8. Γνωρίζω ότι $2<x<3$

ΕΝΟΤΗΤΑ 7

(α) $x+1$

• $2<x<3 \Rightarrow \boxed{3<x+1<4}$

(β) $2x-3$

• $2<x<3 \Rightarrow 4<2x<6 \Rightarrow \boxed{1<2x-3<3}$

(γ) $-3x+1$

• $2<x<3 \Rightarrow -6>-3x>-9$

$$\boxed{-5>-3x+1>-8}$$

(δ) $2-x$

• $2<x<3 \Rightarrow -2>-x>-3$

$$\boxed{0>2-x>-1}$$

(ε) $\frac{x}{3} - 1$

• $2 < x < 3 \Rightarrow \frac{2}{3} < \frac{x}{3} < \frac{3}{3} \Rightarrow \frac{2}{3} < \frac{x}{3} < 1$

$\frac{2}{3} - 1 < \frac{x}{3} - 1 < 0$

$$-\frac{1}{3} < \frac{x}{3} - 1 < 0$$

(52) $\frac{2}{x}$

• $2 < x < 3 \Rightarrow \frac{1}{2} > \frac{1}{x} > \frac{1}{3} \Rightarrow \frac{2}{2} > \frac{2}{x} > \frac{2}{3}$

$$1 > \frac{2}{x} > \frac{2}{3}$$

11. ⓐ $x+y$

- $1 < x \leq 3$
- $2 \leq y < 5$

} ⊕ $\boxed{3 < x+y < 8}$

ⓑ $x \cdot y$

- $1 < x \leq 3$
- $2 \leq y < 5$

} ⊙ $1 \cdot 2 < xy < 15$

$\boxed{2 < xy < 15}$

ⓒ $2x+y$

- $1 < x \leq 3 \Rightarrow 2 < 2x \leq 6$
- $2 \leq y < 5$

⊕

$\boxed{4 < 2x+y < 11}$

(δ) $x-3y$

- $1 < x \leq 3 \Rightarrow 3 \geq x > 1$
- $2 \leq y < 5 \Rightarrow -6 \geq -3y > -15$

$\Big\}$ (+)

$$-3 \geq x-3y > -14$$

(ε) $x-y$

- $1 < x \leq 3 \Rightarrow 3 \geq x > 1$
- $2 \leq y < 5 \Rightarrow -2 \geq -y > 5$

$\Big\}$ (+)

$$1 \geq x-y > 6$$

(ζ) $\frac{x}{y}$

- $1 < x \leq 3 \Rightarrow 3 \geq x > 1$
- $2 \leq y < 5 \Rightarrow \frac{1}{2} \geq \frac{1}{y} > \frac{1}{5}$

$\Big\}$ (·)

$$\frac{3}{2} \geq \frac{x}{y} > \frac{1}{5}$$

12. (a) x^2+1

- $1 < x < 2 \Rightarrow 1^2 < x^2 < 2^2 \Rightarrow 1 < x^2 < 4$

$$\boxed{2 < x^2+1 < 5}$$

(B) $2x^3-1$

- $1 < x < 2 \Rightarrow 1^3 < x^3 < 2^3 \Rightarrow 1 < x^3 < 8$

$$2 < 2x^3 < 16$$

$$\boxed{1 < 2x^3-1 < 15}$$

(γ) $1-\frac{4}{x^2}$

- $1 < x < 2 \Rightarrow 1 < x^2 < 4 \Rightarrow 1 > \frac{1}{x^2} > \frac{1}{4}$

$$-4 < -\frac{4}{x^2} < -\frac{4}{4}$$

$$\boxed{-3 < 1-\frac{4}{x^2} < 0}$$

(8) $(x+1)^2 - 2$

- $1 < x < 2 \Rightarrow 2 < x+1 < 3 \Rightarrow 4 < (x+1)^2 < 9$

$$2 < (x+1)^2 - 2 < 7$$

15.

