

1. $\forall \alpha < \beta < 1 \quad \vee \vee \alpha^2 - \alpha\beta > \alpha - \beta$

$$\alpha^2 - \alpha\beta - \alpha + \beta > 0$$

$$\alpha(\alpha - \beta) - (\alpha - \beta) > 0$$

$$\boxed{(\alpha - \beta)(\alpha - 1) > 0}$$

$\ominus \quad \ominus$

✓

• $\alpha < \beta \Rightarrow \alpha - \beta < 0$

• $\alpha < 1 \Rightarrow \alpha - 1 < 0$

2. $\forall 0 < \alpha < \beta \quad \text{και} \quad \alpha + \beta > 1$

$$\vee \vee \beta^2 - \alpha^2 > \beta - \alpha$$

$$\beta^2 - \alpha^2 - \beta + \alpha > 0$$

$$(\beta - \alpha)(\beta + \alpha) - (\beta - \alpha) > 0$$

$$\boxed{(\beta - \alpha)(\beta + \alpha - 1) > 0}$$

$\oplus \quad \oplus$

✓

• $\alpha < \beta \Rightarrow \beta - \alpha > 0$

• $\alpha + \beta > 1 \Rightarrow \alpha + \beta - 1 > 0$

$$3. \quad \alpha > 1, \beta > 1 \quad \forall \alpha \quad \alpha\beta - (\alpha + \beta) > -1$$

$$\alpha\beta - \alpha - \beta + 1 > 0$$

$$\alpha(\beta - 1) - (\beta - 1) > 0$$

$$\boxed{\begin{array}{cc} (\beta - 1)(\alpha - 1) > 0 \\ \oplus \quad \oplus \end{array}} \quad \checkmark$$

$$\bullet \quad \alpha > 1 \Rightarrow \alpha - 1 > 0$$

$$\bullet \quad \beta > 1 \Rightarrow \beta - 1 > 0$$

$$4. \quad \text{Акоу} \quad (\alpha - \beta)^2 \geq 0 \quad \checkmark$$

$$5. \quad \text{Ав} \quad \alpha, \beta > 0 \quad \forall \alpha \quad (\alpha + \beta)^2 \geq 4\alpha\beta$$

$$\alpha^2 + 2\alpha\beta + \beta^2 \geq 4\alpha\beta$$

$$\alpha^2 - 2\alpha\beta + \beta^2 \geq 0$$

$$(\alpha - \beta)^2 \geq 0 \quad \checkmark$$

$$6. \quad a, b > 0 \quad \text{vs} \quad \frac{a}{b} + \frac{b}{a} \geq 2$$

$$ab \cdot \frac{a}{b} + ab \cdot \frac{b}{a} \geq 2ab$$

$$a^2 + b^2 - 2ab \geq 0$$

$$(a-b)^2 \geq 0.$$

$$7. \quad \text{Av } a, b > 0 \quad \text{vs} \quad a^2 + b^2 \geq 2ab$$

$$a^2 - 2ab + b^2 \geq 0$$

$$(a-b)^2 \geq 0.$$

$$8. \quad \text{Av } a < b < 1 \quad \text{vs} \quad a - a^2 < b - b^2$$

~~$$a - a^2 - b + b^2 < 0$$~~

~~$$a - b - (a^2 - b^2) < 0$$~~

~~$$a - b - (a-b)(a+b) < 0$$~~

~~$$(a-b)(1-a-b) < 0,$$~~

9.

$$\forall 0 < \alpha < \beta < 1 \quad \text{vdo} \quad \alpha^3 - \alpha^2 \beta > \alpha - \beta$$

$$\alpha^3 - \alpha^2 \beta - \alpha + \beta > 0$$

$$\alpha^2(\alpha - \beta) - (\alpha - \beta) > 0$$

$$\boxed{(\alpha - \beta)(\alpha^2 - 1) > 0}$$

⊖ ⊖

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$$\bullet \quad \alpha < \beta \Rightarrow \alpha - \beta < 0$$

$$\bullet \quad \alpha < 1 \Rightarrow \alpha^2 < 1^2 \Rightarrow \alpha^2 < 1$$

$$\alpha^2 - 1 < 0$$

10. $\forall 1 < \alpha < B \quad \text{v} \text{ s} \quad B^2 - \alpha B > B - \alpha$

$$B^2 - \alpha B - B + \alpha > 0$$

$$B(B - \alpha) - (B - \alpha) > 0$$

$$(B - \alpha)(B - 1) > 0$$

$\oplus$
 $\oplus$

✓

• $\alpha < B \Rightarrow 0 < B - \alpha$

• $1 < B \Rightarrow \alpha < B - 1$

Ενότητα 7

27. ① $x^2 + 2y^2 - 2x + 4y + 3 = 0$

$$x^2 - 2x + 1 + 2y^2 + 4y + 2 = 0$$

$$(x-1)^2 + 2(y^2 + 2y + 1) = 0$$

$$(x-1)^2 + 2(y+1)^2 = 0$$

$$\begin{cases} x-1=0 \\ y+1=0 \end{cases} \Rightarrow \begin{aligned} \underline{\underline{x=1}} \\ \underline{\underline{y=-1}} \end{aligned}$$

