Ενότητα 3

13. ⓐ $f(2x-3) = 4x^2 - 14x + 13$

Θέτω $2x-3 = t$

$2x = t+3$

$x = \dfrac{t+3}{2}$

$$f(t) = 4 \cdot \left(\frac{t+3}{2}\right)^2 - 14 \cdot \frac{t+3}{2} + 13$$

$$f(t) = \cancel{4} \cdot \frac{(t+3)^2}{\cancel{4}} - 7(t+3) + 13$$

$$f(t) = t^2 + 6t + 9 - 7t - 21 + 13$$

$$f(t) = t^2 - t + 1$$

ή

$$\boxed{f(x) = x^2 - x + 1}$$

13. (β) $f(e^x) = x^2 - 3x + 1$

Θέτω $e^x = t$

$x = \ln t.$

$f(t) = \ln^2(t) - 3\ln t + 1$

$\boxed{f(x) = \ln^2 x - 3\ln x + 1}$

(γ) $f(1 + \ln x) = x + 2\ln x + 3$

$1 + \ln x = t$

$\ln x = t - 1$

$x = e^{t-1}$

$f(t) = e^{t-1} + 2\ln e^{t-1} + 3$

$f(t) = e^{t-1} + 2(t-1) + 3$

$\underline{f(x) = e^{x-1} + 2x + 1}$

30. (γ) $\lim\limits_{x\to+\infty}\left(x\,ημ\frac{1}{x} + \cancel{\frac{1}{x}\,ημx}^{\,0}\right) = \underline{1}$

$\rightarrow \lim\limits_{x\to+\infty} x\,ημ\frac{1}{x} = \lim\limits_{x\to+\infty} \cancel{x}\,\frac{ημ\frac{1}{x}}{\frac{1}{x}}\,\cancel{\frac{1}{x}} =$

$= \lim\limits_{x\to+\infty} \frac{ημ\frac{1}{x}}{\frac{1}{x}} \overset{\frac{1}{x}=t}{\underset{\substack{x\to+\infty \\ t\to0}}{=\!=}} \lim\limits_{t\to0}\frac{ημt}{t} = 1.$

$\rightarrow \lim\limits_{x\to+\infty} \frac{1}{x}\,ημx \overset{ημ\infty}{\underset{χπισμο}{=\!=}}$

$-1 \le ημx \le 1$

$\boxed{-\frac{1}{x} \le \frac{ημx}{x} \le \frac{1}{x}}$ κοντα στο $+\infty$ το $x>0$

$\left.\begin{array}{l} \lim\limits_{x\to+\infty} -\frac{1}{x} = 0 \\ \lim\limits_{x\to+\infty} \frac{1}{x} = 0 \end{array}\right\}$ Απο Κ.Π $\lim\limits_{x\to+\infty}\frac{ημx}{x} = 0.$

30. (δ) $\lim\limits_{x \to -\infty} \eta\mu x \cdot \eta\mu \frac{1}{x} = 0.$

$$-1 \le \eta\mu x \le 1$$

$$|\eta\mu x| \le 1$$

$$\left|\eta\mu \tfrac{1}{x}\right| \, |\eta\mu x| \le 1 \cdot \left|\eta\mu \tfrac{1}{x}\right|$$

$$\left|\eta\mu \tfrac{1}{x} \, \eta\mu x\right| \le \left|\eta\mu \tfrac{1}{x}\right|$$

$$\boxed{-\left|\eta\mu \tfrac{1}{x}\right| \le \eta\mu \tfrac{1}{x} \, \eta\mu x \le \left|\eta\mu \tfrac{1}{x}\right|}$$

$$\left.\begin{array}{l} \lim\limits_{x \to -\infty} -\left|\eta\mu \tfrac{1}{x}\right| = 0 \\[2em] \lim\limits_{x \to +\infty} \left|\eta\mu \tfrac{1}{x}\right| = 0 \end{array}\right\} \begin{array}{l} \text{Από Κ. Π.} \\[1em] \lim\limits_{x \to +\infty} \eta\mu \tfrac{1}{x} \, \eta\mu x = 0. \end{array}$$

31. (B) $\lim\limits_{x\to+\infty} \dfrac{x^2 + x\,ημx + 1}{x^2 + ημx + 3} =$

$$= \lim_{x\to+\infty} \frac{1 + \cancelto{0}{\frac{ημx}{x}} + \cancelto{0}{\frac{1}{x^2}}}{1 + \cancelto{0}{\frac{ημx}{x^2}} + \cancelto{0}{\frac{3}{x^2}}} = 1$$

$\to \lim\limits_{x\to+\infty} \dfrac{ημx}{x} = 0$

$$-1 \le ημx \le 1$$

$$\boxed{-\frac{1}{x} \le \frac{ημx}{x} \le \frac{1}{x}}$$

$$\left.\begin{aligned} &\lim_{x\to+\infty} -\frac{1}{x} = 0 \\ &\lim_{x\to+\infty} \frac{1}{x} = 0 \end{aligned}\right\} \text{Από κ.π. } \lim_{x\to+\infty} \frac{ημx}{x} = 0$$

