Solutions to PS 9 Physics 201

1. Look at the figure. The answer is $h/2$.

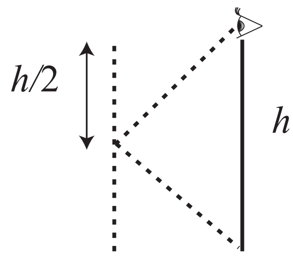

FIG. 1:

2. By applying the Mirror Formula for concave mirrors, we have $1/u + 1/v = 1/f$. To have $u = v$, we need $u = v = 2f = 60$ cm.

3. (i) The position of the ball is given by $z_b = 5 - \frac{1}{2}gt^2 = 5 - 5t^2$. Then, using the Mirror Formula, we have the relation for the postion of the image z_i:

$$\frac{1}{z_i} + \frac{1}{5 - 5t^2} = \frac{1}{2}. \tag{1}$$

Therefore, we get

$$z_i = \frac{2(5 - 5t^2)}{3 - 5t^2} \text{ [m]}. \tag{2}$$

(ii) From $2 = 5 - 3t^2$, we get

$$t = \sqrt{3/5} \text{ [s]} \tag{3}$$

(iii) (We assume elastic collision.) Once it hits the mirror, it will complete a period every 2 seconds since it takes 1 s to come down and another 1 s to go back to the original point.

4. From the Lens Formula,

$$\frac{1}{40} + \frac{1}{10} = \frac{1}{f}. \tag{4}$$

Therefore, $f = 8$ cm. Also,

$$M = -\frac{10}{40} = -\frac{1}{4}. \tag{5}$$

5. Suppose the left lens makes the image v cm to the right of it. Then, from the Lens Formula,

$$\frac{1}{24}+\frac{1}{v}=\frac{1}{-12}. \tag{6}$$

So, we get $v=-8$ cm. (The image is to the left of the lens.) By applying the Lens Formula again to this image and the right lens, we demand

$$\frac{1}{d+8}+\frac{1}{\infty}=\frac{1}{24}. \tag{7}$$

Therefore, $d=16$ cm.

6. Suppose a object is located a distance to the left of the lens and the image is formed v to the right of the lens. (Fig. 2.) Then the optical path length for the ray passing the point which is at the hight of z in the lens is given by

$$\begin{aligned}
l(z) &\approx \sqrt{u^2+z^2}+\sqrt{v^2+z^2}+(n-1)\{(R_1\cos\theta_1-d_1)+(R_2\cos\theta_2-d_2)\} && (8)\\
&= \sqrt{u^2+z^2}+\sqrt{v^2+z^2}+(n-1)\{(\frac{R_1}{\sqrt{1+\tan^2\theta_1}}-d_1)+(R_2\frac{1}{\sqrt{1+\tan^2\theta_2}}-d_2)\} && (9)\\
&= \sqrt{u^2+z^2}+\sqrt{v^2+z^2}+(n-1)\{(\frac{R_1}{\sqrt{1+(\frac{z^2}{R_1^2})}}-d_1)+(\frac{R_2}{\sqrt{1+(\frac{z^2}{R_2^2})}}-d_2)\} && (10)\\
&\approx u+\frac{z^2}{2u}+v+\frac{z^2}{2v}+(n-1)\{(R_1-\frac{z^2}{2R_1}-d_1)+(R_2-\frac{z^2}{2R_2}-d_2)\} && (11)\\
&= u+v+(n-1)(R_1+R_2-d_1-d_2)+\frac{1}{2}\{\frac{1}{u}+\frac{1}{v}-(n-1)(\frac{1}{R_1}+\frac{1}{R_2})\}z^2. && (12)
\end{aligned}$$

From the Principle of Least Time, optical rays go through only the paths with minimum optical path length. So, for the image of the object at u to be formed at v, $l(z)$ should be independent of z. Therefore, we have

$$\frac{1}{u}+\frac{1}{v}=(n-1)(\frac{1}{R_1}+\frac{1}{R_2}). \tag{13}$$

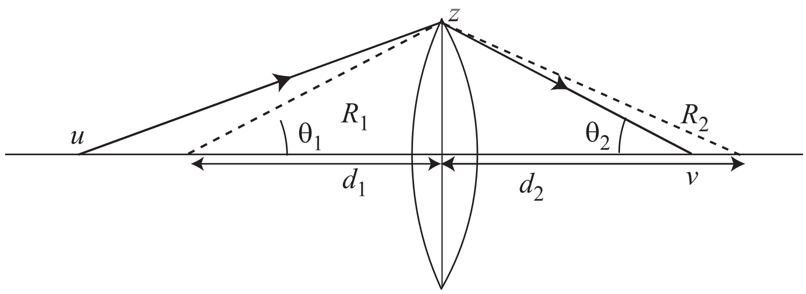

FIG. 2:

7. From the Lens Formula,

$$\frac{1}{u}+\frac{1}{\left(\frac{u}{3}\right)}=\frac{1}{0.6}. \tag{14}$$

Therefore, we get $u = 2.4$ m and $v = 0.8$ m.

