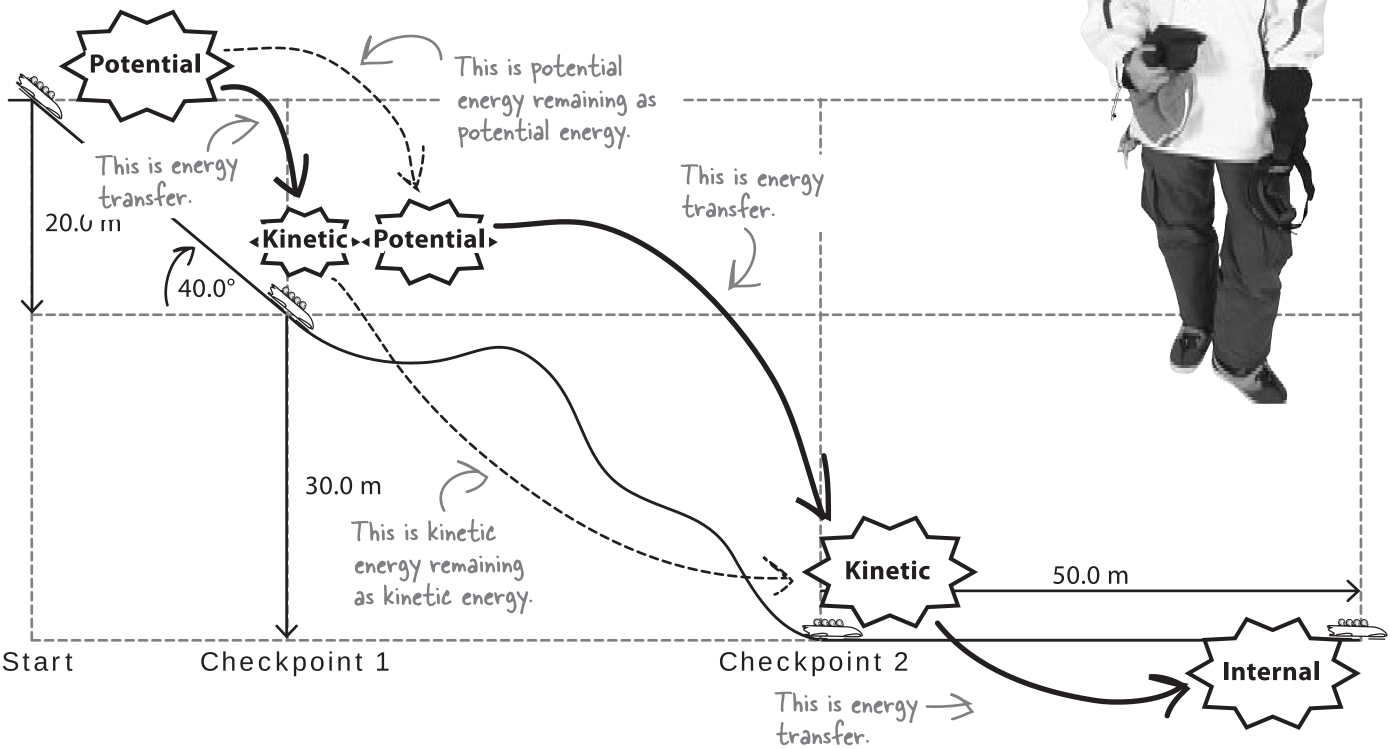
A: Yes ... but when it's appropriate! A lot of scenarios will involve collisions where mechanical energy isn't conserved, and you can't directly calculate the change in internal energy. You'll have to use other tools as well as energy for dealing with problems like that.

Energy conservation helps you to do complicated problems in a simpler way

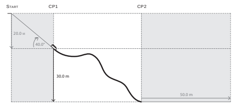
You've just used energy conservation to solve complicated and even impossible-looking problems in a simpler way.

If you have something moving down or up a slope, then you can work out the change in its **potential energy** - and hence the change in its **kinetic energy** and speed - from the change in its **height**, no matter what path it takes.

And if there's friction involved, you can think of it in terms of **energy transfer** rather than forces if you're interested in displacement. This gives you the braking force of 6170 N that you need.



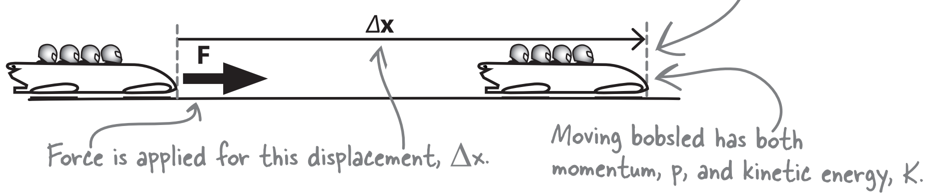
Look out for differences in height, velocity etc. between the start and end of your problem that may allow you to use energy conservation.



I'm not clear on the **difference** between momentum (mv) and kinetic energy ($\frac{1}{2}mv^2$). A moving object has both, their equations look similar and they're both conserved. So aren't they just different ways of saying the same thing?!

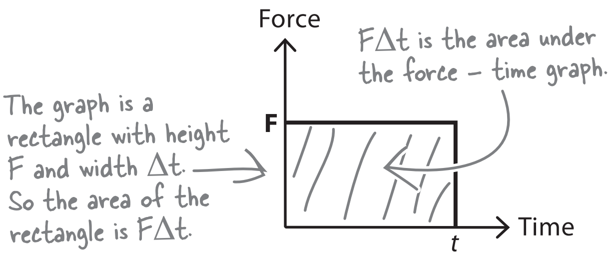
Not quite. Momentum and kinetic energy are different.

If something is stationary, you can get it going by giving it a push, i.e. applying a force.



The **momentum-impulse** equation says that if you apply a net **force** for a period of **time**, it causes a change in momentum, $\mathbf{F}\Delta t = \Delta\mathbf{p} = \Delta(m\mathbf{v})$

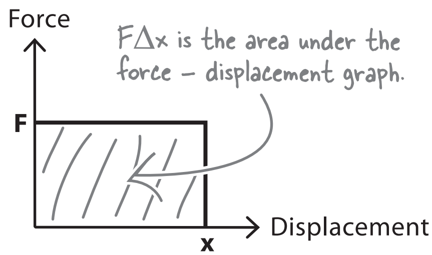
$$F\Delta t = \Delta p$$



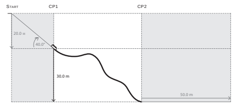
You get something's <u>momentum</u> from the <u>time</u> a force is applied for.

The **kinetic energy** equation says that if you do work on an object by applying a **force** for a certain **displacement** it gains kinetic energy, $\mathbf{F}\Delta\mathbf{x} = \Delta K = \frac{1}{2}m\mathbf{v}^2$

$$F\Delta x = \Delta K$$



You get something's <u>kinetic energy</u> from the <u>displacement</u> a force is applied for.

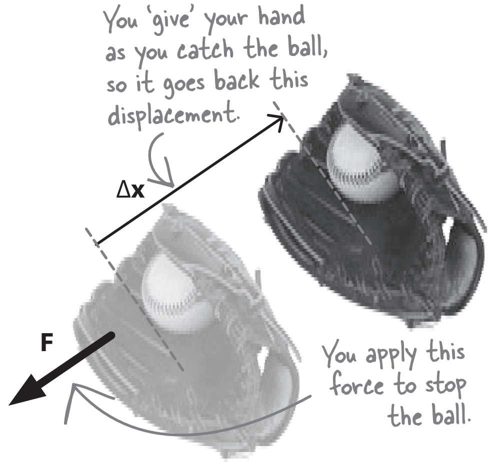


There's a practical difference between momentum and kinetic energy

To think about **pushing** an object to start it off is a bit abstract. A more practical way of seeing the difference between momentum and kinetic energy is to think about **stopping** a moving object.

Imagine catching a ball thrown to you. The ball has a mass and a velocity, and you need to apply a **force** with your hand to stop it. The **time** you need to apply the force for depends on the ball's **momentum**. The **displacement** you need to apply the force over depends on the ball's **kinetic energy** - in stopping the ball, it does work on you to deform the glove and stretch your arm, which reduces the ball's kinetic energy to zero.

The exercise is about the **practical** difference between the **time** and the **displacement** it takes to stop an object - which illustrates the difference between its momentum and its kinetic energy

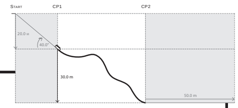


Exercise

a. A baseball with a mass of 145 g is pitched at 35.8 m/s. Calculate (i) its momentum and (ii) its kinetic energy.

b. A bullet with a mass of 3.45 g is fired at 1500 m/s. Calculate (i) its momentum and (ii) its kinetic energy

c. You can catch the baseball by exerting a force on it with your hand. Explain, using physics, why you couldn't practically stop the bullet by exerting the same force. (Assume that by using 'soft hands' as you catch the ball, you apply the force and bring the ball to a stop over a displacement of 30 cm.)



Exercise Solution

a. A baseball with a mass of 145 g is pitched at 35.8 m/s. Calculate (i) its momentum and (ii) its kinetic energy.

(i) $p = mv = 0.145 \times 35.8$

$p = \underline{\underline{5.19 \text{ kg.m/s}}}$ (3 sd)

(ii) $K = \frac{1}{2}mv^2 = 0.5 \times 0.145 \times 38.5^2$

$K = \underline{\underline{107 \text{ J}}}$ (3 sd)

b. A bullet with a mass of 3.45 g is fired at 1500 m/s. Calculate (i) its momentum and (ii) its kinetic energy

(i) $p = mv = 0.00345 \times 1500$

$p = \underline{\underline{5.18 \text{ kg.m/s}}}$ (3 sd)

(ii) $K = \frac{1}{2}mv^2 = 0.5 \times 0.00345 \times 1500^2$

$K = \underline{\underline{3880 \text{ J}}}$ (3 sd)

c. You can catch the baseball by exerting a force on it with your hand. Explain, using physics, why you couldn't practically stop the bullet by exerting the same force. (Assume that by using 'soft hands' as you catch the ball, you apply the force and bring the ball to a stop over a displacement of 30 cm.)

The bullet has around 35 times more kinetic energy than the ball. To stop a moving object, you need to do the same amount of work on it as its current kinetic energy.

So you need to do 35 times more work on the bullet than on the ball.

Work = $F\Delta x$. If you exert the same size of force on the bullet, you need to apply it over a displacement of 30 × 30 cm = 900 cm = 9 meters for the bullet. You can't physically do that, so the bullet goes through your glove (and worse). Not recommended.

The <u>distance</u> required to stop an object depends on its <u>kinetic energy</u>.

there are no Dumb Questions

Q: So even though the equations for momentum and kinetic energy look similar, they're different?

A: Yes. They're different quantities with different units.

Q: But momentum and kinetic energy are both conserved?

A: Momentum is always conserved. Total energy is also always conserved, but may be transferred between potential, kinetic and internal energy - it won't always be kinetic.

Q: If a baseball and a bullet both have the same momentum, how come a bullet does more damage?

A: Momentum depends on v, but kinetic energy depends on v^2. If an object has a high velocity, then v^2 really dominates. If the object is going 3 times faster, it'll have 9 times as much kinetic energy! So the same force requires 9 times as much displacement to stop it. That's part of the reason that a bullet will embed itself a large distance into a piece of wood, but a baseball with the same momentum will only make a small dent.

I've kinda been wondering ... I've been using energy for a couple of chapters now, but I still don't really know what it actually **is**.

Energy is the capacity something has to do work. Total energy is conserved.

The definition of energy is the capacity something has to do work - if all of the energy could be harnessed and used in this way.

You can measure changes in energy as it is transferred, and use energy to do calculations.

So should I think about **changes** in energy rather than about the "total" energy something possesses?

Energy conservation is a law.

Energy conservation is a law of nature. A law isn't an object - you can't pick it up and put it in your pocket and say "this is energy conservation". But you can observe how things behave as a result of a law.

You can observe **changes** in kinetic energy, potential energy, internal energy, etc. When you look at all the changes in energy, you find that they add up to zero and the total energy is conserved. For example a positive change in kinetic energy may be offset by a negative change in potential energy.

Even though you can't "see energy" directly, you can express the law using words or math and use energy conservation to work things out.

You deal with <u>changes</u> in potential, kinetic, internal etc energy - not absolute values.

Question Clinic: The "Show that" Question

The "show that" question is unusual in that it gives you the "answer" and asks you to show how you can get there from a given starting point. The key is to realize that it's OK to **work backwards** as well as forwards! Look at the equation you're supposed to end up with, and figure out which variables you'll need to make substitutions for. Also look back at earlier parts of the question, since you have probably already been asked to "do" something with the equations you'll need to manipulate for the "show that" question.

Be aware that the equations on your equation sheet may use different letters to represent the same thing.

a and g both represent acceleration.

h and $(x - x_0)$ both represent a change in displacement / position.

This is a buzzword which means "mechanical energy conservation".

2. A bobsled moves down a non-uniform, frictionless slope.

a. Using the equations $U = mgh$, and $v^2 = v_0^2 + 2a(x-x_0)$ and the fact that the potential energy, U, at the top is equal to the kinetic energy, K, at the bottom, show that $K = ½mv^2$

b. The second checkpoint is 30.0 m lower than the first. If the bobsled has a speed of 19.8 m/s at the first checkpoint, what is its speed at the second?

This means that the question is about algebra – and you need to manipulate the equations you already have to get this one.

Part b. of a question doesn't exist on its own – it's usually related to part a. in some way. Here, you need to realize that part b. is about using the energy conservation equation you worked out in part a with some numbers included.

Even if you didn't manage the 'show that' part of the question, you're still allowed to USE the equation that you were supposed to derive in the second part.

If you spot a "show that" question, look carefully at the parts of the question that came before to see what it would like you to use. If you get stuck and there's another part of the question that asks you to use the equation in the "show that" part, it's OK to use the equation to do a calculation even if you got stuck deriving the equation.

Question Clinic: The "Energy transfer" Question

Any time you see forces and displacements mentioned, you should immediately think about whether you can use energy as a way of solving the problem. Look out for differences in height - these should immediately scream "gravitational potential energy" at you! And look out for a net force being applied for a certain distance. This may lead to an object gaining kinetic energy, or to work being done against friction, or both, depending on the circumstances.

The buzzword 'brake' means that kinetic energy will be transferred to internal energy when work is done against friction.

You need to explain that energy is conserved, and where it is transferred from and to.

3. A 630 kg bobsled going at 31.3 m/s is to be brought to a halt by a brake applied to the track.

a. Describe this process in terms of energy transfer.

b. If the bobsled is to be stopped in 50.0 m, what force needs to be used?

c. Another way of helping to stop the bobsled is for the final part of the track to be slightly uphill. If the track goes up by 10.0 m during the final 50.0 m, what force would be required from the brake then?

This means you've to use words to put across the physics concepts behind what happens.

Any time you see a force and a displacement, think about whether work is being done.

When you see a change in height, think about gravitational potential energy.

The force will be different, as some of the kinetic energy is transferred to gravitational potential energy because of the change in height.

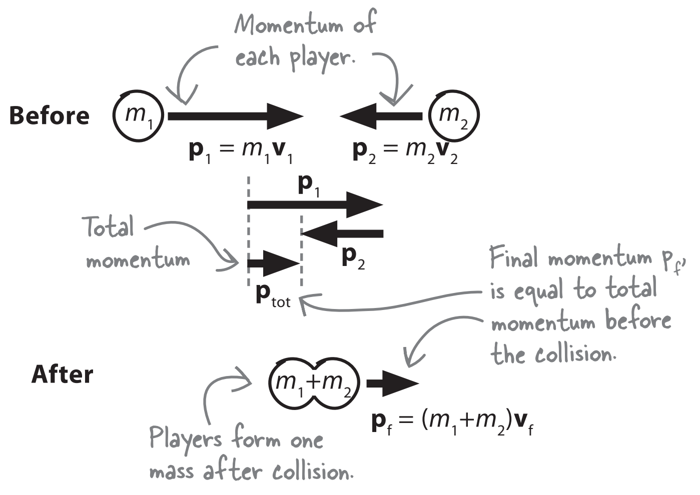
Always remember that it's the **total** energy that's conserved. if the question gives you reason to believe there's **no friction** , then this means the **mechanical** energy (i.e. kinetic + potential) is conserved . If there is friction, then energy may be transferred in more than one way at once, e.g. kinetic to both potential and internal.

After the roaring success of SimFootball, it's time for SimPool

A few weeks after your key role as physics consultant on the multi-million seller SimFootball game, the programmer is back in touch. They're working on a SimPool game - but have run into a problem.

Reusing the old code makes the pool balls stick together!

The pool balls need to bounce when they collide, but the programmer doesn't know how to work out the velocity they move with. His only experience is of the football players in the previous game - but they didn't bounce when they collided.

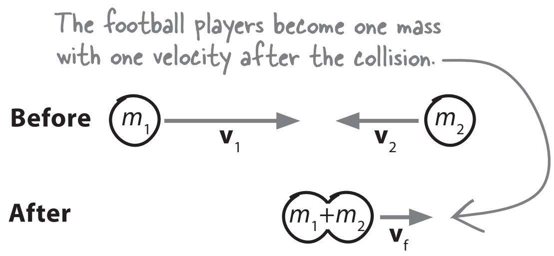


Reusing the football code in the pool game makes the pool balls stick together when they collide - which definitely isn't right!

Momentum conservation will solve an inelastic collision problem

The collision in the football game is called an **inelastic collision**, because the players don't "bounce" off of each other in an elastic way.

Momentum is always conserved in a collision. Since you know the **mass** and **velocity** of each player before the collision, you have **one unknown** (the velocity of the stuck together players after the collision) which you can work out using **one equation** (the momentum conservation equation).



Momentum conservation:

$$m_1\mathbf{v}_1 + m_2\mathbf{v}_2 = (m_1+m_2)\mathbf{v}_f$$

✓ ✓ ✓ ✓ ✓ ✓ ?

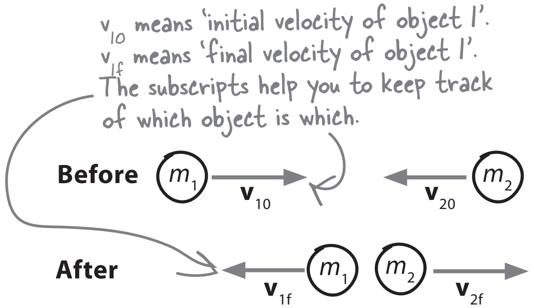
v_f is the only unknown, so you can use this one equation to calculate it.

You need a second equation for an elastic collision

The collision in the pool game is called an **elastic collision** because the pool balls bounce off of each other in an elastic way.

With the completely inelastic collision, you had a single mass moving with a single velocity after the collision. But with this elastic collision you have two masses each with their own velocity. You don't know either of the velocities - so this time there are two unknowns that you need to calculate.

Momentum is always conserved in a collision, but with two unknowns, a single momentum conservation equation is not enough. We'll need to come up with a second equation to fix the pool game - if you have two equations, you can work out two unknowns.



Momentum conservation:

$$m_1\mathbf{v}_{10} + m_2\mathbf{v}_{20} = m_1\mathbf{v}_{1f} + m_2\mathbf{v}_{2f}$$

✓ ✓ ✓ ✓ ✓ ? ✓ ?

You don't know v_{1f} or v_{2f}. As there are two things you don't know, you can't work either of them out when you just have one equation.

To solve for <u>two unknowns</u>, you need <u>two equations</u>.

BRAIN POWER

Where might you get a second equation from, so you can solve for the two unknown velocities?

We soooo need to fix that pool game - the pool balls definitely shouldn't stick together - it's an **elastic** collision!

Jim: Yeah, but that gives us a problem that we didn't have with the **inelastic collision**. We wind up with two **velocities** after the collision instead of only one.

Joe: We can't use **momentum conservation** on its own to solve the problem this time - you can't get the values of two unknowns from one equation.

Frank: Yeah, but momentum is still conserved in an elastic collision - right? So we need to think of a second equation we can use alongside momentum conservation to solve the problem.

Jim: What about **energy conservation**? That's been useful to us before.

Joe: I'm not sure about that. it's not like the pool balls' height changes. We can't do anything with potential energy.

Frank: But the pool balls are moving, right? What about their **kinetic energy**?

Jim: Yeah, the pool balls are moving before the collision, and they're moving afterwards, so they must have kinetic energy before and after.

Joe: How do we know for sure that some of the kinetic energy isn't transferred to internal energy when the pool balls collide?

Frank: Well, it's not like the pool balls get really hot, like a brake would when you apply it to slow down.

Jim: And it's not like the pool balls deform, and the arrangement of the particles inside them gets messed up.

Joe: Yeah, I think you might be right. The collision's elastic, right? And energy must be conserved. Before, there's kinetic energy. After there's kinetic energy. if the change in internal energy is minimal, we can say that there's the same amount of kinetic energy before and after the collision.

Frank: So, momentum is conserved - that's one equation.

Jim: And the kinetic energy's the same before and after because the collision is elastic - that's a second equation!

Joe: And both of the equations have the pool balls' velocities in them, so we can use the two equations to solve the problem!

Momentum is conserved in an inelastic collision.

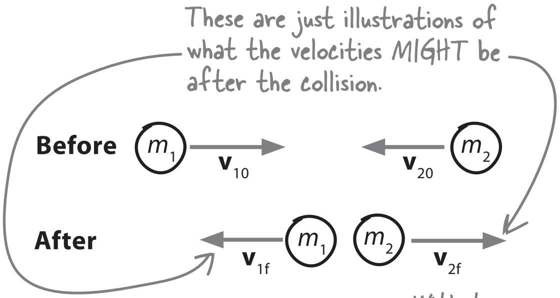
Momentum and kinetic energy are conserved in an elastic collision.

Energy conservation gives you the second equation that you need!

The pool balls hit each other in an **elastic collision**. This means that their internal energies don't increase significantly as a result of the collision, because they don't deform in any way (unlike the football players, whose padding deforms and doesn't bounce back when they collide).

Since the internal energy doesn't change, this means that the **total kinetic energy** of the two pool balls is the same before and after the collision. This gives you a second equation you can use.

If you have **two equations** (momentum conservation and kinetic energy before and after the collision) you can work out **two unknowns** - the final velocities of the two pool balls.



Momentum conservation:

With two equations, you can solve for two unknowns.

$$m_1\mathbf{v}_1 + m_2\mathbf{v}_2 = m_1\mathbf{v}_{1f} + m_2\mathbf{v}_{2f}$$

✓ ✓ ✓ ✓ ✓ ? ✓ ?

Energy conservation:

$$\tfrac{1}{2}m_1\mathbf{v}_1^2 + \tfrac{1}{2}m_2\mathbf{v}_2^2 = \tfrac{1}{2}m_1\mathbf{v}_{1f}^2 + \tfrac{1}{2}m_2\mathbf{v}_{2f}^2$$

✓ ✓ ✓ ✓ ✓ ? ✓ ?

Sharpen your pencil

This means that $m_1 = m_2 = m$

This means that $v_{20} = 0$, which should simplify the problem a bit.

Two pool balls both have the same mass, *m*. A moving ball, velocity $\mathbf{v}_{10}$ hits a second, **stationary**, ball head on. After the collision, the first ball has velocity $\mathbf{v}_{1f}$ and the second $\mathbf{v}_{2f}$.

a. Write down an equation showing that momentum is conserved.

b. Write down an equation showing that kinetic energy is conserved.

c. Eliminate the variable $\mathbf{v}_{2f}$ by rearranging your equation from part a. and substituting it into your equation from part b, so that you only have one unknown, $\mathbf{v}_{1f}$.

d. Multiply out the parentheses and simplify your new equation as much as you can. (Don't worry if you can't find a way to make it say "$\mathbf{v}_{1f}$ = something" yet - we'll do that on the next page.)

Sharpen your pencil Solution

This means that $m_1 = m_2 = m$

This means that $v_{20} = 0$, which should simplify the problem a bit.

Two pool balls both have the same mass, *m*. A moving ball, velocity $\mathbf{v}_{10}$ hits a second, **stationary**, ball head on. After the collision, the first ball has velocity $\mathbf{v}_{1f}$ and the second $\mathbf{v}_{2f}$.

a. Write down an equation showing that momentum is conserved.

$$mv_{10} + 0 = mv_{1f} + mv_{2f}$$

b. Write down an equation showing that kinetic energy is conserved.

$$\tfrac{1}{2}mv_{10}^2 + 0 = \tfrac{1}{2}mv_{1f}^2 + \tfrac{1}{2}mv_{2f}^2$$

$v_{20} = 0$, so it doesn't appear in either equation.

c. Eliminate the variable $\mathbf{v}_{2f}$ by rearranging your equation from part a. and substituting it into your equation from part b, so that you only have one unknown, $\mathbf{v}_{1f}$.

$$mv_{10} = mv_{1f} + mv_{2f} \quad \text{(a)}$$

All terms are multiplied by m, so you can divide it out and cancel it.

$$v_{10} = v_{1f} + v_{2f}$$

$$v_{2f} = v_{10} - v_{1f}$$

$$\tfrac{1}{2}mv_{10}^2 = \tfrac{1}{2}mv_{1f}^2 + \tfrac{1}{2}mv_{2f}^2 \quad \text{(b)}$$

All terms are multiplied by ½m, so you can divide it out and cancel it.

$$v_{10}^2 = v_{1f}^2 + v_{2f}^2$$

Substitute this into (b).

$$v_{10}^2 = v_{1f}^2 + (v_{10} - v_{1f})^2$$

The only unknown in this equation is v_{1f}.

d. Multiply out the parentheses and simplify your new equation as much as you can. (Don't worry if you can't find a way to make it say "$\mathbf{v}_{1f}$ = something" yet - we'll do that on the next page.)

$$v_{10}^2 = v_{1f}^2 + (v_{10} - v_{1f})^2$$

$$v_{10}^2 = v_{1f}^2 + v_{10}^2 - 2v_{10}v_{1f} + v_{1f}^2$$

There's a v_{10}^2 on each side, so it can be subtracted from both sides and canceled.

$$0 = 2v_{1f}^2 - 2v_{10}v_{1f}$$

You can divide every term in the equation by 2.

$$v_{1f}^2 - v_{10}v_{1f} = 0$$

This is the equation where all the terms with v_{1f} in them are on the left hand side.

So I end up with this ... this thing $v_{1f}^2 - v_{10}v_{1f} = 0$ that I'm supposed to rearrange to say "v_{1f} = something". But how am I gonna do that?!

Factoring can also be called factorising.

Factoring your equation will help.

If rearranging your equation to say "$\mathbf{v}_{1f}$ = something" is difficult, then **factoring** it is another way of trying to solve it. Factoring involves spotting where you can put in some parentheses - it's basically the reverse of multiplying out parentheses.

If two things multiplied together = zero, then at least one of the things must be zero. For example, if you have $xy = 0$, then either x or y (or perhaps both) must be 0. You have a zero on the right hand side of your equation, $\mathbf{v}_{1f}^2 - \mathbf{v}_{10}\mathbf{v}_{1f} = 0$.

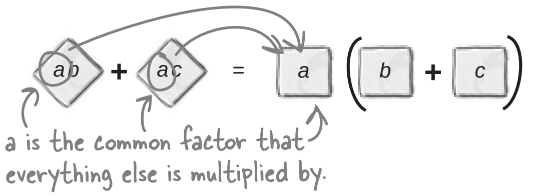
So if you can factor the left hand side of your equation so that it consists of two things multiplied together, you can say for sure that one of them **must be zero**. You can work out from the **context** which of them actually is zero.

If xy = 0 then either x or y (or maybe both) MUST be 0.

Factoring involves putting in parentheses

The terms *ab* and *ac* both have an '*a*' in them that everything else in the term is multiplied by. Because of this, the '*a*' is called a **common factor**, because it is common to both terms.

You can factor your terms by moving the common factor outside a set of parentheses So $ab + ac$ becomes $a(b + c)$.



If you have the equation $a(b+c) = 0$ then either $a = 0$ or $(b+c) = 0$. This is because if two things multiplied together = 0 then one (or both) of the things must be zero.

If more than one term has the same variable multiplying everything else in the term, you can factor your equation.

Sharpen your pencil

a. In your equation $\mathbf{v}_{1f}^2 - \mathbf{v}_{10}\mathbf{v}_{1f} = 0$, which variable appears in both terms, so that everything else in the term is multiplied by it?

b. If you have parentheses, you can multiply them out, e.g. $a(b + c) = ab + ac$. The terms *ab* and *ac* both have the common factor '*a*' in them which everything else in the term is multiplied by.

Use this information to write your equation $\mathbf{v}_{1f}^2 - \mathbf{v}_{10}\mathbf{v}_{1f} = 0$ in a form where the left hand side is a single term that involves parentheses.

c. If two things multiplied together = 0 then one or both of the things must be zero. Use this fact to calculate two values that $\mathbf{v}_{1f}$ may have, then use the context to explain which value you think is correct.

Sharpen your pencil Solution

a. In your equation $\mathbf{v}_{1f}^2 - \mathbf{v}_{10}\mathbf{v}_{1f} = 0$, which variable appears in both terms, so that everything else in the term is multiplied by it?

The variable v_{1f} appears in both terms.

b. If you have parentheses, you can multiply them out, e.g. $a(b + c) = ab + ac$. The terms *ab* and *ac* both have the common factor '*a*' in them which everything else in the term is multiplied by.

Use this information to write your equation $\mathbf{v}_{1f}^2 - \mathbf{v}_{10}\mathbf{v}_{1f} = 0$ in a form where the left hand side is a single term that involves parentheses.

$$v_{1f}^2 - v_{10}v_{1f} = 0$$

$$v_{1f}(v_{1f} - v_{10}) = 0$$

c. If two things multiplied together = 0 then one or both of the things must be zero. Use this fact to calculate two values that $\mathbf{v}_{1f}$ may have, then use the context to explain which value you think is correct.

Either $v_{1f} = 0$ or $(v_{1f} - v_{10}) = 0$

If $(v_{1f} - v_{10}) = 0$ this means that $v_{1f} = v_{10}$, in other words, the 1st pool ball has continued at its original velocity as if it hadn't hit the other one. This is impossible.

The other possible solution is $V_{1f} = 0$. This looks right, as when you hit a pool ball straight at another, the first ball often stops.

You can deal with elastic collisions now

Momentum is always conserved in any collision, whether it's inelastic or elastic. **Energy** is always conserved as well - but if the collision is inelastic, some is transferred as internal energy. If the collision is elastic, then the total kinetic energy is conserved.

A rule of thumb is that any collision that involves an object being deformed in some way is inelastic, as the molecules in the object have been rearranged, which increases the internal energy of the object.

For any collision, use momentum conservation first. If the collision is elastic and you have two unknowns, use kinetic energy as well.

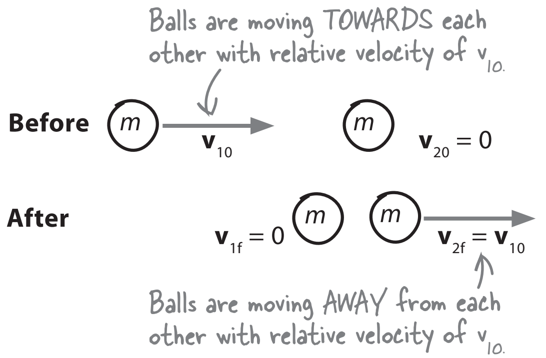
If you have a problem involving an **elastic** collision, you should always use **momentum conservation**. If you only have one unknown, then you're done.

If you have two unknowns, you can use **energy conservation** to give you a second equation that helps you to work out both unknowns.

In an elastic collision, the relative velocity reverses

You've worked out that when one pool ball hits a stationary pool ball head-on, the first pool ball stops and the second pool ball continues with the velocity that the first ball had.

Imagine sitting on top of the second pool ball. You see the first pool ball coming towards you with velocity $\mathbf{v}_{10}$. After the collision, it appears to you that the first pool ball is moving away from you with velocity $-\mathbf{v}_{10}$ (though it's actually the second ball that is moving).



This also works in the opposite direction. If the relative velocity of the objects is reversed after the collision, then the collision must have been elastic.

This is a special case of a general rule for elastic collisions - the **relative velocity** of the two objects is **reversed** after the collision. This is also true if both objects were originally moving.

there are no Dumb Questions

Q: If a collision is elastic, do I always have to use momentum conservation AND energy conservation?

A: Not always. Sometimes you'll only start off with one unknown. Then you'd only need to use one equation.

Q: If I only have one unknown velocity, which equation is it better for me to use - momentum conservation or energy conservation?

A: It's better to use momentum conservation, because direction is important. and the momentum conservation equation tells you the direction of the velocity as well as its size, as momentum is a vector.

Q: Does the kinetic energy equation tell me the direction of the velocity too?

A: No, because kinetic energy is a scalar. An object of a certain mass will always have the same kinetic energy

Q: Can't I get the velocity's direction from the kinetic energy equation?

A: Kinetic energy, $K = ½mv^2$. The velocity is squared. If you multiply two positive numbers together, you get a positive number. And if you multiply two negative numbers together, you also get a positive number.

So whether v is positive or negative (an indication of direction), v^2 will always be positive. This means that you can't work out the direction of the velocity (a vector) from the kinetic energy (a scalar) - though you can work out its size.

Q: In the problem I just did over there, there were two possible answers at the end. How do I pick between them?

A: The reason there were two possible answers is because the kinetic energy conservation equation involves v^2. You then have to pick between them by thinking about the 'k'ontext of the problem. Choose the answer that makes the most physical sense.

Q: What if the collision happens at an angle, instead of along a straight line?

A: The rule of thumb is always to use momentum conservation first. Break down the velocities you're given into components then apply momentum conservation for each component (like you did in chapter 12).

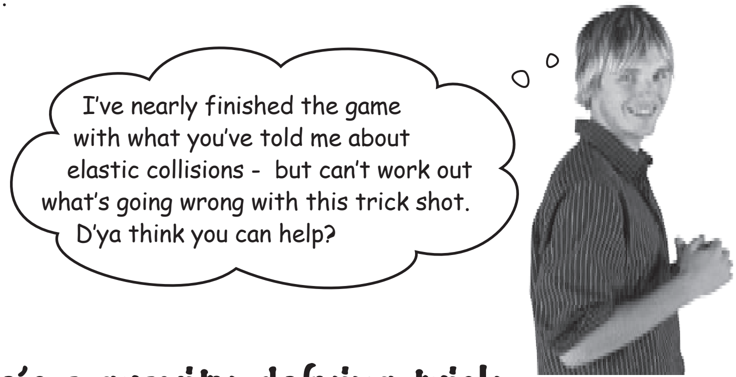
Q: Does the relative velocity always reverse in an elastic collision even if the objects have different masses?

A: Yes. For example, a rubber ball hitting a wall with velocity v will rebound with velocity -v (assuming that the collision is completely elastic). And any two objects with less extreme masses will also have their relative velocity reversed after a collision.

The pool ball collisions work!

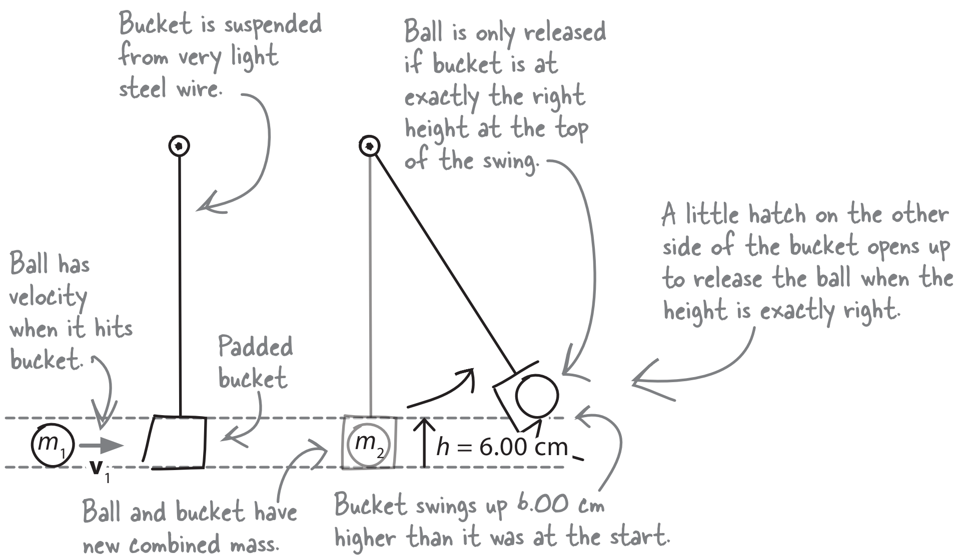
The programmer writes the elastic pool ball collisions into the game, exactly like you describe - and they work!

But a few days before the game is due to be released, he's back with a tricky problem ...



There's a gravity-defying trick shot to sort out ...

The game also has a trick shot mode, which uses some odd items you wouldn't usually see on a pool table. The programmer's stuck on one particular trick shot where the cue ball is hit into a padded 'bucket', which then swings up and releases the cue ball - as long as it reaches exactly the right height at the top of its swing (6.00 cm).

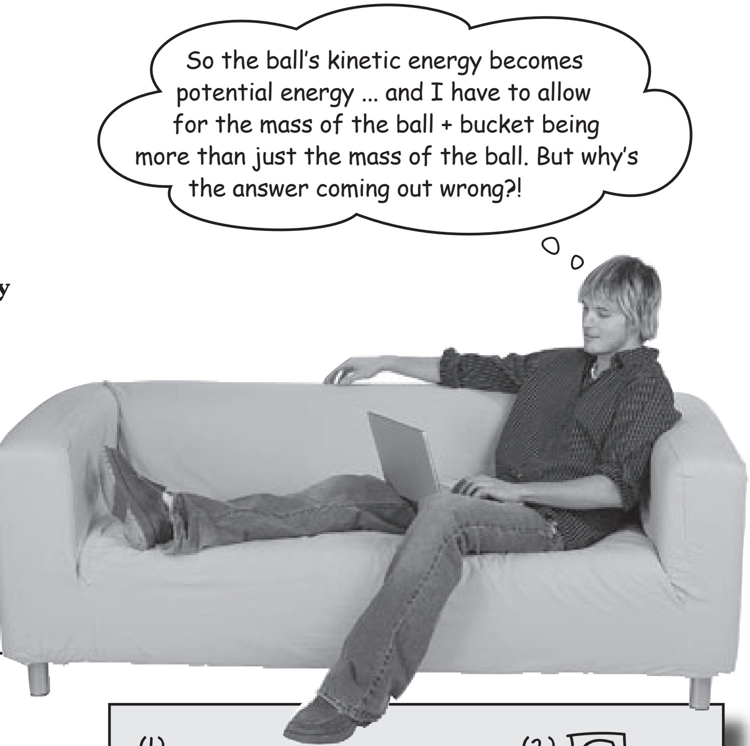


Where is the problem with the programmer's reasoning?

The programmer's been using what you taught him about **energy conservation** to work out the speed the player should hit the ball with.

He's assumed that the initial **kinetic energy** of the ball is being transferred to the final **potential energy** of the ball and bucket, which wind up 6.00 cm higher than they started (a problem that would be very very difficult to do using forces and equations of motion!)

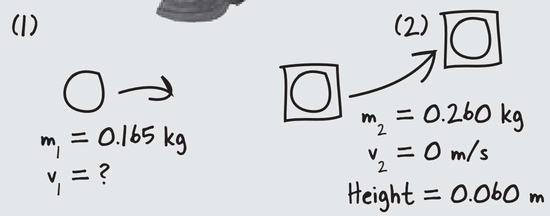
However, the velocity he's calculated is lower than a 'play tester' pool player has to hit the ball with to do the trick in real life. The real pool player needs to hit the ball with a higher velocity - and the programmer doesn't know why.



Sharpen your pencil

Your job is to work out what's going wrong. The programmer's math is to the right and there's space for you to write down what might be going wrong.

The ball has a mass of 165 g and the bucket is 95 g.



K of ball at (1) = U of ball and bucket at (2)

Use energy conservation

$$K = \tfrac{1}{2}m_1 v_1^2 = m_2 g h$$

$$v_1^2 = \frac{2m_2 g h}{m_1}$$

$$v_1 = \sqrt{\frac{2m_2 g h}{m_1}} = \sqrt{\frac{2 \times 0.260 \times 9.8 \times 0.060}{0.165}}$$

$$v_1 = 1.36 \text{ m/s (3 sd)}$$

But the real pool player doing the trick shot in real life has to hit the ball with a higher velocity than this – and I don't know why! Argh!

Sharpen your pencil Solution

Your job is to work out what's going wrong. The programmer's math is to the right AND There's space for you to write down what might be going wrong.

The ball has a mass of 165 g and the bucket is 95 g.

The programmer has assumed that all of the pool ball's kinetic energy is transferred to potential energy.

However, the bucket is padded. This means that the pool ball colliding with it is an inelastic collision, so mechanical energy isn't conserved.

When the pool ball hits the bucket, the padding in the bucket deforms and its internal energy increases. So not all of the ball's kinetic energy is transferred to potential energy.

This is consistent with the real pool player having to hit the ball faster.

(1)

$m_1 = 0.165$ kg

$v_1 = ?$

(2)

$m_2 = 0.260$ kg

$v_2 = 0$ m/s

Height = 0.060 m

K of ball at (1) = U of ball and bucket at (2)

Use energy conservation

$$K = \tfrac{1}{2} m_1 v_1^2 = m_2 g h$$

$$v_1^2 = \frac{2 m_2 g h}{m_1}$$

$$v_1 = \sqrt{\frac{2 m_2 g h}{m_1}} = \sqrt{\frac{2 \times 0.260 \times 9.8 \times 0.060}{0.165}}$$

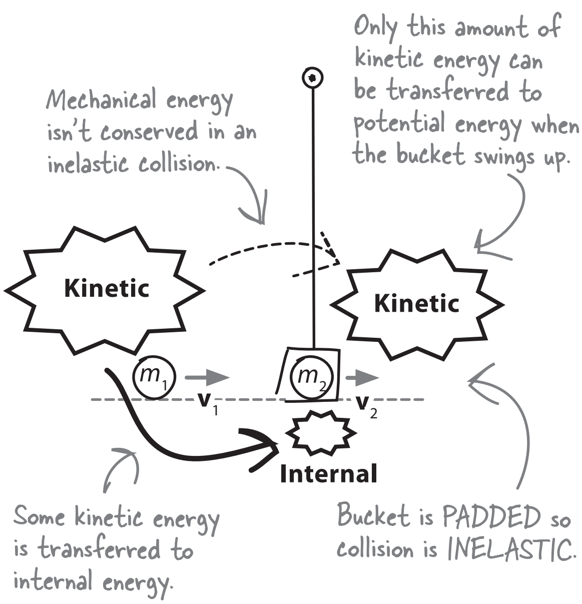
$$v_1 = 1.36 \text{ m/s (3 sd)}$$

But the real pool player doing the trick shot in real life has to hit the ball with a higher velocity than this – and I don't know why! Argh!

The initial collision is inelastic - so mechanical energy isn't conserved

When the ball hits the **padded** bucket, some of its kinetic energy is transferred to the bucket as **internal energy**. The collision is **inelastic**. The bucket is padded ,so the mechanical energy of the system is reduced as the internal energy of the bucket increases due to the padding deforming.

Therefore, the programmer's assumption that the ball's initial kinetic energy will all be transferred to gravitational potential energy is incorrect.

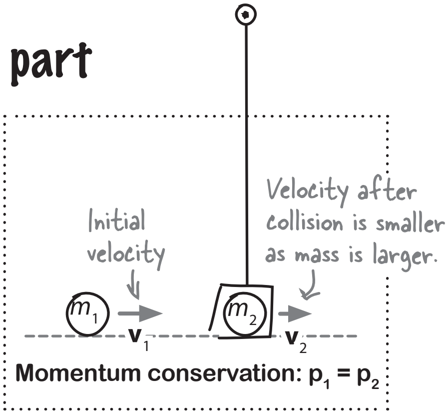


Before you do any math, think: "Is this collision elastic or inelastic?"

Use momentum conservation for the inelastic part

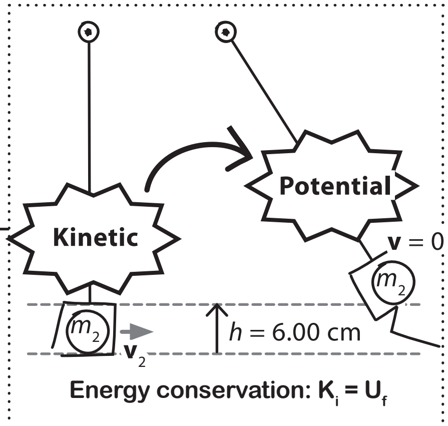
The trick with this trick shot is thinking about it in two stages.

The **first stage** is when the cue ball collides with the padded bucket. This is an **inelastic** collision, so momentum is conserved but mechanical energy isn't. As you know the masses of the ball and bucket, you can use momentum conservation to calculate their new velocity (and hence their kinetic energy) in terms of the cue ball's initial velocity.



The **second stage** is when the bucket containing the ball swings up. This involves the **kinetic energy** of the ball and bucket being transferred to **potential energy**. As you know the **height** that the bucket swings, you can work out its potential energy - and hence the kinetic energy and velocity of the ball and bucket - and hence the initial velocity of the cue ball.

Now it's your turn!



Sharpen your pencil

A pool ball, mass 165 g, is played into a padded bucket, which is arranged so it can swing upwards on the end of some light steel wires. For the shot to work, the ball must be struck so that the top of the bucket's swing is 6.00 cm higher than it started.

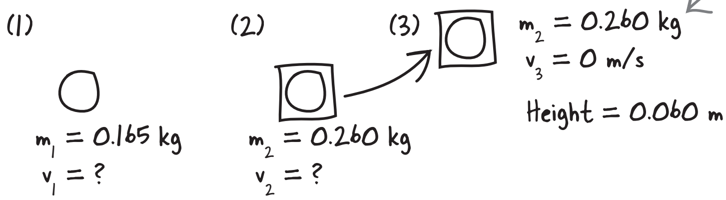
Find the velocity that the ball must be struck with if the bucket has a mass of 95 g.

Hint: The quickest way of doing this is to calculate the velocity that the ball and bucket must have after the collision, then work backwards.

Sharpen your pencil Solution

A pool ball, mass 165 g, is played into a padded bucket, which is arranged so it can swing upwards on the end of some light steel wires. For the shot to work, the ball must be struck so that the top of the bucket's swing is 6 cm higher than it started.

Find the velocity that the ball must be struck with if the bucket has a mass of 95g.



To get a velocity in m/s make sure you work in kg and m, not g and cm.

K of ball and bucket at (2) = U of ball and bucket at (3)

Use energy conservation to get v_2.

$$K = \tfrac{1}{2}m_2 v_2^2 = m_2 gh$$

$$v_2 = \sqrt{2gh} = \sqrt{2 \times 9.8 \times 0.060} = 1.08 \text{ m/s (3 sd)}$$

This is different from the programmer's math, as the ball is already in the bucket before you do the energy conservation part.

Use momentum conservation to get v_1.

$$m_1 v_1 = m_2 v_2$$

$$v_1 = \frac{m_2 v_2}{m_1} = \frac{0.260 \times 1.08}{0.165} = \underline{\underline{1.70 \text{ m/s}}} \text{ (3 sd)}$$

there are no Dumb Questions

Q: So sometimes I can't just use energy conservation to do a problem?

A: That's right - if the internal energy increases in a way that's very difficult to measure (like when padding is deformed) then you know that energy is conserved, but aren't able to do calculations with it!

Q: Does that mean there's a way that the internal energy can change that is easy to quantify?! Surely you can't see what's going on inside an object!

A: If the internal energy increases as a result of work being done entirely against friction, then the change in internal energy is equal to $F\Delta x$, the total quantity of energy transferred as a result of doing work against friction.

Q: What do I do if I can't calculate the increase in internal energy?

A: You need to use momentum conservation to deal with inelastic collisions. This will give you the velocity after the collision - after the inelastic deformation has taken place. It's this velocity that you should use to calculate the kinetic energy that remains in the system.

Question Clinic: The "Ballistic pendulum" Question

The pool ball trick shot is an example of the "ballistic pendulum" question. The name is taken from a technique used to find the velocity of a bullet by firing it into a wooden block suspended from wires, then measuring how high the block swings. The most important thing to remember is that if part of the process is inelastic, mechanical energy (i.e. kinetic + potential) won't be conserved as the internal energies of the block and bullet must increase as the bullet does work by tunnelling into the block.

This is a buzzword that may mean 'elastic collision' depending on what it collides with!

This is a buzzword that means 'inelastic collision'.

You can't use mechanical energy conservation for an inelastic collision, so you need to use momentum conservation for this part

3. A pool ball, mass 165 g, is played into a padded bucket, which is arranged so it can swing upwards on the end of some light steel wires. For the shot to work, the ball must be struck so that the top of the bucket's swing is 6.00 cm higher than it started.

Find the velocity that the ball must be struck with if the bucket has a mass of 95 g.

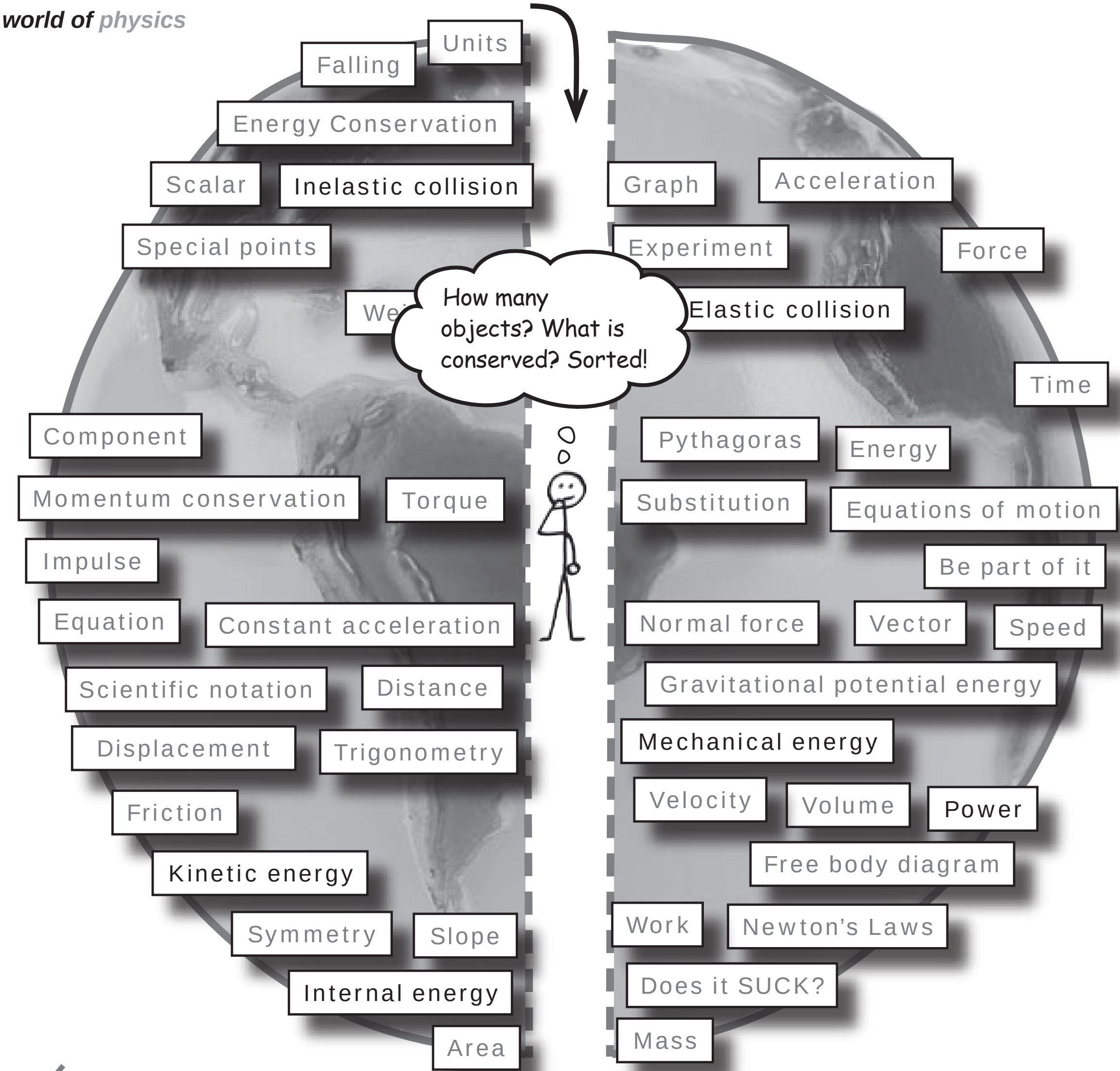
This means that you don't have to take the mass of the wires into account in your calculation.

Some of the ball's initial kinetic energy will be transferred to internal energy.

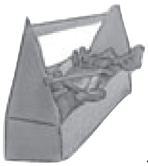
A difference in height should get you thinking about energy conservation.

Find the kinetic energy of the ball and bucket after the collision, since that is what is transferred to potential energy. And to do this, you must first use momentum conservation to work out the initial velocity of the ball.

The secret of doing this type of question is to ask yourself if an inelastic collision is involved before you go rushing into the math. Momentum is conserved in an inelastic collision, but the kinetic energy isn't the same before and after an inelastic collision. First you need to use momentum conservation to calculate the velocity and therefore kinetic energy of the new mass (block + bullet). Then you can use mechanical energy conservation to work out the height.



Kinetic energy	The capacity something has to do work due to its speed.
Internal energy	The total kinetic and potential energy due to the random motion or vibration of particles on a microscopic scale
Mechanical energy	The total kinetic + potential energy of a system on a macroscopic scale.
Power	The rate at which energy is transferred or work is done. Measured in Watts (1 W = 1 Joule per second).
Inelastic collision	A collision where momentum is conserved but kinetic energy is not conserved.
Elastic collision	A collision where both momentum and energy are conserved



Your Physics Toolbox

You've got Chapter 14 under your belt and added some problem solving skills to your toolbox.

Momentum vs kinetic energy

You calculate the change in momentum by thinking about a force applied for a period of time.

You calculate the change in kinetic energy by thinking about a force applied for a displacement.

Inelastic collision

A collision is inelastic if one or both of the objects involved deform in some way, or if the objects stick together.

Momentum is conserved, but kinetic energy isn't conserved because the deformation of the object increases its internal energy.

Elastic collision

A collision is elastic if neither object is deformed, and the objects bounce off of each other.

Mathematically, elastic collisions can be harder to deal with than inelastic collisions, as you still have two separate objects at the end.

However, both momentum and kinetic energy are conserved, which gives you two equations you can use to find two unknowns.

Difference in height

Any time a problem involves a difference in height, it's almost always easier to deal with it using energy conservation than with equations of motion.

The total energy is the same at the start and the end, so any change in potential energy will be accompanied by an equal change in kinetic energy.

Stopping an object

A quick way to determine the force required to stop an object at a certain displacement is to calculate its kinetic energy.

This is equal to the work you need to do against friction to stop it. As work = $F\Delta x$, this allows you to find the force quickly.

15 tension, pulleys and problem solving

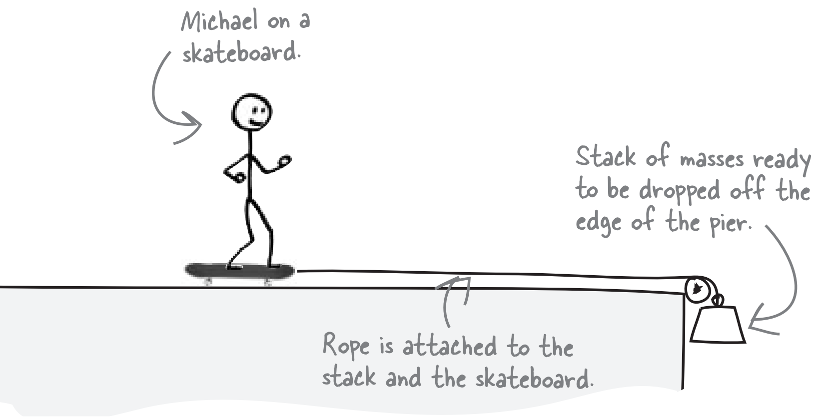
Changing direction

Sometimes you need to deal with the tension in a situation

So far, you've been using forces, free body diagrams and energy conservation to solve problems. In this chapter, you'll take that further as you deal with ropes, **pulleys**, and, yes, **tension**. Along the way, you'll also practice looking for familiar signposts to help navigate your way through complicated situations.

It's a bird... it's plane... ...no, it's... a guy on a skateboard?!

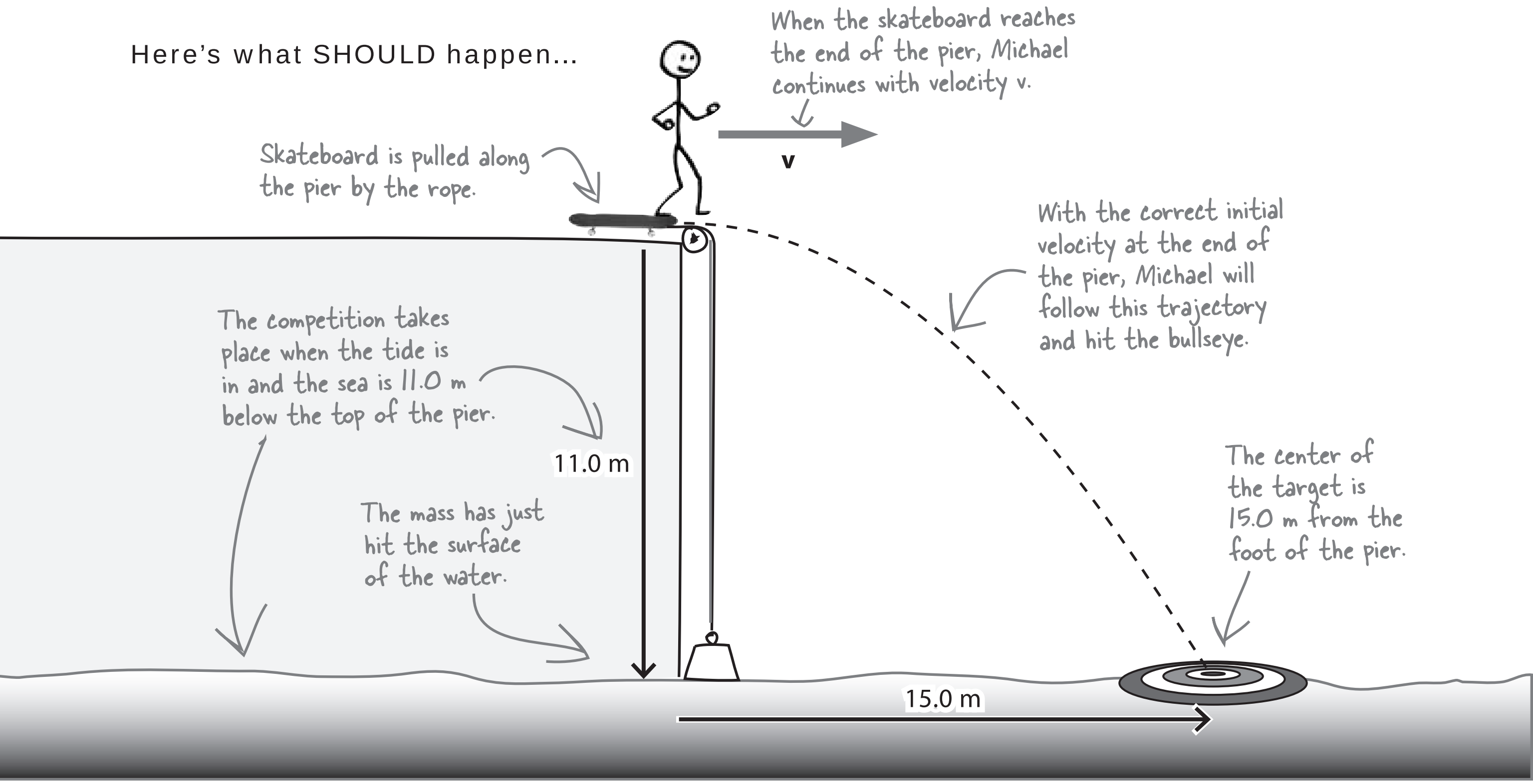
A new high-risk sporting event has come to town. The challenge? Jump off an 11.0 m pier and hit a target floating in the sea 15.0 m from the foot of the pier. Michael, a daredevil and skateboarding fiend, plans to take home the first place prize.



Michael wants to give himself a predictable launch **velocity** so he can be sure of hitting the target in the water. He's attached a skateboard to one end of a rope, put a large stack of **masses** at the other end, and placed a **pulley** in between.

The problem is, Michael's not so great at physics. And that's where you come in... can you help Michael out?

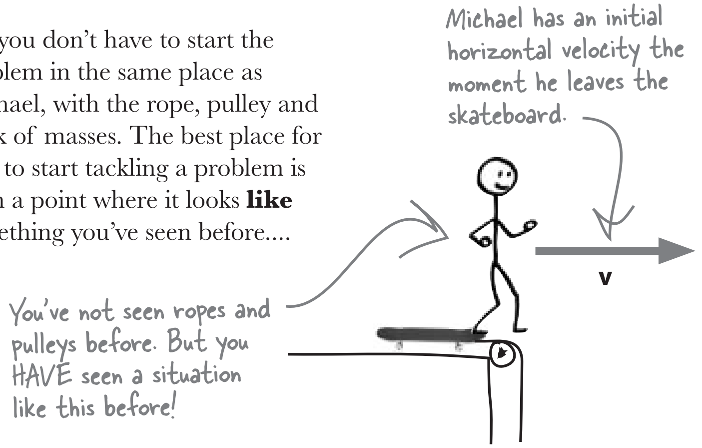
Here's what SHOULD happen...



Always look for something familiar

This problem involves a skateboard, a person, a stack of masses, a rope, a pulley, gravity, the height of the pier, and the distance to the target. Plus it takes place in two dimensions. It's a complicated problem!

But you don't have to start the problem in the same place as Michael, with the rope, pulley and stack of masses. The best place for **you** to start tackling a problem is from a point where it looks **like** something you've seen before....



You can start at the point where Michael flies through the air with velocity **v**. That looks kinda familiar...

Break down a complicated problem into smaller parts, then look for something that's LIKE what you've seen before.

Sharpen your pencil

Suppose Michael is launched horizontally from the end of a pier. What velocity does he need to possess in order to hit a target in the water 15.0 m from the foot of the pier, which is 11.0 m high?

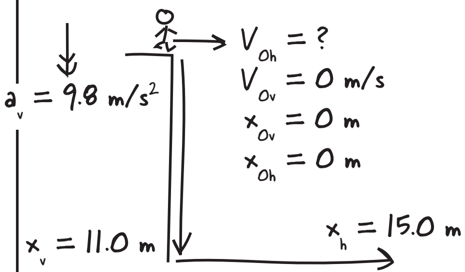
Always start with a sketch to help you figure a problem out.

Sharpen your pencil Solution

This part of the problem uses equations of motion. The key to solving it is to realize that you can do it separately from the rest of the problem, before you know anything about the stack of masses or the pulley.

Suppose Michael is launched horizontally from the end of a pier. What velocity does he need to possess in order to hit a target in the water 15.0 m from the foot of the pier, which is 11.0 m high?

Down is positive direction.



Get time from vertical components:

$$x_v = x_{0v} + v_{0v}t + \frac{1}{2}a_v t^2$$

These terms are both zero.

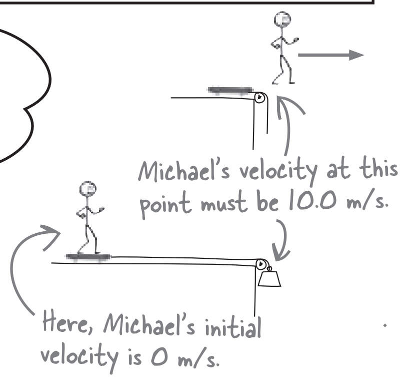
$$\Rightarrow \frac{1}{2}at^2 = x_v$$

$$\Rightarrow \quad t = \sqrt{\frac{2x_v}{a_v}} = \sqrt{\frac{2 \times 11.0}{9.8}} = 1.50 \text{ s (3 sd)}$$

Get horizontal velocity from horizontal components:

$$v_{0h} = \frac{x_h}{t} = \frac{15}{1.50} = \underline{10.0 \text{ m/s (3 sd) left to right.}}$$

For this part of the problem, Michael's **initial** velocity $\mathbf{v}_{0h}$ = 10 m/s. But for the other part, with the rope etc, his initial velocity is zero, and his **final** velocity is 10 m/s. So we need to be careful, right?



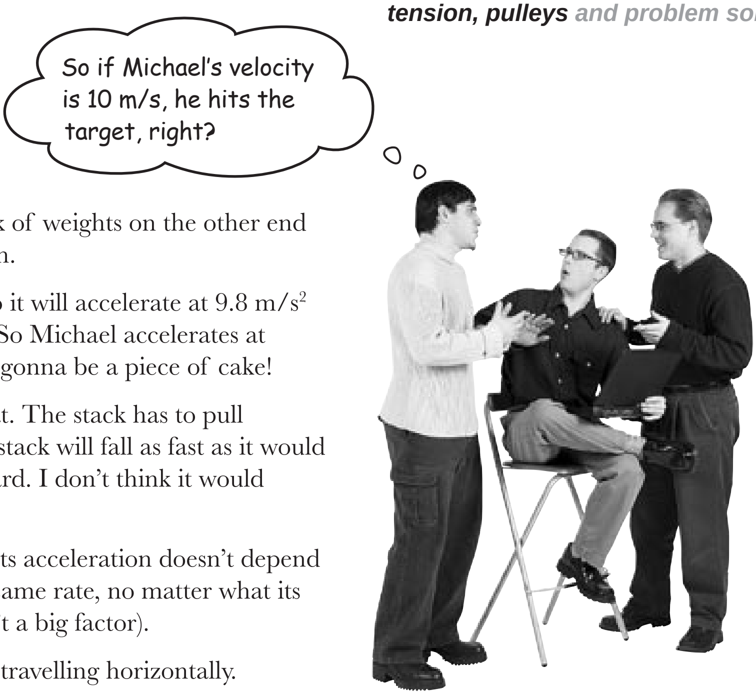
Watch out for what you call your variables in the next part.

Breaking this problem down has made it easier - which is great! But if you forget to redraw your sketch and redefine your variables for the next part, you'll just confuse yourself.

Your starting point this time around was when Michael is launched from the pier, where you've worked out that his **initial velocity** is $\mathbf{v}_{0h}$ = 10.0 m/s.

But the other part of the problem involves him reaching this velocity of 10.0 m/s from a standing start - where it'll be his **final velocity**. So be careful!

When you break up a problem, you might need to redefine variables as you move from part to part.



Jim: Yeah, and we've got that stack of weights on the other end of the rope to **accelerate** him with.

Frank: Yeah, the stack is falling, so it will accelerate at 9.8 m/s^2 and drag Michael along behind it. So Michael accelerates at 9.8 m/s^2, just like the stack. This is gonna be a piece of cake!

Joe: Um, I'm not so sure about that. The stack has to pull Michael along, so I don't think the stack will fall as fast as it would if it wasn't attached to the skateboard. I don't think it would accelerate at 9.8 m/s^2.

Frank: But if something's falling, its acceleration doesn't depend on its mass. Everything falls at the same rate, no matter what its mass is (as long as air resistance isn't a big factor).

Joe: But Michael isn't falling - he's travelling horizontally.

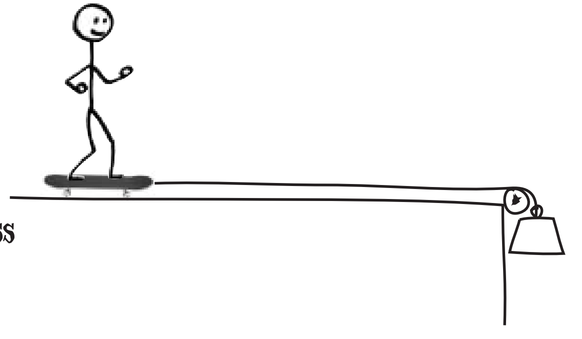
Jim: Oh yeah ... I guess if there was an elephant on the skateboard instead, the board would hardly accelerate at all.

Frank: Yeah, the **force** is due to the weight of the falling stack - but not Michael's weight, as he isn't falling.

Joe: This isn't so straightforward after all.

BE the skater

Your job is to imagine you're Michael on the skateboard. What happens to you as the mass at the other end of the rope falls? How will making the mass larger or smaller affect what happens to you?



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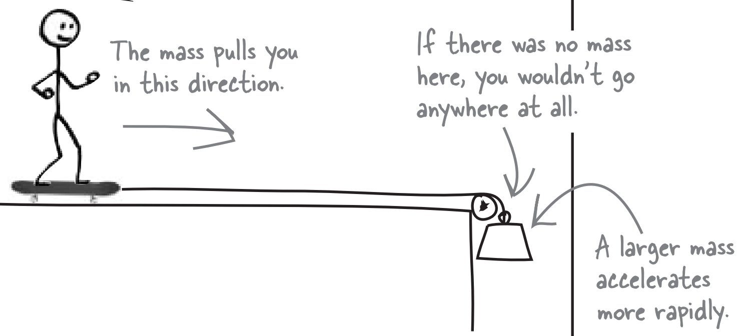
BE the skater - SOLUTION

Your job is to imagine you're Michael on the skateboard. What happens to you as the mass at the other end of the rope falls? How will making the mass larger or smaller affect what happens to you?



As the mass falls vertically, I accelerate horizontally because it pulls the skateboard.

If the mass is larger, I accelerate more quickly.

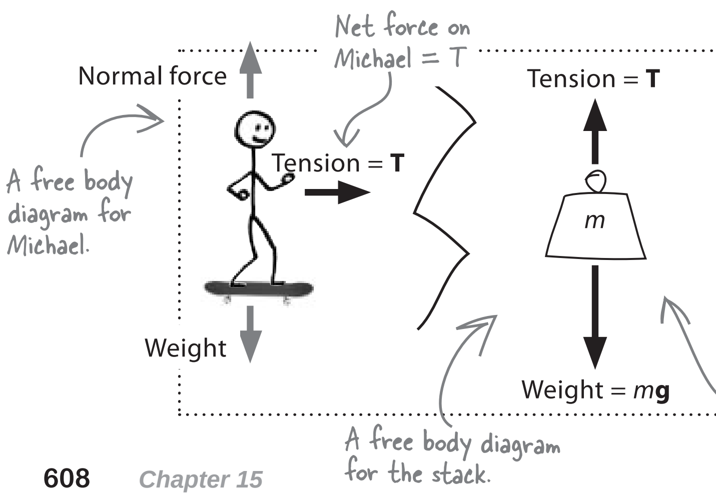
If the mass is smaller, I accelerate more slowly, and if the mass is a really small I might not accelerate at all.

Michael and the stack accelerate at the same rate

Michael and the stack are joined together by the rope, so they both **accelerate** at the same rate. This is because of the **tension** in the rope - the rope is pulled tight. If the rope wasn't there or wasn't pulled tight, there would be no tension and Michael wouldn't accelerate as the stack falls.

A stack with a larger mass on it will accelerate Michael more rapidly, and a smaller mass will accelerate him more slowly. This is something you can analyze by thinking about the tension in the rope.

A rope pulled tight can mediate a TENSION force.



The stack is attached to Michael by a rope. As well as his weight and the normal force (which add to zero), he also experiences a net **tension** force from the rope, **T**, which accelerates him to the right.

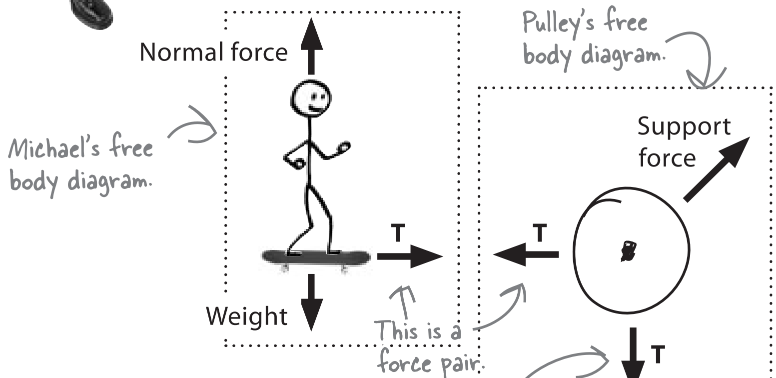
As the stack exerts a force on Michael (via the rope) Michael must exert an equal-sized force on the stack (in this case, via the rope) . So the stack experiences two forces - its **weight** and the **tension**.

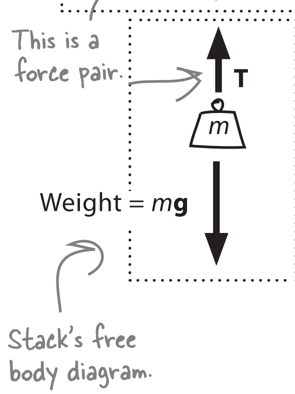
The pulley changes the direction that the tension force acts in.

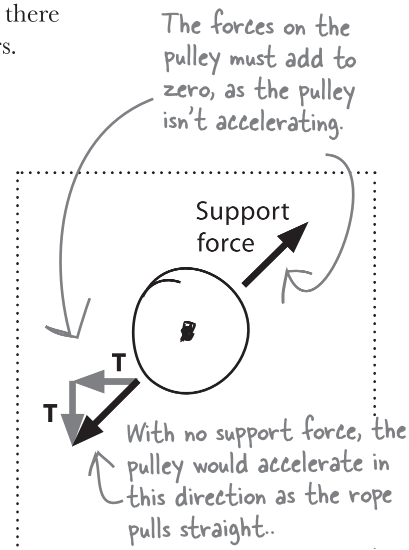
If a rope experiences a pulling force at both ends that makes it tight (like the rope we have here) it's said to be in **tension**.

But a **pulley** changes the **direction** that the tension force acts in. The pulley is able to do this because it's firmly attached to the pier, which is able to provide a support force. Otherwise, the rope would just get pulled straight.

If you just draw the free body diagrams for Michael and the stack, it looks like you have a Newton's 3rd Law force pair that isn't acting in opposite directions. But when you include the free body diagram of the pulley, you can see that there are horizontal and vertical pairs of tension force pairs.







Because the rope is being pulled tight, it exerts a force on the pulley. But the pulley's not accelerating, so the net force on it must be zero. Therefore, the pier that the pulley's attached to must provide a support force.

If there is a pulley, you have to include it when working out force pairs.

there are no Dumb Questions

Q: Why doesn't the stack just accelerate at 9.8 m/s^2, like it usually would if you dropped it?

A: It's attached to Michael (and his skateboard) by a rope. So the force of the stack's weight has to accelerate Michael's mass as well as the stack's own mass.

Q: But don't all falling objects accelerate at the same rate, whatever their mass?

A: Not if they're tied on to something else that isn't falling!

Q: So where does tension come in?

A: Tension is the name given to the force exerted at each end of a rope. For example, if the stack was hanging from a rope attached to the ceiling, the tension would be the same size as the stack's weight.

Q: Is the tension in a rope always the same size as the weight of the object it's supporting?

A: If an object is hanging straight down from a rope, then its acceleration is zero and the net force must be zero. So the tension in the rope must be the same size as the object's weight (in the opposite direction).

But if the object is accelerating downwards, (like the stack is here), the object's weight must be greater than the tension in the rope to produce a net downwards force.

Q: Is it OK to think of a tension force a bit like the normal force in other problems - as it's a support force?

A: As long as you remember that the tension always acts in the direction of the rope, that should be OK conceptually.

Q: Do I have to take into account the mass of the rope as well?

A: Great question! In real life you would, but in practice the mass of the rope has very little effect if it's much smaller than the masses it's attached to. So we're making the approximation that the rope is massless.

Q: How do you know that the two objects attached to the rope both accelerate at the same rate?

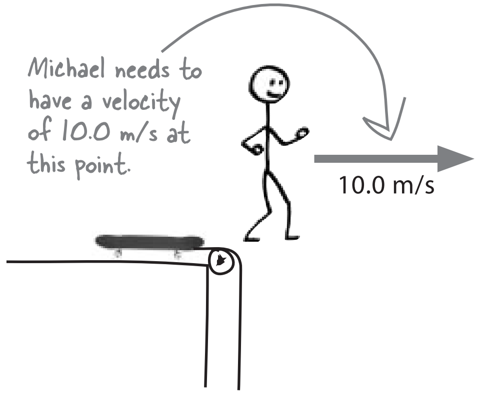
A: If the two objects are at either end of a rope which is pulled tight, they must always move with the same speed in order for the rope to remain tight. Therefore, they must also both accelerate at the same rate (though they may accelerate in different directions depending on how the rope is positioned).

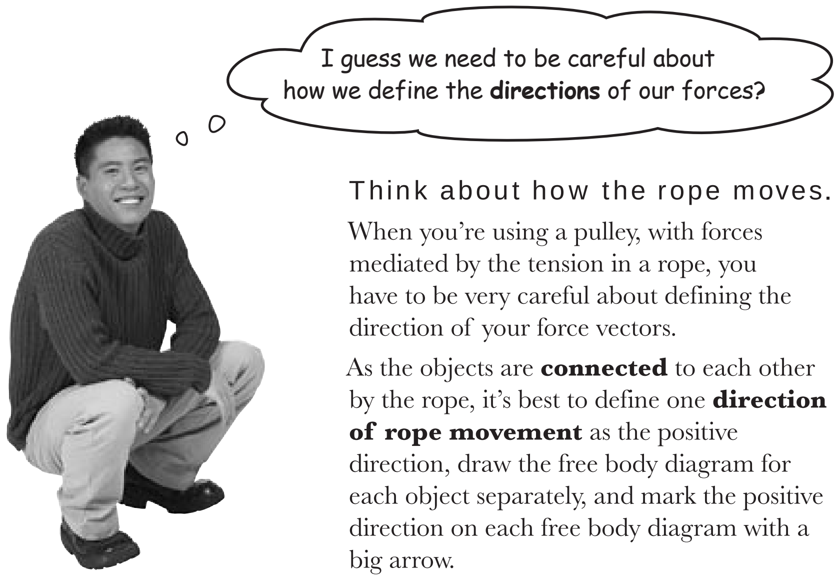
When two objects are joined together and move together, they both accelerate at the SAME rate.

Use tension to tackle the problem

Because the stack and Michael are joined together by a rope under tension, they both accelerate at the same rate .You need to work out what **mass** the stack should have in order for both it and Michael to be going at 10.0 m/s after the stack has fallen 11.0 m from the top to the bottom of the pier.

If you draw separate free body diagrams for the stack and for Michael, showing all the **forces** acting on them, you can use these to work out the mass of the stack.

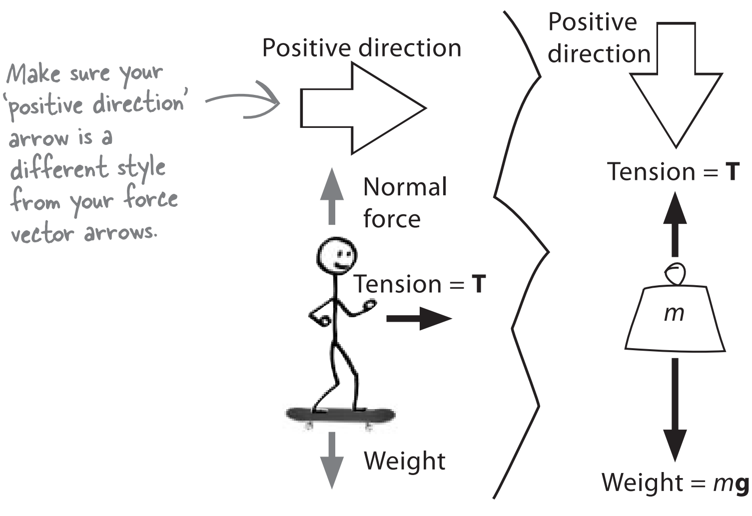




Think about how the rope moves.

When you're using a pulley, with forces mediated by the tension in a rope, you have to be very careful about defining the direction of your force vectors.

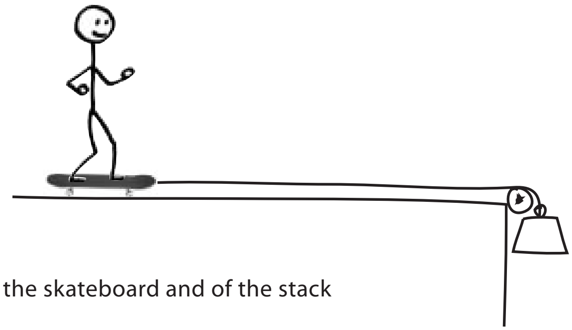
As the objects are **connected** to each other by the rope, it's best to define one **direction of rope movement** as the positive direction, draw the free body diagram for each object separately, and mark the positive direction on each free body diagram with a big arrow.



Define one direction of rope movement as positive and mark it on your free body diagrams.

Sharpen your pencil

Michael on a skateboard, mass *M*, is attached via a rope and a pulley to a stack of masses, mass *m*, as shown in the picture. When the stack is allowed to accelerate downwards in a gravitational field, strength **g**, the rope has tension, **T** and Michael also accelerates.



a. Draw separate free body diagrams of Michael on the skateboard and of the stack

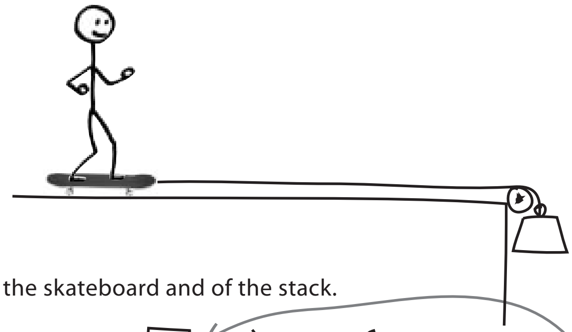
b. Write down the size of the net force on Michael.

c. Write down the size of the net force on the stack.

d. Michael and the stack both accelerate with acceleration, **a**. Use Newton's 2nd Law to write down a separate expression for each of them that relates their mass, their acceleration and the net force on them.

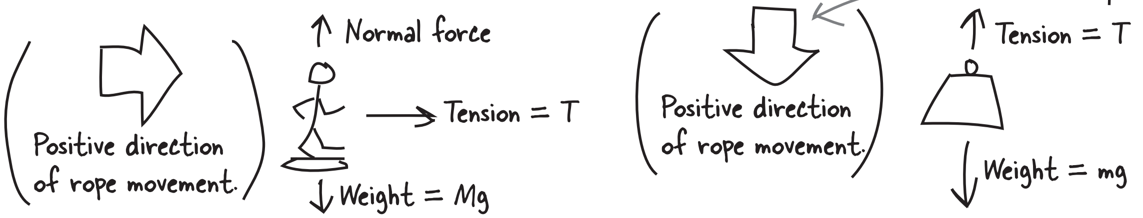
e. The size of the tension in the rope is the same for both Michael and the stack. Make a substitution using your equations from part d. and rearrange your answer so that you have an equation for m, the mass of the stack, in terms of *M*, **g** and **a**.

Sharpen your pencil Solution



Michael on a skateboard, mass *M*, is attached via a rope and a pulley to a stack of masses, mass *m*, as shown in the picture. When the stack is allowed to accelerate downwards in a gravitational field, strength **g**, the rope has tension, **T** and the man also accelerates.

a. Draw separate free body diagrams of Michael on the skateboard and of the stack.



b. Write down the size of the net force on Michael.

Normal force and weight add to zero.

$F_{net} = T$

c. Write down the size of the net force on the stack.

$F_{net} = mg - T$

Drawing on the big arrows helps you get the signs correct in these equations.

d. Michael and the stack both accelerate with acceleration, **a**. Use Newton's 2nd Law to write down a separate expression for each of them that relates their mass, their acceleration and the net force on them.

Man has mass M.

$F_{net} = ma$ becomes $T = Ma$

Stack has mass m.

$F_{net} = ma$ becomes $mg - T = ma$

Make substitutions.

e. The size of the tension in the rope is the same for both Michael and the stack. Make a substitution using your equations from part d. and rearrange your answer so that you have an equation for m, the mass of the stack, in terms of *M*, **g** and **a**.

$T = Ma$ (1)

$mg - T = ma$ (2)

Substitute the expression for T in (1) into (2)

$mg - Ma = ma$

Rearrange equation to say "m = ... "

$mg - Ma = ma$

$mg - ma = Ma$

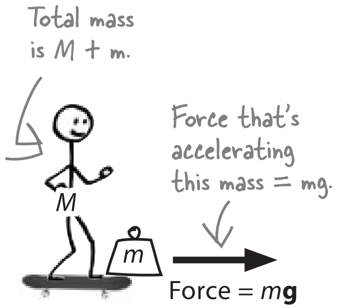
$m(g - a) = Ma$

$$m = \frac{Ma}{g - a}$$

Then you can divide both sides by (g – a) to get "m = ..."

Both the g and the a are multiplied by the m.

So you can introduce some parentheses so that there's only one instance of m on the left hand side.



Could we also have treated the stack and Michael as one 'thing' with mass ($M + m$)? Would that have worked too?

Treating both objects as one single mass works here (but NOT always).

On this occasion, you can also treat Michael and the stack as one object with mass ($M + m$) that's accelerated by the force $m\mathbf{g}$ (the weight of the stack). You'd write down:

$$\text{Force} = \text{mass} \times \text{acceleration}$$

$$m\mathbf{g} = (M + m)\mathbf{a}$$

When you rearrange this, you get the same equation as over there: $m = \frac{M\mathbf{a}}{\mathbf{g} - \mathbf{a}}$

This is NOT a shortcut that will always work. This problem is a special case.

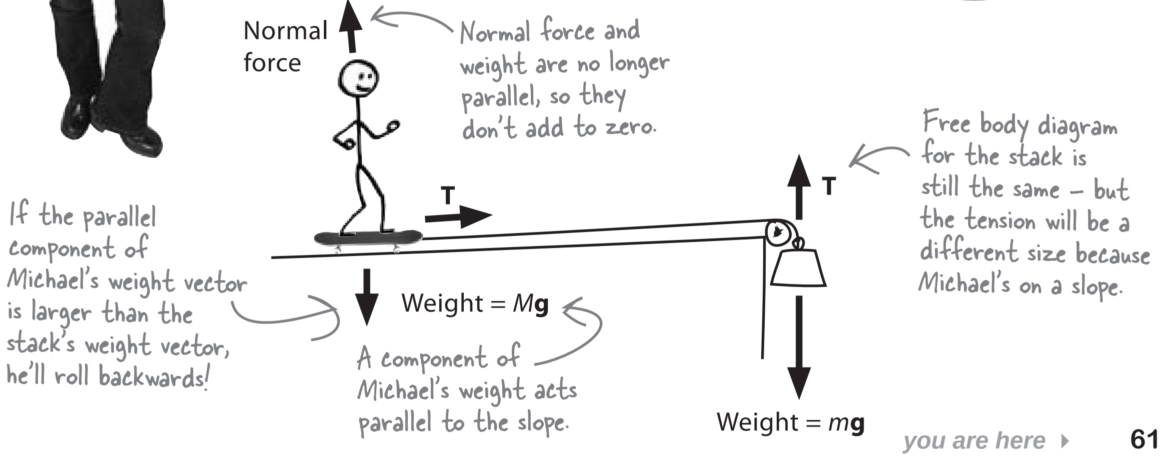
So why bother with the tension at all, if we can do it that way instead?

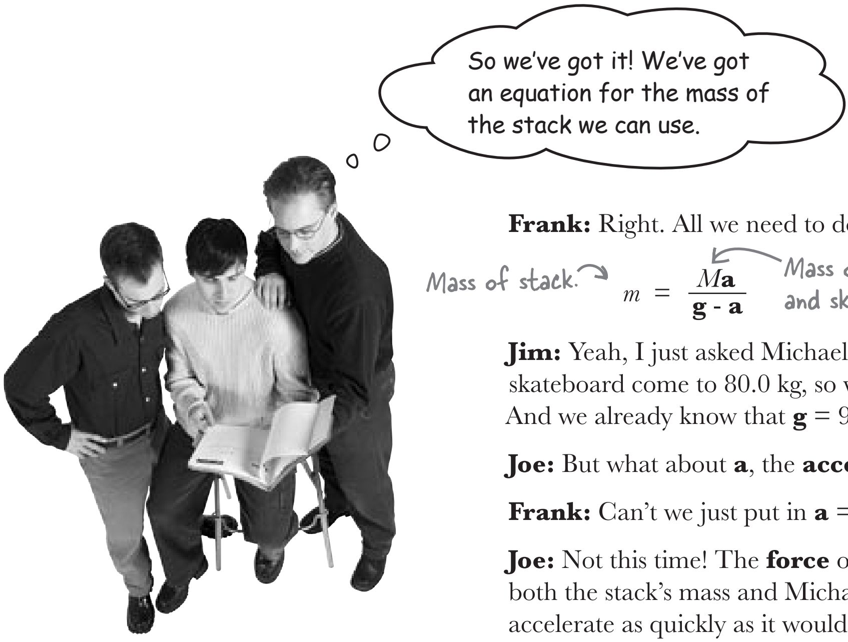
Thinking about the tension in the rope helps you to understand the physics.

If you're able to draw free body diagrams for problems that involve ropes and tension, you'll be able to solve **any** problem, not just this one.

For example, if Michael was being pulled up a ramp by the stack instead of being pulled horizontally, you'd have to think about Michael's weight vector as well. The only way of using forces and Newton's 2nd Law to deal with that is to draw a free body diagram for each object attached to the rope - which is fine if you understand the physics.

If you're analyzing a problem with ropes and pulleys using forces, ALWAYS start with individual free body diagrams, have a go at BEING each object, and go on from there.





Frank: Right. All we need to do it to put the numbers into it:

$$m = \frac{M\mathbf{a}}{\mathbf{g} - \mathbf{a}}$$

Mass of stack.
Mass of Michael and skateboard.

Jim: Yeah, I just asked Michael, and he says that him plus the skateboard come to 80.0 kg, so we can put that in instead of M. And we already know that $\mathbf{g} = 9.8\ \text{m/s}^2$.

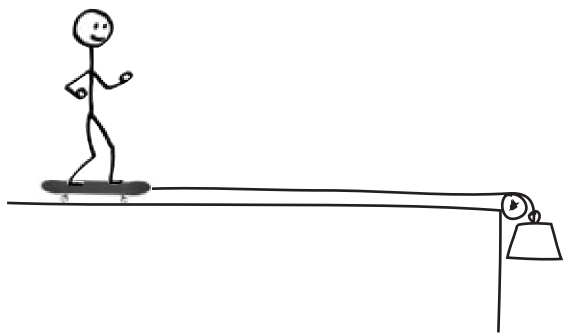
Joe: But what about **a**, the **acceleration**. We don't know that.

Frank: Can't we just put in $\mathbf{a} = 9.8\ \text{m/s}^2$ like we usually do?

Joe: Not this time! The **force** of the stack's weight has to accelerate both the stack's mass and Michael's mass. So the stack won't accelerate as quickly as it would if it was just falling on its own.

Jim: So we gotta work out a value for the acceleration first, before we can get a value for the mass. How are we gonna do that?

Joe: Well, there is the **sketch** we did earlier ...



If you start with a sketch, you can always look back at it for inspiration if you're not sure what to do next.

The sketch acts as an anchor that reminds you of what you're working on - and also of ways you might solve your problem.

Jim: Can we just write things like $\mathbf{v}_0$, $\mathbf{v}$ and $\mathbf{x}$ on it, and use **equations of motion** to calculate **a**?

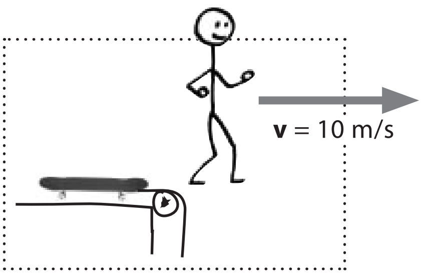
Frank: Oh yeah! I've been getting so wrapped up into working with masses and forces that I forgot we could just look at Michael's velocity, displacement and acceleration, without going into all the reasons behind them.

Joe: Cool, let's do it!

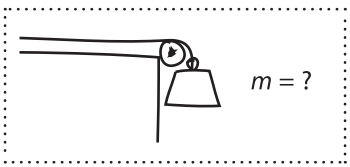
Look at the big picture as well as the parts

Right at the start, you broke down this complicated problem into two parts:

1. Work out the **velocity** Michael needs at the end of the pier to hit the target. You did this first because it involved **equations of motion**, and a scenario similar to others you'd **seen before**.



2. Work out what **mass** the stack needs to be for him to reach this velocity at the end of the pier. You've just used free body diagrams and Newton's 2nd Law to come up with an equation for the mass of the stack, $m = \dfrac{M\mathbf{a}}{\mathbf{g} - \mathbf{a}}$



But now you've discovered that there's **another part** to this problem! You need to work out a value for the acceleration of Michael and the stack. The important thing is not to panic, and to step back and look at the **big picture**.

Then you'll see that you can use your **equations of motion** to calculate the acceleration. Once you have a value, you can calculate the mass of the stack, which is what you really want to know.

Though do be careful with how you define the variables you use.