$\to \lim\limits_{x\to+\infty} \dfrac{ημx}{x^2} = 0$

32. (B) $\lim\limits_{x\to-\infty}\left(\dfrac{x^2}{x+1}+\eta\mu x\right)=$

$-1\leq \eta\mu x\leq 1$

$$\boxed{\frac{x^2}{x+1}-1\leq \eta\mu x+\frac{x^2}{x+1}\leq 1+\frac{x^2}{x+1}}$$

$$\lim_{x\to-\infty}\left(\frac{x^2}{x+1}-1\right)=\lim_{x\to-\infty}\frac{x^2-x-1}{x+1}=$$

$$=\lim_{x\to-\infty}\frac{x\cancel{^2}}{\cancel{x}}=-\infty$$

$$\lim_{x\to-\infty}\left(1+\frac{x^2}{x+1}\right)=\lim_{x\to-\infty}\frac{x^2+x+1}{x+1}=\lim_{x\to-\infty}\frac{x\cancel{^2}}{x}=-\infty$$

Απο κ.π $\lim\limits_{x\to-\infty}\eta\mu x+\dfrac{x^2}{x+1}=-\infty$

33. (B) $\lim\limits_{x\to+\infty} \dfrac{x^2}{2-ημx} =$

$$-1 \le ημx \le 1$$

$$1 \ge -ημx \ge -1$$

$$3 \ge 2-ημx \ge 1$$

$$\frac{1}{3} \le \frac{1}{2-ημx} \le 1$$

$$\boxed{\frac{x^2}{3} \le \frac{x^2}{2-ημx} \le x^2}$$

$$\left.\begin{aligned} &\cdot \lim_{x\to+\infty} \frac{x^2}{3} = +\infty \\ &\lim_{x\to+\infty} x^2 = +\infty \end{aligned}\right\} \text{Απο Κ,Π} \quad \lim_{x\to+\infty} \frac{x^2}{2-ημx} = +\infty$$

34. (γ) $\lim\limits_{x\to+\infty} \dfrac{2^{3x}}{3^{x+1}} = \lim\limits_{x\to+\infty} \dfrac{(2^3)^x}{3^x \cdot 3^1} =$

$= \lim\limits_{x\to+\infty} \dfrac{8^x}{3\cdot 3^x} = \lim\limits_{x\to+\infty} \dfrac{1}{3}\cdot\left(\dfrac{8}{3}\right)^x$

$= \dfrac{1}{3}\cdot(+\infty) = +\infty,$

35. (γ) $\lim\limits_{x\to-\infty} e^x - \pi^x - 3^x = 0-0-0=0$

36. (B) $\lim\limits_{x\to+\infty} \dfrac{e^x - 3\cdot 2^x - 1}{e^x + 3^x - 1}$

$= \lim\limits_{x\to+\infty} \dfrac{e^x\left(1 - 3\dfrac{2^x}{e^x} - \dfrac{1}{e^x}\right)}{3^x\left(\dfrac{e^x}{3^x} + 1 - \dfrac{1}{3^x}\right)} =$

$= \lim\limits_{x\to+\infty} \cancel{\left(\dfrac{e}{3}\right)^x}^{\,0} \dfrac{1 - 3\cancel{\left(\dfrac{2}{e}\right)^x}^{\,0} - \dfrac{1}{e^x}}{\cancel{\left(\dfrac{e}{3}\right)^x}^{\,0} + 1 - \cancel{\dfrac{1}{3^x}}^{\,0}} = 0\cdot\dfrac{1}{1} = 0.$

36. ⓑ $\lim_{x \to -\infty} \dfrac{2^{x+1} - 3^{x+2}}{e^x - 2^{x+1}} =$

$$= \lim_{x \to -\infty} \frac{2^x \cdot 2^1 - 3^x \cdot 3^2}{e^x - 2^x \cdot 2^1} =$$

$$= \lim_{x \to -\infty} \frac{2 \cdot 2^x - 9 \cdot 3^x}{e^x - 2 \cdot 2^x} =$$

$$= \lim_{x \to -\infty} \frac{\cancel{2^x}\left(2 - 9 \cdot \frac{3^x}{2^x}\right)}{\cancel{2^x}\left(\frac{e^x}{2^x} - 2\right)}$$

$$= \lim_{x \to -\infty} \frac{2 - 9 \cdot \overset{0}{\cancel{\left(\frac{3}{2}\right)^x}}}{\underset{0}{\cancel{\left(\frac{e}{2}\right)^x}} - 2} = -1.$$

38. (γ) $\lim\limits_{x\to-\infty} \frac{e^x}{1+x^2} = \lim\limits_{x\to-\infty} e^x \frac{1}{1+x^2}$

$= 0 \cdot 0 = 0$,

39. (γ) $\lim\limits_{x\to+\infty} e^{-x^2} = e^{-\infty} = 0$

(ε) $\lim\limits_{x\to 0^+} e^{\frac{1}{x}} = e^{+\infty} = +\infty$.

