Problem Set 1 Physics 201b January 13, 2010. Due Jan 20

1. I give you a sphere charged with 1 μ C attached to an insulated stick and one more identical sphere similarly insulated. You may discharge any sphere by touching it to the ground. (i) How will you produce a sphere with $\frac{1}{8}$ μ C? (ii) If you cannot ground anything but are given more identical spheres, how many more will you need to get me $\frac{1}{8}$ μ C? (iii) How will you get me $\frac{5}{16}$ μ C?

2. Sketch the lines force when there are charges q and $-2q$ at $x = \pm 1$, using 8 lines for q.

3. At six corners of a hexagon inscribed in a circle of radius $r = 1m$ are placed electrons and at the center is placed a proton. What is the force on the proton? Now remove the electron in the northeast corner and recompute the total force on the proton.

4. Mathematical preliminaries. In this problem set and ones to follow you will need to approximate functions by powers series (Taylor series) keeping a few terms. The Taylor series allows us to write the value of a function f at $x_0 + x$ in terms of f and its derivatives at some other point x_0 as follows:

$$f(x + x_0) = f(x_0) + \left.\frac{df}{dx}\right|_{x_0} x + \frac{1}{2!} \left.\frac{d^2 f}{dx^2}\right|_{x_0} x^2 + \frac{1}{3!} \left.\frac{d^3 f}{dx^3}\right|_{x_0} x^3 + ... \tag{1}$$

   Using this show (up to the order given below)

$$(1 + x)^p = 1 + px + \frac{p(p-1)x^2}{2!} + \frac{p(p-1)(p-2)x^3}{3!} + .. \tag{2}$$

   Write the above result explicitly for the case $p = -1$. Next show

$$\ln(1 + x) = x - \frac{x^2}{2} + \frac{x^3}{3} + ... \tag{3}$$

   It will be assumed from now on that you can derive similar approximations as needed. In each case you must keep the appropriate number of terms in the series.

5. Two sphere of mass m and negligible size are connected to two identical springs of force constant k as shown in Figure 1. The separation is a. When charged to q Coulombs each, the separation doubles. (i) What is k in terms of $q, a,$ and ε_0? (ii) Find k if the separation goes to $a/2$ when the charges are $\pm q$. (iii) In case (i) suppose the charge on the right is held fixed while that on the left is displaced by a tiny amount x and released. Find the resorting force $F = -k_e x$ and the (angular) frequency ω of small oscillations. (I call the effective force constant for oscillations as k_e to distinguish it from the k for the springs.)

6. Imagine four unit charges nailed to four corners of a square of side 2, with the NE corner being at $(x = 1, y = 1)$. Draw pictures whenever appropriate. (i) Show that a charge -1 placed at the origin is in equilibrium, i.e., has no net force on it using

symmetry arguments. (ii) Now consider the stability of this equilibrium by lifting the charge slightly out of the plane by a tiny amount δ. Show that there is a restoring force $-k\delta$ and find k. (Use Taylor series. Since you need the force only for small displacement drop any thing in the formula that goes like $(\text{displacement})^2$ or higher.) (iii) Find ω, the angular frequency of small oscillations if the charge has mass m. (iv) With what speed will it cross the origin if released from $z = \delta$? (v) Establish next the *instability* under displacements in the plane by choosing δ to be along the x-axis and showing $k = -1/(4\pi\varepsilon_0\sqrt{2})$.

7. A rod extends from $x = -a$ to $x = +a$ and carries a charge Q uniformly distributed on it. At the point $x = 2a$ is a point charge Q. Where on the x axis is the field zero?

8. An electron is on a circular orbit around an infinite rod charged with $\lambda = 2\mu\ C/m$. What is its orbital speed? $(1\mu\ C = 10^{-6}C)$ First use symbols to get the velocity v and then put in numbers.

9. A semi-infinite rod extending from the origin up the y-axis carries a linear density $\lambda C/m$. Find the field at the point $x = a, y = 0$. Show how you could relate the x-component to the result for an infinite rod.

10. A semicircular wire of radius a with center at the origin carries a linear density $\lambda C/m$. Find the field at the center. Draw a picture showing orientation of the wire.

11. Take the diagnostic survey covering the topics of electricity and magnetism. To access the survey, go to the P201 classes server, click on the Tests and Quizzes link and then click on the displayed link to start the test. It will help me get a sense of your prior exposure to some of the material we will be covering this semester. Please take the survey in good faith, answering all the questions to the best of your ability without consulting any notes or a textbook. Your absolute score on the test will not count toward your grade. If you complete the survey you will receive full credit for this part of the homework. You have 2 hours to complete the 32 question survey from the time you begin, though it will likely take less time than that. Please make sure you have sufficient time when you start and that your internet connection is stable. You cannot save and continue later.

    If you experience technical problems please email stephen.irons@yale.edu

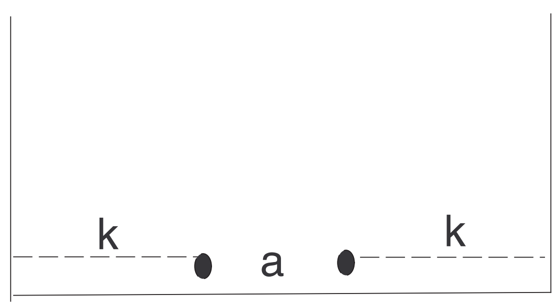

Figure 1: The springs (dotted lines) have a force constant k and the uncharged spheres are separated by a. In part (i) when charged with q Coulombs each, the separation is $2a$. In part (ii) the charges are $\pm q$ and the separation is $a/2$