Sharpen your pencil

Michael is to be launched horizontally from the end of a 11.0 m high pier with a velocity of 10.0 m/s. He stands on a skateboard, which is attached via a rope and pulley to a stack of masses that falls vertically downwards. Michael and the skateboard have a combined mass of 80.0 kg.

a. If Michael reaches the edge of the pier at the same time as the stack hits the water, what is his displacement?

b. Calculate Michael's acceleration.

c. Use the equation $m = \dfrac{M\mathbf{a}}{\mathbf{g} - \mathbf{a}}$ that you worked out before to calculate the mass that the stack needs to be in order to produce this acceleration. (The stack has mass *m*, Michael and his skateboard have mass *M*).

Sharpen your pencil Solution

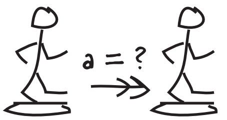
Michael is to be launched horizontally from the end of a 11.0 m high pier with a velocity of 10.0 m/s. He stands on a skateboard, which is attached via a rope and pulley to a stack of masses that falls vertically downwards. Michael and the skateboard have a combined mass of 80.0 kg.

a. If Michael reaches the edge of the pier at the same time as the stack hits the water, what is his displacement?

The stack has fallen 11.0 m vertically.

So Michael has travelled 11.0 m horizontally, towards the edge of the pier.

b. Calculate Michael's acceleration.



$x_0 = 0$ m $x = 11.0$ m

$v_0 = 0$ m/s $v = 10.0$ m/s

$v^2 = v_0^2 + 2a(x - x_0)$

But $v_0 = 0$ and $x_0 = 0$.

$\Rightarrow v^2 = 2ax$

$\Rightarrow a = \frac{v^2}{2x} = \frac{10.0^2}{2 \times 11.0}$

$a = 4.54$ m/s (3

c. Use the equation $m = \frac{M\mathbf{a}}{\mathbf{g - a}}$ that you worked out before to calculate the mass that the stack needs to be in order to produce this acceleration. (The stack has mass *m*, Michael and his skateboard have mass *M*).

$$m = \frac{Ma}{g - a}$$

$$m = \frac{80.0 \times 4.54}{9.8 - 4.54} = \underline{\underline{69.0 \text{ kg}}} \text{ (3 sd)}$$

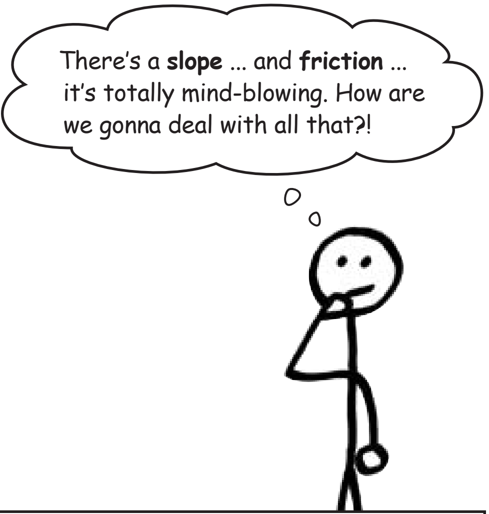
So we should tell Michael to use a 69.0 kg stack – and let him know we want some of the credit when he wins first prize!

But the day before the competition ...

Michael's just been down to the venue to check it out - and he's discovered that the pier slopes up towards the sea at a slight **angle**. The angle is only 5.0° ... but that might be enough to throw out your careful calculations. On the bright side, it levels out just at the end, so at least he's still taking off horizontally, and the end of the pier is still 11.0 m above the water.

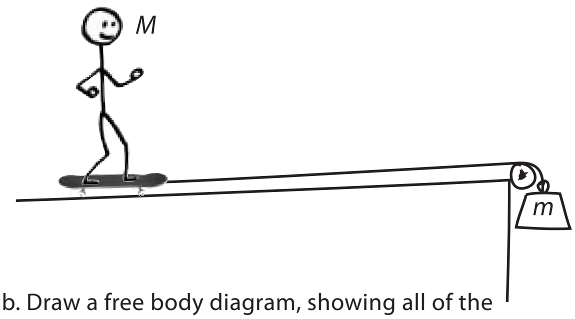
On top of that, Michael's checked the skateboard website and found out that its wheels have a coefficient of **friction**, $\mu = 0.0500$.

Your complicated problem just got a LOT more complicated ...



Sharpen your pencil

Michael on his skateboard, with a total mass M , is attached to a stack of masses, total mass m, via a rope and pulley, as shown in the picture. The wheels of the skateboard have coefficient of friction, μ, and the pier slopes up towards the sea at angle θ from the horizontal.

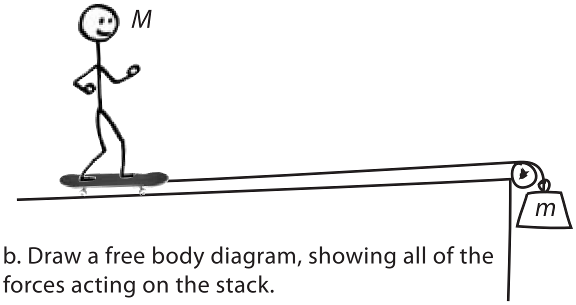


a. Draw a free body diagram, showing all of the forces acting on Michael.

b. Draw a free body diagram, showing all of the forces acting on the stack.

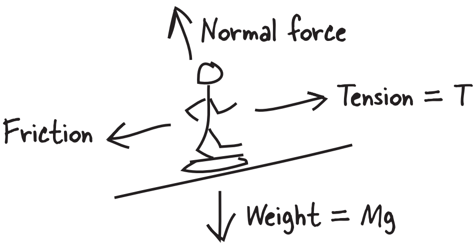
c. Outline what you would do to approach this problem. You don't need to do any math - just explain the process you'd go through.

Sharpen your pencil Solution

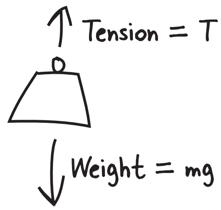


Michael on his skateboard, with a total mass M, is attached to a stack of masses, total mass m, via a rope and pulley, as shown in the picture. The wheels of the skateboard have coefficient of friction, μ, and the pier slopes up towards the sea at angle θ from the horizontal.

a. Draw a free body diagram, showing all of the forces acting on Michael.



b. Draw a free body diagram, showing all of the forces acting on the stack.



c. Outline what you would do to approach this problem. You don't need to do any math - just explain the process you'd go through.

Use triangles to calculate the parallel component of Michael's weight, $F_{\parallel}$. Also calculate the normal force (equal to the perpendicular component of Michael's weight) and use it to calculate the friction force, as $F_{fric} = \mu F_{normal}$. The net force on Michael is parallel to the pier, $F_{net} = T - F_{\parallel} + F_{fric}$. Then do $F_{net} = ma$ for both Michael and the stack. They both have the same acceleration so go on from there by substituting in for the tension, etc, similar to before.

That looks like a **LOT** of work! Is it worth looking for an easier way before we do all that math?

Before you decide to use forces, think about using energy conservation.

Any time you see a problem that involves a **difference in height**, you should look to see if it might be possible to use energy conservation instead of forces, like you did in chapter 14.

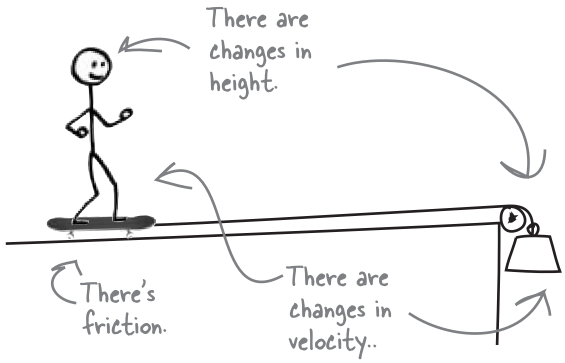
Using **energy conservation** to solve a problem involves fewer intermediate steps and less math than using forces. Which can only be a good thing!

If your problem involves a difference in height, look to see if you can use ENERGY CONSERVATION.

Using energy conservation is simpler than using forces

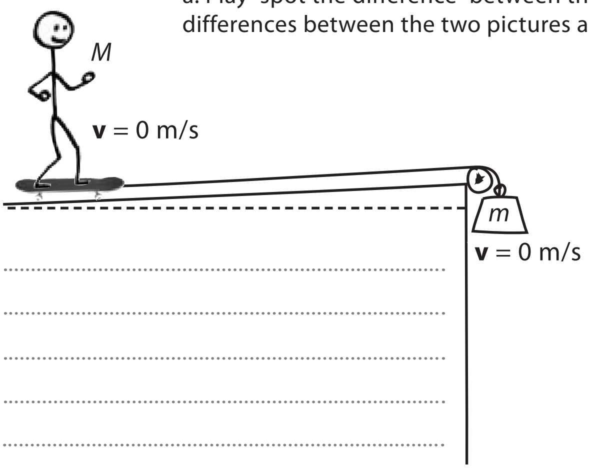
This is the kind of problem that lends itself to using **energy conservation** instead of forces, because it involves masses whose heights and velocities change, and work being done against friction.

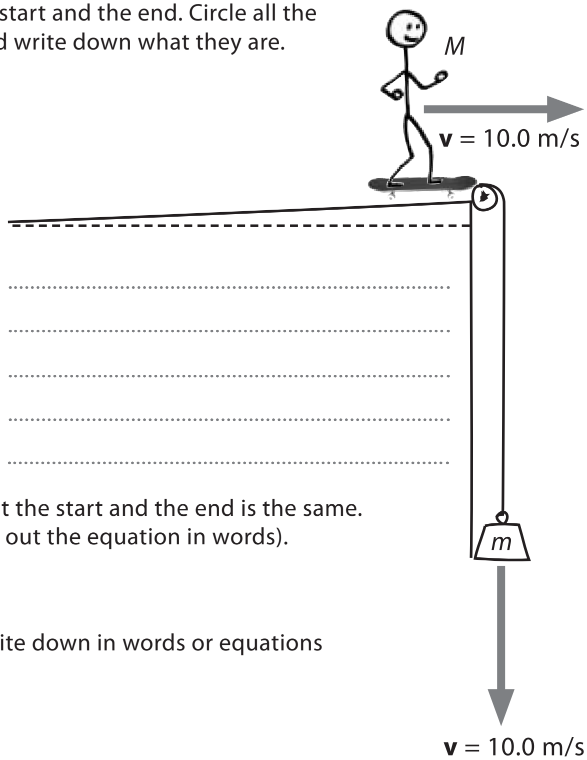
The total energy of the system is always constant, and differences drive changes that lead to energy transfer. So if you **spot the differences** between the start (when you've just dropped the stack) and the end (when Michael has just launched horizontally from the skateboard) you'll know what to include in the energy conservation equation.



Spot the difference

a. Play "spot the difference" between the start and the end. Circle all the differences between the two pictures and write down what they are.



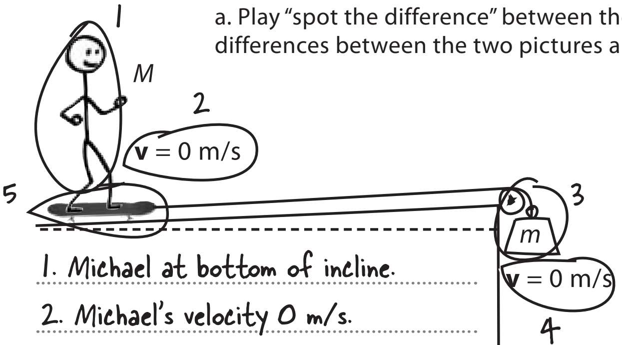


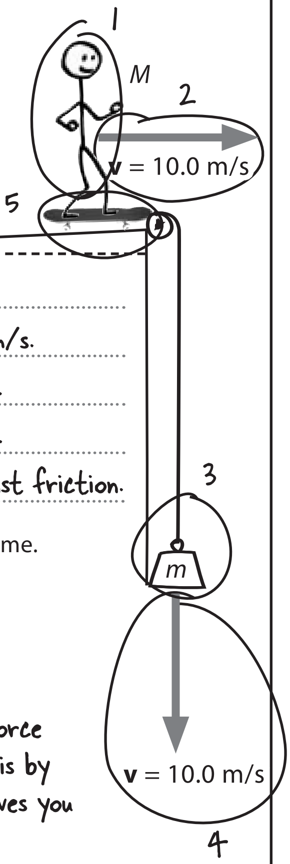
b. Write down an equation to show that the total energy at the start and the end is the same. (You can use any symbols and subscripts you like, or write out the equation in words).

c. For each term in your energy conservation equation, write down in words or equations how you would go about calculating it.

Spot the difference - SOLUTION

a. Play "spot the difference" between the start and the end. Circle all the differences between the two pictures and write down what they are.





Start	End
1. Michael at bottom of incline.	1. Michael at top of incline
2. Michael's velocity 0 m/s.	2. Michael's velocity 10.0 m/s.
3. Stack at top of pier.	3. Stack at bottom of pier.
4. Stack velocity 0 m/s.	4. Stack velocity 10.0 m/s.
5. No work done against friction.	5. Work has been done against friction.

b. Write down an equation to show that the total energy at the start and the end is the same. (You can use any symbols and subscripts you like, or write out the equation in words).

Energy at start = energy at end $\Rightarrow U_{stack} = U_{man} + K_{man} + K_{stack} + W_{friction}$

c. For each term in your energy conservation equation, write down in words or equations how you would go about calculating it.

U = mass × g × h
K = ½ × mass × v^2
} Same method for Michael and stack.

To calculate $W_{friction}$, calculate normal force using angle and weight and multiply this by μ, the coefficient of friction. This gives you the frictional force, $F_{friction}$.

Then $W_{friction} = F_{friction} \times$ displacement

By playing Spot the difference, we know that the **potential** energy that the stack has as a result of being 11.0 m higher at the start than it is at the end is transferred to:

- The **potential** energy Michael gains by going up the incline.
- The **kinetic** energy of Michael and the skateboard.
- The **kinetic** energy of the stack.
- The **work** done against the **friction** of the skateboard wheels.

It's time to put that all together ...

The easiest way to do energy conservation is to play "spot the difference."

Sharpen your pencil

Michael on his skateboard, with a total mass of 80.0 kg, is attached to a stack of masses, total mass *m*, via a rope and pulley. The wheels of the skateboard have coefficient of friction, $\mu = 0.0500$, and the board travels 11.0 m along a pier which slopes up towards the sea at angle $\theta = 5.0°$ from the horizontal as the stack goes 11.0 m straight down.

a. Calculate the difference in the height of Michael on the skateboard at the beginning and end of the pier.

b. Calculate the normal force exerted by the pier on Michael, and hence the work done against friction.

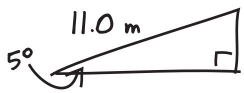
Remember that $F_{fric} = \mu F_N$

c. Michael needs to have a speed of 10.0 m/s at the end of the pier. Use energy conservation to work out the mass that the stack needs to have in order to achieve this.

Sharpen your pencil Solution

Michael on his skateboard, with a total mass of 80.0 kg, is attached to a stack of masses, total mass *m*, via a rope and pulley. The wheels of the skateboard have coefficient of friction, $\mu = 0.0500$, and the board travels 11.0 m along a pier which slopes up towards the sea at angle $\theta = 5.0°$ from the horizontal as the stack goes 11.0 m straight down.

a. Calculate the difference in the height of Michael on the skateboard at the beginning and end of the pier.

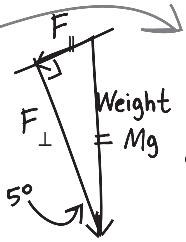


$$\sin(5.0°) = \frac{h_M}{11.0} \Rightarrow h_M = 11.0\sin(5.0°)$$

$$h_M = 0.959 \text{ m (3 sd)}$$

We're using a subscript here so that we don't get mixed up with the height of the pier. h_M is Michael's change in height.

b. Calculate the normal force exerted by the pier on Michael, and hence the work done against friction.



Normal force is same size as perpendicular component of weight.

$$\cos(5°) = \frac{F_{\perp}}{Mg}$$

Use these tips to get your triangles right.

$$\Rightarrow F_{\perp} = mg\cos(5.0°)$$

$$F_{\perp} = 80 \times 9.8 \times \cos(5.0°) = 781 \text{ N}$$

$$F_N = 781 \text{ N}$$

$$W_{friction} = \mu F_N \Delta x = 0.0500 \times 781 \times 11.0$$

$$W_{friction} = 430 \text{ J (3 sd)}$$

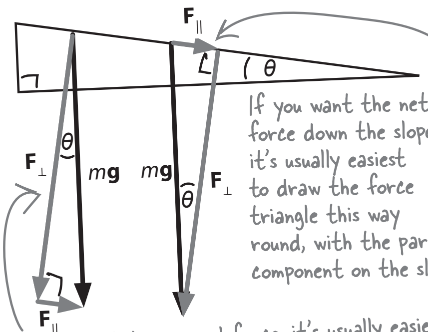
Michael doesn't float in the air or disappear through the ground. So the perpendicular components of the forces acting on him must add to zero.

Triangle Tip: sketch extreme angles

If you're not sure which angle in your force vector triangle corresponds to the angle of your slope, sketch a slope with a **small angle**, θ.

Making this angle small helps you to keep track of similar triangles.

Now draw the force triangle. Draw on the weight pointing straight down. Then draw in the parallel and perpendicular components. It doesn't matter which way round you draw the components, as the triangle's sides will still be the same length.



If you want the net force down the slope, it's usually easiest to draw the force triangle this way round, with the parallel component on the slope.

If you want the normal force, it's usually easiest to draw the triangle this way round, with the perpendicular component below the object.

θ is the small angle in the slope triangle - so θ will also be the small angle in the force triangle.

These also appeared in ch. 13.

c. Michael needs to have a speed of 10.0 m/s at the end of the pier. Use energy conservation to work out the mass that the stack needs to have in order to achieve this.

Energy at start = energy at end

$$U_{stack} = U_{man} + K_{man} + K_{stack} + W_{friction}$$

$$mgh = Mgh_M + \tfrac{1}{2}Mv^2 + \tfrac{1}{2}mv^2 + W_{friction}$$

$$mgh - \tfrac{1}{2}mv^2 = Mgh_M + \tfrac{1}{2}Mv^2 + W_{friction}$$

Introduce parentheses to leave one instance of m on the left hand side.

$$m(gh - \tfrac{1}{2}v^2) = Mgh_M + \tfrac{1}{2}Mv^2 + W_{friction}$$

$$m = \frac{Mgh_M + \tfrac{1}{2}Mv^2 + W_{friction}}{(gh - \tfrac{1}{2}v^2)}$$

$$m = \frac{(80 \times 9.8 \times 0.959) + (0.5 \times 80 \times 10^2) + 430}{(9.8 \times 11) - (0.5 \times 10^2)}$$

$$m = 89.7 \text{ kg (3 sd)}$$

This is larger then the mass you calculated last time, when there was no incline and no friction, which makes sense.

How come we didn't need to calculate the **acceleration** this time? We did when we used forces?

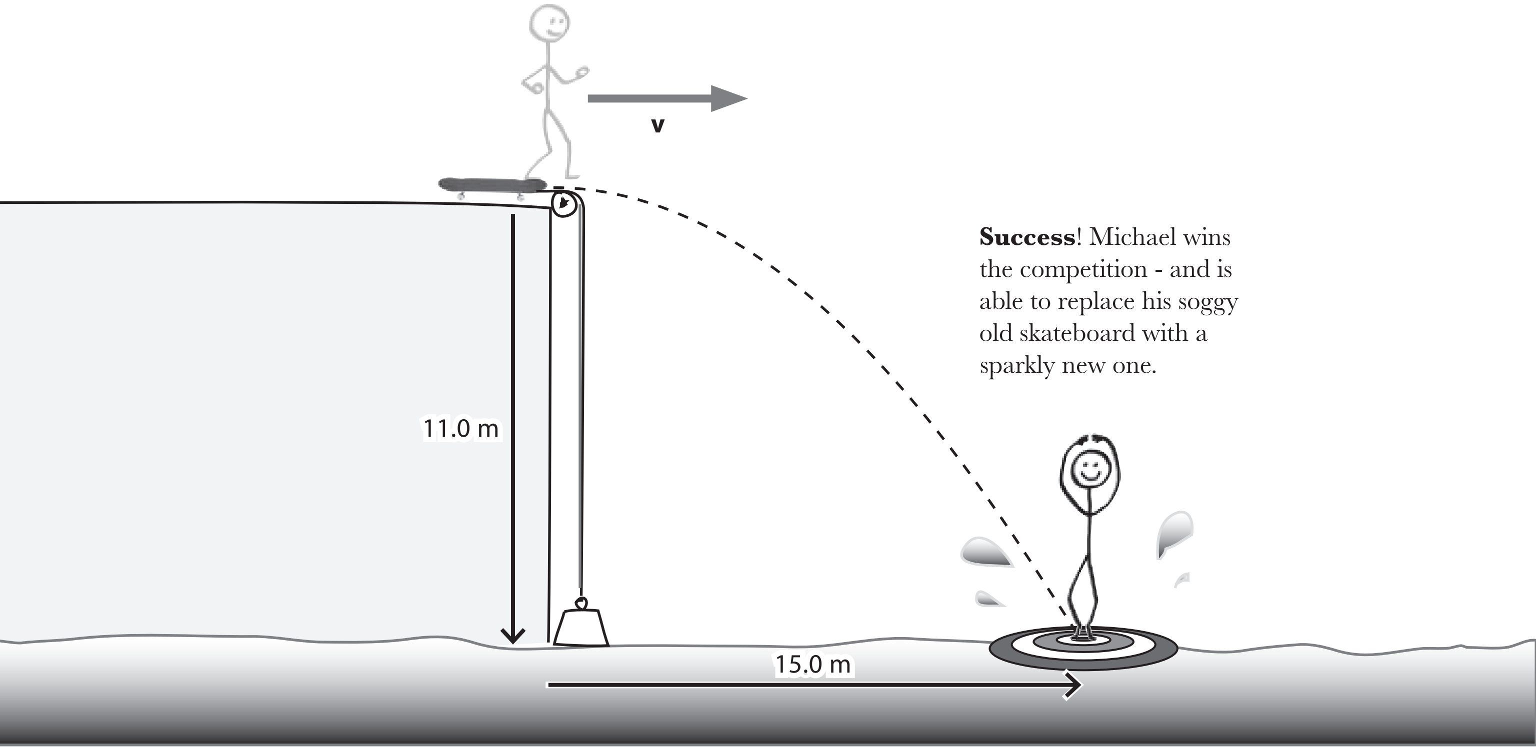
Doing a problem a different way may involve different intermediate steps.

Earlier on, you used **forces** to work out what mass the stack would need to be for a flat pier with no friction. As this method involved Newton's 2nd Law, $\mathbf{F}_{net} = m\mathbf{a}$, the acceleration was important.

This time you used **energy conservation**. You didn't need to calculate the acceleration, as you could get all you needed from the masses, differences in height and differences in velocity.

There goes that skateboard...

Michael's taken your advice, and he's off...
let's see how he does:



Success! Michael wins the competition - and is able to replace his soggy old skateboard with a sparkly new one.

BULLET POINTS

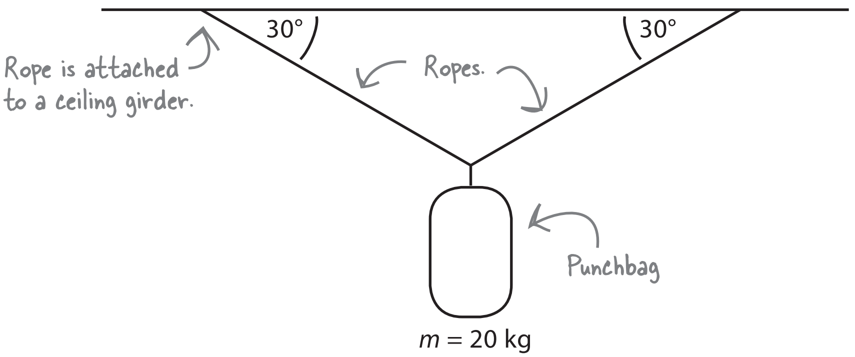
- When two objects are joined together by a rope that's pulled tight, they both accelerate at the same rate in the same direction that the rope moves in.
- The **tension** in a rope is the same at both ends.
- A mass won't accelerate at 9.8 m/s^2 when it falls if it is attached to another mass that isn't also falling with it.
- If you have more than one mass, draw a **free body diagram** for each ...
- ... but do also check to see if you can use **energy conservation** instead!
- Play "spot the difference" to make sure you spot all the **differences** that involve energy transfer before you write down any equations.

Problems with a punchbag

Geoff's Gym was becoming more and more popular - so much so that his customers were having to queue for each piece of apparatus. But being that popular also has its disadvantages - and many of Geoff's customers were talking of going somewhere more quiet once their current memberships had expired.

So Geoff brought in lots of new equipment - bikes, rowing machines, weights machines - some seriously complicated and expensive pieces of kit. But the item Geoff had the most problems with was the humble punchbag.

"I just don't know what went wrong," he explains. "The punchbag's 20 kg. So its weight is 196 N - call it 200 N to be on the safe side. I wasn't able to hang it straight down like I usually would because the girders in the roof are in the wrong place. So I decided to use two ropes to hang it from a couple of girders.



I thought that each rope will be supporting half of the weight. That must be 100 N per rope, as both ropes are at the same angle. The ropes I used to hang the punchbag were guaranteed to cope with a tension of up to 180 N ... or so the manufacturers claimed.

So I put the punchbag on top of a stepladder to set it at the right height, then went up and attached the ropes. But as soon as I took away the stepladder, one of the ropes broke! I couldn't believe it! And then of course the punchbag swung sideways and the other rope broke.

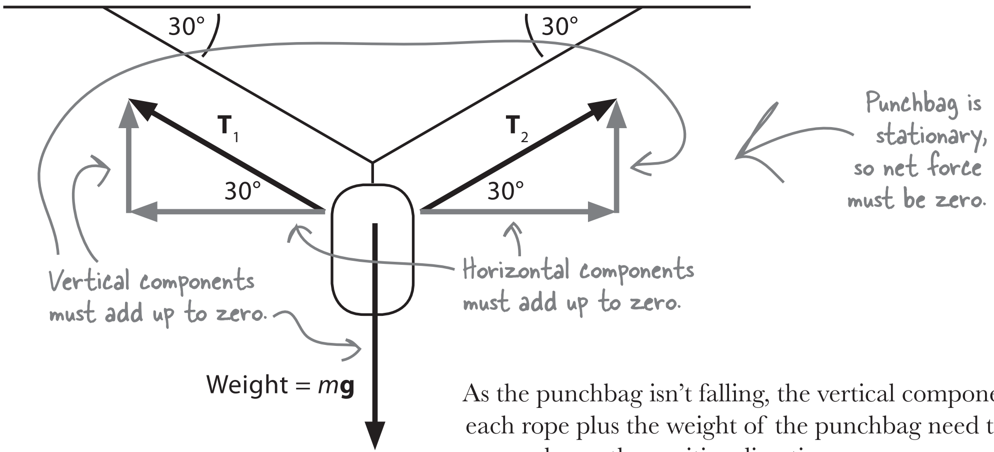
Why did the first rope break?

Problems with a punchbag

Why did the first rope break?

Geoff assumed that each rope would only have to provide enough tension force to prevent the punchbag from **falling**. But the tension force also needs to prevent the punchbag from **swinging**. which is what would happen if the other rope wasn't present.

Each rope exerts a tension force on the punchbag in the direction of the rope:



As the punchbag isn't falling, the vertical components of the tension in each rope plus the weight of the punchbag need to add up to zero. If you make up the positive direction:

$$\mathbf{T}_1\sin(30°) + \mathbf{T}_2\sin(30°) - 196\text{N} = 0$$

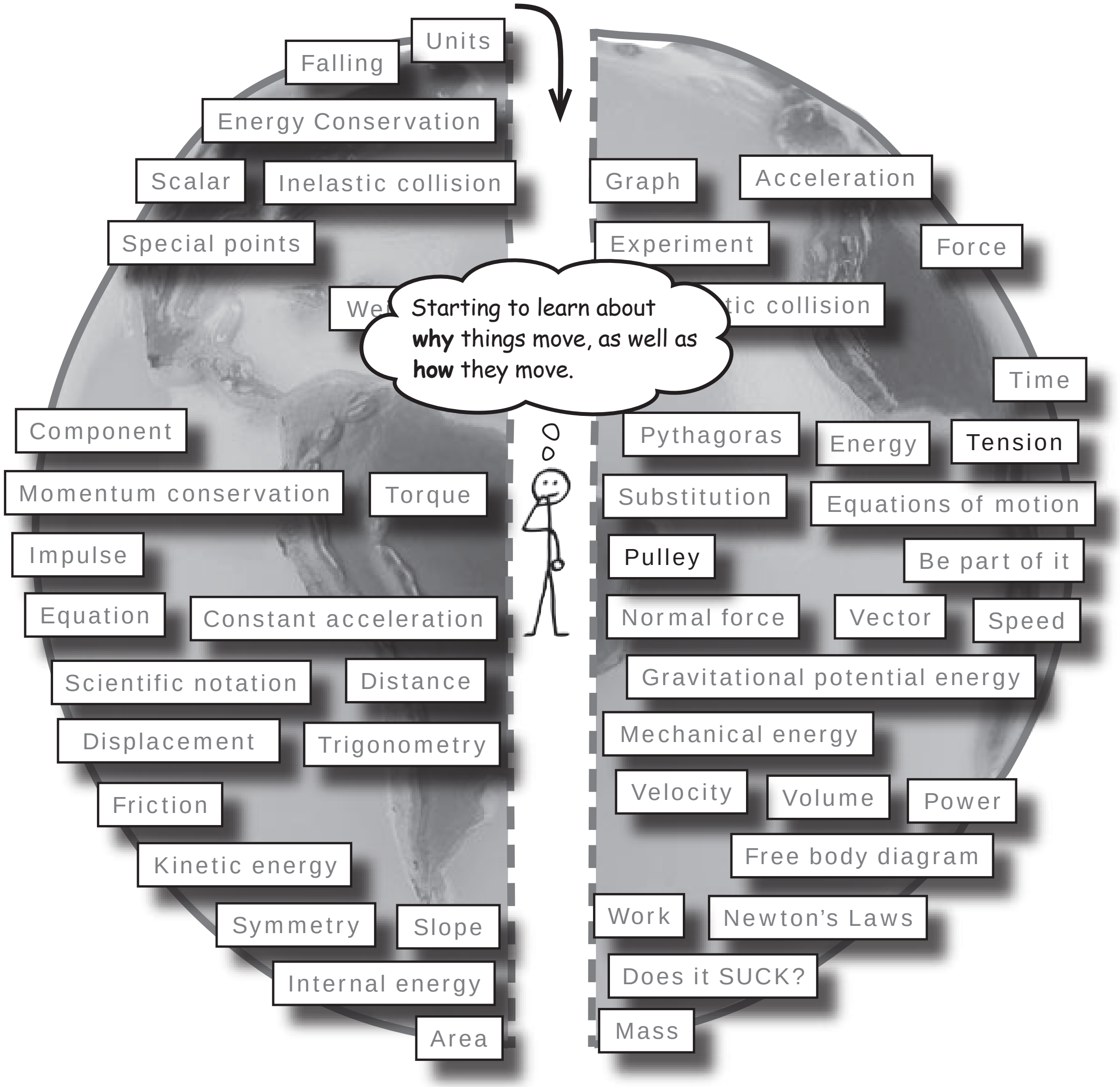
But for the punchbag to be stationary and not to swing, the horizontal components of the tension in each rope must also add up to zero. If you make left the positive direction:

$$\mathbf{T}_1\cos(30°) - \mathbf{T}_2\cos(30°) = 0$$

This gives you two equations to work out two unknowns, $\mathbf{T}_1$ and $\mathbf{T}_2$. When you solve them, the tension in each rope is an enormous 196 N - the same as the tension you'd need to hold up the punchbag using a single vertical rope. As each rope can only cope with a tension of 180N, one of them broke just before the other one would have.

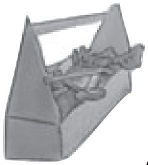
Have a go at solving them yourself!

The high tension is because of the relatively small angle the ropes make with the horizontal - a lot of extra tension is needed to prevent the bag from swinging.



☑ **Tension** — Tension is the force that a rope can mediate when it's pulled tight. The tension force is the same at each end of the rope.

☑ **Pulley** — A pulley allows you to "change" the direction of a force by providing a wheel that a rope can run around and providing a support force to prevent the rope from straightening.



Your Physics Toolbox

You've got Chapter 15 under your belt and added some problem-solving skills to your ever-expanding toolbox.

Break down the problem into parts

When you have a complicated problem, try to break it down into parts.

Then start with the part that looks the easiest for you to solve!

Rope and pulley

If a problem involves a rope and a pulley, look to see if you're asked about forces. If so, you should draw a separate free body diagram for each object attached to the rope.

The size of the tension is the same at each end of the rope.

Both objects must have the same size of acceleration, as they are attached together.

Can you use energy conservation?

If you see a difference in height, you should immediately think about using energy conservation – especially if you're not asked about forces?

This involves less math than using forces – so takes you less time with less chance of slipping up.

Spot the difference

As well as differences in height, look out for differences in speed and work done against friction.

Play "spot the difference" like this before you write down an energy conservation equation to make sure you don't miss anything out.

16 circular motion (part 1)

From a to ω

You say you want a revolution? In this chapter, you'll learn how to deal with **circular motion** with a crash course in **circle** anatomy, including what the **radius** and **circumference** have to do with pies (or should that be πs). After dealing with **frequency** and **period**, you'll need to switch from the **linear** to the **angular**. But once you've learned to use **radians** to measure angles, you'll know it's gonna be alright.

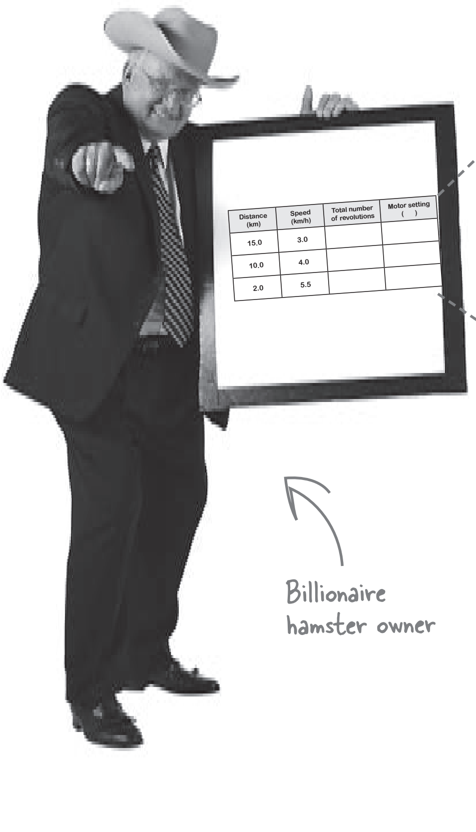
Limber up for the Kentucky Hamster Derby

Universally acclaimed as the most exciting two minutes in a wheel, the Kentucky Hamster Derby is big business! You've been called upon by one of the biggest owners in the business to implement an exacting training programme that's been tailored for the big race.

At the moment, the hamsters aren't following their schedule. Some are slacking off, and others are over-working. It's up to you to make sure that the hamsters train exactly as they should.

Hey kiddo, this Kentucky Hamster Derby is big business, and we gotta get the training schedule absolutely spot on!

Hamsters like to run all night in their wheels. The training schedule is a way of honing the kinds of distances and speeds they typically run at into training for a race.



Distance (km)	Speed (km/h)	Total number of revolutions	Motor setting ( )
15.00	3.00		
10.00	4.00		
2.00	5.50		

Billionaire hamster owner

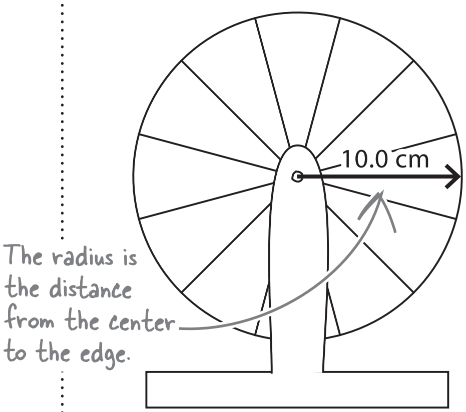
The hamsters are to do three different types of run, depending on which day of their schedule they're on. The hamster owner knows the **distances** he'd like the hamsters to cover and the **speeds** he'd like them to run at.

But he's not great at physics - which is where you come in!

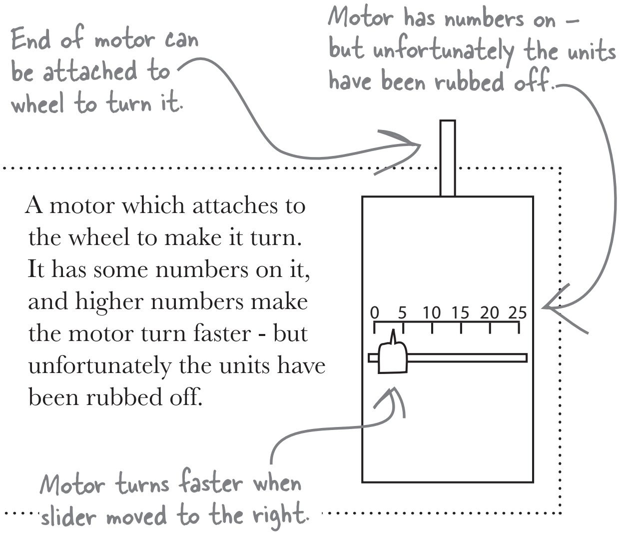
You can revolutionize the hamsters' training

It's time to design the ultimate hamster training tool. The equipment you have to achieve this is:

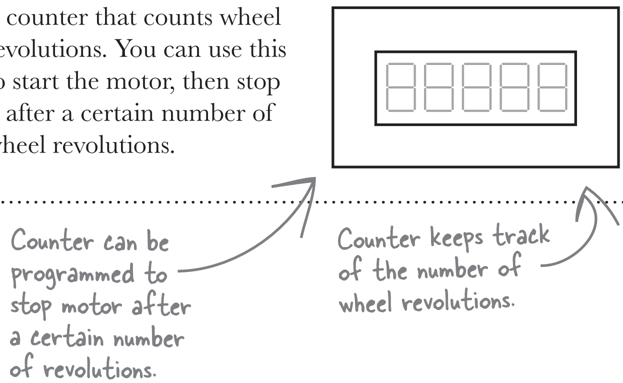
A standard hamster racing wheel with a **radius** of 10.0 cm. This means that it is 10 cm from the center of the wheel to the edge, where the hamster runs.



A motor which attaches to the wheel to make it turn. It has some numbers on it, and higher numbers make the motor turn faster - but unfortunately the units have been rubbed off.



A counter that counts wheel revolutions. You can use this to start the motor, then stop it after a certain number of wheel revolutions.



The schedule involves **linear** distances and speeds, which you'd usually measure along a straight line or by using component vectors.

But the hamster wheel is **circular**, and the counter keeps track of the number of **revolutions**. How are we going to go from the linear schedule to the circular equipment?

Circular motion is different from linear motion.

BRAIN POWER

The schedule is linear, but the equipment is **circular**. What are you going to do next?

Joe: Yeah - it's really important to get this training schedule automated and set up!

Jim: So I guess we use the motor to make the wheel turn at a certain **speed**, and have the timer switch it off when it's covered the right **distance**? That sounds pretty straightforward.

Frank: We already know the wheel's 10.0 cm all the way round ...

Jim: No - the **radius** of the wheel is 10.0 cm, but that's the distance from its center to its edge. We need to know the **distance** the outside of the wheel travels in one **revolution**.

Frank: Yeah, OK, so we need to figure out the distance a hamster runs for each revolution of the wheel. The counter counts the **number** of revolutions the wheel's made. So we work out the number of revolutions required to cover each distance, and we're fine.

Joe: But if we don't know the **circumference** of the wheel, we can't do anything with that.

Frank: What's a circumference?

Joe: The circumference is the special name for the **perimeter** of a circle - the distance all the way round the outside.

Jim: OK, so we need some way of working out the circumference. If we know what distance the hamster covers when the wheel goes round once, we can use that to work out how many **revolutions** the wheel needs to do to cover each distance.

Jim: And if we know that, then we can set the counter to turn the motor off once the wheel's done the correct number of revolutions.

Frank: I guess we need to work out what those numbers on the motor mean as well, so we can get the **speed** right. It's a shame the **units** got rubbed off.

Joe: Yeah, that's a good point.

Jim: Well, speed is distance divided by time, isn't it? So if we get the distance sorted out first, we can think about setting the right speed later.

Frank: Cool, let's go for it!

Circumference is a word that means the 'perimeter' of a circle.

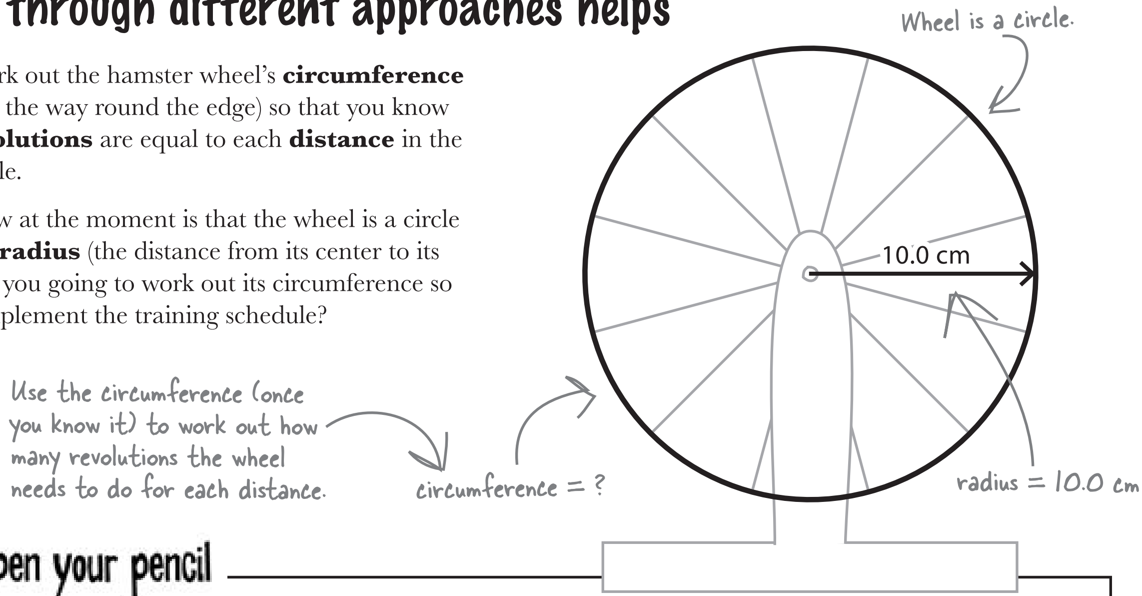
The circumference of a circle is the DISTANCE something at the edge travels in one REVOLUTION.

You can also think of the circumference as the distance around the edge of a stationary circle, but in physics it's more useful to think about it in terms of a rotating circle.

Thinking through different approaches helps

You need to work out the hamster wheel's **circumference** (the distance all the way round the edge) so that you know how many **revolutions** are equal to each **distance** in the training schedule.

But all you know at the moment is that the wheel is a circle with a 10.0 cm **radius** (the distance from its center to its edge). How are you going to work out its circumference so that you can implement the training schedule?



Sharpen your pencil

You have a hamster wheel and want to know the distance that the hamster will cover when the wheel goes round once - i.e., the circumference of the wheel (which has a 10.0 cm radius).

Write down as many methods as you can think of to work this out.

You're **describing** practical methods, not giving a numerical answer.

Sharpen your pencil Solution

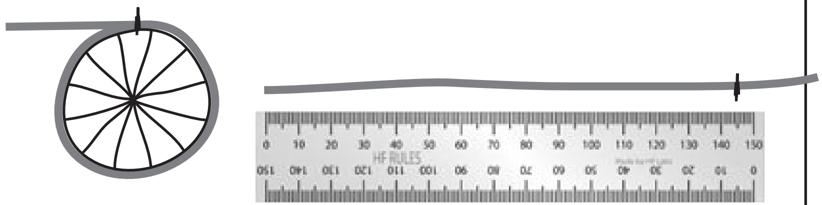
You have a hamster wheel and want to know the distance that the hamster will cover when the wheel goes round once - i.e., the circumference of the wheel.

Write down as many methods as you can think of to work this out.

You're **describing** practical methods, not giving a numerical answer.

Get a piece of string, wrap it once round the wheel.

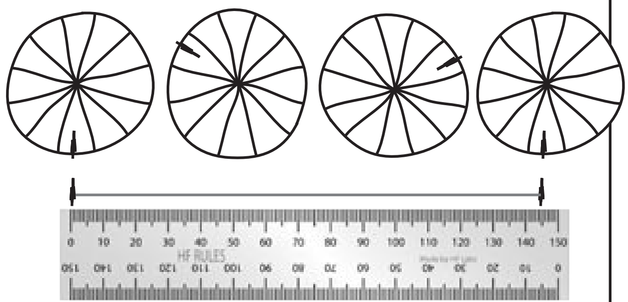
Mark where it comes to, then take the string off the wheel and measure it with a ruler.



Remove the wheel from the stand and make a mark on it. Line up this mark with a mark on the ground.

Then roll the wheel along until the mark is touching the ground again.

Measure the distance between the first mark on the ground and the second mark on the ground.



It'd be a shame to have to do all that again if we decide to use a different wheel, or if we have to deal with other circles in the future. Is there an **equation** we could use instead?

Equations save you time.

You can work out the distance around the hamster wheel by wrapping string around it or rolling it. But what if you need to deal with other circles in the future?

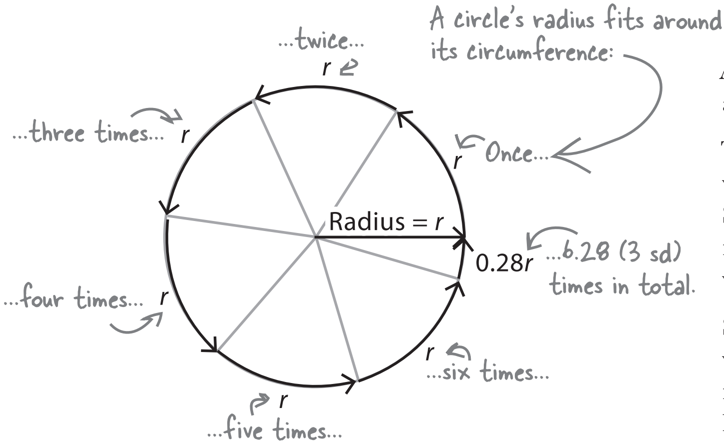
It's definitely easier to use a ruler to measure a **linear** distance (like the radius) than it is to measure a curved distance (like the circumference), so an equation linking the radius and circumference would be useful!

If you have to do a task more than once, try to work out an equation that'll save you time.

A circle's radius and circumference are linked by π

All circles are similar. They're exactly the same shape zoomed in or out. Although they don't have 'sides', you can be sure that **the ratio of a circle's circumference to its radius** will always be the same. (Just like the sides of similar triangles always have the same ratios.)

This is another way of saying that whatever size the circle is, its radius will always fit around its circumference the same number of times.



A circle's radius fits around its circumference approximately 6.28 times (3 sd).

The actual ratio of the lengths is a number with an infinite number of significant digits! So rather than writing "6.28 (3 sd)" as the ratio, there's a special abbreviation for it - 2π, where $\pi = 3.14$ (3 sd).

So you can write down the equation $C = 2\pi r$, where C is the circumference and r is the radius. In other words, if a circle's radius is 1.00 m, its circumference is 6.28 m, and so on.

Circumference → $C = 2\pi r$ ← Radius

$\pi = 3.14$ (3 sd)

But that's just dumb! Why not make π twice as big, i.e. π = 6.28... , so that the ratio is π instead of 2π? If I get to make up a new number, surely it's easier not to have that 2 in there!

π was invented when the diameter was more interesting than the radius.

Mathematicians originally defined π as the ratio between a circle's diameter (the distance across the circle) and its circumference. The diameter is twice the length of the radius, so $\pi = 3.14$ (3 sd) as the diameter fits round the circumference 3.14 (3 sd) times. Once π has been defined, you can't really go changing its value.

In physics, you're often a lot more interested in the radius of a circle than you are in its diameter (for example, torque = radius × force), so the value 2π will often turn up in your equations.

$$\pi = 3.14 \text{ (3 sd)}$$

In physics, the RADIUS is more interesting than the diameter.

For example, torque = radius x force

there are no Dumb Questions

Q: What are the units of π ?

A: π is a ratio of two lengths, the circumference and the diameter. It tells you the NUMBER of times the diameter fits into the circumference - 3.14 (3 sd) times. Length divided by length is dimensionless, so π is a NUMBER and doesn't have any units.

Q: Do I need to remember the value of π for my exam?

A: Knowing that π = 3.14 (3 sd) is useful. If you're doing AP Physics, you won't have a calculator in the multiple choice section, so the approximation π = 3 (1 sd) will help you to choose the option that's in the right ballpark.

Q: Can I use the [π] button on my calculator in the section where I'm allowed a calculator?

A: Yes, that's be fine - though you need to round your answer to an appropriate number of significant digits at the end.

Q: But why is π 3.14 (etc) in the first place, when that makes the ratio of a circle's radius to its circumference 2π?

A: π was originally coined to describe the ratio between a circle's circumference and its **diameter**: $C = \pi d$. Nowadays, the ratio of a circle's circumference to its **radius** is more often used in physics - but the value of π had already been decided.

Q: You said that π has an infinite number of digits? Surely it must end somewhere!

A: Nope! π is an irrational number, which means you can't write it down exactly.

Q: Is that why there's a symbol for it then? To avoid having to write more of it out than you really need to?

A: That's right. You can write the equation: circumference = $2\pi r$ and the '=' sign is true because when you use the symbol π it implies the full irrational number!

Otherwise you'd have to write circumference = $6.28r$ (3 sd) as you'd never be able to write down the exact value of the circumference as an equation.

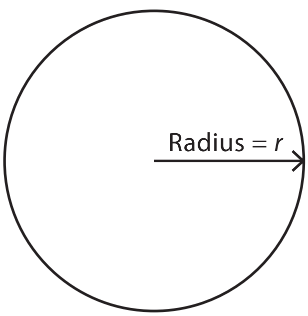
Q: That all sounds a bit philosophical to me. I guess that in practice, I can write down circumference = $2\pi r$ when I'm showing my work, then at the very end I can use the value 3.14 (3 sd) for π when I'm actually putting the numbers in to work out an answer?

A: That's a very good way of thinking about it.

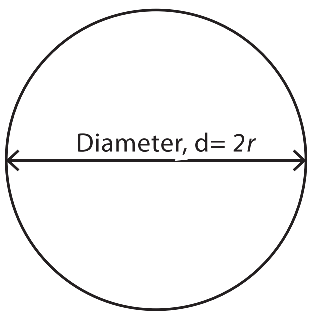
Q: So do I get to use π to design the hamster trainer now?

A: On you go then ...

The ratio of a circle's circumference to its diameter is π. As the radius is half the length of the diameter, the ratio of the circumference to the radius is 2π. So $C = 2\pi r$



$$C = 2\pi r$$

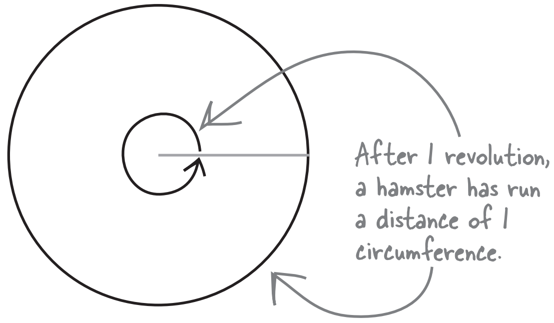


$$C = \pi d$$

π is the ratio of two lengths - it tells you the NUMBER of times the diameter fits into the circumference. Because π is a NUMBER, it has no units.

Convert from linear distance to revolutions

The hamster training schedule contains distances in km. Your job is to implement the schedule using the wheel and a counter that keeps track of the number of **revolutions**. So you need to work out how many circumferences - and therefore how many revolutions - each distance is equivalent to.



Sharpen your pencil

Don't worry about this column yet, you'll fill it in later.

a. Assume you have a hamster wheel, radius *r*. Write down an equation in terms of *r* that gives you the distance that something on the edge of the circle will cover if the circle rotates once.

b. You would like the hamster to cover distance *x*. Write down an equation that gives you *x* in terms of *r*. and the total number of revolutions, *N*.

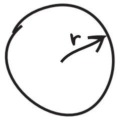
c. Your hamster wheel has a radius of 10.0 cm. Fill in the 'total number of revolutions' column for the distances shown in the hamster training schedule. There's space under the table for you to show your work.

Distance (km)	Speed (km/h)	Total number of revolutions	Motor setting ( )
15.00	3.00		
10.00	4.00		
2.00	5.50		

Sharpen your pencil Solution

Don't worry about this column yet, you'll fill it in later.

a. Assume you have a hamster wheel, radius **r**. Write down an equation in terms of **r** that gives you the distance that something on the edge of the circle will cover if the circle rotates once.



Circumference $C = 2\pi r$

b. You would like the hamster to cover distance *x*. Write down an equation that gives you *x* in terms of *r*. and the total number of revolutions, *N*.

Number of revolutions is the same as the number of circumferences in distance x:

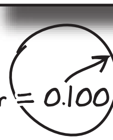
$N = \frac{x}{C}$ and $C = 2\pi r$

$\Rightarrow N = \frac{x}{2\pi r}$

$\Rightarrow \underline{x = 2\pi r N}$

c. Your hamster wheel has a radius of 10.0 cm. Fill in the 'total number of revolutions' column for the distances shown in the hamster training schedule. There's space under the table for you to show your work.

Distance (km)	Speed (km/h)	Total number of revolutions	Motor setting ( )
15.00	3.00	23900 (3 sd)	
10.00	4.00	15900 (3 sd)	
2.00	5.50	3180 (2 sd)	



$r = 0.100$ m

If you have a mixture of distance units, it's usually safest to convert everything to meters.

$x = 2\pi r N \Rightarrow N = \frac{x}{2\pi r}$

15.00 km: $N = \frac{15.00 \times 10^3}{2 \times \pi \times 0.100}$

$\underline{N = 23900 \text{ revolutions}}$ (3 sd)

10.00 km: $N = \frac{10.00 \times 10^3}{2 \times \pi \times 0.100}$

$\underline{N = 15900 \text{ revolutions}}$ (3 sd)

2.00 km: $N = \frac{2.00 \times 10^3}{2 \times \pi \times 0.100}$

$\underline{N = 3180 \text{ revolutions}}$ (3 sd)

How do you get units of revolutions from a distance divided by a distance? Doesn't that fly in the face of everything we've said about units up until now?

Your answer is a NUMBER of revolutions.

To work out the number of wheel revolutions in 15 km, you divide the total distance (15000 m) by the circumference of the wheel ($2\pi r$ = 0.628 m). This gives you an answer of 23900 - and there are no units, as a length divided by a length is dimensionless.

So be careful if you're checking the units of an equation or answer. A quantity that represents a **number** (of things, for example number of revolutions) doesn't have units!

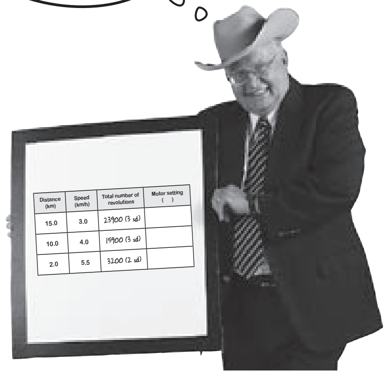
The units divide out and cancel.

$\frac{15000 \text{ m}}{0.628 \text{ m}} = 23900 \text{ (3 sd)}$

The final answer is a NUMBER with no units.

Convert the linear speeds into Hertz

The distances are cool - but the speeds are really important too. I think the motor might be in Hertz ... can you help?



The distances slot into the training schedule brilliantly - but the **speed** the hamsters train at for each session is also vital. You have a motor you can use to turn the wheel ... but the units have been rubbed off. However, the hamster owner thinks that the motor might be marked in **Hertz**.

Hertz is always capitalized, and can be abbreviated to Hz.

Hertz is a unit of **frequency** that describes the number of times **per second** something regular happens. In physics, this is often referred to as the number of cycles per second.

So if the wheel goes round 5 times per second, you can say that it has a frequency, f, of 5 Hz.

The **period** of the wheel is the **time** the wheel takes to do one rotation, and is given the symbol T.

You can calculate the period from the frequency. If something happens 5 times per second (so has a frequency of 5 Hz), then it happens 5 times in 1 second. So its period must be $T = \frac{1}{5} = 0.2$ s.

Frequency

$$f = \frac{1}{T}$$

Period

Period

$$T = \frac{1}{f}$$

Frequency

Sharpen your pencil

Distance (km)	Speed (km/h)	Total number of revolutions	Motor frequency (Hz)
15.00	3.00	23900 (3 sd)	
10.00	4.00	15900 (3 sd)	
2.00	5.50	3180 (3 sd)	

a. What time, in seconds, does it take the hamsters to cover 15.00 km at 3.00 km/h?

b. You already worked out that this training plan involves the wheel doing a total number of 23900 revolutions. Calculate the period, T, and hence the frequency, f, of the wheel, and fill in the table.

c. Do similar calculations to fill in the rest of the table.

The period, T, is the time for one rotation of the wheel.

Sharpen your pencil Solution

Distance (km)	Speed (km/h)	Total number of revolutions	Motor frequency (Hz)
15.00	3.00	23900 (3 sd)	1.33 (3 sd)
10.00	4.00	15900 (3 sd)	1.77 (3 sd)
2.00	5.50	3180 (3 sd)	2.43 (3 sd)

a. What time, in seconds, does it take the hamsters to cover 15.00 km at 3.00 km/h?

$$\text{speed} = \frac{\text{distance}}{\text{time}} \Rightarrow \text{time} = \frac{\text{distance}}{\text{speed}}$$

$$\text{time} = \frac{15.00 \text{ km}}{3.00 \text{ km/hr}} = 5 \text{ hr}$$

$$5 \text{ hr} = 5 \text{ hr} \times \frac{60 \text{ min}}{1 \text{ hr}} \times \frac{60 \text{ s}}{1 \text{ min}} = 18000 \text{ s (3 sd)}$$

b. You already worked out that this training plan involves the wheel doing a total number of 23900 revolutions. Calculate the period, *T*, and hence the frequency, *f*, of the wheel, and fill in the table.

Period is time for 1 revolution.

23900 revolutions take 18000 s.

$$\Rightarrow T = \frac{18000}{23900} = 0.753 \text{ s (3 sd)}$$

$$f = \frac{1}{T} = \frac{1}{0.753} = \underline{1.33 \text{ Hz}} \text{ (3 sd)}$$

c. Do similar calculations to fill in the rest of the table.

10.00 km at 4.00 km/h takes $\frac{10.00}{4.00} \times 60 \times 60 = 9000$ s.

$$T = \frac{9000}{15900} \qquad f = \frac{1}{T} = \frac{15900}{9000} = \underline{1.77 \text{ Hz}} \text{ (3 sd)}$$

2.00 km at 5.50 km/h takes $\frac{2.00}{5.50} \times 60 \times 60 = 1310$ s.

$$T = \frac{1310}{3180} \qquad f = \frac{1}{T} = \frac{3180}{1310} = \underline{2.43 \text{ Hz}} \text{ (3 sd)}$$

This is a shortcut that stops you having to calculate a value for T before you calculate a value for f.

there are no Dumb Questions

Q: What's the difference between the frequency and the period?

A: The frequency is the number of cycles per second - the number of times a regular thing happens per second.

The period is the number of seconds per cycle - the number of seconds that it takes for a regular thing to happen once.

Q: Why is frequency measured in Hz instead of however you'd write "per second" as a unit?

A: "Hz" is easier to write than "1/s" (which is the correct format) and less open to being misinterpreted.

Q: I've seen frequency written in units of s^{-1} in other books. Is that OK?

A: Yes, that's fine, and is another way of representing units using scientific notation. For example, other books may say ms^{-2} instead of m/s^2. As long as you use what you're comfortable with, you'll be fine.

Q: Why aren't the units of frequency written as cycles/s? Why just "1/s"?

A: Good question. The thing to remember is that frequency is the **number** (of things) per second. It could be the number of revolutions, or the number of waves that break on the beach or the number of times a dog barks. A number is dimensionless so has no units.

Q: What if the regular thing happens less than once per second?

A: Then the frequency would be smaller than 1. For example, something that happens every 5 s has a frequency of 0.2 Hz.

Q: Why does $f = \frac{1}{T}$ and $T = \frac{1}{f}$?

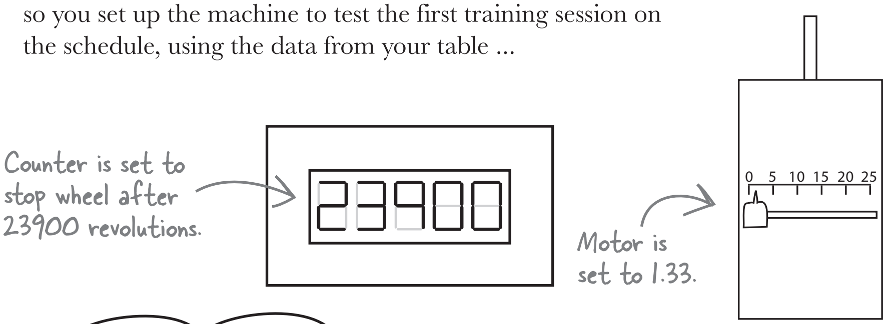
A: Frequency is cycles per second; period is seconds per cycle. "Per" means "divided by", so frequency and period are the "opposite way up" from each other.

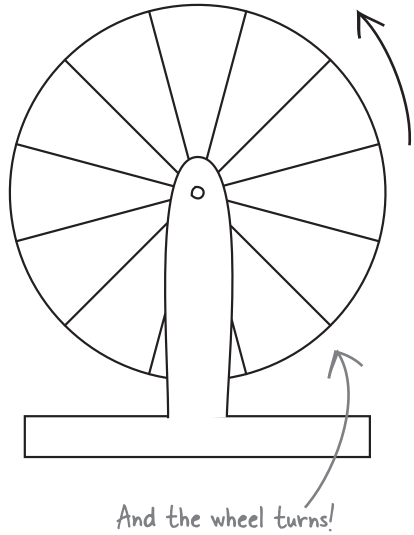
For example, if something happens 5 times per second, it has a frequency of 5 Hz. Another way of putting it is that the thing happens every 0.2 s because $T = \frac{1}{f} = 0.2$ s.

So you set up the machine ...

You've used the fact that the **circumference** of a circle $C = 2\pi r$ to convert the units in the training schedule (km and km/h) to units that are much easier for you to work with - **number of revolutions** and **Hertz**.

The motor has numbers on its dial, but the units have rubbed off. The hamster trainer is sure they must be Hertz, so you set up the machine to test the first training session on the schedule, using the data from your table ...





... but the wheel turns too slowly!

When the billionaire hamster owner comes to inspect your work, there's a problem. He says that the wheel's turning far **too slowly** - something's gone wrong and you need to fix it quickly if you're going to stand any chance in the race!

Sharpen your pencil

a. What do you think might have gone wrong?

b. How would you go about investigating and fixing it?

Sharpen your pencil Solution

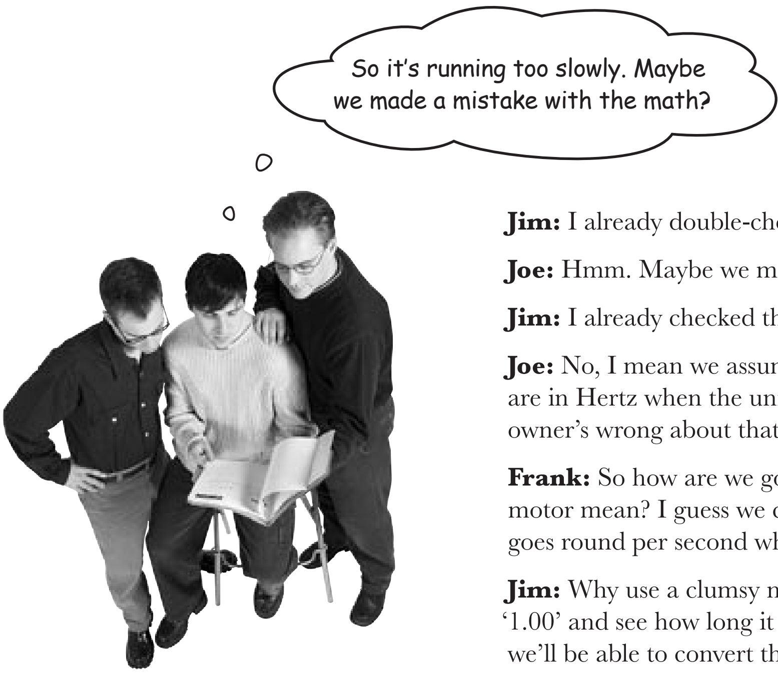
a. What do you think might have gone wrong?

Maybe the owner got the units of the motor wrong, and they're something else instead of Hz.

b. How would you go about investigating and fixing it?

Do some tests or experiments with the motor to count the number of revolutions it does in a certain time when set to a certain number, to try to work out what its units are and how to convert them to what we already know about.

Always make sure you know the UNITS of everything you're dealing with!



Jim: I already double-checked the math - and it's fine.

Joe: Hmm. Maybe we made a mistake with some **units**.

Jim: I already checked the units - they all work out.

Joe: No, I mean we assumed that the numbers written on the motor are in Hertz when the units had rubbed off. But what if the hamster owner's wrong about that?

Frank: So how are we going to work out what the numbers on the motor mean? I guess we could count how many times it actually goes round per second when it's set to 1.33 like it is now?

Jim: Why use a clumsy number like 1.33? Why don't we set it to '1.00' and see how long it takes to do 1 revolution like that. Then we'll be able to convert the frequency in Hertz we worked out to whatever units the numbers are in.

Joe: Yeah, '1' is an easier number to work with. I guess we could also move the slider to try and make the wheel go round exactly once per second. Then we have two values to work with.

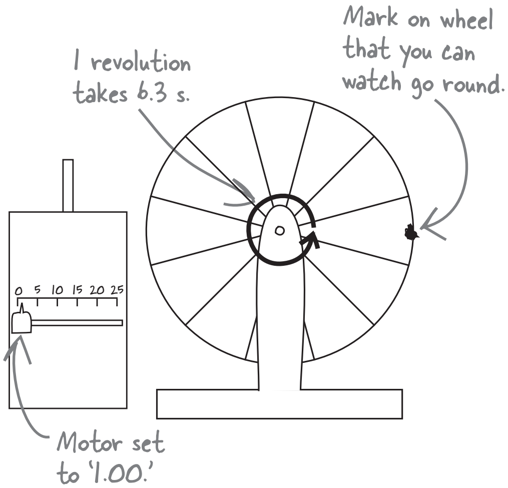
Try some numbers to work out how things relate to each other

We originally assumed that the numbers on the motor represent revolutions per second (i.e. Hz) - but the wheel's going too slowly for that. What you need to do is try some 'nice' values on the control, and count how many revolutions per second they correspond to.

If you want to try things out, choose 'nice' numbers, like 1 or 10, to play with.

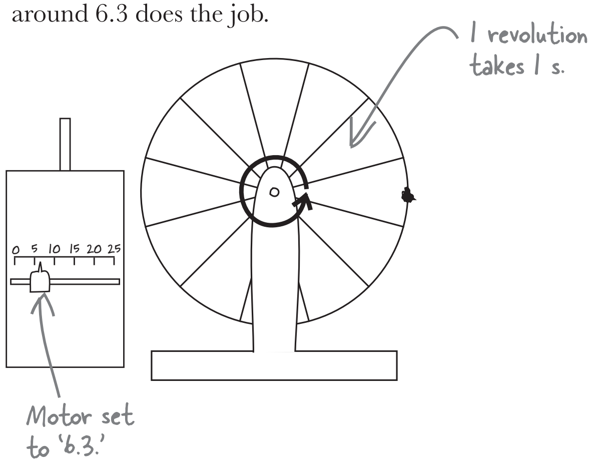
What if you set the motor to '1.0'?

When you set the control on the motor to 1.0, the wheel takes around 6.3 seconds to complete exactly 1 revolution.



What if the wheel goes at 1.0 Hz?

Then you try to find the value on the motor where the wheel actually goes around once per second, i.e., with a frequency of 1.0 Hz. After playing a bit, you find that a setting of around 6.3 does the job.



Sharpen your pencil

With the motor set at 1.0, 1.0 revolution takes 6.3 s. And with the motor set at around 6.3, 1.0 revolution takes 1.0 s.

Have you seen a number close to 6.3 somewhere else recently? Look back through this chapter if you can't remember.

What do you think the numbers on the motor might represent?

Sharpen your pencil Solution

With the motor set at 1.0, 1.0 revolution takes 6.3 s. And with the motor set at around 6.3, 1.0 revolution takes 1.0 s.

Have you seen a number close to 6.3 somewhere else recently? Look back through this chapter if you can't remember.

What do you think the numbers on the motor might represent?

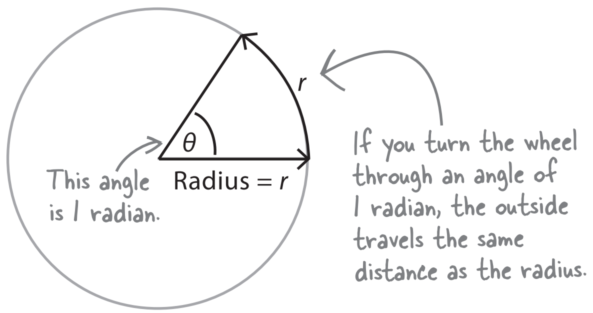
The circumference of a circle is 2π – or 6.28 – times larger than the radius. So 1 revolution is around 6.3 radii. If you set the motor to 1, it takes about 6.3 seconds to do one revolution (i.e. 6.3 radii) And if you set the motor to 6.3, it does 1 revolution (i.e. 6.3 radii) in a second.

So maybe the units on the motor tell you how many radii per second the outside of the wheel does?

The units on the motor are radians per second

You've spotted that there are around 6.3 (2 sd) 'radii' in a circumference. When the motor is set to 6.3, the wheel goes at 1 revolution per second (i.e. 6.3 'radii' per second).

The numbers on the motor should have units of **radians**. A radian is a unit used to measure **angles**. If you rotate the wheel through an angle of 1 radian, the outside of the wheel travels a distance equal to the radius of the wheel.



Radians are a way of measuring angles.

There are 2π radians in one complete revolution.

Radians are very helpful if you're trying to connect the **angle** a wheel's turned through and the **distance** the edge of the wheel has covered.

The edge of a wheel that's turned through an angle of 1 radian has covered a distance the length of the wheel's radius, r. If you turn the wheel an angle of 2 radians, the edge covers a distance of $2r$. If you turn the wheel by 2.4 radians, the edge covers a distance of $2.4r$. You **multiply the angle by the radius to get the distance**.

If you rotate the wheel through an angle of 2π radians, the outside of the wheel travels a distance of $2\pi r$, as the circumference of the circle $= 2\pi r$. Therefore, there are 2π radians in one revolution.

This fits with what you learned by experimenting with the motor. When the motor is set to 6.3 (2π) radians per second, the wheel goes round at a rate of 1 revolution per second.

Radians are particularly useful when you're dealing with circles.

Degrees are only 'familiar' because you're already used to dealing with them. As you practice with radians, they'll become a whole lot more familiar.

Although you'll still use degrees when working with triangles, radians are particularly useful when you're dealing with circles, as you can move between angles and distances more quickly using the equation $x = r\theta$, where x is the distance covered round the edge of the circle as you rotate through angle θ.

$$x = r\boldsymbol{\theta}$$

Linear distance. (→ x)
Radius (→ r)
Angle in radians (→ θ)
This is a scalar equation

Exercise

When you're working with circles, radians help you to move between angles and distances quickly. Here's your chance to practice working with radians and compare it with working with degrees before you go on to do the 'mission-critical' bit with the hamster wheel.

Working with degrees

A hamster wheel with a radius of 0.100 m turns through an angle of 60°. What distance does the outside of the wheel cover?

Hint: What proportion of the circle's circumference is swept out by an angle of 60°?

Working with radians

A hamster wheel with a radius of 0.100 m turns through an angle of $\frac{\pi}{3}$ radians. What distance does the outside of the wheel cover?

Hint: Use the equation $x = r\theta$ where x is the distance covered for the angle θ.

Compare the math you've had to do to calculate a distance from an angle using degrees and using radians.

When you're working with circles, radians help you to move between angles and distances quickly. Here's your chance to practice working with radians and compare it with working with degrees before you go on to do the 'mission-critical' bit with the hamster wheel.

Working with degrees

A hamster wheel with a radius of 0.100 m turns through an angle of 60°. What distance does the outside of the wheel cover?

Angle of 60° is $\frac{60}{360}$ or $\frac{1}{6}$ of the circle.

Total circumference $= 2\pi r$

$\Rightarrow$ Distance covered $= \frac{1}{6} \times 2\pi r$

$= \frac{1}{6} \times 2 \times \pi \times 0.100$

Distance covered $=$ **0.105 m** (3 sd)

Working with radians

A hamster wheel with a radius of 0.100 m turns through an angle of $\frac{\pi}{3}$ radians. What distance does the outside of the wheel cover?

$x = r\theta$

$x = 0.100 \times \frac{\pi}{3}$

$x =$ **0.105 m** (3 sd)

Compare the math you've had to do to calculate a distance from an angle using degrees and using radians.

The final step of the math is more or less the same both times. But it takes much longer to get there from an angle in degrees than it does from an angle in radians.

there are no Dumb Questions

Q: Why would I want to use radians to measure angles?

A: If you're working with circles, radians simplify calculations greatly. In degrees, you first of all have to work out what fraction of the circle the angle is (for example, 60° is a sixth of a full circle), then multiply that by $2\pi r$ to get the distance. But in radians, this has already been done, and you only have to multiply the angle by the radius to get the distance.

Q: Will I be using radians for the rest of this book then?

A: For anything involving circles and regular motion, you'll be using radians. But you'll still be using degrees for triangles.

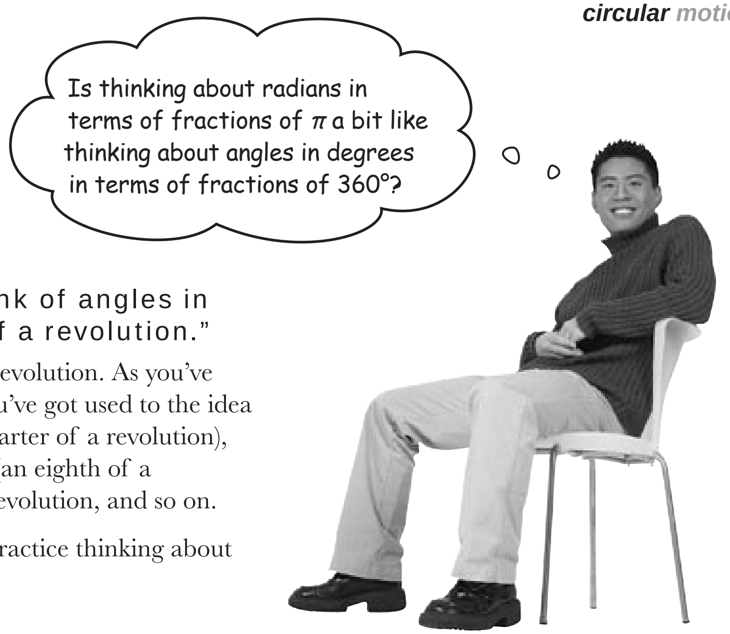
Q: How can I get used to using radians when they look so weird!

A: The big thing to remember is that there are 2π radians in one complete rotation. So there are π radians in half a rotation, $\frac{\pi}{2}$ radians in quarter of a rotation, $\frac{\pi}{3}$ radians in a sixth of a rotation, etc.

Q: Aren't fractions of π very awkward to work with, as π is an irrational number?

A: As long as you keep using fractions, like $\frac{\pi}{3}$ and $\frac{\pi}{2}$, then only put in the value of π at the end of a calculation, the fractions aren't difficult to work with.

Think of angles in radians in terms of fractions of π.



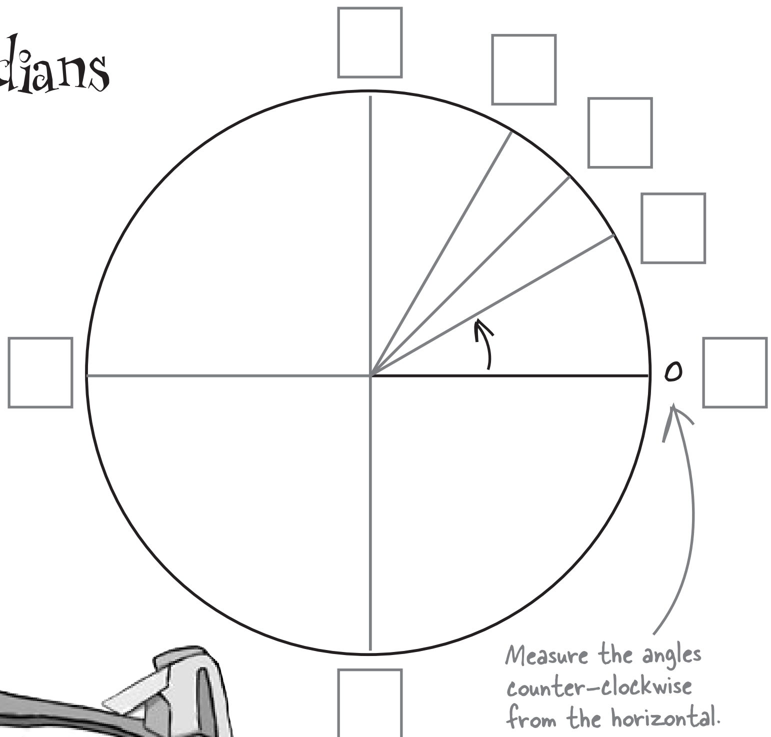
Yes - it's good to think of angles in terms of "fraction of a revolution."

There are 360° in one full revolution. As you've practiced using degrees, you've got used to the idea that 90° is a right angle (quarter of a revolution), 45° is half of a right angle (an eighth of a revolution), 180° is half a revolution, and so on.

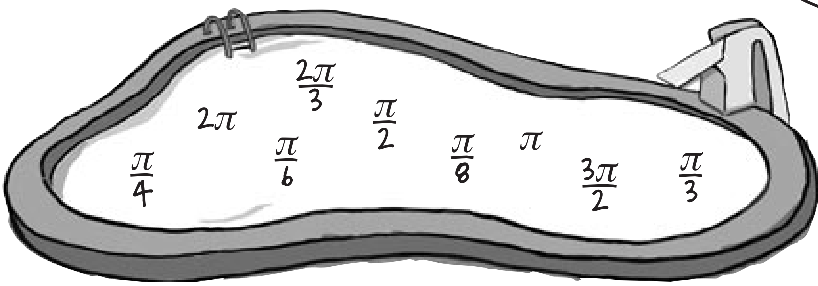
Here's the opportunity to practice thinking about radians in a similar way:

Pool Puzzle - Radians

Your **job** is to take the angles measured in radians from the pool and place them into the boxes around the circle to indicate the size of each angle. You may **not** use the same angle more than once, and you won't need to use all the angles. Your **goal** is to become more familiar with using radians to measure angles.

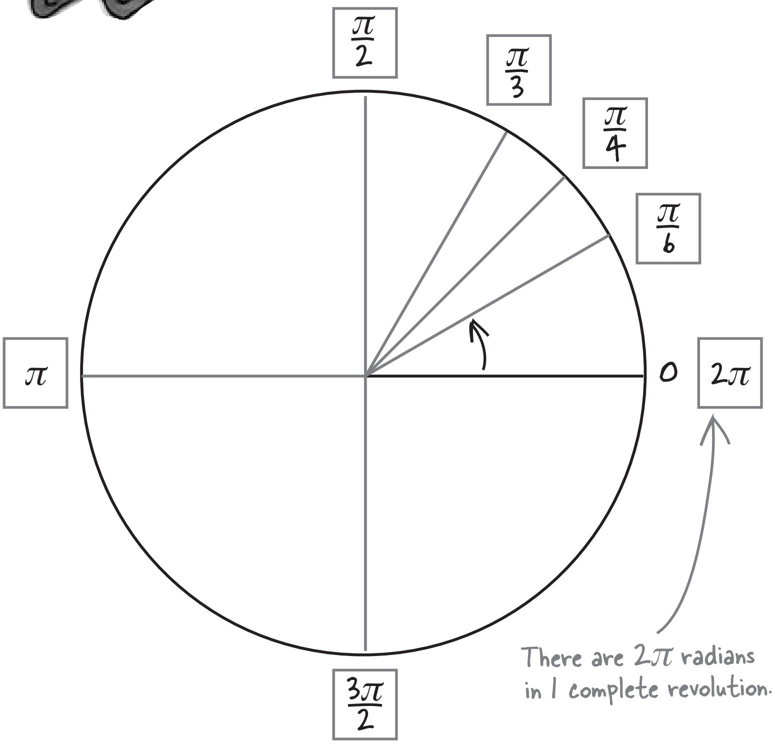


Note: each angle from the pool can only be used once!

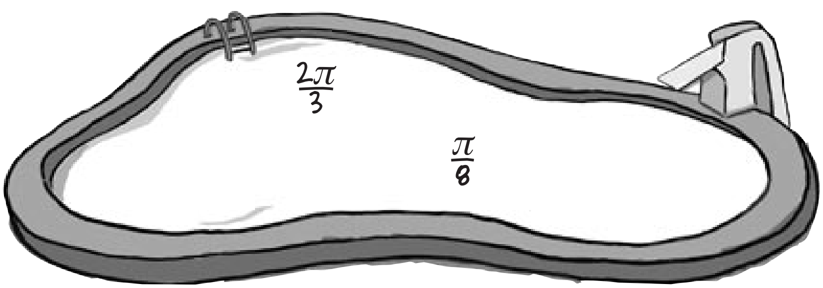


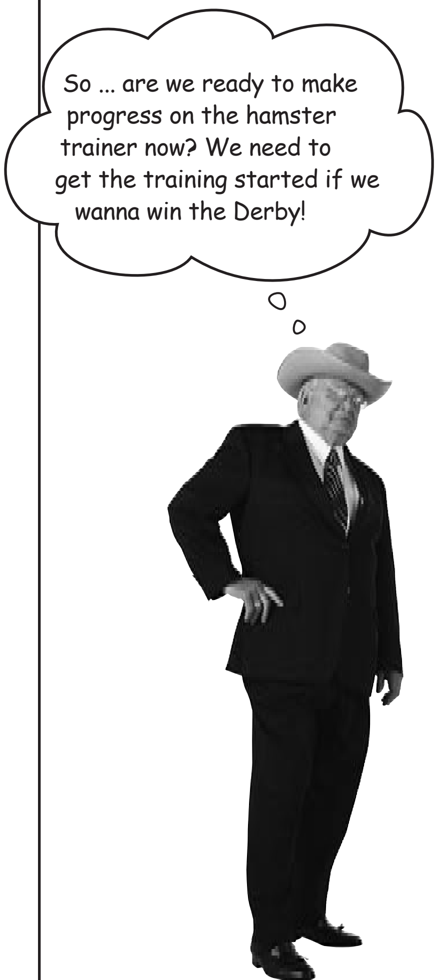
Pool Puzzle - Radians - SOLUTION

Your **job** is to take the angles measured in radians from the pool and place them into the boxes around the circle to indicate the size of each angle. You may **not** use the same angle more than once, and you won't need to use all the angles. Your **goal** is to become more familiar with using radians to measure angles.



Note: each angle from the pool can only be used once!





Convert frequency to angular frequency

You've worked out that the units on the motor are radians per second. This is known as the **angular frequency**, and is given the symbol ω (pronounced "omega"). So if you set the motor to 1.0, it will turn at a rate of 1.0 radian per second.

There are 2π radians in one complete revolution. You already know the frequency, f, that the wheel needs to be set at for each run - this is the number of revolutions per second that the wheel does.

Angular frequency

$$\omega = 2\pi f$$

Frequency

The angular frequency is the number of radians per second.

As there are 2π radians in 1 complete revolution, you can use the equation $\omega = 2\pi f$ to calculate the angular frequency that the motor needs to turn at for each of the hamsters' training sessions.

Sharpen your pencil

Complete the table by calculating the angular frequency that the wheel needs to turn at for each training session. There's space for your work under the table.

Distance (km)	Speed (km/h)	Total number of revolutions	Frequency (Hz)	Angular frequency (rad/s)
15.00	3.00	23900 (3 sd)	1.33 (3 sd)	
10.00	4.00	15900 (3 sd)	1.77 (3 sd)	
2.00	5.50	3180 (3 sd)	2.43 (3 sd)	

Sharpen your pencil Solution

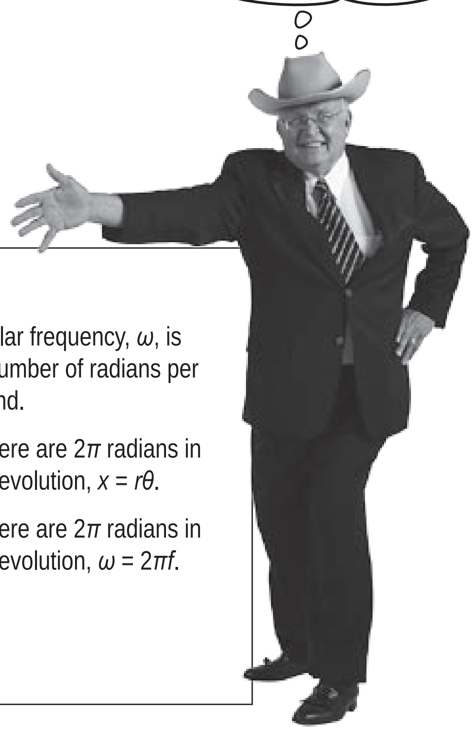
Complete the table by calculating the angular frequency that the wheel needs to turn at for each training session. There's space for your work under the table.

Distance (km)	Speed (km/h)	Total number of revolutions	Frequency (Hz)	Angular frequency (rad/s)
15.00	3.00	23900 (3 sd)	1.33 (3 sd)	8.35 (3 sd)
10.00	4.00	15900 (3 sd)	1.77 (3 sd)	11.3 (3 sd)
2.00	5.50	3180 (3 sd)	2.43 (3 sd)	15.3 (3 sd)

$f = 1.33$ Hz : $\omega = 2\pi f = 2 \times \pi \times 1.33 = \underline{\underline{8.35 \text{ rad/s}}}$ (3 sd)

$f = 1.77$ Hz : $\omega = 2\pi f = 2 \times \pi \times 1.77 = \underline{\underline{11.1 \text{ rad/s}}}$ (3 sd)

$f = 2.43$ Hz : $\omega = 2\pi f = 2 \times \pi \times 2.43 = \underline{\underline{15.3 \text{ rad/s}}}$ (3 sd)



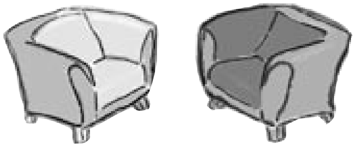
The hamster trainer is complete!

When you put the finishing touches to your hamster trainer by calculating the angular frequency, it's brilliant! After some hearty congratulations from the owner, you find yourself with a 20% stake in the thoroughbred hamster that's favorite to win next month's Kentucky Hamster Derby!

BULLET POINTS

- A circle's circumference is 2π times the size of its radius, $C = 2\pi r$.
- Frequency, f, is measured in cycles per second, or Hertz (Hz).
- Period, T, is the time it takes for one regular thing to happen. Think of it as "seconds per cycle."
- Radians are a way of measuring angles that are especially useful for doing calculations with circles. There are 2π radians in one revolution.
- It can sometimes be helpful to think about radians in terms of fractions of π.
- Angular frequency, ω, is the number of radians per second.
- As there are 2π radians in one revolution, $x = r\theta$.
- As there are 2π radians in one revolution, $\omega = 2\pi f$.

Fireside Chats



Tonight's talk: **Degree and Radian have a barney about who is better at measuring angles.**

Degree: Well, hello radian. You're pretty late to the party, aren't you?!

Radian: I wouldn't quite say that. Sure, you've been around for a bit longer than I have. But that doesn't mean you're inherently better - just over the hill!

Degree: But I'm so easy to understand! None of those messy pies (or should I say πs) - just 360 degrees of goodness in every complete revolution. How much more straightforward can you get?

Radian: OK, so I admit that the πs might take a little bit of getting used to. But once you've got your head around them, you have to admit that I have far greater superpowers than you do!

Degree: Oh yeah?! Such as ... ?

Radian: Well, it's really easy to calculate **distances** from **angles** when something's going round in a circle using me. Suppose a wheel goes round an angle of θ radians, and they want to know the distance the has wheel gone? No problem! Just multiply the angle by the distance you are from the center of the circle (the radius, r) to get the distance, $r\theta$.

Degree: They can do that with me as well though! Suppose a wheel goes round 180°. They can work out that this is half the circle, as 180/360 = 0.5. And they know what the circumference of the circle is - $2\pi r$. So they just multiply those together - $0.5 \times 2\pi r$ to get πr which is the distance the wheel's gone. Sorted!

Radian: Um ... you mean that if you go round π radians (i.e. half a circle) you've travelled a distance of $r\theta = \pi r$ I'm sorry, but my way is just a lot lot quicker!

Degree: But your way is sooo anti-intuitive!

Radian: I think you really mean that my way is much better once you've got used to it!

Degree: Whaaaaatever!

A couple of weeks later ...

The hamsters are ready to move into the sprint training phase of their program. The owner explains that he wants the hamsters to be able to run at 3.00 m/s - and that he also wants an easy way to convert from speeds in m/s to “the numbers on that motor”.

This means that you need to come up with a way of converting between meters per second (**linear speed**) to radians per second (**angular frequency**).

When you did this before, it involved several steps. First of all, you worked out the total number of revolutions for a training run, and the total time the run took. From these figures, you were able to calculate the frequency (revolutions per second) and from there the angular frequency (radians per second).

Sharpen your pencil Solution

Complete the table by calculating the angular frequency that the wheel needs to turn at for each training session. There’s space for your work under the table.

Distance (km)	Speed (km/h)	Total number of revolutions	Frequency (Hz)	Angular frequency (rad/s)
15.00	3.00	23900 (3 sd)	1.33 (3 sd)	8.35 (3 sd)
10.00	4.00	15900 (3 sd)	1.77 (3 sd)	11.1 (3 sd)
2.00	5.50	3180 (3 sd)	2.43 (3 sd)	15.3 (3 sd)

That’s a lot of steps!

But back then, you didn’t know that the numbers on the motor were radians per second - you didn’t know that’s what you were aiming at. Now you know that you’re aiming at radians per second ...

Jim: Yeah - but that took us a loooong time before!

Joe: I've noticed that both the things we're supposed to be working with are "per second". Meters per second and radians per second.

Frank: So if we can convert from meters to radians, we might be able to do it more quickly than we did before.

Jim: We can already go between meters and radians! $x = r\theta$.

Frank: Hmm. So multiplying the angle (in radians) by the radius gives you the distance in meters. $x = r\theta$.

Jim: I wonder if that means multiplying radians *per second* by the radius will give us meters *per second*.

Joe: Yeah ... maybe we can say that $v = r\omega$?

Jim: How do the units check out? If you multiply radians by the radius, you get meters. So if you multiply radians per second by the radius then you get distance per second.

Frank: And "distance per second" is just another way of saying velocity. Brilliant!

- Angular frequency, ω, is the number of radians per second.
- As there are 2π radians in one revolution, $x = r\theta$.

These are some of the things from earlier in the chapter that the guys are talking about.

Sharpen your pencil

a. Your hamster wheel has a radius of 10.0 cm. Use the equation $v = r\omega$ to calculate the angular frequency it would need to turn with to enable the hamster to run with a speed of 3.00 m/s.

b. Your motor can spin at any angular frequency up to 25.0 rad/s. How might you build a speed trainer capable of making a hamster run at 3.00 m/s if you don't have a faster motor available?

Sharpen your pencil Solution

a. Your hamster wheel has a radius of 10.0 cm. Use the equation $v = r\omega$ to calculate the angular frequency it would need to turn with to enable the hamster to run with a speed of 3.00 m/s.

$$v = r\omega$$

$$\Rightarrow \omega = \frac{v}{r} = \frac{3.00}{0.100}$$

$$\omega = \underline{\underline{30.0 \text{ rad/s}}} \text{ (3 sd)}$$

b. Your motor can spin at any angular frequency up to 25.0 rad/s. How might you build a speed trainer capable of making a hamster run at 3.00 m/s if you don't have a faster motor available?

$v = r\omega$. The speed depends on both the radius and the angular frequency.

If it's not possible to increase the angular frequency high enough, another way to get a higher speed is to increase the radius. So you could use a larger wheel with the same motor.

there are no Dumb Questions

Q: So radians help you to go straight from distance to angle, or from speed to angular frequency?

A: Yes - you can use the equations $x = r\theta$ and $v = r\omega$ to move quickly between linear and angular quantities.