(ζ) $\lim\limits_{x\to 1^+} e^{\frac{x}{x-1}} = e^{+\infty} = +\infty$

$\to \lim\limits_{x\to 1^+} \frac{x}{x-1} = \lim\limits_{x\to 1^+} x \cdot \frac{1}{x-1} =$

$= 1 \cdot +\infty = +\infty$

40. (β) $\lim\limits_{x\to-\infty} e^x \eta\mu \frac{1}{x} = 0 \cdot 0 = 0$

(δ) $\lim\limits_{x\to 0^+} \frac{e^{\frac{1}{x}}}{x} = \lim\limits_{x\to 0^+} e^{\frac{1}{x}} \cdot \frac{1}{x} =$

$= +\infty \cdot (+\infty)$

(στ). $\lim\limits_{x\to 0} \frac{e^x}{e^{-x^2}-1} = \lim\limits_{x\to 0} e^x \frac{1}{\underset{\ominus}{e^{-x^2}-1}}$

$= 1 \cdot (-\infty) = -\infty$

• $x^2 \geq 0 \Rightarrow -x^2 \leq 0$

$e^{-x^2} \leq e^0$

$e^{-x^2} \leq 1$

$e^{-x^2} - 1 \leq 0$

41. (B) $\lim\limits_{x\to+\infty} \dfrac{\text{συν}x}{e^x} = 0$,

$$-1 \le \text{συν}x \le 1$$

$$\boxed{-\frac{1}{e^x} \le \frac{\text{συν}x}{e^x} \le \frac{1}{e^x}}$$

$\lim\limits_{x\to+\infty} -\dfrac{1}{e^x} = 0$

$\lim\limits_{x\to+\infty} \dfrac{1}{e^x} = 0$

Απο κ.π $\lim\limits_{x\to+\infty} \dfrac{\text{συν}x}{e^x} = 0$.

42. (B) $\lim\limits_{x\to+\infty} e^{x+\text{ημ}x} = +\infty$.

$$-1 \le \text{ημ}x \le 1$$

$$x-1 \le x+\text{ημ}x \le x+1$$

$$\boxed{e^{x-1} \le e^{x+\text{ημ}x} \le e^{x+1}}$$

$\lim\limits_{x\to+\infty} e^{x-1} = +\infty$

$\lim\limits_{x\to+\infty} e^{x+1} = +\infty$

43. (β) $\lim\limits_{x\to 0} \ln x - \frac{1}{x} + 1 = \lim\limits_{x\to 0^+} \ln x - \frac{1}{x} + 1$

$= -\infty - (+\infty) + 1 = -\infty - \infty + 1 = -\infty$

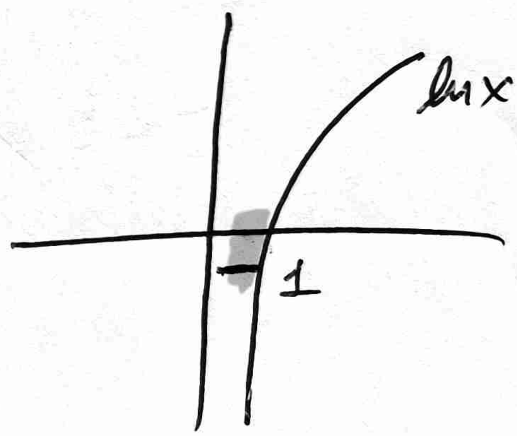
(δ) $\lim\limits_{x\to 0} e^{\frac{1}{x}} \ln x = \lim\limits_{x\to 0^+} e^{\frac{1}{x}} \ln x =$

$= e^{+\infty} \ln 0 = +\infty(-\infty) = -\infty$

44. (γ) $\lim\limits_{x\to 1} \frac{x+\sqrt{1-x}}{\ln x} = \lim\limits_{x\to 1^-} (x+\sqrt{1-x}) \frac{1}{\ln x}$

πρεπει $1-x \geq 0$

$1 \geq x$



$= 1 \cdot (-\infty) = -\infty$

45. ① $\lim\limits_{x\to 0} e^{\frac{\ln x}{x}} = e^{-\infty} = 0$

$\rightarrow \lim\limits_{x\to 0^+} \frac{\ln x}{x} = \lim\limits_{x\to 0^+} \ln x \, \frac{1}{x} = -\infty \cdot (+\infty)$

$= -\infty$

46. Ⓑ $\lim\limits_{x\to +\infty} \left(e^{-x} - \ln x - 1\right) =$

$= e^{-\infty} - \ln(+\infty) - 1 = 0 - (+\infty) - 1$

$= -\infty$,

Ⓓ $\lim\limits_{x\to 0} \left(e^{\frac{1}{x}} - \ln x\right) = \lim\limits_{x\to 0^+} e^{\frac{1}{x}} - \ln x$

$= +\infty - (-\infty) = +\infty + \infty$

$= +\infty$

47. (B) $\lim_{x\to+\infty} \frac{\eta\mu x}{\ln x} = 0$.

