25. (γ) $\lim_{x\to 0}\left(x\eta\mu\frac{1}{x}+\sigma\upsilon\nu x\right)$

ΕΝΟΤΗΤΑ 8

$\rightarrow \lim_{x\to 0}\sigma\upsilon\nu x = 1$

$\rightarrow \lim_{x\to 0} x\eta\mu\frac{1}{x} \xlongequal[\text{ΧΤΙΣΙΜΟ}]{\text{ημ∞}} 0$

$-1 \le \eta\mu\frac{1}{x} \le 1$

$\left|\eta\mu\frac{1}{x}\right| \le 1$

$|x|\left|\eta\mu\frac{1}{x}\right| \le |x|$

$\left|x\eta\mu\frac{1}{x}\right| \le |x|$

$$\boxed{-|x| \le x\eta\mu\frac{1}{x} \le |x|}$$

$0+1=\underline{\underline{1}}$

$\left.\begin{array}{l}\cdot \lim_{x\to 0} -|x| = 0 \\ \cdot \lim_{x\to 0} |x| = 0\end{array}\right\}$ Απο Κ.Π

$\lim_{x\to 0} x\eta\mu\frac{1}{x} = 0.$

(δ) $$\lim_{x\to 0}\left(\frac{\text{ημ}^2 x}{x^2} + x^2\,\text{ημ}\frac{1}{x}\right) = 1 + 0 = 1,$$

$$\to \lim_{x\to 0}\frac{\text{ημ}^2 x}{x^2} = 1^2 = 1$$

$$\to \lim_{x\to 0} x^2\,\text{ημ}\frac{1}{x} = 0$$

$$-1 \le \text{ημ}\frac{1}{x} \le 1$$

$$\boxed{-x^2 \le x^2\,\text{ημ}\frac{1}{x} \le x^2}$$

$$\left.\begin{aligned}&\lim_{x\to 0} -x^2 = 0\\ &\lim_{x\to 0} x^2 = 0\end{aligned}\right\}\ \text{Από Κ.Π.}\quad \lim_{x\to 0} x^2\,\text{ημ}\frac{1}{x} = 0$$

26. δε $\lim_{x\to 0} (\text{συν}x-1)\,\text{ημ}\frac{1}{x} =$

$-1 \le \text{ημ}\frac{1}{x} \le 1$

• $\text{συν}x \le 1$

$\text{συν}x - 1 \le 0$

$$\boxed{-(\text{συν}x-1) \ge (\text{συν}x-1)\,\text{ημ}\frac{1}{x} \ge \text{συν}x-1}$$

$$\left.\begin{array}{l} \bullet\ \lim_{x\to 0} -(\text{συν}x-1) = 0 \\ \\ \bullet\ \lim_{x\to 0} \text{συν}x-1 = 0 \end{array}\right\} \text{Από Κ.Π}$$

$\lim_{x\to 0} (\text{συν}x-1)\,\text{ημ}\frac{1}{x} = 0$

Ενοτητα

10

9. (α) $\lim\limits_{x \to +\infty} \sqrt{x^2 - 3x + 5} = +\infty$

(γ) $\lim\limits_{x \to +\infty} \left(\sqrt{x^2+1} + x \right) = +\infty$

(ε) $\lim\limits_{x \to +\infty} \dfrac{\sqrt{x^2+1} + x}{x} =$

$= \lim\limits_{x \to +\infty} \dfrac{\sqrt{x^2\left(1+\frac{1}{x^2}\right)} + x}{x} = \lim\limits_{x \to +\infty} \dfrac{\cancel{x}\sqrt{1+\frac{1}{x^2}} + \cancel{x}}{\cancel{x}}$

$= \lim\limits_{x \to +\infty} \dfrac{\sqrt{1+\frac{1}{x^2}} + 1}{1} = 1 + 1 = 2.$