8. As shown in the figure, we get virtual, upright images. Actually from the Mirror Formula, we have

$$\frac{1}{u}+\frac{1}{v}=-\frac{1}{f}. \tag{15}$$

and

$$v=-\frac{uf}{u+f}<0, \tag{16}$$

which means that the image is virtual. Also,

$$M=\frac{|v|}{|u|}=\frac{f}{u+f}\leq\frac{f}{f}=1, \tag{17}$$

which means that the image is smaller than object.

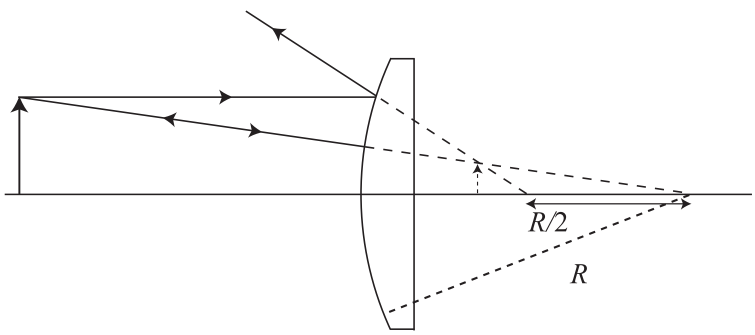

FIG. 3:

9. From the figure, the displacement x is given by

$$x = \overline{AB}\cos\{(\frac{\pi}{2} - \theta_1) + \theta_2\} \tag{18}$$

$$= \overline{AB}\sin(\theta_1 - \theta_2) \tag{19}$$

$$= \frac{d}{\cos\theta_2}\sin(\theta_1 - \theta_2) \tag{20}$$

$$= d\,\frac{\sin(\theta_1 - \theta_2)}{\cos\theta_2}. \tag{21}$$

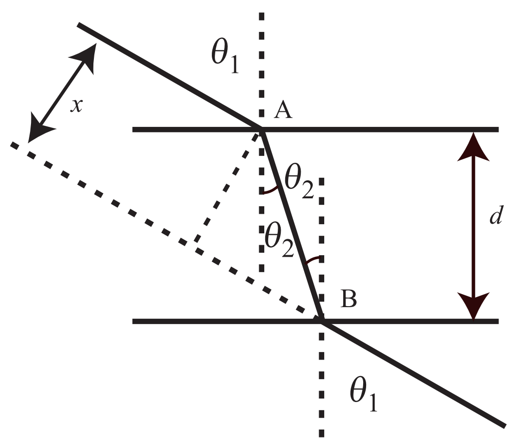

FIG. 4:

10. Applying the Lens Formula to the left lens and the object, we have

$$\frac{1}{0.04} + \frac{1}{v} = \frac{1}{0.08}. \tag{22}$$

Therefore, we get $v = -0.08$ m, that is the image is 0.08 m to the left of the left lens. Next, applying the Lens Formula to this image and the right lens, we have

$$\frac{1}{0.08 + 0.12} + \frac{1}{v'} = \frac{1}{0.08}. \tag{23}$$

So, we get $v' = 0.13$ m. That is, the final image is 0.13 m to the right of the right lens.

11. (i) For P=(a,0), we have

$$r + r' = (a + c) + (a - c) = 2a. \tag{24}$$

Form the definition, $r + r' = 2a$ for any P=(x,y).

(ii) For P=(0,b), we have

$$r + r' = \sqrt{b^2 + c^2} + \sqrt{b^2 + c^2} = 2\sqrt{b^2 + c^2}. \tag{25}$$

Because $r + r' = 2a$ also holds for this point, we have $a = \sqrt{b^2 + c^2}$.

(iii)

$$\begin{aligned}
& r + r' = 2a && (26)\\
\Leftrightarrow\ & \sqrt{(x-c)^2 + y^2)} + \sqrt{(x+c)^2 + y^2} = 2a && (27)\\
\Leftrightarrow\ & 2x^2 + 2c^2 + 2y^2 + 2\sqrt{(x-c)^2 + y^2}\sqrt{(x+c)^2 + y^2} = 4a^2 && (28)\\
\Leftrightarrow\ & \sqrt{(x-c)^2 + y^2}\sqrt{(x+c)^2 + y^2} = 2a^2 - w && (29)\\
\Leftrightarrow\ & ((x-c)^2 + y^2)((x+c)^2 + y^2) = (2a^2 - w)^2 && (30)\\
\Leftrightarrow\ & (w - 2xc)(w + 2xc) = (2a^2 - w)^2 && (31)\\
\Leftrightarrow\ & w^2 - 4x^2c^2 = 4a^4 - 4a^2w + w^2 && (32)\\
\Leftrightarrow\ & (a^2 - c^2)x^2 + a^2y^2 = a^2(a^2 - c^2) && (33)\\
\Leftrightarrow\ & b^2x^2 + a^2y^2 = a^2b^2 && (34)\\
\Leftrightarrow\ & \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 && (35)\\
\Leftrightarrow\ & \frac{x^2}{a^2} + \frac{x^2}{b^2} = 1. && (36)
\end{aligned}$$