$-1 \le \eta\mu x \le 1$

$$\boxed{-\frac{1}{\ln x} \le \frac{\eta\mu x}{\ln x} \le \frac{1}{\ln x}}$$

$$\left.\begin{aligned} &\lim_{x\to+\infty} -\frac{1}{\ln x} = 0 \\ &\lim_{x\to+\infty} \frac{1}{\ln x} = 0 \end{aligned}\right\} \lim_{x\to+\infty} \frac{\eta\mu x}{\ln x} = 0$$

48. (B) $\lim_{x\to-\infty} \ln\left(\sqrt{x^2+1} + x\right) = \ln 0 = -\infty$

$$\to \lim_{x\to-\infty} \left(\sqrt{x^2+1} + x\right) = \lim_{x\to-\infty} \frac{(\sqrt{x^2+1}+x)(\sqrt{x^2+1}-x)}{\sqrt{x^2+1}-x}$$

$$= \lim_{x\to-\infty} \frac{\sqrt{x^2+1}^{\cancel{2}} - x^2}{\sqrt{x^2+1}-x} = \lim_{x\to-\infty} \frac{1}{\sqrt{x^2+1}-x}$$

$$= 0$$

48. ⑥ $\lim_{x\to+\infty} \ln\left(e^{\frac{1}{x}}-1\right) = \ln 0 = -\infty$

⑦ $\lim_{x\to+\infty} \ln(1+e^x) - x =$

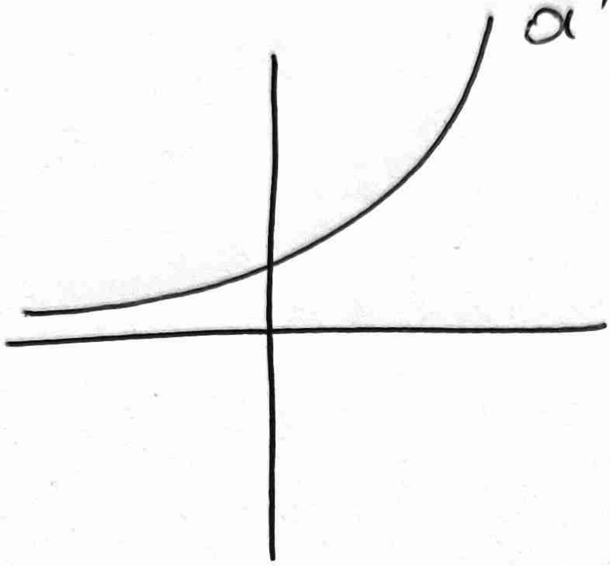
$= \lim_{x\to+\infty} \ln(1+e^x) - \ln e^x$

$= \lim_{x\to+\infty} \ln\left(\frac{1+e^x}{e^x}\right) = \ln 1 = 0.$

$\to \lim_{x\to+\infty} \frac{1+e^x}{e^x} = \lim_{x\to+\infty} \frac{\cancel{e^x}\left(\frac{1}{e^x}+1\right)}{\cancel{e^x}} = 1$

