

## PS 5 Physics 201 February 10 2010 R.Shankar Due February 17.

1. Show that the force on a closed current carrying loop in a uniform magnetic field is zero. From this, or by looking at the integral, deduce that the force on a segment of wire of any shape, starting from A and ending at B in a uniform field is the same as that on a straight wire joining A and B carrying the same current.
2. Show that when the particle in a cyclotron has an orbit of radius  $R$ , its kinetic energy is  $K = q^2 B^2 R^2 / 2m$ . If a cyclotron is to accelerate protons to a kinetic energy  $K = 4MeV$  what must be its radius if the field is  $B = 4T$ ? Argue that relativistic corrections to the kinetic energy are very small at this speed. (One defines  $K = E - mc^2$ , where  $E = mc^2 / \sqrt{1 - v^2/c^2}$ .)
3. The square loop of sides  $.2m$  carries a current  $I = 3A$  and is in a field coming out of the page, as shown in Fig. 1. The field grows in strength at a rate  $.2T/m$  in the horizontal direction. What is the force acting on it?
4. Consider two concentric horizontal circular loops of wire of radius  $R$ , each carrying current  $I$  in the same sense, with one loop a very short vertical distance  $\delta$  below the other. What is the magnitude of the force between them in the limit  $\delta/R \rightarrow 0$ ? Is it attractive or repulsive? Hint: Consider a very tiny segment of one loop and ask how it sees the other loop. Or imagine that the loops are one million miles in radius and spaced 1 cm apart.
5. An infinitely long solid conducting cylinder of radius  $a$  has a cylinder of diameter  $a$  gouged out of it, as shown in Fig. 2. It carries a current  $I$  distributed over its cross section, coming out of the paper. Find  $\mathbf{B}$  at the center of each cylinder. Hint: Use superposition.
6. A solid infinitely long conducting cylinder of radius  $a$  has two cylinders of diameter  $a$  gouged out of it as in Fig. 3. The conductor carries a current  $I$  uniformly over its cross section. Find the field at a point a distance  $r$  from the center and on the axis bisecting the wire as shown. (Note  $I$  is the current in the portion that is not gouged out.)
7. A wire in the  $x - y$  plane is in the form of a square of side  $a$  and carries a counterclockwise current  $I$ . What is  $\mathbf{B}$  at the center of the square?
8. The current comes from  $+\infty$ , makes one and half counterclockwise circles of radius  $a$  and goes back to  $\infty$  as shown in Fig. 4. What is  $\mathbf{B}$  at the center of the circle? Hint: Divide and superpose, feel free to use known results for simple cases.
9. You all know the problem of a charge  $q$  in front of one infinite grounded conducting plane which can be solved by images. Now consider a problem where there are two infinite conducting grounded planes, namely the  $x$ - $y$  and  $x$ - $z$  planes. A charge  $q$  is placed at  $(0, a, a)$  in the  $y$ - $z$  plane, as in Fig. 5. (i) Find  $\mathbf{F}$ , the force of attraction between the charge  $q$  and the planes. The  $x$ -axis comes out of the paper. (ii) How much

- work does it take to bring it in from infinity to its current location? (iii) Suppose all the three principal planes are grounded conductors and  $q$  is placed at the point  $(a, a, a)$  in the first octant. Explain why the image system of eight charges at  $(\pm a, \pm a, \pm a)$  keeps all planes at  $V = 0$  if the sign of the image charge is the product of the signs of the coordinates. (Pick any one plane and pair the charges. The others follow by symmetry.)
10. The wire and strip in Fig. 6 both are infinitely long and carry parallel currents  $I$ , except the latter has it uniformly distributed across its cross section. Show that the force per unit length between them is  $F = \frac{\mu_0 I^2}{2\pi w} \ln \frac{a+w}{a}$ . Show that this answer has the correct limit as  $w \rightarrow 0$ .
  11. A circular loop carrying current  $I$  is partly in a field  $\mathbf{B}$  going into the paper as shown in Fig. 7. Find the force on it. Hint: The force on a closed loop in a uniform field is zero.
  12. A horizontal conductor of length  $L = 2m$  and mass  $m = .12kg$  hangs from a vertical spring (connected to its midpoint) and carries a current  $I = 2A$  running from left to right in a  $\mathbf{B} = .2T$  field going into the paper as shown in Fig. 8. (Do not worry how the current is fed to the rod, these are shown by dotted lines.) What is the force constant  $k$  if the spring stretches by  $\delta = 1.2cm$ ? First use symbols to derive a formula for  $k$  and then put in the numbers.
  13. A toroidal magnet has a square cross section of side  $a$ . The radial distance from the center of the solenoid to the center of the square is  $R$ , Fig. 9. (i) If it has  $N$  turns and carries a current  $I$ , find the magnetic flux crossing the hatched square, assuming  $\mathbf{B}$  comes out of the page. (ii) Find the energy stored in the solenoid by integrating the magnetic energy density. (iii) Find the self-inductance  $L$ . (iv) Find the EMF  $\mathcal{E}$  if the current through it varies as  $I_0 \cos \omega t$ .
  14. In the circuit in Fig. 10, the switch which has been open for a very long time is suddenly closed. (i) What is the current across the inductor? (ii) What is the current after a very long time? (iii) If the switch is opened after this long time, what will be voltage across  $R_2$ ?

