Midterm Physics 201 March 1, 2010 R.Shankar 75 mins
Questions 1-5 BOOK I, 6-9 BOOK II

1. From a sphere of radius R and charge density ρ, I scoop out a sphere of radius $R/2$ as shown in Fig.1. Find the electric field at the center of the big sphere. **5**

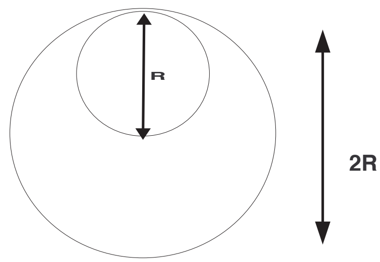

Figure 1: The sphere has radius R, the hole has radius $R/2$.

2. Particles of charge q and mass m are accelerated from rest by a voltage V (between two parallel plates) and enter a region of uniform $\mathbf{B}$ coming out of the page as shown in Fig. 2. They hit a detector after traveling a semicircle of diameter d. Find d in terms of the given parameters. **7**

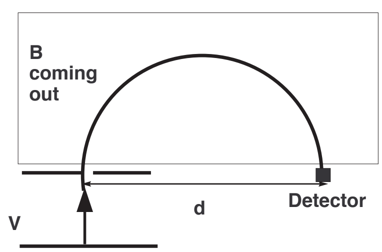

Figure 2: Particles of charge q and mass m are accelerated by a voltage V and fed into a region of uniform B coming out of the page. They travel along a semicircle of diameter d determined by q/m and hit a detector.

3. A capacitor is made of two concentric spheres of radii $a > b$ with charge Q on the inner sphere. Compute the capacitance C. Show by integration that $\frac{Q^2}{2C} = \frac{\varepsilon_0}{2} \int E^2 d^3\mathbf{r}$. **10**

4. Imagine a wire that runs as shown in Fig. 3. What is the field at the center of the circular loop? **8**

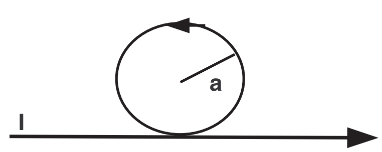

Figure 3: The wire is infinite and straight, except for the circle as shown.

5. The rectangular loop in Fig. 4 shown lying in the x-y plane is massless and free to swing about the axis AA', carries current I, and is in a **B** field parallel to the plane of the loop and perpendicular to the hinge. A mass m is hanging from one of the sides. Find B so that the system is in rotational equilibrium. **5**

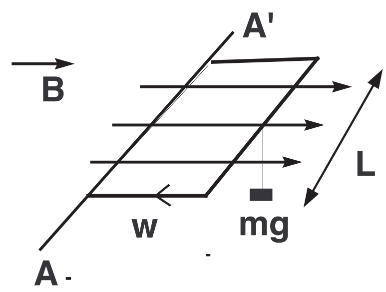

Figure 4: The loop is in rotational equilibrium.

6. (i) Find the magnitude and direction of the magnetic flux penetrating the rectangular loop in Fig 5. You may assume that the **B** field a distance r from an infinite wire has a magnitude $\frac{\mu_0 I}{2\pi r}$. (ii) If I varies as $I_0 \cos \omega t$, what is the maximum EMF that will be induced in the loop? (iii) What is M, their mutual inductance? **10**

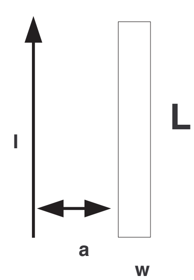

Figure 5: The field from the wire penetrates the rectangular loop.

7. (i) Find the magnetic field at the center of the loop carrying a current I and made of two concentric semicircles of radii a and b and two straight segments along the diameter as shown in Fig. 6. (ii) Find B if $I = 2A,\ a = 1m,\ b = 2m$. **8**

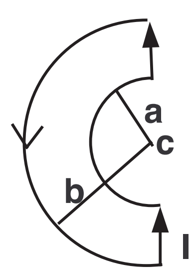

Figure 6: The task is to find **B** at the center C.