Εκθετικά όρια

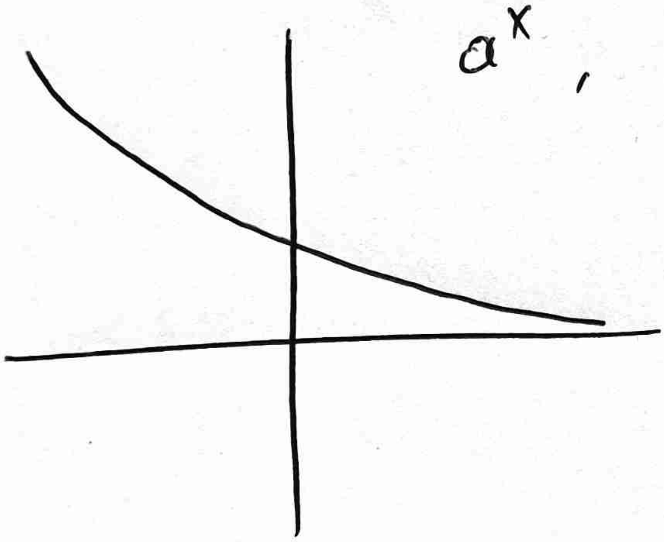
$a^x, \ a>1$



$$\lim_{x \to -\infty} a^x = 0$$

$$\lim_{x \to +\infty} a^x = +\infty$$

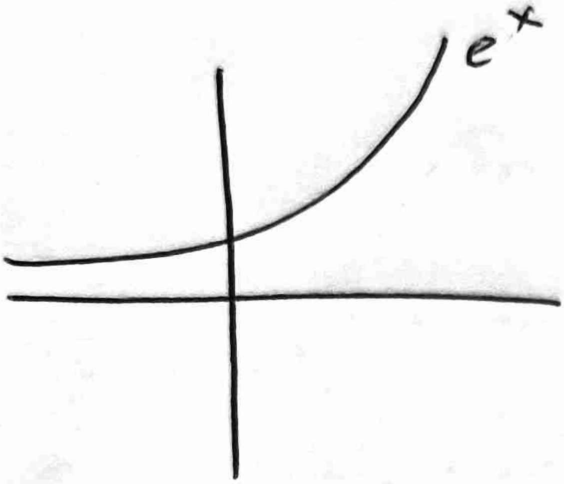
$a^x, \quad 0<a<1$



$$\lim_{x \to -\infty} a^x = +\infty$$

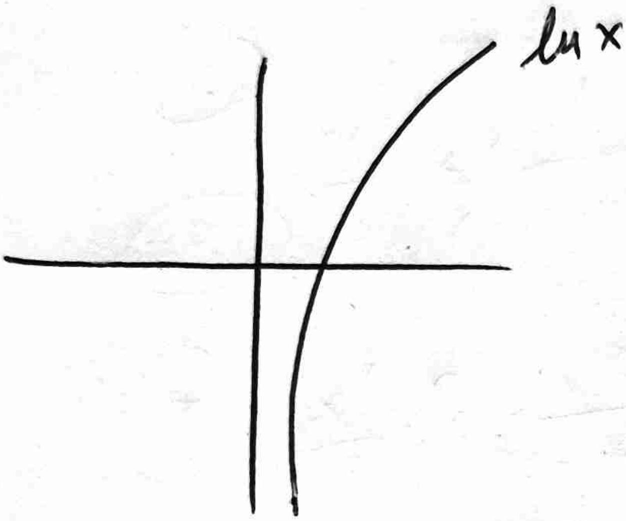
$$\lim_{x \to +\infty} a^x = 0..$$

Προσοχή



$$\lim_{x \to -\infty} e^x = 0$$

$$\lim_{x \to +\infty} e^x = +\infty .$$



$$\lim_{x \to 0^+} \ln x = -\infty$$

$$\lim_{x \to +\infty} \ln x = +\infty .$$

Μνημονικοί κανόνες

$e^{-\infty} = 0$

$e^{+\infty} = +\infty$

$\ln 0 = -\infty$

$\ln +\infty = +\infty$

$e^1 = e$

$e^0 = 1$

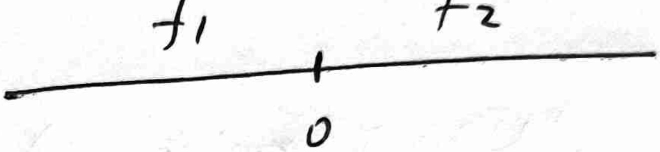
$\ln 1 = 0$

$\ln e = 1.$

Ενότητα 11

4. (Β) $f(x) = \begin{cases} e^x + ημx, & x \leq 0 \\ \ln(x+1), & x > 0 \end{cases}$.

Είναι συνεχής στο 0;



$$\lim_{x \to 0^-} f(x) = \lim_{x \to 0^-} (e^x + ημx) = 1$$

$$\lim_{x \to 0^+} f(x) = \lim_{x \to 0^+} \ln(x+1) = 1$$

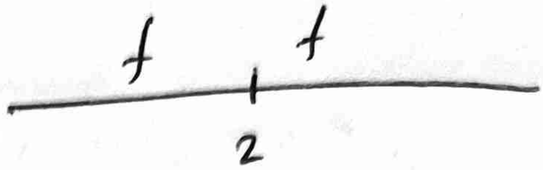
$$\left.\right\} \lim_{x \to 0} f(x) = 1$$

$$f(0) = e^0 + ημ0 = 1$$

Αφού $f(0) = \lim_{x \to 0} f(x) = 1$

Συνεχής στο 0!.

4. (γ) $f(x) = \begin{cases} \dfrac{x^2-4}{x-2}, & x \neq 2 \\ 4, & x = 2 \end{cases}$



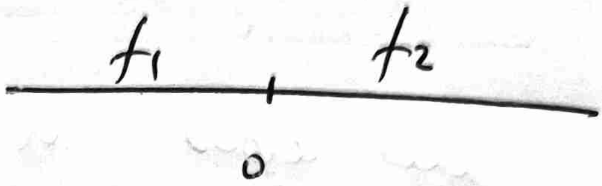
• $\lim\limits_{x\to 2} f(x) = \lim\limits_{x\to 2} \dfrac{x^2-4}{x-2} = \lim\limits_{x\to 2} \dfrac{\cancel{(x-2)}(x+2)}{\cancel{x-2}} = 4$

• $f(2) = 4$

Άρα $f(2) = \lim\limits_{x\to 2} f(x)$ η f συνεχής

στο 2.

22. $f(x)=\begin{cases} x^2 ημ\frac{1}{x}, & x>0 \\ 0, & x=0 \\ x e^{\frac{1}{x}}, & x<0 \end{cases}$



$$\lim_{x\to 0^-} f(x) = \lim_{x\to 0^-} x e^{\frac{1}{x}} = 0 \cdot e^{-\infty} = 0 \cdot 0 = 0$$

$$\lim_{x\to 0^+} f(x) = \lim_{x\to 0^+} x^2 ημ\frac{1}{x} = 0$$

$$-1 \le ημ\frac{1}{x} \le 1$$

$$\boxed{-x^2 \le x^2 ημ\frac{1}{x} \le x^2}$$

$$\left.\begin{array}{l} \lim_{x\to 0^+} -x^2 = 0 \\ \lim_{x\to 0^+} x^2 = 0 \end{array}\right\} \lim_{x\to 0} x^2 ημ\frac{1}{x} = 0.$$

Άρα $\lim_{x\to 0} x^2 ημ\frac{1}{x} = 0$ ✓ $f(0)=0$

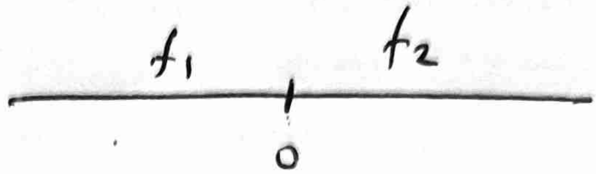
Είναι συνεχής στο $(-\infty, 0)$ και $(0, +\infty)$

ως Π. Σ. Δ

και αφού η f συνεχής στο 0

η f συνεχής γενικά.

