FINALS May 10, 2010 Physics 201 Shankar 180 MINS

1. At $t = 0$ when the capacitor is uncharged the switch S in Fig.1 is closed. (i) What are $I(t = 0^+)$ and $I(t = \infty)$, the currents flowing out of the battery just after the switch is closed and at $t = \infty$? (ii) Describe *qualitatively* the currents in the R_1 and R_2C branches as time goes by from 0 to ∞. (iii) What is the final charge on the capacitor? **10, BOOK I**

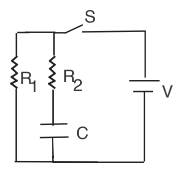

Figure 1: The switch is closed at $t = 0$, at which time the capacitor is uncharged.

2. (i) Find the force exerted by the small loop on the infinite wire shown in Fig. 1. (ii) What is the torque on the loop about an axis that bisects the loop and is parallel to the wire ? **15, BOOK I**

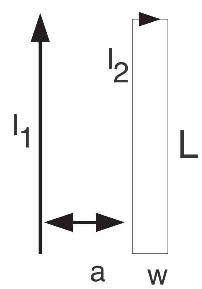

Figure 2: The rectangular loop carries current I_2 and the infinite wire carries current I_1.

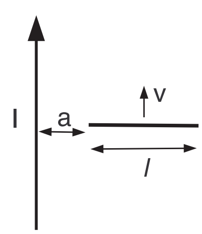

Figure 3: The rod moves parallel to the wire and perpendicular to its own length.

3. In a moving conductor, under equilibrium there should be *no net electromagnetic force* on the charges inside. A conducting rod of length l is oriented perpendicular to an infinite wire carrying current I, with its nearer end a distance a from the wire (Fig. 3). It begins to move with a velocity v parallel to the current. What happens to the charges on this rod when it begins to move ? Which end will be positive? What will be the equilibrium electric field the charges at the ends set up? Show that the EMF due to this field is $\mathcal{E} = \frac{\mu_0 I v}{2\pi} \ln(1 + \frac{l}{a})$. **15, BOOK I**

4. Two infinitely long wires carrying λ C/m lie along the x and y axes. Find the field $\mathbf{E}$ due to each one and the total field at the point $\mathbf{r} = \mathbf{i}x + \mathbf{j}y$. What is $V(1,1) - V(2,2)$, the potential difference between the points $(1,1)$ and $(2,2)$ when $\lambda = 1$ C/m? **15, BOOK II**

5. In the circuit below (Fig. 4) the switch has been closed for a very long time and the voltage across the capacitor is zero. (i) What is the current flowing out of the battery? The switch is opened at $t = 0$. (ii) Write the equation relating the current I flowing through the inductor and the charge Q on the capacitor by going around the loop that includes them. What does this tell you about dI/dt at $t = 0$? (iii) How does the current vary as a function of time in the LC loop? (iv) What is the maximum current in the LC loop? **20, BOOK II**

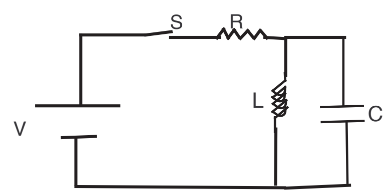

Figure 4:

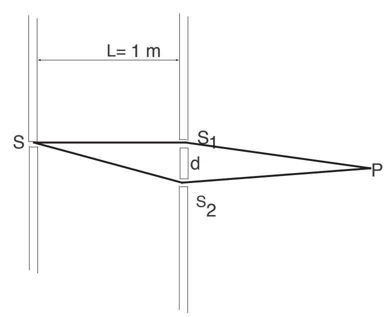

Figure 5:

6. Light of $\lambda = 250nm$ from a point source S reaches a double slit a distance $L = 1m$ away, with the upper slit S_1 in line with the source and the lower one S_2 a distance d below, as shown in Fig. 5. Find the smallest value of d that will ensure that the point P, located equidistant from the slits, is a minimum. Assume $d << L = 1m$. **10, BOOK II**

