Προετοιμασια

Διαγωνισματος

κυριακη

12/10/25.

ωρα: 16:30 – 19.30

Θεωρια

1.1
1.3
1.5
1.6
1.7
1.10
1.11
1.12
1.13
1.14
1.15.
1.18
1.21
1.22
1.23
1.24
1.25
1.26
2.3
2.5.

Αποδειξη

3.4.

Συνθεση Συναρτησεως

1. Εστω $f: (1, +\infty) \longrightarrow \mathbb{R}$ $\quad f(x) = \dfrac{x+1}{x-1}$

Εστω $g: (0, +\infty) \longrightarrow \mathbb{R}$ $\quad g(x) = \ln(x+1)$

$$\longrightarrow (g \circ f)(x) = \ln\left(\frac{x+1}{x-1} + 1\right) = \ln\left(\frac{x+1+x-1}{x-1}\right)$$

$$= \ln\left(\frac{2x}{x-1}\right)$$

$x \in D_f$ και $f(x) \in D_g$

$x > 1$ $\quad \dfrac{x+1}{x-1} > 0$

x		-1		1	
$x+1$	$-$		$+$		$+$
$x-1$	$-$		$-$		$+$
$\frac{x+1}{x-1}$	$+$		$-$		$+$

$x \in (-\infty, -1) \cup (1, +\infty)$

$D_{g \circ f} = (1, +\infty)$,

Αντιστροφη

1. $f(x) = \ln\left(\frac{2x}{x-1}\right)$, $x > 1$

Εστω $f(x_1) = f(x_2)$

$$\ln \frac{2x_1}{x_1 - 1} = \ln \frac{2x_2}{x_2 - 1}$$

$$\frac{2x_1}{x_1 - 1} = \frac{2x_2}{x_2 - 1}$$

$$2x_1(x_2 - 1) = 2x_2(x_1 - 1)$$

$$\cancel{2x_1x_2} - 2x_1 = \cancel{2x_2x_1} - 2x_2$$

$$x_1 = x_2$$

Αρα η f 1-1 αρα αντιστρ.

$$y = \ln \frac{2x}{x-1} \quad , x>1$$

Tclos

$x>1$

$$e^y = \frac{2x}{x-1}$$

$$\frac{e^y}{e^y-2} > 1$$

$$e^y(x-1) = 2x$$

$$\frac{e^y}{e^y-2} - 1 > 0$$

$$e^y x - e^y = 2x$$

$$\frac{\cancel{e^y} - \cancel{e^y} + 2}{e^y - 2} > 0$$

$$e^y x - 2x = e^y$$

$$\Rightarrow e^y - 2 > 0$$

$$e^y > 2$$

$$y > \ln 2$$

$$x(e^y - 2) = e^y$$

$$e^y - 2 \neq 0$$

$$x = \frac{e^y}{e^y-2}$$

$$e^y \neq 2$$

$$y \neq \ln 2 .$$

$$f^{-1}(y) = \frac{e^y}{e^y-2}$$

$$\boxed{f^{-1}(x) = \frac{e^x}{e^x-2}}$$

$$D_{f^{-}} = (\ln 2, +\infty) ,$$

2. $f(x) = (x-1)^2 - 1$, $x \le 1$

$x_1 < x_2 \Rightarrow x_1 - 1 < x_2 - 1 \Rightarrow$

$\Rightarrow (x_1-1)^2 > (x_2-1)^2$

$(x_1-1)^2 - 1 > (x_2-1)^2 - 1$

$f(x_1) > f(x_2)$

$f \searrow \Rightarrow f$ з $1-1$

f odwracalna.

$y = (x-1)^2 - 1$

$y+1 = (x-1)^2$ $\quad\quad$ $y+1 \geq 0$

$y \geq -1$

$\sqrt{y+1}^{\,2} = (x-1)^2$

$|\overset{\oplus}{\sqrt{y+1}}| = |\overset{\ominus}{x-1}|$

$\sqrt{y+1} = 1 - x$

$x = 1 - \sqrt{y+1}$

$\boxed{f^{-1}(x) = 1 - \sqrt{x+1}}$.