8. $f(x) = \begin{cases} e^{\frac{1}{x}}, & x<0 \\ 1, & x=0 \\ x^2+1, & x>0 \end{cases}$



ⓐ Είναι συνεχής;

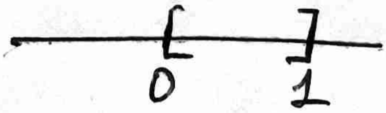
$\lim_{x\to 0^-} f(x) = \lim_{x\to 0^-} e^{\frac{1}{x}} = e^{-\infty} = 0$

$\lim_{x\to 0^+} f(x) = \lim_{x\to 0^+} x^2+1 = 1$

Το $\lim_{x\to 0} f(x)$ δεν υπάρχει άρα η f σx_0 ασυνεχής!

ⓑ Είναι συνεχής στο $[0,1]$

Η f συνεχής $(0,1)$ ως πράξεις συνεχών συναρτήσεων.



$\lim_{x\to 0^+} f(x) = 1$ ✓

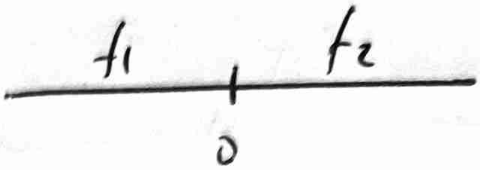
$f(0) = 1$

Συνεχής $[0,1]$.

$\lim_{x\to 1^-} f(x) = 2$ ✓

$f(1) = 2$

5. (β) $f(x) = \begin{cases} x\,ημ\frac{1}{x}, & x<0 \\ συν x, & x \geq 0 \end{cases}$



• $\lim\limits_{x\to 0^-} f(x) = \lim\limits_{x\to 0^-} x\,ημ\frac{1}{x} = 0$

$$-1 \leq ημ\frac{1}{x} \leq 1$$

$$\boxed{-x \geq x\,ημ\frac{1}{x} \geq x}$$

$$\left.\begin{aligned} \lim_{x\to 0} -x = 0 \\ \lim_{x\to 0} x = 0 \end{aligned}\right\} \lim_{x\to 0} x\,ημ\frac{1}{x} = 0$$

• $\lim\limits_{x\to 0^+} f(x) = \lim\limits_{x\to 0^+} συν x = 1$

Το $\lim\limits_{x\to 0} f(x)$ δεν υπάρχει άρα

η $f(x)$ δεν είναι συνεχής στο 0

20. (B) $f(x) = \begin{cases} \dfrac{\text{συν}\,x^2 - 1}{x}, & x \neq 0 \\ 0, & x = 0 \end{cases}$

$$\lim_{x\to 0} \frac{\text{συν}\,x^2 - 1}{x} = \lim_{x\to 0} \frac{\text{συν}\,x^2 - 1}{x^2} \cdot \frac{x^{\cancel{2}}}{\cancel{x}} = 0 \cdot 0 = 0$$

$$\rightarrow \lim_{x\to 0} \frac{\text{συν}\,x^2 - 1}{x^2} \underset{\substack{x\to 0 \\ t\to 0}}{\overset{x^2 = t}{=\!=\!=}} \lim_{t\to 0} \frac{\text{συν}\,t - 1}{t} = 0$$

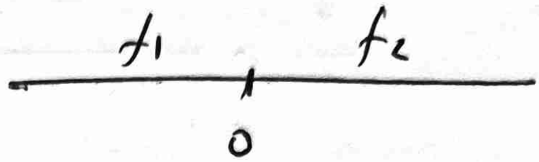
Άρα $\lim_{x\to 0} f(x) = 0$

$f(0) = 0$

Βασικό όριο

$$\lim_{x\to 0} \frac{\text{συν}\,x - 1}{x} = 0$$

21. (δ) $f(x)=\begin{cases} e^{\frac{\ln x}{\sqrt{x}}}, & x>0 \\ \eta\mu x, & x\le 0 \end{cases}$



$$\lim_{x\to 0^-} f(x) = \lim_{x\to 0^-} \eta\mu x = 0$$

$$\lim_{x\to 0^+} f(x) = \lim_{x\to 0^+} e^{\frac{\ln x}{\sqrt{x}}} = e^{-\infty} = 0$$

$$\to \lim_{x\to 0^+} \frac{\ln x}{\sqrt{x}} = \lim_{x\to 0^+} \ln x \cdot \frac{1}{\sqrt{x}} = -\infty\cdot(+\infty)$$

$$= -\infty$$

Άρα $\lim_{x\to 0} f(x) = 0$

$f(0)=0$

Συνεχής στο x_0

$$f(x_0) = \lim_{x \to x_0} f(x).$$

Συνεχής στο $[\alpha, \beta]$.