7. A converging lens of focal length f is placed between an object and a screen located a distance d to the right of the object. Given that $4f < d$, find $u_{\pm}$, the two possible locations for the lens as measured from the object so that a sharp image is formed on the screen. In each case tell me if the image is real or virtual, inverted or right side up and magnified or demagnified? **15, BOOK III**

8. (i) Explain why (with a picture or diagram) $\bar{S}$, the time-averaged intensity of light, defined as the Watts that cross a unit area, is related to the time-averaged energy density $\bar{u}$ by $\bar{S} = \bar{u}c$. **5, BOOK III**

   (ii) A laser with average power $5mW$ makes a spot $1mm$ in radius on a screen. What is $\bar{S}$? (Keep just one significant digit for this and the rest of this problem.) If the light is made of photons of wavelength $\lambda = 400nm$, how many of them hit unit area in one second? If each one has its momentum reversed on hitting the screen, what is the average pressure on the screen? Work with symbols as far as possible. This may save some time on last part (pressure) if you use previous formula for the part just before it. **15, BOOK III**

9. A particle is in the $n = 3$ energy state in a box of length L. What is the probability for finding it in the left half of the box? What is the probability for being in the left one-third of the box? **10, BOOK III**

10. A particle is in the ground state (lowest energy state) of a box that extends from $x = 0$ to $x = L$. Suddenly the box expands to double the size with the right end now at $x = 2L$. Assume that during this instantaneous expansion the wave function of the particle remains unchanged. What is the probability that it will be in the first excited state of the new box? (Draw some pictures and try to avoid hard work.) **15, BOOK IV**

11. There are two particles of mass m in a box of length L and their total energy is $E = \frac{\hbar^2\pi^2}{mL^2}$. What individual energy states are the particles in? Can they be fermions obeying the Pauli Exclusion Principle? **10, BOOK IV**

12. (i) Explain why $\psi(x) = A\cos\frac{\pi x}{L}$ cannot describe a particle of mass m in a ring of circumference L. (ii) Argue that $\psi(x) = A\cos^2\frac{\pi x}{L}$ is however allowed. (iii) When momentum is measured what are the possible outcomes and corresponding relative and absolute probabilities? (iv) Rescale $\psi(x)$ so that these probabilities add up to 1. (v) Taking this $\psi(x)$ to be the initial state $\psi(x,0)$, find the state $\psi(x,t)$ at later times. **25, BOOK IV**

Data Sheet

Electricity and Magnetism

$$\mathbf{F} = q(\mathbf{E} + \mathbf{v} \times \mathbf{B})$$

$$\mathbf{E} = \frac{q}{4\pi\varepsilon_0 r^2}\mathbf{e}_r \qquad (\mathbf{e}_r \text{is unit vector in radial direction})$$

$$d\mathbf{B} = \frac{\mu_0 I}{4\pi}\frac{d\mathbf{l} \times \mathbf{r}}{r^3}$$

$$d\mathbf{F} = I\mathbf{dl} \times \mathbf{B}$$

$$\oint \mathbf{E} \cdot \mathbf{dS} = \frac{Q}{\varepsilon} \qquad \text{Gauss}$$

$$\oint \mathbf{B} \cdot \mathbf{dS} = 0 \qquad \text{No monopoles}$$

$$\oint \mathbf{B} \cdot \mathbf{dl} = \mu_0 I \qquad \text{static case, Ampere}$$

$$\oint \mathbf{B} \cdot \mathbf{dl} = \mu_0 I + \mu_0\varepsilon_0 \frac{\partial \Phi_E}{\partial t} \qquad \text{general case}$$

$$EMF = -\frac{d\Phi_B}{dt} \qquad \text{Lenz's Law}$$

$$V(2) - V(1) = -\int_1^2 \mathbf{E} \cdot \mathbf{dr} \qquad \text{static case}$$

$$u_E = \frac{\varepsilon_0 E^2}{2} \qquad \text{electric energy per unit volume}$$

