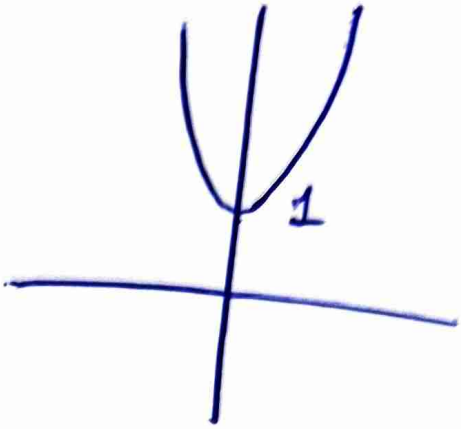
# Δυσκολο SOS

Εστω $f(x) \geq 1$ και

δηλαδη $f(0) = 1$




Να λυθει η εξισωση $f(x) + x^2 = 1$

$f(x) - 1 + x^2 = 0$

Προφανη ριζα
$x = 0$

Αν $x \neq 0$

- $f(x) > 1 \Rightarrow f(x) - 1 > 0$
- $x^2 > 0$




$f(x) - 1 + x^2 > 0$

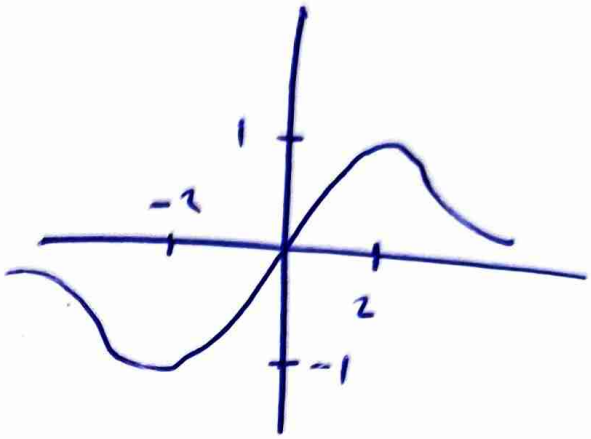
Δεν υπαρχει αλλη ριζα!

# 6. $-1 \le f(x) \le 1$

ΕΝΟΤΗΤΑ 6

(6) iii) $f(x) = |x| + 1$

$0 = |x| + 1 - f(x)$




(7)

- $|x| \ge 0$ Το "=" για $x = 0$
- $f(x) \le 1 \Rightarrow 1 - f(x) \ge 0$ Το "=" για $x = 2$

$$|x| + 1 - f(x) \ge 0$$

0 2

$\nexists\, x_0 \in \mathbb{R}$

Τ.ω $|x_0| + 1 - f(x_0) = 0$

(ε) $|f(\alpha)| + |f(\beta)| = 2$

$|f(x)| \leq 1$ Το "=" $x = 2$

$|f(\alpha)| \leq 1$ Το "=" $\alpha = 2$

$|f(\beta)| \leq 1$ (+) Το "=" $\beta = 2$

$|f(\alpha)| + |f(\beta)| \leq 2$

$f(\alpha) + f(\beta) = 2$

Μαθ αν $\alpha = 2$ $\beta = 2$

8. (α) $f(x) = \dfrac{x^2+5x+1}{x}$

$$f(x) = \frac{x^2}{x} + \frac{5x}{x} + \frac{1}{x}$$

$$f(x) = x + 5 + \frac{1}{x}$$

Για $x>0$ ισχύει ότι $x + \frac{1}{x} \geq 2$

γιατί $x^2 + 1 \geq 2x$

$x^2 - 2x + 1 \geq 0$

$(x-1)^2 \geq 0$ ✓

Άρα $x + \frac{1}{x} \geq 2 \Rightarrow x + \frac{1}{x} + 5 \geq 7$

$f(x) \geq 7$

$\longrightarrow f(x) = 7 \Rightarrow x + \frac{1}{x} + 5 = 7 \Rightarrow x + \frac{1}{x} = 2$

$\Rightarrow x^2 + 1 = 2x \Rightarrow x^2 - 2x + 1 = 0 \Rightarrow (x-1)^2 = 0$

$x = 1$

$A(1,7)$ Ο.Ε.