7. $x^2 + y^2 + z^2 - 6x + 2y - 4z = -14$

$$x^2 - 6x + 9 + y^2 + 2y + 1 + z^2 - 4z + 4 = 0$$

$$(x-3)^2 + (y+1)^2 + (z-2)^2 = 0$$

$$\begin{cases} x-3=0 \\ y+1=0 \\ z-2=0 \end{cases}$$

$$x = -3$$

$$y = -1$$

$$z = 2$$

50. $\forall a > 0, b > 0$

⑥ NSD $a + \frac{4}{a} \geq 4$.

$$a^2 + \cancel{a} \cdot \frac{4}{\cancel{a}} \geq 4a$$

$$a^2 - 4a + 4 \geq 0$$

$$(a-2)^2 \geq 0 \quad \checkmark$$

⑦ vso $\left(a + \frac{4}{a}\right)\left(b + \frac{4}{b}\right) \geq 16$.

$$\left. \begin{array}{l} a + \frac{4}{a} \geq 4 \\ b + \frac{4}{b} \geq 4 \end{array} \right\} \textcircled{\cdot} \quad \forall a > 0$$

$$\left(a + \frac{4}{a}\right)\left(b + \frac{4}{b}\right) \geq 16 \quad \checkmark$$

$$6. \quad (8) \quad x^2 + 4y^2 - 2x - 4y + 2 \leq 0.$$

$$x^2 - 2x + 1 + 4y^2 - 4y + 1 \leq 0$$

$$(x-1)^2 + (2y-1)^2 \leq 0.$$

$$(x-1)^2 + (2y-1)^2 = 0.$$

$$\left\{ \begin{array}{l} x-1=0 \\ 2y-1=0 \end{array} \right.$$

$$(x=1)$$

$$(y=\frac{1}{2})$$

31. $\forall \alpha < \beta$

$$(a) \quad \alpha^3 + \alpha < \beta^3 + \beta$$

$$\cdot \alpha < \beta \Rightarrow \alpha^3 < \beta^3$$

$$\cdot \alpha < \beta$$

$$\quad \quad \quad \quad \quad \oplus$$

$$\alpha^3 + \alpha < \beta^3 + \beta \quad \checkmark$$

$$(b) \quad \text{vfo} \quad 2\alpha^5 + \alpha^3 + \cancel{\beta} < 2\beta^5 + \beta^3 + \cancel{\beta}$$

$$\cdot \alpha < \beta \Rightarrow \alpha^5 < \beta^5 \Rightarrow 2\alpha^5 < 2\beta^5$$

$$\cdot \alpha < \beta \Rightarrow \alpha^3 < \beta^3 \quad \quad \quad \quad \quad \oplus$$

$$2\alpha^5 + \alpha^3 < 2\beta^5 + \beta^3$$

$\checkmark$

$$32. \textcircled{B} \quad \alpha < \beta < 0 \quad \vee \text{ s.t. } \alpha^2 + \frac{2}{\alpha} > \beta^2 + \frac{2}{\beta}$$

$$\in \mathbb{K} \ni \alpha, \beta$$

$$\alpha \beta \alpha^2 + \cancel{\alpha \beta} \frac{2}{\alpha} > \alpha \beta \beta^2 + \cancel{\alpha \beta} \frac{2}{\beta}$$

$$\alpha^3 \beta + 2\beta > \alpha \beta^3 + 2\alpha$$

$$\alpha^3 \beta - 2\beta - \alpha \beta^3 - 2\alpha > 0,$$

$$\alpha \beta (\alpha^2 - \beta^2) - 2(\alpha - \beta) > 0$$

$$\alpha \beta (\alpha - \beta)(\alpha + \beta) - 2(\alpha - \beta) > 0$$

$$\boxed{(\alpha - \beta) (\alpha \beta (\alpha + \beta) - 2) > 0}$$

$\ominus \quad \oplus \quad \ominus$

$$\bullet \alpha < \beta \Rightarrow \alpha - \beta < 0$$

$$\bullet \begin{cases} \alpha < 0 \\ \beta < 0 \end{cases} \Rightarrow \alpha \beta > 0$$

$$\bullet \alpha < 0 \quad \beta < 0 \Rightarrow \alpha + \beta < 0$$

β' трснл

вдс $\alpha < \beta < 0$ вдс $\alpha^2 + \frac{2}{\alpha} > \beta^2 + \frac{2}{\beta}$

• $\alpha < \beta \Rightarrow \alpha^2 > \beta^2$ ⊕

• $\alpha < \beta \Rightarrow \frac{1}{\alpha} > \frac{1}{\beta} = \frac{2}{\alpha} > \frac{2}{\beta}$

$$\alpha^2 + \frac{2}{\alpha} > \beta^2 + \frac{2}{\beta}$$

Επορας Μαθημα

1. Ανισοσύνη 10 and 20.

2. Ενοτήτες 7

(6) α β γ

(23)

(25)

(27) α β.

(32) α.