Tcλɔʃ

$x < 1$

$\cancel{1} - \sqrt{y+1} \cancel{< 1}$

$-\sqrt{y+1} < 0$

Ευρεση Τυπου $f(x)$

1. $f(e^x-1) = \ln x + 1$

Βρει τυπο $f(x)$

Θετω $e^x - 1 = t$

$e^x = t+1$

$x = \ln(t+1)$

$$f(t) = \ln\left(\ln(t+1)\right) + 1$$

$$\boxed{f(x) = \ln\left(\ln(x+1)\right) + 1}$$

2. Δινεται $f: \mathbb{R} \to \mathbb{R}$ συνεχης.

και $(x-1)f(x) = \sqrt{x^2+3} - 2 \quad \forall x \in \mathbb{R}$.

Βρε τυπο $f(x)$.

$$\boxed{f(x) = \frac{\sqrt{x^2+3} - 2}{x-1}, \quad x \neq 1}$$

$f(1) = \lim_{x \to 1} f(x)$ γιατι η f συνεχης!

$$f(1) = \lim_{x \to 1} \frac{\sqrt{x^2+3} - 2}{x-1} = \lim_{x \to 1} \frac{\left(\sqrt{x^2+3} - 2\right)\left(\sqrt{x^2+3} + 2\right)}{(x-1)\left(\sqrt{x^2+3} + 2\right)}$$

$$= \lim_{x \to 1} \frac{\cancel{\sqrt{x^2+3}}^{\,\cancel{2}} - 2^2}{(x-1)\left(\sqrt{x^2+3} + 2\right)} = \lim_{x \to 1} \frac{\cancel{(x-1)}(x+1)}{\cancel{(x-1)}\left(\sqrt{x^2+3} + 2\right)}$$

$$= \frac{2}{4} = \frac{1}{2}.$$

$$f(1) = \frac{1}{2}.$$

$$f(x) = \begin{cases} \dfrac{\sqrt{x^2+3} - 2}{x-1}, & x \neq 1 \\ \dfrac{1}{2}, & x = 1. \end{cases}$$

3. $f: \mathbb{R} \to \mathbb{R}$. $\quad x^2 f\left(\frac{1}{x}\right) = ημx, \; x \neq 0$.

Βρες τύπο $f(x)$.

$$x^2 f\left(\frac{1}{x}\right) = ημx$$

$$\boxed{\text{Θέτω } \frac{1}{x} = t \Rightarrow x = \frac{1}{t}}$$

$$\left(\frac{1}{t}\right)^2 f(t) = ημ\frac{1}{t}$$

$$\frac{1}{t^2} f(t) = ημ\frac{1}{t}$$

$$f(t) = t^2 ημ\frac{1}{t}. \quad ή \quad \boxed{f(x) = x^2 ημ\frac{1}{x}, \; x \neq 0.}$$

$$f(0) \overset{f \text{ συνεχής}}{=\!=} \lim_{x \to 0} f(x) = \lim_{x \to 0} x^2 ημ\frac{1}{x} = 0$$

$$-1 \leq ημ\frac{1}{x} \leq 1$$

$$\boxed{-x^2 \leq x^2 ημ\frac{1}{x} \leq x^2}$$

$$\left.\begin{array}{l} \lim_{x \to 0} -x^2 = 0 \\ \lim_{x \to 0} x^2 = 0 \end{array}\right\} \text{ Από Κ.Π } \underline{\underline{\lim_{x \to 0} x^2 ημ\frac{1}{x} = 0}}$$

$$f(x) = \begin{cases} x^2 \eta\mu \frac{1}{x}, & x \neq 0 \\ 0, & x = 0 \end{cases}.$$