(B) Νδο $f(2021) + \frac{3}{7} f(2022) > 10$

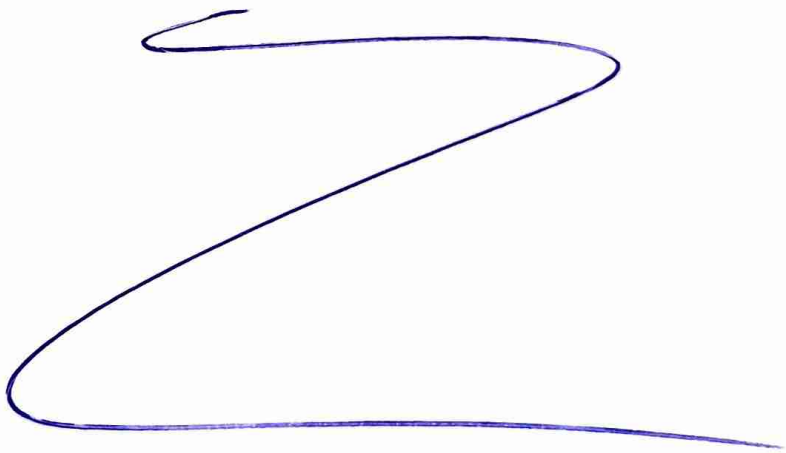
$f(x) \geq 7 \quad \forall x > 0$ Το "=" για $x = 1$

$f(2021) > 7 \Rightarrow 7 f(2021) > 49$ (1)

$f(2022) > 7 \Rightarrow 3 f(2022) > 21$

$7 f(2021) + 3 f(2022) > 70$

$f(2021) + \frac{3}{7} f(2022) > 10$



# 9.

$3f(x)+1 \leq 0$ $\qquad f(2) = -\frac{1}{3}$

(α) $3f(x) \leq -1$

$f(x) \leq -\frac{1}{3}$ Μεγιστη Τιμη $= -\frac{1}{3}$.

(β) $f(x) = 0$

Αφου $f(x) \leq -\frac{1}{3}$ τοτε $f(x) \neq 0$

Αδυνατη.

(γ) $f(x) = (x-2)^2 - \frac{1}{3}$

$0 = (x-2)^2 - \frac{1}{3} - f(x)$

Το $x=2$ προφανως δεν

Εστω $x \neq 2$.

$(x-2)^2 - \frac{1}{3} - f(x) > 0$

- $(x-2)^2 > 0$ $\oplus$
- $f(x) < -\frac{1}{3}$ $\Rightarrow 0 < -\frac{1}{3} - f(x)$

**10.** $f(-3)=1$

$f(1)=2$

$f^2(x)-3f(x)+2 \le 0$

(a)

Θετω $f(x)=t$

$t^2-3t+2 \le 0$

| $t$ | | $1$ | | $2$ |
|---|---|---|---|---|
| $t^2-3t+2$ | $+$ | $0$ | $-$ | $0$ $+$ |

$t \in [1,2]$

$A(-3,1)$ Ο.Ε

$B(1,2)$ Ο.Μ.

$1 \le t \le 2$

$$1 \le f(x) \le 2$$

$$f(-3) \le f(x) \le f(1)$$

(β) $g(x)=|f(x)-1|+|f(x)-2|$ (+) (−)

- $1 \le f(x) \le 2 \Rightarrow 0 \le f(x)-1 \le 1$
- $1 \le f(x) \le 2 \Rightarrow -1 \le f(x)-2 \le 0$