12. ⓐ $\lim\limits_{x \to +\infty} \left(\sqrt{4x^2+x+1} - 2x + 1\right)$

$$= \lim_{x \to +\infty} \sqrt{4x^2+x+1} - (2x-1)$$

$$= \lim_{x \to +\infty} \frac{\left[\sqrt{4x^2+x+1} - (2x-1)\right]\left[\sqrt{4x^2+x+1} + 2x-1\right]}{\sqrt{4x^2+x+1} + 2x-1}$$

$$= \lim_{x \to +\infty} \frac{\sqrt{4x^2+x+1}^{\,2} - (2x-1)^2}{\sqrt{4x^2+x+1} + 2x-1}$$

$$= \lim_{x \to +\infty} \frac{4x^2+x+1 - (4x^2-4x+1)}{\sqrt{4x^2+x+1} + 2x-1}$$

$$= \lim_{x \to +\infty} \frac{5x}{\sqrt{4x^2+x+1} + 2x - 1}$$

$$= \lim_{x \to +\infty} \frac{5x}{\sqrt{x^2\left(4+\frac{1}{x}+\frac{1}{x^2}\right)} + x\left(2-\frac{1}{x}\right)}$$

$$= \lim_{x \to +\infty} \frac{5\cancel{x}}{\cancel{x}\sqrt{4+\frac{1}{x}+\frac{1}{x^2}} + \cancel{x}\left(2-\frac{1}{x}\right)}$$

$$= \frac{5}{2+(2-0)} = \frac{5}{4}.$$

11. ⓐ $\lim\limits_{x \to +\infty} \sqrt{x^2+1} = +\infty$

ⓑ $\lim\limits_{x \to -\infty} \sqrt{x^2+1} = +\infty$

ⓒ $\lim\limits_{x \to +\infty} \dfrac{\sqrt{x^2+1}}{x} = \lim\limits_{x \to +\infty} \dfrac{\sqrt{x^2\left(1+\frac{1}{x^2}\right)}}{x} =$

$= \lim\limits_{x \to +\infty} \dfrac{\cancel{x}\sqrt{1+\frac{1}{x^2}}}{\cancel{x}} = \sqrt{1+0} = 1$

ⓓ $\lim\limits_{x \to +\infty} (f(x)-x) = \lim\limits_{x \to +\infty} \left(\sqrt{x^2+1} - x\right)$

$= \lim\limits_{x \to +\infty} \dfrac{1}{\sqrt{x^2+1}+x} = 0.$

ε) $\lim\limits_{x \to -\infty} \dfrac{\sqrt{x^2+1}}{x} = \lim\limits_{x \to -\infty} \dfrac{\sqrt{x^2\left(1+\frac{1}{x^2}\right)}}{x} =$

$$= \lim_{x \to -\infty} \frac{-\cancel{x}\sqrt{1+\frac{1}{x^2}}}{\cancel{x}} = \underline{\underline{-1}}$$

στ) $\lim\limits_{x \to -\infty} \sqrt{x^2+1} + x =$

$$= \lim_{x \to -\infty} \frac{1}{\sqrt{x^2+1} - x} = 0$$

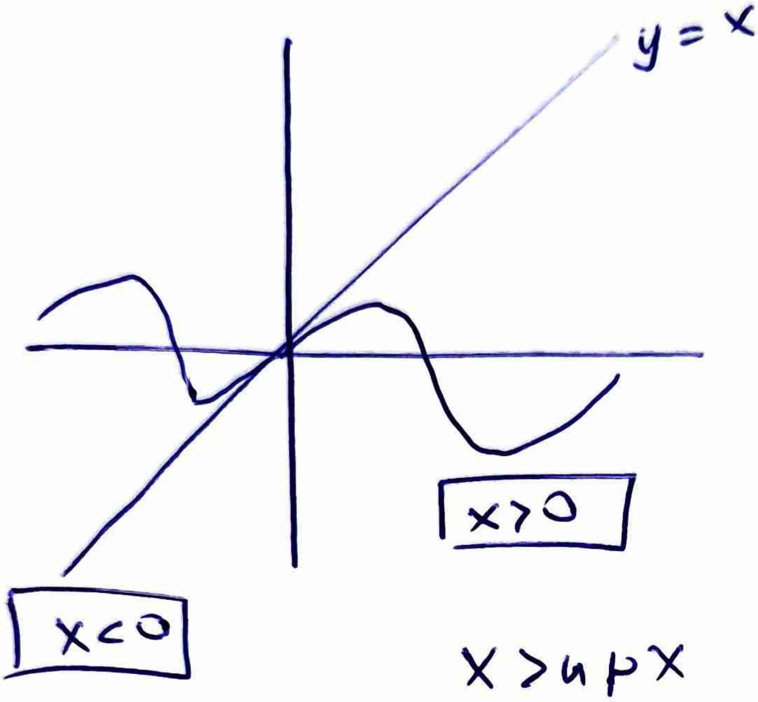
10. (a) $\lim\limits_{x\to+\infty}\left(\sqrt{x^2+1}-x\right)=$

