

## PS 10 Physics 201 March 31, 2010 R.Shankar Due April 7.

1. Consider a slit of width  $D$  in the  $y$ -direction and a beam incident on it from the left along the  $x$ -axis. According to Huygens each point in the slit acts like a source for the region on the right. Replace this continuum of points by  $N$  discrete points spaced  $d = D/N$  apart. Show that the phase difference between two neighboring points is  $\phi = kd \sin \theta = k(D/N) \sin \theta$  for light going in the direction  $\theta$  above the original direction. Using complex numbers argue that the points make contributions  $a, ae^{i\phi}, ae^{2i\phi}, \dots, ae^{i(N-1)\phi}$ , where  $a$  is the contribution from the top most point. Show that the arrows (complex numbers) sum up to

$$A = a \left[ \frac{1 - e^{iN\phi}}{1 - e^{i\phi}} \right]$$

whose real part is the physical answer. You may invoke the geometric series:

$$1 + r + r^2 + \dots + r^{N-1} = \frac{1 - r^N}{1 - r}.$$

Show that we get the first zero when  $D \sin \theta = \lambda$ . Write the expression for  $A$  as a ratio of sines.

2. A double slit experiment is done with light of  $\lambda = 600nm$  and slit separation  $d = .1mm$ .
  - (i) At what angles do the first minimum and first maximum (not counting central maximum) occur? Consider just the maxima and minima above the initial direction and ignore the symmetric points below in this and all parts.
  - (ii) If the light lands on a screen  $2m$  away, where is the first dark dark fringe measured from the central maximum?
  - (iii) Repeat for the second minimum and second maximum.
  - (iv) For the first (noncentral) maximum, what is the spacing on the screen between  $\lambda = 600nm$  and  $\lambda = 500nm$ ?
3. In a double slit experiment done with  $\lambda = 480nm$  the 5-th maximum goes into the second minimum if a transparent material of thickness  $t$  and  $n = 1.56$  is placed in front of one of the slits. What is  $t$ ?
4. Light at  $520nm$  is diffracted by a grating with 300,000 lines per meter. Find the angles for the first and fourth order maxima.
5. Find the minimum thickness  $t$  for which a soap film ( $n = 1.33$ ) will lead to constructive interference on normally incident  $500nm$  light.
6. Find the angular width (angular separation between the two minima closest to central maximum) of beam with  $\lambda = 600nm$  that diffracts from a slit of width  $2000nm$ .
7. Find the lines per cm of a grating which gives a third order line at  $\pi/6$  radians for light with  $\lambda = 500nm$ .

8. Electrons are accelerated by a voltage  $V$  from rest. What is their momentum  $p$  and the de Broglie wavelength  $\lambda$ ? If this beam of electrons are incident on a double slit with spacing  $d$ , what is the smallest angle at which will we get a minimum? If the electrons impinge on a screen  $L$  meters away, at what height  $h$  above the center will the corresponding minimum appear? What is  $V$  if  $d = 50 \text{ nm}$ ,  $h = 300 \text{ nm}$  and  $L = 15 \text{ cm}$ ?
9. A proton at the LHC has an energy  $3.5 \text{ TeV}$ . What is de Broglie wave length? Using  $E^2 = p^2c^2 + m^2c^4$ , first argue that the mass term can be neglected. What will be the wavelength of a photon of the same energy?
10. Light of wavelength  $\lambda$  leaves a source  $S$  that is  $H$  meters above a horizontal mirror as in Fig. 1 and reaches a screen  $D$  meters away, directly and upon reflection from a horizontal mirror. Locate the height  $h$  of the first dark fringe (minimum) above the mirror if  $\lambda = 500 \text{ nm}$ ,  $H = .01 \text{ m}$ ,  $D = 2 \text{ m}$ . Hint: Use images and do not forget a  $\pi$  phase shift on reflection from mirror.

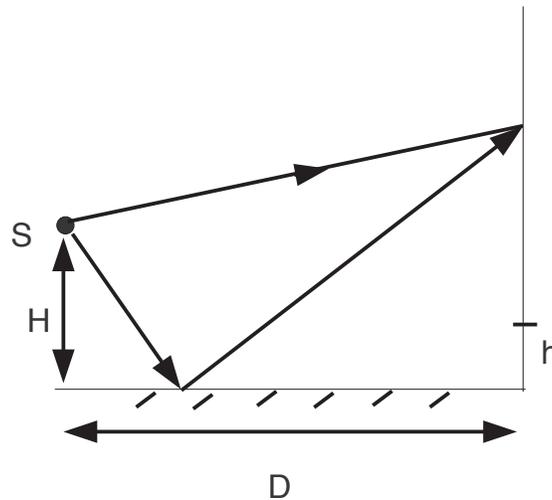


Figure 1: Light goes from source to wall along two paths: one direct and one upon reflecting from the mirror.

11. The work function of a certain metal is  $W = 4 \text{ eV}$ . What is  $\omega_0$  the minimum (angular) frequency of light that can cause photoemission? At double this frequency what will be the velocity of the emitted electron?
12. Free electrons at temperature  $T$  will have a mean kinetic energy  $3kT/2$  where  $k$  is Boltzmann's constant. What is the  $\lambda$  associated with the corresponding momentum? This is called the thermal wave length  $\lambda_T$ . Evaluate this at  $T = 300 \text{ K}$ .
13. **Crystal diffraction** Consider a crystal made of a regular square array of atoms with spacing  $d$  between each row and each column. I show just parts of three rows in Fig. 2. We already know that that atoms in each row will reflect coherently only if the condition  $i = r$  is met (The complementary angles to  $i = r$  is  $\theta$  in the figure. ) The

incident and reflected angles are equal but can take any value. Now we ask under what conditions atoms from *two adjacent* rows will emit coherently. (i) Show that this will happen only if Bragg's condition

$$2d \sin \theta = m\lambda \quad \text{where } m \text{ is an integer}$$

is satisfied. (ii) To show the wave nature of electrons, Davisson and Germer made a beam of electrons accelerated to a kinetic energy  $(p^2/2m) = 54 \text{ eV}$  from a nickel crystal (square lattice with interatomic separation  $a$ ) The diffraction angle in Fig. 2 was  $\theta = 65^\circ$ . What is the lattice spacing  $a$ ?

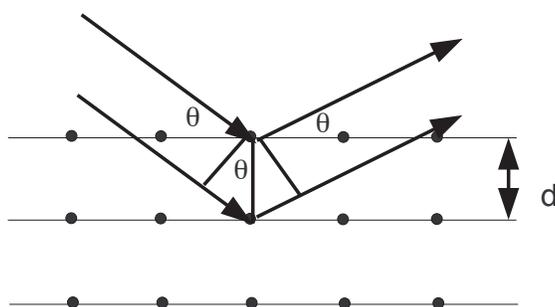


Figure 2: Light is incident at an angle  $\theta$  above the horizontal and gets reflected at the same angle by each layer. But when will each layer interfere constructively with the next?