$g(x)=f(x)-1-f(x)+2$

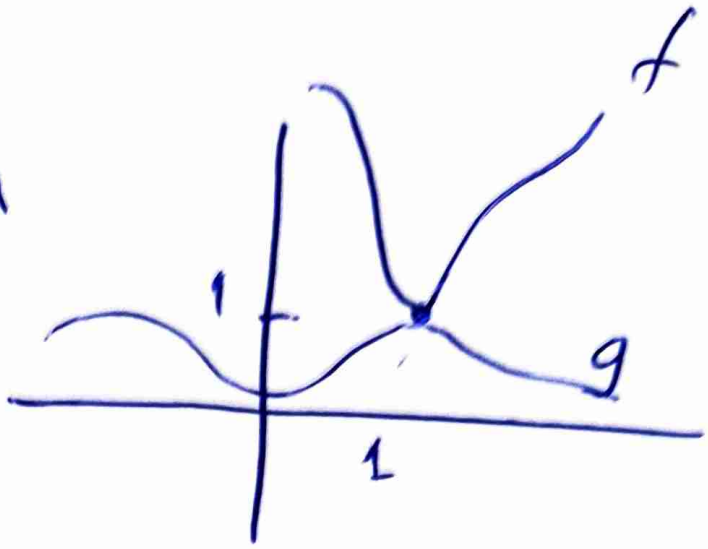
$g(x)=1$

(γ) $2f(x)-5=0$

$f(x)=\frac{5}{2}$

Δεκτή

**11.** (4) Δηο σχημα




$f(x) \geq 1$

$g(x) \leq 1$.

(B) $|x-1|+1 = \frac{2x^2}{x^4+1}$

$f(x) = g(x)$

$\underbrace{f(x)-g(x)}_{h(x)} = 0$

$h(x) = 0$

Προφανή ρίζα $x=1$

γιατί $h(1) = f(1) - g(1) = 1-1 = 0.$

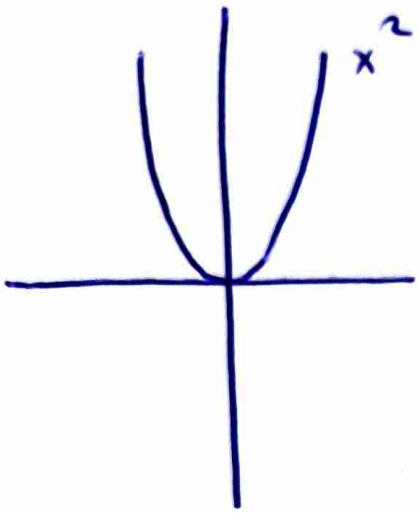
Έστω $x \neq 1$

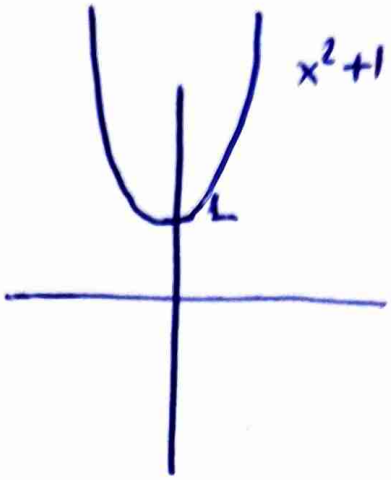
$\cdot f(x) > 1$ ——— (+)