Q: What if I want to know the speed of a point that's on the wheel, but not at the edge?

A: When the wheel spins, that point will 'sketch out' a circle with a radius the same as the distance that the point is from the center.

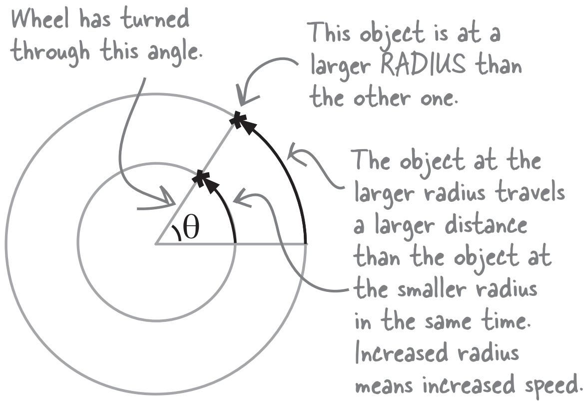
Q: Is it OK to use that value for the radius, even if it's not the radius of the wheel?

A: Yes - in this context, the radius is the distance from the center of the wheel to the point you're interested in.

Q: Will $x = r\theta$ and $v = r\omega$ only work if θ and ω are in terms of radians not degrees?

A: Correct. If θ is in degrees, then you need to work out what proportion of the circle's circumference is included by dividing the angle by 360°, then multiplying by 2π to get the distance the point's travelled. With radians, that's already been done for you.

You can increase the (linear) speed by increasing the wheel's radius



If you have a wheel turning at angular frequency ω, the whole wheel turns through the same **angle** in the same time. But parts of the wheel that are a greater **radius** from the center travel a greater **distance** in the same time, so have a greater **speed**, v.

This means that you can increase the maximum speed that your hamster trainer can cope with by increasing the radius of your wheel - without needing to change the angular frequency that it turns with.

Here, we're written v as a scalar to indicate that it's the size of the object's velocity. The direction of the velocity changes all the time as the wheel rotates.

I guess that ω's also kinda like an angular speed? Because it tells you the number of radians per second - like speed tells you the number of meters per second?

Multiply the linear quantity by the radius to get the equivalent angular quantity.

Yes - ω is sometimes called the angular speed.

ω is also called the angular speed, or sometimes the angular velocity, as it's the "angular equivalent" of the linear velocity.

Just as x and θ are connected by the equation $x = r\theta$, v and ω are connected by the equation $v = r\omega$.

So, because of how radians work, if you have an angular quantity (like θ or ω) you can multiply it by r to get to its linear equivalent.

$$x = r\boldsymbol{\theta}$$

Linear distance (meters). Radius (meters). Angle (radians) – "angular distance"

$$v = r\boldsymbol{\omega}$$

Linear speed (meters per second.) Radius (meters). Angular speed (radians per second) – also called "angular frequency"

Sharpen your pencil

You have a motor capable of turning at 25.0 rad/s, and wish to train a hamster that runs at 3.00 m/s.

What radius of wheel do you need to use?

Sharpen your pencil Solution

You have a motor capable of turning at 25.0 rad/s, and wish to train a hamster that runs at 3.00 m/s.

What radius of wheel do you need to use?

$v = r\omega$

$\Rightarrow r = \frac{v}{\omega} = \frac{3.00}{25.0}$

$r = \underline{\underline{0.120}}$ m (3 sd)

Linear quantity. Radius.

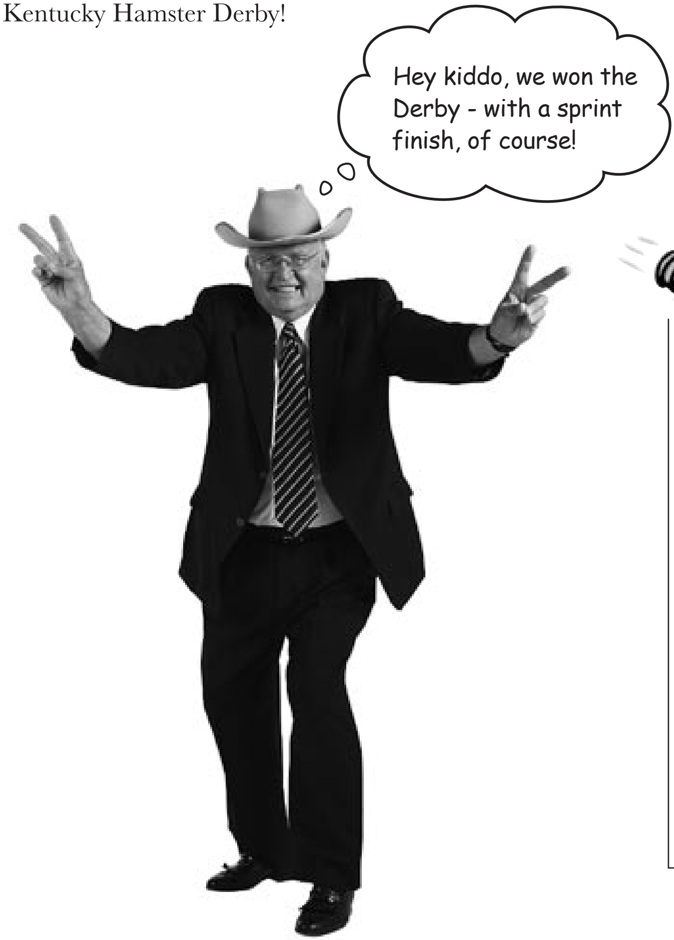
$\mathbf{x = r\theta}$

$\mathbf{v = r\omega}$

Angular quantity.

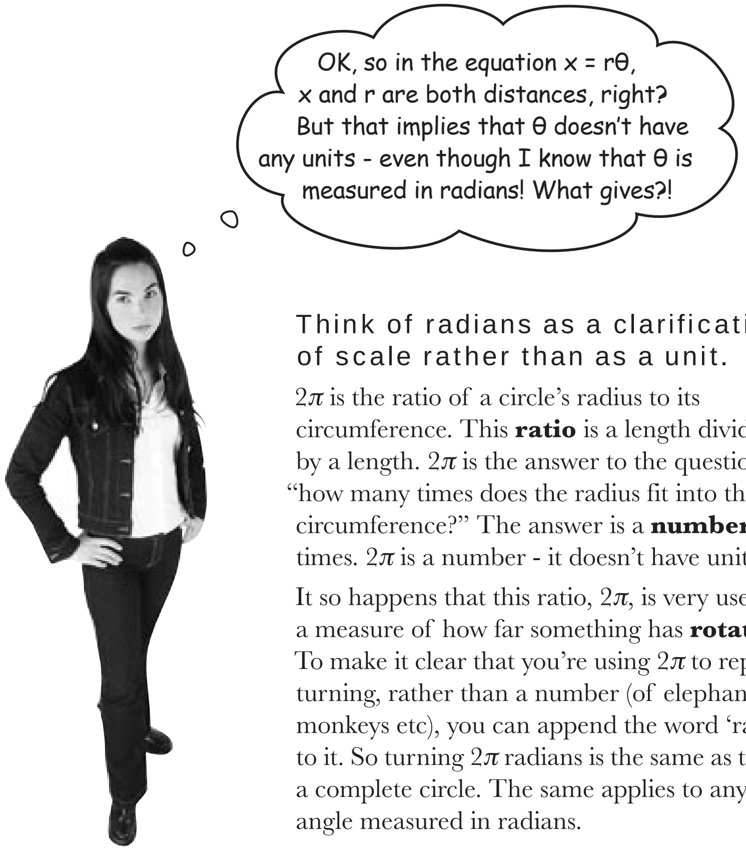
Multiply the angular quantity by the radius to get the linear quantity.

With sprint training in a larger wheel added to their schedule, the hamsters are unbeatable, and win all of their races - including the Kentucky Hamster Derby!



BULLET POINTS

- You can go from an angle, θ, to the distance something r from the center of the circle has travelled by multiplying by the radius: $x = r\theta$.
- You can go from the angular frequency to the speed that something r from the center of the circle is moving at by multiplying by the radius, $v = r\omega$.
- The equation $v = r\omega$ only gives you the size of the velocity vector, not its direction. That's why v is written as a scalar.
- ω is also called the angular speed. This is the same thing as the angular frequency - radians per second.



Think of radians as a clarification of scale rather than as a unit.

2π is the ratio of a circle's radius to its circumference. This **ratio** is a length divided by a length. 2π is the answer to the question "how many times does the radius fit into the circumference?" The answer is a **number** of times. 2π is a number - it doesn't have units.

It so happens that this ratio, 2π, is very useful as a measure of how far something has **rotated**. To make it clear that you're using 2π to represent turning, rather than a number (of elephants, monkeys etc), you can append the word 'radians' to it. So turning 2π radians is the same as turning a complete circle. The same applies to any other angle measured in radians.

This means that when you're thinking about the units of terms in an equation (doing dimensional analysis or units analysis), you should treat radians like a number without any units.

When you're thinking about the units of the terms in an equation, radians are dimensionless so don't have units.

Question Clinic: The "Angular quantities" Question

Any time something's going in a circle, you're almost certain to have to deal with quantities such as angular displacement θ, and angular speed, ω. The main things to remember is that there are 2π radians in one complete revolution and that the circumference of the circle = $2\pi r$. This means you can work out that you get the linear quantity by multiplying the angular quantity (e.g. $\theta = 2\pi$) by the radius, even if you forget the formulae $x = r\theta$ and $v = r\omega$.

'Wheel' is a buzzword that should get you thinking about rotational motion.

This question gives you the diameter. Be careful – all of your equations involve the RADIUS, so you'll need to use a value of 10.0 cm not 20.0 cm.

This is a LINEAR speed.

2. You need to arrange for a hamster wheel (diameter 20.0 cm) to turn so that the outside of the wheel goes at a speed of 3.00 km/h.

a. What angular frequency (in radians per second) does the wheel need to turn with to achieve this?

b. How many revolutions per second is this equivalent to?

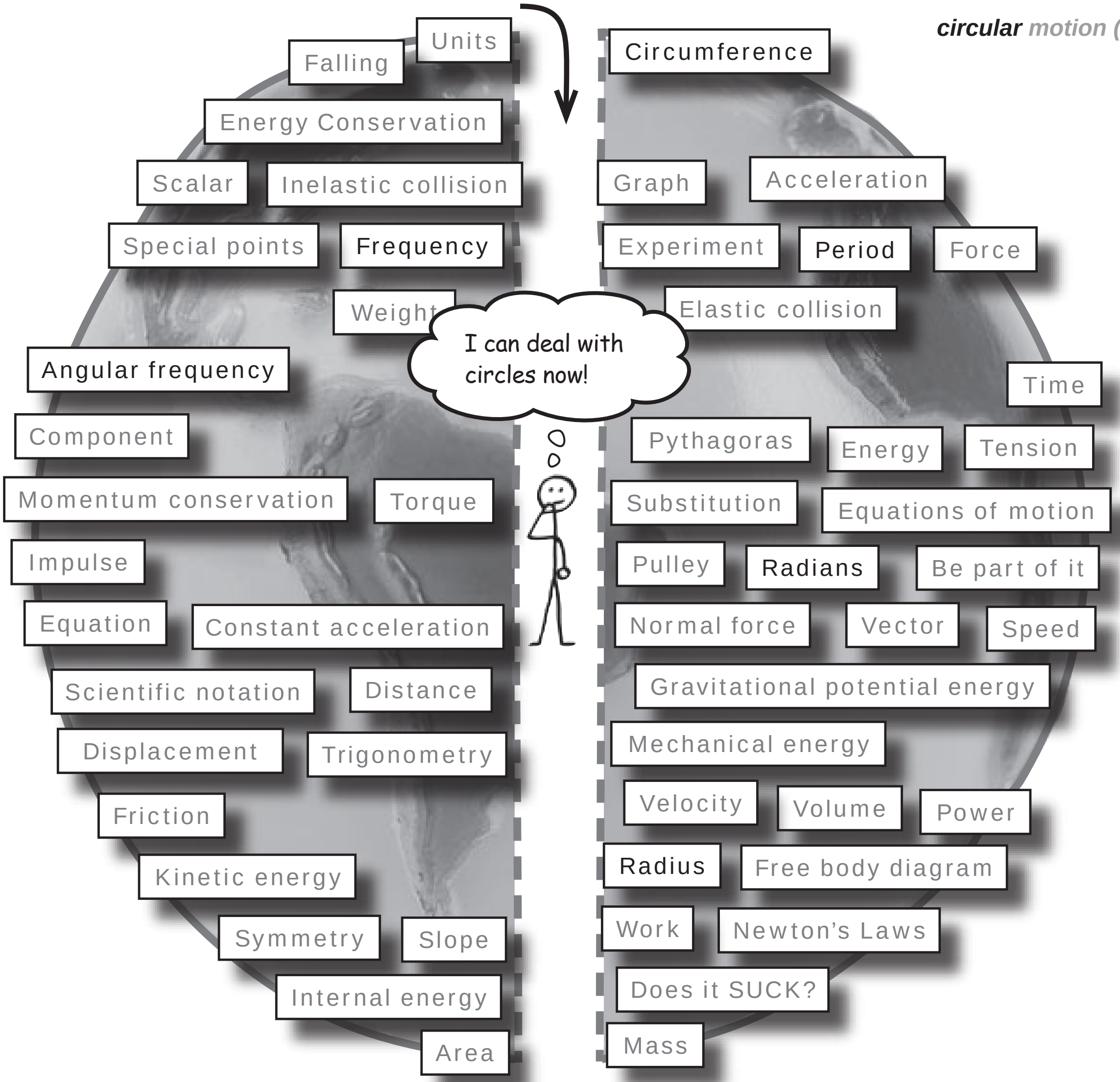
c. If the motor driving the wheel is only capable of rotating it at 25 rad/s, what diameter would the wheel need to have in order to achieve a speed of 3 km/h?

Angular frequency and angular speed are both to do with how fast something is rotating in radians per second.

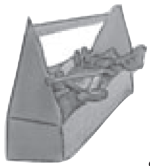
Remember that there are 2π radians in exactly 1 revolution. Ask yourself which number you're expecting to be bigger when you do the conversion.

The question asks you for a diameter when you've been working with a radius throughout. Make sure you give them what they asked for!

One thing to bear in mind with this kind of question is whether it talks about the frequency - number of revolutions per second - or the angular frequency, aka angular speed which is the number of radians per second. As there are 2π radians in one revolution you can convert from one to the other easily enough - as long as you pay attention to which units are involved in the first place!



☑	Radius	The distance from the center to the edge of a circle.
☑	Circumference	The distance round the outside of a circle. $C = 2\pi r$
☑	Frequency	The number of times something regular happens per second.
☑	Period	The number of seconds it takes for something regular to happen once.
☑	Radians	An alternative way of measuring angles. There are 2π radians is one complete revolution.
☑	Angular frequency	The number of radians per second. Also known as angular speed.

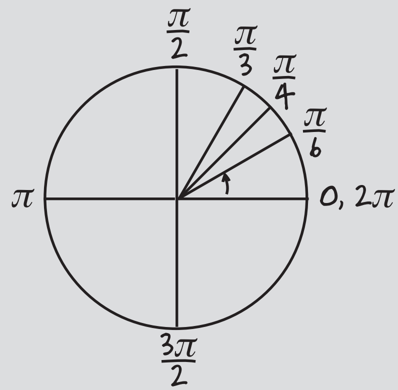


Your Physics Toolbox

You've got Chapter 16 under your belt and added some terminology and problem-solving skills to your tool box.

Radians

Radians are a way of measuring angles. They are especially useful for working with circles. There are 2π radians in 1 revolution. Think of other angles in terms of 'fractions of '2π.



Frequency and period

Frequency, f, is cycles per second.

Period, T, is seconds per cycle.

$$T = \frac{1}{f} \qquad f = \frac{1}{T}$$

Angular frequency and angular speed

Angular frequency and angular speed both have the same size and both have units of radians per second. They're exactly the same thing.

You can get from the frequency, f, to the angular frequency, ω, with the equation:

$$\omega = 2\pi f$$

Linear and angular

You can get from the angle, θ, to the linear distance, x, with the equation:

$$x = r\theta$$

You can get from the angular speed, ω, to the linear speed, v, with the equation:

$$v = r\omega$$

17 circular motion (part 2)

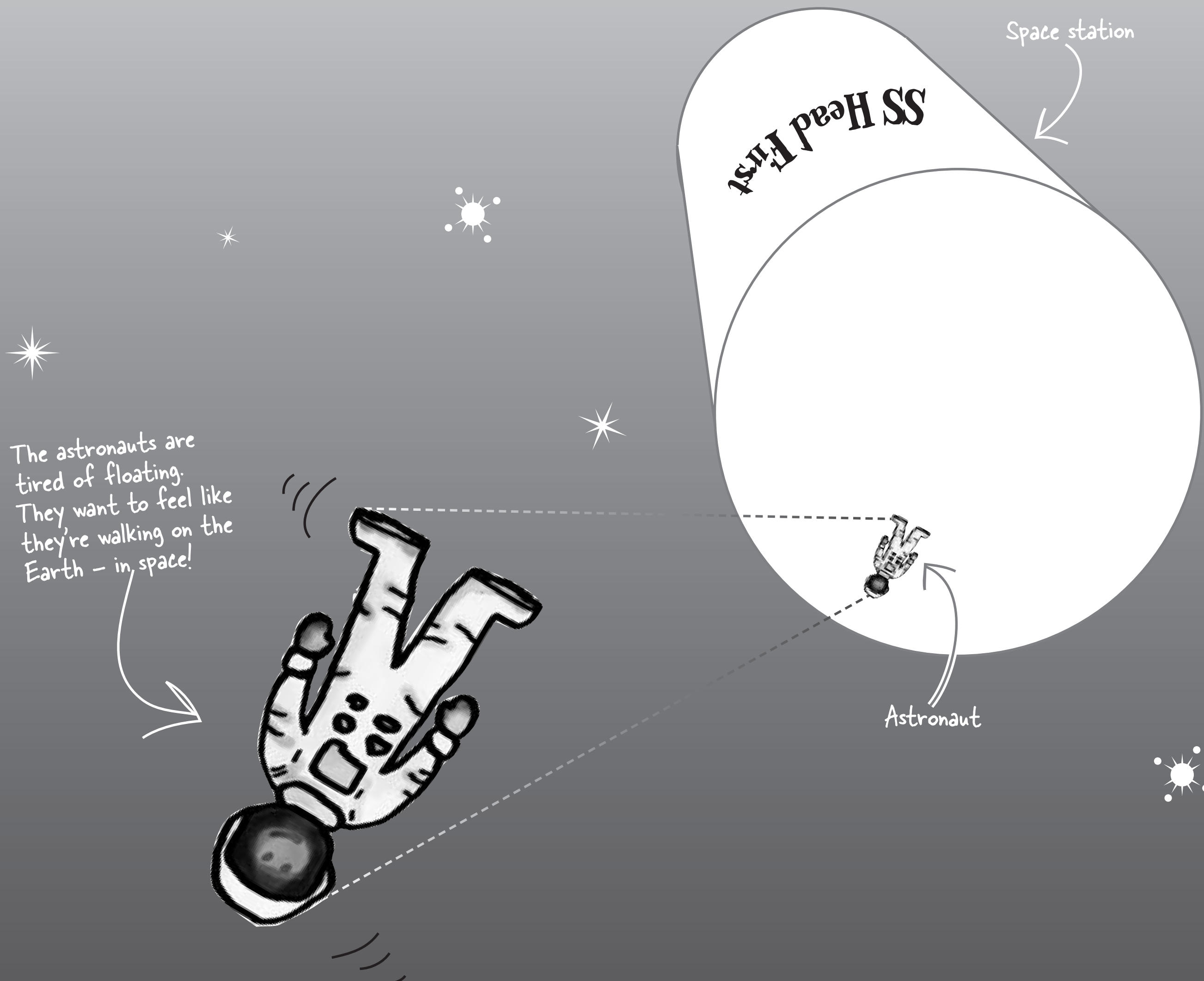
Staying on track

Ever feel like someone's gone off at a tangent? That's exactly what happens when you try to move an object along a **circular** path when there's not enough **centripetal force** to enable this to happen. In this chapter, you'll learn exactly what centripetal force is and how it can keep you on track. Along the way, you'll even solve some pretty serious problems with a certain Head First space station. So what are you waiting for? Turn the page, and let's get started.

Houston ... we have a problem

Astronauts at the Head First space station are threatening to go on strike. They're fed up with floating around all the time. The astronauts want to be able to walk around the station just like they can on Earth.

You've been called in to create artificial gravity for an add-on to the station... and keep those astronauts happy.



How can the space station be in orbit **around** the Earth when gravity attracts it **towards** the Earth?

The space station is in FREEFALL.

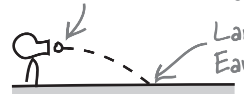
The only force acting on the space station is the force of its **weight** - the **gravitational attraction** it experiences from the Earth. It's not touching anything else, so there are no contact forces on it. And the lack of atmosphere means that there's no friction either as it orbits the Earth.

If an object is in **freefall**, then the only **force** acting on the object is its own weight. So the space station is in freefall. The same is true for the astronauts - the only force acting on each astronaut is that astronaut's weight.

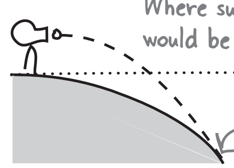
But when the force of an object's weight acts down towards the center of the Earth, how can the object possibly **orbit** the Earth by going around the Earth?!

If an object is in freefall, the only force acting on the object is its own weight.

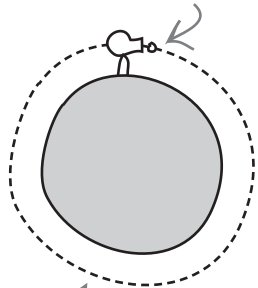
1 Suppose you fire a cannon. The cannonball is in freefall, because the only force acting on the cannonball is its own weight. So the cannonball follows a **curved** path as it falls and hits the Earth.



2 Locally, the Earth appears flat, but it's really round. If the cannonball is very fast or starts off very high, it goes further before it lands because the surface of the Earth curves away from the cannonball as it falls.



3 If the cannon is high enough and the cannonball is fast enough, the surface of the Earth keeps on curving away as the cannonball's flightpath curves down. The cannonball keeps on freefalling round the Earth - in **orbit** - just like the space station and astronauts.



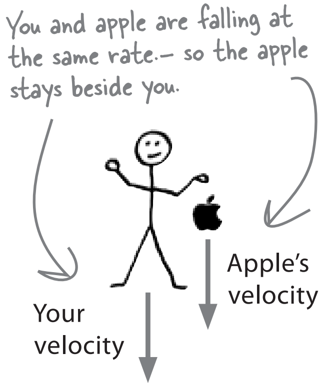
BRAIN POWER

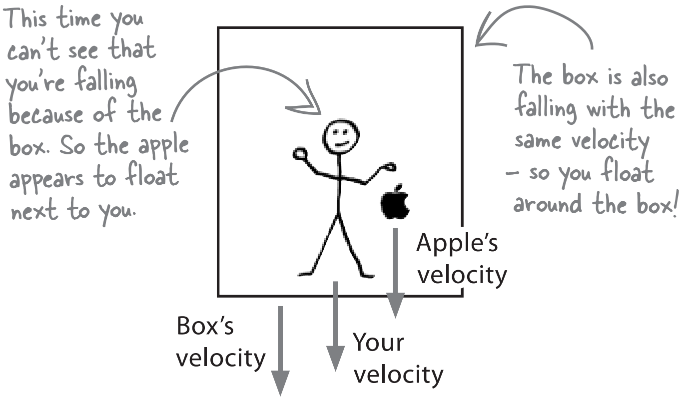
The astronaut is in freefall - the only force acting on him is his weight. So why does he feel weightless in the space station?

When you're in freefall, objects appear to float beside you

Suppose you're midway through a parachute jump, but you haven't opened your chute yet. If you let go of an apple as you fall, the apple will **fall with the same velocity as you**. It looks like the apple's just floating there.

Of course, if you're doing a parachute jump, there are some clues that indicate that you and the apple are both falling. The Earth gradually looks bigger, and you can feel the wind rushing past you!



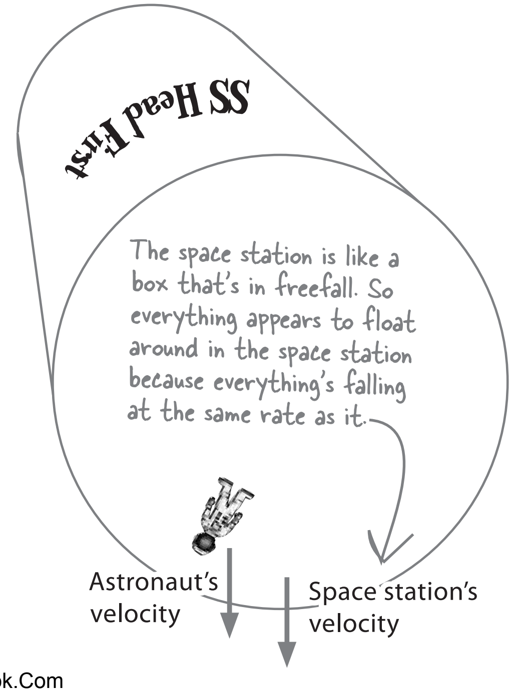


But if you and the apple were in a soundproof windowless box, you wouldn't have any visual or audible clues that you're falling. The apple would appear to float next to you! And as the box is also falling at the same rate as you, you would appear to float around inside the box. If you pushed yourself up off the floor, you wouldn't fall back down again like you would on Earth.

It's the same for an astronaut in the space station. He's continually falling around the Earth. So if he lets go of an apple, it floats around the space station rather than falling towards the ground.

This makes it look as though the apple is weightless - and as though the astronaut is weightless. The astronaut is used to the force of his weight attracting him towards the ground. But here, the "ground" (the wall of the spacestation) is falling at the same rate as the astronaut

This means that the astronaut can't walk around the space station like he can on Earth.



What's the astronaut missing, compared to when he's on Earth?

If an astronaut is in freefall, then he and the objects in the space station will appear to float around because everything's falling at the same rate.

But why does a dropped apple in an orbiting space station appear to act so differently from a dropped apple on Earth - when they're both falling because of the force of their weight, as usual? And why isn't it possible to walk around normally in the space station like you can on Earth?

When you're dealing with forces, always start with a free body diagram...

For problems involving forces, always start with a free body diagram.

Sharpen your pencil

a. Draw a free body diagram of all the forces the astronaut would experience while standing on Earth.

b. Draw a free body diagram of all the forces an astronaut would experience while in freefall.

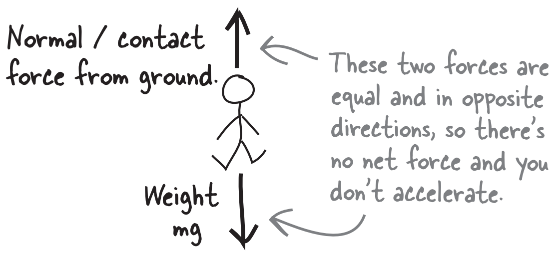
c. When you compare the free body diagrams in parts a and b, which force is missing?

d. How does a person on Earth experience their weight differently from a person in freefall?

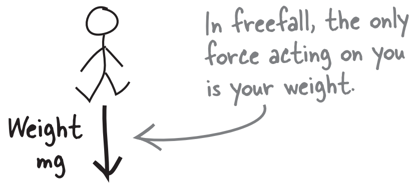
e. How might you introduce a new force to compensate for the missing one you spotted in parts c and d, so the astronaut can walk around as he would on Earth?

Sharpen your pencil Solution

a. Draw a free body diagram of all the forces the astronaut would experience while standing on Earth.



b. Draw a free body diagram of all the forces an astronaut would experience while in freefall.



c. When you compare the free body diagrams in parts a and b, which force is missing?

The astronaut in the space station doesn't experience a contact force from the ground.

d. How does a person on Earth experience their weight differently from a person in freefall?

They don't feel a contact force from the ground – they're in freefall, and the only force they experience is their weight. This means that they can't walk around like they can on Earth, as you need a normal force to have enough friction to walk like they usually do.

e. How might you introduce a new force to compensate for the missing one you spotted in parts c and d, so the astronaut can walk around as he would on Earth?

Newton's 2nd law is $F = ma$. You could make the astronaut experience a force by accelerating the space station.

there are no Dumb Questions

Q: So why do we need artificial gravity when the space station still feels the effect of the Earth's gravity?

A: If you're in freefall, you feel weightless because you - and other objects - appear to float around, as they're not going anywhere with respect to each other.

Q: But you still have a weight, right?

A: Yes - just like someone doing a parachute jump still has a weight. It's the force of their weight that makes them fall!

Q: So why is someone in freefall called "weightless" when they have a weight? Isn't that confusing?

A: Yes, it is confusing! "Weightless" is an everyday way of saying that they aren't experiencing any kind of contact force from a surface as a result of their weight. If the person had scales under their feet, the scales would read zero.

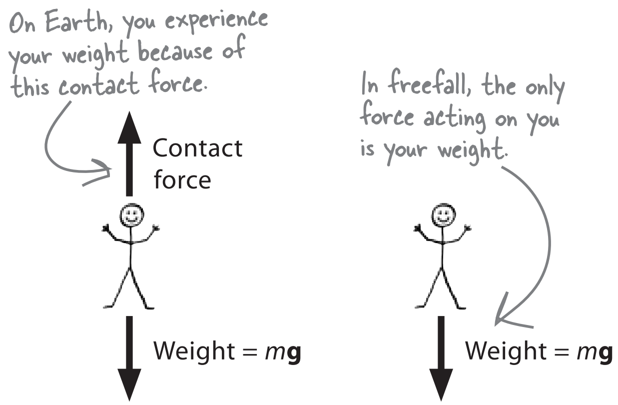
Go back and look at the first WeightBotchers machine in chapter 11 if you're not sure why this is.

Can you mimic the contact force you feel on Earth?

The difference between standing on the ground and freefalling is the **contact force** that the ground exerts on you. This is the force that the astronauts want to experience.

If you can make each astronaut experience a contact force equal to the size of his **weight**, as he does on Earth, he should be able to walk around the space station just like he can on Earth.

But how can you make someone experience a contact force like this?



On Earth, you experience your weight because of a contact force from the ground.

Sharpen your pencil

The key thing is to close your eyes and ask "WHAT DO I FEEL PUSHING ON ME?"

Imagine yourself in these scenarios. Draw the contact force you experience in each situation as a result of the acceleration, and write down what you FEEL. For instance, "Something's pushing me in the back."

All passengers are wearing seatbelts.

Train is sitting still, then accelerates to the right as it pulls out of a station.

Train is moving to the right, then decelerates to a stop as it pulls into a station.

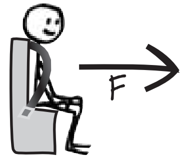
Does this give you an idea about how you might make the astronaut experience a contact force from the inside wall of the space station that would feel similar to the one he experiences on Earth?

Sharpen your pencil Solution

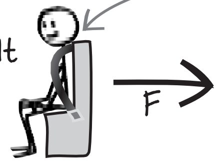
Imagine yourself in these scenarios. Draw the contact force you experience in each situation as a result of the acceleration, and write down what you FEEL. For instance, "Something's pushing me in the back."

Train is sitting still, then accelerates to the right as it pulls out of a station.

The back of the seat is pushing on me.

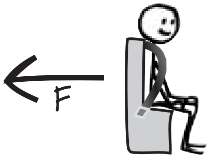


The seatbelt is digging into me.

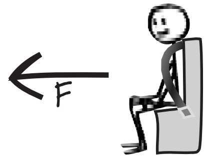


All passengers are wearing seatbelts.

Train is moving to the right, then decelerates to a stop as it pulls into a station.



The seatbelt is digging into me.



The back of the seat is pushing on me.

Does this give you an idea about how you might make the astronaut experience a contact force from the inside wall of the space station that would feel similar to the one he experiences on Earth?

You could accelerate the space station. That would make the astronaut experience a contact force – like I do when a train accelerates.

If you accelerate it at 9.8 m/s^2, then this contact force will be exactly the same size as the one he experiences on Earth.

there are no Dumb Questions

Q: I can imagine the seat pushing into my back when a train accelerates. But why does that happen?

A: Newton's 1st law says that an object will continue to move at a constant velocity unless it's acted on by a net force. If you were sitting on the platform, the fact that the train is accelerating wouldn't affect you, as there's no contact between you and it.

But because you're sitting on the train, the back of your seat is able to mediate a net contact force that causes you to accelerate. You feel the seat pushing into you.

In the context of forces, 'mediate' means 'transmit'.

Q: How large is the contact force?

A: Newton's 2nd Law says that $F = ma$. You can work out the size of the contact force from your mass and your acceleration.

Q: How can I make the astronaut feel a contact force?

A: If you accelerate the space station upward, the astronaut will experience a contact force from its wall because he is inside it - just like you do when you're on the train.

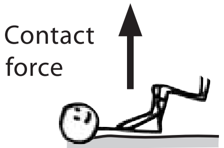
Q: Why will accelerating the space station mean that a dropped apple will fall like it does on Earth?

A: The apple will continue at the velocity it already had (Newton's 1st Law), as there is no contact force on it while it is falling.

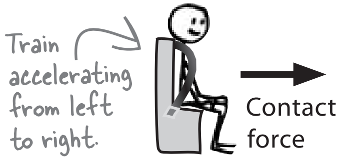
Meanwhile, the space station will accelerate up to meet it at a rate of 9.8 m/s^2. So if you're in the space station, it feels like you're on Earth (because of the contact force you experience), and it looks like you're on Earth because objects accelerate towards the ground at the same rate.

Accelerating the space station allows you to experience a contact force

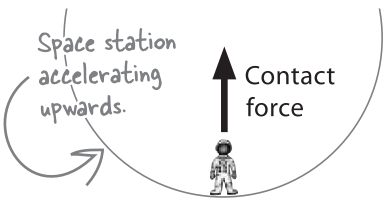
If you lie on the ground with your eyes shut, you can feel a **contact force** from the ground pushing into your back.



If the train pulls away from the station with an acceleration of exactly **9.8 m/s²**, then you would feel exactly the **same size** of contact force as you do when you lie on the ground.



So if you accelerate the space station at a rate of 9.8 m/s², the astronauts will experience the same size of contact force as they usually experience when they're standing on Earth. This creates the artificial gravity that the astronauts want!



But how **practical** is this?

Sharpen your pencil

Don't be intimidated by the first four words of this problem!

a. Einstein's Theory of Relativity says that nothing can move faster than the speed of light, 3.0×10^8 m/s. If you accelerate a space station from rest at a rate of 9.8 m/s², what time would it take it to reach a speed of 3.0×10^8 m/s (assume for a moment that this is possible and there are no relativistic effects)?

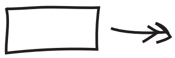
b. What distance would the space station cover in that time?

c. The distance from the Earth to the Moon is 4×10^8 m (1 sd), and the distance to the edge of the Solar System is 5.7×10^{12} m. How does the distance you worked out in part b compare?

d. How practical do you think this idea is for creating artificial gravity in a space station?

Sharpen your pencil Solution

a. Einstein's Theory of Relativity says that nothing can move faster than the speed of light, 3.0×10^8 m/s. If you accelerate a space station from rest at a rate of $9.8\ m/s^2$, what time would it take it to reach a speed of 3.0×10^8 m/s (assume for a moment that this is possible and there are no relativistic effects)?



$v_0 = 0$ m/s

$v = 3.0 \times 10^8$ m/s

$a = 9.8\ m/s^2$

$t = ?\quad x - x_0 = ?$

Work out t: $v = v_0 + at$

But $v_0 = 0$

$$t = \frac{v}{a} = \frac{3.0 \times 10^8}{9.8}$$

$t = 3.1 \times 10^7$ s (2 sd)

b. What distance would the space station cover in that time?

Work out x:

$$x = x_0 + v_0 t + \tfrac{1}{2}at^2$$

$$x = 0 + 0 + 0.5 \times 9.8 \times (3.1 \times 10^7)^2$$

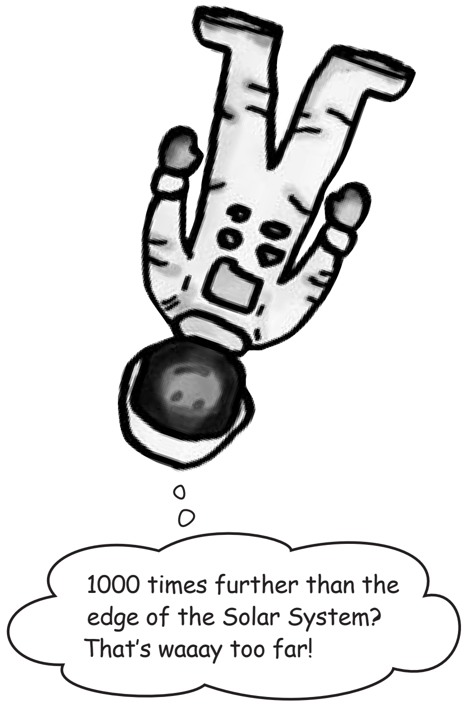
$$x = \underline{4.7 \times 10^{15}\ m}\ (2\ sd)$$

c. The distance from the Earth to the Moon is 4×10^8 m (1 sd), and the distance to the edge of the Solar System is 5.7×10^{12} m. How does the distance you worked out in part b compare?

This distance is around 10 million times greater than the Earth-Moon distance and a thousand times greater than the edge of the Solar System.

d. How practical do you think this idea is for creating artificial gravity in a space station?

It's not practical because it's not possible to sustain it for a good length of time, and you end up very far away from the Earth. It must take a lot of fuel too.

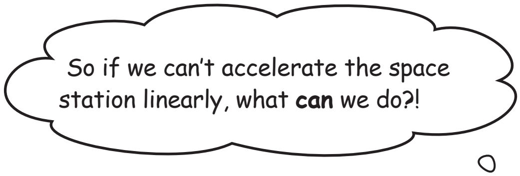


It's (theoretically) possible to make the astronaut experience a contact force similar to the one he experiences on Earth by accelerating the space station along a straight line at $9.8\ m/s^2$.

But it's not **practical**. It's impossible to do this indefinitely, since the space station can't go faster than the speed of light, you'd run out of fuel, and you'd wind up a ridiculously long way away from the Earth.

So if **linear acceleration** isn't practical, what might another option be?

Linear means "along a straight line."



Jim: I wonder if there's another way of experiencing a contact force, apart from accelerating or decelerating along a straight line?

Joe: Hmmm ... what about those carnival rides where you go **around in a circle**? You kinda feel the side of the car pushing on you when they spin really fast ...

Frank: You feel the side of the car pushing you, so there must be a **contact force**. But where does it come from?

Jim: Yeah, it's not like the ride gets faster and faster. It spins at the same rate, so you keep going at a **constant speed**, yet you still feel this contact force from the side of the car. How can you feel a force if your speed is constant - doesn't that break Newton's 1st Law?

Joe: But the **direction** you're traveling in is changing all the time. That means your **velocity** is changing, even though your speed is constant. Velocity is a **vector**. Newton's 1st Law says that you move with a constant velocity unless there's a force acting on you.

Frank: So I guess the contact force **changes your direction** of travel - which changes your velocity - so causes you to **accelerate**.

Jim: But where does the force come from?! It's not like there's a train engine sitting behind you making you accelerate!

Frank: Well, you're thrown to the outside of the ride, aren't you? So there must be some kind of mysterious force pushing you outwards that's only there when you're spinning.

Frank: Hang on! When you're thinking about contact forces, you're meant to shut your eyes and ask, "What do I feel pushing on me?" And when I do that, I feel the side of the car pushing me **inwards**, not a 'ghost force' pushing me outwards.

Jim: But you slide kinda outwards across your seat before you make contact with the side of the car. If there isn't a force pushing you outwards, then why does that happen?!

Joe: If the side of the car wasn't there, you'd go straight on and fly out of the car. You only go in a circle because of the contact force from the side of the car pushing you inwards.

Frank: Ah ... you mean that sliding outwards feeling is just you continuing on at your current velocity (Newton's 1st Law) before you make contact with the side of the car - which exerts a force on you that lets you move in a circle?

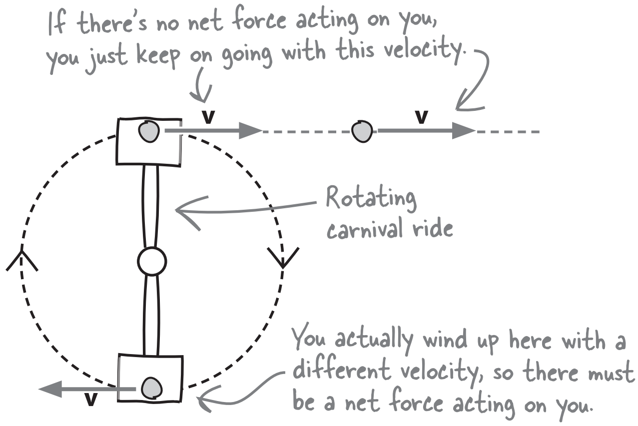
If a contact force is acting on you, you can <u>feel</u> the direction it's pushing you in.

BRAIN POWER

Can you imagine yourself in the spinning carnival ride and the contact force acting on you?

You can only go in a circle because of a centripetal force

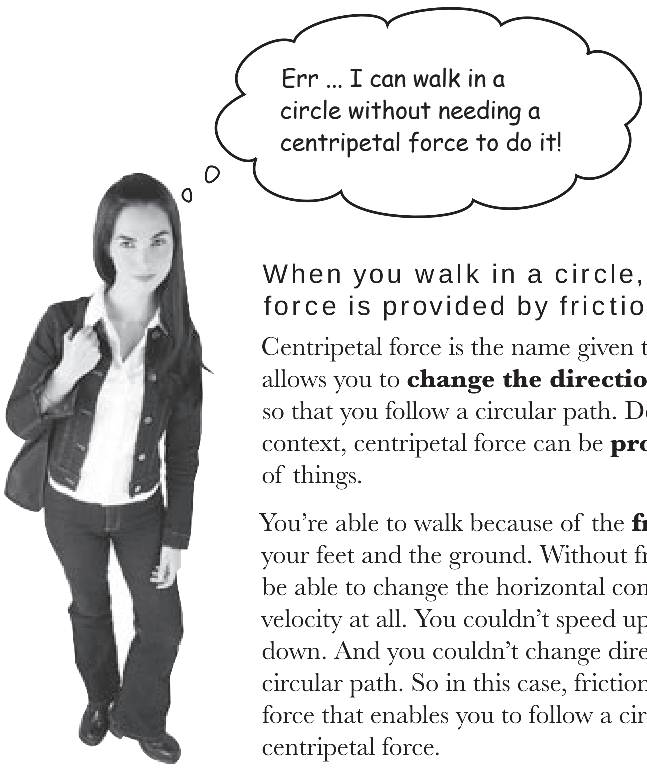
Newton's 1st Law says that you continue with a **constant velocity** unless there's a **net force** acting on you. In other words, you keep on going at the **same speed** in the **same direction.**



If you're going around in a circle, your speed may stay the same, but the direction of your velocity vector changes.

If you're going around in a circle, your speed may be constant, but the **direction** of your velocity is certainly changing! This means that a **force** must be acting on you in order to make you go around in a circle - and stop you from going off along a straight line with the velocity you already have.

A force that allows you to go in a circle like this is called a **centripetal force**.



When you walk in a circle, the centripetal force is provided by friction.

Centripetal force is the name given to a **net force** that allows you to **change the direction of your velocity** so that you follow a circular path. Depending on the context, centripetal force can be **provided** by a number of things.

You're able to walk because of the **friction** between your feet and the ground. Without friction, you wouldn't be able to change the horizontal component of your velocity at all. You couldn't speed up. You couldn't slow down. And you couldn't change direction to follow a circular path. So in this case, friction provides the net force that enables you to follow a circular path - the centripetal force.

If the net force acting on you changes the direction of your velocity so that you travel in a circle, it's called a centripetal force.

there are no Dumb Questions

Q: Is centripetal force another category of force to add to a contact forces and gravitational forces?

A: No, not at all! Centripetal force is the name given to the net force when it enables you to follow a circular path instead of continuing along a straight line.

Q: So if a force is able to change my direction, it might be able to provide a centripetal force?

A: Yes, that's a good way to think about it.

Q: Where does a centripetal force come from?

A: The centripetal force required for you to follow a circular path may come from any source at all. It might be a contact force. It might be a frictional force. It might be a gravitational force.

Q: So it's not a "ghost force" that magically appears from nowhere?

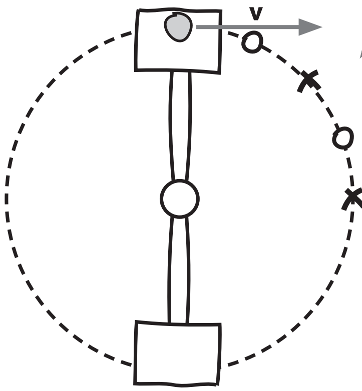
A: That's right. If you draw a free body diagram of an object moving around in a circle, then the net force will be the centripetal force that causes the circular motion.

Q: Which direction does the centripetal force act in?

A: You're about to figure that out ...

Sharpen your pencil

Tip: Draw in the radius of the circle, from the center ot each point. The velocity vector will be at 90° to the radius.



a. Draw the velocity vector at each of the points marked with an 'x' on the ride. Assume that the ride is rotating with a constant angular frequency.

b. In a different color, draw in the left-right and up-down components of the velocity vectors you drew in part a.

c. Describe how the velocity components have changed from one 'x' to the next. At each of the points marked 'o', draw in a vector representing a force that may have caused these changes.

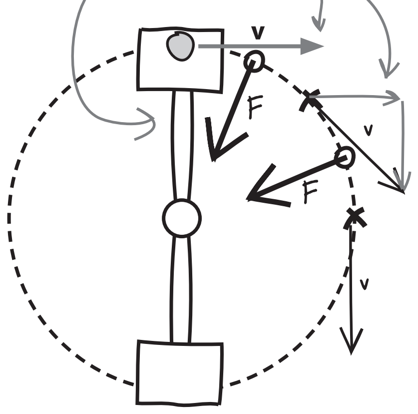
Hint: the direction of the force may be changing as the ride rotates.

d. Which direction do you think the force vector will point in for other positions on the ride?

e. What is the source of this centripetal force that enables you to move in a circle?

Sharpen your pencil Solution

The force needs to have a large down component and a small left component to have made this change.



a. Draw the velocity vector at each of the points marked with an 'x' on the ride. Assume that the ride is rotating with a constant angular frequency.

b. In a different color, draw in the left-right and up-down components of the velocity vectors you drew in part a.

c. Describe how the components have changed from one snapshot to the next, and draw in a vector representing a force that may have caused these changes.

Between the 1st and 2nd snapshots, the down component got much larger and the right component a bit smaller. So the force must be acting down and left (with more down)

Between the 2nd and 3rd snapshots, the right component has disappeared and the down component has got a bit larger. So the force must be acting down and left (with more left).

Centripetal force acts towards the center of the circle.

d. Which direction do you think the force vector will point in for other positions on the ride?

I think the force vector will always point towards the center of the circle.

e. What is the source of this centripetal force that enables you to move in a circle?

The contact force of the outside wall of the car pushing in on me.

It's OK if you said "the arm that goes between the car and the center", as that exerts the centripetal force that stops the car from flying off along a straight line.

Always think "what contact force do I feel pushing me?"

The reason you slide towards the outside is that you want to keep on going in the direction you're already going in. That's Newton's 1st Law.

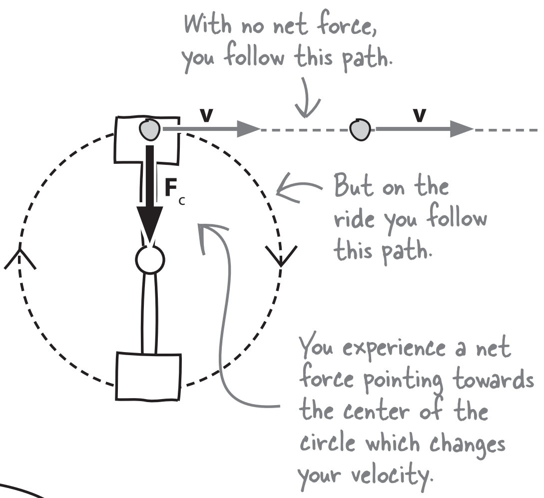
But the side of the car stops you from going any further and pushes you towards the center. This is where the **centripetal force** that makes you go round in a circle comes from.

Being thrown to the outside is an illusion. There's no force moving you in that direction - just your own momentum.

Centripetal force acts towards the center of the circle

If you're sitting in a rotating carnival ride, you experience a centripetal force, $\mathbf{F}_c$, towards the center of the ride. This is the force that enables you to continue traveling in a **circle**, instead of continuing at a constant speed in the same direction as you would if there was no force (Newton's 1st Law).

In the carnival ride, the centripetal force is **provided** by a **contact force** from the side of the car - you feel the side of the car pushing you towards the center of the circle as you go around..



Centripetal force is always provided by the net force on your free body diagram. Otherwise you'd just go along a straight line.

"Centrifugal force" isn't a force.

"Centrifugal force" is the name commonly given to the sensation of being thrown to the outside when you're in something that's rotating (e.g., a centrifuge).

But as you've just learned, what is often referred to as centrifugal force isn't a force! It's just you **continuing at your current velocity** in the absence of a net force.

And once you're in CONTACT with the side of the centrifuge, the contact force can provide a centripetal force that makes you follow a circular path.

Never EVER talk about "centrifugal force."

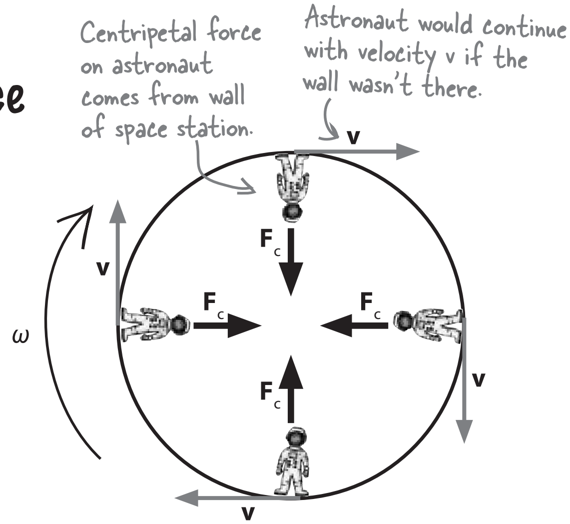
BRAIN POWER

How can you use what you've figured out about centripetal force to help the astronauts?

The astronaut experiences a contact force when you rotate the space station

You can **rotate** the space station just like you rotate the carnival ride. Each astronaut will experience a **centripetal force** acting towards the center of the space station. This will be mediated by the **contact force** between his feet and the side of the space station. This will feel similar to the contact force he experiences on Earth.

As there's now a **contact force**, the astronauts will be able to walk around like they can on Earth - and won't go on strike!



there are no Dumb Questions

Q: So anything that's rotating is subject to a centripetal force?

A: That's right. If the centripetal force wasn't there, the thing wouldn't rotate - it would travel along a straight line at its current speed.

Q: So why didn't the hamster running in the wheel in chapter 16 experience a centripetal force? Or did it?

A: Was the hamster rotating?

Q: No ... the hamster stayed in the same place, and the wheel rotated as its feet pushed it along. I guess the hamster didn't experience a centripetal force.

A: Yeah, that's right.

Q: But the wheel's rotating, so the wheel must experience a centripetal force - yes? But how? The outside of the wheel isn't in contact with anything!

A: Centripetal force doesn't always have to be provided by a contact force from the outside. All you need is a force that points towards the center of the circle.

Q: So I guess the struts in the hamster wheel are mediating the centripetal force?

A: Absolutely! If one of the struts broke, and part of the outside of the wheel flew off, then it wouldn't be moving in a circle anymore. The broken part would fly off along a straight line at a tangent to the circle with whatever velocity it had when it became detached.

Q: That's Newton's 1st Law, right?

A: Yep. It's also how hammer throwing works at the Olympics. The athlete spins round and round with the heavy ball on the end of a chain, then lets go. Without the centripetal force provided by the athlete pulling on the chain, the hammer flies off in a straight line.

If the centripetal force 'disappears', you'll go off at a tangent to the circle.

Q: Yeah, I've seen that before. So are you saying it's the same in the space station? If you rotate the space station, you're giving the astronaut the potential to go flying off into space if a door suddenly opens or something?

A: That's right. If the astronaut didn't experience the centripetal contact force from the side of the space station acting towards its center, he would go along a straight line.

Q: Hmmm. I guess there must be a centripetal force acting on the space station for it to have a circular orbit around the Earth in the first place?

A: Great spot! You'll be learning all about that in chapter 16.

Q: So I guess I need to work out how fast I need to rotate the space station to produce a centripetal force of *m*g, like an astronaut would feel on Earth?

A: Let's look at that now...

What affects the size of centripetal force?

Newton's 2nd Law tells you that the **centripetal force** the astronaut experiences will be equal to his mass × acceleration:

$$\mathbf{F}_c = m\mathbf{a}_c$$

← The centripetal force depends on the centripetal acceleration.

You can write this because the centripetal force is the net force that causes the centripetal acceleration.

The acceleration that the centripetal force causes is called **centripetal acceleration**. Acceleration is the rate of change of velocity and points in the direction of that change:

$$\mathbf{a}_c = \frac{\Delta \mathbf{v}}{\Delta t}$$

← The centripetal acceleration depends on the rate of change of velocity.

You want the centripetal acceleration that the astronauts experience to be equal to 9.8 m/s². This makes the contact force each astronaut experiences in the space station the same size as the contact force he experiences on Earth.

But what affects the **size** of the centripetal acceleration? If you can work that out, you can get an **equation** for the centripetal acceleration and stop the astronauts going on strike!

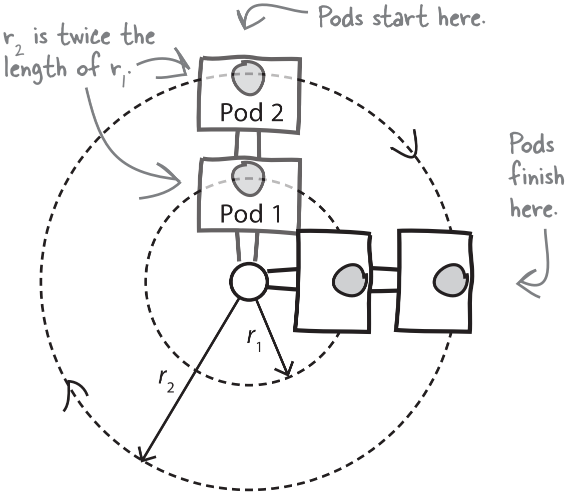
Sharpen your pencil

The equation from chapter 16 shows you what happens to v when you vary r and ω.

The sketch here could either be of a carnival ride or a space station - imagine it in the way that seems most natural to you. There are two 'pods' attached to the circle, one twice as far from the center as the other.

a. Bearing in mind that $v = r\omega$, draw in velocity vectors for the two pods at each position shown.

b. Which pod experiences the larger change in its velocity between the two positions?



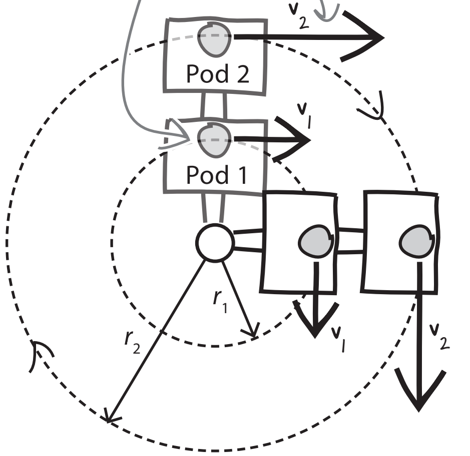
c. Which pod experiences the greater centripetal acceleration between the two positions?

d. Can you think of another way of increasing the centripetal acceleration that the pods experience (assuming each pod's radius is fixed)?

Sharpen your pencil Solution

Twice the radius means double the velocity ($v = r\omega$)

The sketch here could either be of a carnival ride or a space station - imagine it in the way that seems most natural to you. There are two 'pods' attached to the circle, one twice as far from the center as the other.



a. Bearing in mind that $v = r\omega$, draw in velocity vectors for the two pods at each position shown.

b. Which pod experiences the larger change in its velocity between the two positions?

Pod 2 (on the outside) has the bigger change in velocity because its velocity vectors are larger, but the change in direction is the same.

c. Which pod experiences the greater centripetal acceleration between the two positions?

Acceleration is rate of change of velocity. So pod 2 has the greatest centripetal acceleration since it has the greatest change in velocity.

d. Can you think of another way of increasing the centripetal acceleration that the pods experience (assuming each pod's radius is fixed)?

Spin the space station with a faster ω to increase all the velocities (as $v = r\omega$).

We're not looking for an equation for the centripetal acceleration yet – just working out which variables it must depend on.

ω is the number of radians per second. ω can be called the **angular frequency**, or the **angular speed**. It is also sometimes referred to as the **angular velocity**, with the understanding that when the variable is written as a scalar, ω, it only refers to the **size** of the angular velocity, and not its direction. As we will often be moving between v, the size of the linear velocity and ω, we will often refer to ω as the angular velocity to make the connection clearer.

Centripetal acceleration depends on both the radius and the angular velocity, as both of these affect the rate of change of velocity.

For the same angular velocity, a larger RADIUS means a larger centripetal acceleration.

For the same radius, a larger ANGULAR VELOCITY means a larger centripetal acceleration.

Spot the equation for the centripetal acceleration

A larger **centripetal acceleration** is required to move something in a circle when the **radius** is large, and when the **angular velocity** is large, because the rate of change of velocity is larger in both cases.

Deriving the equation for the centripetal acceleration from scratch is tricky and doesn't really help your understanding of the physics. So instead, **spot** the correct equation by **trying out extreme values** and looking at its **units**.

Check an equation by thinking about EXTREMES and working out its UNITS.

You've done this kind of thing before, don't worry!

Equation ID Parade

Here are six equations that all claim to be a formula that gives you the size of the centripetal acceleration of something rotating with angular velocity *ω* at radius *r*.

Annotate the equations to explain what will happen if *r* gets much bigger, or if *ω* gets much bigger. Use this information to cross out any equations that don't behave as you would expect them to.

For your remaining equations, check that the units on both sides are the same. This should leave you with just one equation. (Remember that radians are dimensionless.)

$$a_c = \frac{r}{\omega} \qquad a_c = \frac{\omega}{r} \qquad a_c = r^2\omega$$

$$a_c = r\omega^2 \qquad a_c = \frac{\omega^2}{r} \qquad a_c = \frac{r^2}{\omega}$$

These are all scalar equations as we're only talking about the SIZE of the centripetal acceleration.

Equation ID Parade SOLUTION

Here are six equations that all claim to be a formula that gives you the size of the centripetal acceleration of something rotating with angular velocity ω at radius r.

Annotate the equations to explain what will happen if r gets much bigger, or if ω gets much bigger. Use this information to cross out any equations that don't behave as you would expect them to.

For your remaining equations, check that the units on both sides are the same. This should leave you with just one equation. (Remember that radians are dimensionless.)

a_c gets smaller as ω gets larger. So this is wrong.

~~$$a_c = \frac{r}{\omega}$$~~

a_c gets smaller as r gets larger. So this is wrong.

~~$$a_c = \frac{\omega}{r}$$~~

a_c gets larger as ω and r get larger. So check units.

~~$$a_c = r^2\omega$$~~

RHS: Units = $m^2 \times 1/s$
= m^2/s
Not units of acceleration.

Radians are dimensionless, the units of ω are 1/s.

a_c gets larger as ω and r get larger. So check units.

$$a_c = r\omega^2$$

RHS: Units = $m \times /s^2$
= m/s^2
Units of acceleration! ✓

a_c gets smaller as r gets larger. So this is wrong.

~~$$a_c = \frac{\omega^2}{r}$$~~

~~$$a_c = \frac{r^2}{\omega}$$~~

a_c gets smaller as ω gets larger. So this is wrong.

there are no Dumb Questions

Q: I got the equation '$a_c = r\omega^2$' from the parade. Why doesn't the equation for 'a_c' have 'v' in it, when acceleration is rate of change of velocity?

A: Remember that $v = r\omega$. So you can make a substitution for the 'ω' in '$a_c = r\omega^2$' to express it as an equation that involves v.

ω is the rate of change of the angle θ.

Q: OK ... so I make that substitution and get the equation $a_c = \frac{v^2}{r}$. But that can't be right! a_c should get larger as r gets larger. But you're dividing by r, so a_c would get smaller as r gets larger.

A: But when r gets larger, v gets larger as well, because $v = r\omega$. And because the velocity is squared, the v^2 on the top of the fraction gets larger more rapidly than the r on the bottom of the fraction. So overall, a_c still gets larger as r gets larger.

Q: But what if $r = 0$ in $a_c = \frac{v^2}{r}$? I'm not sure I know how to divide by 0!

A: It's easier to look at the other form of the equation, $a_c = r\omega^2$. If $r = 0$ then $a_c = 0$.

Q: Isn' t acceleration is usually a vector? Why is $a_c = r\omega^2$ a scalar equation gives the size but not the direction?

A: The centripetal acceleration vector always points towards the center of the circle, so its direction is always changing. To avoid getting muddled up with direction, we're just dealing with scalar equations for the size of the acceleration.

Give the astronauts a centripetal force

The equation for the size of the centripetal acceleration is $a_c = r\omega^2$. The vectors $\mathbf{a}_c$ and $\mathbf{r}$ are always in opposite directions, but because the space station is spinning, we're using a scalar equation.

$$a_c = r\omega^2$$

(Size of centripetal acceleration → a_c; Radius → r; Angular velocity → ω)

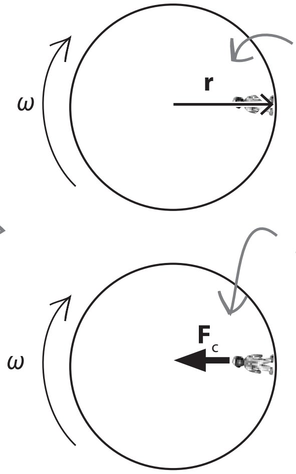
Newton's 2nd Law is $\mathbf{F}_{net} = m\mathbf{a}$. The centripetal force is the net force that must be present for an object to follow a circular path. A substitution gives you the size of the centripetal force:

$$F_c = mr\omega^2$$

(Size of centripetal force → F_c; Mass → m; $F_c = ma_c$)

It's good to appreciate the directions of the vectors even though the equation only deals with the sizes.

Even though F_c and r are in opposite directions, their directions are always changing because the space station is spinning.



r is displacement from center of the circle.

F_c points towards the center of the circle.

Sharpen your pencil

We're designing a spinning module for the existing space station, so that the astronauts can walk around. Two design candidates are cylinders with the same volume, but radii of 10.0 m and 100 m respectively.

a. Calculate the angular velocity required to produce a centripetal acceleration of 9.8 m/s^2 for (i) the 10.0 m radius module and (ii) the 100 m radius module.

ω can be called angular frequency, angular speed or angular velocity! It's always radians per second.

b. If a door in the space station opened, and the astronaut went through, what velocity (size and direction) would he travel AT if he'd been in (i) 10.0 m radius module and (ii) the 100 m radius modules?

Sharpen your pencil Solution

We're designing a spinning module for the existing space station, so that the astronauts can walk around. Two design candidates are cylinders with the same volume, but radii of 10.0 m and 100 m respectively.

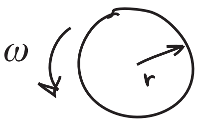
a. Calculate the angular velocity required to produce a centripetal acceleration of 9.8 m/s^2 for (i) the 10.0 m radius module and (ii) the 100 m radius module.

Centripetal force: $F_c = mr\omega^2$

Centripetal acceleration: $F_c = ma_c$

$\Rightarrow a_c = r\omega^2$

$\Rightarrow \omega = \sqrt{\frac{a_c}{r}}$



(i) For 10.0 m radius:

$\omega = \sqrt{\frac{9.8}{10.0}}$

$\omega = 0.990$ rad/s (3 sd)

(ii) For 100 m radius:

$\omega = \sqrt{\frac{9.8}{100}}$

$\omega = 0.313$ rad/s (3 sd)

b. If a door in the space station opened, and the astronaut went through, what velocity (size and direction) would he travel AT if he'd been in (i) 10.0 m radius module and (ii) the 100 m radius modules?

(i) For 10.0 m radius:

$v = r\omega = 10.0 \times 0.990$

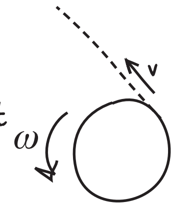
$v = 9.90$ m/s (3 sd)

(ii) For 100 m radius:

$v = r\omega = 100 \times 0.313$

$v = 31.3$ m/s (3 sd)

He'll keep on going with the same velocity he already had, at a tangent to the place he came out of the space station.



The velocity vector is always at a tangent to the circle.

The astronauts want as much floor space as possible

The two space station module designs have the same **volume** (90000 m^3) and need to be spun at similar angular velocities to produce the same centripetal acceleration. The 10.0 m radius space station needs to be spun at a rate of 0.990 rad/s and the 100 m radius station would be spun at a rate of 0.313 rad/s.

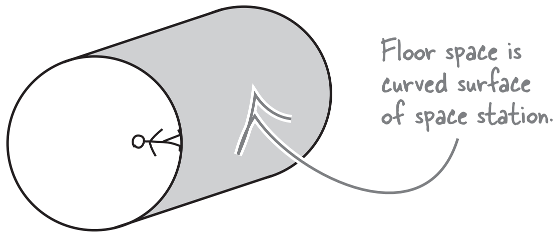
How are you going to choose between the two competing designs? The astronauts can help. They want as much floor **area** to walk around on as possible. They really feel the need to stretch their legs and get some exercise when they've been in the space station for a while!

BRAIN POWER

Which space station do you think will have the most floor space?

Frank: Well, duh, the 100 m radius space station must have a bigger floor area than the 10.0 m radius one, as it's a bigger circle!

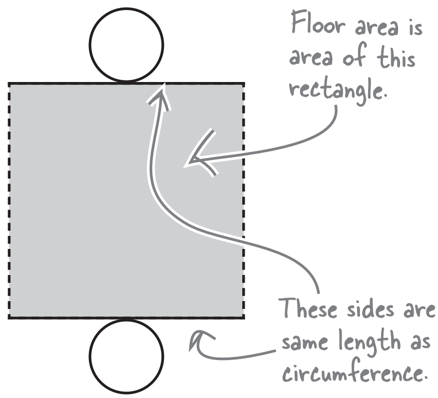
Jim: I'm not so sure. The floor is actually the **side** of the cylinder, not the circular part at the ends.



Frank: Ohhh ... good point. So how are we gonna do that? I don't think I know how to find the **area** of a curved surface!

Joe: Well, let's think this out. If we unroll the cylinder into simpler shapes what would they be?

Jim: OK, the cylinder's basically a **circle** at each end and a **rectangle** rolled around them. And we know how to find the area of a rectangle - job done!



Frank: Hold up ... how do we figure out the **lengths** of the rectangle's sides? We need those to calculate its area.

Jim: Well, one of the sides is easy - it's the same length as the circumference. But there's no mention of how long the other side is for either of the space stations. Houston, we have a problem!

Joe: We DO know that each space station has the same **volume** - 90000 m^3. When something has straight sides and identical ends (like a cylinder), its volume is area of base × height. So we can use that to work out the height?

Jim: That seems right... but the base is a circle. Which means we need to calculate the **area of a circle**. That's gotta be tricky ...

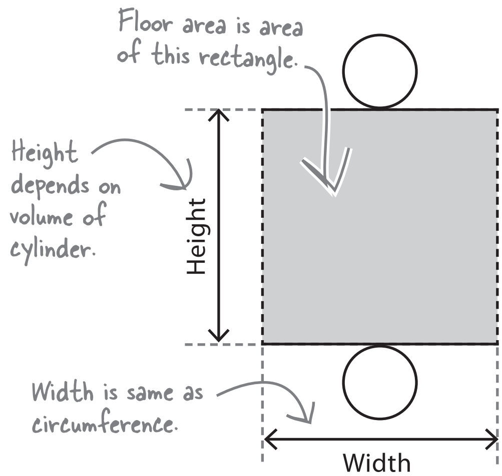
If you're calculating the surface area of a 3D shape, imagine "unrolling" the shape so that the surfaces are flat.

Here, the floor space is the area of a cylinder's curved surface

The astronauts want the design with the larger floor space - which means that you need to calculate the **area** of the curved surface of each cylindrical space station.

If you're calculating a surface area, it's best to 'unroll' the shape so that the surfaces are flat. When you do this, the curved surface becomes a rectangle. That's great, because you already know how to work out the surface area of a rectangle: width × height.

One side of the rectangle is the same length as the circumference, and the other side is the height of the cylinder - so far, so good. But you don't know the cylinder's height, only its volume.



The area of a rectangle is width x height

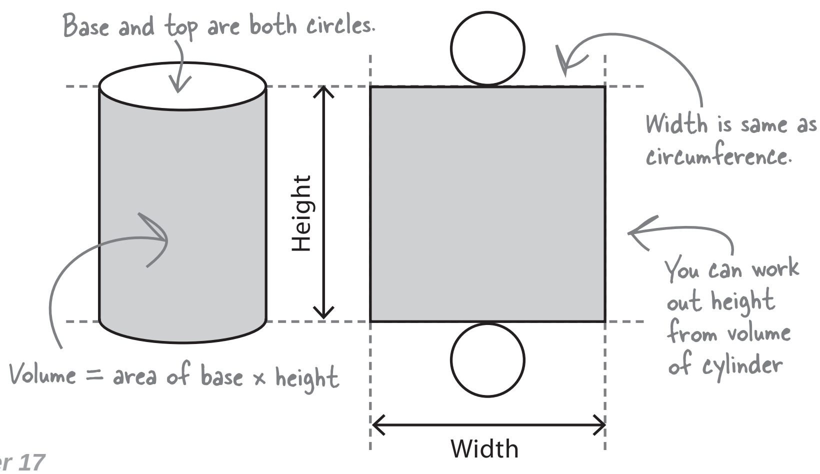
Note the similarities between these two equations.

If you work out the volume, you can calculate the astronauts' floor space

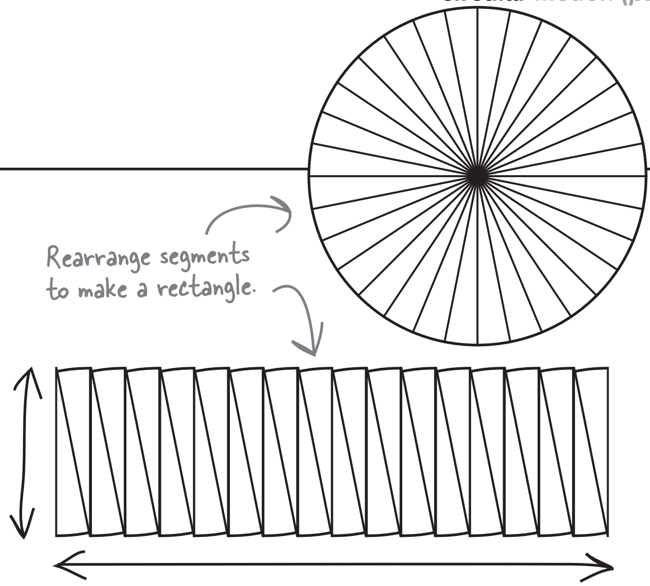
If you have a 3D shape - like a cube or a cylinder - where the base and the top are the same shape with straight sides in between, its volume is **(area of base) × height**.

You already know that both space stations are cylinders with volumes of 90000 m^3. Which means that if you can work out the area of the base (i.e., the area of the circle), you can use the equation to determine the height of the cylinder. Which is great, as that's exactly what you need to calculate the astronauts' floor space.

If a 3D shape has two ends that are the same and straight sides between them, its volume is (area of base) x height



Sharpen your pencil



1. To find a circle's area, you can chop it up into tiny segments and reassemble it into a rectangle. In terms of the circle's radius:

In terms of *r:*

a. What is the height of the rectangle?

b. What is the width of the rectangle?

c. What is the area of the rectangle (and therefore the area of the circle)?

Remember to start with a sketch!

2. Two cylinders each have a volume of 90000 m^3.

a. The first cylinder has a radius of 10.0 m. What is the area of its circular base? What is its height?

b. The second cylinder has a radius of 100 m. What is the area of its circular base? What is its height?

c. What is the area of the curved surface of each cylinder... and which space station has a bigger floor area?

Sharpen your pencil Solution

1. To find a circle's area, you can chop it up into tiny segments and reassemble it into a rectangle. In terms of the circle's radius:

In terms of r:

a. What is the height of the rectangle?

b. What is the width of the rectangle?

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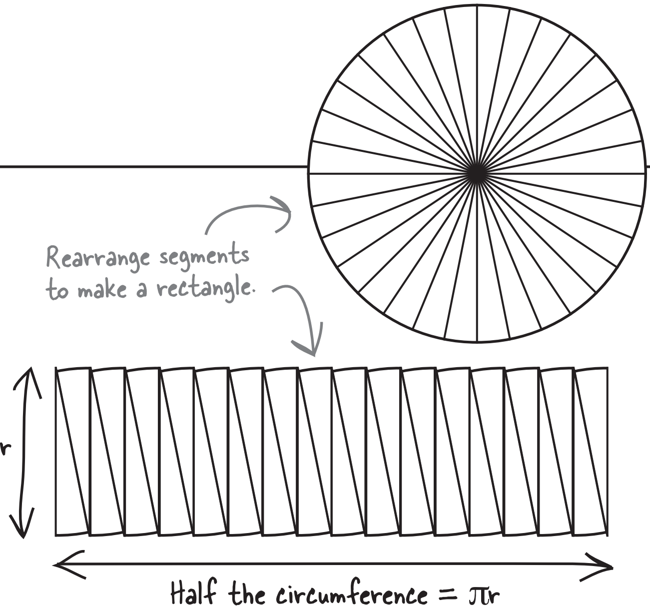


a. The height is the radius = r

b. The width is half the circumference = $\frac{1}{2} \times 2\pi r = \pi r$

c. The area is height × width = $r \times \pi r = \pi r^2$

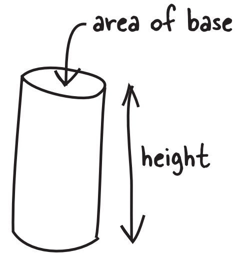
Remember to start with a sketch!

2. Two cylinders each have a volume of 90000 m^3.

a. The first cylinder has a radius of 10.0 m. What is the area of its circular base? What is its height?

b. The second cylinder has a radius of 100 m. What is the area of its circular base? What is its height?

c. What is the area of the curved surface of each cylinder ... and which space station has a bigger 'floor' area?



a. Area of base = $\pi r^2 = 3.14 \times 10.0^2 = 314 \text{ m}^2$ (3 sd)

Get height of cylinder from volume: volume = area of base × height

$$\Rightarrow \text{height} = \frac{\text{volume}}{\text{area of base}} = \frac{90000}{314} = 287 \text{ m} \text{ (3 sd)}$$

b. Area of base = $\pi r^2 = 3.14 \times 100^2 = 31400 \text{ m}^2$ (3 sd)

$$\text{height} = \frac{\text{volume}}{\text{area of base}} = \frac{90000}{31400} = 2.87 \text{ m} \text{ (3 sd)}$$

c. Curved surface is a rectangle: width = circumference = $2\pi r$ height = height of cylinder

For 10 m radius design: Floor area = $2 \times 3.14 \times 10.0 \times 287 = 18000 \text{ m}^2$ (3 sd)

For 100 m radius design: Floor area = $2 \times 3.14 \times 100 \times 2.87 = 1800 \text{ m}^2$ (3 sd)

The 10 m radius design has 10 times the floor space of the 100 m radius one.

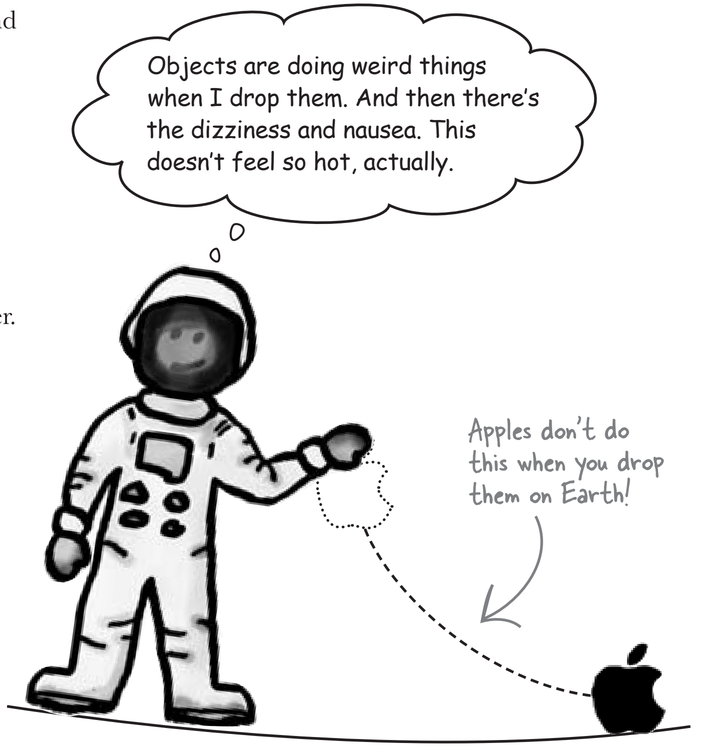
The area of a circle = πr^2

Let's test the space station...

As the tall narrow space station module (the one with the 10.0 m radius that needs to rotate at around 1 rad/s) has the larger floor area, you go ahead and build a test rig for the astronaut to try out.

But weird things are happening! When the astronaut drops an apple, it doesn't fall as you'd expect. And the astronaut's not feeling well either.

What's going on?!



Sharpen your pencil

Jot down some ideas that might help to explain why weird things are going on.

Would using the 100 m radius space station design instead of the 10.0 m space station design help to lessen the weird effects?

Don't worry if you're not sure why these things are happening – just throw some ideas around.

Don't worry if you didn't come up with all of these ideas!

Sharpen your pencil Solution

Jot down some ideas that might help to explain why weird things are going on.

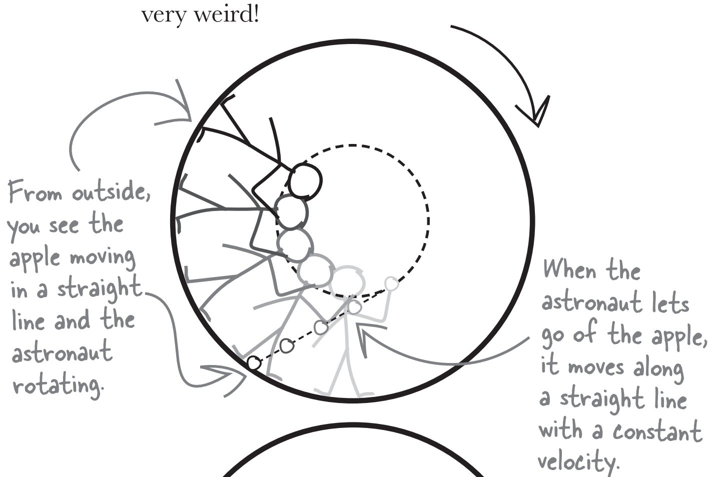
- Maybe his balance or stomach can't cope with high angular velocity.
- His head and feet are at different distances from the center. So they must experience different centripetal accelerations.
- When he drops the apple, the space station keeps rotating, but the apple doesn't. So it looks like it follows a curved path.

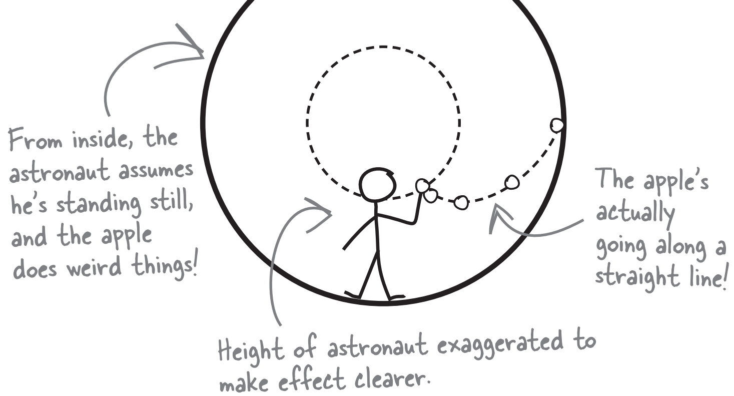
Can't cope with rotation

Your sense of balance isn't designed to work well when you're rotated. Although people can adapt to being spun at low angular velocities, close to 1 rad/s is too fast and will produce nasty symptoms.

Apple falls straight while space station rotates

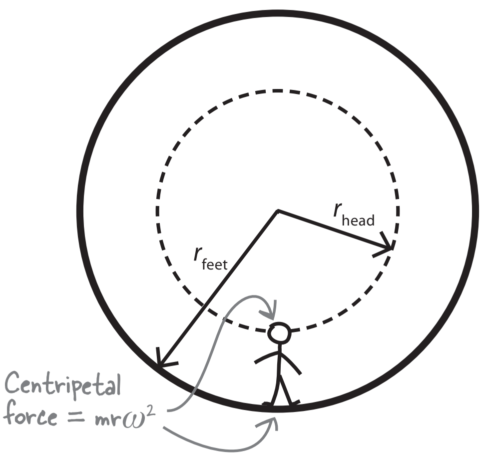
Someone looking at the space station from outside will see the apple moving along a straight line. But to the astronaut in his rotating frame of reference, it looks like the apple follows a curved path. Which looks very weird!





Head and feet at different radii

The centripetal force depends on the radius and is larger the further away something is from the center. So the astronaut will experience a smaller force at his head than he does at his feet. Which will feel a bit weird!

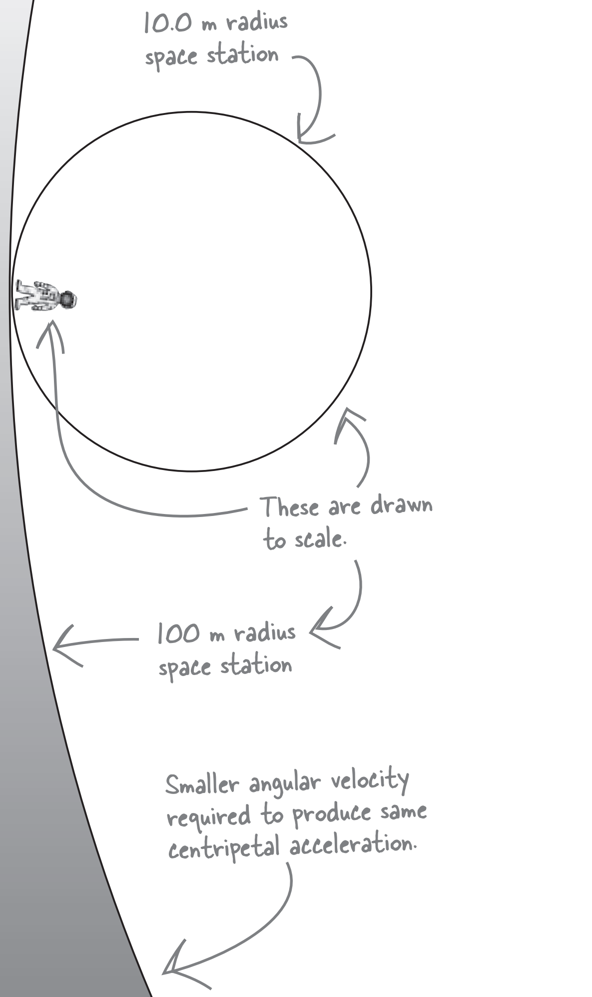


Fewer uncomfortable things happen with the 100 m radius space station

The space station module with the 100 m radius only needs to **rotate** at 0.313 rad/s, compared to the 0.990 rad/s of the module with the 10.0 m radius. This means that the astronaut has a much better chance of getting used to the rotation and feeling fewer ill effects.

The astronaut's head and feet are still **different radii** from the center of the space station, but compared to the radius of the wider space station, this difference is much less.

And when you drop an apple, the 100 m radius space station won't rotate through as great an **angle** as the 10.0 m radius space station, as the angular velocity is lower. Although the apple still won't fall exactly like it would on Earth, the curve in its path will be much less than it was.

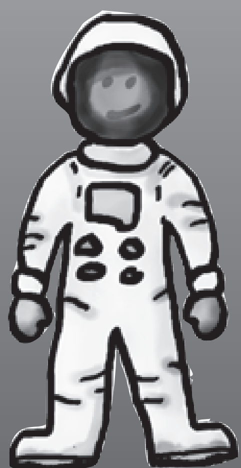


You've sorted out the space station design!

Success! When you spin the 100 m radius test rig, the astronauts are able to walk around on the curved surface without too many ill effects!

Not only have you worked out how to provide 'artificial gravity' in space, you've also managed to understand enough of the physics to choose between two competing space station designs.

Result!



That's it from Head First in Space for now ... though the astronauts are starting to mutter something about going to infinity and beyond?!

Sound exciting? Stay tuned!

Question Clinic: The "Centripetal force" Question

Any time an object follows a curved path, ask yourself whether you may need to think about centripetal force. This is the force required for the object to move in this way: $F_c = mr\omega^2$. You can move between the velocity (v) and the angular velocity (ω) using the equation $v = r\omega$. The larger the radius and the larger the angular velocity, the greater the force required.

This should immediately get you thinking about centripetal force.

Be careful about whether you are being given the RADIUS or the DIAMETER of the circle. Make sure you use the correct value for the radius later on!

2. A carnival ride rotates in a horizontal circle with diameter 20.0 m. People sit in cars on the outside of the circle.

a. Draw a free body diagram for someone sitting on the ride while it isn't moving.

b. Draw a free body diagram for someone sitting on the ride while it is moving. Clearly mark the center of the ride on your diagram.

c. How many revolutions per minute must the ride spin with for the person in the car to experience a horizontal contact force equal to their weight?

Draw all the forces acting ON the person.

Make sure you follow the instructions exactly!

The net force is towards the center of the circle.

Draw all the forces acting ON the person. You know there will be a net centripetal force, but don't label the net force like that. Use a label like "contact force from side of car."

Be careful with units. ω is measured in radians per second.

Don't worry if you don't know the mass. When you say $mr\omega^2 = mg$, the m will divide out.

If you're asked to draw a free body diagram, never EVER label an arrow "centripetal force." The centripetal force is a net force toward the center of the circle that's provided by the forces already on your free body diagram. Oh, and some questions may ask about centripetal acceleration rather than centripetal force. You can just use Newton's 2nd Law, $F_c = ma_c$, to move between F_c and a_c.

there are no Dumb Questions

Q: How should I include the centripetal force when I'm drawing a free body diagram?

A: A free body diagram shows all of the forces acting on a single object. So far, you know about gravitational force (a non-contact force) plus a variety of contact forces - normal force, frictional force, and tension force (exerted by a rope).

Q: None of these forces are the centripetal force though. How do I include the centripetal force on my free body diagram?

A: If you have an object that's moving in a circle, you should include all the forces acting on the object in your free body diagram. When you add these forces together by lining them up nose to tail, there will be a net force on your object that makes it follow a circular path. This centripetal force (the force required to make the object follow a circular path) may be provided by a gravitational force, or a normal force, or a support force, or a tension force.

Q: Are you saying that I shouldn't draw an arrow on my free body diagram and label it centripetal force?

A: Spot on! Your free body diagram should indicate the **origin** of each force vector arrow - gravitational force, normal force, frictional force, tension force, etc.

Centripetal force is the name given to the net force on an object when the net force is making the object follow a circular path. It's not a "ghost force" that appears from nowhere on your free body diagram. So you're right - you shouldn't ever label an arrow on your free body diagram "centripetal force," as this tells you nothing about the origin of the force.

Q: What if the net force doesn't cause the object to follow a circular path?

A: Then it doesn't get called "centripetal force," as this its the term reserved for when the net force does make an object follow a circular path.

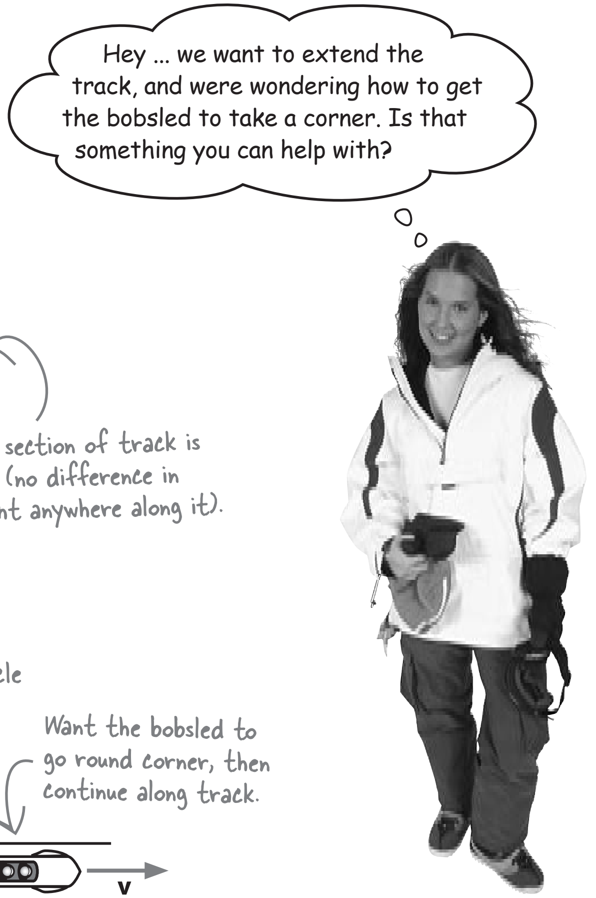
Your free body diagram should indicate the ORIGIN of each of the force vector arrows - gravitational force, normal force, frictional force, tension force, etc.

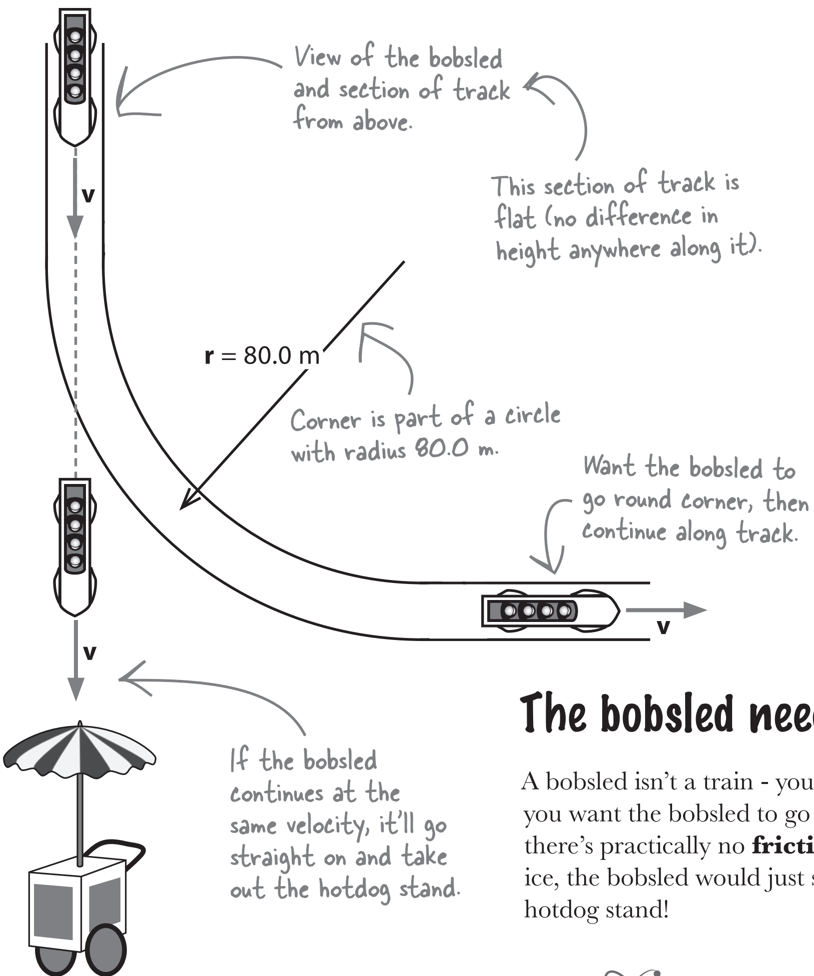
The net force will provide the centripetal force. There shouldn't be any arrows on a free body diagram labelled 'centripetal force'.

Back to the track!

When you last checked in as the track's safety consultant, you were working out the speed the bobsled would have as it passed through various checkpoints.

But they just got a whole lot more ambitious ...





The bobsled needs to turn a corner

A bobsled isn't a train - you can't just point the track where you want the bobsled to go and expect it to follow! Since there's practically no **friction** between the bobsled and the ice, the bobsled would just slide straight on and demolish the hotdog stand!

BRAIN POWER

How might you persuade the bobsled to take the corner instead of going straight on?

Sharpen your pencil

a. The 630 kg bobsled has dropped 50.0 m from a standing start between the beginning of the track and the corner. What is its current speed?

b. The radius of the corner is 80.0 m. What size of centripetal force is required for the bobsled to be able to make it round the corner?