8. At $t = 0$, a capacitor C_1 is charged to a voltage V_0 and connected in series to an uncharged capacitor C_2 and a resistor R as shown in Fig. 7. By using energy and charge conservation show that the total energy dissipated in the resistor between $t = 0$ and $t = \infty$ is

$$W = \frac{V_0^2 C_1 C_2}{2(C_1 + C_2)} \qquad \mathbf{12}.$$

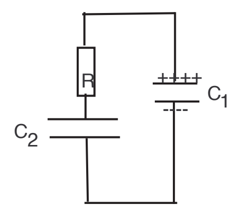

Figure 7: At $t = 0$ the circuit is completed, and C_1, which is charged to V_0 volts, is allowed to discharge via the resistor R and C_2 which is uncharged.

9. In the circuit in Fig. 8, the switch has been closed for a very long time. (i) What is the voltage across the inductor? (ii) What is the current leaving the battery? (iii) How much of it flows across the inductor? (iv) Write the appropriate equation once the switch is suddenly opened. (v) What will be initial voltage across the inductor? (vi) If this is to be less than $10V_0$, what is the allowed range for R_2/R_1? (You do not need L or the actual values of R_1 and R_2 to do this.) **10**

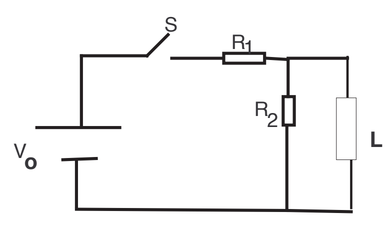

Figure 8: The switch S which has been closed for a long time is suddenly opened. The role of R_2 is to let L vent its energy.

Data Sheet

$$\mathbf{F} = q(\mathbf{E} + \mathbf{v} \times \mathbf{B})$$

$$\mathbf{E} = \frac{q}{4\pi\varepsilon_0 r^2}\mathbf{e}_r \qquad (\mathbf{e}_r \text{is unit vector in radial direction}) \qquad 9$$

$$V = \frac{q}{4\pi\varepsilon_0 r} \qquad \frac{1}{4\pi\varepsilon_0} = 9 \cdot 10^9$$

$$d\mathbf{B} = \frac{\mu_0 I}{4\pi}\frac{d\mathbf{l} \times \mathbf{r}}{r^3} \qquad \frac{\mu_0}{4\pi} = 10^{-7}$$

$$d\mathbf{F} = I\mathbf{dl} \times \mathbf{B}$$

$$\oint \mathbf{E} \cdot \mathbf{dA} = \frac{Q}{\varepsilon_0} \qquad \text{Gauss}$$

$$\oint \mathbf{B} \cdot \mathbf{dl} = \mu_0 I \qquad \text{static case, Ampere}$$

$$EMF = \oint (\mathbf{E} + \mathbf{v} \times \mathbf{B}) \cdot \mathbf{dl} = -\frac{d\Phi}{dt} \qquad \text{Lenz's Law}$$

$$V(1) - V(2) = \int_1^2 \mathbf{E} \cdot \mathbf{dr} \qquad \text{Static case}$$

$$u_E = \frac{\varepsilon_0 E^2}{2} \quad \text{electric energy per unit volume} \quad u_B = \frac{B^2}{2\mu_0} \quad \text{magnetic energy per unit volume}$$

$$L\frac{dI}{dt} + RI + \frac{1}{C}\int I(t)dt = V(t)$$

$$U = \frac{1}{2}CV^2 = \frac{Q^2}{2C} \qquad U = \frac{1}{2}LI^2$$

$$\tau = \mu \times \mathbf{B}$$

$$\mu = I \cdot A \quad \text{magnetic moment of a planar loop of area A}$$