$$= \lim_{x\to+\infty} \frac{\left(\sqrt{x^2+1}-x\right)\left(\sqrt{x^2+1}+x\right)}{\sqrt{x^2+1}+x}$$

$$= \lim_{x\to+\infty} \frac{\sqrt{x^2+1}^{\cancel{2}}-x^2}{\sqrt{x^2+1}+x} =$$

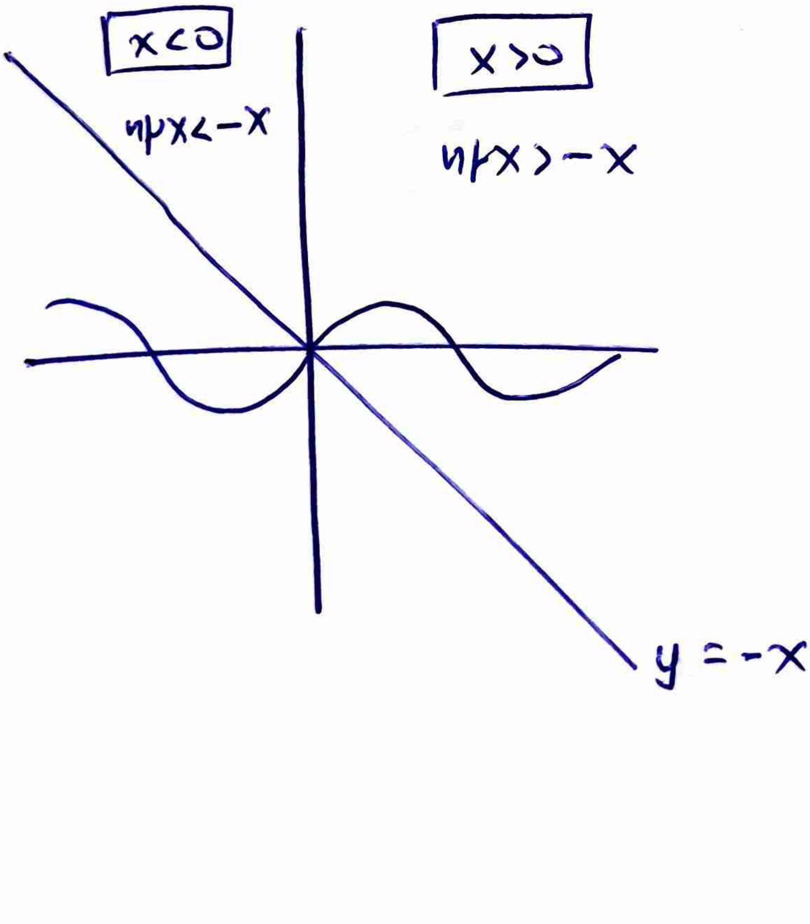
$$= \lim_{x\to+\infty} \frac{\cancel{x^2}+1-\cancel{x^2}}{\sqrt{x^2+1}+x} = 0$$

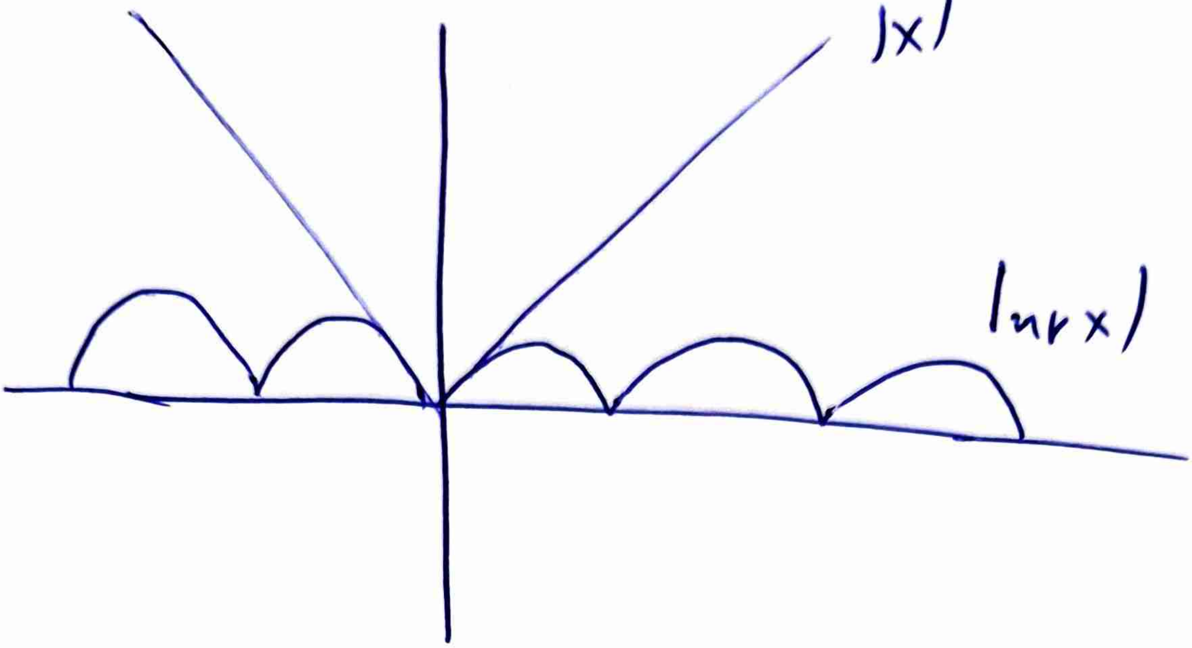
Βασικά Ανισότητα.