Hint: Think about the shape of a cycling velodrome or Indy car oval.

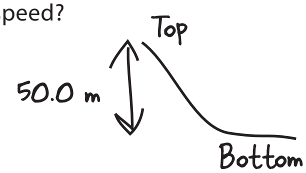
c. In what direction does there need to be a centripetal force to make the bobsled turn the corner?

d. How might the track be modified to provide the force the bobsled needs to make the turn? Draw force vector components to illustrate where the required force could come from if the track was modified.

Sharpen your pencil Solution

a. The 630 kg bobsled has dropped 50.0 m from a standing start between the beginning of the track and the corner. What is its current speed?

Use energy conservation



$$U_{top} = K_{bottom}$$

$$\Rightarrow \cancel{m}gh = \tfrac{1}{2}\cancel{m}v^2$$

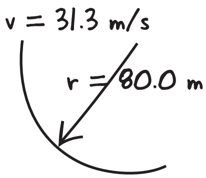
$$\Rightarrow v = \sqrt{2gh} = \sqrt{2 \times 9.8 \times 50.0}$$

$$v = 31.3 \text{ m/s (3 sd)}$$

No need to worry about direction as you're aked about SIZE of force.

b. The radius of the corner is 80.0 m. What size of centripetal force is required for the bobsled to be able to make it round the corner?

Calculate centripetal force, F_c



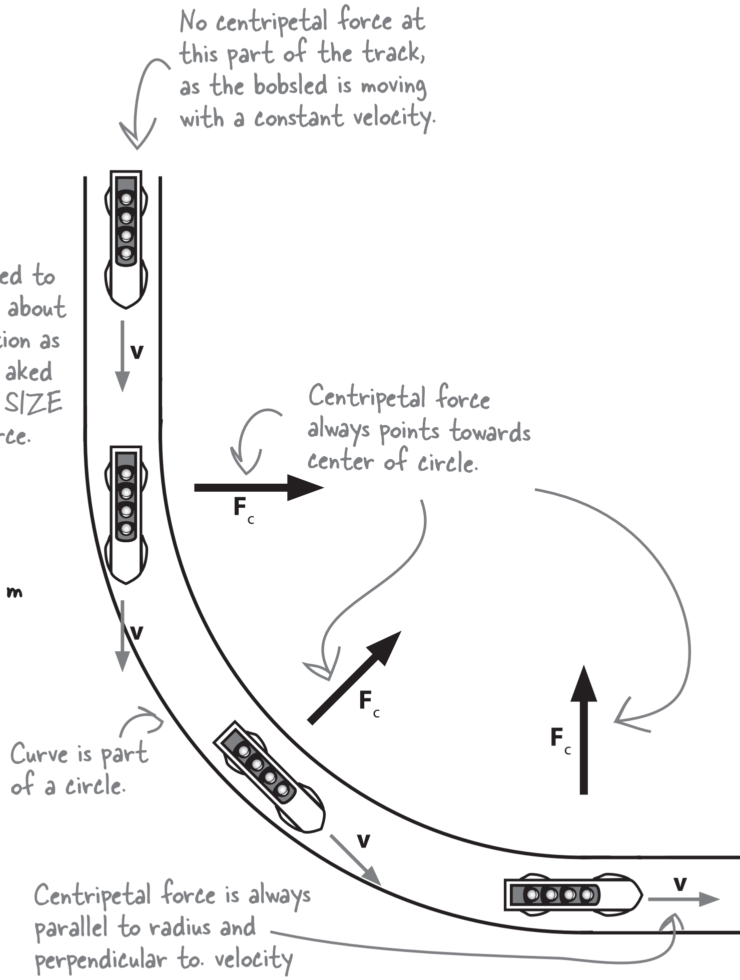
$$F_c = mr\omega^2$$

Use $v = r\omega$ to work out ω

$$\Rightarrow \omega = \frac{v}{r} = \frac{31.3}{80.0} = 0.391 \text{ rad/s}$$

$$F_c = mr\omega^2 = 630 \times 80.0 \times 0.391^2$$

$$F_c = 7710 \text{ N (3 sd)}$$

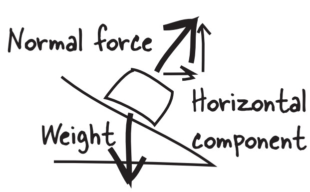


c. In what direction does there need to be a centripetal force to make the bobsled turn the corner?

There needs to be a centripetal force towards the center of the corner.

d. How might the track be modified to provide the force the bobsled needs to make the turn? Draw force vector components to illustrate where the required force could come from if the track was modified.

Banking the track towards the center of the corner to redirect the normal force might work, as there would be a component pointing towards the center of the corner.

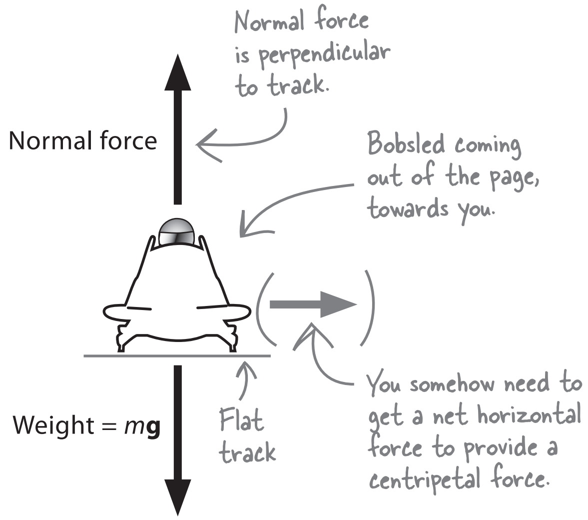


For the bobsled to go around a corner, you need a net force pointing towards the CENTER of the circle to provide a centripetal force.

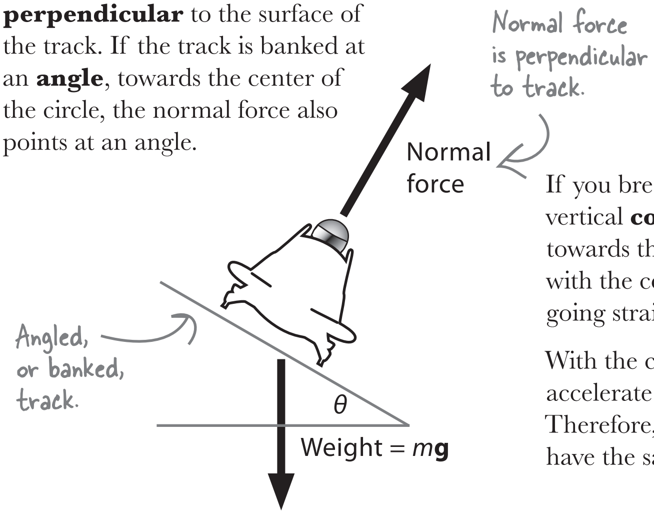
Angling the track gives the normal force a horizontal component

There are two forces acting on the bobsled - its **weight** and the **normal force**. To make the bobsled turn the corner, you need to have a **net force** pointing **horizontally** towards the center of the circle, exerting a **centripetal force** on the bobsled that makes it turn.

The bobsled's weight points vertically downwards, so there's no way that any component of the weight can point horizontally. And if the track is horizontal, the normal force points verticallty upwards, with no horizontal component.



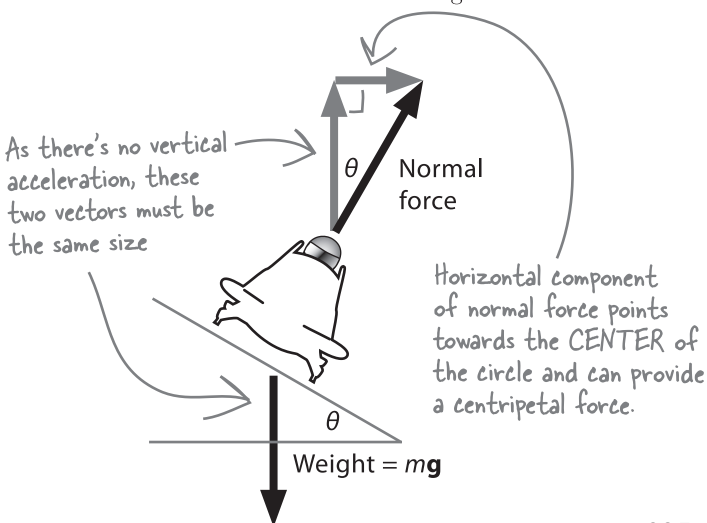
The normal force always acts **perpendicular** to the surface of the track. If the track is banked at an **angle**, towards the center of the circle, the normal force also points at an angle.



If you break the **normal force** down into horizontal and vertical **components**, the horizontal component points towards the **center** of the curve. This can provide the bobsled with the centripetal force it needs to take the corner instead of going straight on.

With the curve banked at the correct angle, the bobsled won't accelerate vertically and the net vertical force must be zero. Therefore, the vertical component of the normal force must have the same size as the bobsled's weight.

The normal force always acts PERPENDICULAR to a surface.

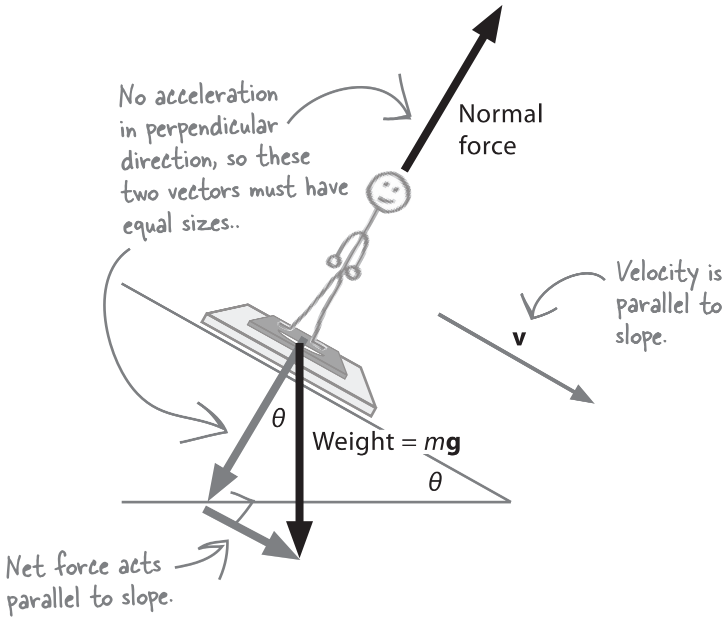


Hang on! Last time we were dealing with angles, the normal force was always a component of the weight. But this time, it looks like it's more than the weight - and that can't be right!

When you slide downhill, there's no perpendicular acceleration

The NET FORCE acts PARALLEL to the slope.

When an object slides down a slope, it accelerates parallel to the slope, **but doesn't accelerate perpendicular to the slope**. This means that the components of the forces perpendicular to the slope must add up to zero.



The only two forces with components perpendicular to the slope are the **weight** and the **normal force**. Therefore, the components of the weight and normal force, which are perpendicular to the slope, must add up to zero so that **the net force is parallel** to the slope.

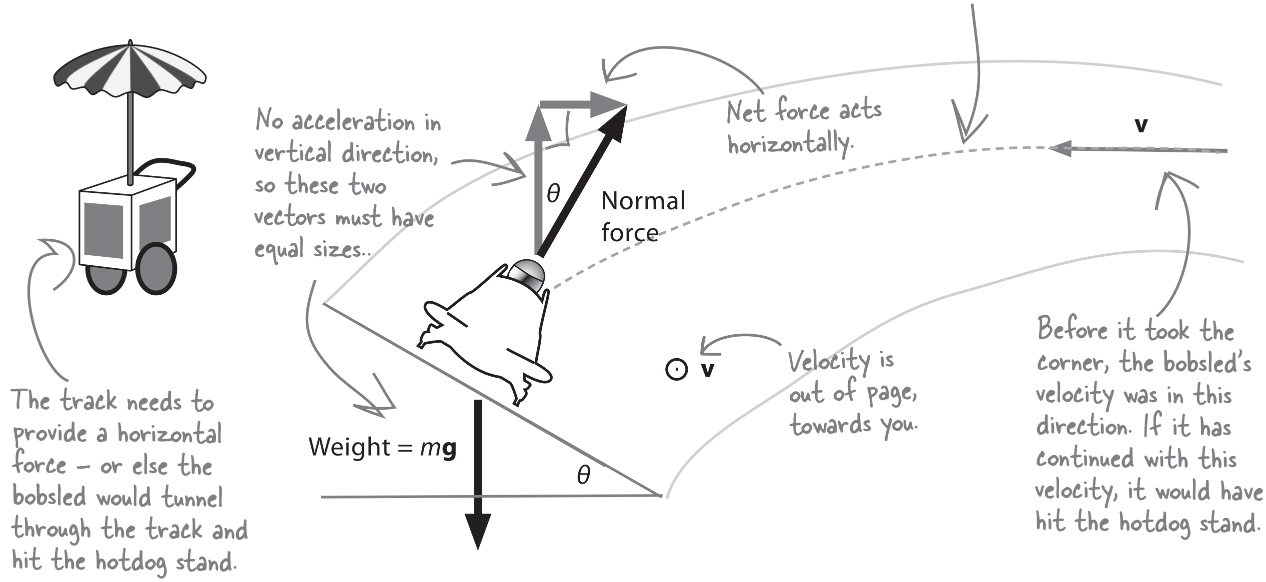
When you turn a corner, there's no vertical acceleration

The NET FORCE acts HORIZONTALLY.

For the bobsled to go round the corner, there must be a net horizontal force acting on it to provide the centripetal force it requires.

This means that the bobsled **doesn't accelerate vertically**, only horizontally. So the vertical components of the forces acting on the bobsled must add up to zero.

The bobsled doesn't float in the air, or go down into the ground. We don't want it to slide UP or DOWN the banking either.



There's no acceleration in the direction 90° to the net force. So the forces in that direction must add to zero.

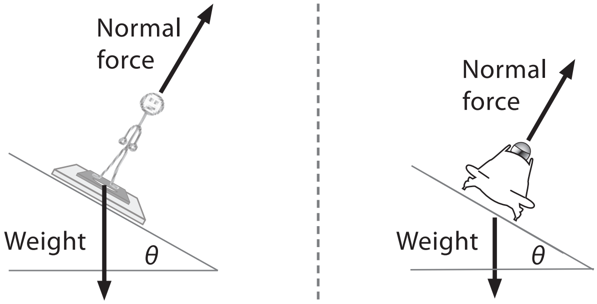
The only two forces with vertical components are the bobsled's **weight** and the **normal force**. Therefore, the vertical component of the normal force and the weight must add to zero so that **the net force is the horizontal centripetal force** the bobsled requires to take the corner.

Not only does the track have to support the bobsled's weight, it also has to prevent the bobsled from tunneling horizontally through the track, and into the hotdog stand. This is why the normal force has a horizontal component. The horizontal centripetal force is provided by the horizontal component of the normal force.

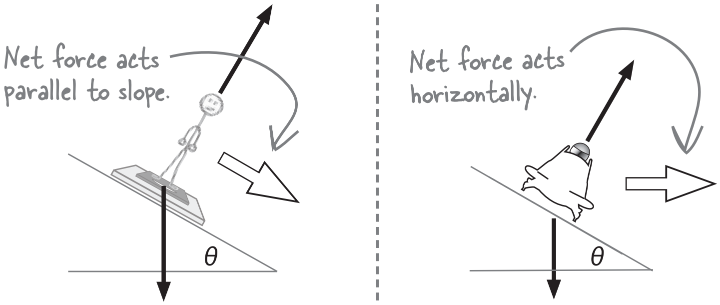
Because the normal force is doing these two jobs (supporting the bobsled's weight and preventing it from tunneling through the track), the size of the normal force must be larger than the size of the bobsled's weight.

How to deal with an object on a slope

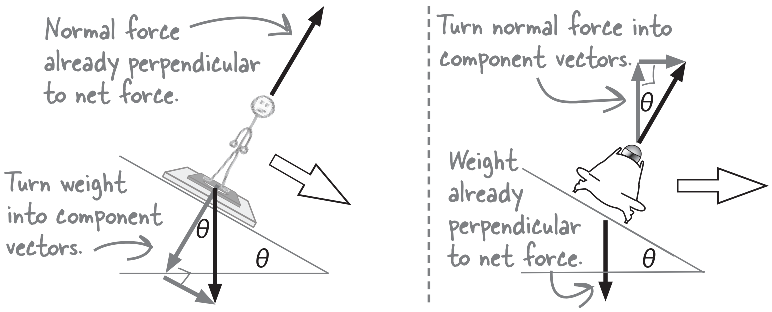
1. Start with a free body diagram



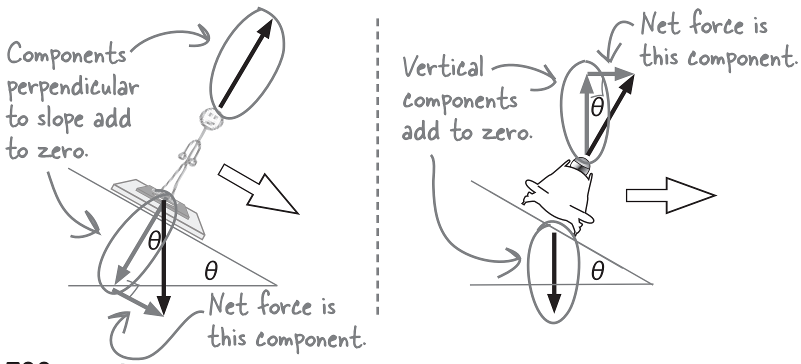
2. Work out the direction of the net force.



3. Draw in components parallel and perpendicular to the net force.



4. The components perpendicular to the net force must add to zero.



there are no Dumb Questions

Q: How can I work out which force components add to zero when an object is on a slope?

A: First of all, work out the direction of the net force. This is the direction that the object accelerates in. Then draw in components of forces parallel and perpendicular to the net force. The components perpendicular to the net force must add to zero.

Q: How can the normal force from a track be larger than an object's weight?

A: This can happen if the track is banked so that the object can take a corner. As well as providing a vertical force to support the object's weight, the track also needs to provide a horizontal force to stop the object from tunneling straight on through the track. So the total normal force - the normal force that the track exerts on the object - will have a vertical 'weight support' component and a horizontal 'anti-tunneling' component.

Q: How do you know the net force is horizontal for an object taking a corner?

A: The net horizontal force provides the centripetal acceleration required for the object to take the corner.

Start off with a free body diagram and work out which components ADD TO ZERO before you draw in the components.

Sharpen your pencil

You already worked this out earlier on.

A 630 kg bobsled traveling at 31.3 m/s requires a horizontal centripetal force of 7710 N to make it go around a corner with a radius of 80.0 m at a constant speed. You can provide the centripetal force by banking the track.

a. Draw a free body diagram of the bobsled taking the corner on an angled track, showing all the forces acting on it.

b. What is the weight of the bobsled?

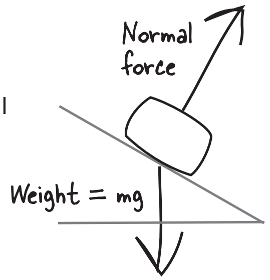
c. What do the horizontal and vertical components of the normal force need to be for the bobsled to go around the corner without sliding up or down the banking?

d. What angle should the track make with the horizontal to achieve this?

Sharpen your pencil Solution

A 630 kg bobsled traveling at 31.3 m/s requires a horizontal centripetal force of 7710 N to make it go around a corner with a radius of 80.0 m at a constant speed. You can provide the centripetal force by banking the track.

a. Draw a free body diagram of the bobsled taking the corner on an angled track, showing all the forces acting on it.



b. What is the weight of the bobsled?

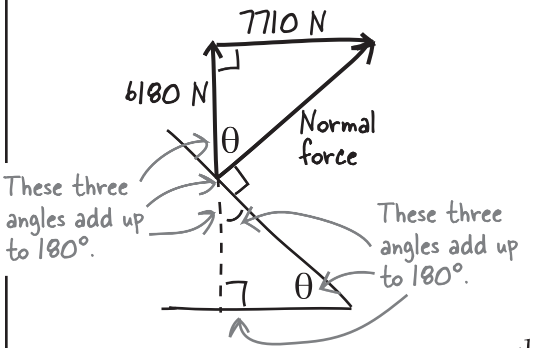
$$\text{Weight} = mg$$
$$\text{Weight} = 630 \times 9.8$$
$$\text{Weight} = \underline{\underline{6180\ \text{N}}}\ (3\ \text{sd})$$

c. What do the horizontal and vertical components of the normal force need to be for the bobsled to go around the corner without sliding up or down the banking?

Horizontal component = 7710 N (provides centripetal force)

Vertical component = 6180 N (must be same size as weight - no acceleration in vertical direction)

d. What angle should the track make with the horizontal to achieve this?



Banking angle of track:

$$\tan\theta = \frac{o}{a} = \frac{7710}{6180} = 1.25\ (3\ \text{sd})$$

$$\Rightarrow \theta = \tan^{-1}(1.25) = \underline{\underline{51.3°}}\ (3\ \text{sd})$$

See page 562 of chapter 14 for help with setting up triangles like this.

there are no Dumb Questions

Q: So - what if the bobsled was going faster or slower? Would the banking angle need to be different?

A: You originally worked out the **required** centripetal force using the bobsled's velocity and the radius of the curve. If the bobsled was going faster or slower, it would require more or less centripetal force.

Q: If the bobsled has a lower mass, will it require a smaller centripetal force?

A: Yes, that's right. But as the bobsled's weight is less, the normal force will also be less, and the banking angle will work out the same. The banking angle doesn't depend on the bobsled's mass - only on the bobsled's speed and the radius of the corner.

Q: So could the bobsled just go round and round forever if I closed off the curve to make a circle?

A: If there was no friction, then it could, but there's always some friction somewhere to transfer kinetic energy to internal energy. So it couldn't keep on going forever.

Banking the track works ...

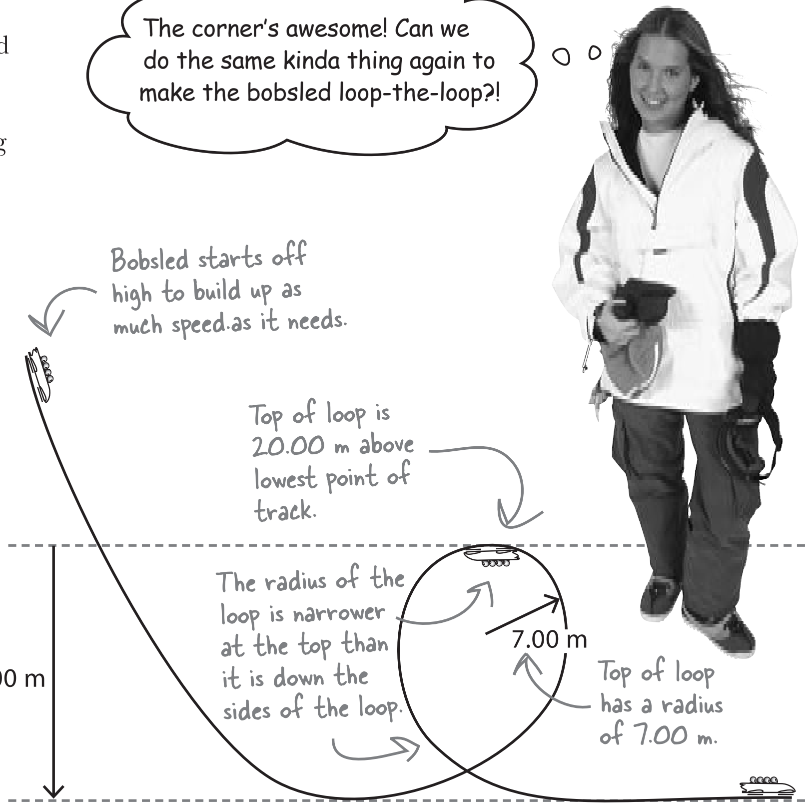
Banking the track at 51.3° like you calculated does the trick! The hotdog stand is saved, and everyone at the bobsled track is happy. Though they've just come up with something even more ambitious ...

The corner's awesome! Can we do the same kinda thing again to make the bobsled loop-the-loop?!

... but now they want it to loop-the-loop!

Inspired by the success of your corner, you've just been asked if you can make the bobsled loop-the-loop at the end of the track!

As the bobsled is traveling in a circle (just like it was for the corner), the track designers are confident that you can make it work ...



Does the fact it's a **vertical** loop change which forces we need to think about?

Try it!

Are horizontal and vertical circles the same? Try it! Tie an object to the end of a piece of string, and swing it **as slowly as possible** in horizontal and vertical circles. Feel what happens to the **tension** in the string as the tension provides a centripetal force.

How does the tension in the string vary at different points as you swing the object in horizontal and vertical circles? Do you need to pull harder or more gently when the object's at the top or bottom of the circle?

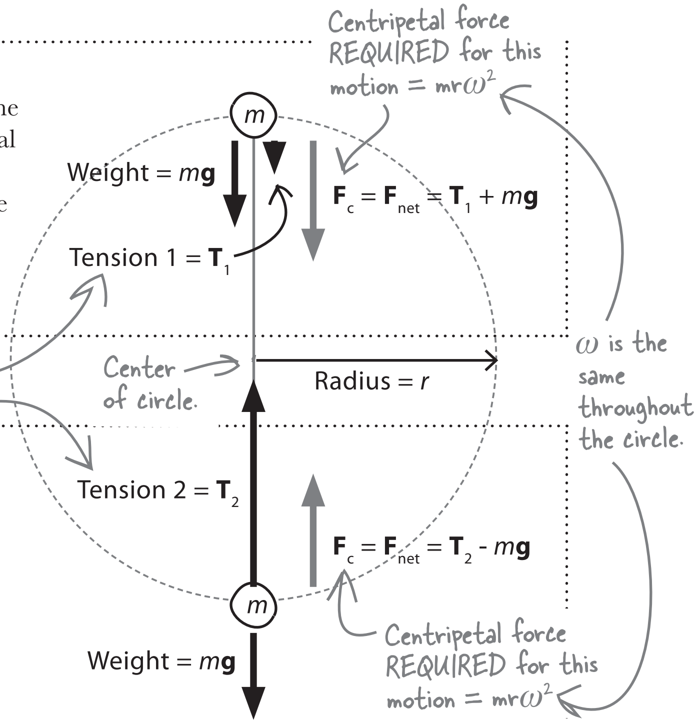
The "support force" (normal force or tension force) required for a vertical circle varies

For an object moving in a **horizontal circle**, the tension in the string provides the centripetal acceleration to keep the object moving. If the object moves at a constant speed, the **tension remains constant**.

This is the same as for the horizontal corner. The banking angle was the same all the way round, providing the same horizontal component of the normal force all the way round.

For a **vertical circle**, the tension in the string is less at the top of the circle than it is at the bottom - as long as the speed of the object remains the same throughout.

At the **top of the circle**, the force of the object's weight points towards the center of the circle, and can provide much of the centripetal force that the object needs to move in a circle (as long as the object's going at an appropriate speed). If the object's going faster than the minimum speed, the centripetal force will be provided by weight + tension in the string.



This is the force that you need to pull the string with.

At the **bottom of the circle**, the object's weight points directly away from the center of the circle, so the centripetal force must all be provided by the tension in the string.

As the object's weight is pointing away from the center of the circle, the tension in the string must be equal to the weight + the required centripetal force.

You might have observed the object on the string kind-of "falling over the top" as you swung it slowly.

You might have noticed the object on the string "pulling against you" when it swung round the bottom of the circle.

At the top of the circle, the object's WEIGHT is able to provide some of the centripetal force, as the weight vector points towards the CENTER of the circle.

I don't agree. Surely there's more force at the top - because gravity's adding to the centripetal force from the string?

Centripetal force is required for circular motion and provided by existing forces

At the top of the circle, the object has a horizontal velocity. Without the string, the object would follow a curved path anyway (just not a circular one).

You mustn't think of centripetal force as a 'ghost force' that appears from nowhere when an object moves in a circle. Circular motion is only possible if a force capable of providing the centripetal force is **already present**.

At the top of the circle, the centripetal force can actually be entirely provided by the gravitational force, as it points towards the center of the circle. At the minimum speed for circular motion, the gravitational force, $m\mathbf{g}$, will **provide** all of the **required** centripetal force, so will be **equal** in size to $mr\omega^2$.

This exercise lets you get the hang of dealing with vertical circles before you tackle the bobsled track.

Exercise

You want to swing a bucket of water, mass m, in a vertical circle at a constant speed, v.

a. Write down an equation that relates the mass of the bucket, the length of your arm and the size of the required centripetal force.

b. If your arm has length r, what's the minimum speed, v, you need to swing it at so that you don't get wet when the bucket passes over your head?

At the minimum speed, the centripetal force is provided entirely by the gravitational force.

c. In terms of m and $\mathbf{g}$, what force does the base of the bucket exert on the water at the top of the circle?

d. In terms of m and $\mathbf{g}$, what force does the base of the bucket exert on the water at the bottom of the circle if you swing it at the same velocity round the whole circle?

Your arm exerts a force on the bucket – and the base of the bucket transmits that force to the water.

Draw a free body diagram to work each of these out! Look at the ones on the opposite page to help you.

Exercise Solution

You want to swing a bucket of water, mass *m*, in a vertical circle at a constant speed, *v*.

a. Write down an equation that relates the mass of the bucket, the length of your arm and the size of the required centripetal force.

$F_c = mr\omega^2$

m is mass of bucket.
r is length of arm.
ω is angular velocity.

b. If your arm has length r, what's the minimum speed, *v*, you need to swing it at so that you don't get wet when the bucket passes over your head?

$F_c = mr\omega^2$

$v = r\omega \Rightarrow \omega = \frac{v}{r}$

$F_c = \frac{mv^2}{r}$

Minimum force = mg = F_c

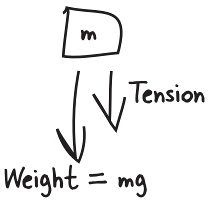
$\Rightarrow mg = \frac{mv^2}{r}$ (m crossed out)

$v^2 = gr$

$v = \sqrt{gr}$

c. In terms of *m* and g, what force does the base of the bucket exert on the water at the top of the circle?

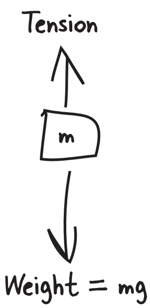
Tension of force of string on bucket transmitted to water by base of bucket.



But net force is F_c = mg.
So tension = 0.

Force exerted by base of bucket on water = 0

d. In terms of *m* and g, what force does the base of the bucket exert on the water at the bottom of the circle if you swing it at the same velocity round the whole circle?



Continues at same speed, so F_c = mg as centripetal force same for same speed.

F_c is net force, so Tension + Weight must be equal to F_c

Up positive direction: $T - mg = F_c$

But $F_c = mg$ $\Rightarrow$ $T - mg = mg$
make substitution

$T = 2\,mg$

I guess that with the bobsled, the normal force from the track does the job of supporting the circular motion, like the bucket does here?

Your free body diagram is key

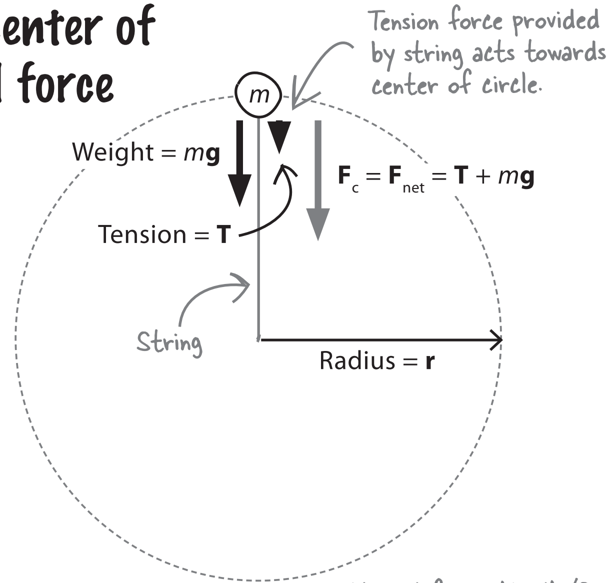
When you draw a free body diagram, take special note of the forces pointing towards the **center** of the circle. These have the potential to contribute towards a net centripetal force that makes an object follow a circular path.

For the bobsled, the normal force from the track will point towards the center of the circle.

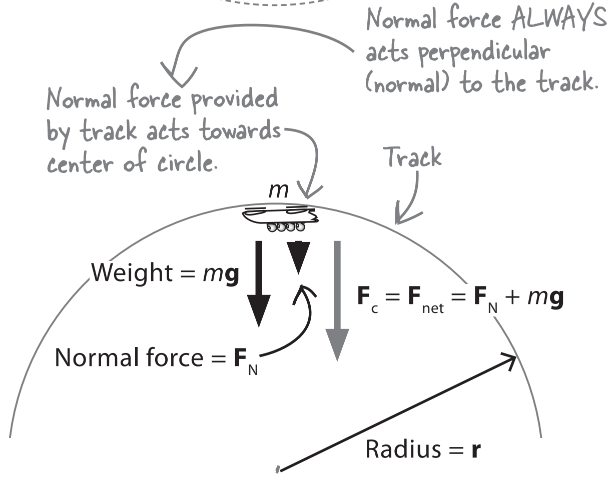
Any force that acts towards the center of the circle can provide a centripetal force

For an object on a string at the top of a circle, there are two forces that can point towards the center of the circle and, therefore, contribute towards a net centripetal force in that direction. One is the **tension** force from the string, and the other is the **gravitational** force - the object's **weight**.

The tension force + the component of the gravitational force that points towards the center of the circle will always add up the centripetal force required to move the object at that speed around a circle with that radius.



The components of all the forces that point towards the center of the circle must add up to the centripetal force.



Think of the normal force provided by the track as an "anti-tunneling" force that stops the bobsled from tunneling through the track.

The bobsled's free body diagram is nearly identical to the object on the string, except that it's the **normal force** from the (circular) track that points towards the center of the circle instead of a tension force.

You can think of the normal force as an "anti tunneling" force. If the bobsled is going upwards with a high speed and the track is soft and can't provide enough normal force, the bobsled will tunnel into - and probably through - the track. The normal force provided by a solid track prevents the bobsled from tunneling, and provides some of the centripetal force that the bobsled requires to follow a circular path.

there are no Dumb Questions

Q: So how can the gravitational force provide a centripetal force?! Surely if that was happening, the bobsled would just fall straight down, off the track.

A: To make it around the loop, the bobsled must already be traveling very fast horizontally when it gets to the top of the circle. So it couldn't possibly fall straight down - even if the track wasn't there, it would move through the air like a projectile.

Q: But what if the bobsled wasn't going fast enough at the top of the loop?

A: Then there wouldn't be enough centripetal force to keep the bobsled on the track - it would curve down towards the ground faster than the curve of the track, and fall off the track.

Q: How do I know if the bobsled's going fast enough or not?

A: The size of the centripetal force is given by the equation $F_c = mr\omega^2$, which can also be written $F_c = \frac{mv^2}{r}$ by making a substitution using the equation $v = r\omega$.

The faster the bobsled's going, the larger the centripetal force required to make it travel in a circle with radius *t*.

Q: Doesn't that equation give you the value of the centripetal force?

A: Not quite. The equation gives you the value of the centripetal force that is **required** for the bobsled to move in that way. If the net force on the free body diagram isn't large enough, then the bobsled won't get round the loop safely.

Q: What else might the centripetal force be provided by, apart from gravity?

A: The normal force from the track always points perpendicular to the track. So when the bobsled is upside-down, the normal force points down.

Think of the normal force as an "anti-burrowing" force that prevents the bobsled from burrowing through the track if it's going really fast.

Q: How can I work out whether the track is safe or not?

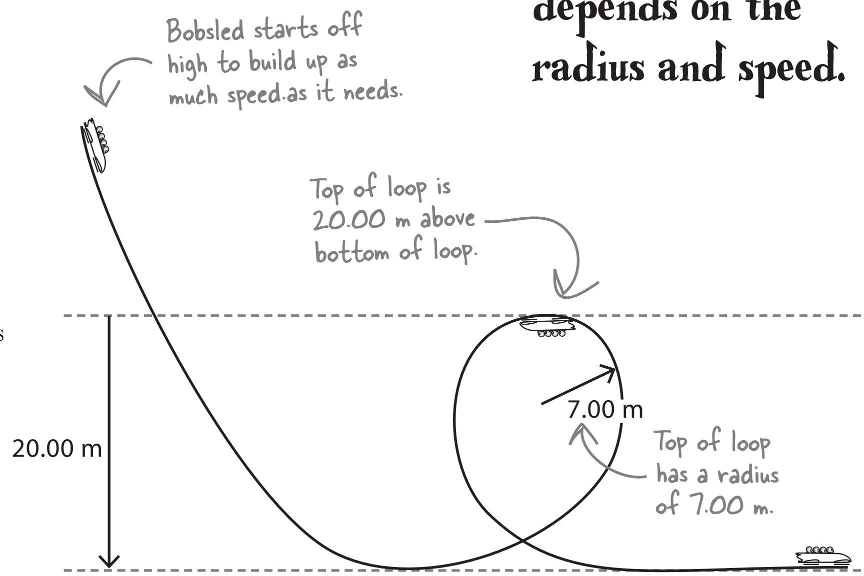
A: Your job is to calculate the minimum speed that the bobsled needs to be going at to get around the loop safely. Which we're just getting on to now ...

The required centripetal force depends on the radius and speed.

How fast does the bobsled need to go?

Your job is to calculate the minimum speed that the bobsled needs to be going at to make it around the 7.00 m radius loop.

At the top of the circle, the bobsled's **weight** points towards the center and can provide all the centripetal force the bobsled needs to go in a circle - as long as it's going at just the right speed.



Sharpen your pencil

Hint: You have various differences in height here...

A 630 kg bobsled is to do a loop-the-loop. The loop has a 7.00 m radius at its top, which is 20.00 m higher than the bottom of the loop. The bobsled can start as high as you want it to above the bottom of the loop.

a. What is the minimum speed the bobsled needs to have at the top of the loop in order to make it around the loop successfully?

b. What speed does the bobsled need to have when it enters bottom of the loop in order to achieve this?

c. What is the minimum height the bobsled needs to start at to achieve these speeds?

d. Now assume that the bobsled is traveling at 10.0 m/s at the top of the loop (instead of the speed you calculated in part a). How large is the normal force that the track exerts on it?

The normal force is like an "anti-tunneling" force that provides a centripetal force and makes the bobsled follow the track instead of tunneling straight through the track.

Sharpen your pencil Solution

A 630 kg bobsled is to do a loop-the-loop. The loop has a 7.00 m radius at its top, which is 20.00 m higher than the bottom of the loop. The bobsled can start as high as you want it to above the bottom of the loop.

a. What is the minimum speed the bobsled needs to have at the top of the loop in order to make it around the loop successfully?

$$F_c = mr\omega^2$$

$$v = r\omega \Rightarrow \omega = \frac{v}{r}$$

$$F_c = \frac{mv^2}{r}$$

At minimum speed, all of F_c provided by weight.

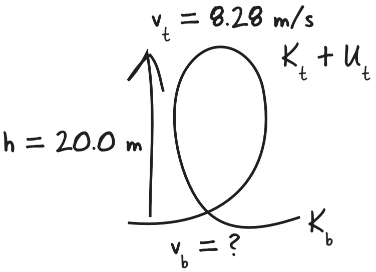
$$F_c = \frac{mv^2}{r}$$

$$\Rightarrow mg = \frac{mv^2}{r}$$

$$v^2 = gr$$

$$v = \sqrt{gr} = \sqrt{9.8 \times 7.00} = \underline{\underline{8.28\ m/s}}\ (3\ sd)$$

b. What speed does the bobsled need to have when it enters bottom of the loop in order to achieve this?



Energy conservation:

$$K_b = K_t + U_t$$

$$\tfrac{1}{2}mv_b^2 = \tfrac{1}{2}mv_t^2 + mgh$$

$$v_b^2 = v_t^2 + 2gh$$

$$v_b = \sqrt{v_t^2 + 2gh} = \sqrt{8.28^2 + 2 \times 9.8 \times 20.00} = \underline{\underline{21.5\ m/s}}\ (3\ sd)$$

We've used the subscripts 't' and 'b' to mean 'top' and 'bottom' of the loop.

c. What is the minimum height the bobsled needs to start at to achieve these speeds?

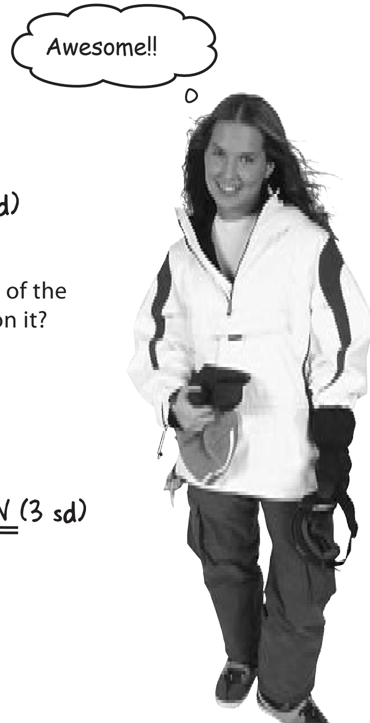
We've used the subscript 'O' to mean the initial conditions atthe start of the track.

Energy conservation:

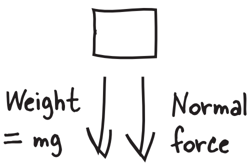
$$U_0 = K_b$$

$$mgh = \tfrac{1}{2}mv_b^2$$

$$h = \frac{\tfrac{1}{2}v_b^2}{g} = \frac{0.5 \times 21.5^2}{9.8} = \underline{\underline{23.6\ m}}\ (3\ sd)$$



d. Now assume that the bobsled is traveling at 10.0 m/s at the top of the loop (instead of the speed you calculated in part a). How large is the normal force that the track exerts on it?



$$F_c = \frac{mv^2}{r} = \frac{630 \times 10^2}{7.00} = 9000\ N\ (3\ sd)$$

$$F_c = \text{weight} + \text{normal force} = mg + F_N$$

$$F_N = F_c - mg = 9000 - 630 \times 9.8 = \underline{\underline{2830\ N}}\ (3\ sd)$$

Normal force stops bobsled burrowing on upwards.

Question Clinic: The "Banked curve" Question

The banked curve question tests your understanding of forces, free body diagrams, components, Newton's laws, circles, and trigonometry - all in all, a good and varied workout! Usually, you'll need to calculate the centripetal force required in order for the object to make it around the curve, then make this equal to the net force that points towards the center of the circle, which comes from the horizontal component of the normal force.

Keep the mass as 'm' for the moment, to avoid numerical errors. It'll divide out later on, when you equate the centripetal force to the horizontal component of the normal force.

These figures give you the information you need to calculate the centripetal force, $F_c = mr\omega^2$

3. A 630 kg bobsled traveling at 31.3 m/s needs to go around a corner with a radius of 80.0 m at a constant speed.

a. What size and direction of centripetal force is required for the bobsled to be able to make it round the corner?

b. You can provide the required centripetal force by banking the track. Explain, using a diagram, why this is the case.

c. What angle should the track make with the horizontal to provide the centripetal force that is required?

Don't forget to say that the direction is horizontal, towards the center of the circle that the bend forms part of.

This means draw a free body diagram and show that the vertical components of the forces add to zero, leaving a net horizontal force that can provide the centripetal force.

Some questions may be about centripetal acceleration instead of centripetal force. Use Newton's 2nd Law, $F = ma$ to move between these two things.

Make sure you calculate the correct angle! Spot similar triangles then think to see if your answer SUCKs.

Any time you're dealing with a slope, start with a free body diagram and work out which direction there's no acceleration in. Then resolve your force vectors into components parallel and perpendicular to this direction. In the banked curve question, there's no acceleration in the vertical direction - which helps you to see that it's the horizontal component that you need to make equal to the centripetal force.

Question Clinic: The "Vertical circle" Question

The vertical circle question can lead to a lot of confusion about what forces are in play and the direction that they point in. The **normal force** is always **perpendicular** to the surface - even if something's upside-down! And the force of something's weight always points downwards. The vertical circle also involves a change in **height** - so be on the lookout for places you can use **energy conservation**.

The mass will divide out when you make the centripetal force equal the net force on your free body diagram.

Difference in height – think potential energy!

This is the value of radius you'll need to use for calculating the required centripetal force.

2. A 630 kg bobsled is to do a loop-the-loop. The loop has a 7.00 m radius at its top, which is 20.00 m higher than the bottom of the loop.

a. What is the minimum speed the bobsled needs to have at the top in order to make it around the loop successfully?

b. How fast does the bobsled need to be going when it enters the bottom of the loop in order to achieve this?

c. What is the minimum height the bobsled needs to start at to achieve these speeds?

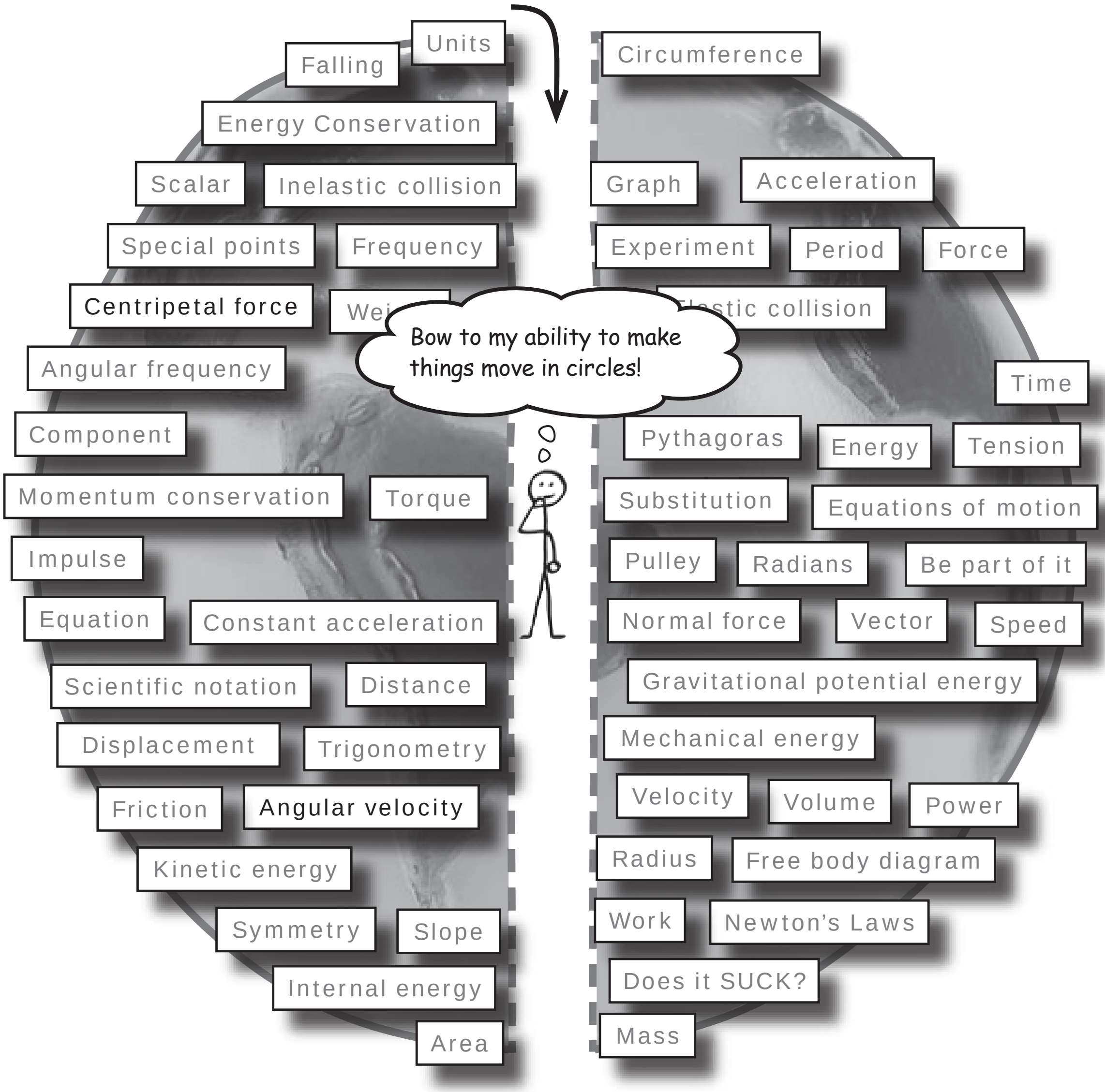
d. Now assume that the bobsled is traveling at 10.0 m/s at the top of the loop (instead of the speed you calculated in part a). How large is the normal force that the track exerts on it?

This is when all the centripetal force is provided by the bobsled's weight.

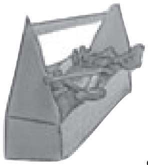
This involves energy conservation.

You need to redo everything as the required centripetal acceleration for this speed will be greater. Part will come from its weight, and part from a normal force exerted by the track.

Always, always, always start with a **free body diagram**! Mark on the real **forces**, and work out which of them contribute towards the **centripetal** force. The **minimum speed** for a successful loop-the-loop is shorthand for saying that the object's **weight** provides all the centripetal force when the thing is upside-down. If it's going faster than that, then some of the centripetal force will be provided by the **normal force** from the track.



☑	Centripetal force	The net force required to make an object travel along a circular path. Always points towards the center of the circle. Size given by $F_c = mr\boldsymbol{\omega}^2$
☑	Angular velocity	The size of the angular velocity is exactly the same as the angular frequency or angular speed.



Your Physics Toolbox

You've got Chapter 17 under your belt and added some terminology and problem-solving skills to your tool box.

Freefall

Something is said to be in freefall when the only force acting on it is its weight.

Examples of objects in freefall are a parachute jumper (if there's no air resistance) and an object orbiting the Earth.

"What's pushing me?"

This is the question you need to ask yourself in order to spot the contact forces that are present.

Shut your eyes and think "What's pushing me?"

You can't experience a non-contact force (such as your weight) in this way.

Volumes and areas

If you have a 3D shape, try to unfold it into 2D shapes you know how to deal with.

Area of circle = πr^2

For a 3D volume with straight sides and the same shape of flat base and top:

Volume = area of base × height

~~Centrifugal force~~

Don't talk about this. Naughty!!

Centripetal force

The force required to move an object round a curved path.

A greater centripetal force is required for a higher speed or a larger radius.

Equation: $F_c = mr\omega^2$

with $v = r\omega$.

Solving problems that involve a slope

Start with a free body diagram

Work out the direction of the net force (the direction that the object accelerates in).

Draw on components parallel and perpendicular to the net force.

The components perpendicular to the net force must add up to zero.

Solving centripetal force problems

Start with a free body diagram

The centripetal force doesn't appear from nowhere – it's the NET FORCE on your free body diagram and it acts towards the CENTER of the circle.

18 gravitation and orbits

Getting away from it all

So far, you've been up close and personal with gravity - but what happens to the attraction as your feet leave the ground? In this chapter, you'll learn that gravitation is an **inverse square law**, and harness the power of **gravitational potential** to take a trip to **infinity**... and beyond. Closer to home, you'll learn how to deal with **orbits** - and learn how they can revolutionize your communication skills.

Party planners, a big event, and lots of cheese

The local party planner is in need of your help. They're catering a big event, and the centerpiece is a huge, innovative globe of cheese.

But there's more... the planner has sent over instructions.

The notes include a cross-section of the cheese globe.

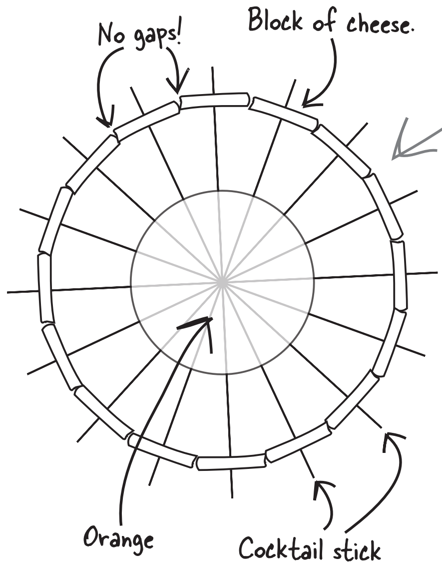
Once-in-a-lifetime cheese globe

The center of the cheese globe is an orange.

Cocktail sticks are inserted into the center of the orange, and radiate outwards. At the outer ends of the sticks are blocks of cheese, all around the outside of the globe. The cheese forms a "shell" around the orange.

Assume you've got a machine that can put the sticks in the orange at regular intervals.

Cheese blocks must be 2.0 cm by 2.0 cm x 0.50 cm, with the square face outwards. Each cheese globe should have 500 blocks.



Each stick should protrude by 2.0 cm from the end of the cheese block, so party-goers can easily remove a block from the cheese globe.

Assume you've got a machine to insert sticks into cheese blocks easily.

THERE MUST BE NO GAPS VISIBLE BETWEEN THE BLOCKS OF CHEESE. THIS IS CRUCIAL!

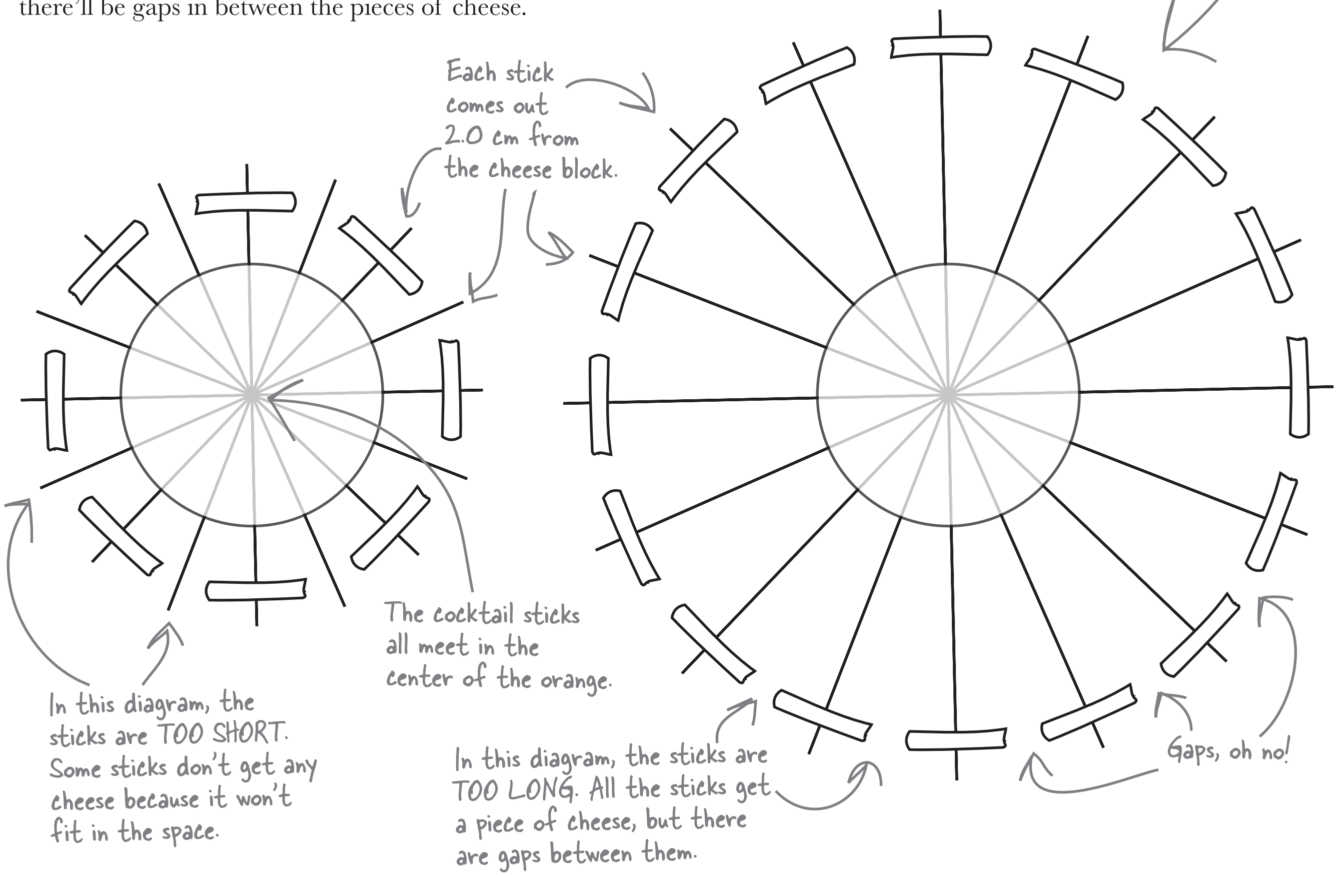
The party planner already has a machine that will put cheese blocks onto cocktail sticks with 2.0 cm sticking out at the end. The machine is also able to put all 500 cocktail sticks into the center of the orange so that they're evenly spaced. You don't need to worry about how to do any of that.

What length should the cocktail sticks be?

Your job is to calculate the **length** that the cocktail sticks should be.

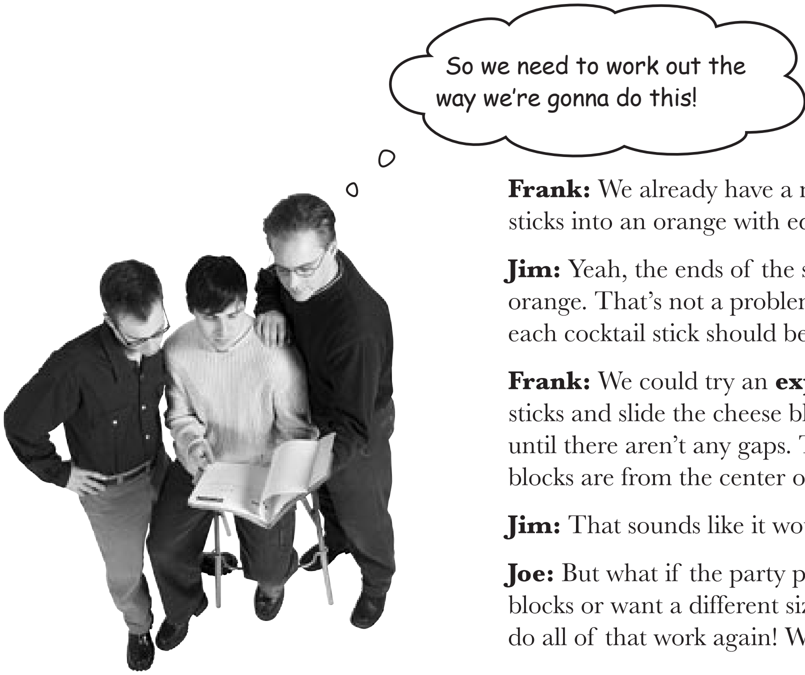
If the sticks are **too short**, there won't be space for you to fit on all 500 pieces of cheese, as the sticks won't spread out enough for the pieces of cheese to fit on next to each other.

But if the sticks are **too long**, they'll spread out too much and there'll be gaps in between the pieces of cheese.



BRAIN POWER

What properties of the cheese blocks might be important when it comes to working out the length that the cocktail sticks should be?



Frank: We already have a machine that can slam 500 cocktail sticks into an orange with equal spacing? Cool or what!

Jim: Yeah, the ends of the sticks all meet in the **center** of the orange. That's not a problem. But we don't know the **length** that each cocktail stick should be.

Frank: We could try an **experiment** - y'know, use really long sticks and slide the cheese blocks up and down the sticks by hand until there aren't any gaps. Then we could measure how far the blocks are from the center of the orange.

Jim: That sounds like it would work, sure.

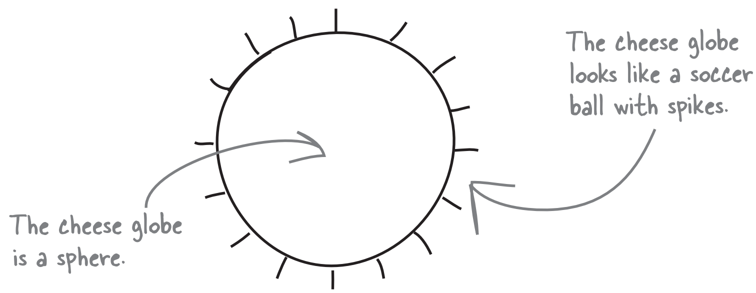
Joe: But what if the party planners change the number of cheese blocks or want a different size of block in the future? We'd have to do all of that work again! We need to future-proof our solution.

Jim: That's true. If this is a success, the planners will want to milk this cheese globe for all it's worth. There must be a butter way.

Joe: Can we come up with an **equation** that tells us what length of cocktail stick to use?

Frank: OK, let's think about that. What's the cheese globe **like**?

Jim: Well, the finished cheese globe kinda looks like the cheese has coated a soccer ball with a bunch of little spikes s



Start solving a new problem by asking yourself, "What is this problem LIKE?"

Joe: Right, the cheese globe's basically a **sphere**.

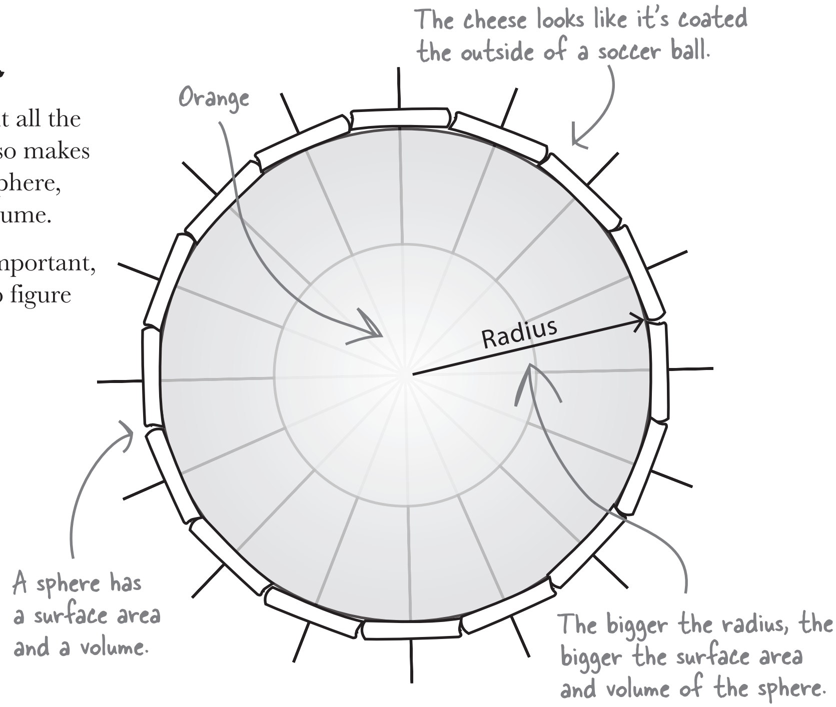
Jim: So maybe the **volume** that each cheese block takes up is important ...

Joe: ... or maybe the **surface area** of the cheese blocks is important ...

The cheese globe is a sphere

Because you start with a spherical orange and put all the cocktail sticks in the same distance, the cheese also makes a **sphere** shape. The larger the **radius** of the sphere, the larger the surface area, and the larger the volume.

But is it the **surface area** of the sphere that's important, or the **volume** ... or both? How are you going to figure that out?



If a sphere has a big radius, then it also has a big surface area and a big volume.

Sharpen your pencil

A single block of cheese has dimensions of 2.0 cm by 2.0 cm by 0.50 cm. You have 500 of them.

a. Calculate the total volume of the cheese blocks in cm^3.

b. Calculate the area in cm^2 that the cheese blocks occupy when you arrange them on a flat surface with the square side of each block facing upwards.

c. Do you think the volume of the cheese blocks or their surface area - or both - is important for working out how long to make the cocktail sticks? Give a reason for your answer.

d. What other information might you need to be able to solve this problem?

Sharpen your pencil Solution

A single block of cheese has dimensions of 2.0 cm by 2.0 cm by 0.5 cm. You have 500 of them.

a. Calculate the total volume of the cheese blocks in cm^3.

Volume of 1 block = length × width × height = 2.0 × 2.0 × 0.50 = 2.0 cm^3 (2 sd)

Volume of 500 blocks = 500 × 2.0 = 1000 cm^3 (2 sd)

b. Calculate the area in cm^2 that the cheese blocks occupy when you arrange them on a flat surface with the square side of each block facing upwards.

Surface area of 1 block = length × width = 2.0 × 2.0 = 4.0 cm^2 (2 sd)

Surface area of 500 blocks = 500 × 4.0 = 2000 cm^2 (2 sd)

c. Do you think the volume of the cheese blocks or their surface area - or both - is important for working out how long to make the cocktail sticks? Give a reason for your answer.

I think the surface area is important, because the blocks have to cover a surface without any gaps in between them, not fill up a volume. The volume isn't important.

d. What other information might you need to be able to solve this problem?

An equation for the surface area of a sphere would be useful.

The surface area of the sphere is the <u>same</u> as the surface area of the cheese

If you were making a solid sphere of cheese, then the volume of the cheese and sphere would need to be equal.

But the cheese makes a spherical 'shell' - not a filled-in sphere. Therefore, the surface area of the cheese will be the same as the **surface area** of the sphere that it's coating.

Your table of information gives you an equation for the surface area of a sphere: $S = 4\pi r^2$.

Ready Bake Equation

Surface area of a sphere

$$S = 4\pi r^2$$

Surface area (S)

Radius (r)

Sharpen your pencil

You have 500 cheese blocks, measuring 2.0 cm by 2.0 cm by 0.50 cm, and are required to make a cheese sphere. You have 500 cocktail sticks, which start in the center of an orange and are equally spread across the orange. 2.0 cm of each stick should be visible on the outside of the sphere.

a. What length should the cocktail sticks be if the 500 pieces of cheese are to be arranged so as to leave no gaps in the surface of the cheese sphere?

b. Now, suppose you have 2000 pieces of cheese the same as before - 4 times as many. Without redoing the calculations you did above, use the equation for the surface area of a sphere to explain what radius of cheese globe you'd be able to make while leaving no gaps.

Sharpen your pencil Solution

You have 500 cheese blocks, measuring 2.0 cm by 2.0 cm by 0.50 cm, and are required to make a cheese sphere. You have 500 cocktail sticks, which start in the center of an orange and are equally spread across the orange. 2.0 cm of each stick should be visible on the outside of the sphere.

a. What length should the cocktail sticks be if the 500 pieces of cheese are to be arranged so as to leave no gaps in the surface of the cheese sphere?

Inside surface of cheese coats a sphere.

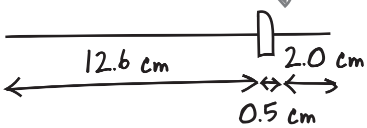
surface area of sphere = surface area of cheese

$\Rightarrow 4\pi r^2 = 2.0 \times 2.0 \times 500$

$r = \sqrt{\frac{2000}{4\pi}} = 12.6$ cm (3 sd)

You need to remember the thickness of the cheese as well as the bit at the end!

Length of cocktail stick:
Need to include 2.0 cm at end, and thickness of cheese.



Length = 12.6 + 0.5 + 2.0 = 15.1 cm (3 sd)

Surface area of sphere.

$S = 4\pi r^2$

Radius of sphere.

Doubling the radius of a sphere QUADRUPLES the surface area of the sphere.

b. Now, suppose you have 2000 pieces of cheese the same as before - 4 times as many. Without redoing the calculations you did above, use the equation for the surface area of a sphere to explain what radius of cheese globe you'd be able to make while leaving no gaps.

The surface area is proportional to r^2, as $S = 4\pi r^2$. If S is 4 times larger, then $4\pi r^2$ will be four times larger. But 4 and π are constants. Therefore r^2 must be 4 times larger, so r must be 2 times larger

Inner radius = 12.6 × 2 = 25.2 cm.

there are no Dumb Questions

Q: How can I tell whether it's the volume or surface area that's important?

A: Ask yourself - "What's it like?" Here, arranging the cheese is like coating the surface of a soccer ball. This means that the surface area of the cheese is important.

If you were melting the cheese and filling the volume of a soccer ball, then the volume of the cheese would be the important thing.

Q: I thought that 3D objects have volumes, not surface areas?

A: Anything you can paint has a surface area - or else there'd be nowhere for the paint to go! A sphere has a surface area. It's a curved surface - and one you can't flatten out properly - but it's a surface nonetheless.

Q: How do you know what happens to the area when you change the radius?

A: In the equation $S = 4\pi r^2$, the area is **proportional** to r^2. If you double the radius, you quadruple the surface area, as $2^2 = 4$
Think about it like this: if the old radius is y and you double it, the new radius is $2y$.

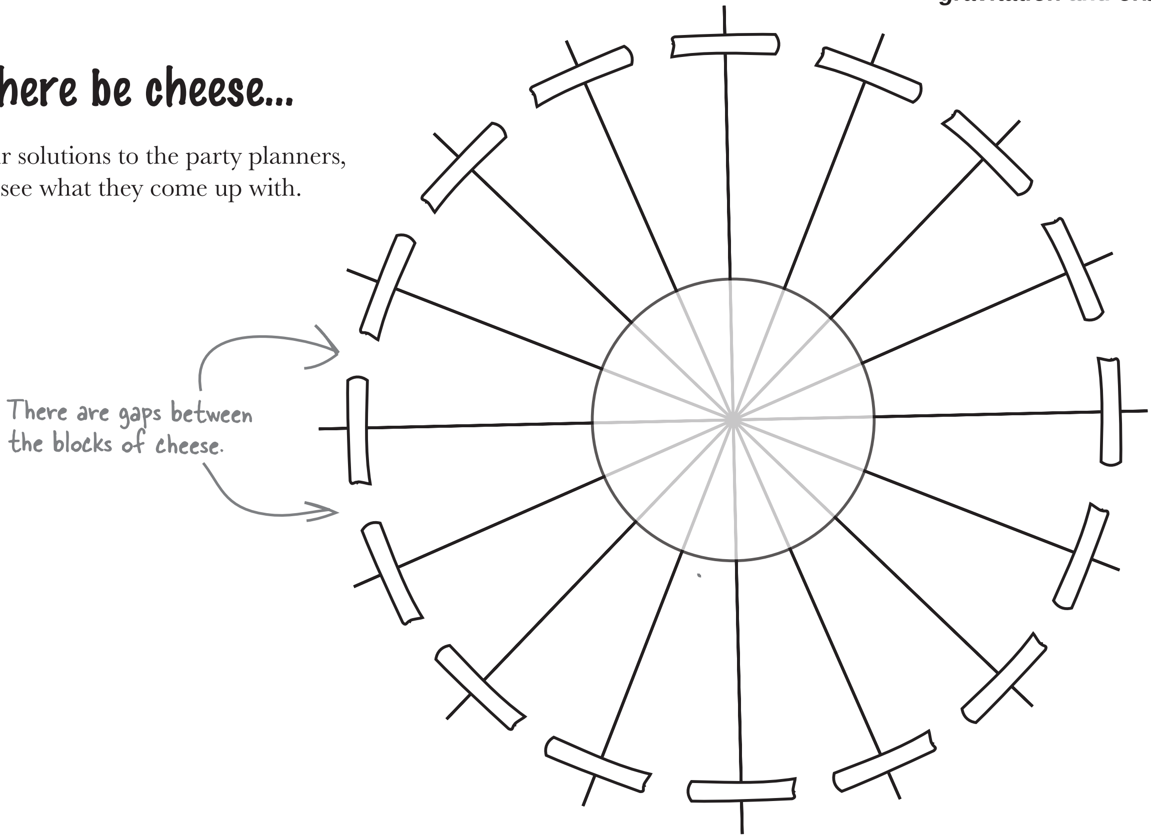
Old: $S = 4\pi y^2$ ← Old surface area.

New: $S = 4\pi(2y)^2 = 4\pi 4y^2 = 4 \times 4\pi y^2$

So when you double the radius, the new surface area is four times the old surface area.

Let there be cheese...

Give your solutions to the party planners, and let's see what they come up with.



...but there are gaps in the globe!

Sharpen your pencil

Your solution on the opposite page is correct, but the party planners have gone wrong somewhere.

Circle the mistake in the notes that the machine programmer made and explain what went wrong.

Cheese globe notes

Set machine to put 500 sticks into orange, evenly-spaced with ends of sticks in center.

Machine puts 2.0 cm by 2.0 cm by 0.5 cm blocks on sticks, square side up, with 2.0 cm of stick protruding.

Set stick length so that 15.1 cm of stick is visible outside orange.

Press go.

Sharpen your pencil Solution

Your solution on the opposite page is correct, but the party planners have gone wrong somewhere.

Circle the mistake in the notes that the machine programmer made and explain what went wrong.

Cheese globe notes

Set machine to put 500 sticks into orange, evenly-spaced with ends of sticks in center.

Machine puts 2.0 cm by 2.0 cm by 0.5 cm blocks on sticks, square side up, with 2.0 cm of stick protruding.

~~Set stick length so that~~ 15.1 cm of stick is visible outside orange.

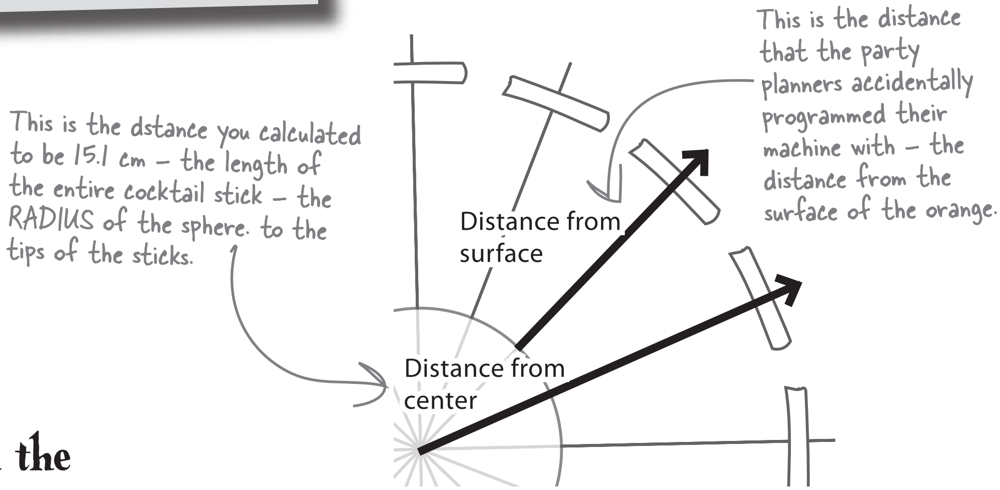
Press go.

I worked out the the sticks need to have a TOTAL length of 15.1 cm.

But the party planner has made 15.1 cm of stick visible on the outside of the orange.

This means that the sticks will be too long, as they will have alength of 15.1 cm + radius of orange.

If the sticks are too long, there will be gaps between the blocks of cheese.



Accidentally working with the distance from the surface instead of the radius - or vice-versa - is a common mistake.

The party planners had programmed their machine incorrectly, using the **distance from the surface** of the orange.

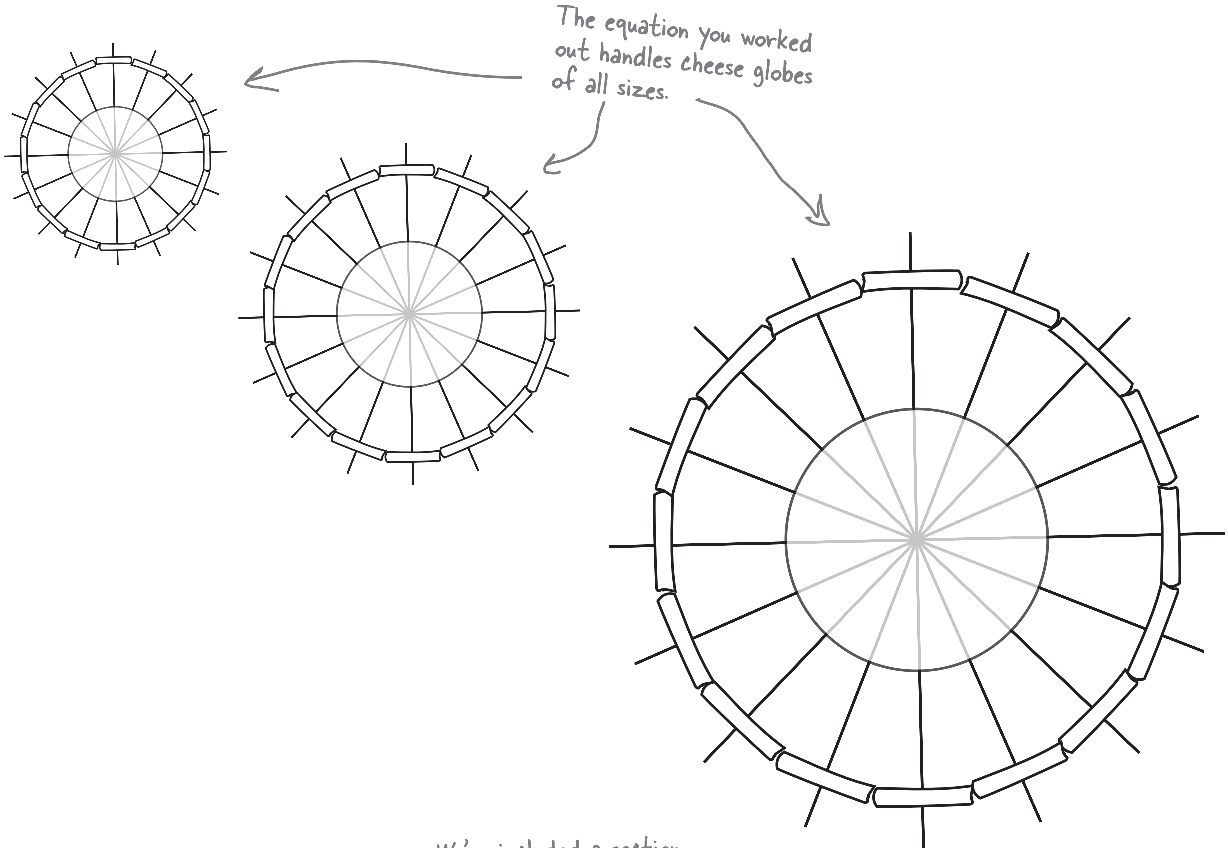
They should have used the **radius** - the **distance from the center**.

Fortunately, it was only a prototype, and you soon have them back on track ...

The party's on!

The party planners are thrilled. In fact, they've already had more orders for different size cheese globes

With an ability to churn out different sizes thanks to your flexible equation for working out cocktail stick sizes, you're ready to turn these unique (and spiky) cheese globes into big business.



BULLET POINTS

We've included a section like this in appendix ii.

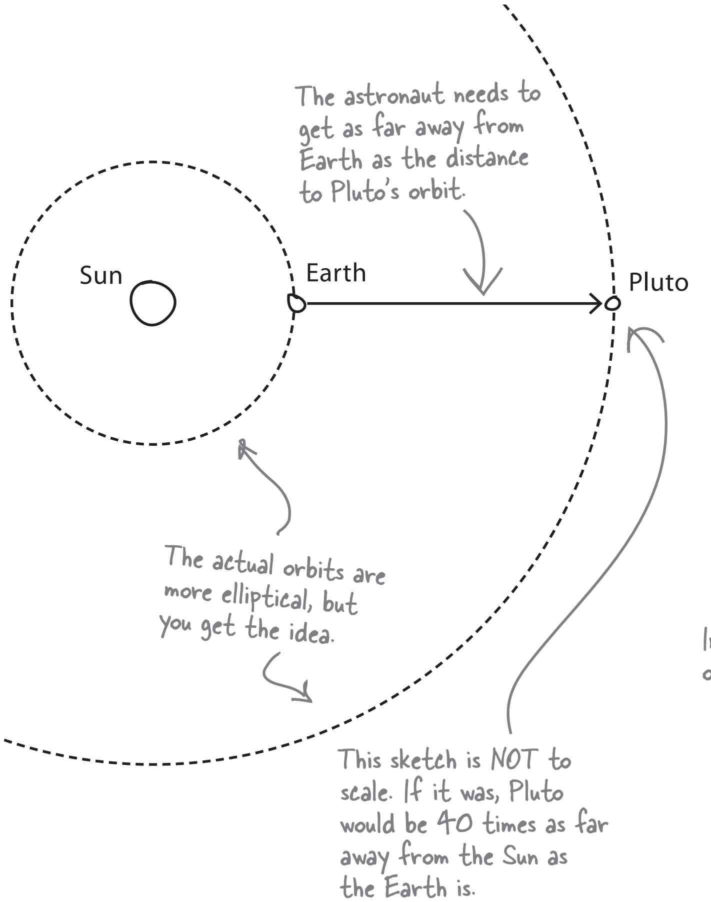
- If you're working with a 3D shape, think about whether it's surface area or volume that's important.
- If the shape will 'unroll' flat, you can work out its surface area by turning it into 2D shapes.

You did this in chapter 17.

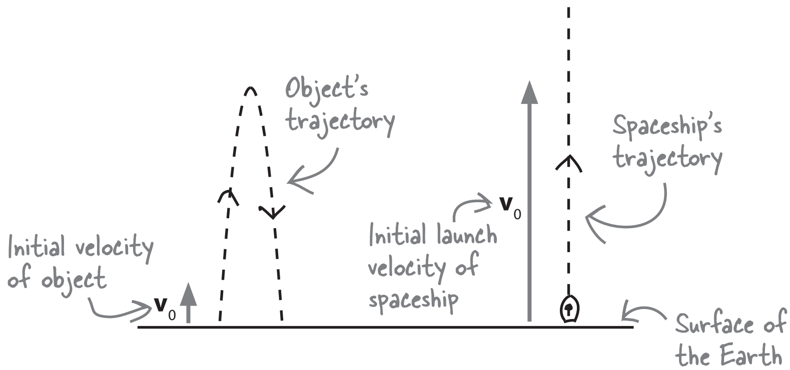
- If the shape won't 'unroll' flat, see if you can look it up in a book or table of information.
- On the AP course, the 'geometry and trigonometry' section of the table of information is the place to look.
- The surface area of a sphere is **proportional** to r^2.

To infinity - and beyond!

The lead astronaut on the Head First space station has been chosen to boldly go where no man has gone before - to the edge of the solar system! Your job is to work out how to get him there.



When you launch an object directly upwards, sooner or later it comes back down. If the astronaut is to make it to the edge of the solar system, he needs to be able to keep on going and going - without being brought back down to Earth!



Since you're able to use a spaceship, this sounds easy at first - but you can't accelerate forever, as you'd run out of fuel! Your best strategy is to blast off to achieve as high a **velocity** as possible - after all, the greater the upwards velocity, the greater the time an object will spend away from the Earth.

The spaceship's acceleration due to gravity doesn't depend on its mass. So you don't need to worry about the mass of the spaceship changing as it burns fuel.

You need to calculate the **escape velocity** for the Earth - the velocity that the astronaut needs to reach at the start when he's close to Earth so as to not fall back down again. (You may need to work out other things for later on in the mission, but the escape velocity for the earth is enough to be getting on with for now!)

Jim: Yeah, we just need to run the spaceship's engine as hard as possible for as long as possible. That gives the astronaut the largest possible **velocity** at the start - and the best chance of making it.

Joe: But how do we guarantee that's going to be good enough? What if the astronaut falls back down to Earth again, like a football does when you throw it up in the air?

Frank: Well, let's work it out. I looked up the distance to Pluto's orbit, and it's around 6×10^{12} m...

Jim: ...and **acceleration** due to the Earth's gravity is 9.8 m/s^2...

Frank: Wait ... you mean **negative** 9.8 m/s^2, right? The acceleration needs to be negative if we're calling "away from the Earth" the positive direction.

Joe: OK, so we know the **displacement** and the **acceleration**, and let's say that he needs to have a **velocity** of 0 m/s when he reaches Pluto's orbit. That's the smallest velocity he could have by that point and still make it.

Jim: Let's get on with the calculation!

This distance is a million times grater than the radius of the Earth, and only has one significant digit. So it doesn't really matter if this distance is measured from the center or the surface of the Earth.

Sharpen your pencil

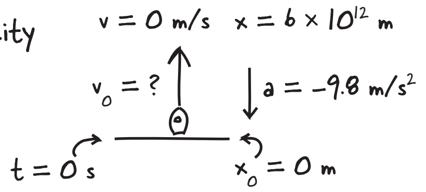
a. Pluto's orbit is 6×10^{12} m (1 sd) from Earth. If the acceleration due to the Earth's gravity is constant throughout the journey, work out the velocity that a spaceship needs to have at the start in order to escape from Earth.

b. How practical does your answer feel to you?

Sharpen your pencil Solution

a. Pluto's orbit is 6×10^{12} m (1 sd) from Earth. If the acceleration due to the Earth's gravity is constant throughout the journey, work out the velocity that a spaceship needs to have at the start in order to escape from Earth.

Suppose spaceship has velocity of 0 m/s by the time it reaches Pluto's orbit:



$$v^2 = v_0^2 + 2a(x - x_0)$$

$$v_0^2 = v^2 - 2a(x - x_0)$$

$$\Rightarrow v_0^2 = 0 - 2 \times (-9.8) \times (6 \times 10^{12} - 0)$$

$$v_0 = \underline{\underline{3 \times 10^{13} \text{ m/s (1 sd)}}}$$

The speed of light is 3×10^8 m/s

b. How practical does your answer feel to you?

This is very, very, very fast. It's 100000 times faster than the speed of light (the largest speed a material body can have) so is just totally impossible!

Looks like the velocity's the wrong **size** - it's way too high!

Jim: Yeah, you can't get a spaceship - or anything else to go faster than the speed of light!

Frank: But real space probes have gone past Pluto and even left the solar system before. So it must be possible ...

Joe: What if the **effect of the Earth's gravity gets less** as you go **further away**?

Frank: How do you mean?

Joe: Well, sounds are quieter if you're further away from their source. And lights are less bright. Maybe it's the same with gravity.

Jim: You mean your **acceleration due to gravity** might get smaller as you move further away?

Frank: Oh yeah ... the equations of motion we used assumed that the acceleration due to gravity was constant for the whole journey

Jim: Good point. If the acceleration changes as you get further away, we can't use those equations of motion - they only work for a situation with constant acceleration.

Joe: Let's try to work out **how** the acceleration changes as you get further away from the Earth ...

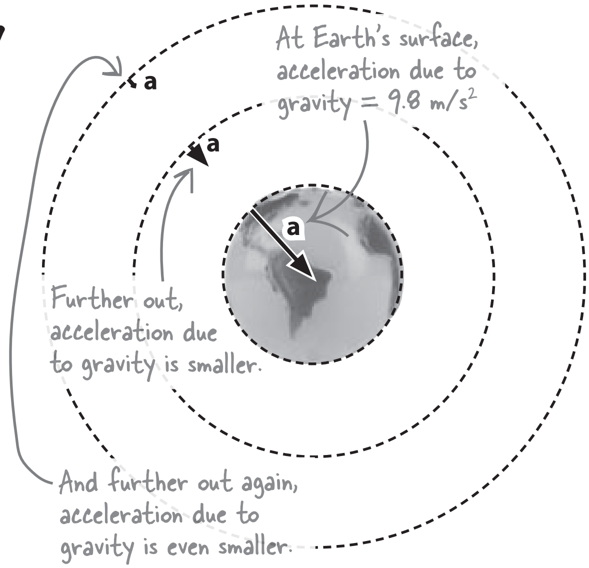
Your equations of motion only work if the acceleration is constant.

Earth's gravitational force on you becomes weaker as you go further away

When you're near the Earth, you experience a **gravitational force** because the stuff you're made of and the stuff the Earth's made of attract each other. But as the **distance** between you and the Earth **increases**, the **gravitational force gets weaker**. In the same way, sounds get quieter and lights get less bright as you increase the distance between yourself and the source of the sound or light.

As the gravitational force on you gets smaller, your **acceleration due to gravity** also decreases (as $\mathbf{F}_{net} = m\mathbf{a}$). So your acceleration isn't constant. That means you can't use your equations of motion here, as they only work when the acceleration is constant.

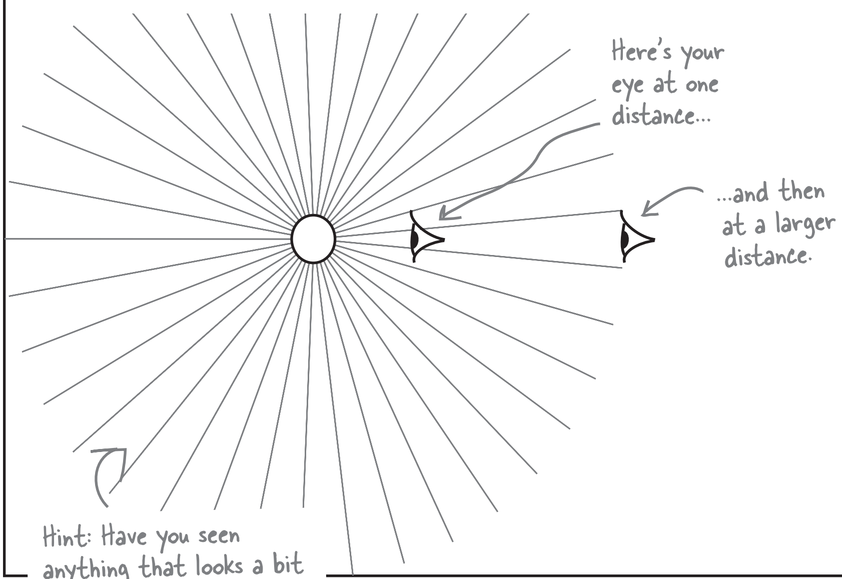
But **how** does the gravitational force change as you get further away?



Sharpen your pencil

The brightness of a light becomes less as you get further away. In this picture, you have a spherical light that emits light equally in all directions, and you look at it with your eye from two different distances. Assume that the pupil of your eye (the bit that lets the light in) has the same area at both distances.

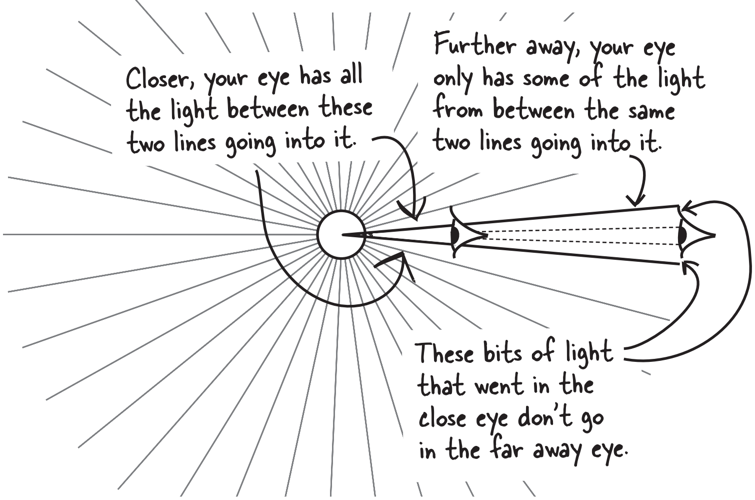
Explain why the light appears dimmer when you are further away.



Sharpen your pencil Solution

The brightness of a light is something that becomes weaker as you get further away. In this picture, you have a spherical light that emits light equally in all directions, and you look at it with your eye from two different distances. Assume that the pupil of your eye (the bit that lets the light in) has the same area at both distances.

Explain why the light appears dimmer when you are further away.



As the light goes away from the bulb, it spreads out more.

The close eye has all the light within the two heavy lines going into it.

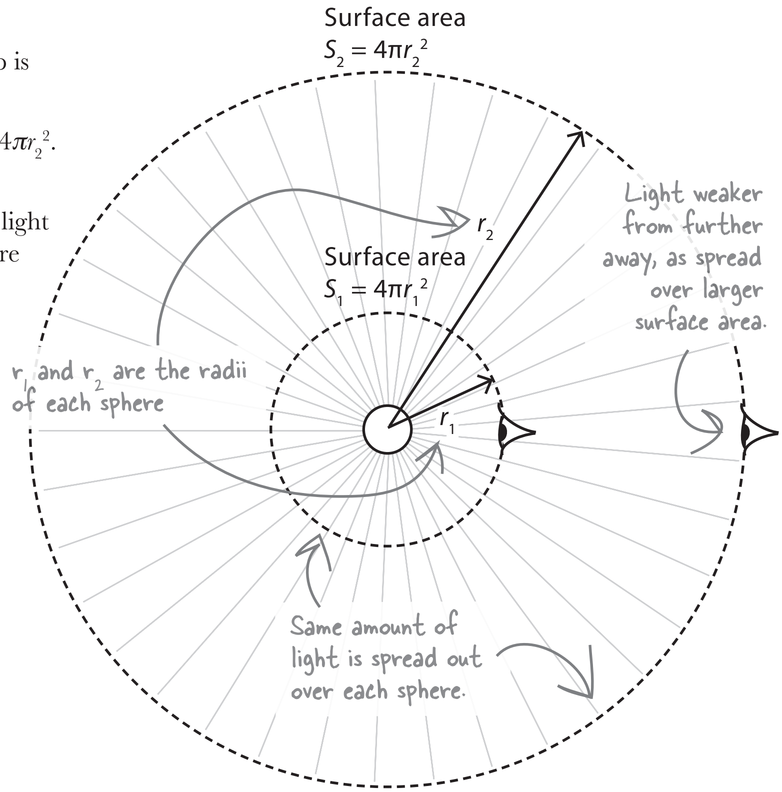
The far away eye only has a part of this light going into it – so the bulb looks dimmer from further away.