$1 \leq x \leq 2$	$x \in [1, 2]$
$1 < x < 2$	$x \in (1, 2)$
$1 \leq x < 2$	$x \in [1, 2)$
$x \geq 1$	$x \in [1, +\infty)$
$x < 1$	$x \in (-\infty, 1)$
$x \leq 2$ ' $x \geq 3$	$x \in (-\infty, 2] \cup [3, +\infty)$
$-5 \leq x \leq 7$	$x \in [-5, 7]$

33. (a) xy

- $-3 < x < -2$
- $-2 < y < -1$

$\Big\}\odot$ $\boxed{6 > xy > 2}$

(b) $\frac{x}{y}$

- $-3 < x < -2 \Rightarrow -2 > x > -3$
- $-2 < y < -1 \Rightarrow -\frac{1}{2} > \frac{1}{y} > -1$

$\Big\}\odot$

$$\boxed{1 < \frac{x}{y} < 3}$$

(c) $x^2 + y^3$

- $-3 < x < -2 \Rightarrow 9 > x^2 > 4$
- $-2 < y < -1 \Rightarrow -8 < y^3 < -1$

$$-1 > y^3 > -8$$

$\Big]\oplus$

$$\boxed{8 > x^2 + y^3 > -4}$$

(δ) $\frac{x}{y}$

- $3 \leq x \leq 5$
- $-2 \leq y \leq -1 \quad \Rightarrow -\frac{1}{2} \geq \frac{1}{y} \geq -1$

$$\frac{1}{2} \leq -\frac{1}{y} \leq 1$$

$$\frac{3}{2} \leq -\frac{x}{y} \leq 5$$

$$-\frac{3}{2} \geq \frac{x}{y} \geq -5$$

36. Αν $x \in [3,5]$ και $y \in [-2,-1]$

(α) $y-x$

- $3 \le x \le 5 \Rightarrow -3 \geqslant -x \geqslant -5$
- $-2 \le y \le -1 \Rightarrow -1 \geqslant y \geqslant -2$

$\Big\}$ (+) $\boxed{-4 \geqslant y-x \geqslant -7}$

(β) $2x-3y$

- $3 \le x \le 5 \Rightarrow 6 \le 2x \le 10 \Rightarrow 10 \geqslant 2x \geqslant 6$
- $-2 \le y \le -1 \Rightarrow 6 \geqslant -3y \geqslant 3$

$\Big|$ (+)

$\boxed{16 \geqslant 2x-3y \geqslant 9}$

(γ) x^2+y^2

- $3 \le x \le 5 \Rightarrow 9 \le x^2 \le 25 \Rightarrow 25 \geqslant x^2 \geqslant 9$
- $-2 \le y \le -1 \Rightarrow 4 \geqslant y^2 \geqslant 1$

$\Big|$ (+)

$29 \geqslant x^2+y^2 \geqslant 10$

31. Αν $\alpha < \beta$ Μονάδικη

Νδο $\alpha^3 + \alpha < \beta^3 + \beta$.

- $\alpha < \beta$ ⊕
- $\alpha < \beta \Rightarrow a^3 < \beta^3$

$a + a^3 < \beta + \beta^3$ ✓

Β' τρόπος

$a^3 + a - \beta^3 - \beta < 0$

$a^3 - \beta^3 + a - \beta < 0$

$(a-\beta)(a^2 + a\beta + \beta^2) + a - \beta < 0$

$(a-\beta)(a^2 + a\beta + \beta^2 + 1) < 0$ (− , +)

- $\alpha < \beta \Rightarrow a - \beta < 0$

$a^2 + Ba + B^2$ Тричлен від

прої x.

$\Delta = B^2 - 4ac$

$\Delta = B^2 - 4 \cdot 1 \cdot B^2$

$\Delta = B^2 - 4B^2$

$\Delta = -3B^2$

Тобто тричлен $a^2 + B \cdot a + B^2 > 0$

44. Αν $\alpha>0$ νδο $\alpha+a^2+\frac{1}{a}+\frac{1}{a^2}\geq 4$

$$a^3+a^4+\alpha+1 \geq 4a^2$$

$$\alpha^4+a^3-4a^2+a+1\geq 0$$

Δυσο που εκανα καιν η

λογικη κινηση που πρεπει να

κανει σαν μαθητη.

