

1. На лоду и сѣновис.

$$\frac{4}{x-2} = \frac{8}{x^2-2x} - \frac{x-2}{x}$$

$$x(x-2) \frac{4}{x-2} = \frac{x(x-2) 8}{x(x-2)} - \frac{x(x-2)(x-2)}{x} \quad \text{Екп: } x(x-2)$$

$$x \cdot 4 = 8 - (x-2) \cdot (x-2)$$

$$x \cdot 4 = 8 - (x^2 - 2x + x^2 - 4)$$

$$0 = 8 - x^2 + 2x - x^2 + 4 - 4x$$

$$0 = -2x - 2x^2 + 12$$

$$0 = 2(-x - x^2 + 6)$$

$\sigma = 2$ решения

$$\Delta = 1 - 4 \cdot (-1) \cdot 6$$

$$\Delta = 1 + 24$$

$$\Delta = 25$$

$$x = \frac{b \pm \sqrt{\Delta}}{2a}$$

$$x = \frac{1 \pm 5}{-2}$$

$$x = -3$$

$$x = 2$$

прим

$$x \neq 0 \quad x \neq 2.$$

2. Να λυθεί η εξίσωση $\lambda^2(x-1) = 2(2x-\lambda)$

$$\lambda^2 x - \lambda^2 = 4x - 2\lambda$$

$$(\lambda^2 - 4)x = (\lambda^2 - 2\lambda)$$

$$(\lambda+2)(\lambda-2)x = \lambda(\lambda-2)$$

Αν το $\lambda = +2$, τότε ~~$(4-4)x = 4-4$~~ $0x = 0$ Αδριστο ή ταυτολογία

Αν το ~~$\lambda = -2$~~ , τότε $0x = -8$... αδύνατο.

Αν το $\lambda \neq \pm 2$, τότε $(\lambda+2)(\lambda-2)x = \lambda(\lambda-2)$

$$x = \frac{\lambda(\lambda-2)}{(\lambda+2)(\lambda-2)} = \frac{\lambda}{(\lambda+2)}$$

