PS 10 Physics 201 April7, 2010 R.Shankar Due April 8.

1. Consider a line extending from $-L/2$ to $L/2$ with the end points glued together to form a ring of circumference L. The wave function is as shown in Fig. 1. (i) Normalize ψ. (ii) What is $P(x > 0)$, the probability the particle has $x > 0$? (iii) What is the probability it has momentum $p = 0$? (iv) If $p = 0$ is obtained in a momentum measurement, what is the normalized ψ just after the measurement? (v) Now what is $P(x > 0)$?

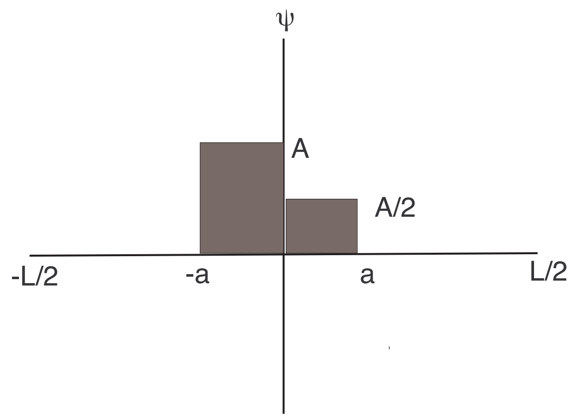

Figure 1: The particle is in a ring of length L obtained by joining $x = \pm L/2$. The initial state ψ has height A for $-a < x < 0$ and $A/2$ for $0 < x < a$.

2. Given

$$\psi(x) = 5\cos^2(2\pi x/L) + 2\sin(4\pi x/L) \tag{1}$$

Find the possible values of p and the corresponding probabilities for obtaining them. Normalizing this is tedious. So use the unnormalized function to read off the *relative* odds. Then rescale them to get the absolute probabilities.

3. A particle in a ring of circumference L extending between $x = \pm L/2$ has a wave function

$$\psi(x) = A \qquad |x| < a \qquad 0 \text{ outside} \tag{2}$$

What is a reasonable estimate for Δx? Normalize ψ and show that

$$|A_p|^2 = \frac{2a}{L}\frac{\sin^2 Z}{Z^2} \qquad \text{where } Z = \frac{pa}{\hbar} \tag{3}$$

Sketch this as a function of Z and show that the first minimum occurs for $p = \pm\pi\hbar/a$. Assuming this is Δp, estimate $\Delta x \Delta p$.

Let us now verify that

$$\sum_p |A_p|^2 = 1 \tag{4}$$

This is hard to do in general since the allowed values of p are discreet and given by

$$p_m = \frac{2\pi m\hbar}{L} \tag{5}$$

Consider now the case where L is very large. The separation dp between one allowed value of p and the next is then

$$dp = p_{m+1} - p_m = \frac{2\pi\hbar}{L} \to 0 \tag{6}$$

Look at Fig. 2, where a few point are shown.

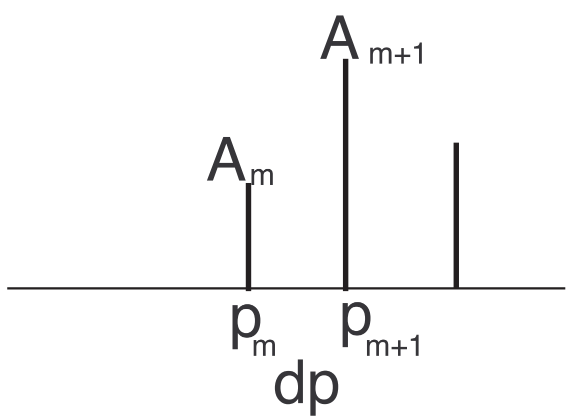

Figure 2: Since $dp = \frac{2\pi\hbar}{L}$ between allowed points is very small, A_p varies pretty much continuously from one p to the next. We can convert the sum to the integral if we multiply it by dp.

Since dp is very small, A_p varies pretty much continuously from one p to the next. *If we multiply $\sum_p |A_p|^2$ by dp, we are simply finding the integral of the continuous function $|A(p)|^2$.* That is

$$(\sum_p |A_p|^2)dp \to \int |A(p)|^2 dp \tag{7}$$

or transferring $dp = \frac{2\pi\hbar}{L}$ to the other side,

$$\sum_p |A_p|^2 = \frac{L}{2\pi\hbar}\int_{-\infty}^{\infty} \frac{2a}{L}\frac{\sin^2}{Z^2}dp \qquad \text{where } Z = \tfrac{pa}{\hbar} \tag{8}$$

Use

$$\int_{-\infty}^{\infty} \frac{\sin^2 Z}{Z^2} = \pi$$

to verify that the $|A_p|^2$ sum to unity.

4. Recall from the last problem that

$$\sum_p f_p = \frac{L}{2\pi\hbar} \int f(p)dp \tag{9}$$

where on the right, the function $f(p)$ is the same function of the continuous variable p as f_p is of the discreet variable p that takes quantized values.

In class we found that for the case $\psi(x) = \sqrt{\alpha}e^{-\alpha|x|}$, the coefficients are given by

$$|A_p|^2 = \frac{4\alpha^3}{L}\left(\frac{1}{\alpha^2 + p^2/\hbar^2}\right)^2 \tag{10}$$

Show that these sum to unity in the large L limit using Eq. 9.