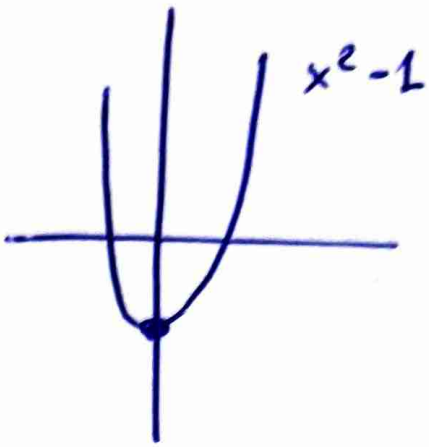
$-g(x) < 1 \Rightarrow -g(x) > -1$

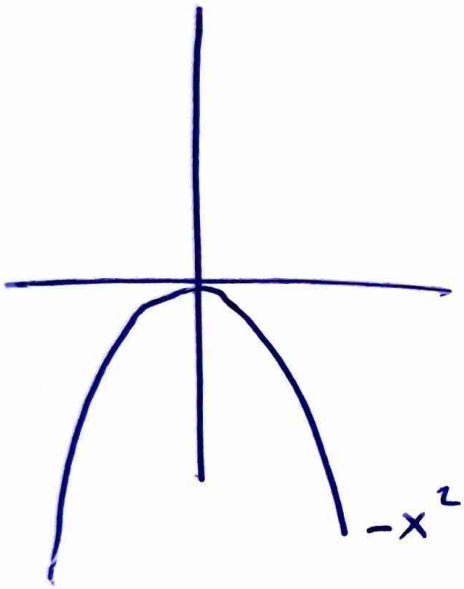
$f(x) - g(x) > 1-1$

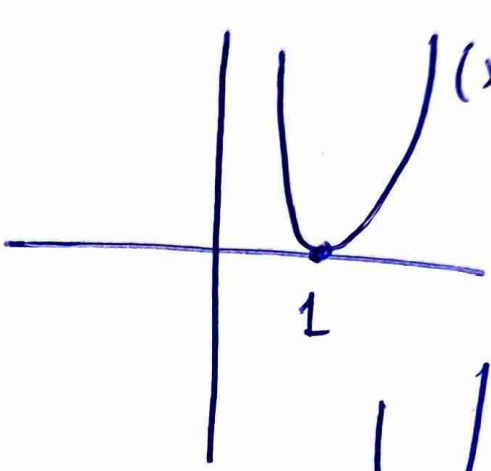
$h(x) > 0$

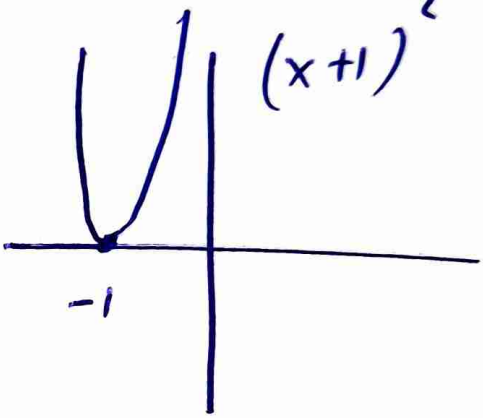