At distance r_1, all of the light coming from the bulb is spread out over the area $4\pi r_1^2$.

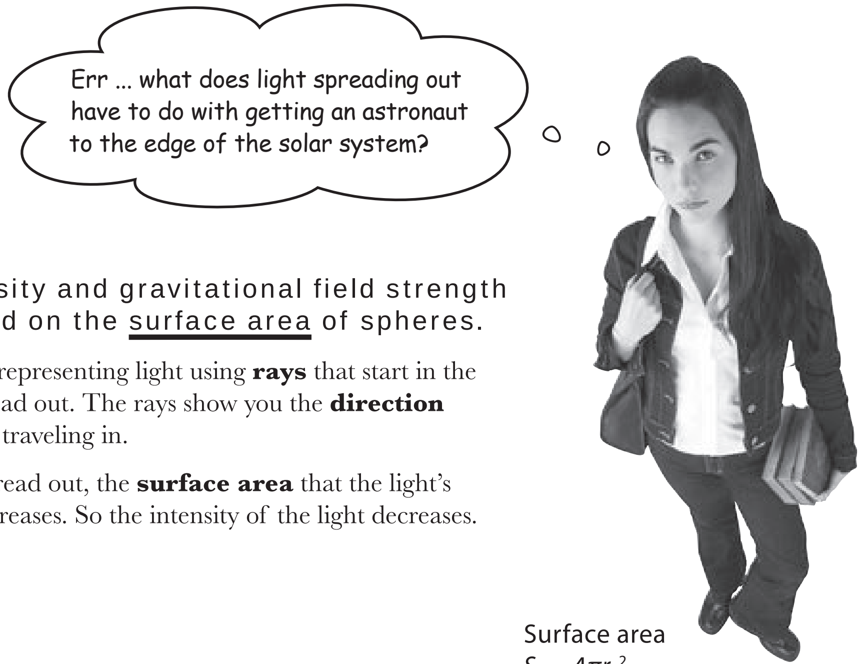
At distance r_2, the light is spread out over the area $4\pi r_2^2$.

The same amount of light it spread out over each surface. However, at a greater distance there's less light available per square meter, as there are more square meters of surface for the light to spread out over.

So the light looks less bright from further away.



The further you are from a light, the greater the SURFACE AREA it's spread over.



Light intensity and gravitational field strength both depend on the surface area of spheres.

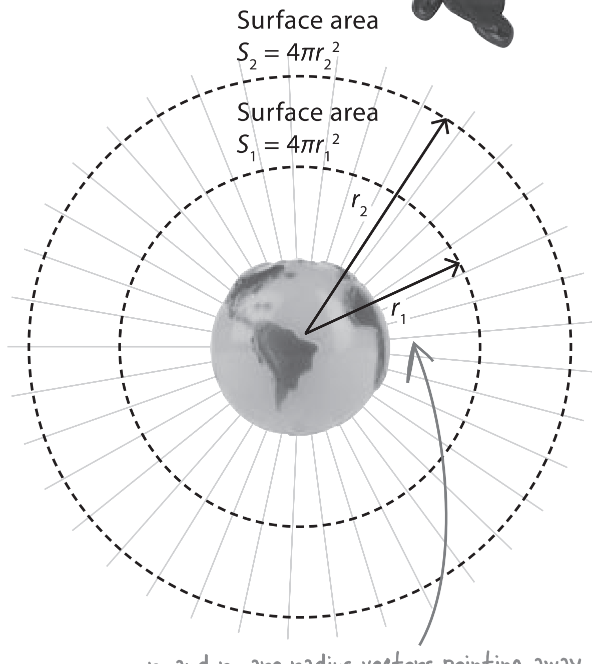
You have been representing light using **rays** that start in the center and spread out. The rays show you the **direction** that the light is traveling in.

As light rays spread out, the **surface area** that the light's spread over increases. So the intensity of the light decreases.

You can represent the Earth's gravitational field in a similar way by using **gravitational field lines**. The lines show you the direction an object will accelerate in if the gravitational force is the only force acting on it. The object will accelerate along a line, moving closer to the Earth.

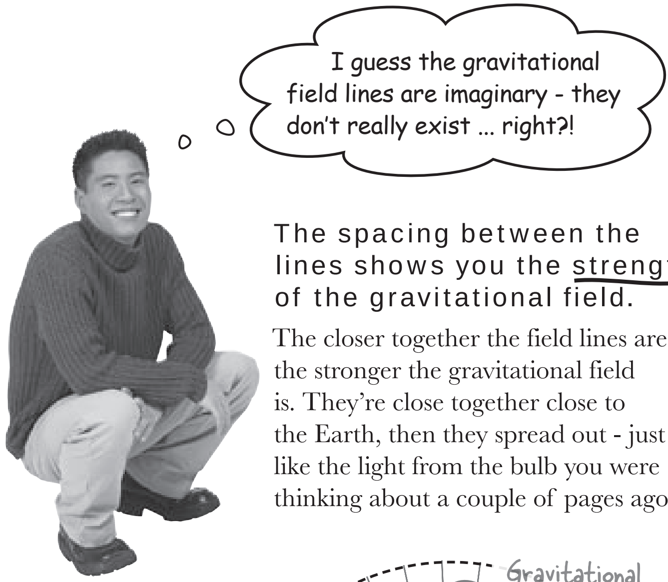
As the gravitational field lines spread out, the surface area of a sphere at that radius increases. So the **gravitational field strength** decreases.

The gravitational field strength is another name for the acceleration due to gravity.



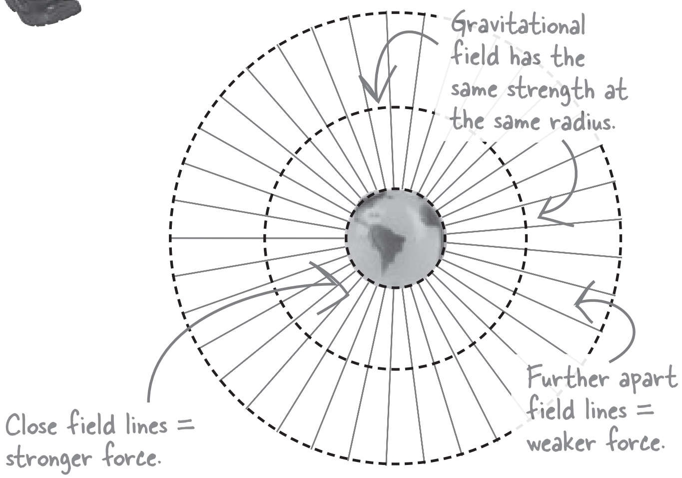
r_1 and r_2 are radius vectors pointing away from the center of the sphere. Most of the time we are interested in the distance, buit sometimes we will be interested in the direction, as r is in the opposite direction from the acceleration caused by the gravitational field.

Gravitational field lines show you the DIRECTION that an object will accelerate in.



The spacing between the lines shows you the strength of the gravitational field.

The closer together the field lines are, the stronger the gravitational field is. They're close together close to the Earth, then they spread out - just like the light from the bulb you were thinking about a couple of pages ago.



The field lines themselves don't physically exist. The field lines are a tool you can use to help you **visualize** what happens to the gravitational field.

The closer together the field lines, the stronger the field (so the greater the acceleration due to gravity).

there are no Dumb Questions

Q: Why is this called a gravitational field? Surely gravitational acceleration or gravitational force is a better?

A: The 'field' is a way of visualizing what's going on with the strength of the acceleration due to gravity. Gravitational field strength has units m/s^2, the same as acceleration. Close to the Earth, the gravitational field strength is 9.8 m/s^2, but further away it's lower.

Q: If I'm drawing gravitational field lines on a picture, is there a standard number of lines that I should draw? Should I always draw 32 lines, like in these pictures?

A: There's no strict rule - you should draw as many field lines as help you visualize what's happening. Probably at least eight is a good rule of thumb.

Q: So what's the connection between gravitational field and gravitational force?

A: The gravitational field represents the acceleration due to gravity that an object would experience if it was placed at that point. You can get from there to the gravitational force using $F_{net} = ma$

Q: And the gravitational force is just the same as an object's weight, right?

A: Absolutely! Though you now know that the force - and therefore the weight - will vary depending on the distance from Earth.

Q: So - what's the equation for that?

A: Funny you should ask ...

Sharpen your pencil

You want to work out an equation for the gravitational force that a spaceship experiences at any distance from the Earth

a. What happens to the surface area of a sphere if you double its radius?

b. The gravitational force depends on the distance something is from the Earth. If you're further away, will the gravitational force be larger or smaller?

c. The gravitational field gets 'spread out' over the surface area of a sphere as you get further away, in the same way as light does. What do you think will happen to the gravitational field strength if you double the distance between you and the Earth (i.e. double the radius)?

d. What do you think would happen to the gravitational force if you double your mass?

Hint: F=ma.

e. What do you think would happen to the gravitational force if you doubled the mass of the Earth?

f. Here are four equations. In each equation, G is a constant (and a conversion factor) that will make the numbers and units work out later on. For each equation, write down whether what happens to the size of the gravitational force, F_G, when you change r (the distance of the object from the Earth), m_1 (the mass of the Earth) or m_2 (the mass of the object) is what you'd expect in real life. Circle the equation you think is correct.

F_G points towards the Earth and r points away. In gravitation, it's conventional to include a minus sign to show the directions of F_G and r are opposite even though this is a scalar equation for the size of F_G.

$$F_G = -Gm_1m_2r^2$$

$$F_G = -\frac{Gm_1r}{m_2}$$

In each equation, G is a constant that makes the units work out. So don't worry about units right now.

$$F_G = -\frac{Gm_1m_2}{r}$$

$$F_G = -\frac{Gm_1m_2}{r^2}$$

Sharpen your pencil Solution

You want to work out an equation for the gravitational force that a spaceship experiences at any distance from the Earth

a. What happens to the surface area of a sphere if you double its radius?

If you double the radius, you quadruple the surface area, as $S = 4\pi r^2$ and $2^2 = 4$.

Look back at the No Dumb Questions on page 718 for an explanation.

b. The gravitational force depends on the distance something is from the Earth. If you're further away, will the gravitational force be larger or smaller?

If you're further away, the gravitational force will be smaller.

c. The gravitational field gets 'spread out' over the surface area of a sphere as you get further away, in the same way as light does. What do you think will happen to the gravitational field strength if you double the distance between you and the Earth (i.e. double the radius)?

If you double the radius, you quadruple the surface area that the field will be spread out over. So I think that the field strength would decrease by a factor of four.

d. What do you think would happen to the gravitational force if you double your mass?

If you double your mass, you double the force, as $F = ma$

Hint: $F=ma$.

e. What do you think would happen to the gravitational force if you doubled the mass of the Earth?

If you double the mass of the Earth, you double its gravitational attraction to you ($F = ma$). As gravitational forces exist in Newton's 3rd Law pairs, the force that the Earth exerts on you must also double.

f. Here are four equations. In each equation, G is a constant (and a conversion factor) that will make the numbers and units work out later on. For each equation, write down whether what happens to the size of the gravitational force, F_G, when you change r (the distance of the object from the Earth), m_1 (the mass of the Earth) or m_2 (the mass of the object) is what you'd expect in real life. Circle the equation you think is correct.

$$F_G = -G m_1 m_2 r^2$$

Wrong – if you increase r, then size of F_G gets larger.

$$F_G = -\frac{G m_1 r}{m_2}$$

Wrong – if you increase m_2, then size of F_G gets smaller.

$$F_G = -\frac{G m_1 m_2}{r}$$

Wrong – if you double r, then the size of F_G only halves.

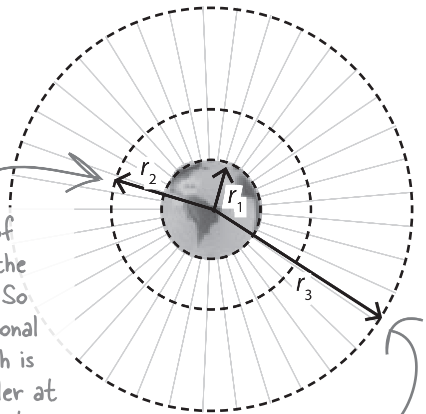
$$F_G = -\frac{G m_1 m_2}{r^2}$$

Right – Doubling m_1 or m_2 doubles size of F_G. Doubling r quarters F_G.

Gravitation is an inverse square law

The gravitational force between two objects depends on $\frac{1}{r^2}$. If you double the distance, you decrease the force by a factor of 4. Something that behaves like this is said to follow an **inverse square law**.

This is because an inverse is 1 divided by something - in this case $\frac{1}{r^2}$ - and the square part comes from the fact that the thing doing the dividing is squared.



there are no Dumb Questions

Q: But the surface of the Earth is the zero point for distance, right? Doesn't that equation go all weird with a zero on the bottom when you're at the surface?

A: *r* is the distance from the CENTER of the Earth, not the distance from the surface of the Earth. When you're at the surface of the Earth, *r* is just the radius of the Earth. And when you're further away, *r* is your distance from the center.

Q: Doesn't the force depend on $\frac{1}{4\pi r^2}$ rather than $\frac{1}{r^2}$, as the surface area is $4\pi r^2$?

A: The 4π is a constant. Whatever you do to *r* (double it, half it, etc), the 4π doesn't change. So it's not right to say that the gravitational force 'depends' on the 4π.

Q: What are the units of *G*?

A: You can work that out ...

Sharpen your pencil

You've realized that equation below does all the right things when you change the masses and the radius.

Work out what SI units *G* must have to make **both sides** of the equation have the **same units**.

$$F_G = -\frac{G m_1 m_2}{r^2}$$

Hint: Don't use Newtons for force. Work out more fundamental units for force (based on kg, m and s) using F = ma

Be very careful with m for mass and m for meters!

Sharpen your pencil Solution

You've realized that equation below does all the right things when you change the masses and the radius.

Work out what SI units *G* must have to make **both sides** of the equation have the **same units**.

$$F_G = -\frac{G m_1 m_2}{r^2}$$

Work out the units of each side individually first.

Write the units out using fractions rather than in "m/s" form, as it makes the algebra easier.

Units of left hand side:

$F = ma$

$[F] = [m][a] = \frac{kg.m}{s^2}$

Units of right hand side:

Equation is $\frac{Gm_1m_2}{r^2}$

Units are $\frac{[G][m_1][m_2]}{[r^2]} = \frac{[G].kg^2}{m^2}$

Units are same on both sides, so rearrange to solve for [G]:

$$\frac{kg.m}{s^2} = \frac{[G].kg^2}{m^2} \Rightarrow [G] = \frac{kg.m.m^2}{s^2.kg^2} = \frac{m^3}{kg.s^2} = m^3/kg.s^2$$

When you're working with units, it's safest to put dots between them, as you may treat 'kg' like k × g, as if the k and the g are different variables.

[G] means 'units of G'

You can rearrange units equations just like you can rearrange other equations.

But what IS G?! I don't get what it's there for!

G is like a conversion factor which makes the units and numbers correct.

If you're working in SI units, F_G will be in Newtons, m_1 and m_2 in kg and *r* in meters. The first job *G* does is to give both sides of the equation the same **units** - as you've just worked out.

The second job that *G* does is to make the **numbers** correct. If you just multiply m_1 and m_2 together then divide by r^2, the numerical answer you get won't be equal to the force in Newtons.

G is kinda like a conversion factor - a **constant** that makes the numbers correct as well. For this reason, *G* is called the **gravitational constant**.

G is the gravitational constant. It makes both the units and the numbers work out.

Jim: Um, not quite. We don't know what *G* is in the equation. Well I mean, we know its **units**, but we don't know its **size**. If only we had a way of working that out.

Joe: Well, we've got one equation, and *G* is an unknown. As long as *G* is the only unknown, we can use the equation to calculate the value of *G*.

Frank: I guess we already know that **acceleration due to gravity at the Earth's surface** is 9.8 m/s^2. That's gotta give us a few things we can use in that equation.

Joe: And I just found the **radius** and **mass** of the Earth in a textbook. So we know those two things as well. I think that should be enough to work out the value of *G*...

Sharpen your pencil

a. By considering the acceleration due to gravity at the Earth's surface, use the equation and the ready-bake facts to work out the value of *G*, the gravitational constant.

Ready Bake Facts

Mass of the Earth:

5.97×10^{24} kg

Radius of the Earth:

6.38×10^{6} m

Gravitational force between two masses:

$$F_G = -\frac{G m_1 m_2}{r^2}$$

The minus sign is just a convention to show that F_G and r are in opposite directions. If you're working with the SIZE of F_G, you can leave the minus sign out of your calculations.

b. Use the value for *G* you worked out in part a to calculate the acceleration due to the Earth's gravity that a spaceship experiences at Pluto's orbit, which is 6×10^{12} m (1 sd) from Earth.

Sharpen your pencil Solution

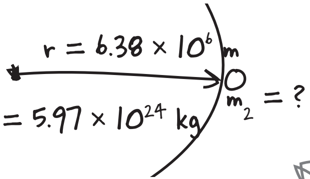
a. By considering the acceleration due to gravity at the Earth's surface, use the equation and the ready-bake facts to work out the value of *G*, the gravitational constant.

Make m_1 the mass of an object and m_2 the mass of the Earth. $a = 9.8\ m/s^2$ at Earth's surface.

$$F = m_1 a = \frac{G m_1 m_2}{r^2}$$

$$a = \frac{G m_2}{r^2}$$

$$\Rightarrow G = \frac{a r^2}{m_2}$$



$$G = \frac{9.8 \times (6.38 \times 10^6)^2}{5.97 \times 10^{24}} = \underline{\underline{6.68 \times 10^{-11}\ m^3/kg.s^2}}$$

Ready Bake Facts

Mass of the Earth:

5.97×10^{24} kg

Radius of the Earth:

6.38×10^{6} m

Gravitational force between two masses:

$$F_G = -\frac{G m_1 m_2}{r^2}$$

The minus sign shows that F_G and r are in opposite directions. Here, we've used the equation without the minus sign to calculate the SIZE of F_G

b. Use the value for *G* you worked out in part a to calculate the acceleration due to the Earth's gravity that a spaceship experiences at Pluto's orbit, which is 6×10^{12} m (1 sd) from Earth.

$$F = m_1 a = \frac{G m_1 m_2}{r^2} \Rightarrow a = \frac{G m_2}{r^2} = \frac{6.68 \times 10^{-11} \times 5.97 \times 10^{24}}{(6 \times 10^{12})^2} = 1 \times 10^{-11}\ m/s^2\ (1\ sd)$$

Do I have to calculate the value of *G* like this every time I want to use it? That stinks!

G is a <u>constant</u> that you can look up when you need it.

G is a fundamental **physical constant**. It's not a number you have to remember or calculate every time, and we've included its value in the appendix, so you can look it up if you need it.

If you're taking an exam, the value of *G* will be given in your table of information - so you can just look it up there and use the value in your equation.

there are no Dumb Questions

Q: How did people work out the mass of the Earth in the first place?

A: Historically, the mass of the Earth was worked out from *G*. *G* was originally worked out from experiments that measured the gravitational attraction between large masses, like cannonballs.

Q: You mean that it's not just the Earth that attracts things gravitationally?

A: Precisely! Everything that's made of stuff attracts everything else gravitationally. It's just that the effect isn't very large unless one of the objects you're dealing with has a very large mass (like the Earth), so you don't usually notice.

Sharpen your pencil

The minus sign shows that F_G and r are in opposite directions. Here, we've used the equation without the minus sign to calculate the SIZE of F_G, and therefore the size of a.

$F = m_1 a = \frac{G m_1 m_E}{r^2}$

Gravitation is an **inverse square law.** This means that if you double the distance that an object is from the Earth, it will only experience a quarter of the gravitational force that it did in its previous position.

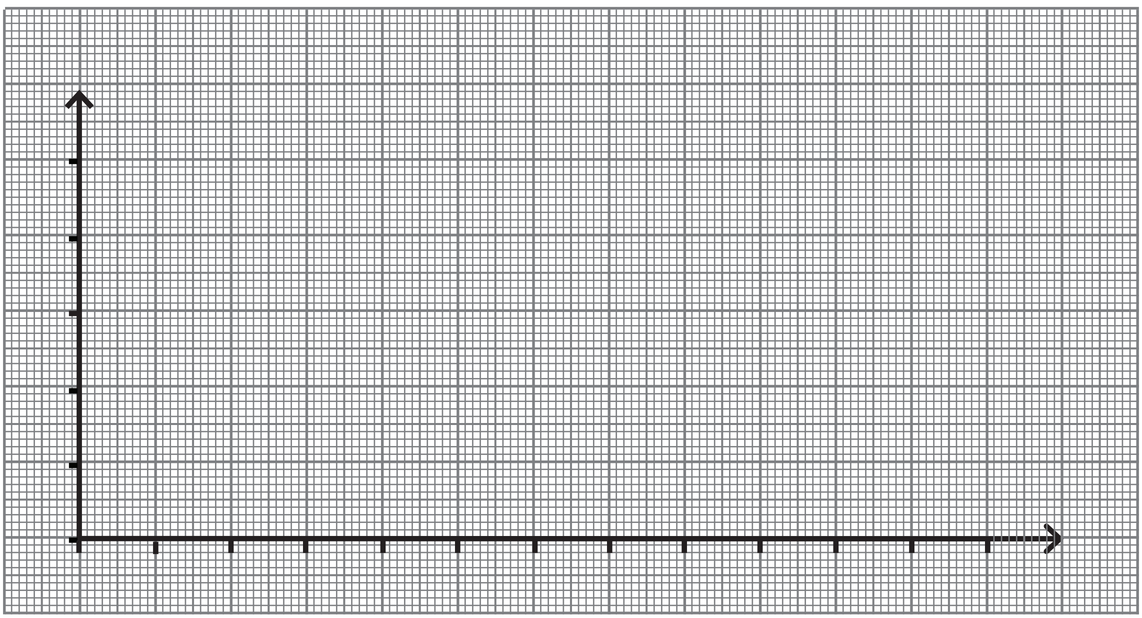
But what does an inverse square law look like? A **graph** will help you visualize it.

a. Fill in the values of the gravitational field strength at a variety of distances from Earth. *G*, the gravitational constant, is 6.67×10^{-11} $m^3/kg.s^2$. m_E, the mass of the Earth, is 5.97×10^{24} kg. The radius of the Earth is 6.4×10^6 m (rounded to 2 sd) to make the figures easier.

Hint: look at how the values in the table increase...

b. Draw a graph with the distance from the Earth on the horizontal axis and the gravitational force on the vertical axis.

Distance from Earth (m)	$a = \frac{Gm_E}{r^2}$ ← m_E is the mass of the Earth.
6.4×10^6	
1.28×10^7	
1.92×10^7	
2.56×10^7	
5.12×10^7	



You need to remember to square r.

You can either calculate each of these values individually, or use the inverse square relationship (double the distance, quarter the force) once you've calculated the first value.

Sharpen your pencil Solution

Gravitation is an **inverse square law.** This means that if you double the distance that an object is from the Earth, it will only experience a quarter of the gravitational force that it did in its previous position.

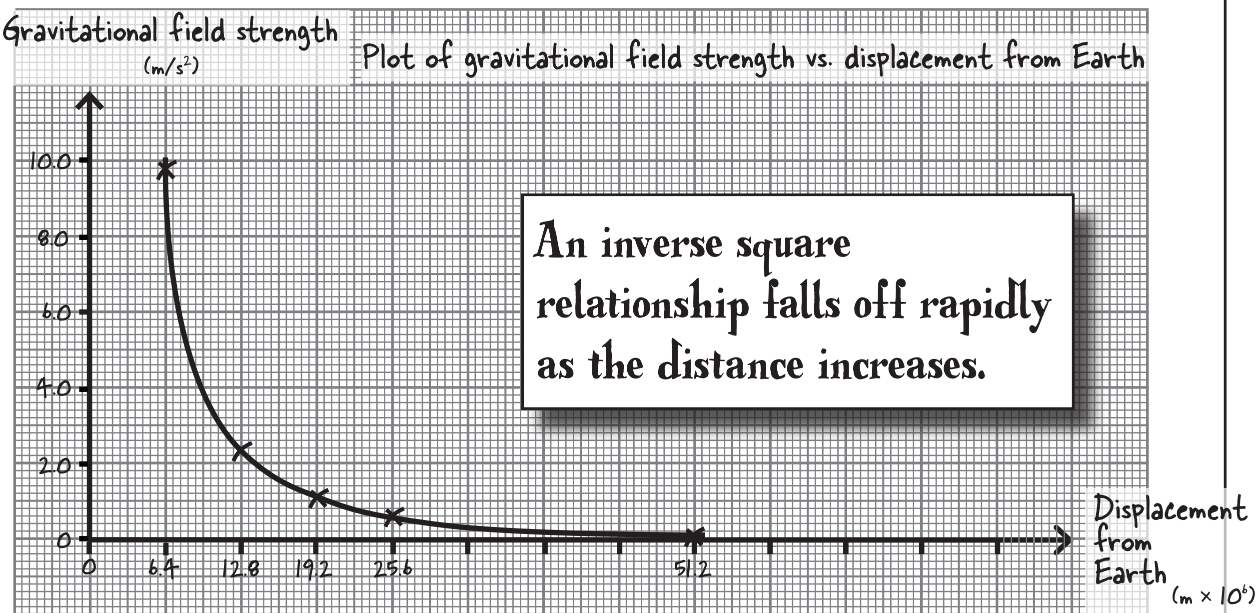
But what does an inverse square law look like? A **graph** will help you visualize it.

a. Fill in the values of the gravitational field strength at a variety of distances from Earth. *G*, the gravitational constant, is 6.67×10^{-11} $m^3/kg.s^2$. m_E, the mass of the Earth, is 5.97×10^{24} kg. The radius of the Earth is 6.4×10^6 m (rounded to 2 sd) to make the figures easier.

Hint: look at how the values in the table increase...

Distance from Earth (m)	$a = \frac{Gm_E}{r^2}$ ← m_E is the mass of the Earth.
6.40×10^6	9.72 m/s^2 (3 sd)
1.28×10^7	2.43 m/s^2 (3 sd)
1.92×10^7	1.08 m/s^2 (3 sd)
2.56×10^7	0.608 m/s^2 (3 sd)
5.12×10^7	0.152 m/s^2 (3 sd)

b. Draw a graph with the distance from the Earth on the horizontal axis and the gravitational force on the vertical axis.



1 Earth radius

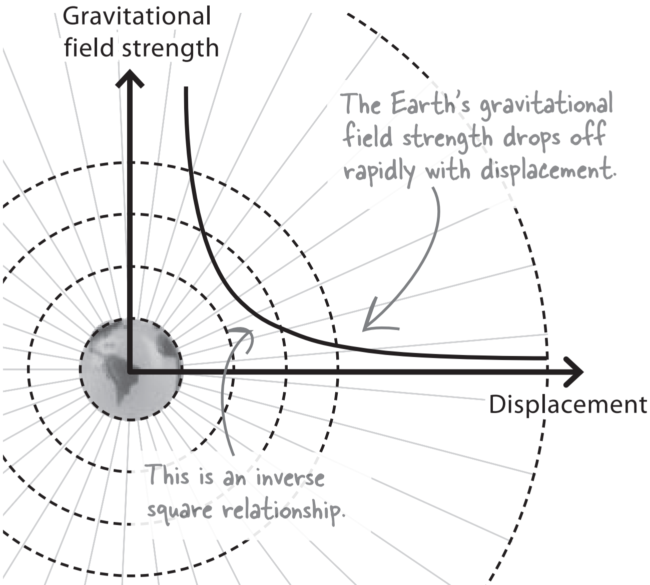
2 Earth radii

It's OK to do this with the scale on a graph if the numbers are very large or very small.

Now you can calculate the force on the spaceship at <u>any</u> distance from the Earth

An astronaut is going from the Earth to the outer reaches of the solar system. We're trying to calculate his **escape velocity** - the speed he needs to be going at so as to not fall back down to Earth again.

You've realized that the Earth's gravitational field must get smaller as you move further away from the Earth. This means that the **acceleration** of an astronaut's spaceship due to the gravitational field isn't constant - and, therefore, the **force** that the spaceship experiences isn't constant either.



You've worked out that the gravitational field strength follows an **inverse square** relationship. If you double the displacement, you quarter the strength of the gravitational field - and also quarter the gravitational force that the spaceship experiences.

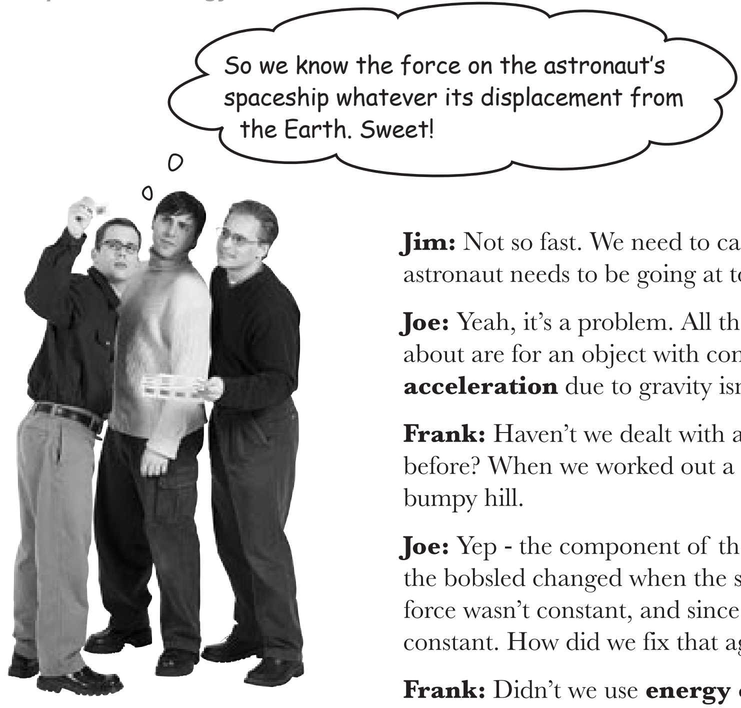
As the force (and acceleration) aren't constant, you can't use equations of motion to calculate the escape velocity.

You can only use equations of motion when the acceleration is constant.

BRAIN POWER

You have a problem where the **force** acting on the spaceship isn't constant - so its **acceleration** isn't constant and you can't use equations of motion.

Can you think of a different method you can use to calculate the escape velocity?



Jim: Not so fast. We need to calculate the **velocity** that the astronaut needs to be going at to escape from the Earth.

Joe: Yeah, it's a problem. All the equations of motion we know about are for an object with constant acceleration. But the **acceleration** due to gravity isn't constant

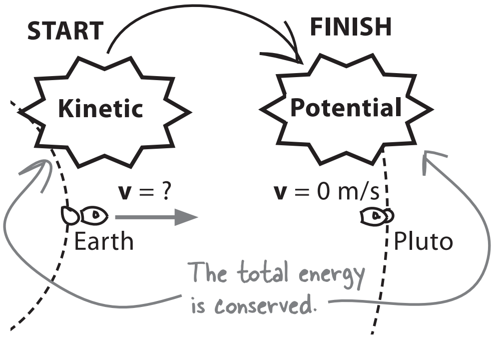
Frank: Haven't we dealt with a **non-constant acceleration** before? When we worked out a bobsled's velocity at the bottom of a bumpy hill.

Joe: Yep - the component of the gravitational **force** accelerating the bobsled changed when the steepness of the hill changed. So the force wasn't constant, and since $\mathbf{F}_{net} = m\mathbf{a}$, the acceleration wasn't constant. How did we fix that again?

Frank: Didn't we use **energy conservation**?

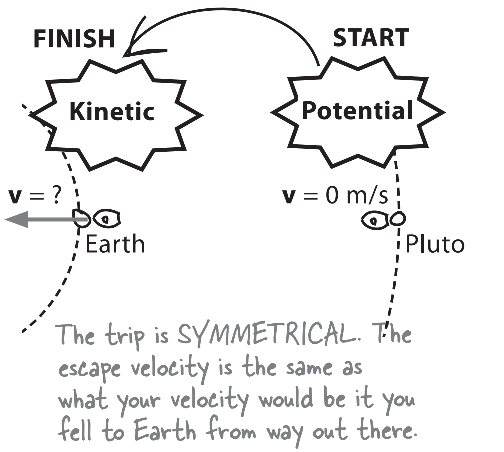
Joe: Yeah, we said that the bobsled's **potential energy** at the top of the slope was transferred to **kinetic energy** at the bottom of the slope - whatever the shape of the slope in between.

Frank: It was just the **difference in height** that was important.



Jim: Can we do the same for the spaceship? Can we pretend that it starts out very far away with lots of potential energy, then "falls" back down to Earth? It'd be **symmetrical**, right? **The velocity you need to escape from the Earth would be the same as the velocity you end up with when you fall to Earth**.

Joe: The change in potential energy is the same size as the change in kinetic energy, so we can use $K = \frac{1}{2}m\mathbf{v}^2$ to calculate the velocity!



Frank: Cool. So we just say $U_g = m\mathbf{gh}$ to calculate the change in potential energy, and we're nearly done!

Jim: Um ... I'm not so sure. The '**g**' in that equation is the acceleration due to gravity, isn't it? But that isn't constant here!

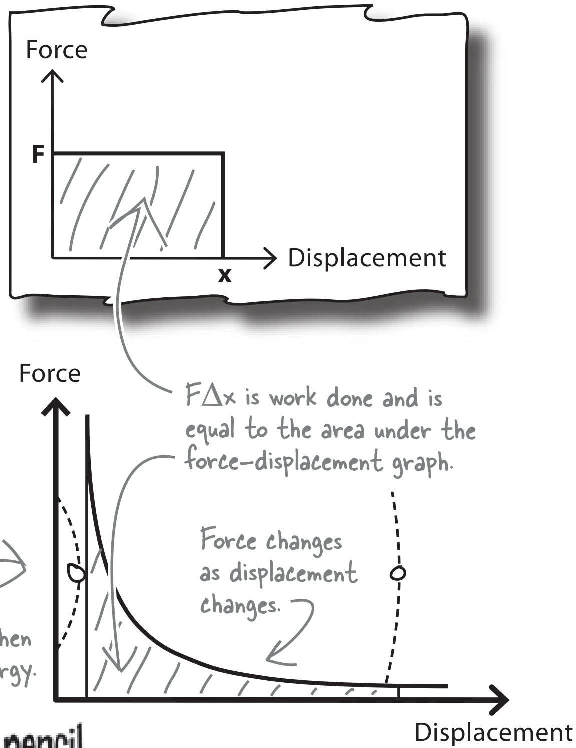
Joe: Hmm. The equation $U_g = m\mathbf{gh}$ originally comes from work = force × displacement, doesn't it. And it works over displacements on a scale of a few hundred meters. So can we break down the force-displacement graph into little portions of a few hundred meters each, calculate the work done for each portion, then add them together to get the total work done?

Frank: Or can we try to look up a book to get an equation for the gravitational potential energy when you're far away?

The potential energy is the area under the force-displacement graph

You give an object potential energy, U, by doing work on the object against the force of gravity. As Work = $\mathbf{F}\Delta\mathbf{x}$, close to the Earth you can say $U = m\mathbf{gh}$, where $m\mathbf{g}$ is the gravitational force and $\mathbf{h}$ is the object's change in height.

A visual way of showing this is to say that the potential energy is equal to the **area under the force-displacement graph**.



But if you're going a long way from the Earth, the force changes as the displacement changes. This means that it's not so easy to calculate the area under the force-displacement graph - and, therefore, not so easy to calculate the change in potential energy.

If the force is a gravitational force, then work done = change in gravitational potential energy.

To save you having to calculate the area under the curved graph (difficult!) we've provided a ready-bake **equation** for the gravitational potential energy.

Ready Bake Equation

$$U_G = -\frac{G m_1 m_2}{r}$$

If you're taking an exam, you'll see this on your equation sheet.

Sharpen your pencil

a. According to the ready bake equation, what is the value of U when r is very, very large (i.e., infinite)?

b. According to the ready bake equation, what is the value of U at the Earth's surface for a 10.0 kg mass? (The radius of the Earth is 6.38×10^6 m and its mass is 5.97×10^{24} kg. G is 6.67×10^{-11} m^3/kg.s^2)

c. Comment on your answers.

Sharpen your pencil Solution

Ready Bake Equation

$$U_G = -\frac{G m_1 m_2}{r}$$

a. According to the ready bake equation, what is the value of *U* when *r* is very, very large (i.e., infinite)?

When r is very very large, U = 0 J, as you are dividing by a very very large number and r dominates.

b. According to the ready bake equation, what is the value of *U* at the Earth's surface for a 10.0 kg mass? (The radius of the Earth is 6.38×10^6 m and its mass is 5.97×10^{24} kg. *G* is 6.67×10^{-11} $m^3/kg.s^2$)

At Earth's surface, r is radius of Earth:

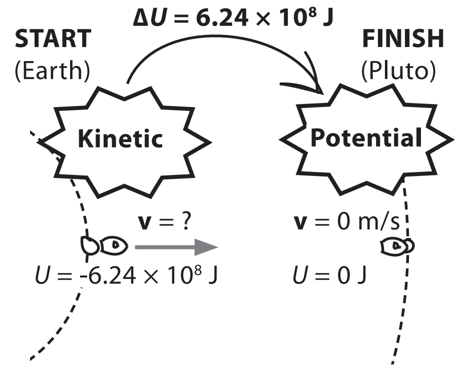
$$U = -\frac{Gm_1m_2}{r} = -\frac{6.67 \times 10^{-11} \times 10.0 \times 5.97 \times 10^{24}}{6.38 \times 10^6}$$

$$U = -6.24 \times 10^8 \text{ J (3 sd)}$$

c. Comment on your answers.

Weird! Last time, U = 0 at the surface, not at infinity. And what's with the negative number at the Earth's surface?!

It's the CHANGE in gravitational potential energy that allows you to do calculations, not the absolute values.

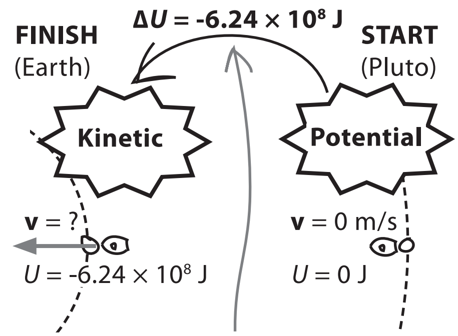


You can only measure changes in energy.

If you know the **change in potential energy** between the surface of the Earth and the edge of the solar system, you can say that an identical amount of **kinetic energy** must be transferred to get there - then use this to work out the escape velocity, as $K = \frac{1}{2}m\mathbf{v}^2$.

As it's the **change** in potential energy that's important, it doesn't matter what the absolute values you calculate are - as long as the change is the same.

It's like putting an object part-way along a ruler. If one end of the object is at the mark that says 2.0 cm and the other end is at the 4.0 cm mark, you can see that the object is 2.0 cm long. Same goes if your object is between the 28.0 cm and 30.0 cm marks. The object is still 2.0 cm, long, and where you choose to count from on your **scale** doesn't affect this.



ΔU will have the same SIZE whichever direction you make the trip in, but in one direction the change will be positive (as you lose kinetic energy) and in the other direction the change will be negative (as you gain kinetic energy).

If $U = 0$ at infinity, the equation works for any star or planet

Saying that $U = 0$ at the Earth's surface is great if you're close to the Earth's surface, when you can take $\mathbf{g} = 9.8\ m/s^2$ to be a constant. In that context, you'd almost always define zero potential energy to be at ground level, as this is a common reference point for everything.

But if you're moving away from the Sun or another planet, the Earth's surface isn't a good reference point!

It's a bit like temperature. On Earth, it's convenient to have a scale defined by the freezing and boiling points of water on Earth, 0°C and 100°C. But that's not a very universal scale, as the reference point depends on you being on Earth.

Your have your maximum possible potential energy when you're at infinity.

For temperature, scientists use the Kelvin scale, where "absolute zero" is defined as the lowest temperature theoretically possible. No one has ever reached absolute zero - just like no one has ever reached infinity - but defining an unchanging common reference point like this gives you a benchmark you can measure everything else against.

So rather than having a different reference point for every one of the (estimated) 10^{22} stars and planets in the universe, it's better to use a **common reference point** that is the same for any star or planet, regardless of its mass and the distance an object is from it.

That common reference point is the place where you have **maximum gravitational potential energy**. This has to be when you have the **maximum possible displacement** from the Earth - and the Sun - and all the other stars. We call this reference point "being at **infinity**".

So ... you have your maximum potential energy when r is very large. And the equation tells us that U = 0 when r is very large. So your **maximum** potential energy is zero?! Doesn't that mean your potential energy will be **negative** if you're not at infinity?

$U = 0$ at infinity. Total energy is conserved, so as you gain kinetic energy by falling, your potential energy becomes negative.

The values you calculate for the potential energy will be negative.

The further away you are from the surface of a planet, the greater your potential energy. Defining $U = 0$ at infinity - the furthest you can possibly be - means that as you gain kinetic energy by falling towards a star or planet, your gravitational potential energy becomes negative.

But that's OK - the **scale** hasn't changed. The **change** in potential energy between two points is still the same. Where you choose to count from on your scale doesn't matter. You just have to be careful with minus signs!

Potential Energy Exposed

This week's interview:
Potential energy answers charges of inconsistency.

Head First: So, potential energy, you've been accused of inconsistency. What's the bottom line?

Potential energy: The bottom line (or zero potential energy) is wherever you want it to be.

Head First: Um... are you implying that even you don't know when you're equal to zero?!

Potential energy: Yes ...

Head First: And if you don't know, how on Earth am I - or anyone else - supposed to know?

Potential energy: I think you've got the wrong end of the stick there. You can only measure **changes** in potential energy, not absolute values And that means you get to **choose where zero potential energy is**.

Head First: But why's that useful?

Potential energy: It gives you a **reference point** to measure **changes** in potential energy against.

Head First: Right. But why suddenly put that reference point at **infinity**? I was perfectly happy before, when zero was at the surface of the Earth.

Potential energy: You have to think about the bigger picture. When you're close to the Earth's surface, you can use the simple form of the equation $U_g = m\mathbf{gh}$, because the acceleration due to gravity - and therefore the gravitational force - is constant.

Head First: But why can't you do the same further away from the Earth?

Potential energy: The gravitational force drops off in an inverse square way. So it's not constant - and you need to do more complicated math to calculate the change in potential energy.

Head First: I can see that - but it still doesn't tell me why you suddenly want to put zero at infinity instead of the Earth's surface!

Potential energy: What if you're launching your spaceship from the moon, or from Mars, not Earth?

Head First: Um, I guess I put the zero of potential energy at the surface of the Moon, or Mars ...

Potential energy: But that's very inconsistent, isn't it? How are you supposed to compare all these values when you keep on changing where your zero is?

Head First: Hmmm. You may have a point. But I'm finding the concept of negative potential energy difficult. I didn't think energy was a vector.

Potential energy: I'm not a vector - just like temperature isn't a vector. A temperature of –2°C doesn't point in the opposite direction from a temperature of 2°C. The negative sign is just to indicate its **position on a scale**, not a direction.

Head First: What does that mean in practice?

Potential energy: OK, try thinking about it like this. Suppose you start at infinity with zero total energy, then fall towards the Earth (or another star or planet) What happens to your kinetic energy?

Head First: I guess your velocity increases so your kinetic energy increases.

Potential energy: Right! But the total energy needs to stay the same. So if you start off with zero potential energy and gain, say 100 J of kinetic energy, your new potential energy must be -100 J.

Head First: OK ... but all these messy negative numbers make the math tricker, right?!

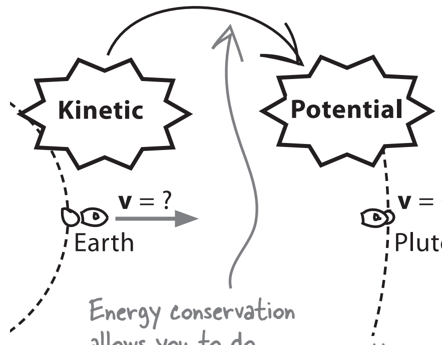
Potential energy: I do admit, that's the down side of putting zero at infinity. But as long as you're careful you'll be OK. You're dealing with **changes** in potential energy - so going further away from a planet is still a positive change - like going from a temperature of -40°C to -2°C is a change of 38°C.

Head First: Thank you - that's much clearer now.

Use energy conservation to calculate the astronaut's escape velocity

If the gravitational force on the astronaut's spaceship was constant and didn't drop off with distance, you could just use forces and equations of motion to work out his **escape velocity**.

But the gravitational force on the spaceship changes as he gets further away, so it's difficult to do the calculation using forces. But you can use **energy conservation**, as the spaceship's kinetic energy at the start must have been transferred to potential energy by the end.



Sharpen your pencil

Ready Bake Equation

$$U_G = -\frac{G m_1 m_2}{r}$$

After a spaceship blasts off from the Earth's surface, its propulsion system is switched off.

What is the minimum velocity that the spaceship must have to successfully escape from the Earth's gravitational field so that it can reach infinity without falling back down again?

If you can escape to infinity, you can definitely make it to Pluto!

The radius of the Earth is 6.38×10^6 m, and its mass is 5.97×10^{24} kg. G, the gravitational constant, is 6.67×10^{-11} $m^3/kg \cdot s^2$

Sharpen your pencil Solution

After a spaceship blasts off from the Earth's surface, its propulsion system is switched off.

What is the minimum velocity that the spaceship must have to successfully escape from the Earth's gravitational field so that it can reach infinity without falling back down again?

The radius of the Earth is 6.38×10^6 m, and its mass is 5.97×10^{24} kg. G, the gravitational constant, is 6.67×10^{-11} m^3/kg·s^2

Ready Bake Equation

$$U_G = -\frac{Gm_1m_2}{r}$$

This equation for the gravitational potential energy (with $U = 0$ at infinity) has a similar form to the equation for the gravitational force.

Earth's surface: $v_0 = ?$

Infinity: $v = 0$

m_1 = spaceship mass

m_2 = Earth mass

Kinetic at Earth's surface transferred to potential at infinity.

$$\Delta U = U_{infinity} - U_{earth}$$

$$\Delta U = 0 - U_{earth}$$

Zero minus a negative number is a positive number, so $\Delta U = \frac{Gm_1m_2}{r}$

$$\Delta K = \Delta U$$

$$\tfrac{1}{2}\cancel{m_1}v_0^2 = \frac{G\cancel{m_1}m_2}{r}$$

As the mass of the spaceship divides out, the escape velocity doesn't depend on the mass.

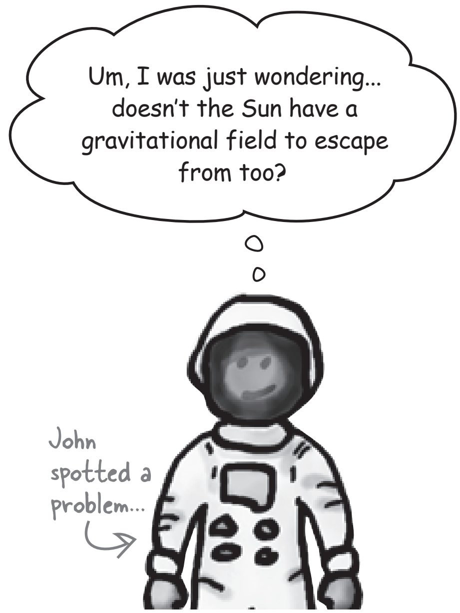
It's OK if you used the value for the gravitational potential energy at the Earth's surface that you calculated on page ??, but dividing out the mass of the spaceship is usually easier for these types of questions.

$$\Rightarrow \quad v_0 = \sqrt{\frac{2Gm_2}{r}} = \sqrt{\frac{2 \times 6.67 \times 10^{-11} \times 5.97 \times 10^{24}}{6.38 \times 10^6}}$$

$$v_0 = 1.12 \times 10^4 \text{ m/s} = \underline{\underline{11.2 \text{ km/s}}} \text{ (3 sd)}$$

The escape velocity that ensures you won't fall back down to Earth again after you switch the engine off is 11.2 km/s. Pretty fast - but in space there's no kind of atmosphere, and no friction to slow you down!

But the astronaut's not totally happy with this. He thinks that you also need to escape from the Sun's gravitational field! The Sun has around 300000 times more mass than the Earth - which will really tell over large distances! You'll have to factor in the change in potential energy due to the astronaut's distance from the Sun changing, as well as his distance from the Earth.



Sharpen your pencil

Some time after a spaceship blasts off from the Earth's surface, its propulsion system is switched off.

What is the minimum velocity that the spaceship must have to successfully escape from both the the Earth's gravitational field and the Sun's gravitational field so that it can reach infinity without falling back down again?

The radius of the Earth is 6.38×10^{6} m, and the mass of the Earth is 5.97×10^{24} kg. The distance from the Earth to the Sun is 1.50×10^{11} m. The radius of the Sun is 6.96×10^{8} m, and the mass of the Sun is 1.99×10^{30} kg. *G*, the gravitational constant, is 6.67×10^{-11} $m^3/kg.s^2$

Hint: Start with two sketches, one for the change in potential energy due to the Earth's gravitational field, and the other for the change in potential energy due to the Sun's gravitational field.

Be VERY careful with your variable names, as you will have various masses, radii and displacements floating around.

Hint: The total change in potential energy is equal to the changes due to the Earth's gravitational field and the Sun's gravitational field added together. Then you can use the same method as on the opposite page.

Sharpen your pencil Solution

Some time after a spaceship blasts off from the Earth's surface, its propulsion system is switched off.

What is the minimum velocity that the spaceship must have to successfully escape from both the the Earth's gravitational field and the Sun's gravitational field so that it can reach infinity without falling back down again?

The radius of the Earth is 6.38×10^6 m, and the mass of the Earth is 5.97×10^{24} kg. The distance from the Earth to the Sun is 1.50×10^{11} m. The radius of the Sun is 6.96×10^8 m, and the mass of the Sun is 1.99×10^{30} kg. *G*, the gravitational constant, is 6.67×10^{-11} m^3/kg.s^2

Change in potential energy due to Earth's gravitational field

m_1 = spaceship mass

m_E = Earth mass

r_E = Radius of Earth

Earth's surface: $U = -\frac{Gm_1m_E}{r_E}$

Infinity: $U = 0$

Change in potential energy due to Sun's gravitational field

m_1 = spaceship mass

m_S = Sun mass

r_{fromS} = Distance from Sun

Earth's surface: $U = -\frac{Gm_1m_S}{r_{fromS}}$

Infinity: $U = 0$

The radius of the sun is irrelevant! It's the distance you are from the Sun that's important.

Kinetic at Earth's surface transferred to potential at infinity.

$$\Delta U = U_{infinity} - U_{earth}$$

$$\Delta U = 0 - U_{earth}$$

Zero minus a negative number is a positive number, so $\Delta U = \frac{Gm_1m_E}{r_E} + \frac{Gm_1m_S}{r_{fromS}}$

$$\Delta K = \Delta U$$

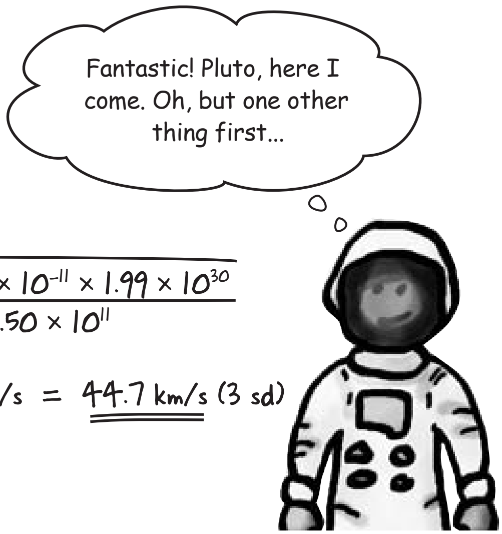
$$\frac{1}{2}m_1v_0^2 = \frac{Gm_1m_E}{r_E} + \frac{Gm_1m_S}{r_{fromS}}$$

The mass of the spaceship divides out.

$$\Rightarrow \quad v_0 = \sqrt{\frac{2Gm_E}{r_E} + \frac{2Gm_S}{r_{fromS}}}$$

$$v_0 = \sqrt{\frac{2 \times 6.67 \times 10^{-11} \times 5.97 \times 10^{24}}{6.38 \times 10^6} + \frac{2 \times 6.67 \times 10^{-11} \times 1.99 \times 10^{30}}{1.50 \times 10^{11}}}$$

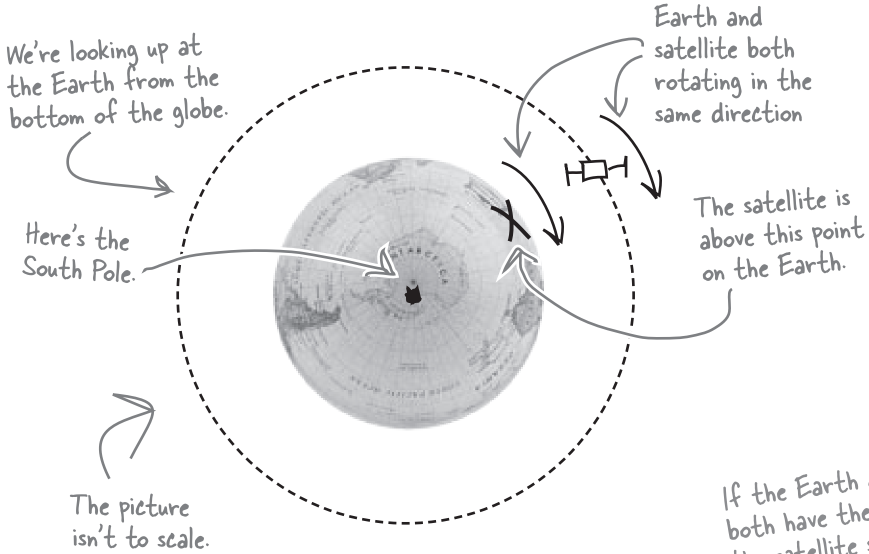
$$v_0 = \sqrt{1.44 \times 10^8 + 1.77 \times 10^9} = 4.37 \times 10^4 \text{ m/s} = \underline{\underline{44.7 \text{ km/s}}} \text{ (3 sd)}$$



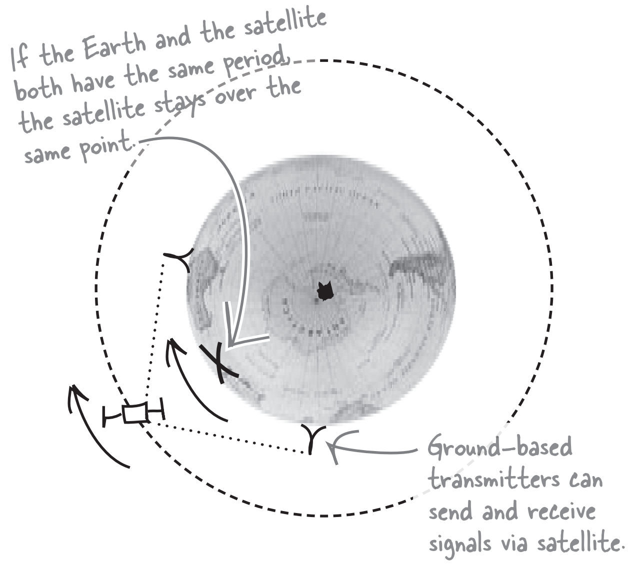
We need to keep up with our astronaut

Before sending our astronaut off to Pluto, we need to get some communication **satellites** in place to keep in contact with him. These satellites need to be in a certain type of orbit around the Earth called a **geostationary orbit**.

A geostationary orbit is one where the satellite always stays over the same point on the Earth's surface.



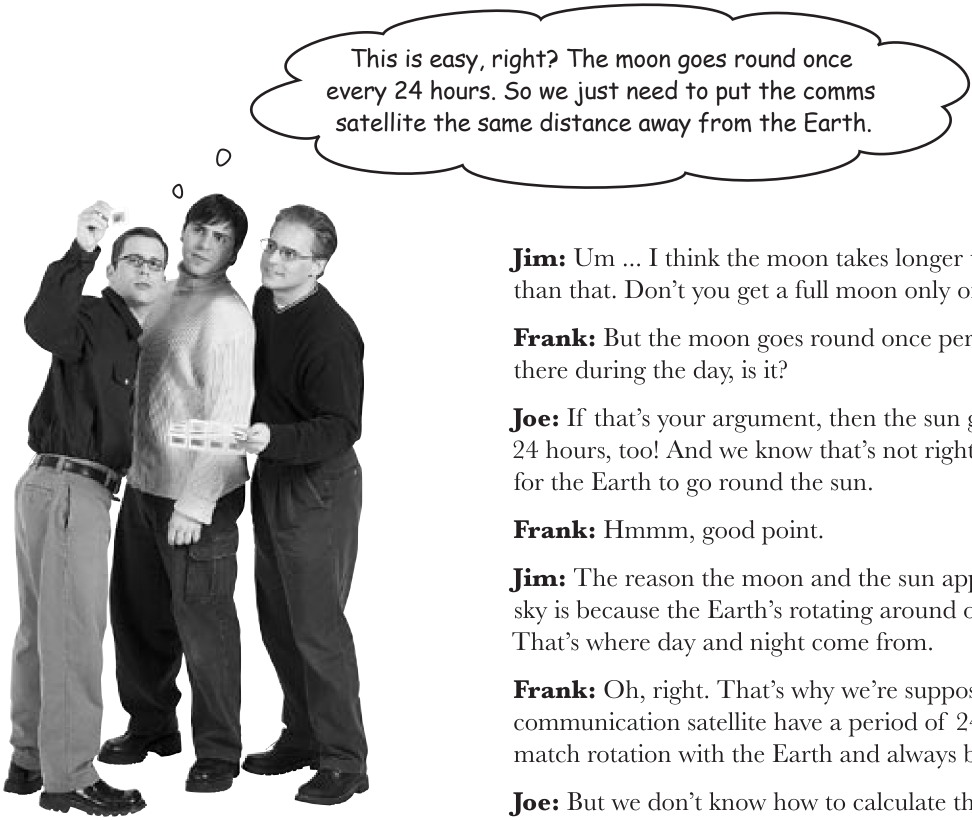
A GEOSTATIONARY orbit has a period of 24 hours, the same as the period of the Earth's rotation.



The satellite is moving... it just remains above the same point on the Earth's surface because the Earth is rotating with the **same period** as the satellite.

This allows you to use the satellite for Earth communications by aiming your transmitters and receivers at the same point in the sky.

We need the **period** of the satellite to be **24 hours** since that's the **time** it takes for the Earth to rotate once.



Jim: Um ... I think the moon takes longer to go around the Earth than that. Don't you get a full moon only once a month?

Frank: But the moon goes round once per 24 hours, right? It's not there during the day, is it?

Joe: If that's your argument, then the sun goes round once every 24 hours, too! And we know that's not right - it takes a whole year for the Earth to go round the sun.

Frank: Hmmm, good point.

Jim: The reason the moon and the sun appear to move across the sky is because the Earth's rotating around on its axis once a day. That's where day and night come from.

Frank: Oh, right. That's why we're supposed to make the communication satellite have a period of 24 hours, isn't it? So it can match rotation with the Earth and always be above the same point?

Joe: But we don't know how to calculate the period of the orbit. Actually, we don't know how to calculate ***anything*** for orbits!

Jim: Hmm. I'm sure that Pluto must take longer to orbit the sun than the Earth does. Maybe the period of the orbit depends on your **displacement** from the thing you're orbiting.

Frank: I'm not so sure. Remember when we were working with the hamster wheel? Didn't every part of the wheel rotate with the same angular velocity? That means every part had the same period, too, right? So the distance from the center of the wheel didn't affect the period of the wheel.

Joe: But all the parts of the hamster wheel were joined together, so they *have* to go around with the same period. Planets and satellites aren't joined together. So the period might depend on the distance from the center after all.

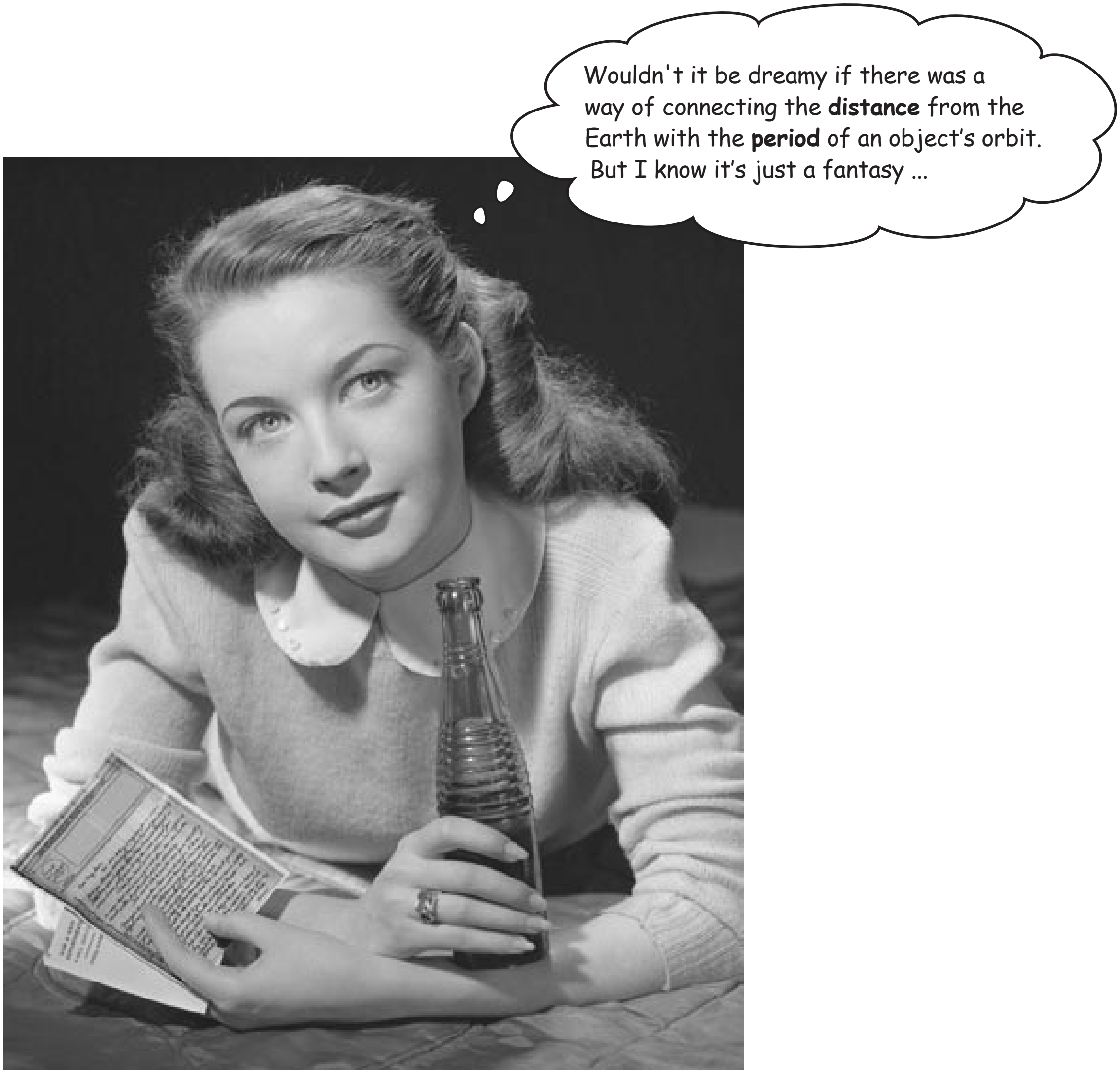
Frank: Hmm, that's a good point, too.

Jim: I'm sure that far away objects must take longer to orbit.

Joe: But how can we work that out?

A day is the time it takes the Earth to spin once on its own axis

A year is the time it takes the Earth to orbit once round the sun.


Wouldn't it be dreamy if there was a way of connecting the **distance** from the Earth with the **period** of an object's orbit. But I know it's just a fantasy ...

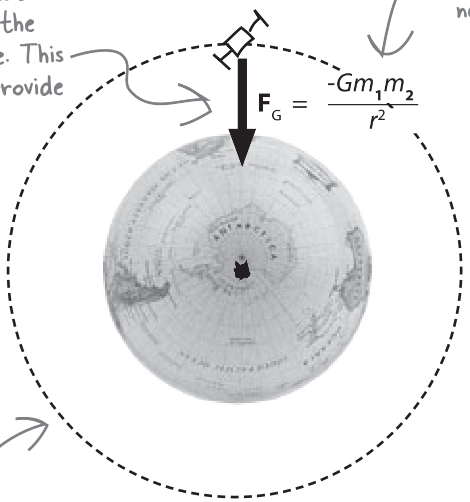
The centripetal force is provided by gravity

An object that travels along a circular path must have a **net force** acting on it to provide a **centripetal force**. This force must act towards the center of the circle.

In chapter 15, you learned to equate the size of the centripetal force, $F_c = mr\omega^2$ with the net force. Here, the net force on the satellite is provided by the gravitational force from the Earth.

SIZE is given by the equation (remember the negative sign is convention).

DIRECTION: Gravitational force acts towards the center of the circle. This means that it can provide a centripetal force.



There will be only one radius where the satellite has a period of 24 h and a geostationary orbit.

When you've equated the centripetal force with the gravitational force, you'll have the job of making some substitutions so that your equation is in terms of T, the **period** instead of ω, the angular velocity.

Then you can calculate the radius of the satellite's orbit for any period - including the 24 hour geostationary period you're interested in.

Use the ready-bake facts to help you solve the problem.

If you make sure both are pointing towards the center of the circle, then they'll both have the same sign.

Equate the CENTRIPETAL FORCE with the GRAVITATIONAL FORCE to solve orbit problems.

Ready Bake Facts

Mass of the Earth:

5.97×10^{24} kg

Radius of the Earth:

6.38×10^{6} m

Gravitational constant

$G = 6.67 \times 10^{-11}$ m^3/kg·s^2

Sharpen your pencil

You can look up equations involving T in the appendix.

a. Work out a general equation for the size of *r*, the radius of a satellite's orbit in terms of *T*, the period of the satellite, *G* the gravitational constant and m_e the mass of the Earth (don't insert any values, just work out an equation).

b. Calculate the height that the satellite must be above the Earth's surface to have a period of 24 hours.

c. If the radius of a satellite's orbit is doubled, what happens to the period of its orbit? (Please do this question using proportion.)

d. What happens to a satellite's kinetic and potential energy as it goes round a circular orbit?

Sharpen your pencil Solution

a. Work out a general equation for the size of *r*, the radius of a satellite's orbit in terms of *T*, the period of the satellite, *G* the gravitational constant and m_e the mass of the Earth (don't insert any values, just work out an equation).

Gravitational force provides centripetal force for orbit.

$$F_c = F_{net}$$

$$m_s r\omega^2 = \frac{G m_s m_e}{r^2}$$

Both forces points towards the center of the circle, so you can lose the 'conventional' minus sign.

We've used m_s for the saatellite's mass, but the symbol doesn't really matter, as the mass divides out!

$$\omega^2 = \frac{G m_e}{r^3}$$

Make a substitution for ω.

$$\omega = 2\pi f \quad \text{and} \quad f = \frac{1}{T}$$

$$\Rightarrow \omega = \frac{2\pi}{T}$$

$$\left(\frac{2\pi}{T}\right)^2 = \frac{G m_e}{r^3}$$

$$\frac{4\pi^2}{T^2} = \frac{G m_e}{r^3}$$

$$r^3 = \frac{G m_e T^2}{4\pi^2}$$

This is called Kepler's 3rd Law. It shows you how the radius and period of an orbit are related.

The radius in the gravitational force equation is always the distance from the center of the Earth, not the surface. So you need to calculate the difference at the end.

b. Calculate the height that the satellite must be above the Earth's surface to have a period of 24 hours.

$$r = \sqrt[3]{\frac{G m_e T^2}{4\pi^2}} = \sqrt[3]{\frac{6.67 \times 10^{-11} \times 5.97 \times 10^{24} \times 86400^2}{4 \times \pi^2}} = 4.22 \times 10^7 \text{ m (3 sd)}$$

24 hours is $24 \times 60 \times 60 = 86400$ s

Distance from surface $= r - r_e = 4.22 \times 10^7 - 6.38 \times 10^6 = \underline{3.59 \times 10^7 \text{ m}}$ (3 sd)

c. If the radius of a satellite's orbit is doubled, what happens to the period of its orbit? (Please do this question using proportion.)

$$r^3 = \frac{G m_e T^2}{4\pi^2}$$

Everything in the equation is constant apart from r and T.

If r is doubled, left hand side of equation increases $2 \times 2 \times 2 = 8$ times.

This means that T^2 is 8 times larger than before.

So T is $\sqrt{8} = 2.83$ (3 sd) times larger than before.

d. What happens to a satellite's kinetic and potential energy as it goes round a circular orbit?

They remain constant because the satellite's height and speed remain constant (even though its velocity is changing direction all the time)..

there are no Dumb Questions

Q: What was that ∛ thing all about in the math bit over there?

A: It's a cube root symbol. You know how a square root is what you do to find out what number you need to square to get the number you started with?

Q: Yeah ...

A: Well, a cube root is what you type into your calculator to work out what number you'd need to cube to get the number you started off with.

Q: So if I have r^3 = something then I can say that $r = \sqrt[3]{\text{something}}$

A: Yes. It works in the same way that a square root does.

Q: So ... I noticed that the mass of the satellite divided out when I made the centripetal force equal to the gravitational force. That kinda thing has happened a few times now.

A: Yeah, good spot. That's because the gravitational force depends on the object's mass, *m*. The gravitational force provides the centripetal force, $F_c = ma_c$. So when you equate them, you get an '*m*' on each side of your equation, which divides out.

Q: So why can't I just equate the gravitational field strength with the centripetal acceleration and skip the step of dividing out the mass?

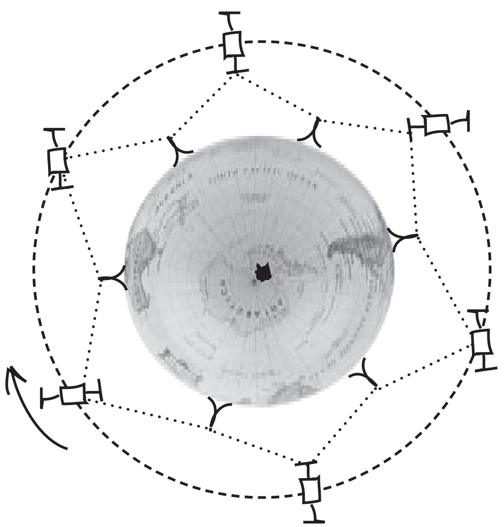
A: You could ... but dealing with forces is a good habit to get into.

Q: Why should I deal with forces, not accelerations?

A: There are a lot of other sources of centripetal force - and the size of the force may not depend on the mass of the object.

Q: When might the size of the force not depend on the mass of the object?

A: Well, an electron will follow a curved path when it goes through a magnetic field because of its electric charge, not its mass. You don't have to worry about that right now because it's in the electromagnetism part of your course. But if you get used to equating the centripetal force with the source of the net force, you'll find dealing with that a whole lot easier when you get there.



With the comms satellites in place, it's Pluto (and beyond)

The communications satellites are in place in their geostationary orbits nearly 4000 km above the Earth's surface.

And the astronaut it good to go.

Stand by for blasting off to Pluto - and beyond!

Question Clinic: The "gravitational force = centripetal force" Question

Another classic way of getting you to do something with centripetal force is to give you a problem to do with orbits to solve. The big thing here is to equate the centripetal force with the gravitational force that provides it - then follow through with the math.

Buzzwords to get you thinking about circular motion

Make sure you use the mass of the Earth – not the mass of the sun or anything else on your table of information!

This tells you which letters you should include.

2. A communications satellite is to be put into orbit around the Earth.

a. Work out a general equation for T, the period of a satellite in terms of r the radius of its orbit, G the gravitational constant and m_e the mass of the Earth.

b. Work out the height that the satellite must be above the Earth's surface to have a period of 24 hours.

c. If the radius of the satellite's orbit is doubled, what happens to the period of its orbit?

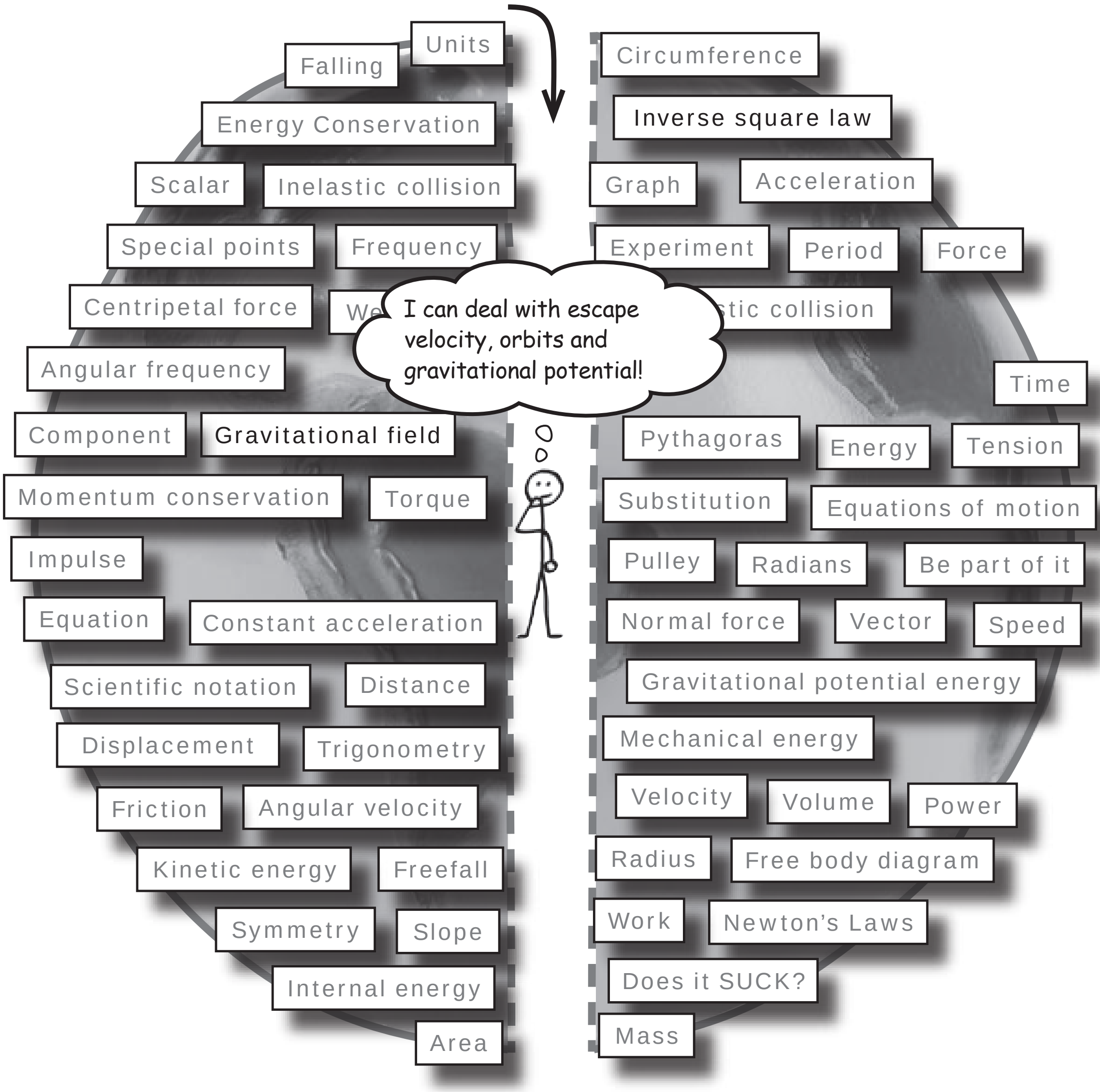
This means don't use numbers yet.

You'll need to convert this time to seconds to make all the units work through correctly.

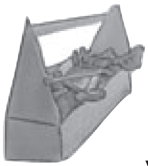
Notice that you'll have to subtract the radius of the Earth from the radius of the orbit to get the distance above the surface!

This is a typical question where you need to use proportion in an equation you already know (in this case, the equation from part a).

Be careful not to use the wrong equation! The equations for the gravitational force, gravitational field (i.e., acceleration due to gravity) and the gravitational potential energy (with 0 at infinity) all look very similar. Remember that gravitational force follows an inverse square law - and that its equation involves the masses of both the objects involved.



☑	Gravitational field	The strength of a gravitational field at a point is the same as the acceleration of an object in freefall at that point. Gravitational field lines help you to visualize the gravitational field strength.
☑	Inverse square law	If a quantity (for example the gravitational field strength) is proportional to $\frac{1}{r^2}$ then the quantity follows an inverse square law.



Your Physics Toolbox

You've got Chapter 18 under your belt and added some problem-solving concepts to your toolbox.

Gravitational field

The gravitational field strength tells you the acceleration something would experience if it was allowed to freefall at that distance from the Earth.

The gravitational field strength follows an inverse square law.

Gravitational field lines

Gravitational field lines point in the direction something would move in if it was allowed to freefall from that point.

The closer together they are, the stronger the gravitational field is at that point.

Inverse square law

If a quantity follows an inverse square law, it drops off by $1/r^2$ as you increase the distance from its source. This means that if you double the distance, there's a fourfold decrease in the quantity.

Examples: light, sound, gravitational field strength.

Geostationary orbit

An orbit with a period of 24 hours. So-called because if the satellite is over the equator and rotating in the same direction as the Earth, it stays over the same spot on the ground.

Gravitational potential

Another name for gravitational potential energy. The term is most commonly used when you've defined $U = 0$ to be at infinity.

The gravitational potential drops off as $1/r$ – if you double the distance, you halve the gravitational potential.

Calculations with gravitational potential

The change in potential energy is equal to the change in kinetic energy.

Be very, very careful with minus signs! Because $U = 0$ at infinity, all other values of gravitational potential will be negative. But the CHANGE in potential energy is the same wherever you define zero to be.

Calculations with orbits

The net force that provides the centripetal force for an orbit is the gravitational force.

Equate the gravitational force with the centripetal force, and the answers will drop out of your equation.

Ready Bake Equations

$$F_G = -\frac{G m_1 m_2}{r^2}$$

$$U_G = -\frac{G m_1 m_2}{r}$$

19 Oscillations (part 1)

Round and round

Things can look very different when you see them from another angle. So far you've been looking at circular motion from above - but what does it look like from the side? In this chapter, you'll tie together your **circular motion** and **trigonometry** superpowers as you learn extended definitions of **sine** and **cosine**. Once you're done, you'll be able to deal with anything that's moving around a circle - whichever way you look at it.

Welcome to the fair!

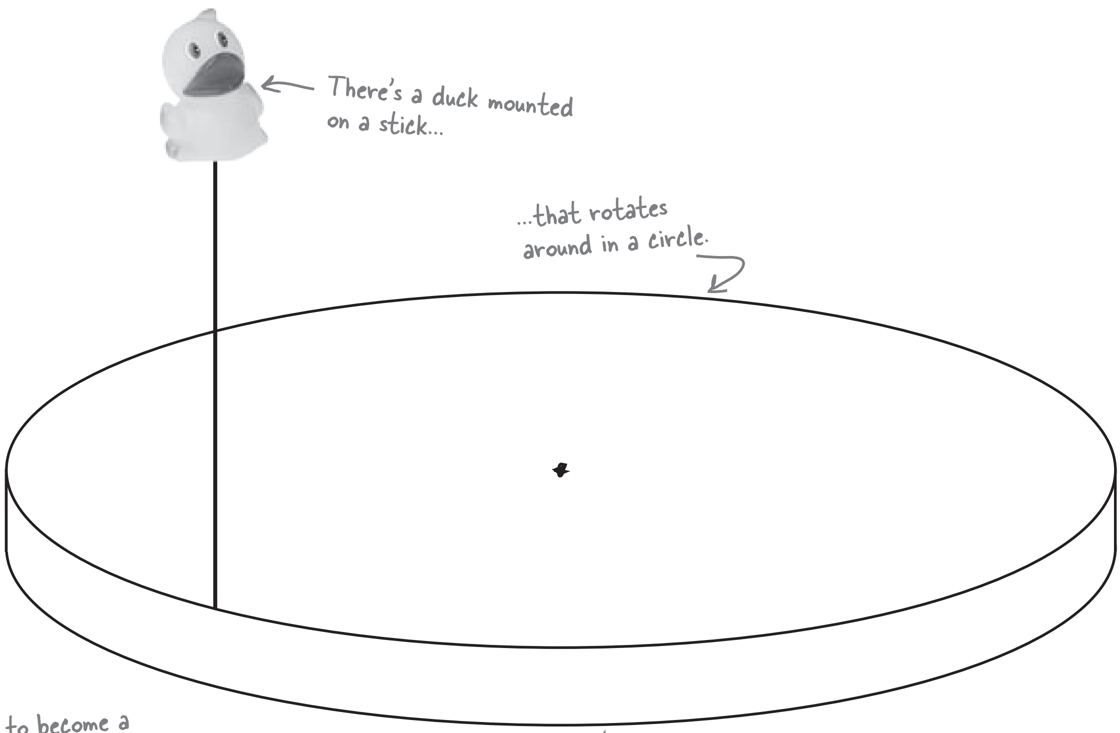
After your success at hamster training, you've got another client who needs your help. Jane's opening up a new booth at the local fair, and wants to build a duck-shooting competition - with a twist.

This chapter builds on what we did in chapter 16, with the hamster trainer.

At the moment, she's got a duck moving around in a **circle**. But she expects the game to be popular, and the rotating duck takes up space that she could use to pack in more paying customers.

So instead, Jane wants to let players shoot at a digital version of the duck, displayed on a giant **flat screen**. Each player feels like they're shooting at a real rotating duck, but Jane can just monitor the screen to run the game.

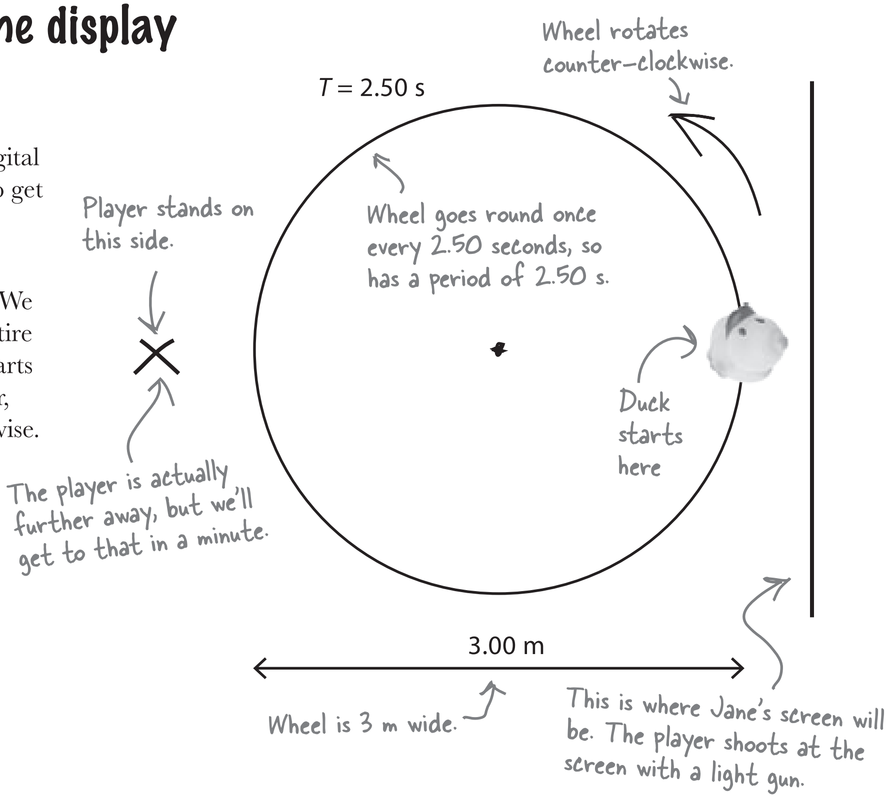
Jane's got all the equipment: a duck mounted on a rotating stick, a screen, and light guns that can shoot at the screen. The screen even registers where the gun hits, and which player shot the duck. Your job is the hard part, though: where should the screen display the duck as it moves around in a circle?



Reproduce the duck on the display

The real duck sits on a pole at eye level and travels around in a circle. But your job is to figure out where on Jane's big screen the digital duck should appear. Every time Jane tries to get the screen working, things look funny.

We know the real duck's moving along the edge of a circle, which is 3.00 meters wide. We also know that the duck goes around the entire circle once every 2.50 seconds. The duck starts in the middle on the far side from the player, and travels around the circle counter-clockwise.



Sharpen your pencil

Draw a player's-eye view of the game, and describe what the player sees from their side perspective as the duck goes around in a circle.

The player is much further away than there was space to show in the picture. The player is actually standing somewhere off the left hand page!

Make sure to write down any important **times** and **distances** from the moment the game starts that might help you draw the digital duck on the screen.

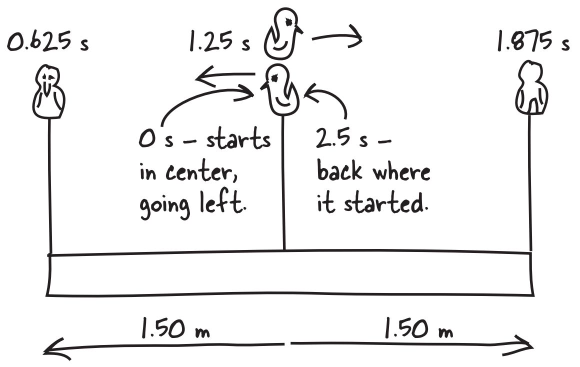
Space for your drawing.

Sharpen your pencil Solution

Draw a player's-eye view of the game, and describe what the player sees from their side perspective as the duck goes around in a circle.

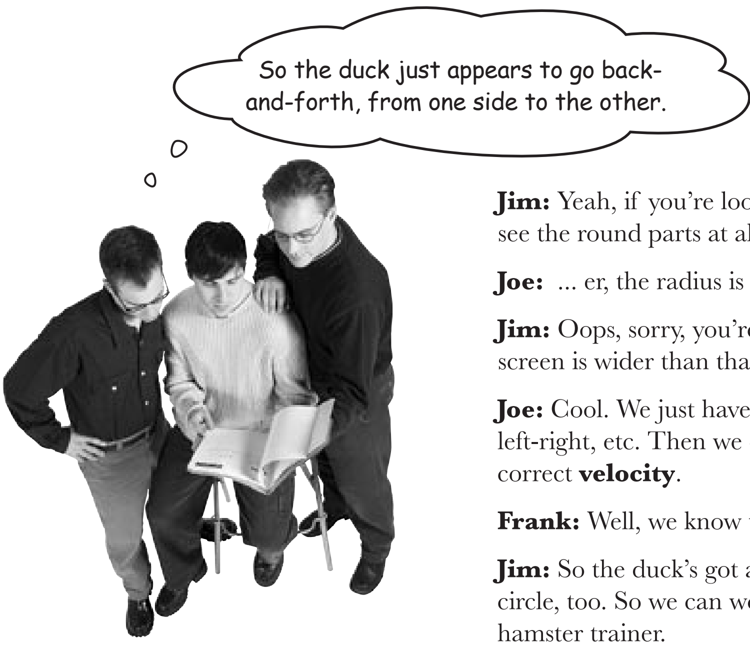
The player is much further away than there was space to show in the picture. The player is actually standing somewhere off the left hand page!

Make sure to write down any important **times** and **distances** from the moment the game starts that might help you draw the digital duck on the screen.



The player sees the duck starting out in the center of their view. Then after 0.625 s, the duck's on their left side. After 1.25 s the duck is back in the center. After 1.875 s, the duck's on the right. Then after 2.50 s, it's back where it started in the center.

The player doesn't see a circle at all. The duck just appears to move side to side (though the player would see different sides of the duck as the duck turns around the circle).



Jim: Yeah, if you're looking at a circle from the side, you don't really see the round parts at all. And the **radius** of the circle is 3.00 meters ...

Joe: ... er, the radius is 1.50 meters. The *diameter's* 3.00 meters.

Jim: Oops, sorry, you're right. The diameter's 3.00 meters. Jane's screen is wider than that, so we can show the duck at its actual size.

Joe: Cool. We just have to work out how fast the duck's going, left-right-left-right, etc. Then we can move the duck across the screen at the correct **velocity**.

Frank: Well, we know the duck takes 2.50 seconds to go around once.

Jim: So the duck's got a **period** of 2.50 s. We know the **radius** of the circle, too. So we can work out the duck's speed, just like we did for the hamster trainer.

Joe: You know, I'm a bit worried about the light gun. There might be some problems there. Here's what I'm thinking...

Sharpen your pencil

Joe's worried about registering hits on the screen. The distance from the player to the duck is smaller when the duck is at the front of the circle than it is when the duck is at the back of the circle. So the distance that the light from the light gun travels is different depending on the duck's position.

This might mean that a hit when the real duck's at the front of the circle may register before a hit at the back of the circle. But the screen for the digital duck is flat, so will register all hits at the same time. Do you think that's a problem we'll need to worry about?

a. A duck is on the edge of a roundabout with a diameter of 3.00 meters. The roundabout goes round once every 2.50 seconds. What is the duck's speed?

The equations you'll need to use for this are in chapter 16.

b. The duck is to be zapped with a light gun by a player standing 40.00 m away from the front of the roundabout. How much longer does it take the light to cover the 43.00 m to the back of the roundabout compared to the 40.00 m to the front of the roundabout? (The speed of light is 3×10^8 m/s.)

Ready Bake Fact

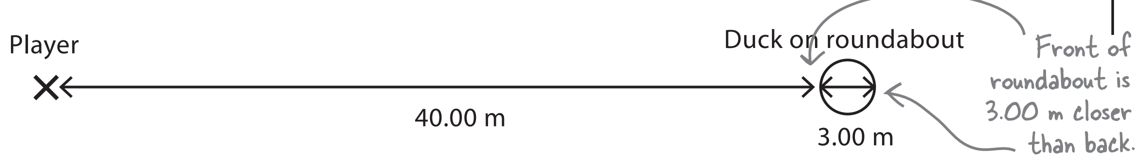
Speed of light:

3.00×10^8 m/s

c. Do you think the player will notice this extra time that it takes for a hit to register?

Sharpen your pencil Solution

Joe's worried about registering hits on the screen. The distance from the player to the duck is smaller when the duck is at the front of the circle than it is when the duck is at the back of the circle. So the distance that the light from the light gun travels is different depending on the duck's position.



This might mean that a hit when the real duck's at the front of the circle may register before a hit at the back of the circle. But the screen for the digital duck is flat, so will register all hits at the same time. Do you think that's a problem we'll need to worry about?

a. A duck is on the edge of a roundabout with a diameter of 3.00 meters. The roundabout goes round once every 2.50 seconds. What is the duck's speed?

Use equation $v = r\omega$; work out ω from frequency

Period of circle, $T = 2.50$ s

Frequency, $f = \frac{1}{T} = \frac{1}{2.50} = 0.400$ Hz

$$v = r\omega = r \times 2\pi f$$
$$= 1.50 \times 2 \times \pi \times 0.400$$
$$= 3.77 \text{ m/s (3 sd)}$$

You can also do this by using $C = 2\pi r$ to calculate the distance the duck travels in 1 reolution, then dividing the distance by the period

b. The duck is to be zapped with a light gun by a player standing 40.00 m away from the front of the roundabout. How much longer does it take the light to cover the 43.00 m to the back of the roundabout compared to the 40.00 m to the front of the roundabout? (The speed of light is 3×10^8 m/s.)

Difference in distance = 43.00 – 40.00 = 3.00 m

Time it takes light to cover this:

$$\text{speed} = \frac{\text{distance}}{\text{time}}$$

$$\Rightarrow \text{time} = \frac{\text{distance}}{\text{speed}} = \frac{3.00}{3.00 \times 10^8}$$

$$= \underline{\underline{1.00 \times 10^{-8}}} \text{ s (3 sd)}$$

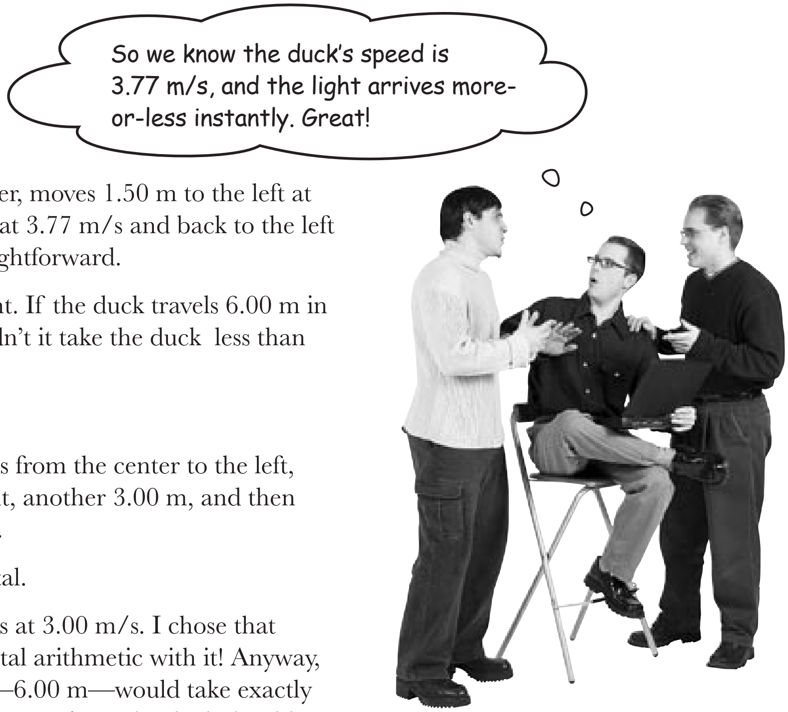
Ready Bake Fact

Speed of light:

3.00×10^8 m/s

c. Do you think the player will notice this extra time that it takes for a hit to register?