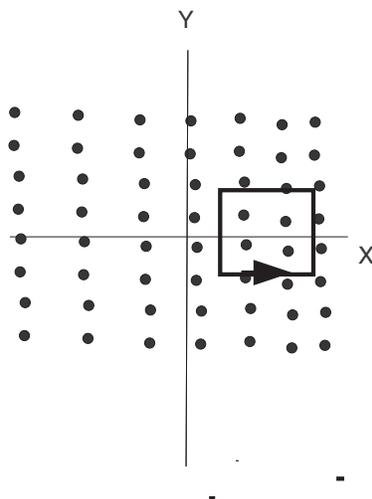


Figure 1: The square current carrying loop is in a  $B$  field which grows linearly along the x-axis.

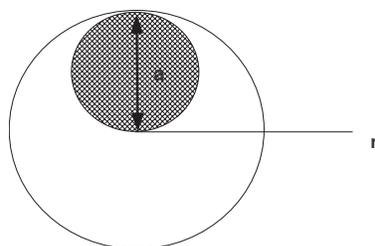


Figure 2: A solid infinitely long conducting cylinder of radius  $a$  has a cylinder of diameter  $a$  gouged out of it. It carries a current  $I$  distributed uniformly over its cross section and coming out of the page.

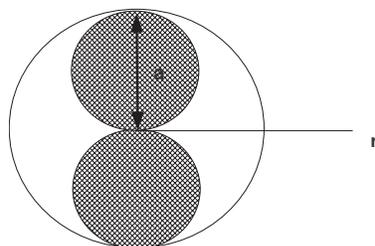


Figure 3: A solid conducting cylinder of radius  $a$  has two cylinders gouged out of it, each of diameter  $a$ . It carries a current  $I$  distributed uniformly over its cross section.

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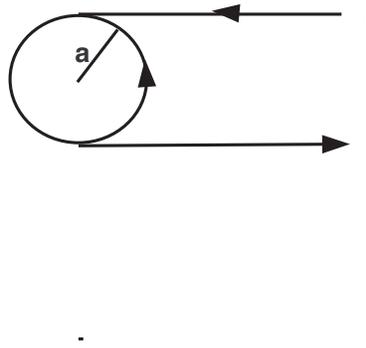


Figure 4: The current comes from  $+\infty$ , makes one and half full circles and goes back to  $\infty$ .

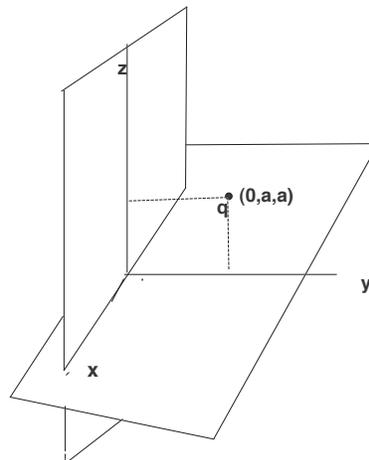


Figure 5: The grounded L-shaped conductor is actually the cross section of the x-y and x-z planes as they cross the  $x=0$  plane.

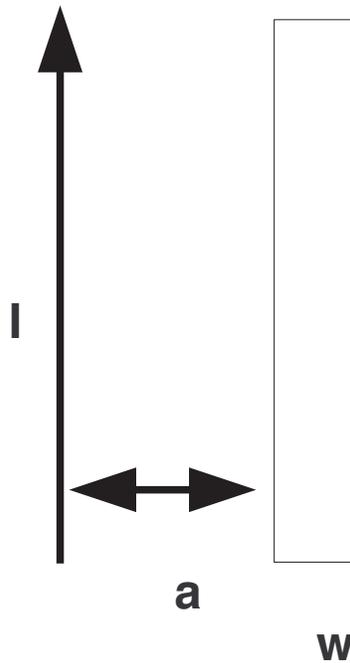


Figure 6: The wire and strip both carry parallel currents  $I$ , except the latter has it uniformly distributed across its cross section.

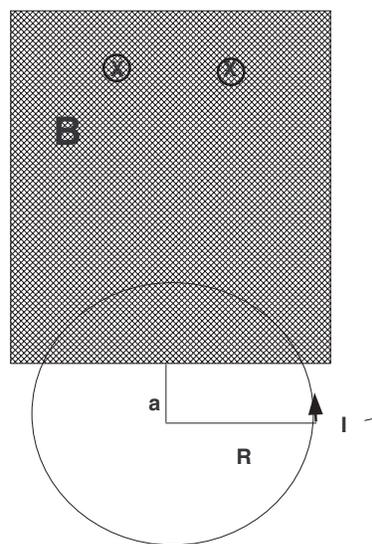


Figure 7: The circular loop carrying counterclockwise current  $I$  is partly in the region of uniform  $B$  going into the paper.

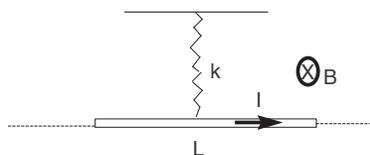


Figure 8: The rod carries a current  $I$  as shown. There is a uniform  $\mathbf{B}$  going into the paper.

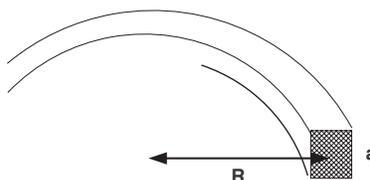


Figure 9: The toroidal solenoid has a square cross section (hatched region) of side  $a$ . The center of the circle is  $R$  meters from the center of the square.

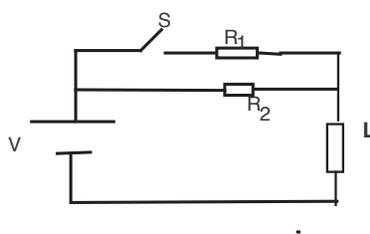


Figure 10: The switch  $S$  which has been open for a long time is suddenly closed and then opened after a long time.