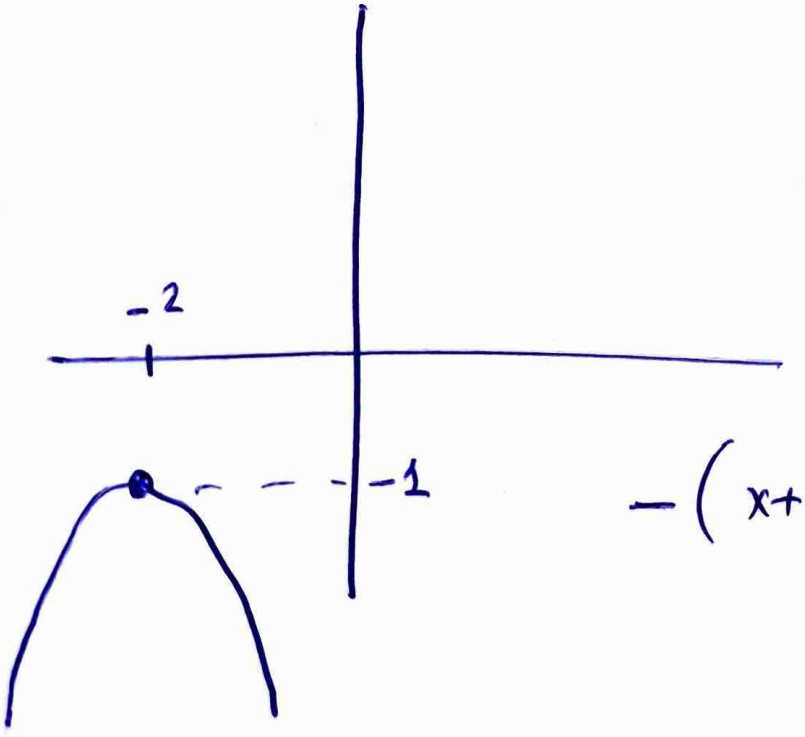



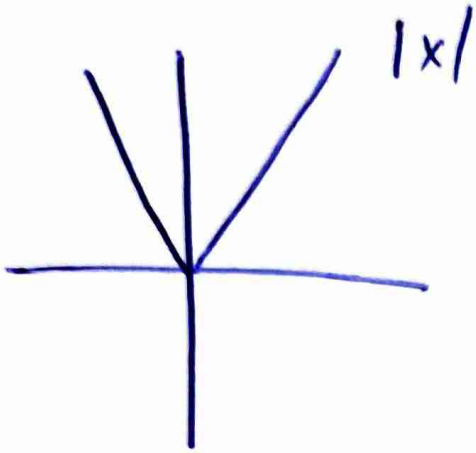



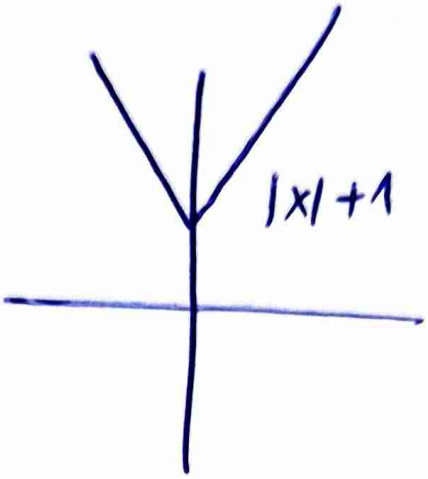



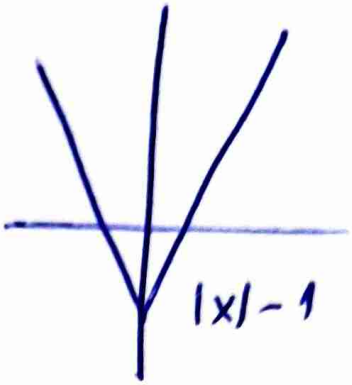



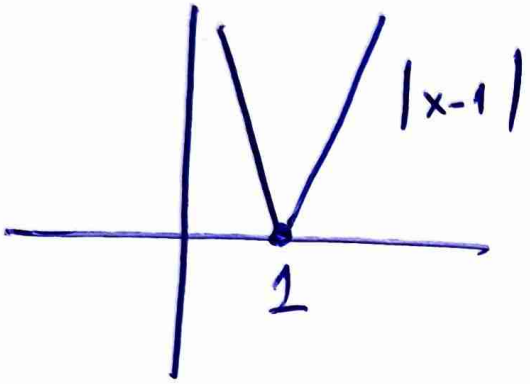



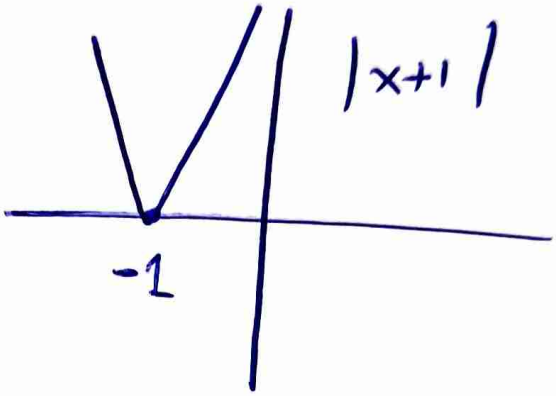

$-(x+2)^2-1$.