3. Έστω $A = |x+3|$

Να λυθεί η εξίσωση

$$A^2 + A - 6 = 0$$

$$\Delta = B^2 - 4\alpha\gamma$$

$$\Delta = 1^2 - 4 \cdot (-6)$$

$$\Delta = 25$$

$$A = \frac{-1 \pm 5}{2}$$

2

-3

$$A = 2 \quad \text{ή} \quad A = -3$$

$$|x+3| = 2 \quad \text{ή} \quad |x+3| = -3$$

Αδυσχ

$$x+3 = 2$$

$$\text{ή} \quad x+3 = -3$$

$x = -1$

$x = -5$

4. Na λούδα η ερίσων.

$$\left| |7 - |2x - 3|| - 6 \right| = 4$$

$$|7 - |2x - 3|| - 6 = 4 \quad \vee \quad |7 - |2x - 3|| - 6 = -4$$

$$|7 - |2x - 3|| = 10$$

$$|7 - |2x - 3|| = 2$$

$$7 - |2x - 3| = 10 \quad \vee$$

$$7 - |2x - 3| = -10$$

$$-|2x - 3| = 3$$

$$-|2x - 3| = -10 - 7$$

$$-|2x - 3| = -17$$

$$|2x - 3| = -3$$

$$|2x - 3| = 17$$

Αδυνατά

$$2x - 3 = 17 \quad \vee \quad 2x - 3 = -17$$

$$2x = 20$$

$$2x = -14$$

$$x = 10$$

$$x = -7$$

$$7 - |2x - 3| = 2 \quad \vee$$

$$7 - |2x - 3| = -2$$

$$5 = |2x - 3|$$

$$2x - 3 = 5 \quad \vee \quad 2x - 3 = -5$$

$$2x = 8$$

$$2x = -2$$

$$x = 4$$

$$x = -1$$

$$|x - 3| = 9$$

$$2x - 3 = 9$$

$$2x = 12 \quad x = 6$$

η

$$2x - 3 = -9 \quad \vee$$

$$2x = -6$$

$$x = -3$$

5. Να λυθεί η εξίσωση

$$| |x| - 3 | = | 2|x| - 1 |$$

~~$$| |x| - 3 | - | 2|x| - 1 | = 0$$~~

$$|x| - 3 = 2|x| - 1 \quad \text{ή} \quad |x| - 3 = -2|x| + 1$$

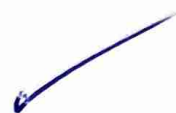
$$-|x| = 2$$

Αδυνατεί

$$3|x| = 4$$

$$|x| = \frac{4}{3}$$

$$x = \frac{4}{3} \quad \text{ή} \quad x = -\frac{4}{3}$$



6. Να λυθεί η εξίσωση

$$(12x-11-6)^3 + 27 = 0$$

$$(12x-11-6)^3 = -27$$

$$(12x-11-6)^3 = (-3)^3$$

$$|2x-1|-6 = -3$$

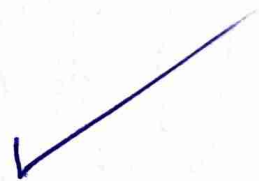
$$12x-11 = 3$$

~~$$2x-1 = 3$$~~

$$2x-1 = 3 \quad \vee \quad 2x-1 = -3$$

$$2x = 4 \quad \vee \quad 2x = -2$$

$$\underline{x = 2} \quad \vee \quad \underline{x = -1}$$



7. Δίνεται η εξίσωση

$$(x-2)^2 - 6(1+x) = (2-2x)(x-1)(x+1) - 2x$$

η οποία έχει παραδική λύση

α) Βρλ το λ .

β) Αν η παραδική λύση ~~ρ ∈ [-1, 1]~~
Βρλ το λ .

γ) Αν η παραδική λύση είναι
 $\rho = \frac{1}{2}$ Βρλ το λ .

$$(2-2)^2 - 6(1+x) = (2-2x)(2-1)(2+1) - 22$$

$$2^2 - 42 + 4 - 6 - 6x = (2-2x)(2^2 - 1) - 22$$

$$2^2 - 42 - 2 - 6x = 22^2 - 2 - 2x2^2 + 2x - 22$$

~~$$2^2 - 42 - 2 - 6x - 22^2$$~~

$$-6x + 2x2^2 - 2x = 22^2 - 2 - 22 - 2^2 + 42 + 2$$

$$x(-6 + 22^2 - 2) = 2^2 + 22$$

$$x(-8 + 22^2) = 2^2 + 22$$

$$x \cdot 2(2^2 - 4) = 2(2 + 2)$$

$$2(2-2)(2+2) \cdot x = 2(2+2)$$

$$1. \quad 2-2=0 \Rightarrow 2=2 \quad 0x=8 \quad \text{Αδύνατη}$$

$$2. \quad 2+2=0 \Rightarrow 2=-2 \quad 0x=0 \quad \text{ταυτολογία}$$

$$3. \quad 2 \neq 2, \quad 2 \neq -2 \quad x = \frac{2(2+2)}{2(2-2)(2+2)} = \frac{2}{2(2-2)}$$

$$a. \quad 2 \neq -2 \quad 1 \neq 2$$

$$(B) \quad -1 < p < 1 \quad \Rightarrow \quad -1 < \frac{1}{2(2-2)} < 1$$

$$\textcircled{8} \quad \rho = \frac{1}{2}$$

$$\frac{\lambda}{2(\lambda-2)} = \frac{1}{2}$$

$$2\lambda = 2(\lambda-2)$$

$$2\cancel{\lambda} = \cancel{2\lambda} - 4$$

$$0 = -4$$

Δα υπάρχει $\lambda \in \mathbb{R}$ ώστε

το $\frac{1}{2}$ να είναι

η ποσότητα ρ_{max} .

8. ^α Να υπολογιστούν τα
 $(\sqrt{3}+2)^2$ και $(\sqrt{3}-2)^2$

β) $A = (\sqrt{7+4\sqrt{3}} - \sqrt{7-4\sqrt{3}}) \sqrt{3}.$

α) $(\sqrt{3}+2)^2 = \sqrt{3}^2 + 2 \cdot \sqrt{3} \cdot 2 + 2^2 = 3 + 4\sqrt{3} + 4 = 7 + 4\sqrt{3}$

$(\sqrt{3}-2)^2 = \sqrt{3}^2 - 2 \cdot \sqrt{3} \cdot 2 + 2^2 = 3 - 4\sqrt{3} + 4 = 7 - 4\sqrt{3}.$

