# PS 6 Physics 201 February 17, 2010 R.Shankar Due February 24.

1. Consider the betatron again using relativistically correct formulas. . Both the Lorentz formula $\mathbf{F} = q(\mathbf{E} + \mathbf{v} \times \mathbf{B})$ and $\mathbf{F} = d\mathbf{p}/dt$ are valid in the final relativistic theory, but now momentum $\mathbf{p} = m\mathbf{v}/\sqrt{1 - v^2/c^2}$. Assuming $\mathbf{p} = 0$ at $t = 0$, show that once again the magnitude of tangential momentum at time $t$ is is $p_T(t) = \frac{qr}{2}\bar{B}(t)$. The available radial force is again $qvB_0$, where $B_0$ is the field at the orbit radius. Show by drawing diagrams for two times very near each other on a circular orbit that the requisite radial force, the rate of change of radial momentum, is $dp_r/dt = \omega p_T$. (The same ideas are invoked in nonrelativistic mechanics where $mv^2/r$ can be viewed as $mv \times v/r$.) Show that we get the same condition $\bar{B}(t) = 2B_0$.

2. A tiny loop of radius $R_1 << R_2$ is concentric with a loop of radius $R_2$, both lying in the x-y plane. Find their mutual inductance, assuming the field of the bigger one is constant over the tiny loop.

3. A loop of resistance $R$, width $w$ (horizontal) and very long length $L$ (vertical) is falling under gravity in a $B$ field perpendicular to its plane and nonzero only below some height, as shown in Fig 1. Assuming some part of the loop is always outside the field region, show that the terminal speed is $v_T = \frac{MgR}{B^2w^2}$. Which way is the current flowing? (Get this from Lenz's Law.)

4. A circular loop of radius $A$ in the plane of the paper contains a resistance $R$ and capacitor $C$. It is in a field $B(t) = B_0\, t$ going into the paper. What is the maximum charge on the capacitor? Draw a picture to indicate the sign of the charges on the capacitor.

5. A rod of mass $m$ and length $w$ and resistance $R$ starts from rest and slides on two parallel rails of zero resistance as shown in Fig.2. A battery of voltage $V$ is connected as shown. (i) Argue that the net EMF in the loop is $V - Bvw$ when the rod has speed $v$. (ii) Write down $F = m\frac{dv}{dt}$ and integrate it so show that

$$v(t) = \frac{V}{Bw}\left[1 - e^{-B^2w^2t/mR}\right].$$

   Hint: Find the limiting speed and separate that out from the total $v$.

6. Consider the circuit in Fig. 3 whose switch is closed at $t = 0$. Show that the current in the inductor is given by

$$I(t) = \frac{V_0}{R_1}\left[1 - e^{-R't/L}\right]$$

   where $R' = R_1R_2/(R_1 + R_2)$. Hint: Write the voltage equations for two loops. Let $I_1$ and $I_2$ flow in $L$ and $R_2$ respectively. Eliminate $I_2$ in favor of $dI_1/dt$. Solve the $I_1$ equation by pulling out asymptotic part.

7. An LCR circuit has $R = 100\Omega$, $L = 10mH$ and a resonant frequency of 3 kHz. (i) What is $C$? (ii) What is $Z$ at 5 kHz? (iii) What is the current in response to a voltage

$V(t) = 200\cos(10000\pi t)$? (iv) What is the average power consumption? (v) What is the maximum voltage on each of the three circuit elements? Why do these numbers not add up to $200V$?

8. A capacitor $C = 20\mu F$ is connected in series with a resistor $R = 100\Omega$ and inductance $L$ and is driven by a voltage $V = 110\cos 100\pi t$. What is the maximum voltage drop across the $R - L$ segment (measured from the beginning of $R$ to the end of $L$) if the circuit is at resonance?

9. Consider an LCR circuit with no external voltage source. Write the equation satisfied by the current, writing the charge on the capacitor as $\int I(t)dt$. Assume $I(t) = I_0 e^{-\alpha t}$ and find the two allowed values for $\alpha$ and write the general answer as a linear combination of the two solutions with coefficients $I_\pm$. Consider the case of small $R$ when the two $\alpha$'s form a complex conjugate pair. What restriction can you place on the two coefficients $I_\pm$ by demanding that the solution be real? Show that in the end $I(t)$ assumes the form
$$I(t) = Ae^{-at}\cos(\omega' t - \phi)$$
and relate the real parameters $a$ and $\omega'$ to $R, L$and $C$. What determines the free parameters $A$ and $\phi$?

10. Here is some practice with AC circuits. You can treat them like DC circuits, add impedances in series or parallel, just assign frequency dependent impedances $i\omega L$ and $1/(i\omega C)$ to inductors and capacitors and of course $R$ to resistors. Consider the following circuit shown in Fig. 4. Show that when $\frac{L}{C} = R_1 R_2$, $V_A - V_B = 0\ \forall\omega$. (When current comes to a fork, the current in each branch will be proportional to the impedance of the other.)

    I hope to post some notes that should help with AC circuits.

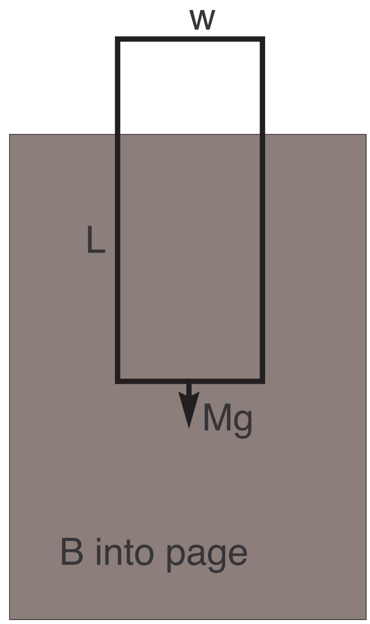


Figure 1: The loop has reached terminal velocity in a $B$ field which goes into paper.

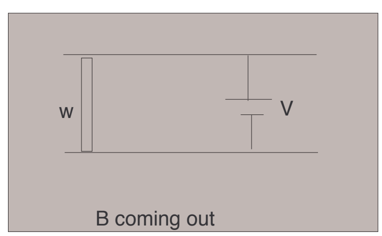


Figure 2: The loop is made of a sliding rod and two fixed rails and powered by a battery with voltage $V$.

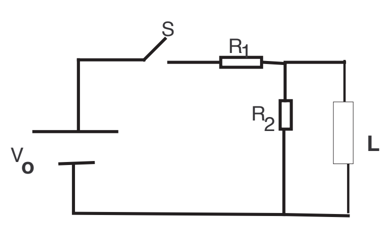


Figure 3: The switch is closed at $t = 0$.

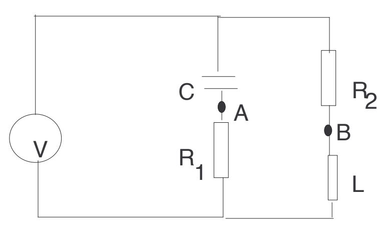


Figure 4: When is $V_A - V_B = 0\ \forall\omega$?