1×10^{-8} s is <u>very</u> short – 10 nanoseconds! Players won't notice the time difference between hitting the duck at the front of the circle and hitting the duck at the back of the circle. So we can just make the flat screen register hits instantly.



Jim: So the duck starts in the center, moves 1.50 m to the left at 3.77 m/s, goes 3.00 m to the right at 3.77 m/s and back to the left at 3.77 m/s. Should be pretty straightforward.

Joe: Wait... that doesn't sound right. If the duck travels 6.00 m in total at a speed of 3.77 m/s, wouldn't it take the duck less than 2.00 s to get back to the start?

Jim: How do you mean?

Joe: Well, we know the duck moves from the center to the left, 1.50 m, then all the way to the right, another 3.00 m, and then back to the center, another 1.50 m.

Frank: Right. So that's 6.00 m total.

Joe: But let's say that the duck goes at 3.00 m/s. I chose that speed because it's easier to do mental arithmetic with it! Anyway, the trip across the circle and back—6.00 m—would take exactly 2.00 s. But 3.77 m/s is *faster* than 3.00 m/s, so the duck should take *less* than 2.00 s to go around once.

Jim: But the duck actually takes 2.50 s to go round once - that's *more* than 2.00 s, not less! Something weird's going on ...

Does your answer make sense? Always check your work!

Sharpen your pencil

The real duck that you are reproducing on the screen takes 2.50 seconds for exactly 1 revolution. That's a total distance of 6.00 m from the player's perspective. But we just calculated that the duck's speed is 3.77 m/s.

a. How can it possibly take the duck 2.50 s to do a round trip of 6.00 m from the player's perspective when the duck has a speed of 3.77 m/s?

b. Describe qualitatively the speed that the player observes the duck moving at for any 'special points' you can spot on the circle. Illustrate this with a sketch.

Sharpen your pencil Solution

The real duck that you are reproducing on the screen takes 2.50 seconds for exactly 1 revolution. That's a total distance of 6.00 m from the player's perspective. But we just calculated that the duck's speed is 3.77 m/s.

a. How can it possibly take the duck 2.50 s to do a round trip of 6.00 m from the player's perspective when the duck has a speed of 3.77 m/s?

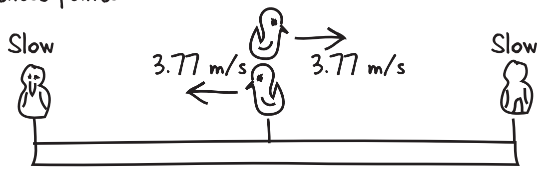
b. Describe qualitatively the speed that the player observes the duck moving at for any 'special points' you can spot on the circle. Illustrate this with a sketch.

a. The duck is going at 3.77 m/s round the circumference of the circle, not across the circle's diameter
Circumference $= 2\pi r = 2 \times 3.14 \times 1.50 = 9.42$ m

The round trip is 9.42 m, not 6.00 m, and takes 2.50 s for a duck traveling at 3.77 m/s.

b. The player viewing the duck from a side view. When the duck's in the middle, it looks like the duck is moving faster – at 3.77 m/s.

But when the duck is at the ends of the circle, most of its motion is along the player's line of sight: forward or backward. So to the player, it looks like the duck's speed is closer to 0 m/s at those points.



So although the duck goes at a constant speed, the player only sees the left-right component of its velocity. Because the player's got a side-view, right?

The player only sees one component of the duck's position and velocity.

If you are the player, the side-on view that you have makes it look like the duck's moving left and right along a straight line. You don't notice the duck going forward and backward at all. You only sees the left-right **component** of the duck's displacement and velocity vectors.

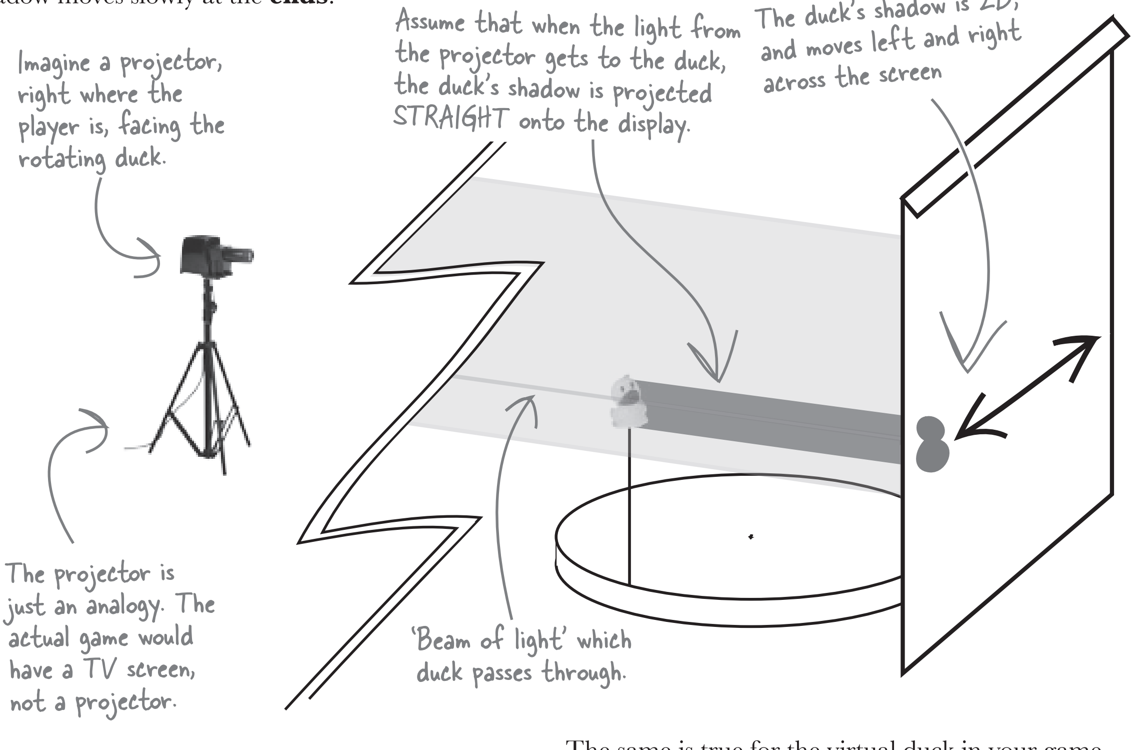
So from your perspective, the velocity the duck appears to have only depends on the left-right component of its velocity vector. This means that the duck appears to move rapidly across the center of its path, but slowly at each end of the circle.

The screen for the game is TWO-DIMENSIONAL

Even though the duck is three-dimensional, a **projection** of that duck is only two-dimensional. Imagine you're standing next to a projector, and you can only watch the shadow of the duck cast upon a screen.

As the duck moves across the center of the circle, it's mostly moving left to right (or right to left). The duck's shadow moves quickly across the **center** of the screen.

As the duck turns around at each end of the circle, it's moving mostly back-to-front (or front-to-back). But the screen only shows you left-to-right movement, so the duck's shadow moves slowly at the **ends**.



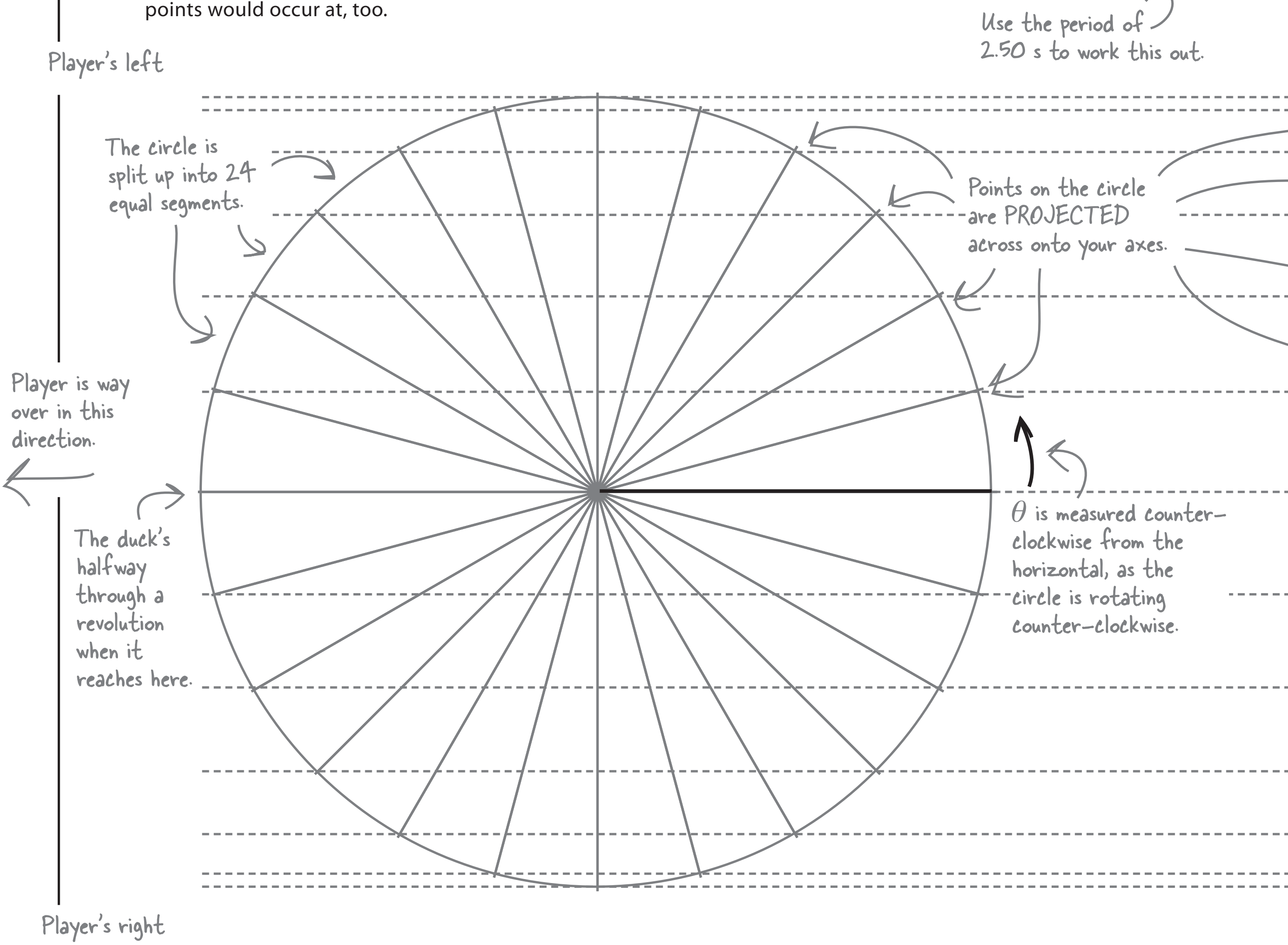
The same is true for the virtual duck in your game. The duck is displayed on a two dimensional screen. You don't notice the forward-backward component of the duck's displacement, only the left-right **component**. You see a **projection** of the duck's **displacement vector**.

Sharpen your pencil

Your job is to work out what the duck's displacement vector does as time goes on, so you know where to plot the duck on the screen. The circle the duck's attached to has a diameter of 3.00 m and a period of 2.50 s.

Use the pictures below, plus the fact that the angular velocity of the roundabout is constant, to **sketch** a graph of how the duck's left-right component (plotted on the vertical axis) varies with the angle θ (measured in radians and plotted on the horizontal axis) for one complete revolution (2π radians).

Mark the values at special points in the duck's motion on your sketch. Look for points where the duck is at an **extreme** in displacement - for instance, at zero, or at the maximum in either direction. Write the times that these points would occur at, too.

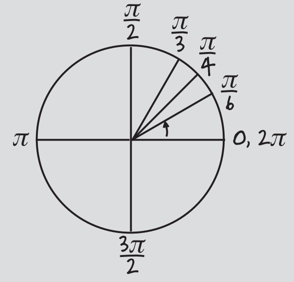


You'll probably find the note you made in chapter 16 helpful when you're thinking about this exercise.

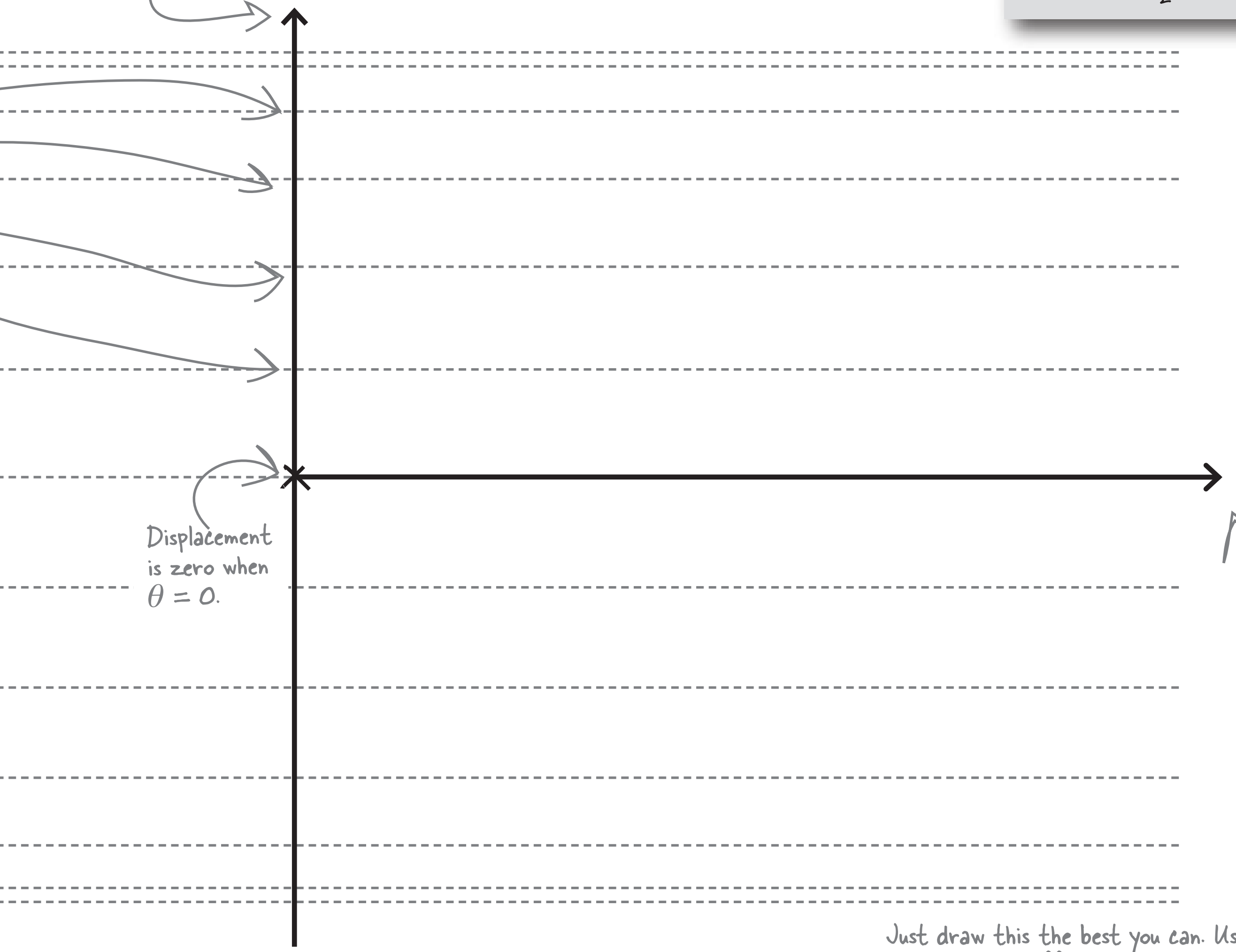
You need to do the graph title, axis labels, etc. yourself.

Radians

Radians are a way of measuring angles that's especially useful for working with circles. There are 2π radians in 1 revolution. Think of other angles in terms of 'fractions of '2π.



Make the player's left the positive direction.



Displacement is zero when $\theta = 0$.

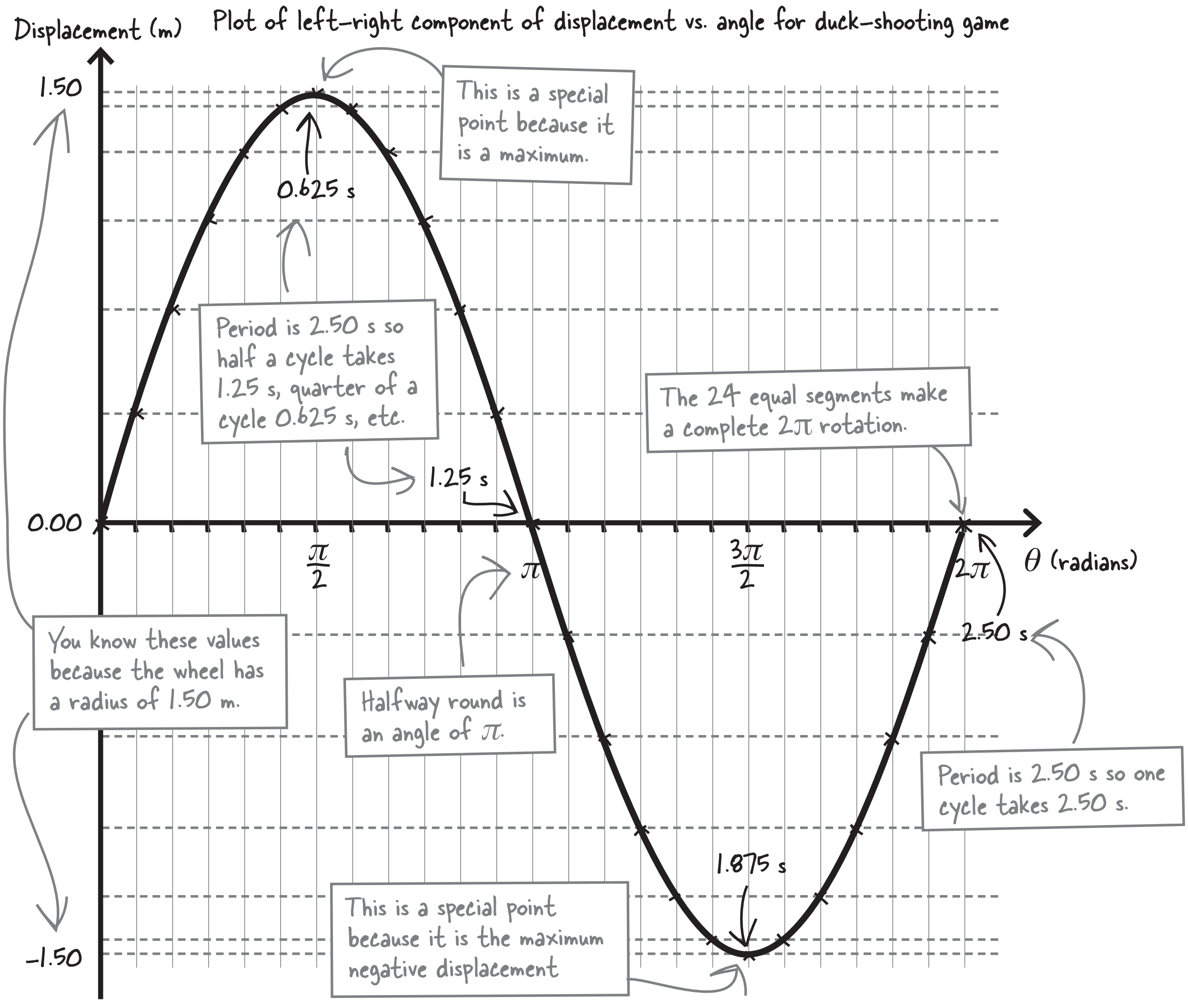
Just draw this the best you can. Use your ruler to mark off the horizontal-axis in 24 equal segments, like the circle.

Sharpen your pencil Solution

Your job is to work out what the duck's displacement vector does as time goes on, so you know where to plot the duck on the screen. The circle the duck's attached to has a diameter of 3.00 m and a period of 2.50 s.

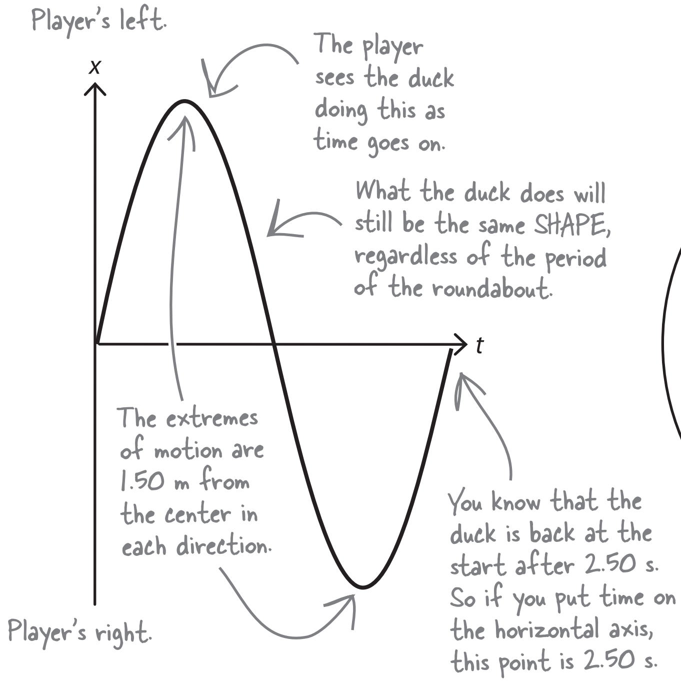
Use the pictures below, plus the fact that the angular velocity of the roundabout is constant, to **sketch** a graph of how the duck's left-right component (plotted on the vertical axis) varies with the angle θ (measured in radians and plotted on the horizontal axis) for one complete revolution (2π radians).

Mark the values at special points in the duck's motion on your sketch. Look for points where the duck is at an **extreme** in displacement - for instance, at zero, or at the maximum in either direction. Write the times that these points would occur at, too.



So we know what the duck does...

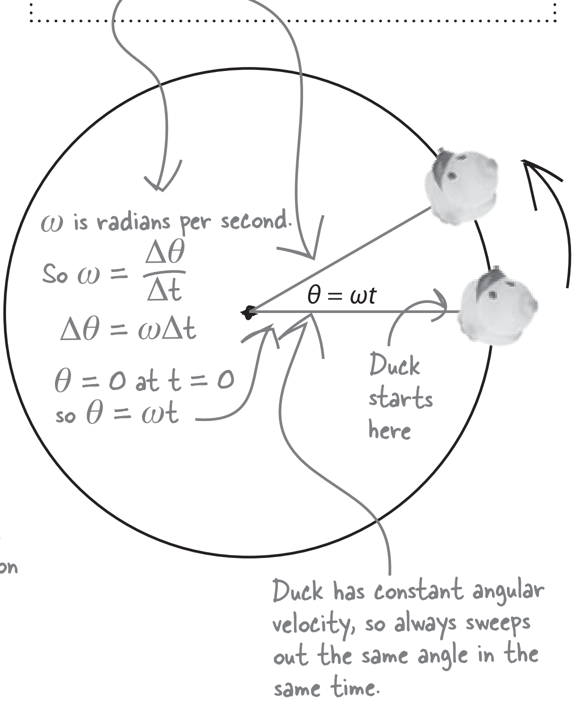
When you're playing the game, you observe only the duck going left and right, because you see only the left-right **component** of the duck's displacement.



θ is the angle, ω is the angular speed.

With linear quantities, you can write:
distance = speed × time

So with angular quantities, you can write an equivalent equation for the angle, θ:
$\theta = \omega t$



...but where exactly is the duck?

Although the **shape** of our graph is correct, the only **exact values** we know are at the **extremes**, when the duck is at its maximum left or right displacement, or when the duck has zero displacement.

Though as your graph is drawn fairly accurately, you can read off values from it. But it would be better to have an equation to give an exact value for the displacement at any time.

To get the screen working, we need to know exactly where the duck is at any given time... and that means we need an **equation** for the duck's **displacement**.

BRAIN POWER

We're about to use right-angled triangles to figure out the duck's displacement at any time. Can you think of how right-angled triangles might relate to circles?

Any time you're dealing with a component vector, try to spot a right-angled triangle

ω is radians per second.

So $\omega = \frac{\Delta\theta}{\Delta t}$

$\Delta\theta = \omega\Delta t$

$\theta = 0$ at $t = 0$

so $\theta = \omega t$

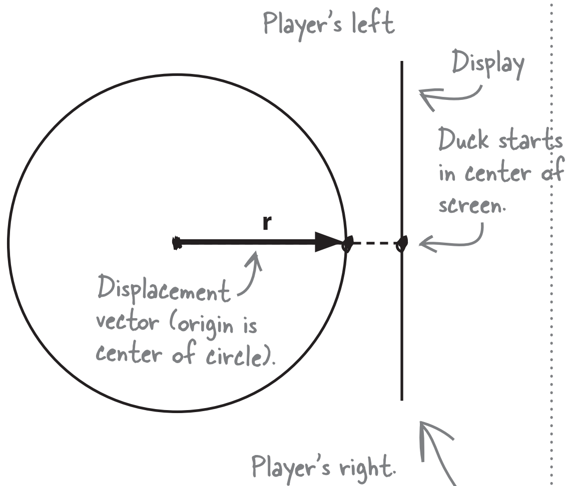
This is the "angular equivalent" of distance = speed x time.

When you play the game, you see only the **component** of the duck's **displacement** vector that's **parallel** to the screen. If you also draw in a **perpendicular** component, you can form a **right-angled triangle** with the parallel component, perpendicular component, and the radius.

1

The duck is always **distance** *r* away from the center of the circle. The duck rotates counter-clockwise with a constant angular velocity ω.

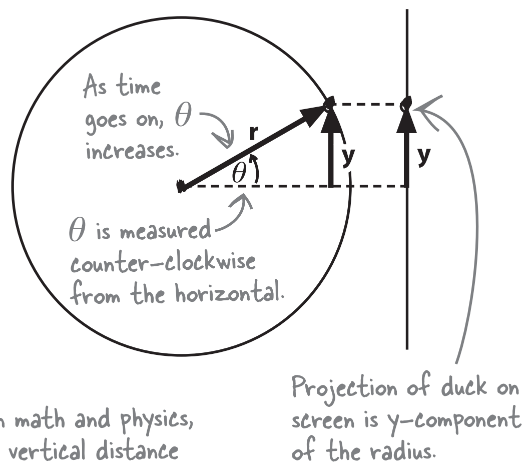
The duck's image on the screen is the projection of the **component** of its **displacement** parallel to the screen.



2

You can say what the angle θ is at any **time**, using the equation $\theta = \omega t$. But what you really want to know is the duck's displacement from the center at any time.

The **projection** of the duck's displacement from the center is always vertical (the way we've drawn it here).We'll call this displacement vector from the horizontal axis **y**, as shown below. Its projection on the screen is the **y-component** of the radius.



In math and physics, a vertical distance or displacement is sometimes called y, to distinguish it from x.

Here, we're looking DOWN on the duck from above. The player is over to the left a long way away, and we're projecting what they see onto this display.

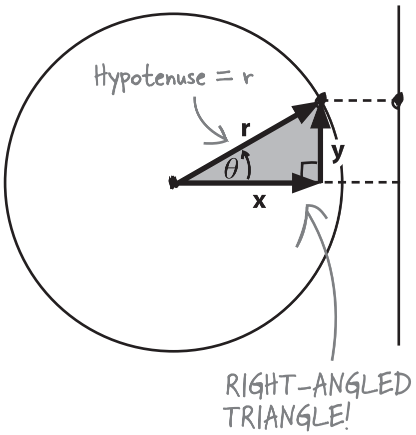
Because of the way this is drawn, the screen looks vertical. That's because the player's left is at the "top" and the player's right is at the "bottom" – just like the "sharpen your pencil" you did on page 771.

When you do a projection by drawing a right-angled triangle inside a circle like this, the triangle's HYPOTENUSE is always the RADIUS of the circle.

3

If we also draw in **x**, the **x**-component of the displacement, we can form a **right-angled triangle** using the **y**-component, the **x**-component and the radius.

When you draw a right-angled triangle like this, its hypotenuse is always the radius.

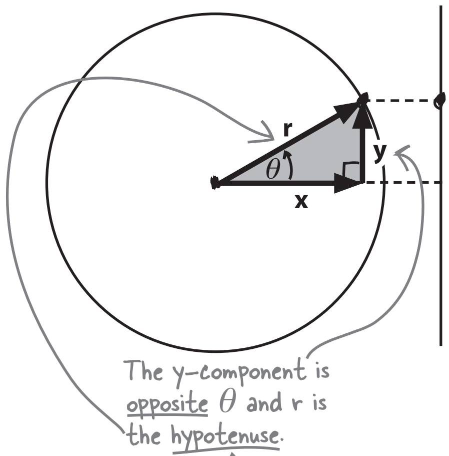


4

The sine function outputs a scalar, not a vector, so the equation gives you the length of y

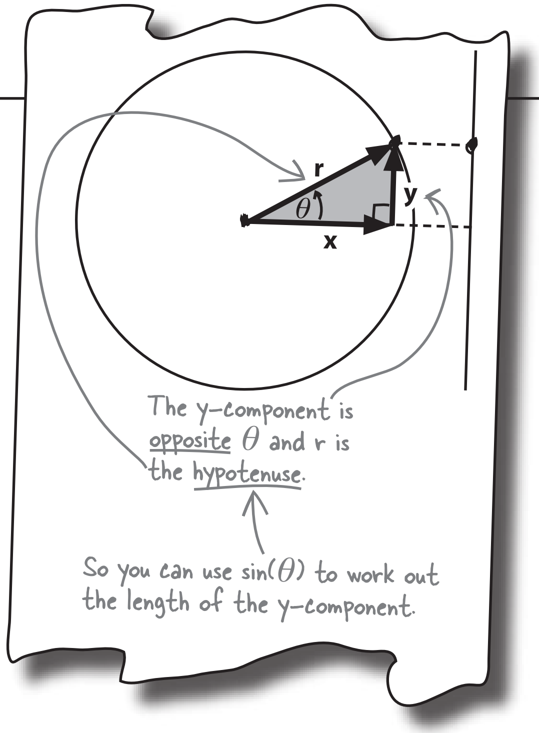
Now we can use trigonometry to **calculate** the length of y in terms of r and θ, because $\sin(\theta) = \frac{y}{r}$

And if you know what the length of y is in terms of θ, you can work out what y is at any time, by making the substitution $\theta = \omega t$. This tells you exactly where on the screen to draw the duck at any **time**.



So you can use sin(θ) to work out the length of the y-component.

Sharpen your pencil



Your job is to use the right-angled triangle you've spotted to calculate the size of the y-component of the duck's displacement at any time. This gives you the position of the duck on the screen at any time.

a. The roundabout's diameter is 3.00 m and the duck's velocity is 3.77 m/s.

What is the duck's angular velocity, ω?

b. Assume that at $t = 0$ s, $\theta = 0$. Write down an equation for θ in terms of ω and t (where θ is measured in radians). Use this to fill in the 't' column of the table on the opposite page.

c. Write down an equation for the length of the y-component of the duck's displacement with respect to the angle θ, where θ is measured in radians.

Use this equation and your answer to part b. to write down an equation for the length of y with respect to t.

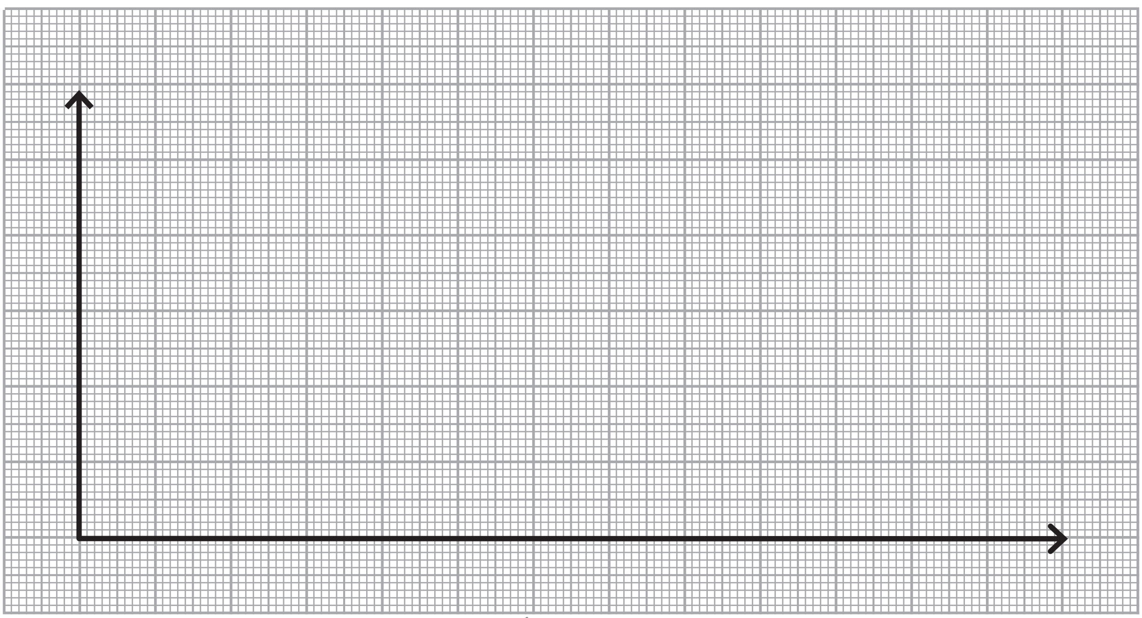
θ is in radians. This means your calculator should be in radians mode too.

d. Fill in the table with values for t and the y-component of the duck's displacement for the given angles, where θ is in radians.

Angle, θ (radians)	Time, t (s)	Length of duck's y-component (m)
0		
$\frac{\pi}{12}$		
$\frac{\pi}{8}$		
$\frac{\pi}{6}$		
$\frac{\pi}{4}$		
$\frac{\pi}{3}$		
$\frac{\pi}{2}$		

e. Use your table to draw a graph of the y-component of the duck's displacement vs. time.

You should give your graph a title and label its axes.



You should choose a suitable scale for each axis.

Sharpen your pencil Solution

Your job is to use the right-angled triangle you've spotted to calculate the y-component of the duck's displacement at any time. This gives you the position of the duck on the screen at any time.

a. The roundabout's diameter is 3.00 m and the duck's velocity is 3.77 m/s. What is the duck's angular velocity, ω?

b. Assume that at $t = 0$ s, $\theta = 0$. Write down an equation for θ in terms of ω and t (where θ is measured in radians). Use this to fill in the 't' column of the table.

c. Write down an equation for **y**, the y-component of the duck's displacement with respect to the angle θ, where θ is measured in radians. Use this equation and your answer to part b. to write down an equation for **y** with respect to t.

d. Fill in the table with values for t and the y-component of the duck's displacement for the given angles, where θ is in radians.

e. Use your table to draw a graph of the y-component of the duck's displacement vs. time.

Angle, θ (radians)	Time, t (s)	Length of duck's *y*-component (m)
0	0	0
$\frac{\pi}{12}$	0.104	0.388
$\frac{\pi}{8}$	0.156	0.574
$\frac{\pi}{6}$	0.208	0.750
$\frac{\pi}{4}$	0.313	1.06
$\frac{\pi}{3}$	0.417	1.30
$\frac{\pi}{2}$	0.625	1.50

a. $v = r\omega$

$\Rightarrow \omega = \frac{v}{r} = \frac{3.77}{1.50} = 2.51$ rad/s (3 sd)

b. ω is radians per second. So $\omega = \frac{\Delta\theta}{\Delta t}$

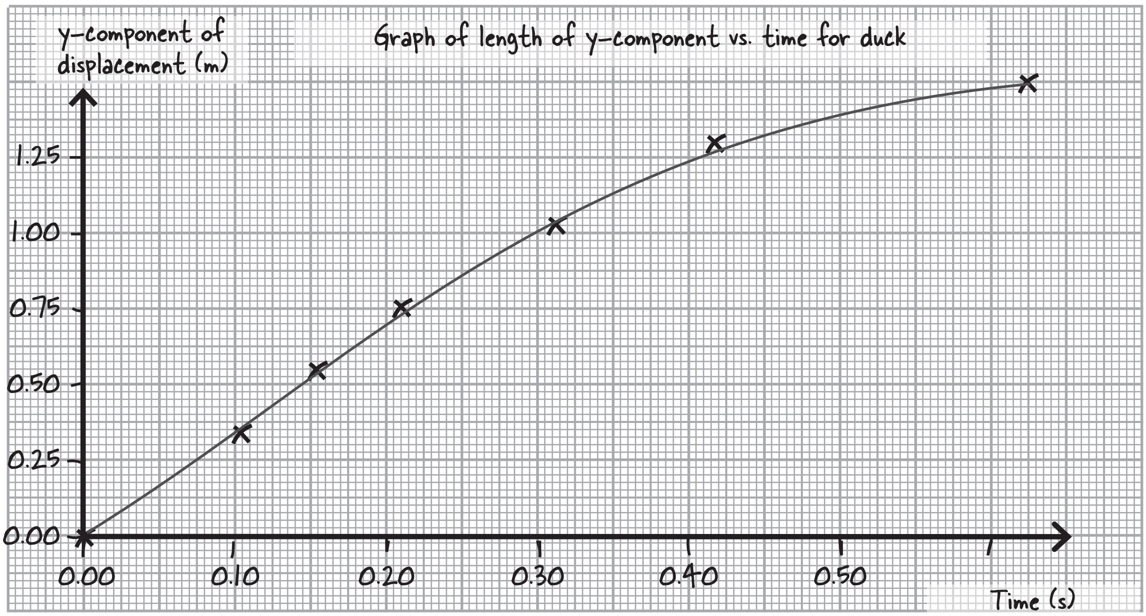
$\Rightarrow \Delta\theta = \omega\Delta t$

$\theta = 0$ at $t = 0$ so $\theta = \omega t$

c. $\sin(\theta) = \frac{o}{h} = \frac{y}{r}$

$\Rightarrow y = r\sin(\theta)$ and substitute in $\theta = \omega t$

$y = r\sin(\omega t)$



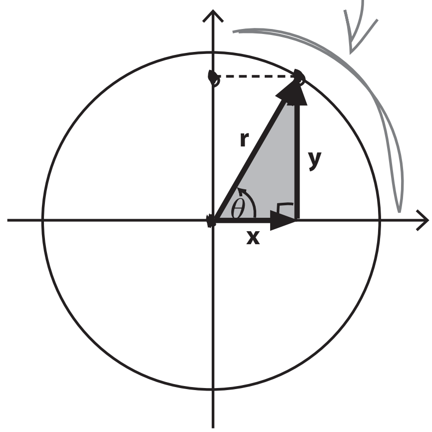
OK, so the duck's position is y = rsin(ωt). But we've only drawn quarter of the graph! What about bigger angles further round the circle? They can't be part of a right-angled triangle!

Because $\theta = \omega t$

We need wider definitions for sine and cosine.

Up until now, we've thought of sine and cosine applying only to **angles** that are part of a right-angled triangle. Now we need to extend their definitions to cover *all* the angles that can be swept out inside a **circle**.

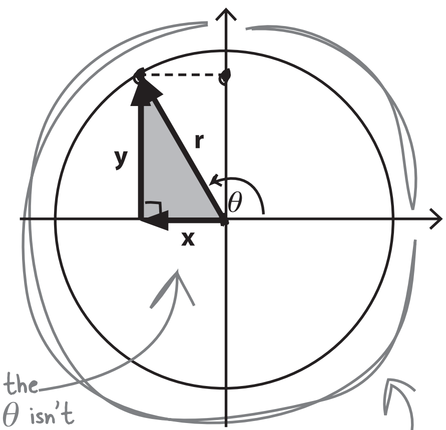
Our original definition of sin(θ) lets us deal with only this quarter of the duck's circle.



For angles smaller than a right-angle, we formed a right-angled triangle using **r**, the **radius** of the circle **y**, the y-component of the radius and **x**, the x-component of the radius.

Then we **projected** the radius vector onto a line parallel to the *y*-axis and used the equation $\sin(\theta) = \frac{y}{r}$ as usual to calculate *y*, the duck's distance from the center of the screen,

But now we need to deal with the other angles further round the circle, which are larger than $\frac{\pi}{2}$ radians (90°). If we can't deal with these angles, we don't know where to draw the duck on the screen.



The **definition of sine** still comes from the ratio of the *y*-component to the length of the radius, and is the same equation as before:

$$\sin(\theta) = \frac{y}{r}$$

New definition:

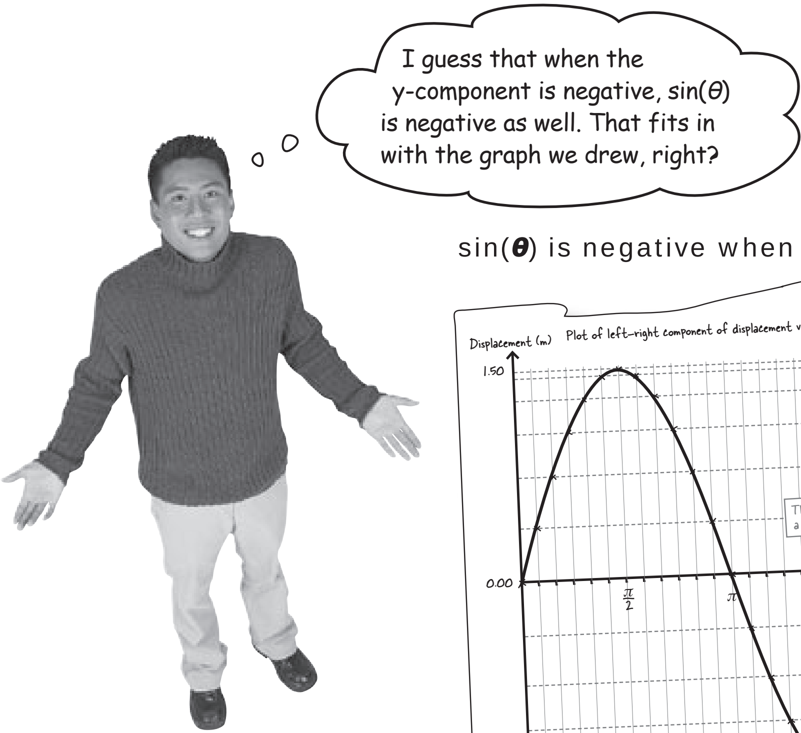
$$\sin(\theta) = \frac{\mathbf{y}}{\mathbf{r}}$$

The definition of sin(θ) is the RATIO of y and r, even if θ isn't part of a right-angled triangle.

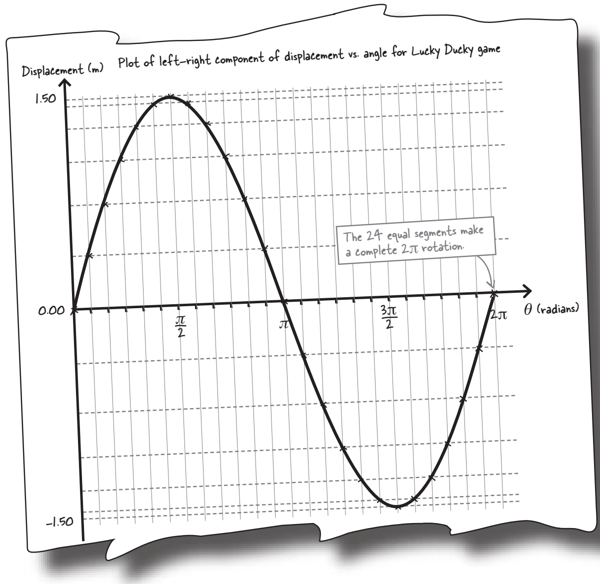
The new definition lets us handle the rest of the circle as well.

This works for ANY angle in a circle, not just an angle that's part of a right-angled triangle.

The method for calculating the length of *y* hasn't changed. You still form a right-angled triangle using the *y*-component, *x*-component and length of the radius. Then you can calculate the length of *y* using the new definition of sin(θ) - which tells you the duck's position on the screen for any angle.



sin(θ) is negative when the y-component is negative.



This is our graph from page 768.

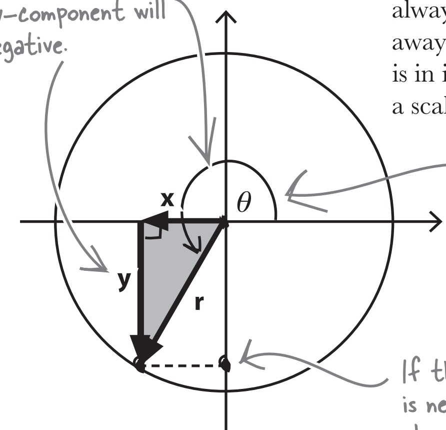
These values of the y-component are all negative. That's because we defined the player's left as the positive direction, and their right as negative.

For angles larger than π (larger than 180°), the y-component of the displacement points in the other direction, so has a negative sign. This means that sin(θ) is also negative.

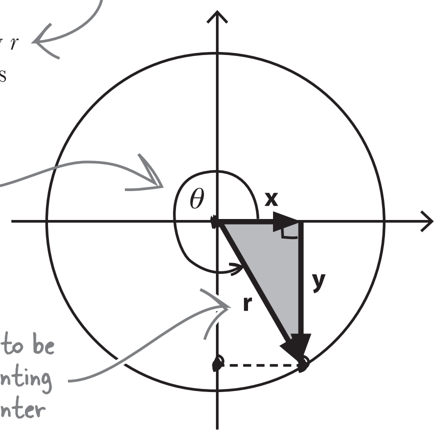
To make sure the signs work out OK, *r* is always treated as positive because it's pointing away from the center of the circle. This is why *r* is in italics - in the context of this definition it's a scalar with a (positive) size and no direction.

r is still a vector that you can draw on your circle diagram – but only its size is used in the definition of sine.

If θ is more than halfway around, then the y-component will be negative.



The sines of these two angles have the same value, as the y-components have the same size and sign.



If the y-component is negative, sin(θ) is also negative.

r is always taken to be positive as it's pointing away from the center of the circle.

there are no Dumb Questions

Q: So an angle doesn't need to be in a right-angled triangle for it to have a sine?

A: That's right. Our new extended definition of sine says that if you have a vector that starts at the origin, and you measure the angle θ that it makes with the *x*-axis, $\sin(\theta)$ is the *y*-component of the vector divided by the radius.

Q: But why is it useful to be able to calculate the sine of any angle? Isn't the sine only interesting when there's a right-angled triangle involved?

A: The sine of larger angles is crucial when you have circular motion, like in the duck-shooting game. In the game, we need to know the *y*-component of the duck's displacement vector. So we need to calculate the sine of the angle that the duck makes with the x-axis.

Q: So how did you decide which way around to put the axes?

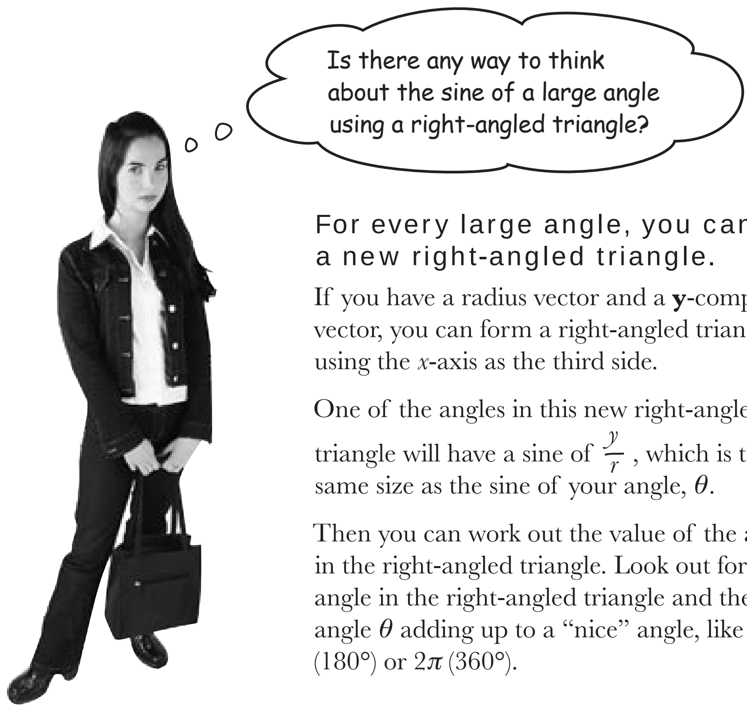
A: It was just the way it came out when we drew the aerial view of the player and the duck on the circle. We made the player's left the positive direction, and their right the negative direction. You could have done it the other way, and the effect would be the same. You'd just have positive values for the player's right, instead of their left.

Q: I've heard people talking about a "sine wave" before, but never knew what they meant. Did I just draw a sine wave?

A: Yes, you did! And it's called a wave because the pattern repeats itself again and again if you keep on going around and around for more than one revolution.

Q: In the "Sharpen your pencil" on page 771, I didn't have to calculate any sines for angles larger than a right-angle, because I just projected across from the points on the circle. How would I actually calculate values for the *y*-components?

A: Your calculator is able to work out the sine of any angle you give it. Just make sure the calculator's measuring the angle the same way as you are (in degrees or radians)

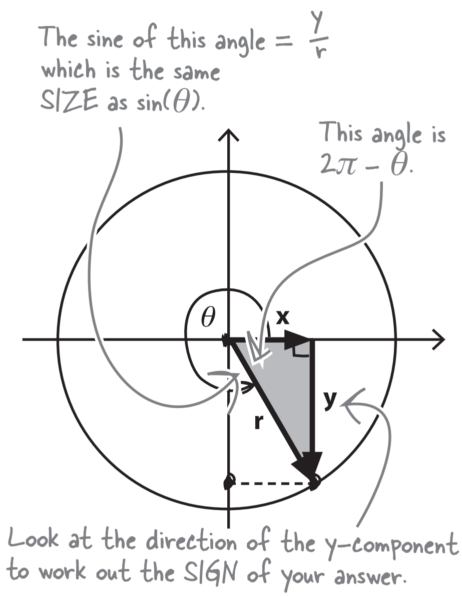


For every large angle, you can find a new right-angled triangle.

If you have a radius vector and a **y**-component vector, you can form a right-angled triangle using the *x*-axis as the third side.

One of the angles in this new right-angled triangle will have a sine of $\frac{y}{r}$, which is the same size as the sine of your angle, θ.

Then you can work out the value of the angle in the right-angled triangle. Look out for the angle in the right-angled triangle and the large angle θ adding up to a "nice" angle, like π (180°) or 2π (360°).

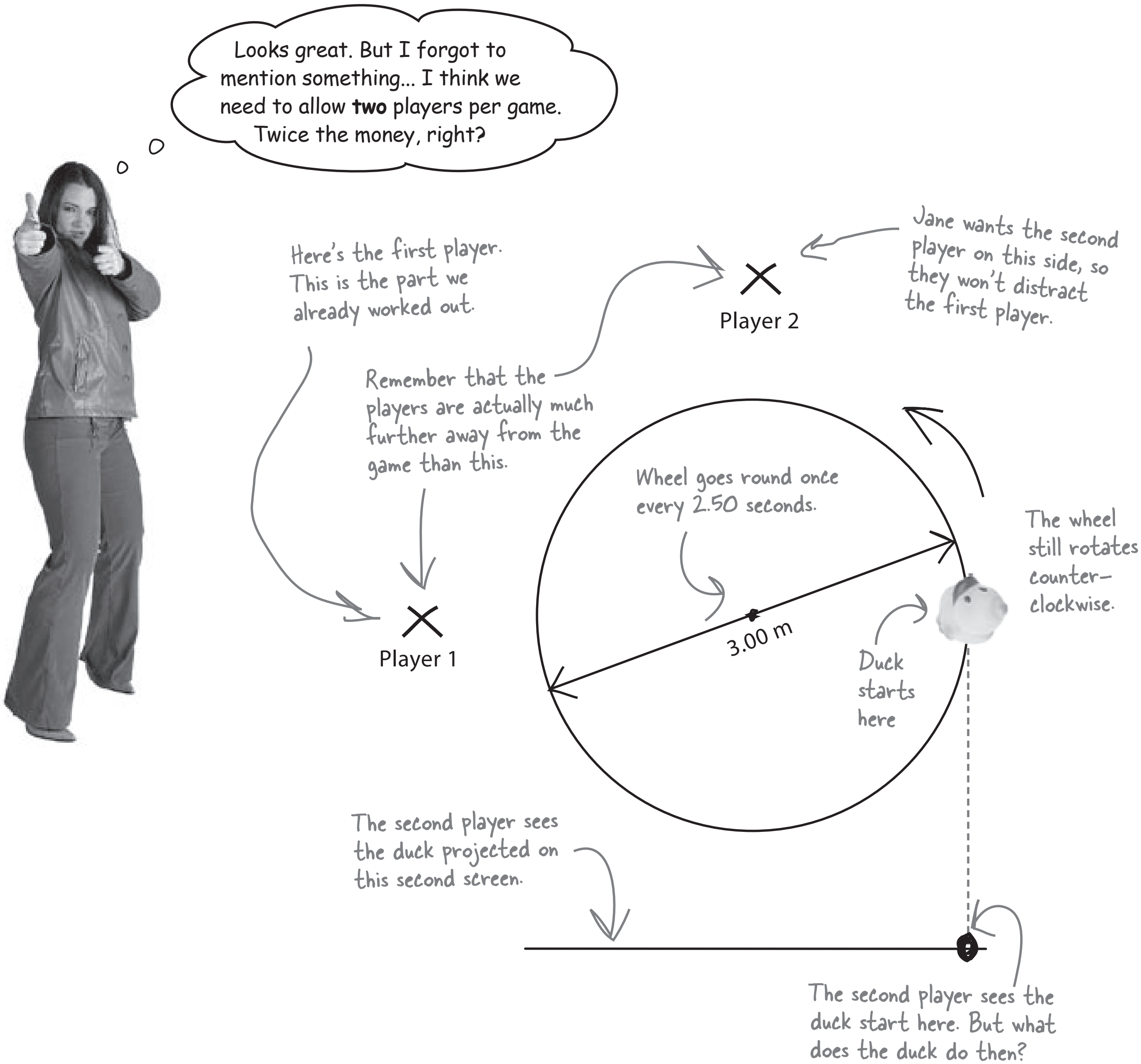


Let's show Jane the display

We're all set to give Jane our equation, $y = r\sin(\omega t)$ and help her get her screen working. Time to start raking in the profits...

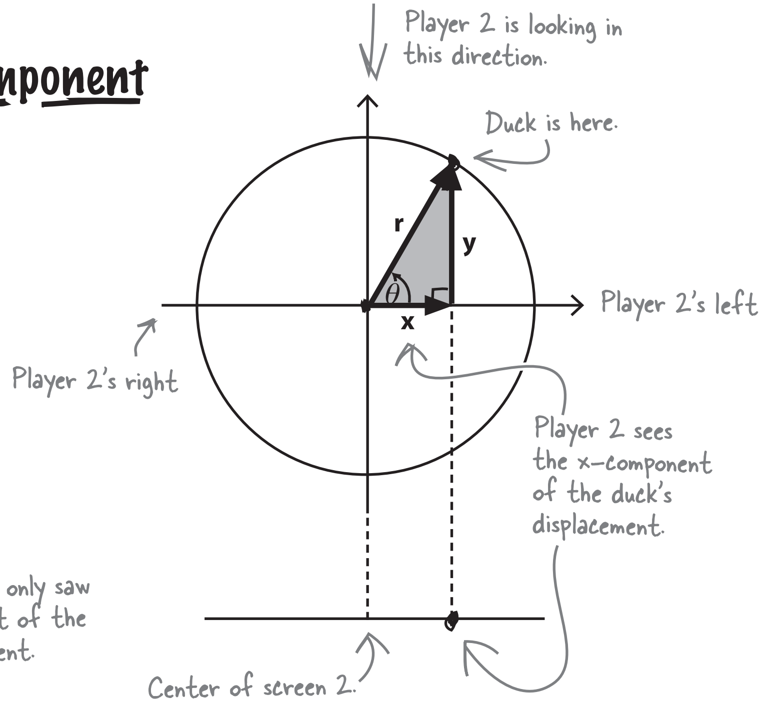
...but Jane's got a new idea, and it's going to take some more work.

$$y = r\sin(\boldsymbol{\omega} t)$$



The second player sees the x-component of the duck's displacement

The second player is facing in a different direction. They're standing $\frac{\pi}{2}$ radians (90°) further around the circle, clockwise, than the first player.



This means that the second player doesn't see the **y**-component of the duck's displacement. Instead, the second player only sees the **x-component** of the duck's displacement.

The first player only saw the y-component of the duck's displacement.

Sharpen your pencil

Make Player 2's left the positive direction, like we did for Player 1.

a. This time, you're interested in x, the length of the **x**-component of the duck's displacement. Write down an equation for x in terms of r and θ. (The duck starts at $\theta = 0$).

You may find it helpful to draw a sketch.

b. Sketch a graph of how x varies with θ for one complete revolution of the circle, radius r. Be sure to mark any special points.

This is the projection that player 1 sees.

c. Sketch a graph that shows how y, the length of the **y**-component of the duck's displacement varies with θ. Compare and contrast this graph with the one you drew in part b, and note any similarities and differences.

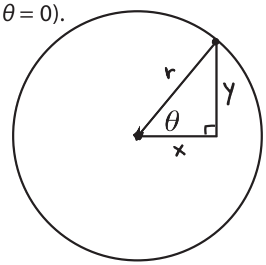
Sketch these two graphs one above the other with the same scale.

Use this space to explain the similarities and differences.

Sharpen your pencil Solution

Make Player 2's left the positive direction, like we did for Player 1.

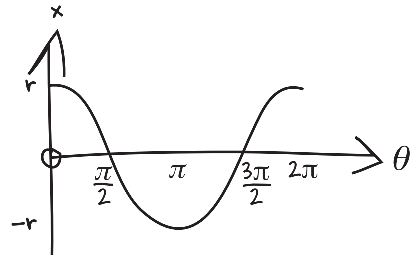
a. This time, you're interested in *x*, the length of the **x**-component of the duck's displacement. Write down an equation for *x* in terms of *r* and *θ*. (The duck starts at *θ* = 0).



$\cos(\theta) = \frac{a}{h} = \frac{x}{r}$

$\Rightarrow x = r\cos(\theta)$

b. Sketch a graph of how *x* varies with *θ* for one complete revolution of the circle, radius *r*. Be sure to mark any special points.

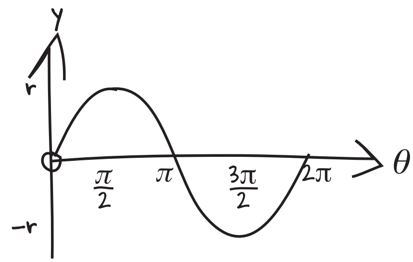


c. Sketch a graph that shows how *y*, the length of the **y**-component of the duck's displacement varies with *θ*. Compare and contrast this graph with the one you drew in part b, and note any similarities and differences.

The graphs are similar, except that player 2's is shifted along by $\frac{\pi}{2}$ because they are further round the circle by an angle of $\frac{\pi}{2}$.

The duck starts in the center for the first player, but at a maximum for the second player.

Sine and cosine are related because they both involve one component and the hypotenuse.

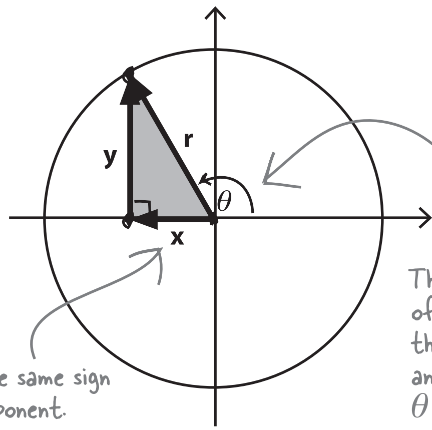


Sketch these two graphs one above the other with the same scale.

We need a wider definition of cosine, too

Just like there's a wider definition of sine, there's also a wider definition of cosine, which you've just worked out. It's very similar to the wider definition of sine, except that it involves the x-component: $\cos(\theta) = \frac{x}{r}$

The sign of cos(*θ*) is the same as the sign of the x-component.



cos(θ) has the same sign as the x-component.

The definition of cos(θ) is the ratio of x and r, even if θ isn't part of a right-angled triangle.

New definition:

$$\cos(\theta) = \frac{x}{r}$$

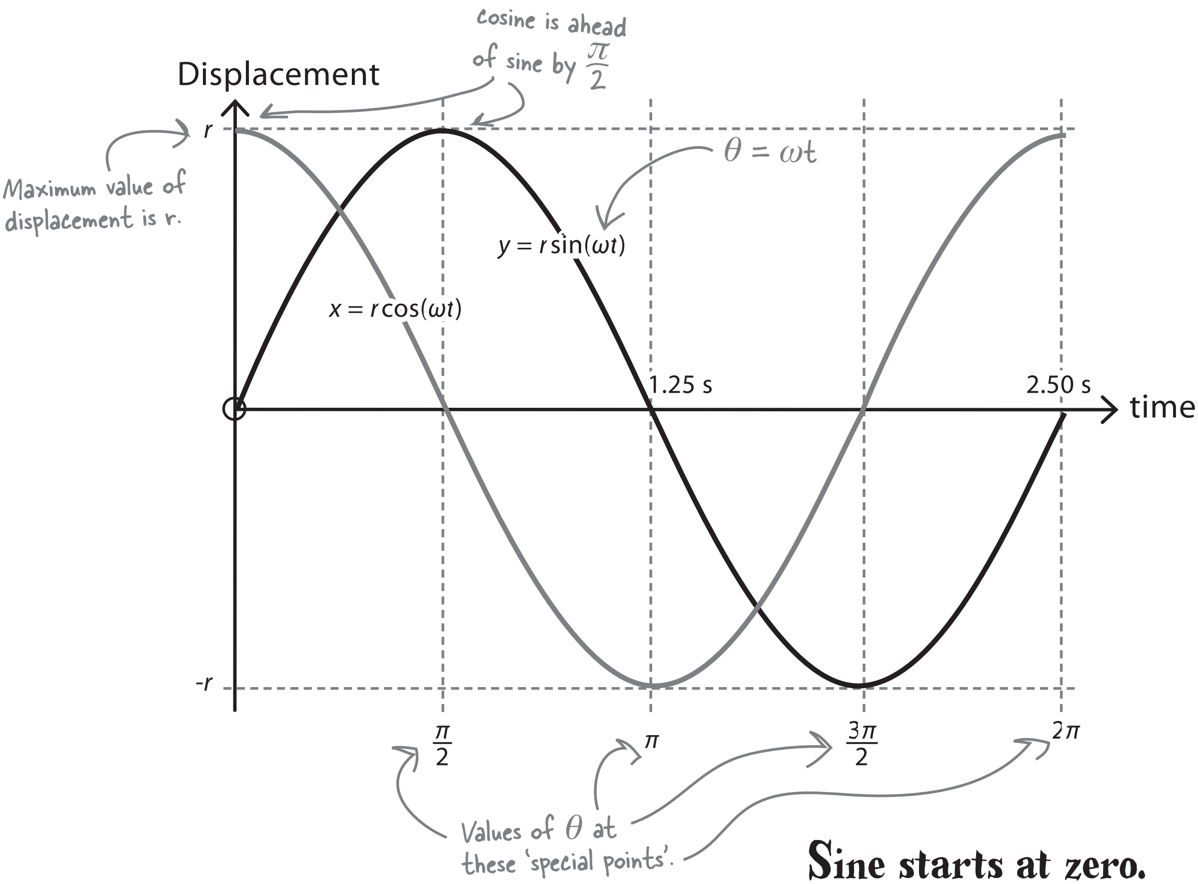
sine and cosine are related to each other

The second player sees the *x*-component of the duck's displacement, $x = r\cos(\omega t)$, as the duck moves round the circle, while the first player sees the *y*-component: $y = r\sin(\omega t)$.

The graphs of sine and cosine are closely related. They're exactly the same shape, in fact, except that cosine is "ahead" of sine by $\frac{\pi}{2}$. In our game, that's because the second player's vantage point is $\frac{\pi}{2}$ ahead of the first player's.

A cosine graph starts with the maximum value of the variable at the origin, and a sine graph has a value of 0 at the origin.

Sine and cosine have the same wave shape, but start in different places.



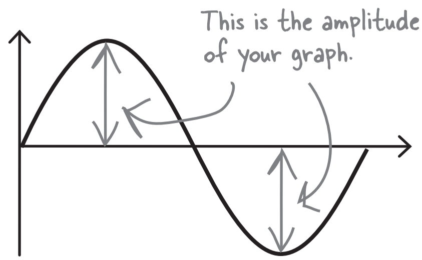
Sine starts at zero.

Cosine starts at its maximum value.

Looks like the sine and cosine graphs are both the same height when you draw them next to each other.

The maximum value of your graph is called the amplitude.

The maximum deviation from the center that your variable can have is called the **amplitude** of the graph or equation.



The AMPLITUDE is the maximum deviation from the center of your sine or cosine graph.

Here, you know that the amplitude of the displacement-time **graph** is r, the radius of the circle. So you know that the extremes of the graph have to be $+r$ and $-r$.

You can also use the amplitude to work out an **equation** for your graph. The maximum value that sine or cosine can have is 1. So to have a graph where the maximum value is r, you have to multiply the sine or cosine by r.

That's why your equations have the form $\mathbf{y} = r\sin(\omega t)$ and $\mathbf{x} = r\cos(\omega t)$.

Maximum value of r.

Maximum value of 1.

This also means that your two graphs both have the same height - because they have the same amplitude.

Sine Exposed

This week's interview:
Sine revisits our studio.

Head First: So, sine, it's been a while since we first met back in chapter 9, but it's good to have you back today to discuss these latest revelations. Or should that be revolutions?!

sine: Ha, that's right. I do cover far more angles than you thought I could... but I don't see why that's such a big deal.

Head First: It's not a big deal, not really, just a bit... unexpected! I'd never have thought that an angle larger than a right angle could even have a sine.

sine: Yeah, I kinda understand how you feel. It's like finding out that an old friend has a whole other secret life. Though my extended definition isn't really all that different from the one you had before.

Head First: Hmm... before, I thought of you as having to do with the ratio of particular sides in a right-angled triangle. But now I guess we know that angles that can't possibly be in a right-angled triangle can still have a sine.

sine: Well, I'm still the **ratio** of two lengths. That part of your definition hasn't changed.

Head First: But those lengths aren't part of a right-angled triangle, anymore.

sine: But they are very well-defined lengths, and you can use them to form a right-angled triangle! If you're dealing with an angle in a circle, **t**: a vector going from the center to the edge of the circle. Then the *x*-component is one side, and the *y*-component is the other side.

Head First: That's all fine, I suppose... but how do I work out which side is the opposite and which is the adjacent?

sine: Well, angles in physics are always measured **counter-clockwise from the horizontal**. If your angle's smaller than a right angle, sine is the ratio of the *y*-component and the radius. And my friend cosine is the ratio of the *x*-component and the radius.

Head First: Well, sure, that's what we already knew. But when you start having the sine of angles larger than a right angle... that's when it gets a bit hairy.

sine: But I'm still the same! I'm still the ratio of the *y*-component and the radius.

Head First: I guess you are... though that means you're not as exclusive as you once were, doesn't it?

sine: Yeah, sadly, there will always be two different angles that have the same value of sine.

Head First: And that can be a downer, can't it? I mean, we've seen you looking negative recently.

sine: That's true. I'll be negative for any angle larger than halfway around, as the *y*-component of the radius vector will point down, not up.

Head First: And then there's your friend cosine. He's negative at different times from you, isn't he? Doesn't that make it difficult for you to work together sometimes?

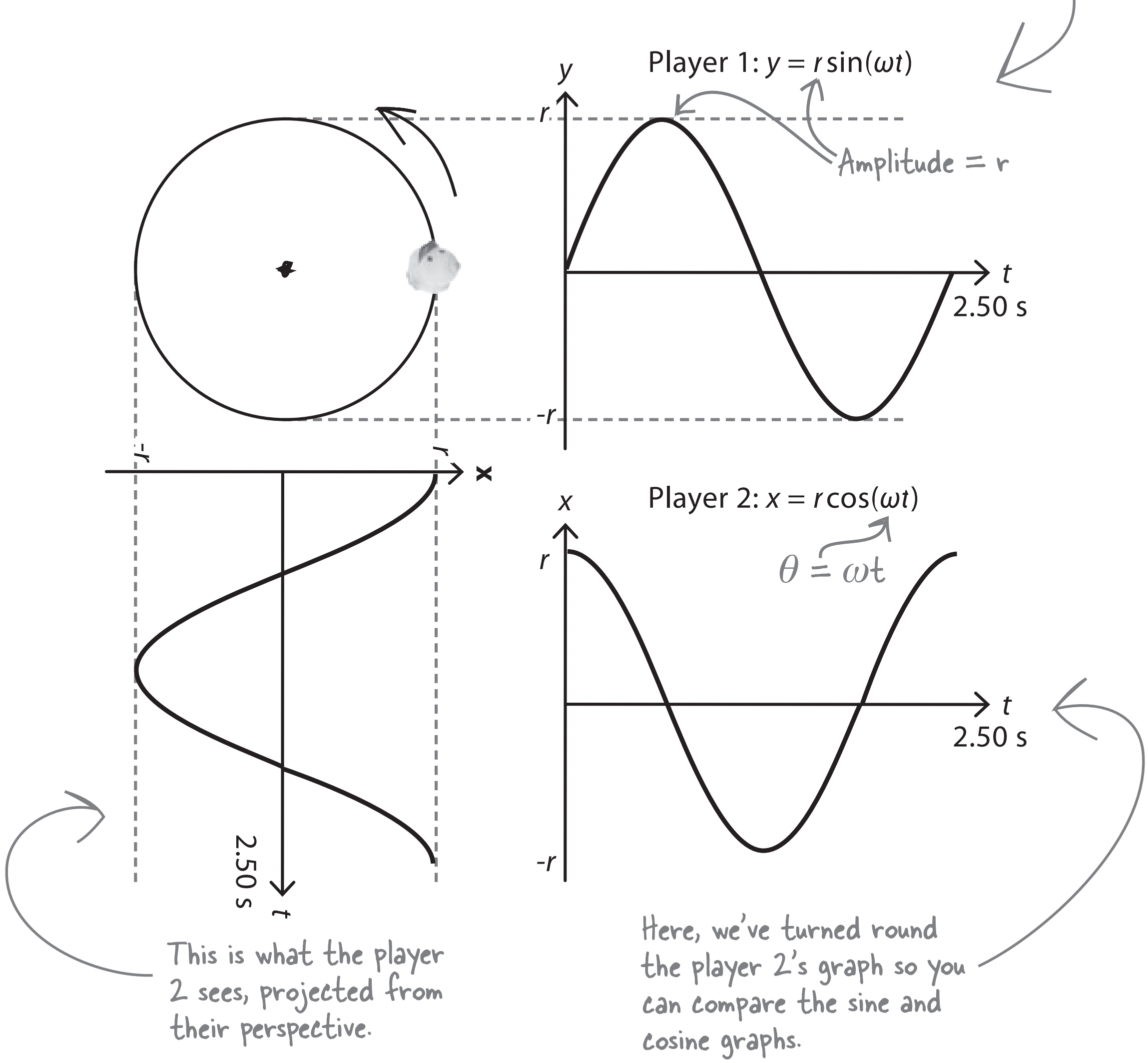
sine: Not really. We're both aware of when the other is going to be positive or negative. Just remember that in the top-right quadrant (where the angle is less than a right angle) we're both positive. Then you can work out which direction is positive for each of us and take it from there.

Head First: Thank you, sine. I think we already knew more about you than we thought we did.

Let the games begin!

The **equations** $y = r\sin(\omega t)$ and $x = r\cos(\omega t)$ work perfectly. Jane plugs them into her screens, and the customers are already lining up. Who doesn't love rotating ducks and light guns?

Even better, you've come up with **general equations**. If Jane ever wants to change the game, maybe by using a different size of wheel that spins at a different rate, the equations will still work. She can just change the values for r and ω, and she's good to go.



Hey, I wanna be able to track the duck better. Can we have its current **velocity** from each player's point of view displayed on their screens?

Jane's got another request: What's the duck's velocity from each player's point of view?

Just when you thought the duck-shooting odyssey was over, Jane's come up with another feature request. Now she wants to show the value of the duck's velocity from each player's point of view on the display.

Looks like it's time to get out our calculators and pencils again.

Before, you calculated that the duck's linear velocity as it travels round the circle is 3.77 m/s. So that's got to come into it somewhere.

BRAIN POWER

How would you sketch a velocity-time graph and calculate the value of the duck's velocity from each player's point of view?

Get the shape of the velocity-time graph from the slope of the displacement-time graph

The easiest way to draw a sketch of the velocity-time graph is to use the slope of the displacement-time graph.

This works because velocity is rate of change of displacement, $\mathbf{v} = \frac{d\mathbf{x}}{dt}$. ← We talked about this at the end of chapter 6.

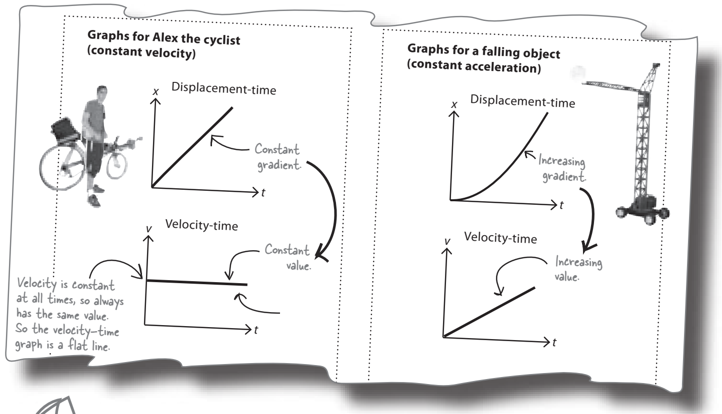
Therefore:

A large positive slope on the displacement-time graph corresponds to a large positive velocity.

Zero slope on the displacement-time graph corresponds to zero velocity.

A small negative slope on the displacement-time graph corresponds to a small negative velocity.

And so on.



You already moved between the displacement and velocity in chapter 6, for graphs of an object with a constant velocity, and an object with constant acceleration.

You can get an **equation** for the velocity by thinking about component vectors - right?

Component vectors are fine here...

You could work out an equation for the duck's velocity using component vectors. Each player sees only one component of the duck's velocity. If the duck is moving directly towards you or away from you, you don't notice its velocity at all. And if the duck is in the center, you see it moving with its linear velocity, v.

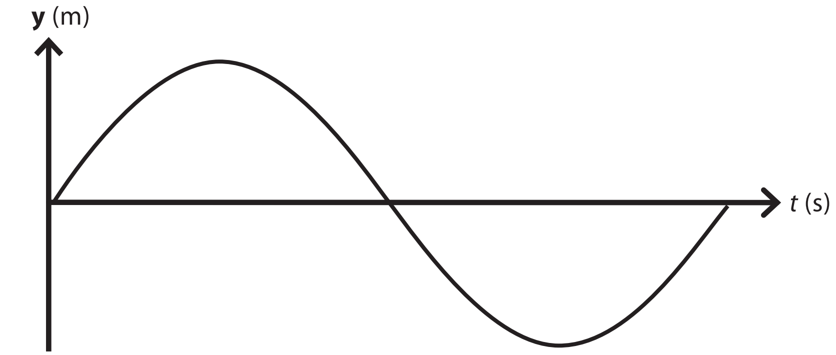
...but it's quicker to get an equation for the velocity directly from your graph!

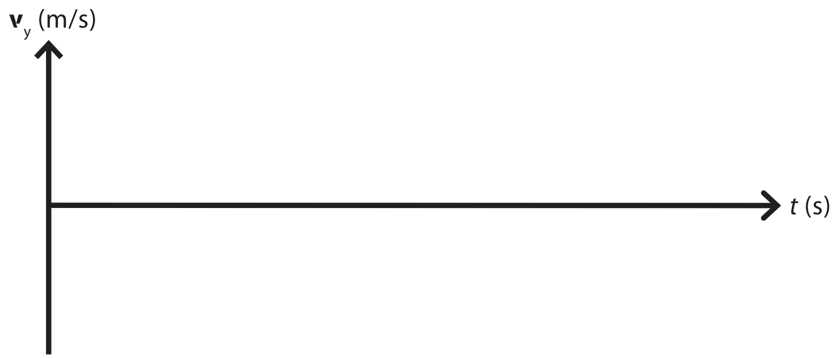
If your velocity-time graph has a "standard shape" and you know its period, you'll be able to work out an equation for the velocity using the **amplitude** of the graph...

Sharpen your pencil

a. The displacement-time graph for player 1 is shown on the top graph. Use the slope of this displacement-time graph to sketch the duck's velocity-time graph underneath it. Add extreme values to both graphs where appropriate (the circle's diameter is 3.00 m, its period is 2.50 s, and the duck's linear velocity is 3.77 m/s).

b. Annotate any special points on your graphs to explain why these points occur where they do.





c. What kind of graph does your velocity-time graph resemble in terms of its shape? Use this to write down an equation for v_y, the *y*-component of the velocity in terms of *v* (the duck's linear velocity), *ω*, and *t*.

Look back at how we worked out the equation for r earlier in this chapter if you get stuck.

You'll need to express θ in terms of ω and t.

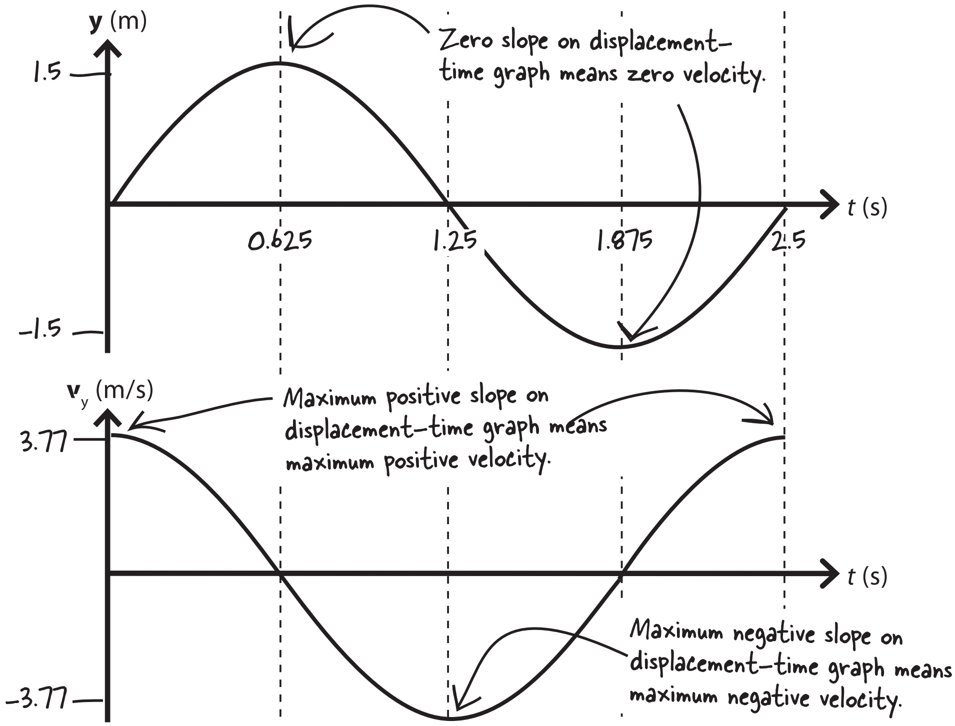
d. What is the value of the duck's velocity from player 1's point of view at *t* = 0.90 s?

You'll need to think about the amplitude of your graph (see page 786).

Sharpen your pencil Solution

a. The displacement-time graph for player 1 is shown on the top graph. Use the slope of this displacement-time graph to sketch the duck's velocity-time graph underneath it. Add extreme values to both graphs where appropriate (the circle's diameter is 3.00 m, its period is 2.50 s, and the duck's linear velocity is 3.77 m/s).

b. Annotate any special points on your graphs to explain why these points occur where they do.



c. What kind of graph does your velocity-time graph resemble in terms of its shape? Use this to write down an equation for v_y, the y-component of the velocity in terms of v (the duck's linear velocity), ω, and t.

The shape looks like a graph of $\cos(\theta)$, because the graph starts at the maximum value.

The maximum value of cosine is 1, but the maximum value of my graph is $v = 3.77$ m/s.
So my graph has an amplitude of v.

$v_y = v\cos(\theta)$ And also, $\theta = \omega t$, which I can substitute into my equation:

$v_y = v\cos(\omega t)$

This equation only works when θ is in radians, so make sure your calculator is in the correct mode.

d. What is the value of the duck's velocity from player 1's point of view at $t = 0.90$ s?

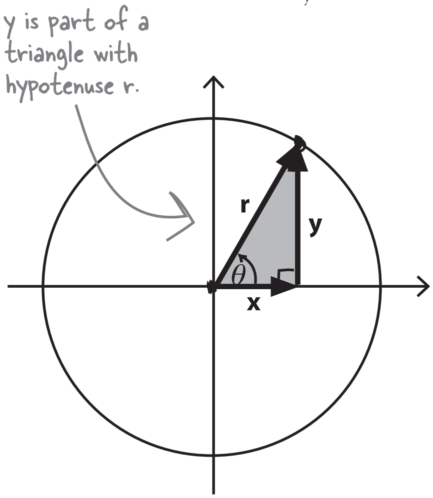
$v = r\omega \quad \Rightarrow \omega = \frac{v}{r} = \frac{3.77}{1.50} = 2.51$ rad/s (3 sd)

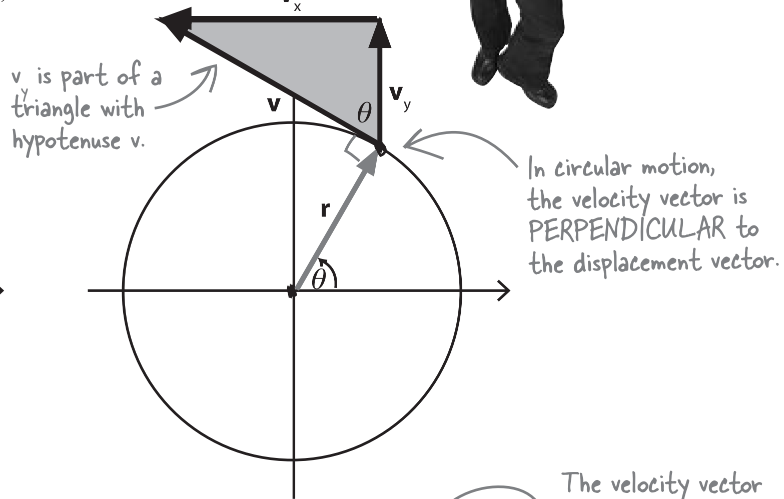
$v_y = v\cos(\omega t) = 3.77 \times \cos(2.51 \times 0.90) = -2.39$ m/s (3 sd)

But didn't we say before that the definition of **sine** involved the y-component? How come we've ended up with a **cosine** graph from the y-component of the velocity?

The velocity vector is part of a different triangle.

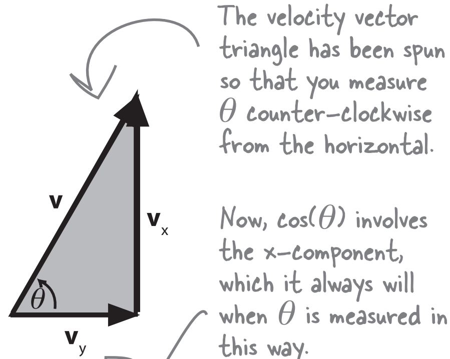
If you work out the value of v_y, the y-component of the velocity vector, using a right-angled triangle, you get the answer $v_y = v\cos(\theta)$. That works out the same as $v_y = v\cos(\omega t)$, as $\theta = \omega t$.





The wider definitions of sine and cosine only hold true if the angle θ is measured counter-clockwise from the horizontal. So to check the velocity component vector triangle, we have to rotate it so that θ is measured counter-clockwise from the horizontal.

This makes $\mathbf{v}_y$ lie along the x-axis as the side adjacent to θ. As cosine involves the ratio of the side that lies along the x-axis and the hypotenuse, the equation $v_y = v\cos(\omega t)$ makes sense.

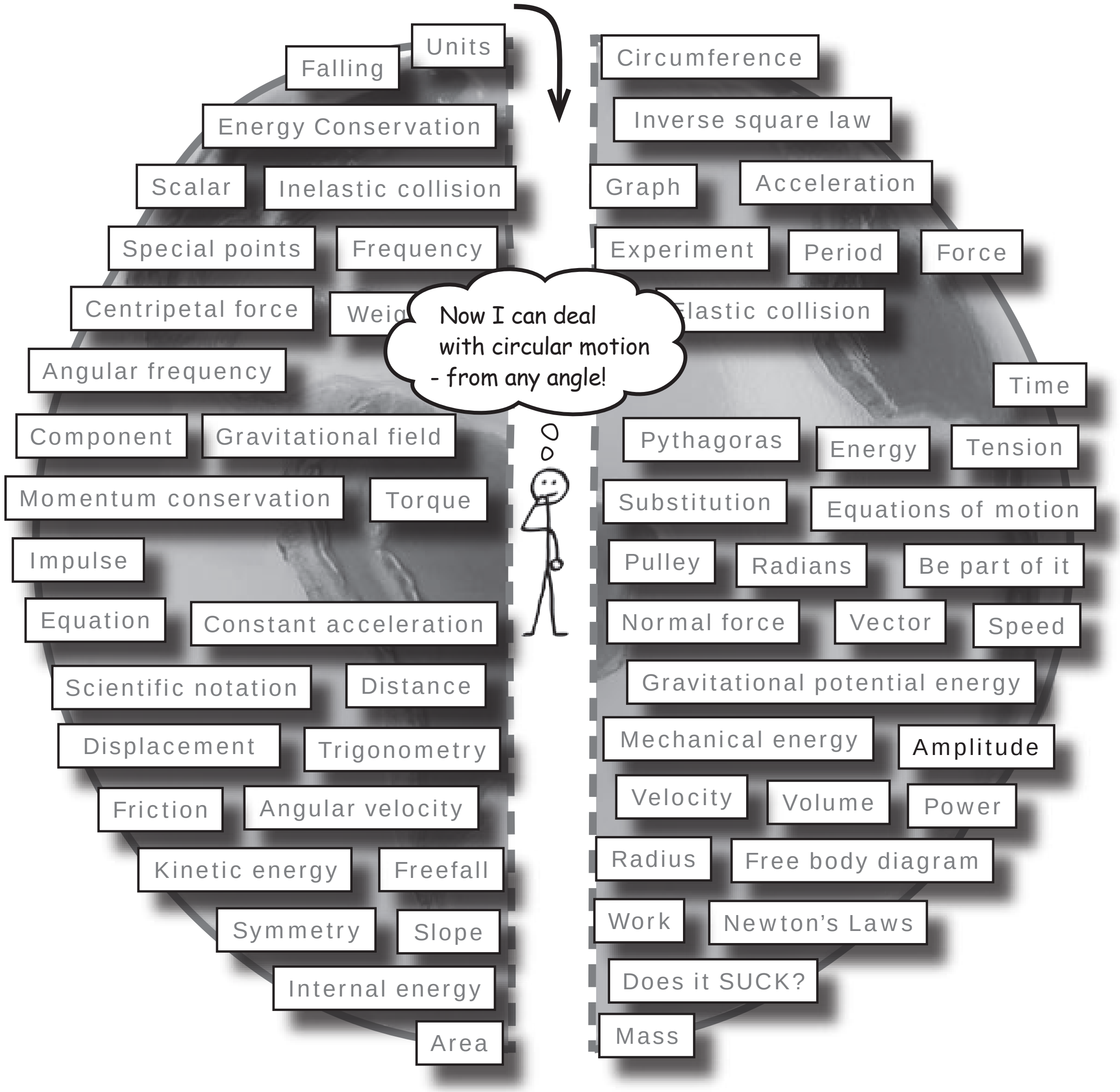


$$\sin(\theta) = \frac{y}{r}$$
$$\cos(\theta) = \frac{x}{r}$$

only apply when θ is measured counter-clockwise from the horizontal.

The game is complete!

This time, the duck-shooting odyssey really is over. The game is an overnight fairground success, and Jane's even starting to opening franchises around the country, with thousands and thousands of people playing the game every day!



☑ Amplitude — The maximum deviation from the center that your variable can have is called the amplitude of your graph or equation.

Your Physics Toolbox

You've got Chapter 19 under your belt and added some problem-solving concepts to your toolbox.

New definitions for sine and cosine

If you measure θ counter-clockwise from the horizontal:

$$\sin(\theta) = \frac{y}{r}$$

$$\cos(\theta) = \frac{x}{r}$$

Equation of a sine or cosine graph

General form is:

$$x = A\sin(\theta)$$

$$x = A\cos(\theta)$$

where A is the amplitude (the maximum value that x can have).

Working out an equation or graph

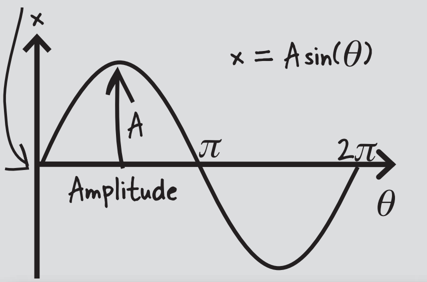
The amplitude is the maximum value that the variable can have.

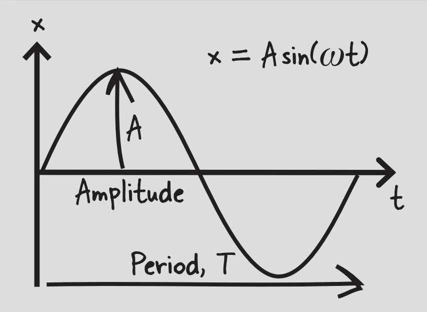
Use the substitution $\theta = \omega t$.

The thing returns to the start after 2π radians, or 1 period.

Sine graph

Value is 0 at $\theta = 0$.

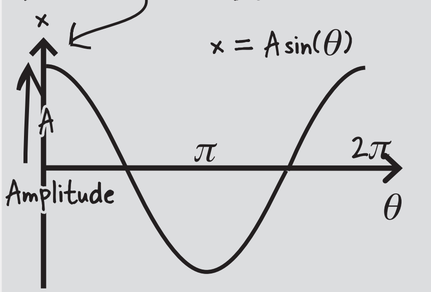


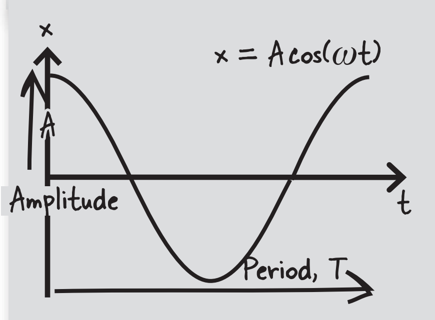


You can plot sine and cosine graphs againt θ or against t.

Cosine graph

Value is maximum at $\theta = 0$.





20 Oscillations (part 2)

Springs 'n' swings

What do you do when something just happens over and over? This chapter is about dealing with **oscillations**, and helps you see the big picture. You'll put together what you know about graphs, equations, forces, energy conservation and periodic motion as you tackle springs and pendulums that move with **simple harmonic motion** to get the ultimate "I rule" experience ... without having to repeat yourself too much.

Get rocking, not talking

You've heard of talking to your plants, but you ain't seen nothing yet! Anne's been in touch to tell you about the latest sensation that's rocking the gardening world - her newly-patented Plant Rocker.

Anne has only patented the idea - the design is up to you!

Here's the patent - d'ya think you can help?

Patent number 910 - Plant Rocker

A spring-operated horticultural device.

Removes the need to talk to plants.

Rocks the plant gently with a frequency of 0.750 Hz.

Direction of rocking motion doesn't matter.

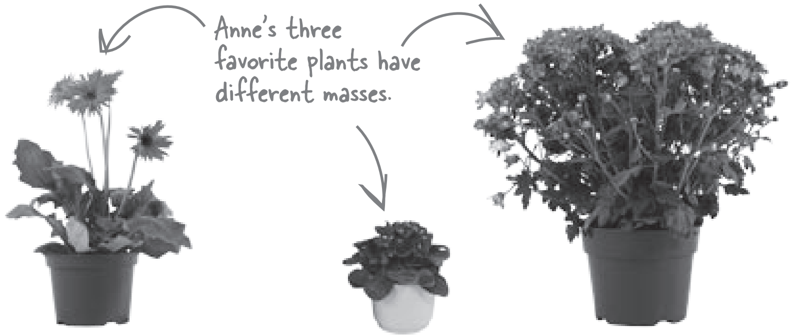
Amplitude / size of rocking motion doesn't matter.

