

Επαγωγή για το Διαγώνισμα.

1. Να λυθούν οι εξισώσεις.

α) $|2x-1|=3$

$$2x-1=3$$

$$2x=4$$

$$x=2$$

ή $2x-1=-3$

$$2x=-2$$

$$x=-1$$

β) $|2x-1|=-3$

Αδυναμία

γ) $|2x-1|=0$

$$\Rightarrow 2x-1=0$$

$$2x=1$$

$$x=\frac{1}{2}$$

$$\textcircled{\delta} \quad |2x-1| = x-2$$

$$\left\{ \begin{array}{l} \text{при } x-2 \geq 0 \Rightarrow x \geq 2 \\ \downarrow \end{array} \right.$$

$$2x-1 = x-2$$

$$2x-x = 1-2$$

$$\underline{\underline{x = -1}}$$

$$\text{и} \quad 2x-1 = -x+2$$

$$2x+x = 2+1$$

$$3x = 3$$

$$\underline{\underline{x = 1}}$$

$$\textcircled{\varepsilon} \quad |2x-1| = |x-1|$$

$$2x-1 = x-1 \quad \text{и} \quad 2x-1 = -x+1$$

$$2x-x = 0$$

$$\textcircled{x = 0}$$

$$2x+x = 2$$

$$3x = 2$$

$$\textcircled{x = \frac{2}{3}}$$

$$\textcircled{3} |x-2| + |x^2-4| = 0$$

$$x-2=0$$

$$\textcircled{x=2} \checkmark$$

or

$$x^2-4=0$$

$$x^2=4$$

$$\textcircled{x=2}$$

$$\textcircled{x=-2}$$

2. κλασματική εξίσωση

$$\frac{x-1}{x} - \frac{1}{x+1} + \frac{1}{x^2+x} = 0$$

$$\frac{x-1}{x} - \frac{1}{x+1} + \frac{1}{x(x+1)} = 0$$

Προσδιορισμοί

• $\boxed{x=0}$

• $x+1=0 \Rightarrow \boxed{x=-1}$

• $x(x+1)=0 \Rightarrow \boxed{x=0} \vee x+1=0$

$\boxed{x=-1}$

ΕΚΛΟ: $x(x+1)$

Προτι $x \neq 0$

$x \neq -1$

$$x(x+1) \frac{x-1}{x} - x(x+1) \frac{1}{x+1} + x(x+1) \frac{1}{x(x+1)} = 0$$

$$(x+1)(x-1) - x + 1 = 0$$

$$(x+1)(x-1) - (x-1) = 0$$

$$(x-1)(x+1-1) = 0$$

$(x-1)x = 0$

$\boxed{x=1}$ ✓

~~$\boxed{x=0}$~~

$$B. \quad \lambda^2(x-1) = \lambda + x - 2.$$

παράμετροι του Βελγίου

$$\lambda^2 x - \lambda^2 = \lambda + x - 2$$

$$\lambda^2 x - x = \lambda^2 + \lambda - 2$$

$$(\lambda^2 - 1)x = \lambda^2 + \lambda - 2$$

$$(\lambda - 1)(\lambda + 1)x = (\lambda + 2)(\lambda - 1)$$

1. Αν $\lambda = 1$ τότε $0x = 0$ Αόριστη.

2. Αν $\lambda = -1$ τότε $0x = -1 \cdot (-2) \Rightarrow 0x = 2$ Αδύνατη

3. Αν $\lambda \neq 1, \lambda \neq -1$ $x = \frac{(\lambda+2)(\cancel{\lambda-1})}{(\cancel{\lambda-1})(\lambda+1)} \Rightarrow x = \frac{\lambda+2}{\lambda+1}$

$$\lambda^2 + \lambda - 2 = (\lambda + 2)(\lambda - 1)$$

$$x = \frac{-B \pm \sqrt{\Delta}}{2a} = \frac{-1 \pm 3}{2} \begin{matrix} \textcircled{1} \\ \textcircled{-2} \end{matrix}$$

$$\Delta = B^2 - 4ac$$

$$\Delta = 1 - 4 \cdot 1 \cdot (-2)$$

$$\Delta = 9$$

4. $\hookrightarrow$ $x^2 + 2y^2 - 2x + 4y + 3 = 0$

ψα xvw εα x, y

$$x^2 - 2x + 1 + 2y^2 + 4y + 2 = 0$$

$$(x-1)^2 + 2(y^2 + 2y + 1) = 0$$

$$(x-1)^2 + 2(y+1)^2 = 0$$

πρστυ

$$x-1=0$$

κλμ

$$y+1=0$$

$$x=1$$

$$y=-1$$

5. Ρητοροίηση παραστάσεων.