β) $A = (\sqrt{7+4\sqrt{3}} - \sqrt{7-4\sqrt{3}}) \sqrt{3}$

$A = (\sqrt{(\sqrt{3}+2)^2} - \sqrt{(\sqrt{3}-2)^2}) \sqrt{3}.$

$(|\sqrt{3}+2| - |\sqrt{3}-2|) \sqrt{3}$

$(\sqrt{3}+2 - \sqrt{3}+2) \sqrt{3}$

$4\sqrt{3}$

$$9. A = \sqrt[3]{\sqrt{a}} \cdot \sqrt[9]{a}$$

$$B = a^{\frac{1}{2}} \sqrt[3]{a^2} =$$

$$A = \sqrt[3]{a^{\frac{1}{2}}} \cdot a^{\frac{1}{9}}$$

$$B = a^{\frac{1}{2}} \cdot a^{\frac{2}{3}}$$

$$A = a^{\frac{\frac{1}{2}}{3}} \cdot a^{\frac{1}{9}}$$

$$B = a^{\frac{3}{6}} \cdot a^{\frac{4}{6}}$$

$$B = a^{\frac{7}{6}}$$

$$A = a^{\frac{1}{6}} \cdot a^{\frac{1}{9}}$$

$$A = a^{\frac{3}{18}} \cdot a^{\frac{2}{18}}$$

$$A = a^{\frac{5}{18}}$$

$$\sqrt[p]{a^v} = a^{\frac{v}{p}}$$

$$10. A = \frac{\sqrt{5}}{\sqrt{5} + \sqrt{3}} + \frac{\sqrt{3}}{\sqrt{5} - \sqrt{3}}.$$

$$A = \frac{\sqrt{5}(\sqrt{5} - \sqrt{3})}{(\sqrt{5} + \sqrt{3})(\sqrt{5} - \sqrt{3})} + \frac{\sqrt{3}(\sqrt{5} + \sqrt{3})}{(\sqrt{5} - \sqrt{3})(\sqrt{5} + \sqrt{3})} \quad \text{①}$$

$$A = \frac{\sqrt{5}^2 - \sqrt{5}\sqrt{3}}{\sqrt{5}^2 - \sqrt{3}^2} + \frac{\sqrt{3}\sqrt{5} + \sqrt{3}^2}{\sqrt{5}^2 - \sqrt{3}^2}$$

$$A = \frac{5 - \cancel{\sqrt{5}\sqrt{3}} + \cancel{\sqrt{3}\sqrt{5}} + 3}{2} \quad \Rightarrow$$

$$A = \frac{8}{2}$$

$$\boxed{A = 4}$$

11. a) $|x-1|=4$

d) $x^4 - 1 = 0$

b) $|2x-4|=|x-2|$

e) $x^3 + 8 = 0$

f) $x^4 = x^3$

f) $x^4 + 8 = 0$

a) $x-1=4$ in $x-1=-4$
 $x=5$ $x=-3$ ✓

b) $|2x-4|=|x-2|$

$2x-4=x-2$ in $2x-4=-x+2$

$2x-x=2$

$3x=6$

$x=2$

$x=2$ ✓

$x^3 = -8$
 $x^3 = (-2)^3$

g) $x^4 = x^3$

e) $x^3 + 8 = 0$

$x^4 - x^3 = 0$

~~$x^3 + 2^3 = 0$~~

$x^3(x-1)=0$

~~$(x+2)^3 = 0$~~

$x=0$ in $x-1=0$
 $x=1$ ✓

~~$x^3 + 3x^2 \cdot 2 + 3x \cdot 2 + 2^3 = 0$~~

~~$x^3 + 6x^2 + 6x + 8 = 0$~~

~~$x^2(x+6) + 2(4x+2) = 0$~~

$x = -2$

h) $x^4 - 1 = 0$

$(x-1)(x+1) = 0$

$x-1=0$ in $x+1=0$

$x=1$

$x=-1$

$$12. \textcircled{a} |x-3| = 2x-4$$

$$\textcircled{b} |x-3| = 3-x$$

$$a. \quad x-3 = 2x-4 \quad \text{и} \quad x-3 = -2x+4$$

$$-x = -1$$

$$\text{и} \quad 3x = 7$$

$$\underline{x=1}$$

$$\text{и} \quad \underline{x = \frac{7}{3}}$$

$$b. \quad x-3 = 3-x \quad \text{и} \quad x-3 = -3+x$$

$$2x = 6$$

$$\underline{x=3}$$

$$\text{и} \quad ?$$

$$\textcircled{a} |x-3| = 2x-4$$

$$\text{при } 2x-4 \geq 0$$

$$x-3 = 2x-4 \quad \text{и} \quad x-3 = -2x+4$$

$$-x = -1$$

$$\cancel{x=1}$$

$$3x = 7$$

$$\textcircled{x = \frac{7}{3}} \quad \checkmark$$

$$\textcircled{b} |x-3| = 3-x$$

$$\text{при } 3-x \geq 0$$

$$x-3 = 3-x \quad \text{и} \quad \cancel{x-3 = -3+x}$$

$$2x = 6$$

$$\textcircled{x=3}$$

$$\text{Априори}$$

$$\underline{\underline{x \leq 3}}$$