The plant rocker needs to work for three different masses of plant

Anne has three favorite plants that she wants to rock, but they're all different sizes (and different masses).

Your design will need to work for all three plants - and Anne insists that each of the plants needs its very own rocker.

The plants must be rocked individually in three separate plant rockers.

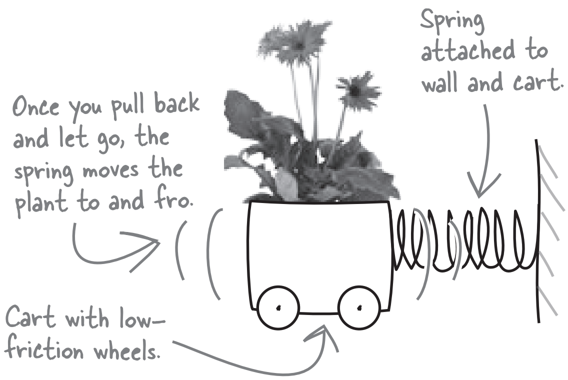


A spring will produce regular oscillations

Anne would like to use a **spring** to make the plant rocker. She'd start it rocking by pulling back the spring then letting go so that it **oscillates**. The **direction** that the plant rocks in doesn't matter and the **amplitude** of the rocking (the maximum displacement of the plant from the equilibrium position in the center) isn't important either.

However, Anne is very insistent that the plant rocker must have a **frequency** of 0.750 Hz.

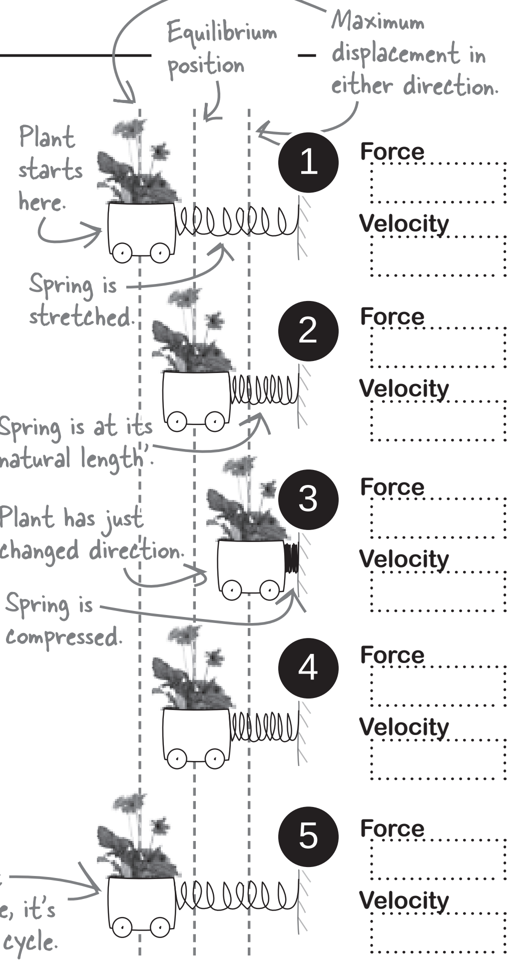
Time to imagine what it's like to ...



BE the plant

Your job is to imagine you're the plant, to get an 'inside perspective' on how the spring-powered plant rocker works. Each picture is taken from a different part of the plant rocker's 'cycle'.

Draw the force and velocity vectors in the boxes and write a brief description of what's going on beside each picture.



1. Force / Velocity / Brief description
2. Force / Velocity / Brief description
3. Force / Velocity / Brief description
4. Force / Velocity / Brief description
5. Force / Velocity / Brief description

If the force or velocity is zero, write 'zero' in the box.

BE the plant - SOLUTION

Your job is to imagine you're the plant, to get an 'inside perspective' on how the spring-powered plant rocker works. Each picture is taken from a different part of the plant rocker's 'cycle'.

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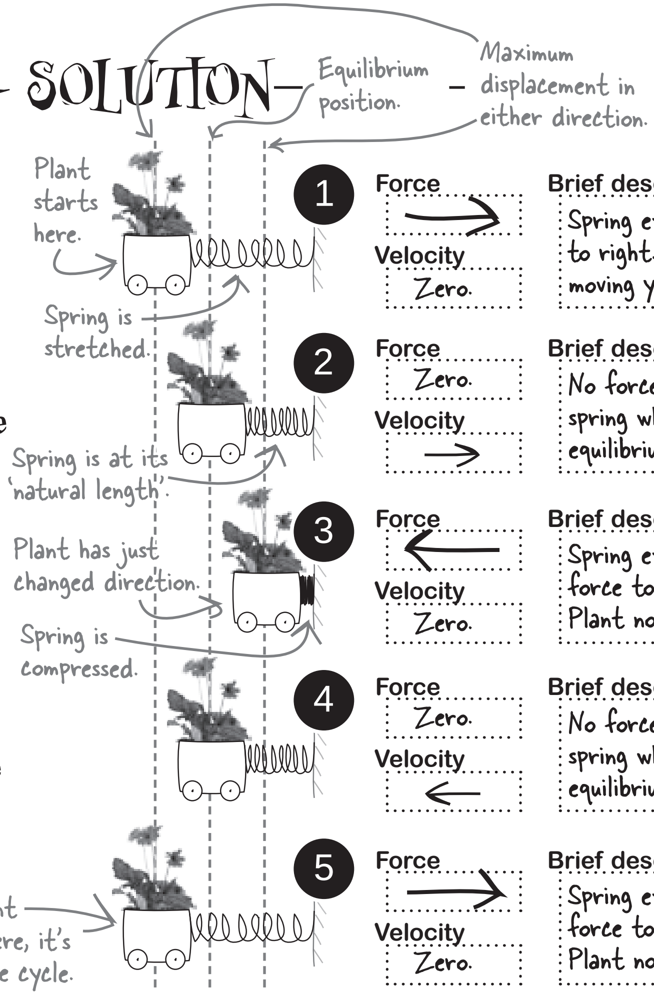


1
Force: →
Velocity: Zero.
Brief description: Spring exerts force to right. Plant not moving yet.

2
Force: Zero.
Velocity: →
Brief description: No force from spring when plant in equilibrium position.

3
Force: ←
Velocity: Zero.
Brief description: Spring exerts force to left. Plant not moving.

4
Force: Zero.
Velocity: ←
Brief description: No force from spring when plant in equilibrium position.

5
Force: →
Velocity: Zero.
Brief description: Spring exerts force to right. Plant not moving.

Any time you stretch or compress a spring away from its **equilibrium** position, the **spring exerts a force in the opposite direction from the displacement**.

When you pull the plant to the left and let go, the **force** that the spring exerts on it **accelerates** the plant to the right. As the spring becomes less stretched, it exerts a smaller force. This continues until the spring is back at its equilibrium position. There is no net force on the plant when the spring is in the equilibrium position.

With no net force, the plant continues with the same **velocity**, overshoots the equilibrium position and begins to compress the spring. As the spring gets shorter and shorter, it exerts a larger and larger leftwards force on the plant, slowing the plant down. The plant is briefly stationary at the **maximum displacement**.

Then the plant accelerates to the left, passes through the equilibrium position, overshoots and ends up back where it started - a complete cycle of motion.

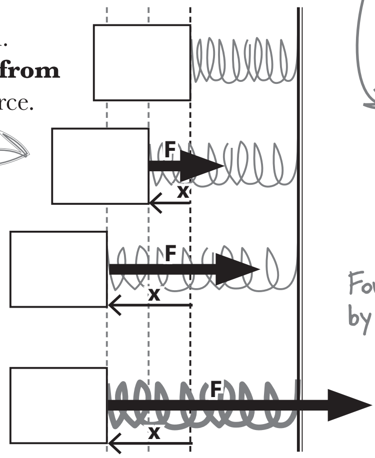
Displacement from equilibrium and strength of spring affect the force

You know how the plant rocker works qualitatively. Now it's time to be quantitative and start working out some values.

The **force** that the spring exerts depends on:

The change in the spring's length. The greater the **displacement from equilibrium**, the greater the force.

If you double the displacement from equilibrium, you double the force.



The strength of the spring. The **stronger the spring**, the greater the force.

Kinetic energy is a capital K; the spring constant is a small k. Take care not to mix them up.

This is summed up by the equation $\mathbf{F}_s = -k\mathbf{x}$, where **F** is the force exerted by the spring, **x** is the displacement from the equilibrium position and k is the spring constant, a measure of the strength of the spring. There's a minus sign in the equation because the force is in the opposite direction from the displacement. This relationship is known as **Hooke's Law**.

$$F_s = -kx$$

Force exerted by spring.

Spring constant

Displacement from equilibrium position.

Use a minus sign to show that the force the spring exerts in in the opposite direction from the displacement.

Sharpen your pencil

a. Use the equation $\mathbf{F}_s = -k\mathbf{x}$ to work out the units of k, the spring constant (in kg, m, s, etc).

b. The plant rocker is to have a frequency of 0.750 Hz. What is its period?

c. Use the equation $\mathbf{F}_s = -k\mathbf{x}$ to explain whether you think using a stronger spring (with a larger spring constant) will have an effect on the period.

d. Do you think there are any other variables that would change the period?

Sharpen your pencil Solution

a. Use the equation $\mathbf{F}_s = -k\mathbf{x}$ to work out the units of k, the spring constant (in kg, m, s, etc).

$F = -kx \Rightarrow ma = -kx$

$\Rightarrow k = -\frac{ma}{x} \Rightarrow [k] = \frac{[m][a]}{[x]}$

$[k] = \frac{kg \cdot m/s^2}{m}$ $\quad [k] = kg/s^2$

b. The plant rocker is to have a frequency of 0.750 Hz. What is its period?

Period, T, is number of seconds per cycle.
Frequency, f, is number of cycles per second.

$T = \frac{1}{f} = \frac{1}{0.750} = 1.33\text{ s}$ (3 sd)

c. Use the equation $\mathbf{F}_s = -k\mathbf{x}$ to explain whether you think using a stronger spring (with a larger spring constant) will have an effect on the period.

If you have a strong spring and a weaker spring and pull them both back the same displacement at the start, the strong spring will accelerate the plant more rapidly, because a large k means a large force.. Onvce the plant's moved through the equilibrium position, it'll also decelerate the plant and then pull it back in the opposite direction more quickly. So I think a stronger spring will lead to a shorter period.

d. Do you think there are any other variables that would change the period?

F = ma, so if the plant is more massive it'll accelerate less when you pull it back. So it won't move so quickly and the period will be shorter. The amount you pull the plant back may also affect the period?

there are no Dumb Questions

Q: Why do we want to calculate the force that the spring exerts on the plant?

A: Any time you're dealing with forces, it's good to start with a free body diagram and work out the net force (like you did when you were 'being' the plant).

Q: How do you know that doubling the spring's displacement from equilibrium doubles the force?

A: By experimenting with springs! There was a bit of this back in chapter 11 when we were dealing with how scales measure your weight.

Q: So how would you measure a spring constant? Surely springs don't come with one written on?

A: A strong spring with a large spring constant will stretch less than a weaker spring when you apply the same force to it by hanging the same mass from it.

So if you measure the spring's displacement from equilibrium (i.e. its change in length) for a variety of masses, you can plot a graph. You can use the graph to calculate the spring constant. You'll calculate a spring constant later on in this chapter.

Q: Are the extremes and the equilibrium position 'special points'?

A: Yes. When the displacement is at its maximum, the force is also at its maximum (though they're in opposite directions). The velocity is zero at the extremes when the plant is changing direction.

In the equilibrium position, there's no net force on the plant, so it continues at its current (and maximum) velocity).

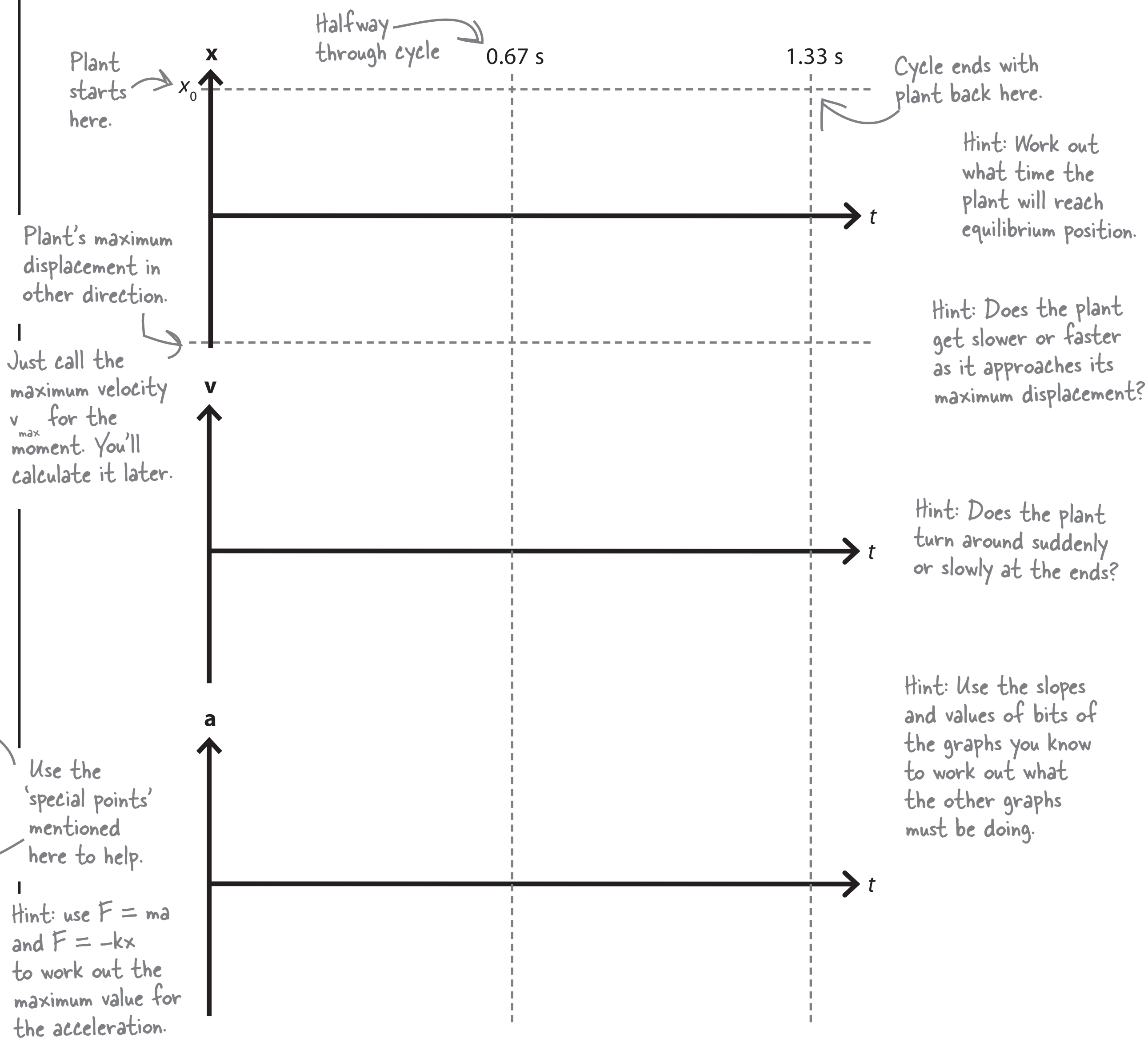
Q: Will thinking about the force help us with the frequency of the oscillations?

A: We're just getting on to that ...

Sharpen your pencil

Graphs of the plant rocker's motion will help you to work out an equation that connects the spring constant with the frequency of oscillations. The plant starts off at $x = x_0$.

Sketch graphs of the plant's displacement, velocity and acceleration vs time for one cycle of its motion with a period of 1.33 s (same as a frequency of 0.750 Hz). Start off by marking the 'special points' where you'd find the maximum of each variable, and sketch on from there. We've already put some of them on for you.



Sharpen your pencil Solution

Graphs of the plant rocker's motion will help you to work out an equation that connects the spring constant with the frequency of oscillations. The plant starts off at $x = x_0$.

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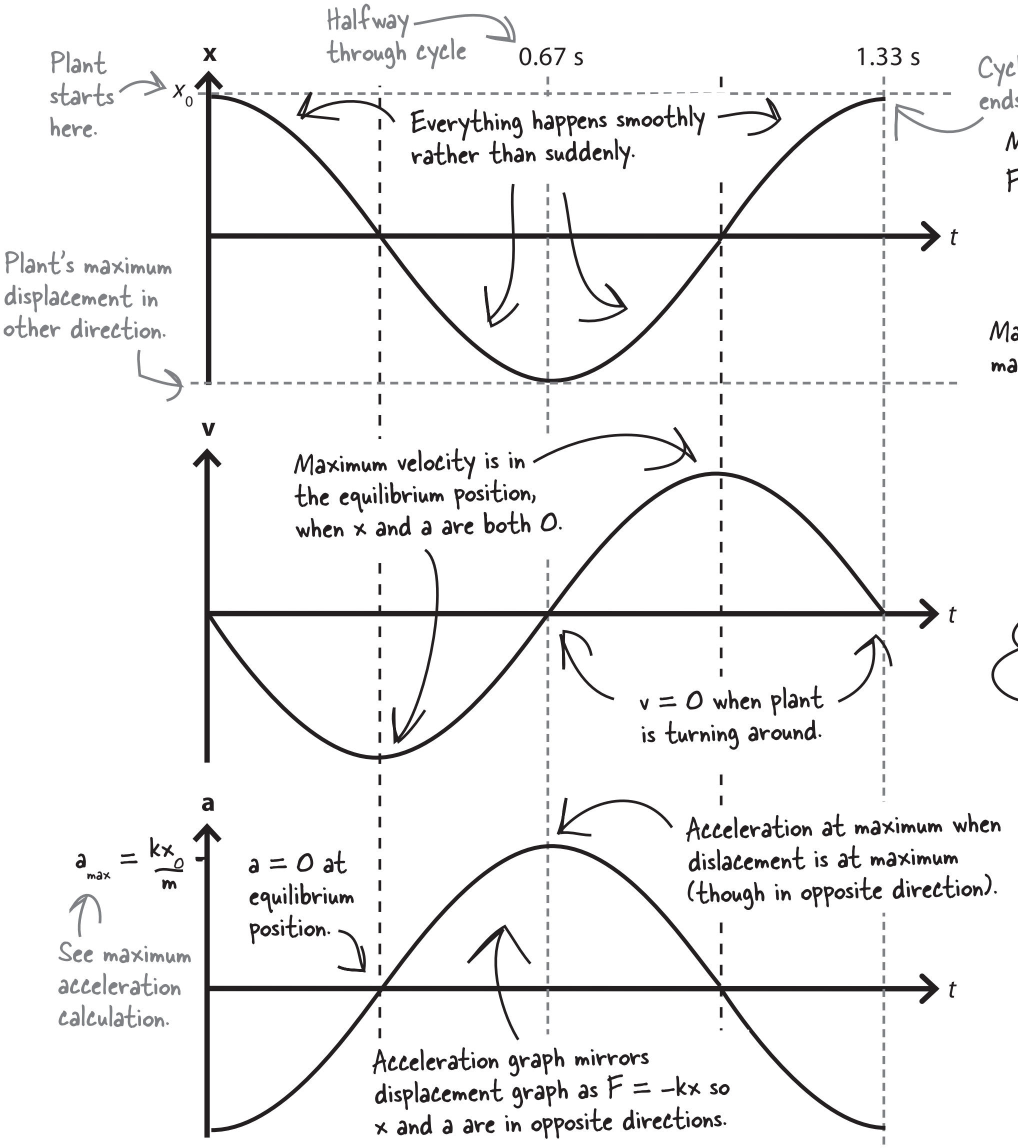


Maximum acceleration calculation:

$$F = ma = -kx$$

$$\Rightarrow ma = -kx$$

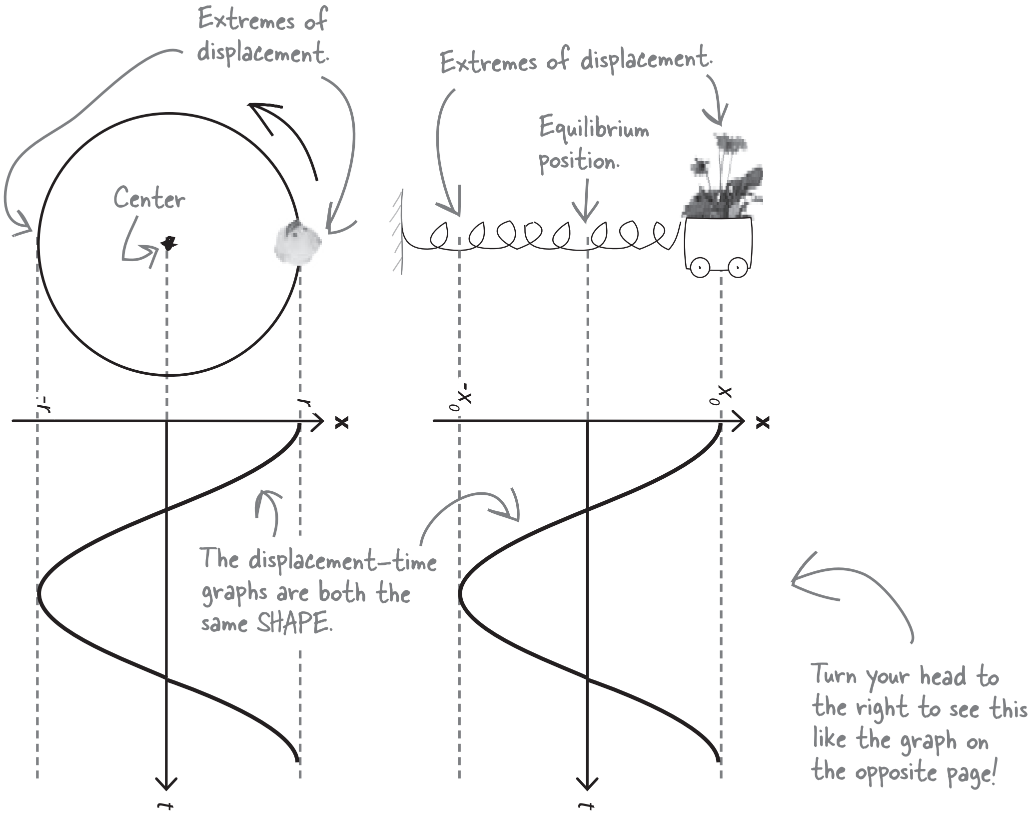
$$a = \frac{-kx}{m}$$

Maximum acceleration when there's maximum displacement.

$$\Rightarrow a_{max} = \frac{-kx_0}{m}$$

A mass on a spring moves like a side-on view of circular motion

Looking at the plant rocker - a mass on a spring - from side on is identical to looking at **circular motion** from side on.



The displacement, velocity and acceleration-time graphs of a mass on a spring are all sinusoids.

This means that the graphs for the plant's **displacement**, velocity and **acceleration** are the same **shapes** as the equivalent graphs for circular motion viewed from side on.

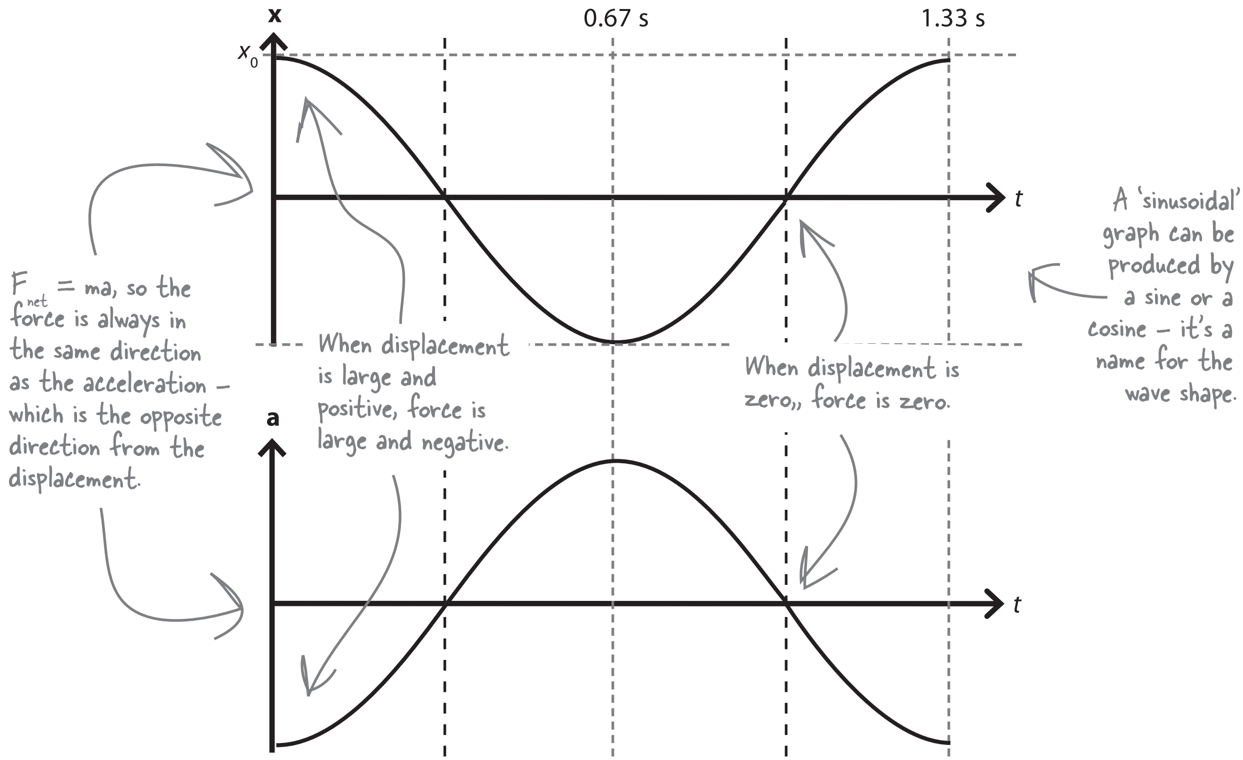
So the plant rocker must use the **same types of equations** as the side-on view of circular motion. This means that the equations for the plant rocker's displacement, velocity and acceleration will involve **sines** and **cosines**.

A mass on a spring moves with simple harmonic motion

The plant rocker oscillates to-and-fro because of a '**restoring force**' from the spring, $\mathbf{F}_s = -k\mathbf{x}$.

The force is directly proportional to the spring's **displacement** from the equilibrium position and acts in the opposite direction from the displacement. The graphs you sketched of the plant rocker confirm that the acceleration - and therefore the force - is always in the **opposite direction** from the displacement.

You get simple harmonic motion when the restoring force is directly proportional to the displacement, but in the opposite direction.



This kind of situation, where the restoring force is proportional to the displacement from the equlibrium position, is common enough in physics for it to be given its own name, **simple harmonic motion**, or SHM.

SHM always produces these kinds of **sinusoidal** displacement-time, velocity - time and acceleration-time graphs. A "simple harmonic" is another name for a sinusoid.

Yes - the equations for the frequency, period, maximum speed, etc are the same

If you set up a mass on a spring and a side-on circular motion on a turntable next to each other, they look identical. The shapes of their displacement-time, velocity-time and acceleration-time graphs are identical.

This means you can use all the equations you already know for the frequency, period and angular frequency of circular motion for simple harmonic motion.

Angular frequency and angular speed

Angular frequency and angular speed both have the same size and both have units of radians per second. They're exactly the same thing.

You can get from the frequency, f, to the angular frequency, ω, with the equation:

$$\omega = 2\pi f$$

SHM has a frequency and a period, so you can use all the equations you already know from circular motion.

Frequency and period

Frequency, f, is cycles per second.

Period, T, is seconds per cycle.

Because they're related like this:

$$T = \frac{1}{f} \qquad f = \frac{1}{T}$$

You can do all sorts of combining and rearranging equations with these two postit notes.

there are no Dumb Questions

Q: It seems like a big jump from a mass on a spring to three sinusoidal graphs. Can you run some of that by me again?

A: The key thing is that you have a situation where the force is directly proportional to the displacement from the equilibrium position, and in the opposite direction from the displacement. Any time this is the case - whether the force is provided by a spring or something else - you have simple harmonic motion.

Q: What does simple harmonic motion look like?

A: If you watch the mass on the spring from side-on, you'll see that the mass moves quickly through the equilibrium position but slowly at each end of the motion, and transitions smoothly between these velocitites.

Q: Is simple harmonic motion exactly like looking at circular motion from side on?

A: Yes! If you have simple harmonic motion (SHM) and circular motion with the same period and look at them from side-on, they appear identical.

Q: With SHM there's an acceleration produced by the force of the spring which I can plot on a graph. Where's the acceleration in circular motion?

A: If an object's moving around a circle, there must be a centripetal acceleration provided by something in order for the circular motion to be possible. So the acceleration-time graph of circular motion viewed from side on would be the component of the centripetal acceleration that you can see from your vantage point.

Actually proving this requires calculus

You did a substitution to work out the maximum value of the acceleration to put on your graph: $\mathbf{F} = -k\mathbf{x}$ and also $\mathbf{F} = m\mathbf{a}$, so if you do a substitution for $\mathbf{F}$ you get $-k\mathbf{x} = m\mathbf{a}$; rearranged, this is $\mathbf{a} = -\frac{k}{m}\mathbf{x}$

At the moment, this is an equation containing two quantities that are constant throughout the plant's motion (k and m) and two that vary ($\mathbf{a}$ and $\mathbf{x}$). You can't use one equation to work out two unknowns.

However, acceleration is rate of change of displacement, $\mathbf{a} = \frac{d\mathbf{v}}{dt}$. You can substitute this in the equation $\mathbf{a} = -\frac{k}{m}\mathbf{x}$ to get $\frac{d\mathbf{v}}{dt} = -\frac{k}{m}\mathbf{x}$ This doesn't appear to help, as there are still two unknowns ($\mathbf{v}$ and $\mathbf{x}$) in the equation. But as displacement is rate of change of velocity, $\mathbf{v} = \frac{d\mathbf{x}}{dt}$, you can make another substitution to get $\frac{d}{dt}\left(\frac{d\mathbf{x}}{dt}\right) = -\frac{k}{m}\mathbf{x}$.

This equation only has one unknown, $\mathbf{x}$, but you need calculus to solve it. Don't worry - we've given you the ready-bake solution on the opposite page

You don't have to do calculus!

You don't need to be able to go line by line from the nasty-looking equation $\frac{d}{dt}\left(\frac{d\boldsymbol{x}}{dt}\right) = -\frac{k}{m}\boldsymbol{x}$ to the ready-bake equation on the opposite page. On a non-calculus course, you'll only need to **apply** the ready-bake equation to solve problems.

Simple harmonic motion is sinusoidal

The mass starts off with its maximum displacement at $t = 0$. This means that the equation must be some kind of cosine, as cosine is also at its maximum when $t = 0$.

The exact form of the equation is $\mathbf{x} = \mathbf{x}_0 \cos\left(\sqrt{\frac{k}{m}}\, t\right)$.

Ready Bake Equation

Mass on spring starts at $x = x_0$:

$$x = x_0 \cos\left(\sqrt{\frac{k}{m}}\, t\right)$$

cosine is at a maximum when $t = 0$

x_0 is the maximum value of x, and therefore the amplitude of the motion.

It looks like you don't need to have an angle to have a sinusoid shape, even though sine and cosine usually involve angles?

The value you 'put into' the cosine function is equal to the angle in an equivalent side-on circular set-up.

If you set up a plant on a spring and a duck on a turntable next to each other so that they both have the same amplitude and frequency, their motion looks identical from side on.

You can describe circular motion viewed from side on in terms of the angle θ that the turntable has moved through, using the equation $\mathbf{x} = \mathbf{x}_0\cos(\theta)$. You can describe circular motion in terms of **time** using the same equation with the substitution $\theta = \omega t$ to get $\mathbf{x} = \mathbf{x}_0\cos(\omega t)$.

Here, θ and ω are "mathematical tools" rather than actual physical angles.

As the circular and simple harmonic motion **look** identical from side on, the equation for SHM has the same **form** as the equation for side-on circular motion, despite the fact that there are no physical angles involved with the plant on the spring.

Hey ... are you close to working out how to rock my plants at the right **frequency**?

BRAIN POWER

How can you use the equation for SHM to work out the **frequency** of the plant rocker?

Work out constants by comparing a situation-specific equation with a standard equation

The 'standard' equation for simple harmonic motion (and circular motion from side on) that starts at a maximum is $\mathbf{x} = \mathbf{A}\cos(\omega t)$.

The equation for the plant's displacement is $\mathbf{x} = \mathbf{x}_0\cos\left(\sqrt{\frac{k}{m}}\,t\right)$.

If you 'line up' these two equations, you can see that they're identical in 'form' but that there are different variables in 'important places'.

$$x = A\cos(\omega t)$$

$$x = x_0\cos\left(\sqrt{\frac{k}{m}}\,t\right)$$

x and t are variables.

You met this equation in ch 19.

x_0, k and m are constants for any one 'run' of the plant rocker.

Look at the amplitude

Both equations consist of a cosine multiplied by something. In the 'standard' equation, that something is **A**, the **amplitude**, which gives you the maximum value that the equation can have (since the maximum value of cosine is 1).

In your displacement equation, the amplitude is $\mathbf{x}_0$, which is how far you pulled the plant rocker back at the start - it's the maximum value the displacement can have.

Look at the argument of the cosine

Both equations contain the **cosine** of a quantity. In the 'standard' equation, this quantity is ωt, the angular frequency multiplied by the time. As the definition of angular frequency is $\omega = 2\pi f$, you can work out the frequency and period if you know the value of ω.

In your equation, you have the cosine of $\sqrt{\frac{k}{m}}\,t$. So $\sqrt{\frac{k}{m}}\,t$ in this equation is equivalent to ωt in the 'standard equation. You can write down the new equation $\omega t = \sqrt{\frac{k}{m}}\,t$ and use this to calculate the frequency of the plant rocker.

Here, the variable ω is a tool you can use to get what you want.

The 'standard' equation could also be written $\mathbf{x} = \mathbf{x}_0\cos(2\pi f t)$ (as $\omega = 2\pi f$) which doesn't involve ω at all, only f.

The mass on a spring has a **frequency** (number of cycles per second). You want to know how to produce oscillations with this frequency, and can use ω as a mathematical tool to get there.

Compare your equation with the 'standard' one to work out the amplitude and frequency of your SHM.

Sharpen your pencil

You have three plants with masses of 100, 250 and 500 grams respectively. You wish to attach each of them to an individual spring so that they can be rocked horizontally with a frequency of 0.750 Hz.

a. Compare the equation $\mathbf{x} = \mathbf{x}_0\cos\left(\sqrt{\frac{k}{m}}t\right)$ with the standard equation for simple harmonic motion to work out an equation for the frequency of the plant rocker.

b. Check your equation over by imagining what would happen if k and m were altered one at a time. Jot down your thoughts about whether your equation behaves as the plant would in real life.

c. Calculate the spring constant required to rock each plant with a frequency of 0.750 Hz when the plant rocker is pulled back to $\mathbf{x}_0$ = 10.0 cm at t = 0 to start off with.

Hint: You'll need to work out its units as well.

d. Would it be possible to use the same strength of spring for all three plants if you pulled the plants back different distances to start off the plant rocker? Why / why not?

Sharpen your pencil Solution

You have three plants with masses of 100, 250 and 500 grams respectively. You wish to attach each of them to an individual spring so that they can be rocked horizontally with a frequency of 0.750 Hz.

a. Compare the equation $\boldsymbol{x} = \boldsymbol{x}_0 \cos\left(\sqrt{\frac{k}{m}}\, t\right)$ with the standard equation for simple harmonic motion to work out an equation for the frequency of the plant rocker.

'Standard' equation: $x = A\cos(\omega t)$

Mass on a spring: $x = x_0 \cos\left(\sqrt{\frac{k}{m}}\, t\right)$

$$\omega = \sqrt{\frac{k}{m}} \Rightarrow 2\pi f = \sqrt{\frac{k}{m}}$$

$$f = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$$

This works because $\omega = 2\pi f$

b. Check your equation over by imagining what would happen if *k* and *m* were altered one at a time. Jot down your thoughts about whether your equation behaves as the plant would in real life.

If k is bigger, the equation says that the frequency gets higher. This makes sense, as the force would be larger so the plant would accelerate more and move more quickly.

If m is bigger, the equation says that the frequency gets lower. This makes sense as the force would be the same but the mass would be larger, so the plant wouldn't accelerate as much and would move more slowly.

c. Calculate the spring constant required to rock each plant with a frequency of 0.750 Hz when the plant rocker is pulled back to $\mathbf{x}_0 = 10.0$ cm at $t = 0$ to start off with.

$$f = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$$

$$\Rightarrow f^2 = \frac{1}{4\pi^2} \cdot \frac{k}{m}$$

$$\Rightarrow k = 4\pi^2 m f^2$$

You can get the units of the spring constant from the units of the right hand side of your equation.

100 g plant: $k = 4\pi^2 \times 0.100 \times 0.750^2 = \underline{2.22 \text{ kg/s}^2}$ (3 sd)

250 g plant: $k = 4\pi^2 \times 0.250 \times 0.750^2 = \underline{5.55 \text{ kg/s}^2}$ (3 sd)

500 g plant: $k = 4\pi^2 \times 0.500 \times 0.750^2 = \underline{11.1 \text{ kg/s}^2}$ (3 sd)

You need the masses to be in kilograms, not grams.

d. Would it be possible to use the same strength of spring for all three plants if you pulled the plants back different distances to start off the plant rocker? Why / why not?

No, because the frequency doesn't depend on the amplitude, only the spring constant and the mass. So the frequency will be the same however far back you pull the plant at the start.

$$x = A\cos(\boldsymbol{\omega} t)$$

is the standard equation for any SHM. For a mass on a spring,

$$\boldsymbol{\omega} = \sqrt{\frac{k}{m}}$$

Use this equation to solve ANY SHM problem.

ω is a tool that gives you what you want to find out.

Question Clinic: The "This equation is like that one" Question

In physics, you sometimes come across specific instances of general equations. One example of this is the general equation for simple harmonic motion, x = Acos(ωt). ANY object moving with simple harmonic motion will have an equation of this form (or an equation of the form x = Asin(ωt) if the motion starts at x = 0. In the equation, A represents the amplitude - the maximum value of x. And the argument of the cosine is always equal to ωt. This means that you can calculate ω, and from it f and T.

See also the equation y = mx + c, covered in appendix i.

Each of the variables in the equation is defined here.

12. The displacement of a mass, *m*, oscillating on a spring with spring constant *k*, after time, *t*, is given by the equation $x = x_0\cos\left(\sqrt{\frac{k}{m}}t\right)$

a. By comparing this equation with the standard equation for simple harmonic motion, work out an equation for *f*, the frequency of the oscillator.

b. Calculate the spring constant required to rock a 100 gram plant with a frequency of 0.750 Hz.

This is the amplitude

This means that you compare it with the equation x = Acos(ωt) to get the amplitude and angular frequency.

This is the angular frequency, ω

So $\omega = \sqrt{\frac{k}{m}}$

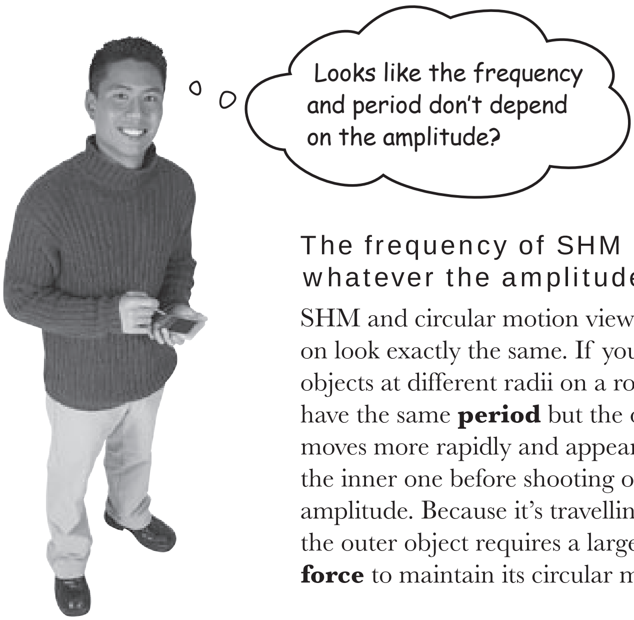
Here, $\omega = 2\pi f$

So $2\pi f = \sqrt{\frac{k}{m}}$

You will often have to use the equation you work out to do a calculation. Don't worry - if you get the equation wrong then work the numbers through OK, you'll get partial credit.

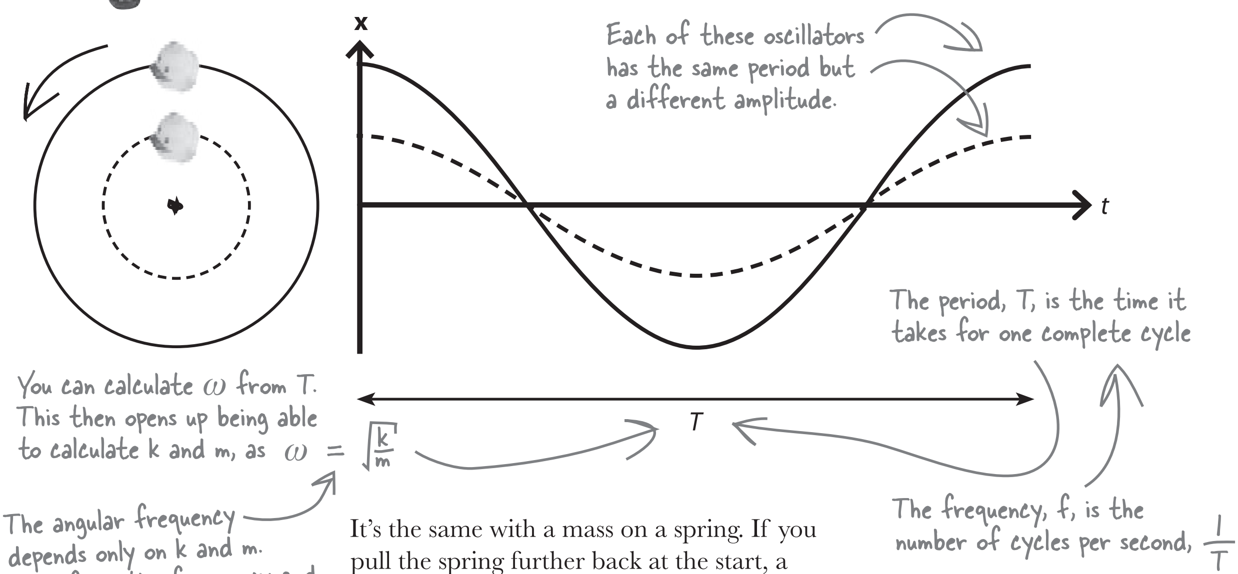
You can solve problems by spotting that an equation is like one you already know about, and making substitutions.

The key thing in a question like this is keeping track of what is plotted on each axis in each equation, especially if they use different letters for the things plotted on the horizontal and vertical axes. Do a sketch of each graph and write the letters on to help you keep track.



The frequency of SHM is the same, whatever the amplitude.

SHM and circular motion viewed from side-on look exactly the same. If you follow two objects at different radii on a rotating disc, they have the same **period** but the outer object moves more rapidly and appears to 'overtake' the inner one before shooting out to a larger amplitude. Because it's travelling more rapidly, the outer object requires a larger centripetal **force** to maintain its circular motion.



The angular frequency depends only on k and m. Therefore, the frequency and period depend only on k and m, but not on the amplitude.

It's the same with a mass on a spring. If you pull the spring further back at the start, a larger **force** acts on the mass, and the mass achieves a higher **velocity** through the equilibrium position. But this means that the mass has more momentum, so it 'overshoots' further than an oscillation with the same mass and a smaller amplitude.

However large the **amplitude** of the mass's oscillations, the SHM will always have the same **period**, and the same **frequency**.

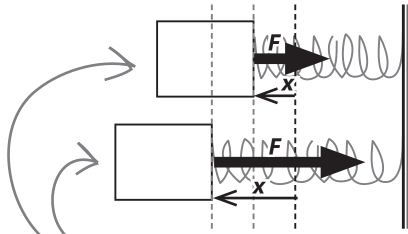
Unless you damage the spring by over-stretching it when you pull it back – then you won't get SHM at all, just a repair bill!

The frequency and period of SHM don't depend on the amplitude.

You rock! Or at least Anne's plants do

You've designed Anne's patented horticultural talk-free device - and it rocks! First of all, you worked out how the spring makes the plant oscillate and found out about Hooke's Law, $\mathbf{F} = -k\mathbf{x}$.

Then you sketched graphs of the plant's **displacement**, **velocity** and **acceleration** and spotted that they have similar shapes to graphs of circular motion when the motion is viewed from side on. This is **simple harmonic motion** - and you always get these shapes of graphs when the restoring **force** is proportional to the displacement from the equilibrium position.



Simple harmonic motion is sinusoidal. This lets you calculate the frequency, period, amplitude, etc, as the equation has a standard format.

After getting the ready-bake (calculus-derived) equation for the displacment, $\mathbf{x} = \mathbf{x}_0\cos\left(\sqrt{\frac{k}{m}}\,t\right)$, you compared it with the 'standard' equation for simple harmonic motion $\mathbf{x} = \mathbf{A}\cos(\omega t)$. Comparing the arguments of the cosines in the equations let you write down the equation $\omega t = \sqrt{\frac{k}{m}}\,t$ and then $\omega = \sqrt{\frac{k}{m}}$, which enabled you to calculate the spring constant each plant would need.

And it doesn't matter how far back you pull the plant to start it off, as the frequency and period of SHM don't depend on the amplitude of the motion.

But Anne forgot to mention someting ...

BRAIN POWER

What might the maximum velocity of a mass on a spring depend on?

Our design's brilliant - but Anne wants the plant's maximum velocity to be exactly 1.50 m/s.

Jim: I guess that the maximum velocity's gonna depend on how far back we pull the plant at the start.

Frank: Yeah, the larger the **displacement** from equilibrium, the larger the **force** from the spring and the larger the **acceleration**. The plant has its **maximum velocity** as it goes through the equilibrium position, before the spring starts to slow it down again.

Joe: But if the initial displacement is too large, the plant will go too fast. We need to calculate exactly how far back we need to pull the plant to give it a velocity of 1.50 m/s in the center.

Frank: I guess we could go take a calculus class then come back and have a go at solving those sinusoidal equations. NOT!!

Jim: I'm sure there must be another way - if we draw enough free body diagrams and force vectors, maybe we'll spot something.

Joe: Hey ... we're only thinking about using forces. But isn't it usually easier to use **energy** in problems where that's possible?

Frank: Perhaps ... differences drive change that lead to energy transfer. Well, the velocity of the plant is changing all the time.

Joe: And so's the length of the spring. You start off with **elastic potential energy** in the stretched spring, then have entirely **kinetic energy** in the equilibrium position, and then entirely potential again at the other extreme.

Jim: Yeah when the velocity is at its maximum, all of the potential energy the spring had at the start is now kinetic energy $K = \frac{1}{2}m\mathbf{v}^2$.

Frank: Which has **v** in it! If we know how large the spring's potential energy store is at the start, we can use energy conservation to calculate the plant's velocity in the center, when the elastic potential energy is zero. That works!

Joe: So we give the spring potential energy at the start ...

Jim: Yeah, we do **work** on the spring, to do that, right? And work = force × displacement. Sorted!

Joe: Um ... but which force do we use? As you stretch the spring, the force we're doing work against gets larger and larger.

Frank: The work is the **area under the force-displacement graph**. Can we calculate that?

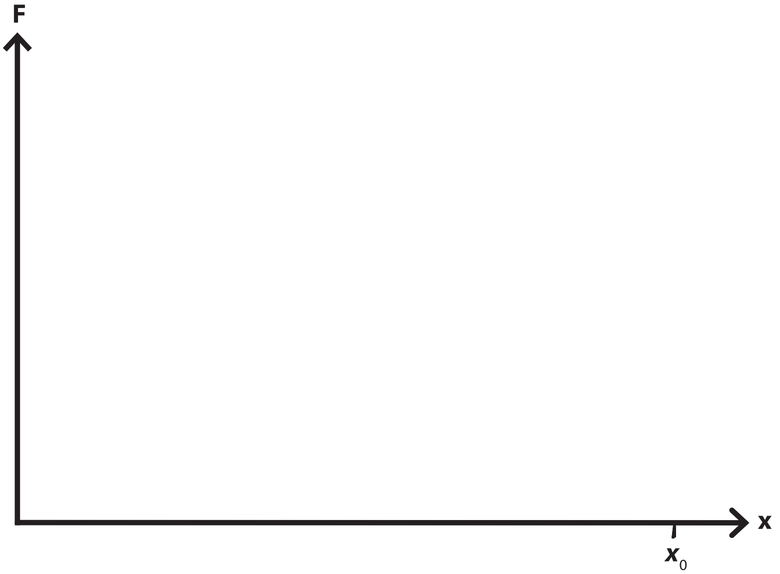
A compressed or stretched spring has elastic potential energy. You can use energy conservation to solve problems that involve springs.

To use force, acceleration, velocity and displacement here, you'd have to go and take a calculus course first. Best to use energy ...

Sharpen your pencil

This is a hard exercise. Take time to get your head around it, and don't be worried if it takes you a while.

a. Use the axes below to sketch a graph of force applied vs displacement as you stretch a spring with spring constant k to displacement $\mathbf{x}_0$. As your graph is of the force you need to apply to extend the spring, rather than the force that the spring exerts on you, the force and displacement lie in the same direction, and the value of the force is $\mathbf{F} = k\mathbf{x}$.



b. Mark on the value of **F** when the displacement = $\mathbf{x}_0$.

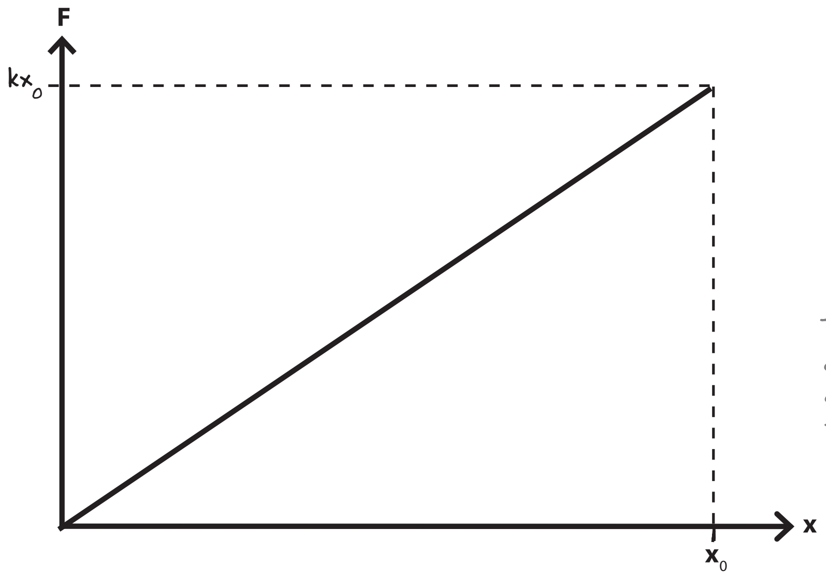
c. The total work done is the total area between your graph and the horizontal axis. Calculate this area and hence write down an equation for the work done in stretching the spring to displacement $\mathbf{x}_0$.

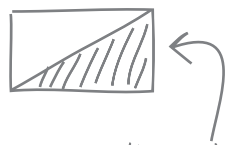
The subscript 's' indicates that a spring is involved.

d. How much elastic potential energy, U_s, is transferred to a spring with spring constant k by stretching it displacement $\mathbf{x}_0$ from its equilibrium position?

Sharpen your pencil Solution

a. Use the axes below to sketch a graph of force applied vs displacement as you stretch a spring with spring constant k to displacement $\mathbf{x}_0$. As your graph is of the force you need to apply to extend the spring, rather than the force that the spring exerts on you, the force and displacement lie in the same direction, and the value of the force is $\mathbf{F} = k\mathbf{x}$.





The area under the graph is a triangle. This has half the area that a rectangle with the same side lengths would.

Kinetic

b. Mark on the value of **F** when the displacement = $\mathbf{x}_0$.

c. The total work done is the total area between your graph and the horizontal axis. Calculate this area and hence write down an equation for the work done in stretching the spring to displacement $\mathbf{x}_0$.

Work done = area under F–x graph.

Work done = area of triangle

The area of the triangle is half the area of a rectangle with the same horizontal and vertical sides.

$$\text{Work done} = \text{Area} = \tfrac{1}{2} \times \text{base} \times \text{height}$$

$$= \tfrac{1}{2} \times x_0 \times kx_0$$

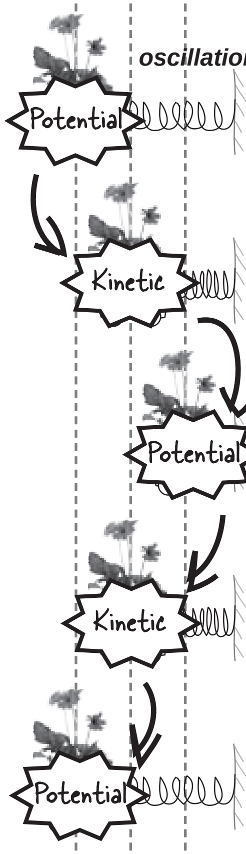
$$\text{Work done} = \tfrac{1}{2}kx_0^2$$

The work done against a force is equal to the area under the force - displacement graph.

d. How much elastic potential energy, U_s, is transferred to a spring with spring constant k by stretching it displacement $\mathbf{x}_0$ from its equilibrium position?

$$U_s = \tfrac{1}{2}kx_0^2$$

But how can the plant's total energy be $\frac{1}{2}kx_0^2$ when it's hardly ever at x_0?



The total energy of a mass on a spring depends on the spring constant and amplitude.

When you pull the plant back, you do **work** on the spring by extending it. Doing work involves **energy transfer** and the elastic potential energy of the spring increases.

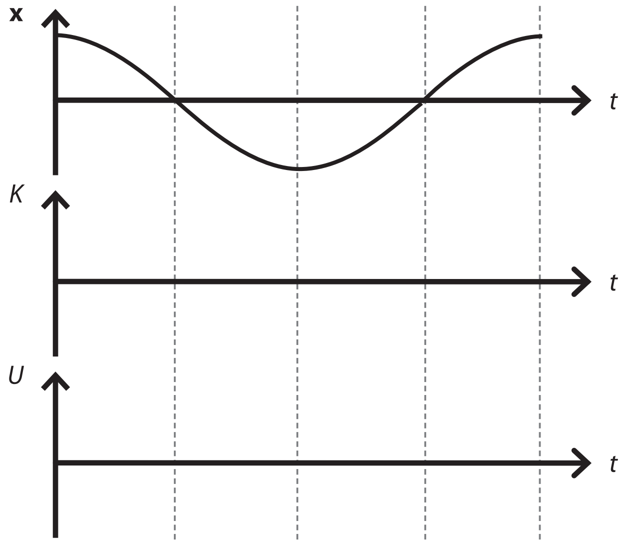
When you let go, this potential energy is gradually transferred to kinetic energy as the plant's velocity increases. In the equilibrium position, there's only kinetic energy. As the plant moves past the equilibrium position, the kinetic energy is transferred to elastic potential energy once again ... etc!

The potential energy at any displacement is $\frac{1}{2}k\mathbf{x}^2$ and the kinetic energy is $\frac{1}{2}m\mathbf{v}^2$. But the **total** energy of the system is always $\frac{1}{2}k\mathbf{x}_0^2$.

Potential

Sharpen your pencil

a. Sketch graphs of *K*, the kinetic energy and *U*, the potential energy, for one complete cycle of the plant rocker.

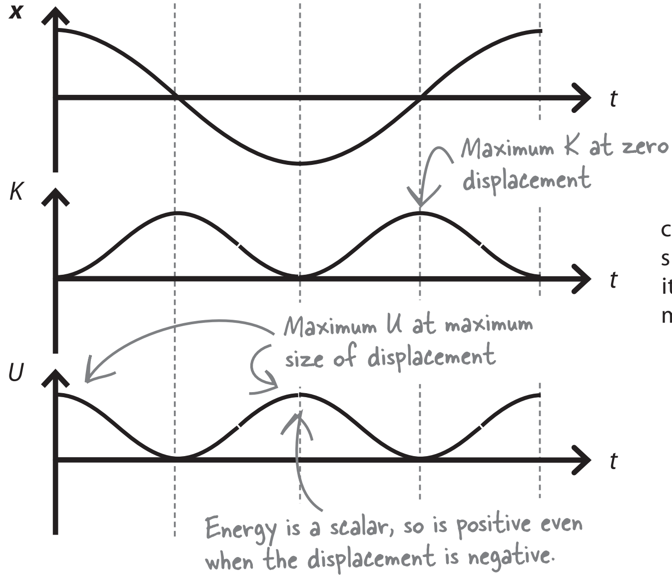


b. Use conservation of energy to come up with an equation for $\mathbf{v}_{max}$, the maximum velocity of the plant, in terms of *k*, *m* and $\mathbf{x}_0$, the initial (and maximum) displacement.

c. A plant, mass 100 g, is attached to a horizontal spring, spring constant 2.22 Nm. What should its initial displacement, $\mathbf{x}_0$, be if the plant's maximum velocity is to be 1.50 m/s?

Sharpen your pencil Solution

a. Sketch graphs of *K*, the kinetic energy and *U*, the potential energy, for one complete cycle of the plant rocker.



b. Use conservation of energy to come up with an equation for $\mathbf{v}_{max}$, the maximum velocity of the plant, in terms of *k*, *m* and $\mathbf{x}_0$, the initial (and maximum) displacement.

Maximum K = Maximum U

$$\tfrac{1}{2}kx_0^2 = \tfrac{1}{2}mv_{max}^2$$

$$v_{max} = \sqrt{\frac{k}{m}}\,x_0$$

c. A plant, mass 100 g, is attached to a horizontal spring, spring constant 2.22 Nm. What should its initial displacement, $\boldsymbol{x}_0$, be if the plant's maximum velocity is to be 1.50 m/s?

$$v_{max} = \sqrt{\frac{k}{m}}\,x_0 \Rightarrow x_0 = \sqrt{\frac{m}{k}}\,v_{max}$$

$$x_0 = \sqrt{\frac{0.100}{2.22}} \times 1.50$$

$$x_0 = 0.318 \text{ m (3 sd)}$$

$$U_s = \tfrac{1}{2}kx^2$$

The elastic potential energy of a spring depends on the amplitude and the spring constant.

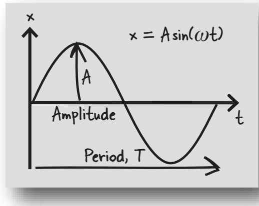
there are no Dumb Questions

Q: But what about the mass of the spring? We didn't include that in the energy calculation, just the mass of the plant.

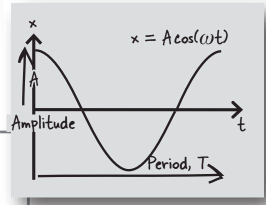
A: We made an approximation that the spring is massless - both here and before, when we were dealing with forces. If the mass of the spring is negligible compared to the mass of the plant, this is a reasonable simplifying assumption to make. If the mass of the spring wasn't negligible, we'd have to take into account the fact that different parts of the spring have different displacements - which is hard!

The plants rock - and you rule!

You've tied together your knowledge from lots of different areas of physics to give the plant rocker a known frequency and maximum velocity. Excellent!

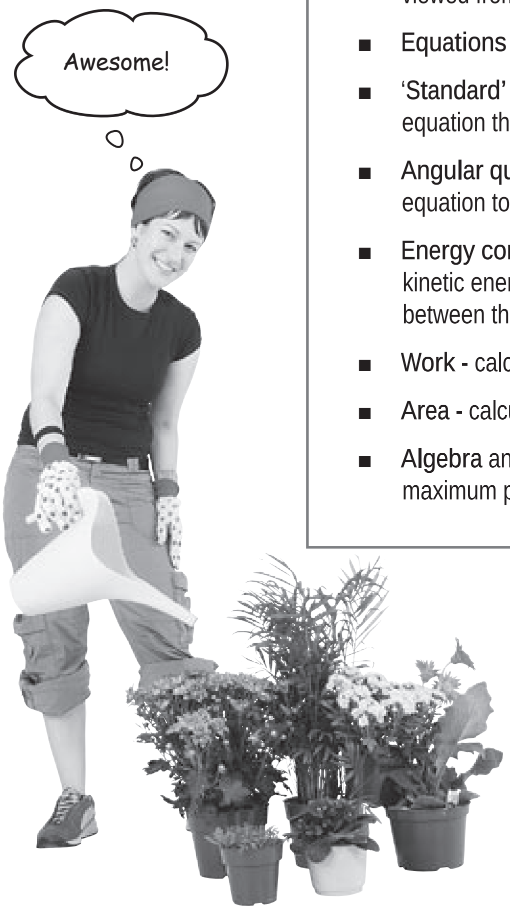


You've also been using these graph shapes.



Physics Superpowers

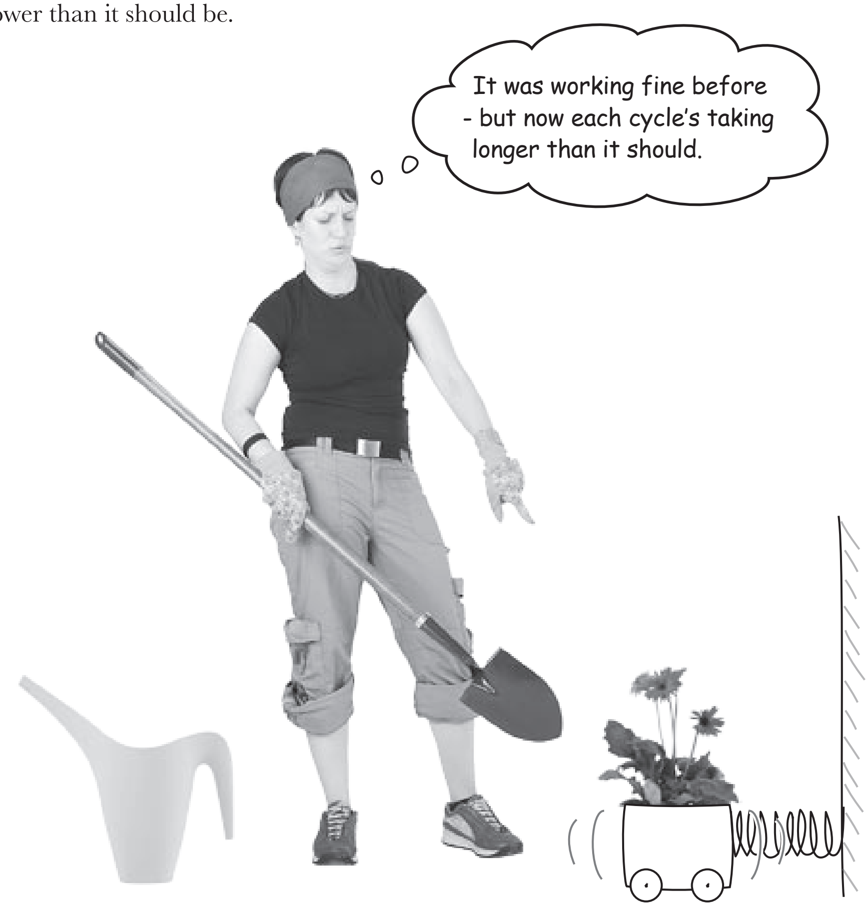
- **Forces** - analysing a problem using a free body diagram.
- **Displacement**, **velocity** and **acceleration** - using the relationships between them.
- **Graphs** - showing what the motion's like visually.
- **Circular motion** - spotting that the motion is like circular motion viewed from side on.
- **Equations** - showing what the motion's like symbolically.
- **'Standard' equations** - spotting that your equation is like an equation that you already know.
- **Angular quantities** - using the analogy with the standard equation to move from ω to f, and also to k and m.
- **Energy conservation** - spotting that there are potential and kinetic energy stores and energy is continually transferred between them.
- **Work** - calculating the work done on the spring.
- **Area** - calculating the area under the force - displacement graph
- **Algebra** and **substitution** - calculating the velocity from the maximum potential and kinetic energy.



When you combine superpowers like this, you're really thinking like a physicist!

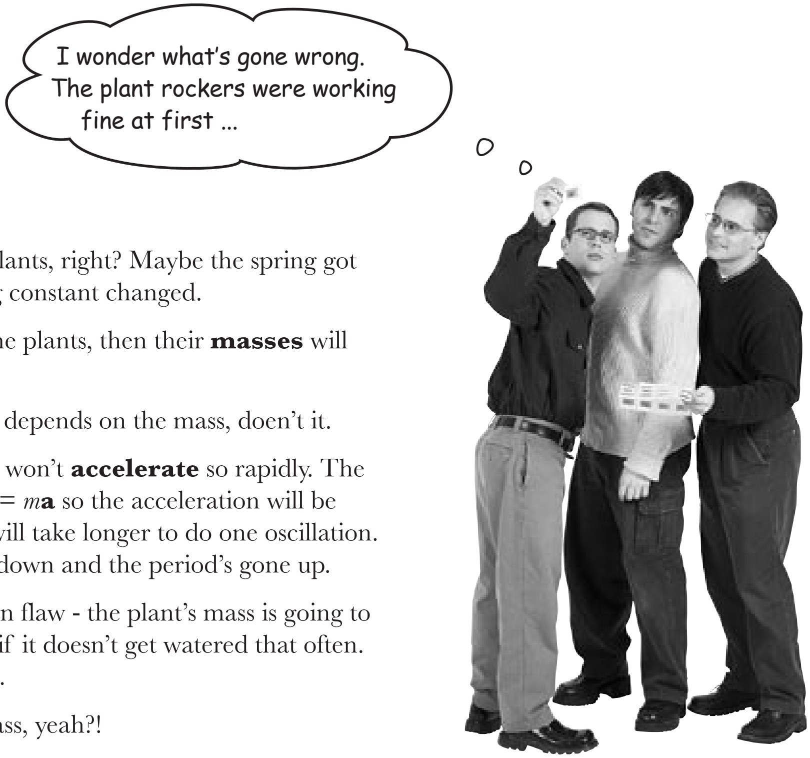
But now the plant rocker's frequency has changed ...

Although Anne is initially pleased with your solution, she's soon back in touch with a problem - the plant rocker's frequency has changed. Each cycle's taking longer than it did before, so the frequency is lower than it should be.



BRAIN POWER

What could have changed the frequency of the plant rocker?



Jim: Anne's been watering the plants, right? Maybe the spring got rusty or something, and its spring constant changed.

Joe: Actually - if she's watered the plants, then their **masses** will have changed.

Frank: Oh yeah - the frequency depends on the mass, doen't it.

Jim: Yeah, a more massive plant won't **accelerate** so rapidly. The force on it is still the same, but $\mathbf{F} = m\mathbf{a}$ so the acceleration will be smaller if m is larger. The plant will take longer to do one oscillation. That's why the frequency's gone down and the period's gone up.

Joe: This is a pretty serious design flaw - the plant's mass is going to change anyway as it grows, even if it doesn't get watered that often. I wonder what we can do to fix it.

Frank: So the problem is the mass, yeah?!

Jim: Yeah - if the mass was constant, then the frequency would be constant (as long as the spring didn't weaken or anything).

Frank: So can we make the mass **divide out** somehow? We've worked with equations before where that happened.

Joe: Ooh, I think I see what you mean. When we've done calculations involving **gravity**, like orbits and stuff, then the mass has sometimes divided out completely because it appeared on both sides of the equation.

Jim: Maybe if we hang the spring **vertically**, it'll be OK. Then gravity will be acting on the plant as well.

Joe: If the mass did divide out, we wouldn't need a different spring for each plant. Actually, this feels a bit wrong. Surely an elephant bouncing vertically on a spring would have a different frequency of oscillation from a mouse on the same spring?

Frank: We might as well test it out with math before we try building it. That shouldn't take too long ...

Think about the physics behind what's going on - either by 'being' the thing in the problem, or by seeing what its equation would do if you changed the values of the variables.

Watering a plant increases its mass.

The frequency of a horizontal spring depends on the mass

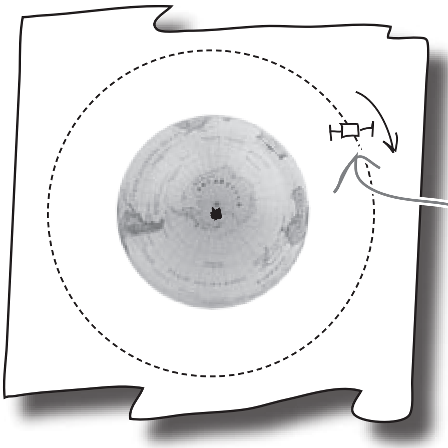
The equation for the frequency of the plant attached to the horizontal spring is $f = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$. If you increase the mass of the plant, the frequency becomes smaller, as you are dividing by the m on the right hand side of the equatuon. So the plant takes longer to do one cycle.

This is because $\boldsymbol{F} = m\boldsymbol{a}$. If the plant's mass is larger it accelerates less when acted on by the same force of the stretched spring. Watering the plants increases the mass. Anne is adamant that the plant rocker should always have a frequency of 0.750 Hz. So this design with a horizontal spring won't work, as the frequency changes when the mass changes.

Will using a vertical spring make a difference?

When gravity is the only force acting on an object, the object's acceleration doesn't depend on its mass.

In chapter 18, you worked out that the frequency and period of a satellite's orbit are completely independent of its mass.



The frequency and a period of a satellite in orbit don't depend on its mass.

The guys have had the idea that perhaps by hanging the plant from a **vertical spring**, where gravity has an influence, the physics will work out differently from the horizontal spring.

You need to work out whether they're right!

Perhaps this is also the case for a vertical spring ... perhaps not.

Sharpen your pencil

a. A spring hangs from the ceiling. When you attach a plant to it, it extends to an new equilibrium position where the plant is at rest. Draw a free body diagram showing all of the forces acting on the plant in this new equilibrium position.

Hint: Look back at page 807 for an equation for the force from a spring.

b. If the plant has a mass of 0.100 kg and the spring extends by 44.1 cm, what is the spring constant?

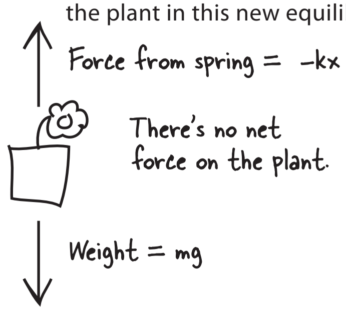
c. You pull the plant down a further 4.00 cm and let go. Draw a free body diagram of the forces acting on the plant at the moment you let go, and calculate the net force on the plant.

d. What would be the net force on a plant attached to a horizontal spring that was extended 4.00 cm from its equilibrium position at the moment it's released?

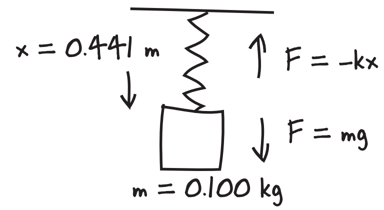
e. Do you think that the spring being vertical will affect the frequency and period of the plant's oscillations compared with the horizontal spring? Why / why not?

Sharpen your pencil Solution

a. A spring hangs from the ceiling. When you attach a plant to it, it extends to an new equilibrium position where the plant is at rest. Draw a free body diagram showing all of the forces acting on the plant in this new equilibrium position.



b. If the plant has a mass of 0.100 kg and the spring extends by 44.1 cm, what is the spring constant?



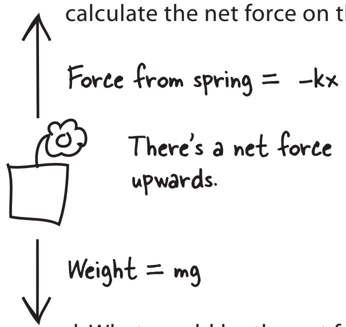
No net force, so: $mg - kx = 0$

$kx = mg$

$k = \frac{mg}{x} = \frac{0.100 \times 9.8}{0.441}$

$k = 2.22\ \text{kg/s}^2$ (3 sd)

c. You pull the plant down a further 4.00 cm and let go. Draw a free body diagram of the forces acting on the plant at the moment you let go, and calculate the net force on the plant.



$F_{net} = mg - kx$

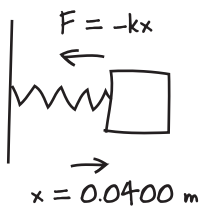
$F_{net} = 0.100 \times 9.8 - 2.22 \times 0.481$

$F_{net} = -0.878$ N (3 sd)

This is 44.1 cm + 4.00 cm.

Looks like my spring idea doesn't work out. What else can I use to rock my plants? Having to do it myself really stinks!

d. What would be the net force on a plant attached to a horizontal spring that was extended 4.00 cm from its equilibrium position at the moment it's released?



$F_{net} = -kx$

$F_{net} = -2.22 \times 0.0400$

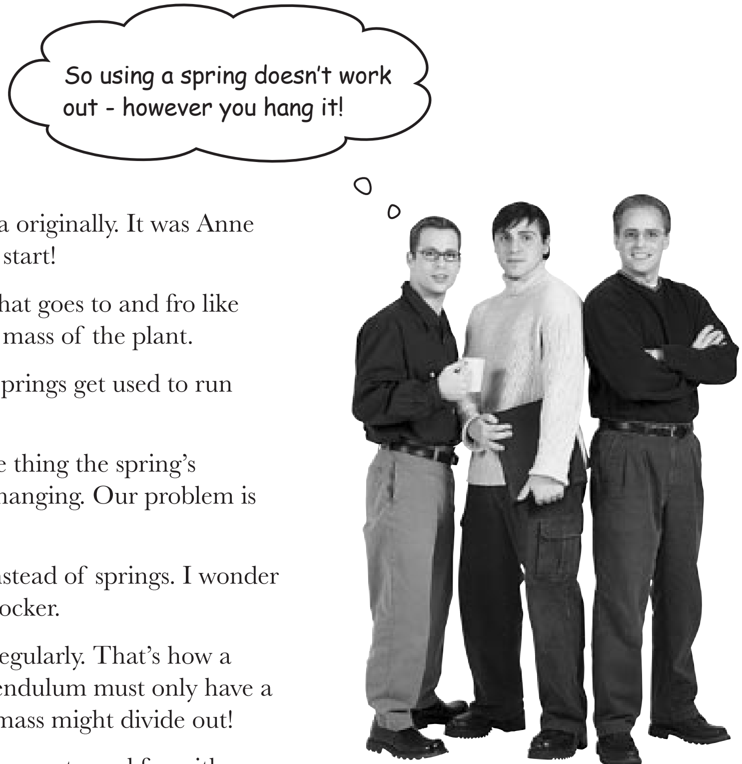
$F_{net} = -0.888$ N (3 sd)

The small difference between these two values is because of rounding. They're basically the same net force.

e. Do you think that the spring being vertical will affect the frequency and period of the plant's oscillations compared with the horizontal spring? Why / why not?

When you pull the plant the same distance from its equlibrium, the force on it is the same. All that's changed by hanging it vertically is the equilibrium position.

I think the frequency will be the same as for the horizontal spring, as the restoring force is unchanged.



Jim: But at least the spring wasn't our idea originally. It was Anne who suggested we use a spring right at the start!

Joe: So we gotta think of something else that goes to and fro like clockwork, but that doesn't depend on the mass of the plant.

Frank: Like clockwork you say, hmmm. Springs get used to run some watches and clocks, don't they?

Jim: Yes, but in the clocks, the mass of the thing the spring's attached to inside the clock doesn't keep changing. Our problem is that the mass of the plant does change.

Frank: But some clocks use pendulums instead of springs. I wonder if we can use a **pendulum** for the plant rocker.

Joe: Yeah ... a pendulum goes to and fro regularly. That's how a Grandfather clock works, isn't it? And a pendulum must only have a **gravitational force** acting on it, so the mass might divide out!

Jim: But the Grandfather clock pendulum goes to and fro with a period of 1 s (or 2 s if it's one tick at each end of the swing - I'm not sure!). That's too short! We need the plant rocker to have a frequency of 0.750 Hz, which we already said is a period of 1.33 s.

Frank: Maybe giving a pendulum plant rocker a larger **amplitude** by pulling it back further at the start will change the period. The plant will have more distance to cover for each swing.

Joe: And maybe we could change the **length** of the pendulum - the distance from the ceiling to the plant. That might affect the frequency and period too.

Jim: And we're still not sure if the frequency and period of the pendulum depend on the **mass** of the plant. Though gravity is the only force acting on the plant (apart from the tension in the string it's attached to) so it could be more promising than the spring.

Frank: Yeah ... I'm just trying to imagine whether an adult will swing slower or faster or just the same as a child if they're sitting on a swing. I'm not sure.

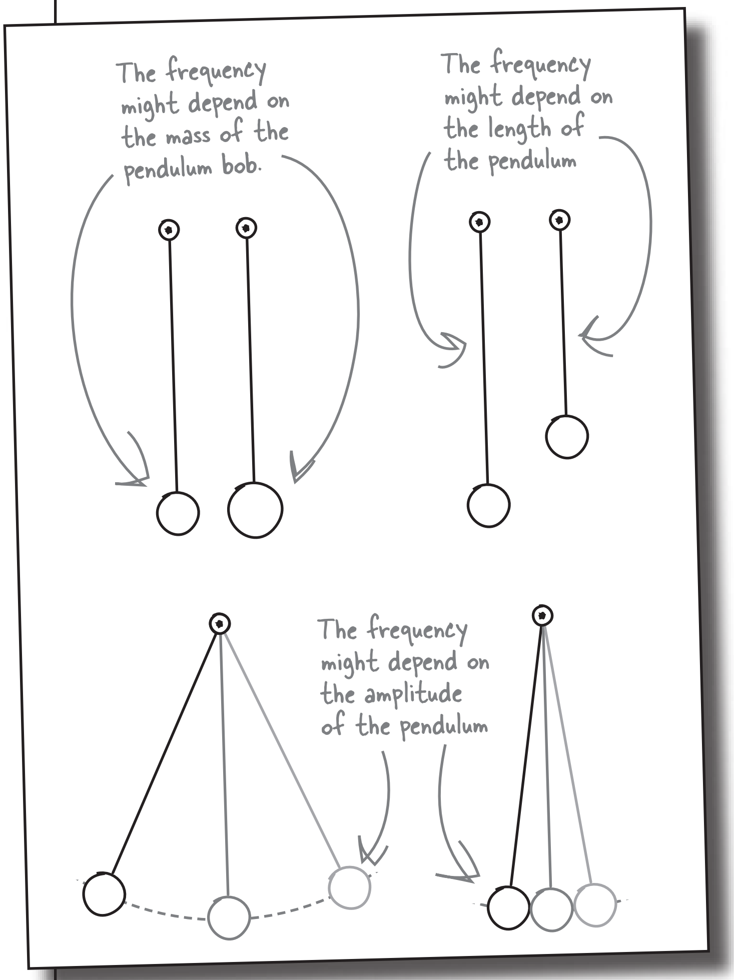
Joe: I guess we ought to do an **experiment** to work out whether the **mass**, **length** or **amplitude** affect the period of the pendulum - and if so, how they affect them.

A pendulum swings to and fro with a regular frequency and period.

Try it!

Your job is to work out which variables (mass of pendulum bob, length of string, amplitude of swing) affect the frequency and period of a pendulum. This is a completely open-ended investigation - you can go about it however you like, designing and doing your own experiments, drawing your own graphs and writing up your conclusions.

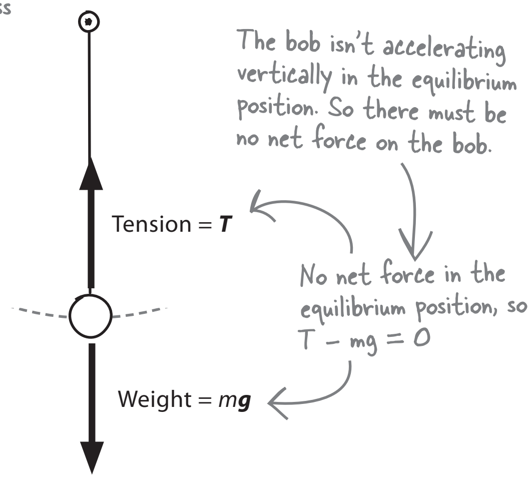
There'll be a competition page on the Head First Physics website where you can submit your write-up, with prizes for the best entries.

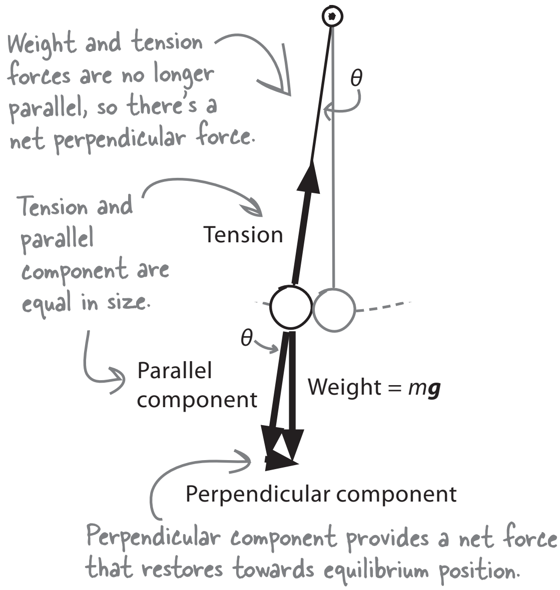


(for angles less than 10°.)

A pendulum swings with simple harmonic motion

A pendulum's **equilibrium** position is when the bob is hanging straight down. When the bob hangs straight down, its weight (due to gravitational force) and the tension in the string are both vertical. As the bob doesn't accelerate vertically, there must be no net force on the bob.





If the bob is pulled back a short displacement, through the angle θ, the bob's weight vector is no longer parallel to the tension in the string. A component of the bob's weight is **perpendicular** to the string, and provides a **net force** that causes the bob to **accelerate** towards the equilibrium position. As the bob nears equilibirum, the angle becomes smaller and the net force also becomes smaller.

Through the equilibrium position, the net force is zero, so the bob continues with a constant velocity. And as the bob swings the other way, the net force becomes larger again, slowing the bob down until it reaches the top of its swing on the other side.

Just like the mass on the spring, the **net force is proportional to the displacement** of the mass from the equilibrium position (as long as the angle it's moved through is less than around 10°). This satisfies the requirement for **simple harmonic motion**, The equation for the displacement of a pendulum is given to the right.

g is the acceleration due to gravity.

Ready Bake Equation

Pendulum starts at $x = x_0$:

$$x = x_0 \cos\left(\sqrt{\frac{g}{l}}\, t\right)$$

$$x = x_0 \cos(\boldsymbol{\omega} t)$$

You can compare this equation with the 'standard' SHM equation to work out what you want to know.

x_0 is the maximum value of x, and therefore the amplitude of the motion.

l (the letter 'l', not the number '1') is the length of the pendulum

What does the frequency of a pendulum depend on?

The equation for the **displacement of a simple pendulum** (a pendulum that moves with SHM) that starts at a maximum is $\boldsymbol{x} = \boldsymbol{x}_0 \cos\left(\sqrt{\frac{g}{l}}\, t\right)$. Like the equation for the period of the mass on a spring, this equation is derived using calculus, but is provided here as a ready-bake equation.

Your experiment and the ready bake equation both tell you the same thing: the period of the pendulum depends on the **length** of the string that attaches the bob to the celing, but not on the mass or the amplitude.

As well as this, the equation says that the period of the pendulum depends on the **acceleration** due to gravity. Practically speaking, this isn't something you have to worry about with the plant rocker, but it does mean that your favorite Grandfather clock won't keep time on the moon!

The frequency and period of a pendulum depend on its length, but not on its mass.

Sharpen your pencil

Hint: Use the equations on the post-its on page 807.

It's time to work out the length of the pendulum you'll need for the plant rocker.

a. Use the ready-bake equation for the plant's displacement, $\boldsymbol{x} = \boldsymbol{x}_0 \cos\left(\sqrt{\frac{g}{l}}\, t\right)$, to get an equation for the frequency of the plant rocker.

Hint: Compare the equation with the "standard" equation for SHM.

b. Calculate the length of pendulum you require for the plant rocker to have a frequency of 0.750 Hz.

c. Use the equation you worked out in part a. to explain what will happen to the period of the plant rocker if you double the length of the pendulum.

d. If you took the pendulum to the moon, where acceleration due to gravity is a sixth of its value on earth, what effect would this have on the period of the pendulum?

Sharpen your pencil Solution

It's time to work out the length of the pendulum you'll need for the plant rocker.

a. Use the ready-bake equation for the plant's displacement, $x = x_0 \cos\left(\sqrt{\frac{g}{l}}\,t\right)$, to get an equation for the frequency of the plant rocker.

'Standard' equation: $x = A\cos(\omega t)$

Simple pendulum: $x = x_0 \cos\left(\sqrt{\frac{g}{l}}\,t\right)$

$$\omega = \sqrt{\frac{g}{l}} \Rightarrow 2\pi f = \sqrt{\frac{g}{l}}$$

$$f = \frac{1}{2\pi}\sqrt{\frac{g}{l}}$$

b. Calculate the length of pendulum you require for the plant rocker to have a frequency of 0.750 Hz.

$$f = \frac{1}{2\pi}\sqrt{\frac{g}{l}}$$

$$\Rightarrow f^2 = \frac{1}{4\pi^2}\cdot\frac{g}{l}$$

$$\Rightarrow l = \frac{1}{4\pi^2}\cdot\frac{g}{f^2} = \frac{1}{4\pi^2}\cdot\frac{9.8}{0.750^2}$$

$$l = 0.442 \text{ m (3 sd)}$$

c. Use the equation you worked out in part a. to explain what will happen to the period of the plant rocker if you double the length of the pendulum.

$$T = \frac{1}{f} \quad \text{and} \quad f = \frac{1}{2\pi}\sqrt{\frac{g}{l}}$$

$$\Rightarrow T = 2\pi\sqrt{\frac{l}{g}}$$

If you double the length of the pendulum, the part under the square root becomes twice as large as it was before., so T becomes $\sqrt{2}$ larger than it was before.

d. If you took the pendulum to the moon, where acceleration due to gravity is a sixth of its value on earth, what effect would this have on the period of the pendulum?

$$T = 2\pi\sqrt{\frac{l}{g}}$$

If you make g a sixth of its value, the part under the square root becomes six times larger (since g is on the bottom). So T becomes $\sqrt{6}$ larger than it was before.

It's really important to be able to move between f, T and quickly and comfortably - so practise doing that!

Angular frequency and angular speed

Angular frequency and angular speed both have the same size and both have units of radians per second. They're exactly the same thing,

You can get from the frequency, f, to the angular frequency, ω, with the equation:

$$\omega = 2\pi f$$

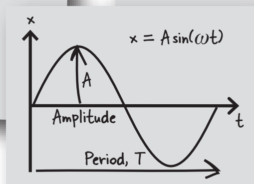
Frequency and period

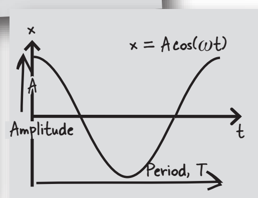
Frequency, f, is cycles per second.

Period, T, is seconds per cycle.

Because they're related like this:

$$T = \frac{1}{f} \qquad f = \frac{1}{T}$$

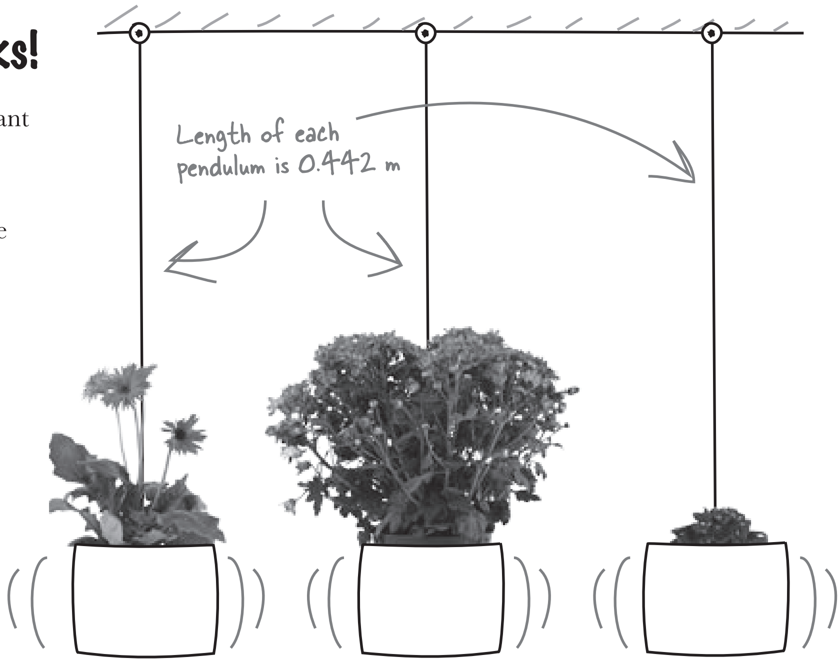




The pendulum design works!

You use your answers to make a pendulum plant rocker for Anne - and it works perfectly.

Even better, the frequency doesn't depend on the mass of the plant, so you can use the same design for all three of Anne's favorites!



BULLET POINTS

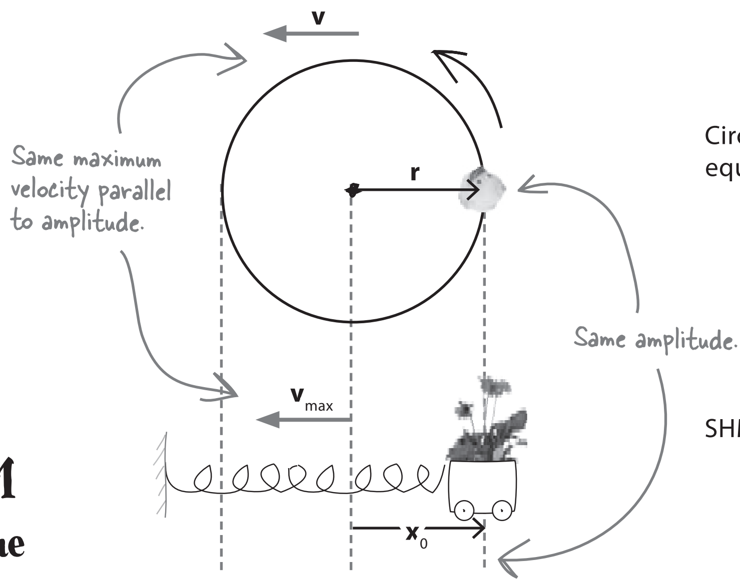
- If the restoring force is proportional to the displacement, you have simple harmonic motion (abbreviated to SHM)
- SHM looks like circular motion from side on, and the equations for the displacement, velocity and acceleration are all sinusoidal (shaped like a sine or cosine graph).
- For a spring, the period depends on the mass and the spring constant, but not the amplitude.
- For a pendulum with small amplitudes, the period depends on the length and the gravitational field strength, but not on the mass.
- It's fine to use forces to analyse SHM - but you reach a point where you require calculus. So use energy to solve SHM problems where you can.
- The kinetic energy in the equilibrium position (where the force and displacement are zero) is equal to the potential energy at an extreme.

You can use this to solve problems with pendulums as well as with springs.

I was just wondering... if SHM is like side-on circular motion, can we use something like $v = r\omega$ to work out the maximum speed instead of using energy conservation?

Yes, you can apply your circular motion equations here too!

Simple harmonic motion looks like side-on circular motion. This means that if you have SHM and circular motion that both have the same amplitude and frequency, the maximum speed of the SHM will be the same as the speed of the circular motion.



Circular motion equation:

$v = r\omega$

SHM equation:

$v_{max} = x_0\omega$

The MAXIMUM speed is the same for circular motion and SHM if they both have the same amplitude and frequency.

Thuis means that you can adapt the equation $v = r\omega$ to give you the maximum speed of the SHM.

The equation would become $v_{max} = x_0\omega$

This equation has angular frequency in it – just use $\omega = 2\pi f$ to calculate it.

vmax depends on the amplitude and the frequency.

Question Clinic: The "Vertical spring" Question

The vertical spring question is a common way of testing your understanding of simple harmonic motion. First of all, something is hung from the spring to stretch it to a new equilibrium position. Then the spring is stretched or compressed further and released. The question asks you to work out the frequency or period of the oscillations

If you see a spring, think 'simple harmonic motion'.

This wording indicates that the spring is vertical, so you need to take the object's weight into account.

If the spring extends, it must be exerting a force as $F = -kx$.

So the force from the spring should be on your free body diagram.

54. A spring hangs downwards from the ceiling.

a. When you attach a plant to the spring, it extends until the plant is at rest. Draw a free body diagram showing all of the forces acting on the plant at this point.

b. If the plant has a mass of 0.100 kg and the spring extends by 44.1 cm, what is the spring constant, *k*?

c. You pull the plant down a further 4.00 cm and let go. Calculate the period of the plant's oscillations.

d. What distance does the plant cover in one complete cycle?