Δεν βγηκε!

Αφου $\alpha>0$ τοτε $\alpha+\frac{1}{\alpha}\geq 2$ $\forall \alpha>0$

$\oplus$

$$a^2+\frac{1}{a^2}\geq 2$$

$$\alpha+\frac{1}{a}+a^2+\frac{1}{a^2}\geq 4$$ ✓

$$x + \frac{1}{x} \geq 2 \qquad \forall x > 0$$

Αποδεικνύεται

$x^2 + 1 \geq 2x$

$x^2 - 2x + 1 \geq 0$

$(x-1)^2 \geq 0$ ✓

$$x + \frac{1}{x} \leq -2 \qquad \forall x < 0$$

Αποδεικνύεται

$x^2 + 1 \geq -2x$

$x^2 + 2x + 1 \geq 0$

$(x+1)^2 \geq 0$ ✓

45. (α) Νδο $\frac{\alpha}{\beta}+\frac{\beta}{\alpha}\geq 2$ αν $\alpha,\beta>0$.

$$\alpha\beta\,\frac{\alpha}{\beta}+\alpha\beta\,\frac{\beta}{\alpha}\geq 2\alpha\beta$$

$$\alpha^2+\beta^2\geq 2\alpha\beta$$

$$\alpha^2-2\alpha\beta+\beta^2\geq 0$$

$$(\alpha-\beta)^2\geq 0 \quad \checkmark$$

(β) Νδο αν $x,y,z>0$ $\frac{x+y}{z}+\frac{y+z}{x}+\frac{z+x}{y}\geq 6$

$$\frac{x}{z}+\frac{y}{z}+\frac{y}{x}+\frac{z}{x}+\frac{z}{y}+\frac{x}{z}\geq 6$$

$$\boxed{\frac{x}{z}+\frac{z}{x}+\frac{y}{x}+\frac{x}{y}+\frac{z}{y}+\frac{y}{z}\geq 6}$$

- $\frac{x}{z}+\frac{z}{x}\geq 2$ (+)
- $\frac{y}{x}+\frac{x}{y}\geq 2$ $\frac{z}{y}+\frac{y}{z}\geq 2$

$\checkmark$

50. $\alpha > 0 \quad , \beta > 0$

(α) Να δ.ο $\alpha + \frac{4}{\alpha} \geq 4$

$$\alpha^2 + 4 \geq 4\alpha$$

$$\alpha^2 - 4\alpha + 4 \geq 0$$

$$(\alpha - 2)^2 \geq 0 \, .$$

(β) Να δ.ο $\left(\alpha + \frac{4}{\alpha}\right)\left(\beta + \frac{4}{\beta}\right) \geq 16$

$$\left.\begin{array}{l} \alpha + \frac{4}{\alpha} \geq 4 \\ \beta + \frac{4}{\beta} \geq 4 \end{array}\right\} (\bullet)$$

$$\left(\alpha + \frac{4}{\alpha}\right)\left(\beta + \frac{4}{\beta}\right) \geq 16$$

49. Αν $a>2$ να δ. $a^3-2>2a^2-a$

$$a^3-2-2a^2+a>0$$

$$a^2(a-2)-2+a>0$$

$$a^2(a-2)+(a-2)>0$$

$$(a-2)(a^2+1)>0$$

(+) (+)

- $a>2 \Rightarrow a-2>0$
- $a^2+1>0$

51. Αν $x \in (0,1)$

Να συγκρίνετε οι αριθμοί

x^3 και x

Έστω $x^3 < x$

$x^3 - x < 0$

$x(x^2-1) < 0$

$x(x-1)(x+1) < 0$ ✓

• $0 < x < 1 \Rightarrow -1 < x-1 < 0$

$x \in (0,1) \Rightarrow 0 < x < 1$