$x > ημx$

$x < ημx$





$$|x| \geq |\eta\mu x|$$

Το "=" για $x=0$.

Σύνοψη

Αν $x<0 \Rightarrow x<\eta\mu x$
Αν $x>0 \Rightarrow x>\eta\mu x$
} Το "=" για $x=0$

Αν $x<0 \Rightarrow \eta\mu x<-x$
Αν $x>0 \Rightarrow \eta\mu x>-x$
} Το "=" για $x=0$

$|x| \geq |\eta\mu x|$ Το "=" για $x=0$

17. α $\lim_{x \to +\infty} \left((\sigma\upsilon\nu\theta - 2)\, x^3 + 5x - 1 \right)$

$= \lim_{x \to +\infty} (\sigma\upsilon\nu\theta - 2)\, x^3 =$

$= (\alpha\rho\nu\eta\tau\iota\kappa\acute{o})(+\infty) = -\infty$

$-1 \le \sigma\upsilon\nu\theta \le 1$

$-3 \le \sigma\upsilon\nu\theta - 2 \le -1$

β $\lim_{x \to -\infty} (\lambda^2+1)\, x^3 - 3x + 1 =$

$= \lim_{x \to -\infty} (\lambda^2+1)\, x^3 = (\theta\varepsilon\tau\iota\kappa\acute{o}) \cdot (-\infty)$

$= -\infty$

$\alpha \neq 0$

1) $\lim_{x \to -\infty} \left(|\alpha| - |\eta\mu\alpha| \right) x^3 - x - 1 \ , \ \alpha \neq 0$

$(+)$

$= \lim_{x \to -\infty} \left(|\alpha| - |\eta\mu\alpha| \right) x^3 = -\infty$

- $|\alpha| \geq |\eta\mu\alpha|$

$|\alpha| - |\eta\mu\alpha| \geq 0$

18.

(Β) $\lim\limits_{x\to+\infty} (\lambda^2-2)x^4-\lambda x+1$

$$=\lim_{x\to+\infty}(\lambda^2-2)x^4 = (\lambda^2-2)\cdot(+\infty)$$

λ		$-\sqrt{2}$	$\sqrt{2}$
λ^2-2	+	−	+

1. Αν $\lambda\in(-\infty,-\sqrt{2})\cup(\sqrt{2},+\infty)$ $\quad (+)\cdot(+\infty)=+\infty$.

2. Αν $\lambda\in(-\sqrt{2},\sqrt{2})$ $\quad (-)(+\infty)=-\infty$.

3. Αν $\lambda=-\sqrt{2}$ ή $\lambda=\sqrt{2}$ τότε $0\cdot\infty$

$\xrightarrow{\lambda=\sqrt{2}} \lim\limits_{x\to+\infty} -\sqrt{2}x+1 = \lim\limits_{x\to+\infty} -\sqrt{2}x=-\infty$

$\xrightarrow{\lambda=-\sqrt{2}} \lim\limits_{x\to+\infty} \sqrt{2}x+1 = \lim\limits_{x\to+\infty} \sqrt{2}x=+\infty$, (δ) $\lim\limits_{x \to +\infty} x^3 \ln a + a x^2 - x$ $\underline{\underline{a>0}}$

$= \lim\limits_{x \to +\infty} \left(\ln a \ x^3 \right)$

a	0		1	
$\ln a$		$-$		$+$

1. Αν $a>1$ τότε $\ln a > 0$ άρα $(\theta\varepsilon\tau\iota\kappa\acute{o}) \cdot +\infty = +\infty$.