This indicates that the plant is in equilibrium, with no net force acting on it.

The forces should add to zero.

Use the diagram you drew in part a. with these values.

Note the word distance, and the complete cycle. It'll travel 4.00 cm to the center, another 4.00 cm to the far side, then the same again: a total of 16.00 cm.

Make sure you use the displacement from the NEW equilibrium position when you do the SHM calculation.

The key thing in this question is defining the displacement correctly. The initial displacement creates a new equlibrium position, and also allows you to calculate the spring constant. When you do the SHM calculation, you should redefine the new equilibrium position as x = 0 and use the value for the spring constant that you calculated earltier to work out the frequency or period.

Question Clinic: The "How does this depend on that" Question

Many questions have no numbers in them at all, and are designed to test how well you understand the physics, and the language of equations that can be used to describe it. You're told what the original situation is, then asked what would happen if a variable became larger or smaller, for example by doubling or halving.

This question doesn't contain any numbers, only letters to represent each quantity.

55. A mass, *m*, on a spring oscillates with amplitude *A* and period *T*. Starting from this set-up each time:

a. What happens to the period if you double the amplitude to 2*A*?

b. What happens to the total energy of the oscillator if you double the amplitude to 2*A*?

c. What happens to the maximum speed of the mass if you double the amplitude to 2*A*?

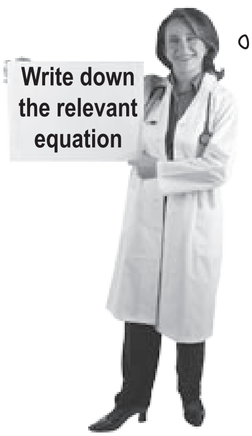
d. What happens to the period if you double the mass to 2*m*?

a. You can do this part without an equation – the period of SHM doesn't depend on its amplitude.

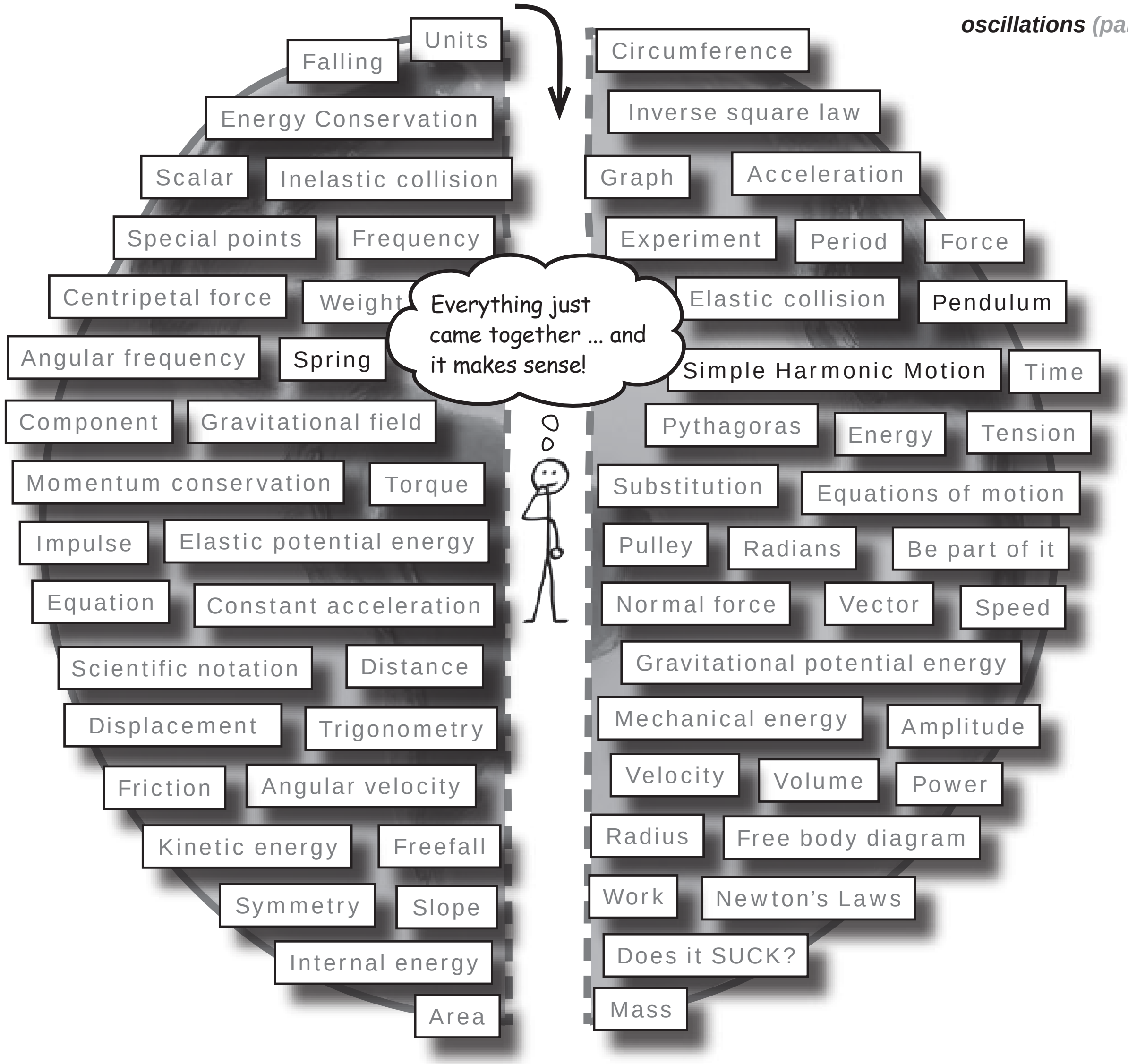
b. You need the equation for the total energy, $E = \frac{1}{2}kA^2$ As the 'A' is squared, if you double A, E becomes four times as large.

c. You need the equation $\frac{1}{2}mv^2 = \frac{1}{2}kA^2$ to see that if you double A, then v will also double.

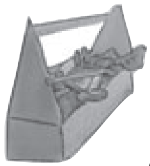
d. You need the equation $T = 2\pi\sqrt{\frac{m}{k}}$ to see that the period becomes $\sqrt{2}T$.



Make sure you write down the relevant equation so you can see how changing one variable will affect the other variables. Then 'insert' the factor the variable has changed by next to it (for example, m might become 2m) and look to see how this affects the size of the other variable you're asked about. If your equation has m^2 in it, then inserting '2m' instead of 'm' gives you $(2m)^2$, or $4m^2$, so the answer will be four times larger than it was before. Try it with the question above!



☑	Simple harmonic motion	Motion where a restoring force to an equilibrium position is directly proportional to the displacement from the equilibrium position.
☑	Spring	A spring exerts a restoring force proportional to the displacement from its equilibrium position.
☑	Pendulum	For small angles, something swinging undergoes a restoring force proportional to the displacement from its equilibrium position.



Your Physics Toolbox

You've got Chapter 20 under your belt and you've added some problem-solving concepts to your toolbox.

Hooke's Law

Hooke's Law says that the force a spring exerts is proportional to its displacement from equilibrium, and in the opposite direction to the displacement.

$$F = -kx$$

(k is the spring constant)

Simple harmonic motion

Oscillation you get when the restoring force is directly proportional to the displacement from the equilibrium position.

Abbreviated to SHM.

SHM graphs

Displacement, velocity and acceleration–time graphs are all sinusoidal.

Start by drawing the displacement–time graph, starting at x_0.

Acceleration–time graph exactly mirrors displacement–time graph.

Get velocity – time graph from slope of displacement–time graph

Mass on a spring

Frequency and period depend on the mass and the spring constant.

Frequency and period are independent of the amplitude.

Frequency and period independent of the gravitational field strength (horizontal and vertical springs have the same frequency).

Simple pendulum

Moves with SHM for small angles only (θ less than around 10°).

Frequency and period depend on the length of the pendulum and the gravitational field strength.

Frequency and period are independent of mass.

Frequency and period are independent of amplitude for small angles.

Comparing equations

If your equation has the same form as a 'standard' equation, you can write them one above each other and say that terms are equivalent.

This is how you can get the equation for the frequency of an oscillator – by comparing its equation with the 'standard' equation that describes SHM.

ω is your FRIEND!

The most simple form of the SHM equation uses ω.

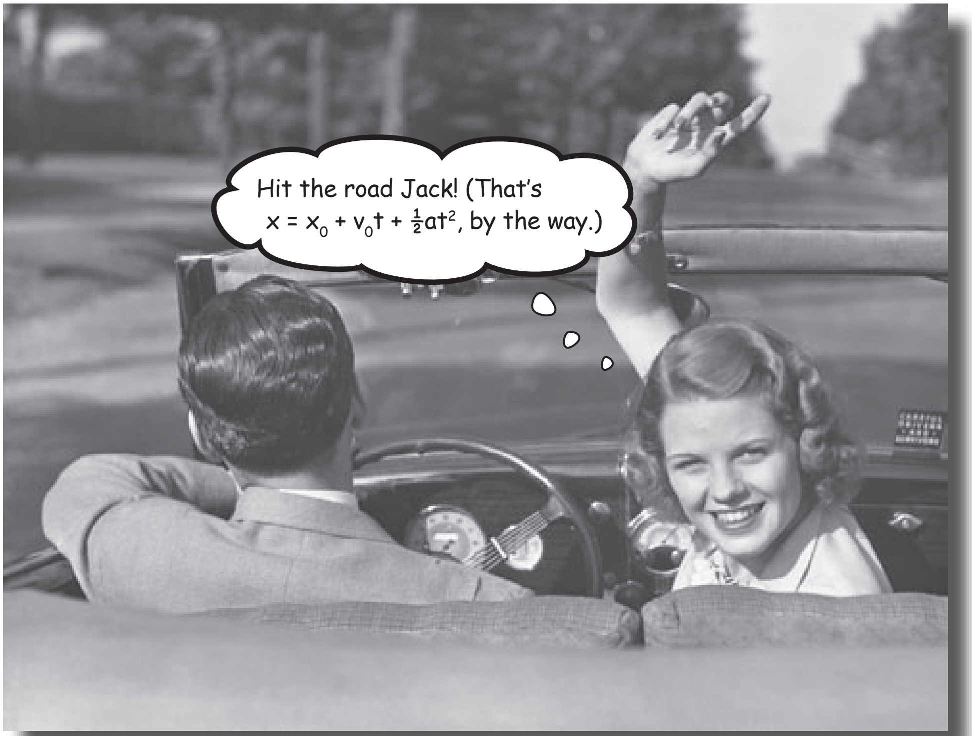
ω is the "link" between simple harmonic morion and circular motion.

Circular motion and SHM

Try thinking of SHM in terms of circular motion to see which equations you can adapt or reuse.

21 think like a physicist

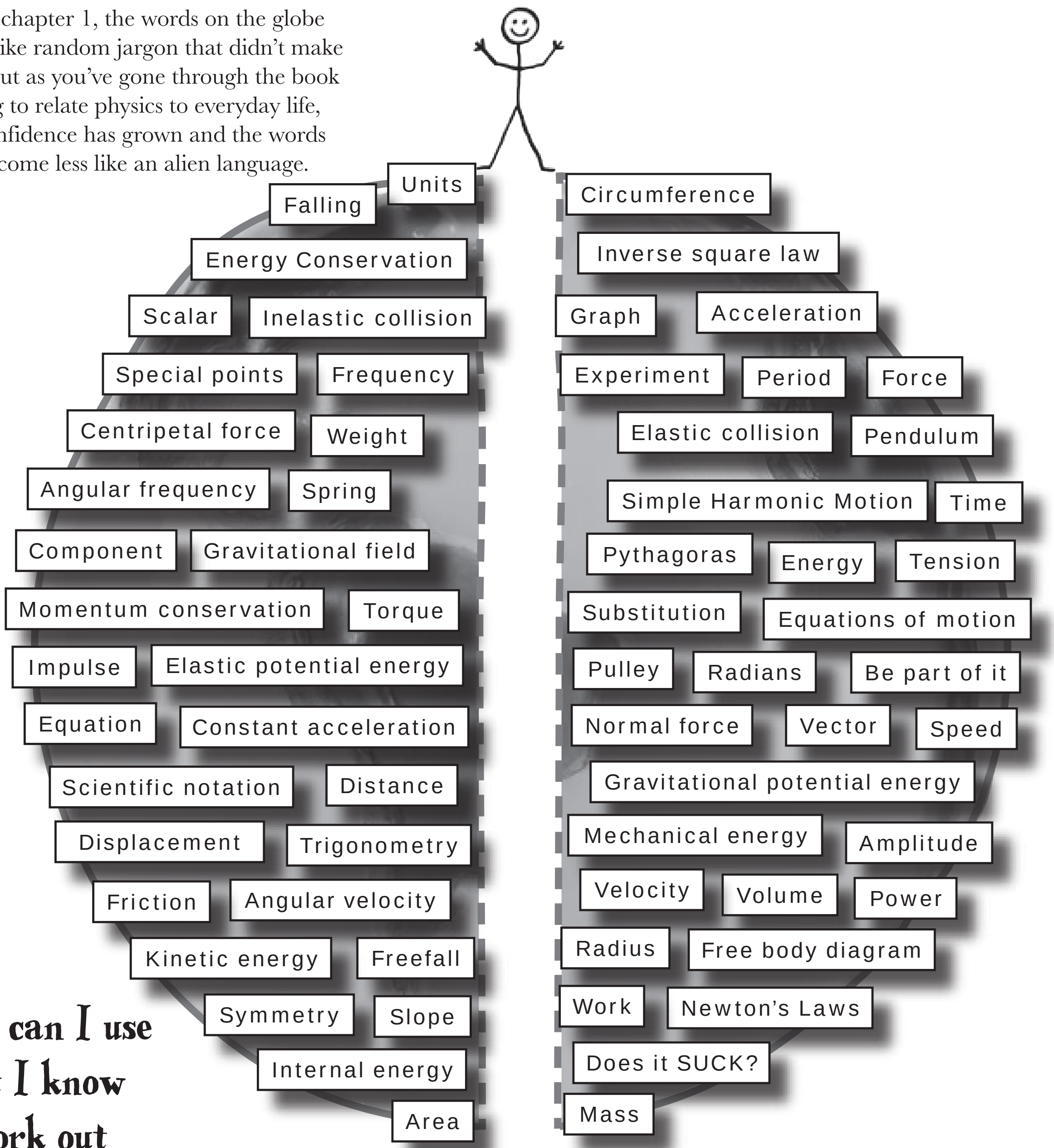
It's the final chapter



It's time to hit the ground running. Throughout this book, you've been learning to **relate** physics to **everyday life** and have absorbed **problem solving** skills along the way. In this final chapter, you'll use your new set of **physics tools** to dig into the problem we started off with - the bottomless pit through the center of the Earth. The key is the question: "How can I use what I know to work out what I don't know (yet)?"

You've come a long way!

Back in chapter 1, the words on the globe looked like random jargon that didn't make sense. But as you've gone through the book learning to relate physics to everyday life, your confidence has grown and the words have become less like an alien language.



"How can I use what I know to work out what I don't know (yet)?"

Now you're in chapter 21 - and you're able to use these same words to help you think through and **solve problems**. You've learned to ask "How can I use what I know to work out what I don't know (yet)?"

Now you can finish off the globe

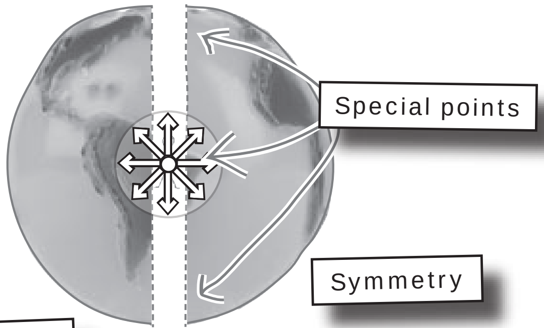
What better way to use your physics superpowers than to revisit the tunnel through the center of the Earth and really get to grips with what happens there.

Back in chapter 1, you learned to **be part of it** by putting yourself at the heart of the problem and asking "What happens next?" and "What's it like?" Now you can also describe what happens using physics terms and concepts.

Be part of it

You can use physics terms and concepts to describe what happens next.

You spotted that there's a **special point** in the center where there's no net force on you because everything is symmetrical - you're equally attracted in all directions, and as gravity is a non-contact force, you don't feel crushed. You also realized that you're always attracted towards the center unless you're already in the center.



Acceleration

Force

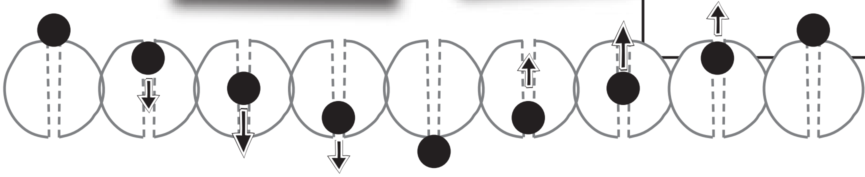
This means that you initially **accelerate** as you fall due to the **net force** on you at the top of the tunnel, briefly move with a **constant velocity** through the center, then decelerate as the gravitational **force** continues to attract you towards the center. After briefly emerging at the other end, you do the same thing again in reverse.

Newton's 2nd Law

Newton's 1st Law

Newton's Laws

Velocity



Sharpen your pencil

a. Use your increased physics knowledge to revisit the question "What's it like?" What does the trip through the Earth **now** remind you of? Be sure to mention all the parallels you can see.

b. Are there any requirements that need to be met in order for your answer to part a to make sense?

Sharpen your pencil Solution

a. Use your increased physics knowledge to revisit the question "What's it like?" What does the trip through the Earth **now** remind you of? Be sure to mention all the parallels you can see.

It looks like simple harmonic motion.

There's an equilibrium point in the center where there's no net force on you.

The force on you always acts towards the equilibrium position in the center, so is in the opposite direction from the displacement.

You move slowly at the edges and quickly through the center.

b. Are there any requirements that need to be met in order for your answer to part a to make sense?

The restoring force would have to be proportional to your displacement from the equilibrium position, and in the opposite direction from the displacement.

That's the requirement for simple harmonic motion.

Displacement

Force

Simple Harmonic Motion

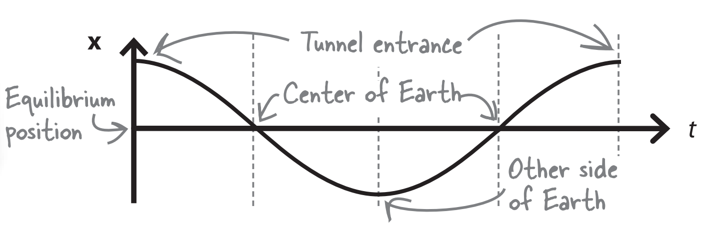
The round-trip looks like simple harmonic motion

Way back in chapter 1, you figured out that you'd fall into the tunnel, travel through the Earth, and appear at the other end of the tunnel. Then you'd fall back in again, go through the tunnel in the opposite direction, and end up where you started... and so on.

That sounds a lot like **simple harmonic motion**, something you learned about in chapter 20. Now that you know how to tackle simple harmonic motion problems, we can add a lot of detail to what we figured out before.

What's it LIKE?

If you come across a situation you haven't seen before, think about whether you've seen something similar in the past by asking yourself: "What's it like?"



But what time does the round-trip take?

A trip through the center of the Earth can be pretty tiring. Suppose you want to place a pizza order so that once you've gone through the Earth and come back again, you can have a nice snack.

Break Neck Pizza promises delivery in 45 minutes... but will you be able to get through the Earth and then back again in time to meet Alex the delivery guy?

What **time** does your journey through the Earth take?

First of all, you'll have to work out if your journey through the Earth IS simple harmonic motion. The journey might only **look** like simple harmonic motion without actually **being** simple harmonic motion.

Then, if your journey is simple harmonic motion, you can use the equations you worked out in chapter 20 to calculate the time that the round-trip takes - and whether you'll be back in time for pizza.

Back in time for pizza?

- Trip through Earth looks like SHM.
- Is it SHM? Is the restoring force proportional to the displacement?
- If it's SHM, can use SHM equations to calculate time that trip takes.

So that's that! Going through the Earth and back again is simple harmonic motion.

simple harmonic motion

The net force on the object that always points in the direction of the equilibrium position.

Joe: Hang on. It's only SHM if the **restoring force** is directly **proportional** to the **displacement** from the equilibrium position of the object that's moving to and fro. We don't know whether the restoring force follows that pattern or not yet.

Frank: Yeah, if the trip through the Earth isn't SHM, we can't use our SHM equations to calculate the **time** it takes.

Joe: The gravitational force is an **inverse square law**, isn't it? $\mathbf{F}_G = -\frac{Gm_1m_2}{\mathbf{r}^2}$. So if you double the displacement, the force is only a quarter of what it was before. The force isn't directly proportional to the displacement. The force gets smaller as the displacement gets larger!

Frank: Oh yeah. For SHM, the restoring force needs to get larger as the displacement gets larger.

Jim: Hey - didn't we say before that the force is zero in the center of the Earth?! If the equation $\mathbf{F}_G = -\frac{Gm_1m_2}{\mathbf{r}^2}$ works at the center of the Earth (where $\mathbf{r} = 0$), you're dividing by zero. If you divide by a very small number, you get a very large answer. And if you divide by zero, you get an answer of infinity! Computer says no!

Joe: Yes ... maybe $\mathbf{F}_G = -\frac{Gm_1m_2}{\mathbf{r}^2}$ only works when you're **outside** the Earth. When you're outside, all of the Earth is below you.

Frank: But when you're inside the tunnel, some of the Earth is **below** you and attracts you downwards. The rest of the Earth is **above** you and attracts you upwards.

Jim: So the net force on you in the center is zero, as there are equal masses of Earth above and below you. And the net force on you somewhere else in the tunnel depends on how much Earth is above you and how much Earth is below you.

Joe: Looks like we need to work out a different equation for when you're inside the Earth then, if $\mathbf{F}_G = -\frac{Gm_1m_2}{\mathbf{r}^2}$ isn't going to work.

Anytime you want to use an equation, think about the CONTEXT. Is it OK to use the equation here?

You can only use your simple harmonic motion equations from chapter 20 if the restoring force is proportional to the displacement.

BRAIN POWER

Calculating the net force on you when you're **inside** the Earth is a complicated problem. How might you break the problem down into smaller parts?

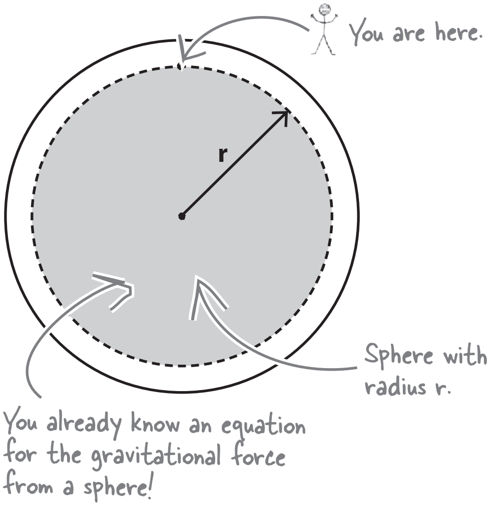
You can treat the Earth like a sphere and a shell

The gravitational force between two spheres is $\boldsymbol{F}_G = -\frac{Gm_1m_2}{\mathbf{r}^2}$. You can treat each sphere as if its entire mass was concentrated at a single point in the center of the sphere.

If you treat the human body like a very small sphere, you can use this equation to calculate the **gravitational force** that the Earth exerts on you - as long as you're **outside** the Earth.

But when you're **inside** the tunnel, there's Earth above you and below you. Calculating the gravitational force that the Earth exerts on you when you're inside it is a complicated problem!

Try to break down complicated problems into smaller parts.



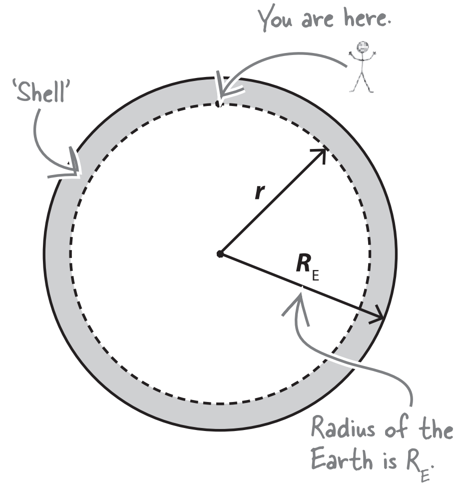
But you can **break down** this problem into two parts by thinking of the Earth in two parts. Anytime you're inside the Earth, you're a certain distance (let's call this distance **r**) from the equilibrium position in the center of the Earth.

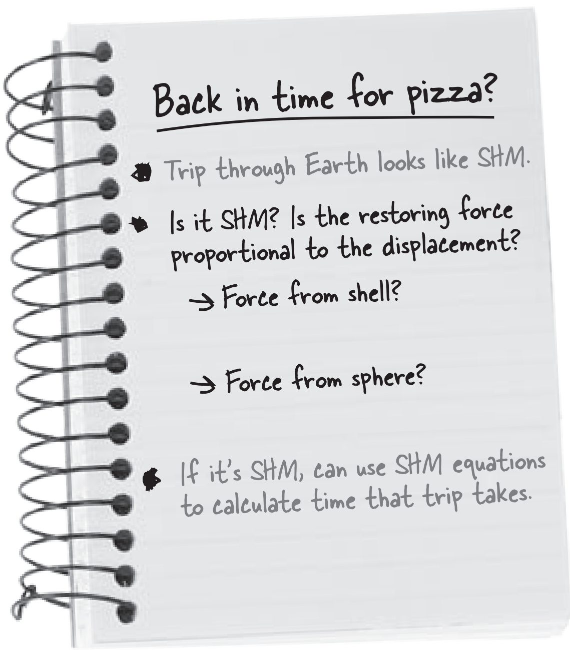
So beneath you, there's a **sphere** with radius **r**. You already know how to calculate the gravitational force exerted on you by a sphere if you know its radius and mass: $\mathbf{F}_G = -\frac{Gm_1m_2}{\mathbf{r}^2}$

We're defining the radius as the displacement away from the center of the Earth. The force acts towards the center of the Earth – hence the minus sign.

The rest of the Earth forms a '**shell**.' Some of the shell is below you and some of the shell is above you.

If you can work out an equation for the force exerted on you by the shell, you can add it to the force exerted on you by the sphere. This gives you the **net force** that the Earth exerts on you while you're inside the tunnel.

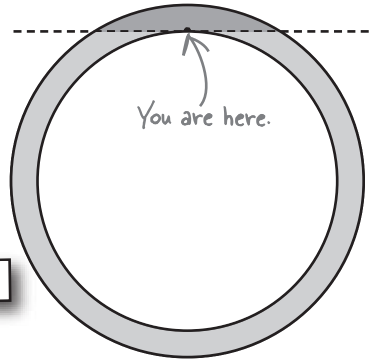




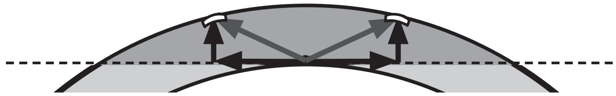
You can deal with the sphere - but what about the shell?

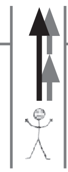
Use the magnets to work out the force from the shell. Annotate the diagrams as you go.

There's more shell ________ you than there is ________ you, so there's more ________ ________ you than there is ________ you. But the shell ________ you is (on average) a much larger ________ away from you than the shell ________ you.



distance | below | mass | above





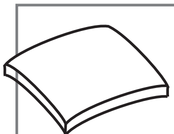
below | left | upwards | above | add to zero | right

The shell above you is totally ________ . So for every small piece of Earth to the ________ of you there's an equivalent piece to the ________ .

The ________ components of the gravitational force from these two pieces of Earth are equal but in ________ directions, so they ________ .

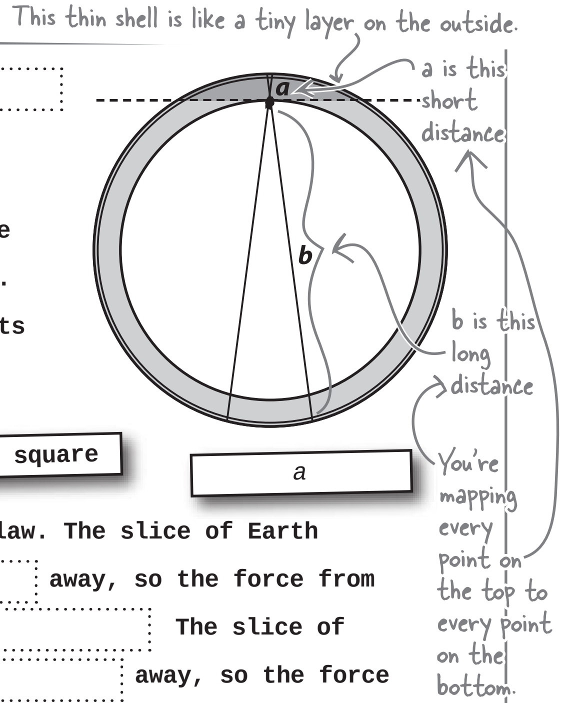
The ________ components add together, so the net force on you from the shell above you is ________ . The same argument applies to the shell below you, which exerts a net ________ force on you once the ________ components ________ . So the part of the shell ________ you attracts you ________ and the part ________ you attracts you ________

symmetrical | downwards | opposite | horizontal | vertical



If you take a piece of very thin shell, its ________ will be its ________ multiplied by its ________ . So the ________ of the very thin shell will depend on its surface area.

mass | thickness | surface area | volume



If you take a small slice of thin shell from __________ and the equivalent from __________, you can think of them as being tiny slices from spheres with radius __________ and __________ with surface areas __________ and __________ respectively. And as the __________ of a thin shell depends on its surface area, the masses of the slices are proportional to __________ and __________

Gravitation is an __________ law. The slice of Earth above you is distance __________ away, so the force from 1 kg of it is proportional to __________ The slice of Earth below you is distance __________ away, so the force from 1 kg of it is proportional to __________ Therefore, the force from the slice at distance '*a*' is proportional to __________ × __________ and the force from the slice at distance '*b*' is proportional to __________ × __________

inverse square

a

below

a^2

b^2

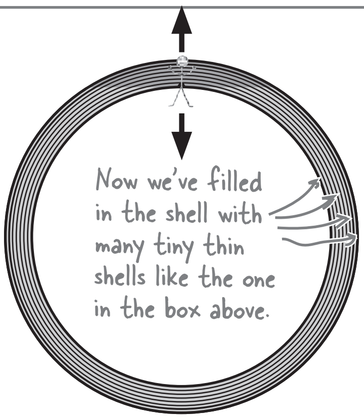
$\frac{1}{a^2}$

b

volume

$\frac{1}{b^2}$

above



So the force from the slice above you is __________ to the force from the slice below you. And the net force from the whole thin shell is __________ If you're further inside the Earth, you can think of the thick shell being made up of many many thin shells. So the net force from the thick shell is also __________

zero

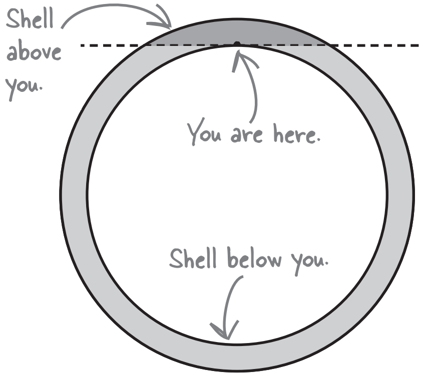
equal

volume

Use this space to sum up what you've discovered: ....

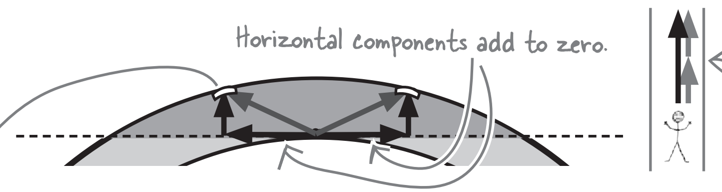
Magnets Solution

There's more shell **below** you than there is **above** you, so there's more **mass below** you than there is **above** you. But the shell **below** you is (on average) a much larger **distance** away from you than the shell **above** you.



In problems that involve SYMMETRY, there are often force components that add to zero.

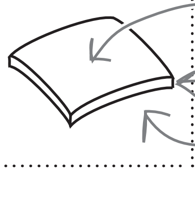
The shell above you is totally **symmetrical**. So for every small piece of Earth to the **left** of you, there's an equivalent piece to the **right**. The **horizontal** components of the gravitational force from these two pieces of Earth are equal but in **opposite** directions, so they **add to zero**.



The **vertical** components add together, so the net force on you from the shell above you is **upwards**. The same argument applies to the shell below you, which exerts a net **downwards** force on you once the **horizontal** components **add to zero**. So the part of the shell **above** you attracts you **upwards,** and the part **below** you attracts you **downwards**.

Vertical components add to produce a net force.

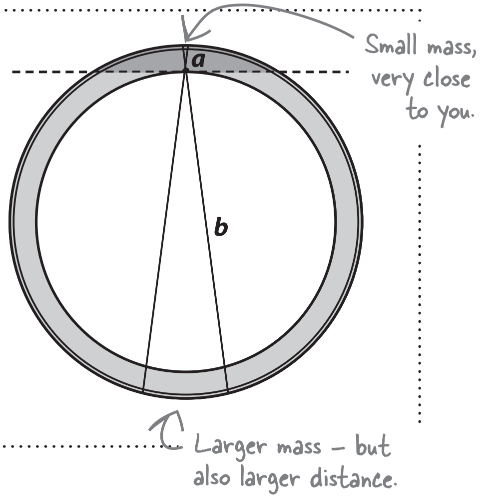
If you take a piece of very thin shell, its **volume** will be its **surface area** multiplied by its **thickness**. So the **mass** of the very thin shell will depend on its surface area.

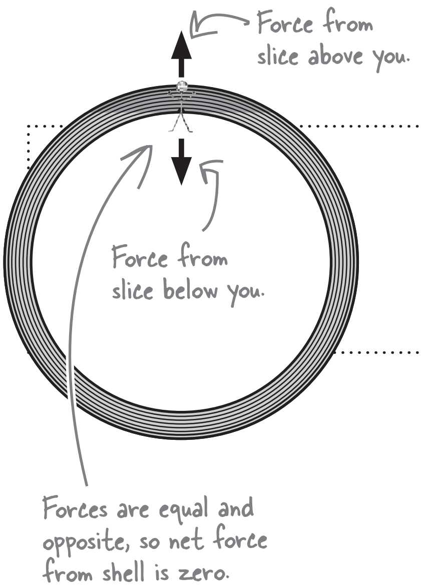


Here, we're using bold to show you where the magnets went. The distances a and b are both scalars.

If you take a small slice of thin shell from **above** and the equivalent from **below**, you can think of them as being tiny slices from spheres with radius $\boldsymbol{a}$ and $\boldsymbol{b}$ with surface areas $\boldsymbol{a}^2$ and $\boldsymbol{b}^2$ respectively. And as the mass of a thin shell depends on its surface area, the masses of the slices are proportional to $\boldsymbol{a}^2$ and $\boldsymbol{b}^2$.

Gravitation is an inverse square law. The slice of Earth above you is distance $\boldsymbol{a}$ away, so the force from 1 kg of it is proportional to $\frac{1}{\boldsymbol{a}^2}$. The slice of Earth below you is distance $\boldsymbol{b}$ away, so the force from 1 kg of it is proportional to $\frac{1}{\boldsymbol{b}^2}$. Therefore, the force from the slice at distance '*a*' is proportional to $\boldsymbol{a}^2 \times \frac{1}{\boldsymbol{a}^2}$ and the force from the slice at distance '*b*' is proportional to $\boldsymbol{b}^2 \times \frac{1}{\boldsymbol{b}^2}$.





So the force from the slice above you is **equal** to the force from the slice below you. And the net force from the whole thin shell is **zero**. If you're further inside the Earth, you can think of the thick shell being made up of many many thin shells. So the net force from the thick shell is also **zero**.

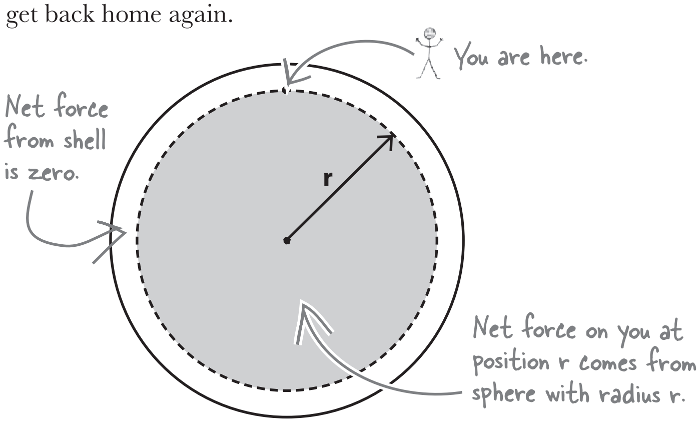
Use this space to sum up what you've discovered: The net force from the shell is zero. This is because the forces from the small, close mass above you and the large, far away mass below you are the same size, but in opposite directions.

The net force from the shell is zero!

The net force from the shell is zero

This means that the net force that the shell exerts on you is zero. So the net force exerted on you when you're inside the tunnel must come entirely from the sphere, radius r.

If that force is proportional to r, then you'll move through the Earth with simple harmonic motion and can use the equations you already know to calculate the time you take to get back home again.



there are no Dumb Questions

Q: Do I need to understand and reproduce all of that?!

A: Don't worry - you won't be asked to do something that difficult in an exam. The big take-away is that the net force from the shell is zero because the forces from a small mass close by and a large mass far away added to zero. If you got that, you're great!

Q: But the Earth isn't a sphere and a shell ... is it?!

A: Treating the Earth like a sphere and a shell is a **mathematical tool**. In the same way, moving objects don't have velocity vector arrows and components drawn on them in real life, but vector arrows are very useful tools in physics.

Q: What's so special about a shell?

A: You've worked out that the force the shell exerts on you is zero. So the net force on you must come entirely from the sphere.

Q: What's so special about a sphere?

A: You already know how to calculate the gravitational force an object experiences as a result of being outside a sphere.

Q: Doesn't the equation for the gravitational force exerted by a sphere only work if I'm outside the sphere?

A: When your displacement is r from the center of the Earth, then you're outside the sphere with radius **r**.

Q: Why choose that particular radius, *r*, as the place to draw the boundary between the sphere and shell?

A: **r** is your displacement from the equilibrium position in the center of the Earth. When we did SHM in chapter 18, we called this displacement **x**. Here it's better to use **r** so that you remember that the displacement from the equilibrium position is also a **radius**.

If, when your displacement is r, the force exerted on you by the Earth is proportional to **r**, then your trip through the Earth is SHM. The period of the SHM is the same as the time it takes you to get back to where you started - the time you want to calculate!

Sharpen your pencil

Your job is to work out whether you move through the Earth with simple harmonic motion. For it to be simple harmonic motion, the force on you must be proportional to your displacement from the equilibrium position in the center of the Earth.

So is the net force on you proportional to **r** or not?

Ready Bake Equation

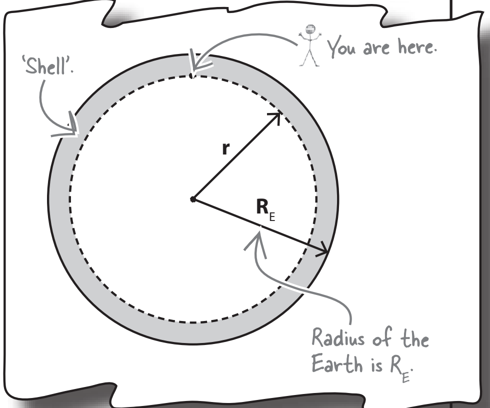
Volume of a sphere:

$$V = \frac{4}{3}\pi r^3$$

a. Use the ready bake equation for the volume of a sphere to write down equations for V_E , the volume of the Earth, and V_s , the volume of the sphere inside the Earth, radius **r**. (Use the symbol $\mathbf{R}_E$ for the radius of the Earth.)

b. Use the fact that the small sphere, radius **r**, is part of the Earth to work out an equation for the mass of the sphere. (Use the symbols M_E and m_s for the mass of the Earth and sphere, respectively.)

Hint: The mass of each sphere is proportional to its volume.



c. Use the mass of the small sphere from part b to work out an equation for the gravitational force, $\mathbf{F}_G$, that the small sphere of radius **r** exerts on you (your mass is *m*). Is $\mathbf{F}_G$ proportional to **r**?

Hint: Think about which quantities in your equation are variables and which are constants.

Sharpen your pencil Solution

Your job is to work out whether you move through the Earth with simple harmonic motion. For it to be simple harmonic motion, the force on you must be proportional to your displacement from the equilibrium position in the center of the Earth.

So is the net force on you proportional to **r** or not?

Ready Bake Equation

Volume of a sphere:

$$V = \frac{4}{3}\pi r^3$$

a. Use the ready bake equation for the volume of a sphere to write down equations for V_E, the volume of the Earth, and V_s, the volume of the sphere inside the Earth, radius **r**. (Use the symbol $\mathbf{R}_E$ for the radius of the Earth.)

Volume of Earth: $V_E = \frac{4}{3}\pi R_E^3$

Volume of sphere: $V_s = \frac{4}{3}\pi r^3$

Volume

b. Use the fact that the small sphere, radius **r**, is part of the Earth to work out an equation for the mass of the sphere. (Use the symbols M_E and m_s for the mass of the Earth and sphere respectively.)

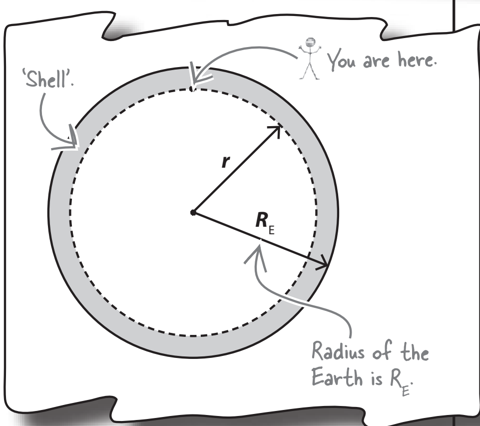
Hint: The mass of each sphere is proportional to its volume.

Mass is proportional to volume.

$$\frac{m_s}{V_s} = \frac{M_E}{V_E} \Rightarrow m_s = \frac{M_E V_s}{V_E}$$

$$\Rightarrow m_s = \frac{M_E \cdot \frac{4}{3}\pi r^3}{\frac{4}{3}\pi R_E^3} = \frac{M_E r^3}{R_E^3}$$

Mass



c. Use the mass of the small sphere from part b to work out an equation for the gravitational force, $\mathbf{F}_G$, that the small sphere of radius **r** exerts on you (you have mass *m*). Is $\mathbf{F}_G$ proportional to **r**?

$$F_G = -\frac{G m_s m}{r^2} = -\frac{G M_E r^3 m}{R_E^3 r^2} = -\frac{G M_E r m}{R_E^3} = -\frac{G M_E m}{R_E^3} r$$

But G, M_E, R_E and m are all constants, so $F_G = -\text{constant} \times r$

F_G is directly proportional to r and in the opposite direction. So it's simple harmonic motion.

There are a lot of letters multiplying or dividing the 'r,' but they're all constants. So the entire circle thing is a constant.

The force is proportional to the displacement, so your trip is SHM

You just worked out that $\mathbf{F}_G$, the force from the sphere, is proportional to $\mathbf{r}$, your displacement from the center of the Earth. As $\mathbf{F}_G$ always points towards the equilibrium position in the center, your trip through the Earth is definitely **simple harmonic motion**!

Displacement

Force

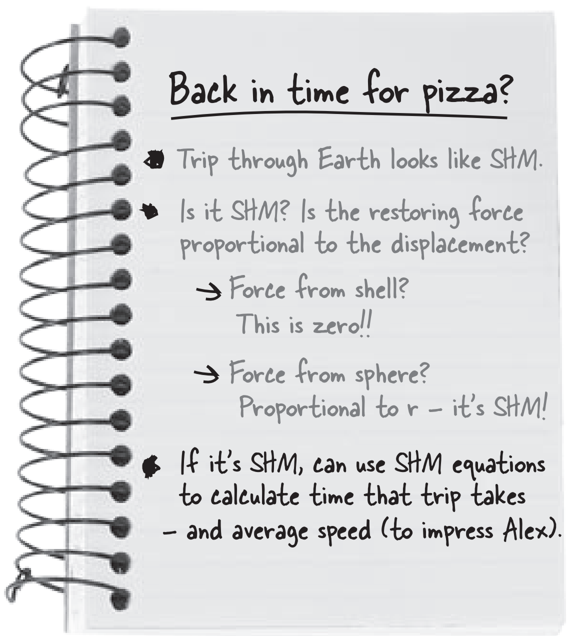
Simple Harmonic Motion

Yeah, but are you gonna arrive before I do? And are you faster than I am?

Speed cycling legend, Alex, wants to know how fast you are as well as the time you take. Let's impress him...

Now you can use what you already know about simple harmonic motion to fill in some of the details. The **period** of the SHM is the time it takes you to go through the Earth and back again.

Alex also wants to know your average speed - which you can calculate as well ...



Time

Speed

Period

You'll do this on the next page

Hint: Use the question clinic to help you answer this question!

Sharpen your pencil

Simple harmonic motion can be described using the equation $\mathbf{a} = -\omega^2\mathbf{x}$, where the symbols have their usual meanings.

a. Use Newton's 2nd Law to rewrite your equation for the force you experience as you pass through the Earth in terms of **a** instead of $\mathbf{F}_G$. Hence compare your equation with the $\mathbf{a} = -\omega^2\mathbf{x}$ form, and rearrange to give an expression for ω.

$$F_G = -\frac{GM_E m r}{R_E^3}$$

This is the equation you worked out on the previous page.

b. The mass of the Earth is 5.97×10^{24} kg, the radius of the Earth is 6.38×10^6 m, and G, the gravitational constant, is 6.67×10^{-11} $m^3/kg.s^2$. Calculate the time (in minutes and seconds) that it would take for you to return to your starting point after stepping into a tunnel that goes through the center of the Earth. Does this take you less than 45 minutes? The time that Alex takes to arrive with your pizza.

c. What is your average speed during your trip through the Earth? What is your average velocity?

Hint: Use the question clinic to help you answer this question!

Question Clinic: The "Equation you've never seen before" Question

Sometimes, a question will present you with an equation you've never ever seen before. But don't just assume you can't do it just because it's unfamiliar. Answering a question like this is sometimes a case of combining the equation you're given with another you already know so that you can solve a problem. And sometimes it's a case of interpreting some other new information you're given in the question as well.

If the equation is unfamiliar, don't panic! Write it down and annotate it with what each symbol represents.

Make sure you look up any symbols you're not familiar with. In an exam, you'll have an equation sheet you can use. Also remember that the same quantity (e.g., displacement) may be represented by more than one symbol (r, x, ... etc)

4. Simple harmonic motion can be described using the equation $a = -\omega^2 x$, where the symbols have their usual meanings.

a. Rewrite your equation for the force you experience as you pass through the Earth using this form, and rearrange it to give an expression for ω.

b. The mass of the Earth is 5.97×10^{24} kg and the radius of the Earth is 6.38×10^6 m. Calculate the time (in minutes and seconds) it would take for you to return to your starting point after stepping into a tunnel that goes through the center of the Earth.

c. What is your average speed during your trip through the Earth? And what is your average velocity?

In this exam version of the question, the value for G isn't given. You'd be expected to look it up in your table of information.

This indicates that you'll need to use an answer from an earlier part of the problem.

This jargon means you'll have to do substituting and rearranging.

If you found part a difficult, look at part b. It gives you some hints about some values you're expected to have in your part a answer. You might be able to work backwards!

Note the difference between speed (which involves distance) and velocity (which involves displacement).

Make sure you use the correct units and the correct start and end points when you do the calculation.

In a question like this, you need to be especially clear about what each variable represents. There are several different letters that are conventionally used to represent length in physics equations depending on the context - x (displacement), r (radius), l (length), and h (height). When you look at equations, think about what each variable means, as you may be able to make a substitution that isn't immediately obvious.

Sharpen your pencil Solution

Simple harmonic motion can be described using the equation $\boldsymbol{a} = -\omega^2\mathbf{x}$, where the symbols have their usual meanings.

a. Use Newton's 2nd Law to rewrite your equation for the force you experience as you pass through the Earth in terms of **a** instead of $\mathbf{F}_G$. Hence compare your equation with the $\mathbf{a} = -\omega^2\mathbf{x}$ form, and rearrange to give an expression for ω.

$$F_G = -\frac{GM_E r m}{R_E^3}$$

This is the equation you worked out on the previous page.

I've been using r as displacement from equilibrium, this equation uses x. I'll continue to use r to be consistent.

$$F_G = -\frac{GM_E r m}{R_E^3} \text{ and } F_G = ma \Rightarrow \cancel{m}a = -\frac{GM_E r \cancel{m}}{R_E^3}$$

This is of the form: $a = -\omega^2 r$ except that $\omega^2 = \frac{GM_E}{R_E^3} \Rightarrow \omega = \sqrt{\frac{GM_E}{R_E^3}}$

b. The mass of the Earth is 5.97×10^{24} kg, the radius of the Earth is 6.38×10^6 m and G, the gravitational constant, is 6.67×10^{-11} $m^3/kg.s^2$. Calculate the time (in minutes and seconds) that it would take for you to return to your starting point after stepping into a tunnel that goes through the center of the Earth. Does this take you less than 45 minutes?

Time it takes is equal to the period, T. Need to get from ω to T.

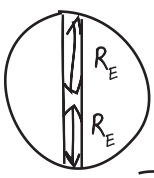
$$\omega = 2\pi f \quad \text{and} \quad f = \frac{1}{T} \Rightarrow \omega = \frac{2\pi}{T}$$

$$\Rightarrow T = \frac{2\pi}{\omega} = \frac{2\pi}{\sqrt{\frac{GM_E}{R_E^3}}} = \frac{2 \times \pi}{\sqrt{\frac{6.67 \times 10^{-11} \times 5.97 \times 10^{24}}{(6.38 \times 10^6)^3}}} = 5070 \text{ s (3 sd)}$$

Need time in minutes and seconds. $5070 \text{ s} = 5070 \cancel{\text{s}} \times \frac{1 \text{ min}}{60 \cancel{\text{s}}} = 84.5 \text{ min} = 84 \text{ min } 30 \text{ s.}$

This is more than 45 minutes, so Alex gets there first.

c. What is your average speed during your trip through the Earth? What is your average velocity?



$$\text{Average speed} = \frac{\text{Total distance}}{\text{Total time}}$$

$$\text{Total distance} = 4R_E$$

$$\Rightarrow \text{Average speed} = \frac{4R_E}{T} = \frac{4 \times 6.38 \times 10^6}{5070}$$

Average speed = 5030 m/s (3 sd)

$$\text{Average velocity} = \frac{\text{Total displacement}}{\text{Total time}}$$

But the total displacement is zero, as you start and finish in the same place.

Average velocity = 0 m/s

You know your average speed - but what's your top speed?

Wow, an average speed of over 5 km per second?! That totally rocks! What's your top speed?

OK, he's impressed!

You just used what you know about simple harmonic motion to work out that your round-trip time is 84 mins 30 s, and your average speed on your trip through the Earth is over 5 km per second!

Even though you won't beat Alex to your house, he is really impressed and wants to know what your **maximum speed** is.

A mass on a spring moves with SHM. When you worked out the maximum speed of a mass on a spring, you used energy conservation. But working out the potential energy of an object inside the Earth is going to be tough ...

You already know one way of calculating the maximum speed. But can you use the same way here?

So we gotta calculate the gravitational potential energy inside the Earth?! Man, that was tough enough from outside!

Joe: Not necessarily - there's something I've been thinking about for the last few pages. When we said, "What's it like?" right at the start, I said I thought it looked like **circular motion** from side on.

Frank: Oh yeah. Circular motion from side on and **simple harmonic motion** both use the **same type of equations**.

Jim: How does that help us?

Joe: I was thinking that if we looked at an orbit from side on, it might look like the trip through the Earth and back. We know how to calculate the **velocity** of an object that's in orbit around the Earth. That velocity will be the same as the maximum velocity when you look at the orbit from side on.

Joe: Yeah, I was thinking that the **amplitude** of an orbit just over the Earth's surface and the amplitude of the trip through the Earth would be the same.

Frank: But what if the orbit and the simple harmonic motion don't have the same **period**?

Jim: We know how to calculate the period of an orbit as well. If the orbit and the SHM have the same period as well as the same amplitude, then they must look identical from side on...

Circular motion from side on looks like simple harmonic motion

If you could orbit the Earth close to its surface, you'd follow a circular path. When you look at an **circular motion** from side on, you only observe one component of the displacement, velocity and acceleration.

Circular motion from side on and simple harmonic motion both use the same type of equation, so their graphs are the same shape.

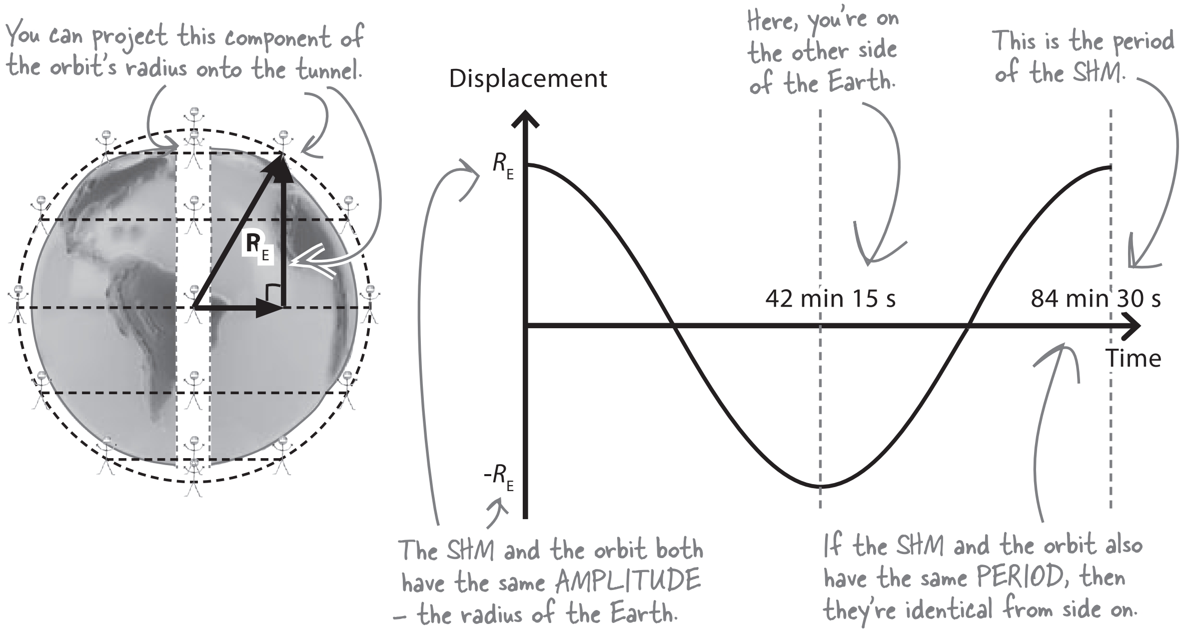
Trigonometry

Component

Angle

Radius

Amplitude



There's often more than one way of looking at a problem.

If the circular orbit has the same **amplitude** and the same **period** as the simple harmonic motion trip through the Earth, then the two journeys will look identical from side on.

This means that the **maximum speed** of the trip through the Earth will be the same as the **linear speed** of the orbit (as long as the periods are the same). And you can already calculate the linear speed of an orbit...

We drew the displacement–time graph up there, but the two velocity–time graphs would also be identical.

Sharpen your pencil

If the orbit has the same period as the SHM, then they look identical from side on.

a. Calculate the period of a circular orbit at the Earth's surface. (The mass of the Earth is 5.97×10^{24} kg, and its radius is 6.38×10^{6} m.) How does this compare to the period of SHM through the center of the Earth?

b. What is the maximum velocity of something (or someone) falling through the Earth?

there are no Dumb Questions

Q: But surely an orbit at the surface of the Earth wouldn't work because of air resistance?

A: That's absolutely right ... but then again, we made some assumptions about oscillating to and fro through the center of the Earth already! Originally, we said that we'd ignore air resistance (which would slow you down) and the Earth's rotation (which would make you hit the sides of the tunnel) - so we can safely ignore them both for the orbit as well!

Q: But how do you know that the period of the orbit and the period of the SHM are the same? They might be different even though the amplitudes are the same.

A: That's right - we don't know that the periods are the same ... yet. But as you've dealt with orbits before, you'll be able to work out whether they are soon enough ...

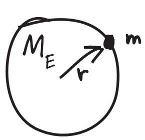
Hint: Think about what the centripetal force is provided by. You can use your equation appendix, if you like!

Sharpen your pencil Solution

If the orbit has the same period as the SHM, then they look identical from side on.

a. Calculate the period of a circular orbit at the Earth's surface. (The mass of the Earth is 5.97×10^{24} kg, and its radius is 6.38×10^{6} m.) How does this compare to the period of SHM through the center of the Earth?

$$F_c = \cancel{m}r\omega^2 = \frac{GM_E\cancel{m}}{r^2}$$



$$\omega = \sqrt{\frac{GM_E}{R^3}}$$

$$\omega = 2\pi f \quad \text{and} \quad f = \frac{1}{T} \Rightarrow \omega = \frac{2\pi}{T}$$

$$\Rightarrow T = \frac{2\pi}{\omega} = \frac{2\pi}{\sqrt{\frac{GM_E}{R_E^3}}}$$

$$= \frac{2 \times \pi}{\sqrt{\frac{6.67 \times 10^{-11} \times 5.97 \times 10^{24}}{(6.38 \times 10^6)^3}}}$$

$$T = 5070 \text{ s (3 sd)}$$

This is the same period as you oscillating through the center of the Earth.

b. What is the maximum velocity of something (or someone) falling through the Earth?

Maximum velocity is same as velocity of circular motion.

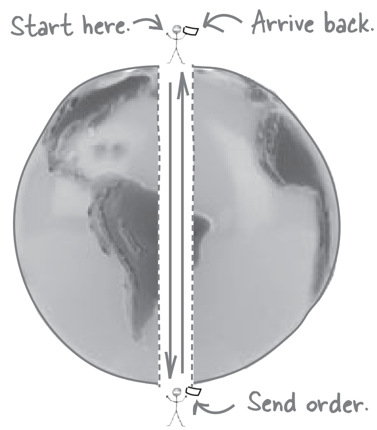
$$v = r\omega \quad \text{and} \quad \omega = \frac{2\pi}{T}$$

$$v = \frac{2\pi r}{T} = \frac{2 \times \pi \times 6.38 \times 10^6}{5070}$$

$$v = 7900 \text{ m/s (3 sd)}$$

Your top speed is a blistering pace of just under 8 km per SECOND!

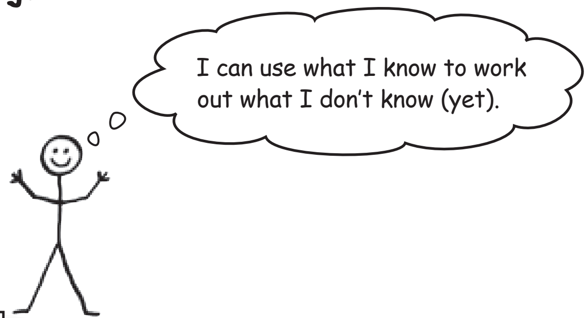
As long as you send your Break Neck Pizza order when you pop out on the other side of the world (with 42 min and 15 s still to go), you arrive home just before Alex does!

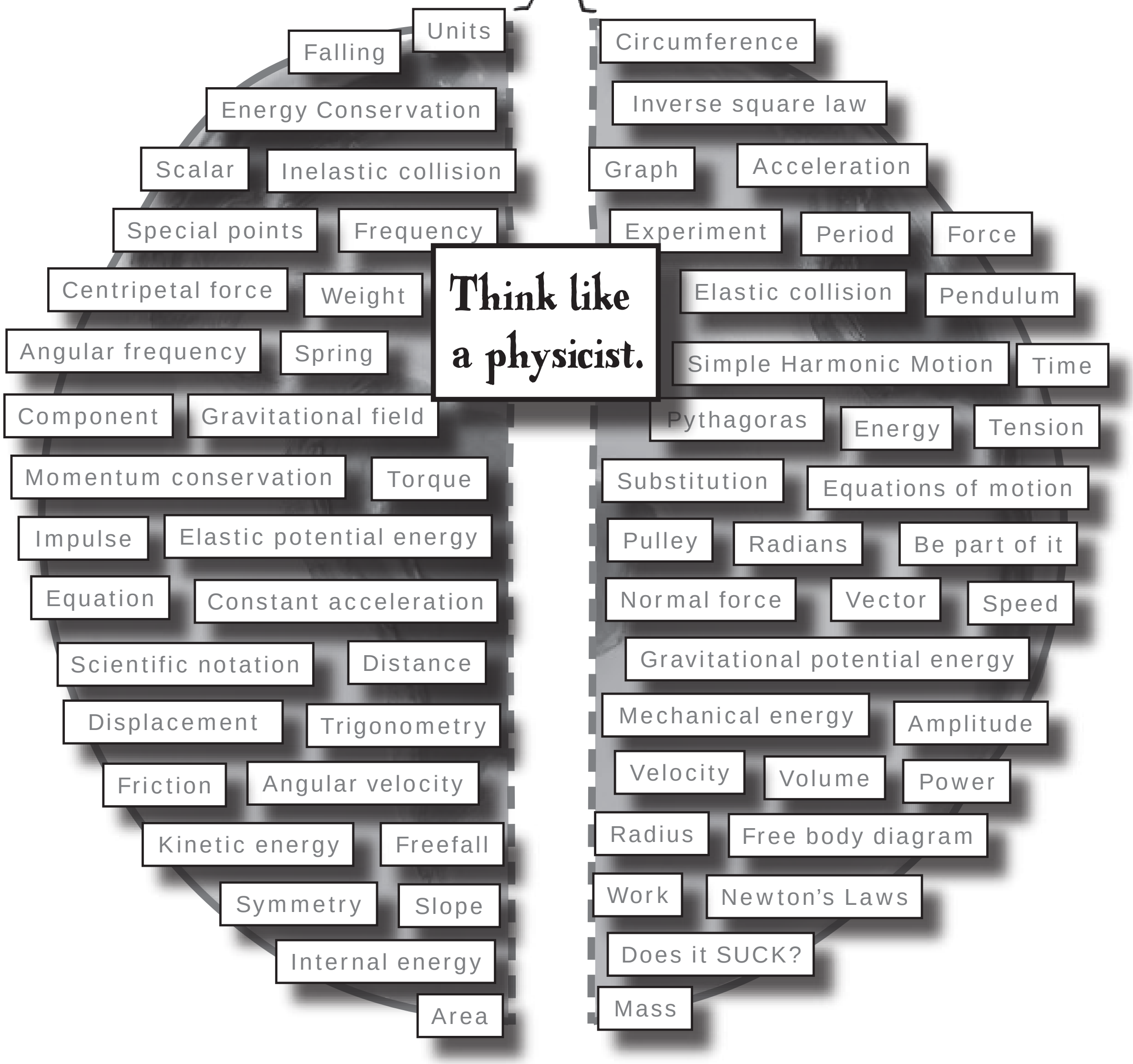


You can do (just about) anything!

You've finished your trip through the Earth - and your trip through this book. You've learned how physics works in the real world and absorbed problem solving strategies that you can use (just about) anywhere.

Think like a physicist!





appendix i: leftovers

The top 6 things (that we didn't cover before, but are covering now)

No book can ever tell you everything about everything.

We've covered a lot of ground, and given you some great thinking skills and physics knowledge that will help you in the future, whether you're taking an exam or are just curious about how the world works. We had to make some really tough choices about what to include and what to leave out. Here are some topics that we didn't look at as we went along, but are still **important** and **useful**.

#1 Equation of a straight line graph, y = mx + c

In chapter 20, you learned to compare the equation for a specific case of simple harmonic motion (for example, a mass on a spring or a simple pendulum) with a **standard equation** for simple harmonic motion, $\mathbf{x} = \mathbf{A}\cos(\omega t)$.

When you line up your SHM equation with this standard equation, you can use it to work out the amplitude and angular frequency of your system.

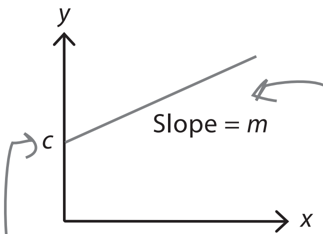
$$x = A\cos(\omega t)$$

$$x = x_0\cos\left(\sqrt{\frac{k}{m}}\,t\right)$$

The amplitude is x_0, because the x_0 in the specific equation corresponds to A in the standard equation.

The angular frequency is $\sqrt{\frac{k}{m}}$ because $\sqrt{\frac{k}{m}}$ in the specific equation corresponds to ω in the standard equation.

There's an even more fundamental equation you can do this with - the **standard equation for a straight line**. This is given by $y = mx + c$, which you can plot on a graph of y vs x.



m is the slope of the graph

c is the intercept, where the graph cuts the horizontal axis.

When x = 0, the equation $y = mx + c$ becomes $y = 0 + c$, which is just $y = c$.

Therefore, c is the **intercept** - the value of y when $x = 0$.

If you increase the value of x by 1, then value of y increases by m, because of the $y = mx$ part of the equation. This means that the **slope** of the graph = m.

$$y = mx + c$$

Vertical axis → y; Slope → m; Horizontal axis → x; Intercept → c

Every equation for a straight line graph follows this pattern, or **form**. It means that if you work out which variable to plot on which axis, you can work out the values of the other variables in your equation from the slope and the intercept.

Every equation for a straight line graph follows the pattern y = mx + c

Equation	Vertical axis	Horizontal axis	Slope	Intercept
$y = mx + c$	y	x	m	c
$x = x_0 + vt$	x	t	v	x_0
$v = v_0 + at$	v	t	a	v_0

Plot a graph of v against t and calculate the acceleration from the slope of the graph.

Why would I want to compare an equation to the graph y = mx + c when I can just plot a straight line graph anyway, using the variables I already measured?

(just about)

You can turn ANY equation into a straight line graph and measure its slope

Suppose you're doing an experiment to calculate the acceleration of a block down a slope (perhaps to eventually work out the coefficient of friction).

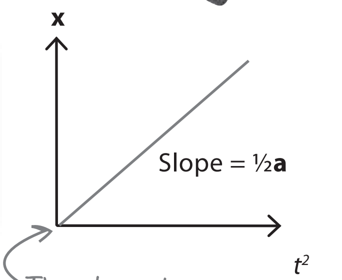
You'd expect your graph of displacement vs time to be of the form $\mathbf{x} = \mathbf{x}_0 + \mathbf{v}_0 t + \frac{1}{2}\mathbf{a}t^2$.

If you set things up so that $\mathbf{x}_0 = 0$ and $\mathbf{v}_0 = 0$, the equation becomes $\mathbf{x} = 0 + 0 + \frac{1}{2}\mathbf{a}t^2$, or just $\mathbf{x} = \frac{1}{2}\mathbf{a}t^2$

You know from experience that $\mathbf{x} = \frac{1}{2}\mathbf{a}t^2$ will produce a curved graph... so what does it have to do with $y = mx + c$, which is the equation for a straight line graph?

Line up the variables in your equation with the variables in the standard equation so that you keep track of which is which.

Equation	Vertical axis	Horizontal axis	Slope	Intercept
y = mx + c	y	x	m	c
x = ½at²	x	t²	½a	0



The intercept is zero because x = 0 when t² = 0

If you work out the value of t^2 for each of your data points and plot t^2 along the horizontal axis and **x** along the vertical axis, you'll end up with a straight line graph with a slope of ½**a**. Draw the graph, measure the slope, and you get the value for **a**, which is what you want!

Another way of thinking about it is to say "Let $z = t^2$" and substitute that in to your equation $\mathbf{x} = \frac{1}{2}\mathbf{a}t^2$. You then have the equation $\mathbf{x} = \frac{1}{2}\mathbf{a}z$. So if you plot a graph with **x** on the vertical axis and z on the horizontal axis, the slope of the graph will be ½**a**.

If you already know the **form of the equation** you expect your experimental results to take, this can be a very powerful tool.

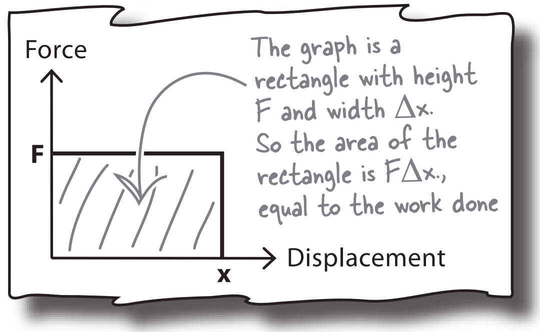
Arrange things so that the quantity you want to know winds up as the slope or intercept of a straight line graph.

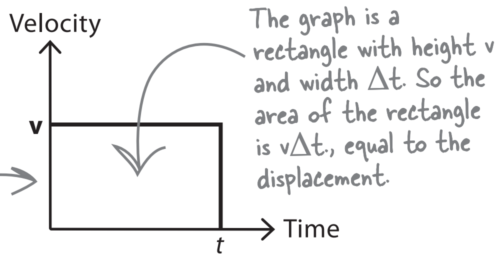
#2 Displacement is the area under the velocity-time graph

You've already learned that the work done is the area under the force-displacement graph, and used this several times to calculate energy transfer.

This is possible because the quantity of work done = $F\Delta x$. This equation has the same form as the equation for the area of a rectangle: area = height × width. If you have a rectangle on a graph where the height is F and the width is Δx, then the area of the rectangle will be equal to the work done.

If you have any equation of the form A = bc, then A will be the area under a graph of b plotted against c.



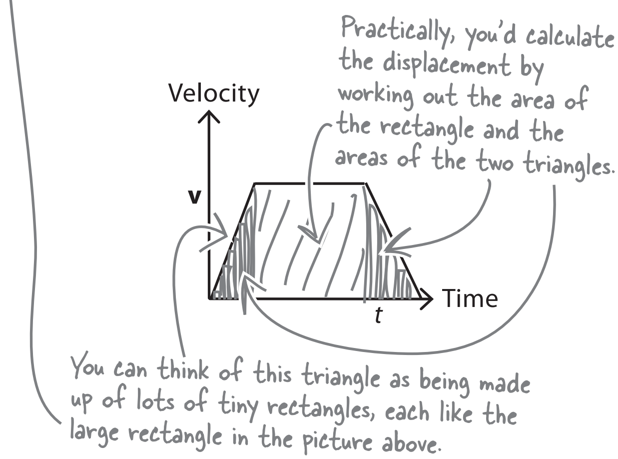


The most common example of this kind of graph is a **velocity-time graph**.

This is because $\mathbf{v} = \frac{\Delta \mathbf{x}}{\Delta t}$, and so $\Delta \mathbf{x} = \mathbf{v}\Delta t$.

Therefore, if you plot a graph of $\mathbf{v}$ against t, the **area under the graph** will be equal to $\Delta \mathbf{x}$

This even works for graphs that aren't rectangles. For **any shape of graph**, you could theoretically split up the area between the graph and the horizontal axis into lots and lots of tiny rectangles. Then you can add up the areas of the rectangles - to get the total area under the graph, and the total displacement.



If A = bc then A will be the area under a graph of b plotted againt c.

For example, x = vt, so x is the area under a v-t graph.

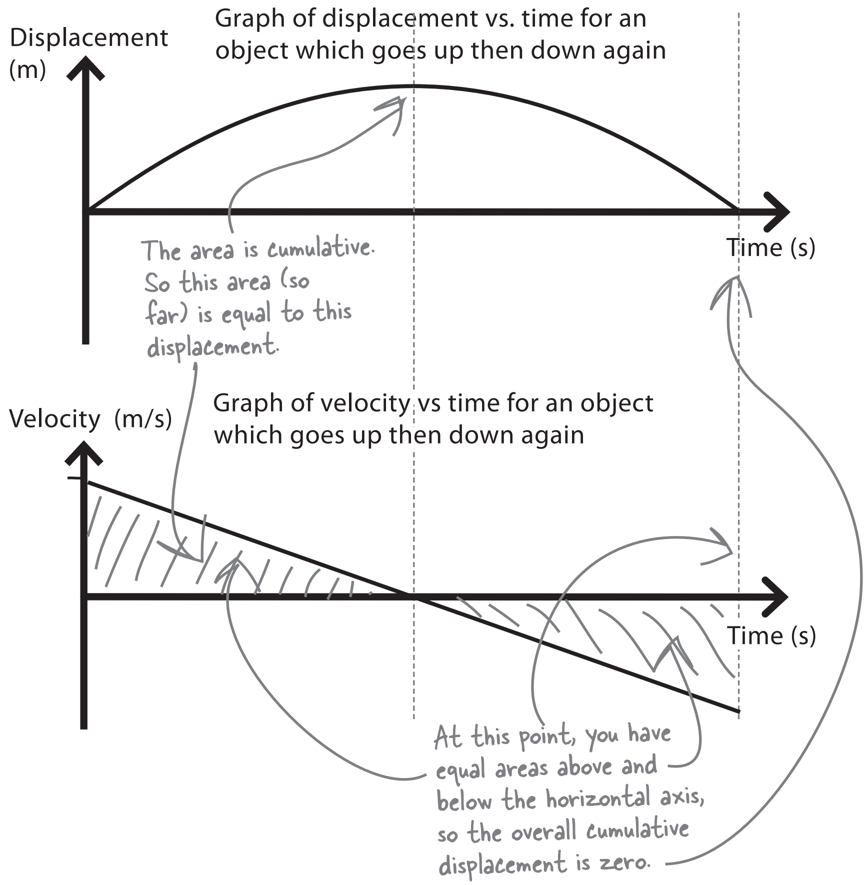
I guess that if the velocity is negative, then you rack up the area underneath the horizontal axis instead of above it?

Yes - if the velocity is negative, the displacement is changing in the opposite direction.

The area between the velocity-time graph and the x-axis tells you the **total displacement so far**. If the velocity has only ever been positive, the total displacement must also be positive.

However, if the velocity later becomes negative, the object must be retracing its steps and traveling in the opposite direction. This corresponds to the velocity-time graph being **below** the horizontal axis, and a change of displacement in the negative direction.

If there are equal areas above and below the horizontal axis of the velocity-time graph, the net displacement must be zero.



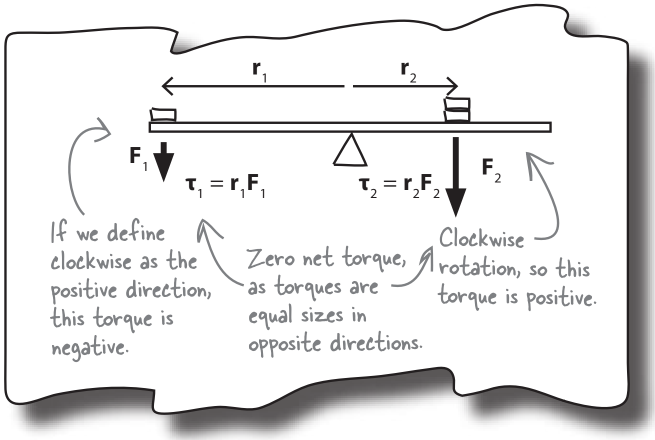
Area above horizontal axis is positive displacement.

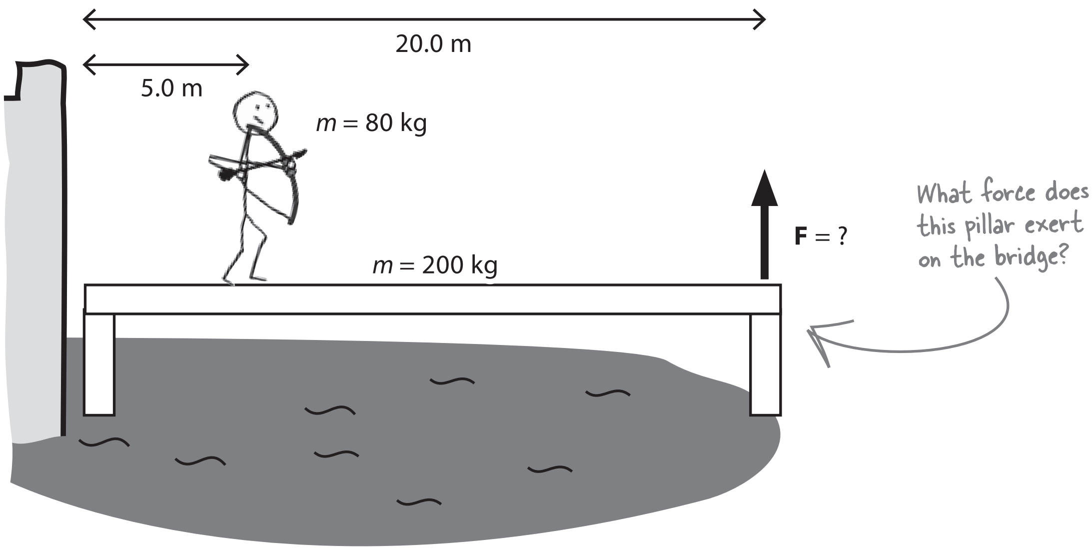
Area below horizontal axis is negative displacement.

#3 Torque on a bridge

We've previously defined torque as a "turning force." When you identify a fulcrum, torque is defined as the **displacement** from the fulcrum a force is applied at × the component of the **force** perpendicular to the lever, $\tau = rF_\perp$

Torque is a **vector** - you define clockwise as positive and counter-clockwise as negative.





In any problem that asks you about forces, look to see if all the forces act through the center of the object. If they don't, then a TORQUE must be present.

In some problems, you're asked to calculate the **force** that a pillar or string exerts on a bridge to support it. For example:

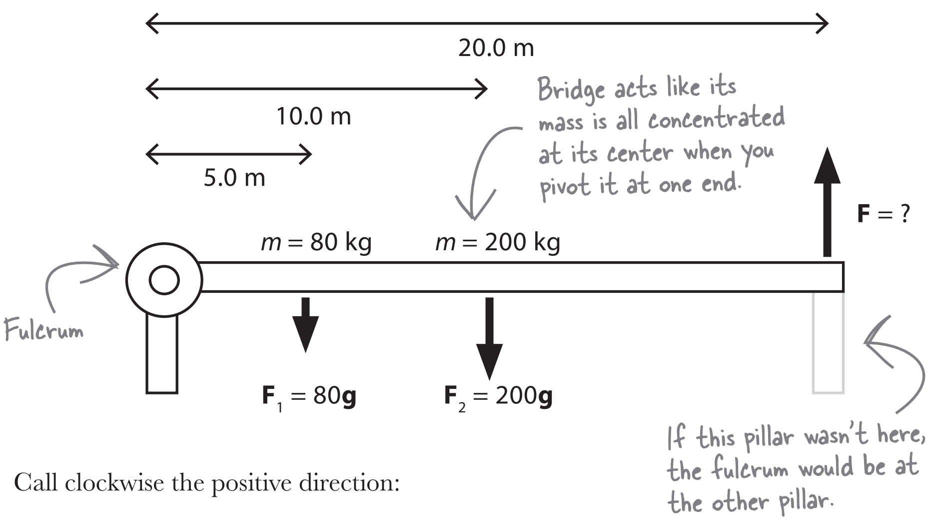
"Imhotep, mass 80 kg, is standing 5.0 m from the end of a bridge that's 20.0 m wide and has a mass of 200 kg. The bridge is horizontal and in equilibrium What force will the support at the far end of the bridge exert on the bridge?"

The key to solving a problem like this is recognizing that it's actually about **torque**

The key question in working this out is to ask yourself: "if the pillar wasn't there, where would the **fulcrum** be?

In this case, the bridge would rotate around the pillar that supports it at the other end. So the force that our pillar provides must provide a torque that makes the total torque on the bridge around the fulcrum add to zero.

As well as the torque provided by our pillar, we need to think about the torque from Imhotep and the torque from the bridge itself. The second key is realizing that the bridge acts like all its mass is **concentrated in the center**. Even if Imhotep was invisible, there would still be a torque.



Call clockwise the positive direction:

$$\mathbf{r}_1\mathbf{F}_1 + \mathbf{r}_2\mathbf{F}_2 - \mathbf{r}_3\mathbf{F} = 0$$

$$\mathbf{F} = \frac{\mathbf{r}_1\mathbf{F}_1 + \mathbf{r}_2\mathbf{F}_2}{\mathbf{r}_3}$$

$$\mathbf{F} = \frac{5.0 \times 80 \times 9.8 + 10.0 \times 200 \times 9.8}{20.0}$$

$$\mathbf{F} = \underline{\underline{1176 \text{ N}}}$$

Finally, check to see if your answer SUCKs. The units are correct, but what about the size? The total weight of the bridge plus Imhotep is (200 + 80) × 9.8 = 2744 N. But the mass is mostly concentrated at the other end of the bridge, so this pillar should support less than half the combined weight.

1176 N is less than half of 2744 N, so the answer is plausible.

If you want to know the force that a pillar (or string) exerts on a bridge, ask yourself "where would the FULCRUM be if that pillar (or string) wasn't there?"

#4 Power

Power is the rate at which you do work, and is measured in Joules per second. Sometimes you're asked to calculate the time it takes a machine with a certain power to transfer a certain amount of energy.

So for example, a 1.0 kW engine produces 1.0 kJ per second, and would do a job requiring 10 kJ of energy transfer in 10 seconds.

There's also another equation you can use as a shortcut, which involves the **velocity** that the object you're doing work on moves at:

$$\text{Power} = \frac{\Delta W}{\Delta t}$$

$$\Rightarrow \text{Power} = \frac{\Delta(\mathbf{F}_{\parallel}\mathbf{x})}{\Delta t} = \mathbf{F}_{\parallel}\frac{\Delta \mathbf{x}}{\Delta t}$$

$$\Rightarrow \text{Power} = \mathbf{F}_{\parallel}\mathbf{v}$$

This is rate of change of displacement, or velocity.

(Remember, we're using the component of the force parallel to the displacement. So the velocity must also be parallel to the force and displacement)

Power = Fv

Work is measured in Joules.

Power is the rate at which you do work.

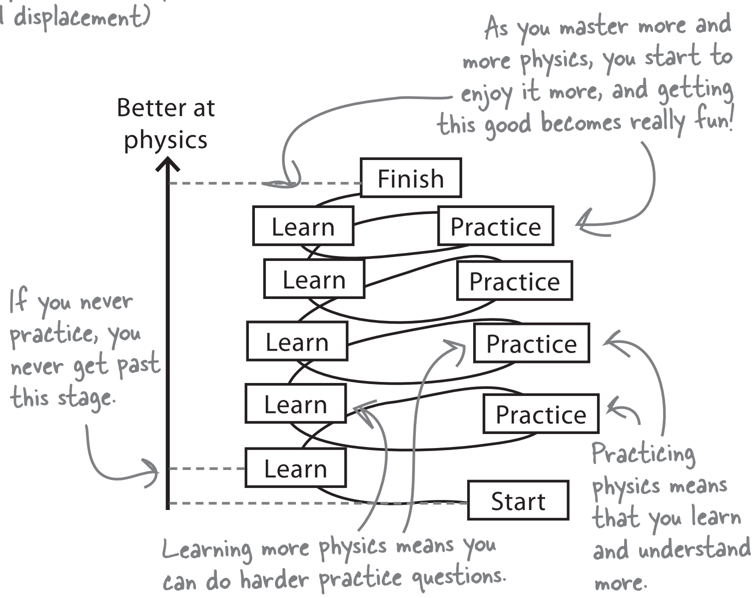
Power's measured in Joules per second, J/s

#5 Lots of practice questions

This book is fundamentally a learning book, not a textbook or a question bank. The questions and exercises have been carefully chosen to help you grasp the physics concepts you're learning about.

This is because to fully **understand** physics, you need to **do physics**, not just read about it. You need to be able to **use** the concepts to do calculations as well being able to explain them.

You can't get good at tennis just by reading a book about it - and in a similar way, you need to practice lots of exercises, problems and questions to get good at physics.



Do as many practice questions as you can - especially using past exam papers from your course.

You can never do enough practice questions - but page count had to kick in somewhere, so you'll have to find more elsewhere. A good source is your exam board, which will have **past papers**, or the kind of revision guide that contains practice exams.

If you're taking AP Physics, you can download the past papers from their website - together with sample worked answers and marking schemes!

#6 Exam tips

Although not everyone reading this book will be studying for an exam, a lot of people probably will be! We've based the content of this book on the mechanics and experimental parts of the AP Physics B course (an American College course). The syllabus is also largely the same as the mechanics content of an English A-Level exam (also taken internationally). But these exam tips apply right across the board!

Arrive at your exam fresh so that you're ready to be inventive and solve problems. Trying to cram tires out the creative parts of your brain.

BULLET POINTS

- Find a procrastination-free place to work. If necessary, hide your internet cable or wireless card!
- Remember that different people structure their work in different ways. Some are happier with a timetable written out in advance, and some with writing down what they have revised and how long they spent on it as they go along.
- Don't be psyched out by what other people on your course claim they are or aren't doing. Just get on with what you're doing.
- Start off with a mixture of reading through your notes or this book to make sure you understand the concepts, while doing (or redoing) exercises from the book.
- Read through your homework from each section of your course to remind yourself of how you did problems before. If you've forgotten how to do any, redo them to remind yourself of the method.
- Get a good stock of previous exam questions. Once you've been through your notes and your homework, do all the questions in two or three papers "open book" (referring to your notes when you need them), then do all the questions in the most recent paper or two without using your notes. Do as many past papers as is physically possible!
- In the exam, find out whether you are allowed a calculator in advance, and make sure you have spare batteries. Download or photocopy the equation table you'll have in your exam, and make sure you use it when doing past papers.
- Get a good night's sleep and don't cram – physics exams test how you can think on your feet, not what you have learned by rote.
- Read the question. Underline the important parts. Read the question again. You get zero credit for answering a question you've not been asked!
- Start with a sketch, and by asking yourself what it's LIKE.
- Try to give an explanation of what you're doing at each stage of the problem. This helps you get things straight in your head - and helps your examiner give you points for showing that you understand.
- Show your work! A numerical answer with no work usually only scores half points even if it's correct.
- Never cross anything out. If you change your mind part-way through, only cross out your first answer when you've completed your second.
- If you have a multiple choice exam, find out if it is negatively marked. In the AP Physics B multiple choice paper, you lose quarter of a mark for every question you get wrong. So if you can reduce the possibilities to two or three answers, it's worth taking a guess. But not if you're guessing between four or five answers.

appendix ii: equation table

Point of Reference

It's difficult to remember something when you've only seen it once.

Equations are a major way of describing what's going on in physics. Every time you use equations to help **solve a problem**, you naturally start to become familiar with them without the need to spend time doing rote memorization. But before you get to that stage, it's good to have a place you can **look up** the equation you want to use. That's what this **equation table** appendix is for - it's a point of reference that you can turn to at any time.

Mechanics equation table

Equations of motion

"No displacement"	$\mathbf{v} = \mathbf{v}_0 + \mathbf{a}t$
"No final velocity"	$\mathbf{x} = \mathbf{x}_0 + \mathbf{v}_0 t + \frac{1}{2}\mathbf{a}t^2$
"No time"	$\mathbf{v}^2 = \mathbf{v}_0^{\,2} + 2\mathbf{a}(\mathbf{x} - \mathbf{x}_0)$

If the force varies with time, you need to use the area under the force–time graph on the left hand side of the equation.

Forces

Momentum	$\mathbf{p} = m\mathbf{v}$
Newton's 2nd Law - momentum version ($\mathbf{F}_{net}\Delta t$ is also called impulse)	$\mathbf{F}_{net}\Delta t = \Delta\mathbf{p}$
Newton's 2nd Law - acceleration version	$\mathbf{F}_{net} = m\mathbf{a}$
Friction	$F_{fric} = \mu F_N$
Torque	$\boldsymbol{\tau} = \mathbf{r}\mathbf{F}_\perp$

τ is perpendicular to r and $F_\perp$. Clockwise is positive, counter-clockwise is negative.

This is a scalar equation. Even though F_{fric} and F_N are perpendicular, the direction of F_{fric} depends on the direction an object is being moved in, not on F_N.

Work and energy

Work done on a system	$W = \mathbf{F}_{\|\|}\Delta\mathbf{x}$
Gravitational potential energy	$U_g = m\mathbf{gh}$
Kinetic energy	$K = \frac{1}{2}m\mathbf{v}^2$
Average power	$P_{avg} = \dfrac{\Delta W}{\Delta t}$
Power used to do work on a system	$P = \mathbf{F}_{\|\|}\Delta\mathbf{v}$

Power is the rate at which work is done.

These are all scalar equations because when you multiply a vector by a vector parallel to it, the result is a scalar.

Circular Motion

These equations also work for simple harmonic motion.

Period and frequency	$T = \dfrac{1}{f}$
Angular frequency (aka angular speed or size of angular velocity)	$\omega = 2\pi f$
Linear and angular distance	$x = r\theta$
Linear and angular velocity	$v = r\omega$
Centripetal acceleration (using angluar frequency)	$a_c = r\omega^2$
Centripetal acceleration (using linear speed)	$a_c = \dfrac{v^2}{r}$

In simple harmonic motion, these give you the maximum values of x and v.

Gravitation

Gravitational force between two spheres	$F_G = -\dfrac{Gm_1m_2}{r^2}$
Gravitational potential between two spheres	$U_G = -\dfrac{Gm_1m_2}{r}$

Although this is a scalar equation, it is useful to put in the minus sign to remember that F and r are in opposite directions.

There's a minus sign in this equation because U_G is defined as zero when r is infinite.

Simple Harmonic Motion

Force and spring constant	$\mathbf{F}_s = -k\mathbf{x}\mathbf{v}$
Elastic potential energy of a spring	$U_s = \frac{1}{2}\mathrm{k}\mathbf{x}^2$
Standard SHM equations take one of these forms for **x**, **v** and **a** (depending on itital conditions)	$\mathbf{x} = \mathbf{x}_0\sin(\omega t)$ $\mathbf{x} = \mathbf{x}_0\cos(\omega t)$
Angular frequency for mass on a spring	$\omega = \sqrt{\dfrac{k}{m}}$
Angular frequency for simple pendulum	$\omega = \sqrt{\dfrac{g}{l}}$

Also called gravitational field strength.

These are all "base" SI units – you can rewrite the other units in this table in terms of them. For example, $F = ma$, so in base units, force has units of kg·m/s²

Constants

Acceleration due to gravity near the Earth's surface	$\mathbf{g}$ =	9.8 m/s²
Gravitational constant	G =	6.67×10^{-11} m³/kg·s²
Speed of light	c =	3.00×10^{8} m/s

Other useful values, such as the radius of the Earth and the mass of the Earth, are given in some equation tables but not others. The AP Physice B table omits them.

Units

Length	meter	m
Mass	kilogram	kg
Time	second	s
Frequency	Hertz	Hz
Force	Newton	N
Energy	Joule	J
Power	Watt	W

Geometry

Area of a rectangle	A =	base × height
Area of a triangle	A =	½ × base × height
Circumference of a circle	C =	$2\pi r$
Area of a circle	A =	πr^2
Surface area of a sphere	S =	$4\pi r^2$
Volume of a sphere	V =	$\frac{4}{3}\pi r^3$
Volume of a prism (3D shape with same shape of base and top, and straight sides)	V =	area of base × height

If you have a 3D shape, try "unrolling" or "unfolding" it flat to see which 2D shapes it's constructed from.

Prefixes

10^{9}	giga	G	10^{-3}	milli	m
10^{6}	mega	M	10^{-6}	micro	μ
10^{3}	kilo	k	10^{-9}	nano	n
10^{-2}	centi	c	10^{-12}	pico	p

Letters used in equations

Distance	x, r, l
Displacement	$\mathbf{x}, \mathbf{r}$
Velocity	$\mathbf{v}$
Acceleration	$\mathbf{a}$
Time	t
Mass	m
Momentum	$\mathbf{p}$
Force	$\mathbf{F}$
Torque	$\boldsymbol{\tau}$
Work	W
Potential energy	U
Kinetic energy	K
Power	P
Period	T
Frequency	f
Angle	θ
Angular frequency	ω
Spring constant	k

r always means radius.

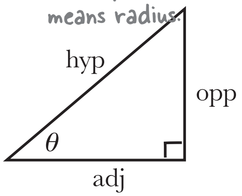
Bold means a vector quantity.

Italic means a scalar quantity.

If an equation contains a variable that is usually a vector but is written in italics, then you just use the size of the variable in the equation.

Trigonometry

r always means radius.



Pythagoras	$\text{hyp}^2 = \text{opp}^2 + \text{adj}^2$
Sine	$\sin(\theta) = \frac{\text{opp}}{\text{hyp}}$
Cosine	$\cos(\theta) = \frac{\text{adj}}{\text{hyp}}$
Tangent	$\tan(\theta) = \frac{\text{opp}}{\text{adj}}$

The extended definitions of sine, cosine and tangent for angles greater than 90° aren't given in the AP Physics B equation table.

Index

Symbols

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