Η f συνεχής (α, β)

και ισχύουν $\lim_{x \to \alpha^+} f(x) = f(\alpha)$

$$\lim_{x \to \beta^-} f(x) = f(\beta).$$

Όλες οι γνωστές συναρτήσεις.

e^x, $\ln x$, $\eta\mu x$, $\sigma\upsilon\nu x$, α^x

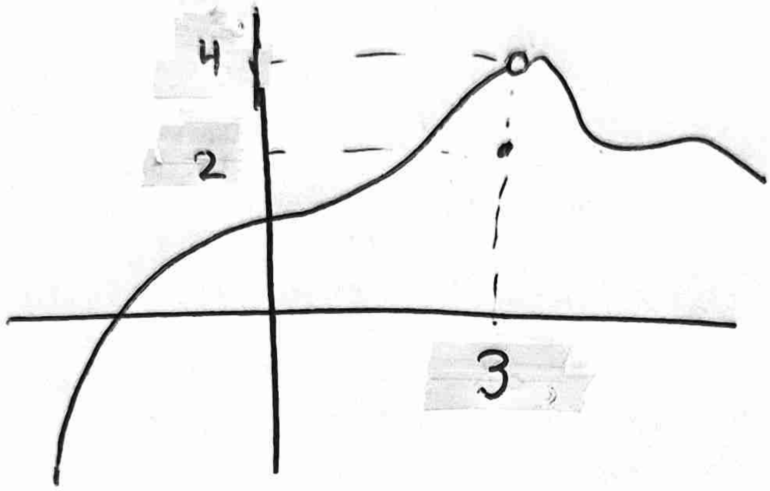
x^2-5x+6, x, x^8, $\sqrt{x}$ κτλ

και οι πράξεις μεταξύ αυτών.

Είναι συνεχείς συναρτήσεις.

Συνέχεια Συνάρτησης

1.



$$\lim_{x \to 3^-} f(x) = 4$$

$$\lim_{x \to 3^+} f(x) = 4$$

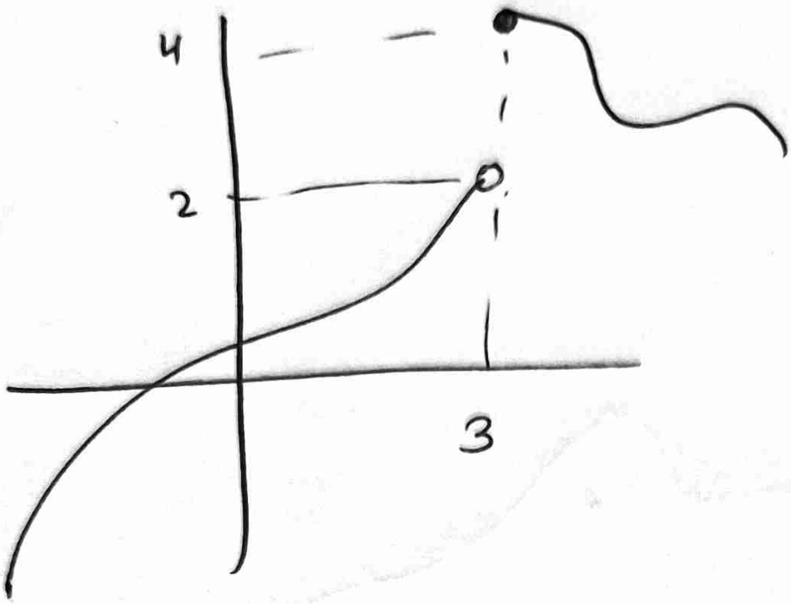
$$\lim_{x \to 3} f(x) = 4$$

$$f(3) = 2$$

Η f όχι συνεχής στο 3

γιατί $f(3) \neq \lim_{x \to 3} f(x)$

2.



$\lim_{x \to 3^-} f(x) = 2$

$\lim_{x \to 3^+} f(x) = 4$

Το όριο δεν υπάρχει.

Η f όχι συνεχής στο 3.

25. $f(x) = \begin{cases} 2x^2 - 1 + \ln a, & x \le 1 \\ \sqrt{x} + 1 - \sqrt{a}, & x > 1 \end{cases}$ Συνεχής!

$\lim_{x \to 1^-} f(x) = 1 + \ln a$

$\lim_{x \to 1^+} f(x) = 2 - \sqrt{a}$

$1 + \ln a = 2 - \sqrt{a}$

$\ln a = 1 - \sqrt{a}$

$\ln a + \sqrt{a} - 1 = 0$

$\boxed{\varphi(x) = \ln x + \sqrt{x} - 1}$

$\varphi(a) = 0$

$\varphi(a) = \varphi(1)$

φ 1-1

$\varphi \nearrow$

$\underline{\underline{a = 1}}$

14. Εστω $f: \mathbb{R} \to \mathbb{R}$ συνεχης.