2. Αν $a<1$ τότε $-\infty$

3. Αν $a=1$ τότε $\lim\limits_{x \to +\infty} x^2 - x =$

$= \lim\limits_{x \to +\infty} x^2 = +\infty.$

19. (B) $\lim_{x\to-\infty} \dfrac{(2\mu-1)x^3-3x+1}{(\mu+1)x^2+2} = l \in \mathbb{R}$

$$= \lim_{x\to-\infty} \frac{(2\mu-1)x^3}{(\mu+1)x^2} = l$$

$$\lim_{x\to-\infty} \frac{(2\mu-1)x}{\mu+1} = l$$

$$\frac{2\mu-1}{\mu+1}(-\infty) = l \in \mathbb{R}.$$

1. Αν $\dfrac{2\mu-1}{\mu+1} \neq 0$ Άτοπο $\Rightarrow \dfrac{2\mu-1}{\mu+1} = 0$

$2\mu-1=0$

$\mu = \frac{1}{2}$

2. Αν $\mu=-1$ τότε

$\lim_{x\to-\infty} \dfrac{-3x^3-3x+1}{2} = +\infty$ Άτοπο!

Το $\mu=-1$ Απορρίπτεται

20. (B) $\lim_{x \to -\infty} \left(\mu^2 x + \sqrt{x^2+1} \right) = \ell \in \mathbb{R}$

$$= \lim_{x \to -\infty} \mu^2 x + \sqrt{x^2\left(1+\frac{1}{x^2}\right)}$$

$$= \lim_{x \to -\infty} \mu^2 x - x\sqrt{1+\frac{1}{x^2}}$$

$$= \lim_{x \to -\infty} x\left(\mu^2 - \sqrt{1+\frac{1}{x^2}}\right)$$

$$= -\infty\left(\mu^2 - 1\right)$$

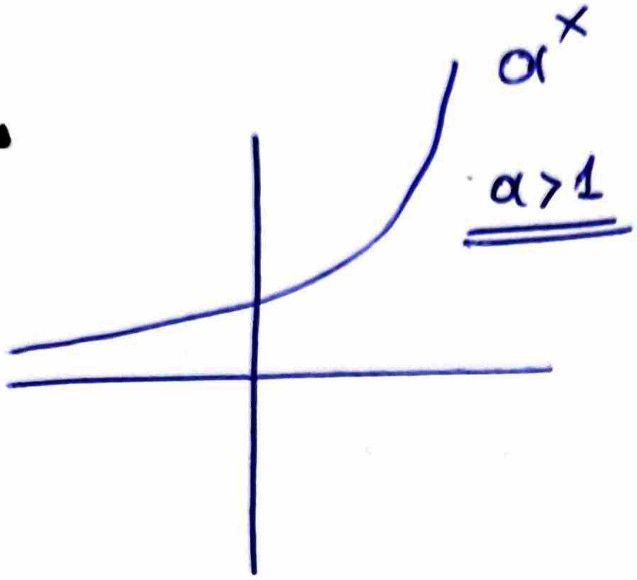
Αν $\mu^2 - 1 \neq 0$ Άτοπο.