$$u_B = \frac{B^2}{2\mu_0} \qquad \text{magnetic energy per unit volume}$$

$$E = cB \quad \text{for an EM wave}$$

$$c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}}$$

Optics

$$n_1 \sin\theta_1 = n_2 \sin\theta_2$$

$$\frac{1}{u} + \frac{1}{v} = \frac{1}{f}$$

$$d\sin\theta = m\lambda \quad \text{diffraction grating or double slit}$$

Quantum

$$E = \hbar\omega = hf \qquad p = \hbar k = \frac{h}{\lambda} \quad \text{photon}$$

$$i\hbar\frac{\partial\psi(x,t)}{\partial t} = -\frac{\hbar^2}{2m}\frac{\partial^2\psi(x,t)}{\partial x^2} + V(x)\psi(x,t)$$

$$-\frac{\hbar^2}{2m}\frac{d^2\psi_E(x)}{dx^2} + V(x)\psi_E(x) = E\psi_E(x)$$

$$\psi(x) = \sum_E \psi_E(x)A_E \qquad A_E = \int \psi_E^*(x)\psi(x)dx \qquad P(E) = |A_E|^2$$

$$\psi_p(x) = \frac{e^{ipx/\hbar}}{\sqrt{L}}$$

$$\psi(x) = \sum_p A_p\psi_p(x) \qquad A_p = \int \psi_p^*(x)\psi(x)dx \qquad P(p) = |A_p|^2$$

$$p = \frac{2\pi\hbar}{\lambda}$$

$$\psi_n(x) = \sqrt{\frac{2}{L}}\sin\frac{n\pi x}{L} \qquad \text{particle in a box from } x = 0 \text{ to } x = L$$

$$\psi(x,t) = \sum_E \psi_E(x)A_E(0)e^{-iEt/\hbar} \qquad \text{where } A_E(0) \text{ are coefficients at } t = 0$$

Circuits

$$L\frac{dI}{dt} + RI + \frac{1}{C}\int I(t)dt = V(t)$$

$$L\frac{d^2Q}{dt^2} + R\frac{dQ}{dt} + \frac{1}{C}Q = V(t)$$

$$Z = R + i(\omega L - \frac{1}{\omega C}) = |Z|e^{i\phi}$$

$$I_0 = \frac{V_0}{Z}$$

$$\text{Average Power} = \frac{V_0 I_0}{2}\cos\phi$$

$$\text{Energy stored} = \frac{1}{2}CV^2 = \frac{Q^2}{2C}$$

$$\text{Energy stored} = \frac{1}{2}LI^2$$

Constants and math formulas

$$\mathbf{(1+x)^n = 1 + nx + ...} \qquad \text{if } |x| << 1$$

$$c = 3 \cdot 10^8 m/s$$

$$\hbar = 10^{-34} J \cdot s \qquad h = 6.6\ 10^{-34} J \cdot s$$

$e = 1.6\ 10^{-19}$coulombs is the magnitude of charge of electron or proton.

$$\frac{1}{4\pi\varepsilon_0} = 9 \cdot 10^9 Nm^2/C^2$$

$$\frac{\mu_o}{4\pi} = 10^{-7} N/A^2$$

$$e^{i\theta} = \cos\theta + i\sin\theta$$

$$z = x + iy = |z|e^{i\theta} \qquad |z| = \sqrt{x^2+y^2} = \sqrt{zz^*} \qquad z^* = x - iy = |z|e^{-i\theta}$$

$$\sin^2\theta = \frac{1-\cos 2\theta}{2}$$

$$\cos^2\theta = \frac{1+\cos 2\theta}{2}$$

$$Volume(\text{sphere}) = \frac{4\pi r^3}{3}$$

$$Area(\text{sphere}) = 4\pi r^2$$

$$Area(\text{circle}) = \pi r^2$$

$$\int \sin ax dx = -\frac{\cos ax}{a}$$

$$\int \cos ax dx = \frac{\sin ax}{a}$$

$$\int e^{ax} dx = \frac{e^{ax}}{a}$$

$$\int \frac{dx}{x} = \ln x$$