Solutions to PS 8 Physics 201

1. (a)

$$\omega = kc = 6 \times 10^9 \frac{rad}{s} \tag{1}$$

(b)

$$f = \frac{\omega}{2\pi} = 9.55 \times 10^8 Hz \tag{2}$$

(c) Since the argument of the sin function is of the form $ky + \omega t$, we know that

$$\mathbf{k} = -20\mathbf{j} \tag{3}$$

and therefore the wave is propagating in the $-y$ direction

(d)

$$\begin{aligned} \mathbf{B} &= -\frac{1}{c}\mathbf{j} \times \mathbf{k} 1000 \sin\left(20y + 6 \times 10^9 t\right) && (4) \\ &= -3.34 \times 10^{-6} \mathbf{i} \sin\left(20y + 6 \times 10^9 t\right) && (5) \end{aligned}$$

(e)

$$\begin{aligned} \bar{u} &= \epsilon_0 \bar{E^2} && (6) \\ &= \epsilon_0 (1000)^2 \overline{\sin^2 (20y + 6 \times 10^9 t)} && (7) \\ &= \frac{\epsilon_0}{2} (1000)^2 && (8) \\ &= 4.43 \times 10^{-6} \frac{J}{m^3} && (9) \end{aligned}$$

Similarly,

$$\begin{aligned} \bar{S} &= c\bar{u} && (10) \\ &= 1329 \frac{W}{m^2} && (11) \end{aligned}$$

2. The average power P transmitted by the station is equal to the intensity $\bar{S}$ at a distance r times the surface area of a sphere of radius r. Thus,

$$\begin{aligned} P &= 4\pi r^2 \bar{S} && (12) \\ \bar{S} &= \frac{P}{4\pi r^2} && (13) \\ \frac{1}{2} c\epsilon_0 E_{max}^2 &= \frac{P}{4\pi r^2} && (14) \\ E_{max} &= \sqrt{\frac{P}{2\pi r^2 c\epsilon_0}} && (15) \end{aligned}$$

With $P = 50kW$ and $r = 10km$, this gives

$$E_{max} = 0.173\frac{N}{C} \tag{16}$$

and therefore

$$B_{max} = \frac{E_{max}}{c} = 5.78 \times 10^{-10}T \tag{17}$$

3.

$$f = \frac{c}{400nm} = 7.5 \times 10^{14}Hz \tag{18}$$

4. We know that the amplitude of the magnetic field must be $\frac{1}{c}$ times the amplitude of the electric field. Furthermore, we also know that by the right hand rule, the direction of the magnetic field must be perpendicular to the direction of propagation and $(\mathbf{E}, \mathbf{B}, \mathbf{k})$ form a right handed coordinate system. Finally, $\mathbf{E}$ and $\mathbf{B}$ must be in phase. All of this leads to the conclusion that

$$\mathbf{B} = \frac{\mathbf{E_0}}{c}(\mathbf{i} - \mathbf{k})\sin(ky - \omega t) \tag{19}$$

5. (a) For the surface integral over a closed surface, it suffices to compute the surface integral over a cube, since any closed surface can be approximated arbitrarily well by a large number of infinitesimally small cubes. Considering the integral of $\mathbf{E}$ over a cube , we find

$$\int\int \mathbf{E} \cdot d\mathbf{A} = \tag{20}$$

$$\int\int \mathbf{E} \cdot \hat{\mathbf{n}}dA = \tag{21}$$

$$\int\int (E_x(right) - E_x(left))dydz = 0 \tag{22}$$

since E_x depends only on z

Similarly for $\mathbf{B}$ we find

$$\int\int \mathbf{B} \cdot d\mathbf{A} = \tag{23}$$

$$\int\int \mathbf{B} \cdot \hat{\mathbf{n}}dA = \tag{24}$$

$$\int\int (B_y(top) - B_y(bottom))dxdz = 0 \tag{25}$$

(b) Modifying the discussion in class, we get from the line integrals of E that

$$\frac{\partial E_x}{\partial z} = -\frac{\partial B_y}{\partial t} \tag{26}$$

and from the line integrals of B that

$$-\frac{\partial B_y}{\partial z} = \frac{1}{c^2}\frac{\partial E_x}{\partial t} \tag{27}$$

Plugging in the function forms of $\mathbf{E}$ and $\mathbf{B}$ gives

$$kE_0 = \omega B_0 \tag{28}$$

$$kB_0 = \frac{\omega}{c^2}E_0 \tag{29}$$

Thus, we find

$$\omega = |k|\, c \tag{30}$$

$$B_0 = \mathrm{sign}(k)\frac{E_0}{c} \tag{31}$$