Problem Set 2 Physics 201b January 20, 2010. Due Jan 27

1. A rod has charge density $\lambda(x) = \frac{\lambda_0 x}{L}$ in the interval $-L < x < L$. Find the field at a point $x = x_0 > L$. Examine this result for $x_0 \to \infty$ and show that it falls off like a dipole field $\mathbf{E} = \mathbf{i}\frac{\lambda_0 L^2}{3\pi\varepsilon_0 x_0^3}$ and find the associated dipole moment. Hint: Expand in a Taylor series to an order that yields a nonzero result. Hint for doing integral: $x/(...) = (x - a + a)/(...)$.

2. A dipole with moment $p = 10^{-29} C \cdot m$ and of length $10^{-10} m$ is at an angle of $+\pi/6$ with respect to a uniform electric field along the x-axis $\mathbf{E} = \mathbf{i}\ 0.5 N/C$. What is the torque on it? What work will it take to align it an angle π? If disturbed from the position of stable equilibrium, what will be the (angular) frequency (ω) of small oscillations if the dipole has a mass $10^{-27} kg$ at each end?

3. A solid nonconducting sphere of uniform charge density and total charge $-Q$ and radius $r = a$ is surrounded by a concentric conducting spherical shell of inner radius $r = b$ and outer radius $r = c$ with $c > b > a$. The outer shell has charge $2Q$. Use Gauss' law to find the field for all r. Show with a sketch where the charges reside and some field lines.

4. Consider a hollow conducting cylinder of radius a and charge λ per unit length surrounded by an outer hollow conducting cylinder of radius b with charge $-\lambda$ per unit length. Find the field for all r. What is σ, the charge per unit area in the inner cylinder? Consider the field between two cylinders when $b - a << a$ is very small and compare the field to that inside a parallel plate capacitor.

5. A charge of one Coulomb is at the center of a unit cube. What is the flux through one of its faces?

6. A charge density distribution is given by $\rho(r) = Ar^2 \quad C/m^3 \quad 0 \le r \le R$. Remember that volume integrals in spherical coordinates are given by $\int\int\int r^2 \sin\theta dr d\theta d\phi f(r, \theta, \phi)$. Find the total charge Q and the field as for all r, expressed in terms of Q.

7. Find the volume of a sphere of radius R centered at the origin by slicing it parallel to the x-y plane into discs of thickness dz and appropriate radius. You may assume the formula for the area of a circle.

8. The gravitational field $\mathbf{G}$, defined as force on a unit mass, is very much like the electric field, with a magnitude $G = Gm/r^2$ for a point mass m at the origin. Write down Gauss' Law for this field in terms of the mass density ρ_m.

9. A point charge $1\mu C$ is at the center of a spherical shell of radius $1m$ and negligible thickness carrying $-2\mu C$. Find the electric field at $r = .5m$ and $r = 2m$.

10. A solid sphere of radius R has uniform charge density ρ. A hole of radius $R/2$ is scooped out of it as shown in Figure 10. Show that the field inside the hole is uniform and along the x-axis and of magnitude $\rho R/6\varepsilon_0$. Hint: Think of the hole as a superposition of positive and negative charges.

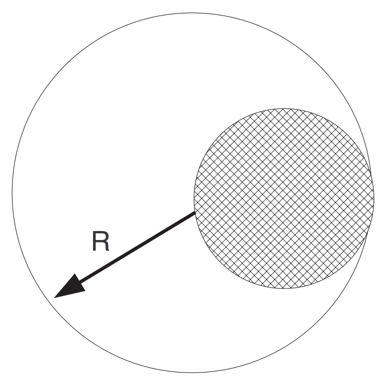

Figure 1: A solid sphere of radius R and charge density ρ with a hole of radius $R/2$.