Av $\mu = 1$ $\lim\limits_{x \to -\infty} x + \sqrt{x^2+1}$

$\mu = -1$

$$= \lim_{x \to -\infty} \frac{1}{x - \sqrt{x^2+1}} = 0$$

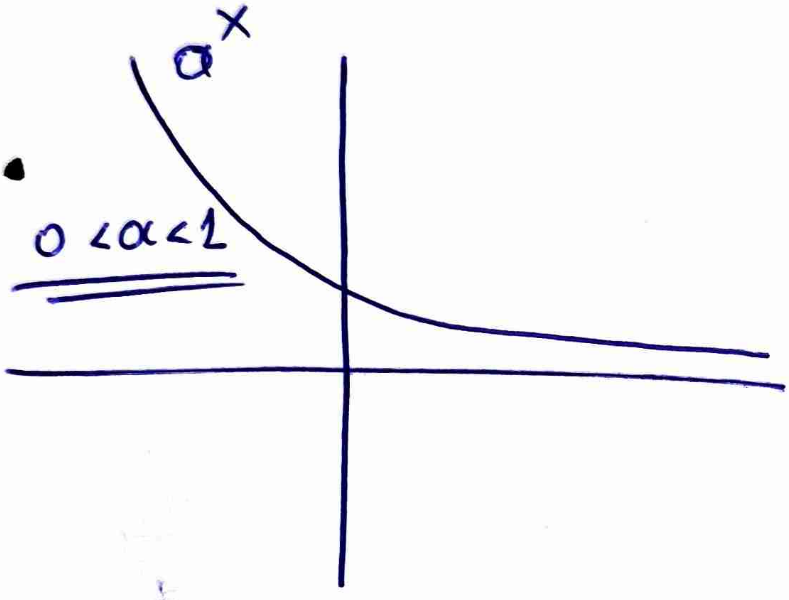
1.



$$\lim_{x\to+\infty} a^x = +\infty$$

$$\lim_{x\to-\infty} a^x = 0$$

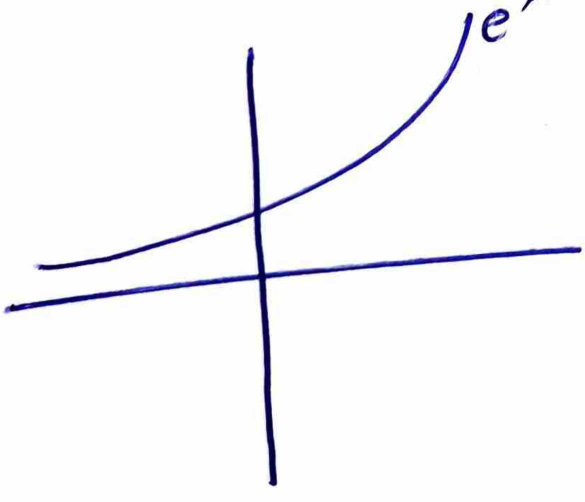
2.



$$\lim_{x\to+\infty} a^x = 0$$

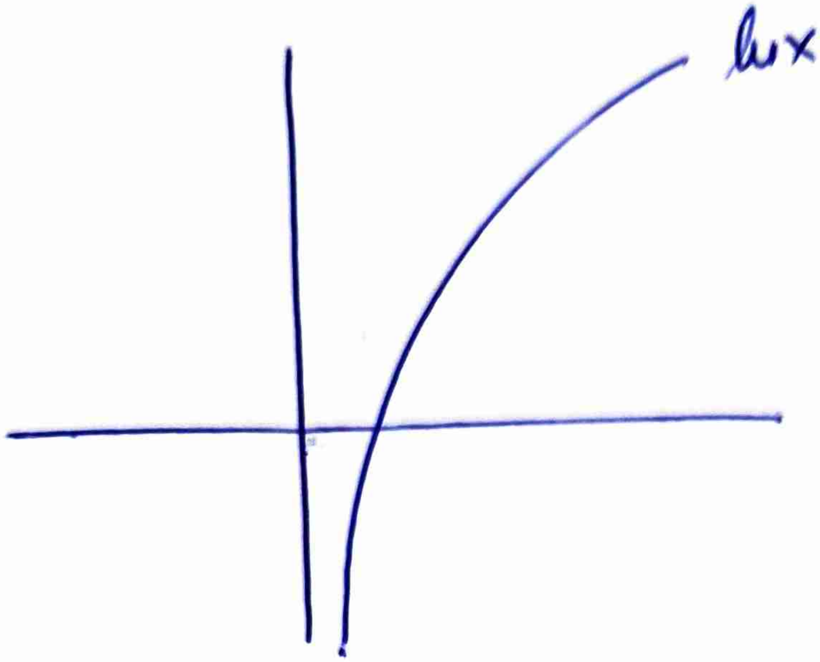
$$\lim_{x\to-\infty} a^x = +\infty$$

Διασημότητα



$$\lim_{x\to-\infty} e^x = 0$$

$$\lim_{x\to+\infty} e^x = +\infty.$$



$\lim_{x \to +\infty} \ln x = +\infty$

$\lim_{x \to 0^+} \ln x = -\infty$

Μαθηματικα της Πλατσας

$e^{-\infty} = 0$ $\ln 0 = -\infty$

$e^{+\infty} = +\infty$ $\ln +\infty = +\infty$.