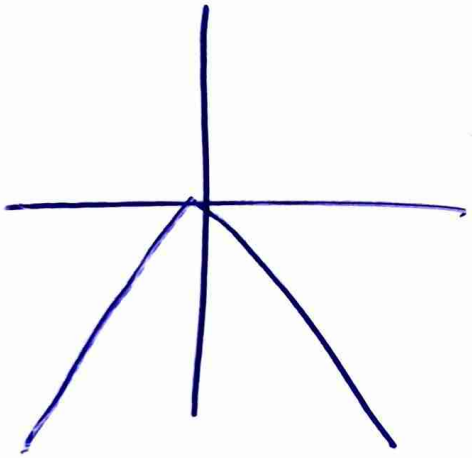

|x|

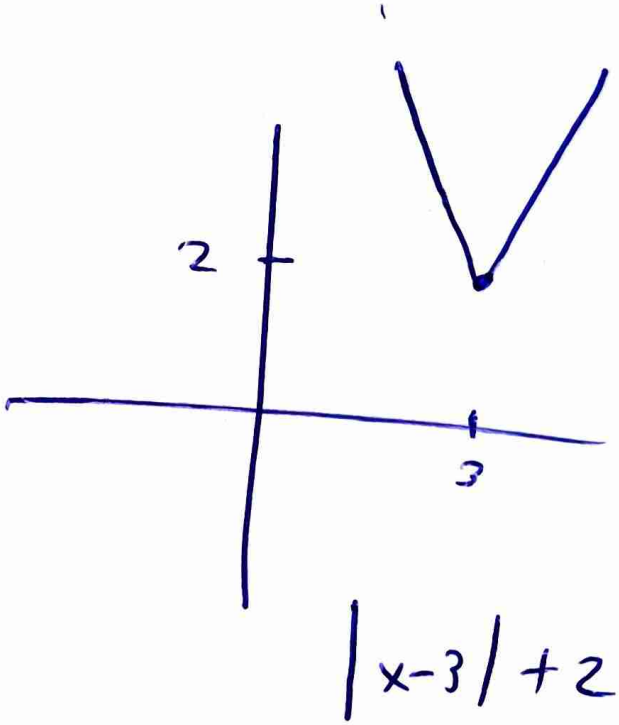

|x|+1


|x|-1


|x-1|
1


|x+1|
-1


-|x|

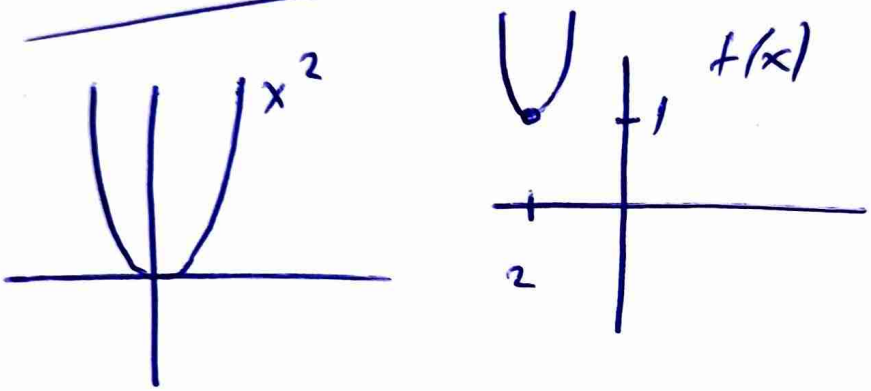

2
3
|x-3|+2

# Пробни

1. $f(x) = x^2 + 4x + 5$

$f(x) = x^2 + 4x + 4 + 1$

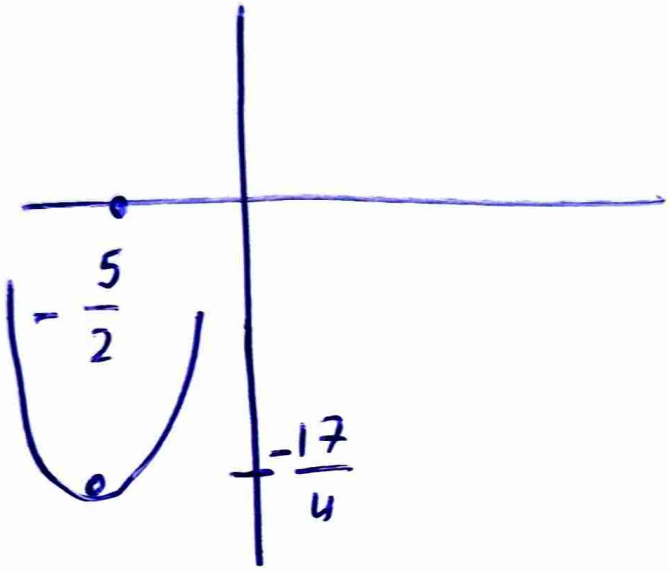
$\boxed{f(x) = (x+2)^2 + 1}$



2. $f(x) = x^2 + 5x + 2$

$$f(x) = x^2 + 2 \cdot \frac{5}{2} x + \frac{25}{4} - \frac{25}{4} + 2$$

$$\boxed{f(x) = \left(x + \frac{5}{2}\right)^2 - \frac{17}{4}}$$




В'троw/

$$f(x) = x^2 + 5x + 2 = \left(x + \frac{5}{2}\right)^2 - \frac{17}{4}$$

$$K\left(-\frac{B}{2a}, -\frac{\Delta}{4a}\right)$$

$$K\left(-\frac{5}{2}, -\frac{17}{4}\right)$$

3. $f(x) = 3x^2 - 2x + 1$

$f(x) = 3\left(x^2 - \frac{2}{3}x + \frac{1}{3}\right)$

$f(x) = 3\left(x^2 - 2\cdot\frac{1}{3}\cdot x + \frac{1}{9} - \frac{1}{9} + \frac{1}{3}\right)$

$f(x) = 3\left((x - \frac{1}{3})^2 + \frac{2}{9}\right)$

$$\boxed{f(x) = 3\left(x - \tfrac{1}{3}\right)^2 + \frac{2}{3}}$$