$$x f(x) + 2 = f(x) + \sqrt{x^2+3}$$

Αν η f συνεχης Τοτε $f(x)$

$$x f(x) - f(x) = \sqrt{x^2+3} - 2$$

$$f(x)(x-1) = \sqrt{x^2+3} - 2$$

$$\boxed{f(x) = \frac{\sqrt{x^2+3} - 2}{x-1}, \quad x \neq 1}$$

$$f(1) = \lim_{x\to 1} f(x) = \lim_{x \to 1} \frac{\sqrt{x^2+3} - 2}{x-1}$$

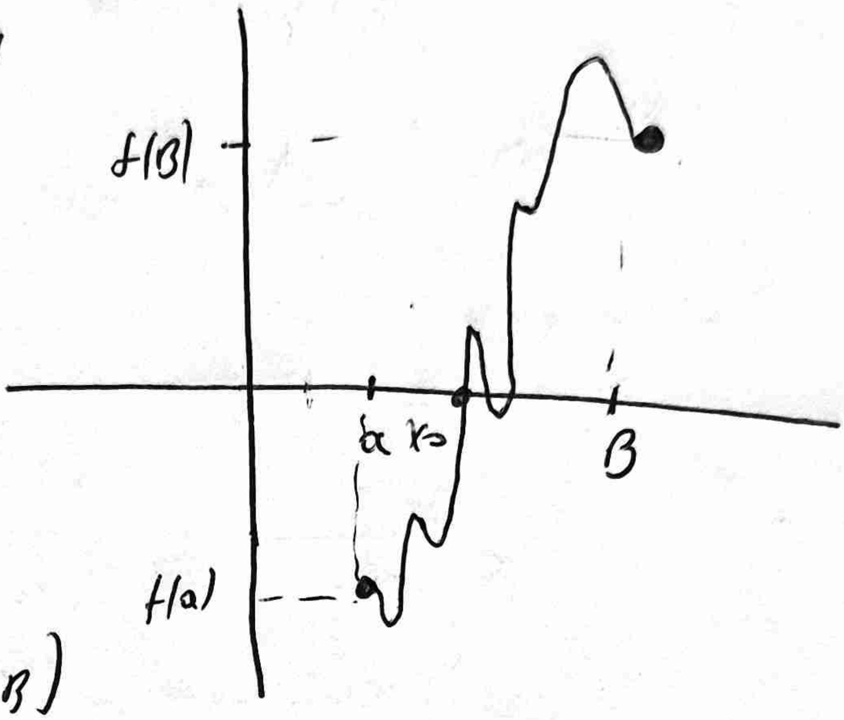
$$= \lim_{x\to 1} \frac{\cancel{(x-1)}(x+1)}{\cancel{(x-1)}\left(\sqrt{x^2+3}+2\right)} = \frac{2}{4} = \frac{1}{2}$$

$$f(x) = \begin{cases} \dfrac{\sqrt{x^2+3} - 2}{x-1}, & x \neq 1 \\ \dfrac{1}{2}, & x = 1. \end{cases}$$

Bolzano

1. f συνεχης $[a, \beta]$

2. $f(a) \cdot f(\beta) < 0$



Τοτε υπαρχει τουλαχιστον ενα αριθμος $x_0 \in (a, \beta)$ τω $f(x_0) = 0$

Η C_f τεμνει τον $x'x$ τουλαχιστον μια φορα.

4. (B) Νδο η εξίσωση $e^{x-2} = 2 - x$

έχει μία τουλάχιστον ρίζα στο $(1, 2)$

$$\underbrace{e^{x-2} - 2 + x}_{f(x)} = 0$$

• Η $f(x)$ συνεχής $[1, 2]$ ως π.σ.σ

$f(1) = e^{1-2} - 2 + 1 = e^{-1} - 1 = \frac{1}{e} - 1 < 0$

$f(2) = e^{2-2} - 2 + 2 = e^{0} = 1 > 0$

• $f(1) f(2) < 0$

Από Bolzano $\exists\, x_0 \in (1, 2)$ τ.ω $f(x_0) = 0$

$\downarrow$

τέτοιο ώστε

$e^{x_0-2} - 2 + x_0 = 0$

$e^{x_0-2} = 2 - x_0 \qquad x_0 \in (1, 2)$

11. (δ) Νδο η εξίσωση $e^x - 1 = συνx$

έχει μοναδική ρίζα. $(0, \frac{\pi}{2})$

$\underbrace{e^x - 1 - συνx = 0}_{f(x)}$

Η f συνεχής $[0, \frac{\pi}{2}]$ ως π.σ.σ

$f(0) = e^0 - 1 - συν0 = -1 < 0$

$f(\frac{\pi}{2}) = e^{\frac{\pi}{2}} - 1 > 0$

• $\frac{\pi}{2} > 0 \Rightarrow e^{\frac{\pi}{2}} > e^0 \Rightarrow e^{\frac{\pi}{2}} > 1 \Rightarrow e^{\frac{\pi}{2}} - 1 > 0$.

$f(0) f(\frac{\pi}{2}) < 0$

Bolzano $\exists\, \xi \in (0, \frac{\pi}{2})$ τ.ω $f(\xi) = 0$.

$e^{\xi} - 1 = συν\xi$

Μονοτονία

• $x_1 < x_2 \Rightarrow e^{x_1} - 1 < e^{x_2} - 1$

• $x_1 < x_2 \Rightarrow συνx_1 > συνx_2$ (+)

$-συνx_1 < -συνx_2$

$f \uparrow$

ξ μοναδικό.