35. (β) $\lim\limits_{x \to +\infty} \left(e^x - 3^{x+1} - 2\right) =$

$$= \lim_{x \to +\infty} \left(e^x - 3 \cdot 3^x - 2\right)$$

Όταν το $x \to +\infty$ κοινό παράγοντα
το μεγαλύτερο εκθετικό

$$= \lim_{x \to +\infty} 3^x \left(\frac{e^x}{3^x} - 3 \cdot \frac{2}{3^x} \right)$$

$$= \lim_{x \to +\infty} 3^x \left(\left(\frac{e}{3}\right)^x - 3 \cdot \frac{2}{3^x} \right)$$

$$= +\infty \left(0 - 3 - 0 \right) = -\infty.$$

(δ) $\lim\limits_{x \to -\infty} \left(e^x - \pi^x - 3^x\right) = 0 - 0 - 0 = 0$

34. (a) $\lim_{x \to +\infty} \left(\frac{e}{3}\right)^x = 0$

$\downarrow$

$\frac{e}{3} < 1$

(β) $\lim_{x \to -\infty} \left(\frac{\pi}{3}\right)^x = 0$

$\frac{\pi}{3} > 1$

(γ) $\lim_{x \to +\infty} \frac{2^{3x}}{3^{x+1}} = \lim_{x \to +\infty} \frac{(2^3)^x}{3^x \cdot 3^1}$

$= \lim_{x \to +\infty} \frac{8^x}{3^x \cdot 3^1} = \lim_{x \to +\infty} \frac{1}{3} \cdot \left(\frac{8}{3}\right)^x$

$= \frac{1}{3} \cdot (+\infty) = +\infty.$

36. (B) $\lim_{x \to +\infty} \dfrac{e^x - 3 \cdot 2^x - 1}{e^x + 3^x - 1}$

$$= \lim_{x \to +\infty} \frac{e^x\left(1 - 3 \cdot \frac{2^x}{e^x} - \frac{1}{e^x}\right)}{3^x\left(\frac{e^x}{3^x} + 1 - \frac{1}{3^x}\right)} =$$

$$= \lim_{x \to +\infty} \left(\frac{e}{3}\right)^x \cdot \frac{1 - 3 \cdot \left(\frac{2}{3}\right)^x - \frac{1}{e^x}}{\left(\frac{e}{3}\right)^x + 1 - \frac{1}{3^x}}$$

$$= 0 \cdot \frac{1 - 3 \cdot 0 - 0}{0 + 1 - 0} = 0 \cdot 1 = 0.$$

(8) $\lim\limits_{x \to -\infty} \dfrac{2^{x+1} - 3^{x+2}}{e^x - 2^{x+1}} =$

$= \lim\limits_{x \to -\infty} \dfrac{2^x \cdot 2 - 3^x \cdot 3^2}{e^x - 2^x \cdot 2^1} =$

$= \lim\limits_{x \to -\infty} \dfrac{\cancel{2^x}\left(2 - 9 \frac{3^x}{2^x}\right)}{\cancel{2^x}\left(\frac{e^x}{2^x} - 2\right)}$

$= \lim\limits_{x \to -\infty} \dfrac{2 - 9 \cdot \left(\frac{3}{2}\right)^x}{\left(\frac{e}{2}\right)^x - 2} =$

$= \dfrac{2 - 9 \cdot 0}{0 - 2} = -1$

38. (γ) $\lim_{x \to -\infty} \frac{e^x}{1+x^2} = \frac{0}{+\infty} = 0$

$\left(\frac{0}{\infty} = 0 \cdot \frac{1}{\infty} = 0 \cdot 0 = 0 \right)$

39. (δ) $\lim_{x \to -\infty} e^{-x^2} = e^{-\infty} = 0$

Επισημω

$\lim_{x \to -\infty} e^{-x^2} = \lim_{u \to -\infty} e^u = 0$

Θετω $-x^2 = u$

$x \to -\infty$

$u \to -\infty$

(ε) $\lim_{x \to 0^+} e^{\frac{1}{x}} = e^{+\infty} = +\infty$

(ζ) $\lim_{x \to 1^+} e^{\frac{x}{x-1}} = e^{+\infty} = +\infty$.

$\to \lim_{x \to 1^+} \frac{x}{x-1} = +\infty$