4. Έστω $g:(0,+\infty) \longrightarrow R$

$$g^2(x) + x^2 = x^4 - 2x\,g(x)$$

Η C_g διέρχεται $A(1,-2)$. $\quad \hookrightarrow g(1) = -2$

Βρείτε τύπο $g(x)$

$$g^2(x) + 2x\,g(x) + x^2 = x^4$$

$$(g(x) - x)^2 = (x^2)^2$$

$$|g(x) - x| = |\overset{+}{x^2}|$$

$$|\overset{-}{h(x)}| = x^2$$

Ρίζες $h(x)$

$h(x) = 0$

$|h(x)| = 0$

$x^2 = 0$

$\boxed{x = 0}$

x		0	
$h(x)$			$-$

$-h(x) = x^2$

$h(x) = -x^2$

$g(x) - x = -x^2$

$$\boxed{g(x) = x - x^2}$$

$h(1) = g(1) - 1 = -2 - 1 = -3.$

Ασκηση 5ος

Εστω $f(x) = e^x + x$

1. Βρει το συνολο τιμων.

- $\dfrac{D_f}{\mathbb{R}}$

- Μονοτονια

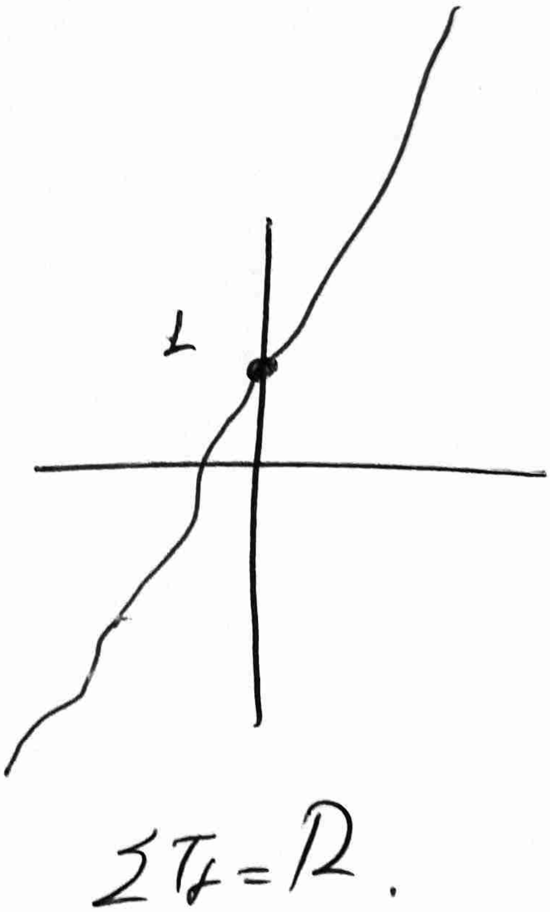
$\circ\; x_1 < x_2 \Rightarrow e^{x_1} < e^{x_2}$

$\cdot\; x_1 < x_2 \quad \rfloor \; (+)$

$$e^{x_1} + x_1 < e^{x_2} + x_2$$

$$f(x_1) < f(x_2)$$

$$f \nearrow$$



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x	$-\infty$	$+\infty$
$f(x)$	$-\infty$ $\nearrow$	$+\infty$

$$\lim_{x\to -\infty} f(x) = \lim_{x\to -\infty} e^x + x = -\infty$$

$$\lim_{x\to +\infty} f(x) = \lim_{x\to +\infty} e^x + x = +\infty$$

$\Sigma T_f = \mathbb{R}$.

2. Νδο η εξίσωση $f(x)=0$ έχει ακριβώς μία αρνητική ρίζα.

- f συνεχής
- $f \nearrow$
- $ΣT_f = \mathbb{R}$

Το $0 \in ΣT_f$ άρα

$$\exists!\ \xi \text{ τ.ω } f(\xi)=0.$$

Έστω $\xi \geq 0$

$f \nearrow$

$f(\xi) \geq f(0)$

$0 \geq 1$

Άτοπο! $\xi < 0$.