$$\textcircled{\alpha} \frac{\sqrt{2}}{\sqrt{2}-\sqrt{3}} = \frac{\sqrt{2}(\sqrt{2}+\sqrt{3})}{(\sqrt{2}-\sqrt{3})(\sqrt{2}+\sqrt{3})} =$$

$$= \frac{\sqrt{2}^2 + \sqrt{2}\sqrt{3}}{\sqrt{2}^2 - \sqrt{3}^2} = \frac{2 + \sqrt{6}}{2 - 3} = \frac{2 + \sqrt{6}}{-1}$$

$$= -2 - \sqrt{6}.$$

$$\textcircled{\beta} \frac{\sqrt{2}-\sqrt{3}}{\sqrt{2}} = \frac{\sqrt{2}(\sqrt{2}-\sqrt{3})}{\sqrt{2}^2} = \frac{\sqrt{4}-\sqrt{6}}{2}$$

$$= \frac{2 - \sqrt{6}}{2}$$

10. ⑥ $|x^2 - 3| = 1$

$$x^2 - 3 = 1$$

$$x^2 = 4$$

$$x^2 = 2^2$$

$$x = 2 \quad x = -2$$

$$\therefore x^2 - 3 = -1$$

$$x^2 = 2$$

$$x = \pm\sqrt{2}$$

⑦ $|x^4 - 1| = 3$

$$x^4 - 1 = 3$$

$$x^4 = 4$$

$$(x^2)^2 = 2^2$$

$$x^2 = 2$$

$$x = \pm\sqrt{2}$$

$$\therefore x^4 - 1 = -3$$

$$x^4 = -2$$

Answer.

$$\therefore x^2 = -2$$

Answer

$$\textcircled{1} |x^3 - 1| = 7$$

$$x^3 - 1 = 7 \quad \vee \quad x^3 - 1 = -7$$

$$x^3 = 8$$

$$x^3 = 2^3$$

$$\textcircled{x = 2}$$

$$x^3 = -6$$

$$\underline{\underline{x = -\sqrt[3]{6}}}$$

$$\textcircled{8} |x^5 - 2| = 30$$

$$x^5 - 2 = 30 \quad \vee \quad x^5 - 2 = -30$$

$$x^5 = 32$$

$$x^5 = 2^5$$

$$\textcircled{x = 2}$$

$$x^5 = 2 - 30$$

$$x^5 = -28$$

$$\textcircled{x = -\sqrt[5]{28}}$$

Евотута 13

$$(a) \quad x^2 - 2(x-1) = 2x - 1$$

$$x^2 - 2x + 2 - 2x + 1 = 0$$

$$x^2 - 4x + 3 = 0$$

$$(x=1) \quad (x=3)$$

$$(b) \quad (x-1)^2 = 3x(x-2) - 1$$

$$x^2 - 2x + 1 = 3x^2 - 6x - 1$$

$$x^2 - 3x^2 + 4x + 2 = 0$$

$$-2x^2 + 4x + 2 = 0$$

$$\Delta = 32$$

$$x = \frac{-4 \pm \sqrt{32}}{-4} = \frac{-4 \pm 4\sqrt{2}}{-4}$$

$$x = \frac{-1 \pm \sqrt{2}}{-1}$$

$$\begin{aligned} & (1 - \sqrt{2}) \\ & (\sqrt{2} + 1) \end{aligned}$$

$$\textcircled{8} \quad (2x-1)^2 - 3(x-1) = 1$$

$$4x^2 - 4x + 1 - 3x + 3 = 1$$

$$4x^2 - 7x + 3 = 0$$

$$\Delta = 49 - 48 = 1$$

$$x = \frac{7 \pm 1}{8} \quad \begin{array}{l} \textcircled{1} \\ \textcircled{\frac{6}{8}} \end{array}$$

9. ⑥ $x^3 - (3x+1)^3 = 0$

$$x^3 = (3x+1)^3$$

$$x = 3x+1$$

$$x - 3x = 1$$

$$-2x = 1$$

$$\underline{\underline{x = -\frac{1}{2}}}$$

⑬ $27x^3 - (x-1)^3 = 0$

$$27x^3 = (x-1)^3$$

$$(3x)^3 = (x-1)^3$$

$$3x = x-1$$

$$2x = -1$$

$$\underline{\underline{x = -\frac{1}{2}}}$$

Ενότητα 12

5. (a) $x^4 = x \Rightarrow x^4 - x = 0$

$$x(x^3 - 1) = 0$$

$$x = 0$$

$$x^3 - 1 = 0$$

$$x^3 = 1$$

$$x = 1$$

(b) $81x^5 - 16x = 0$

$$x(81x^4 - 16) = 0$$

$$x = 0$$

$$81x^4 - 16 = 0$$

$$81x^4 = 16$$

$$(3x)^4 = 2^4$$

$$3x = 2$$

ή

$$3x = -2$$

$$x = \frac{2}{3}$$

$$x = -\frac{2}{3}$$

$$\textcircled{1} \quad 2x^7 = 32x^3$$

$$2x^7 - 32x^3 = 0$$

$$x^7 - 16x^3 = 0$$

$$x^3 (x^4 - 16) = 0$$

$$x^3 = 0$$

$$\textcircled{x=0}$$

$$\vee \quad x^4 - 16 = 0$$

$$x^4 = 16$$

$$x^4 = 2^4$$

$$\textcircled{x=2}$$

$$\textcircled{x=-2}$$

$$\textcircled{8} \quad 24x^5 - 3x^2 = 0$$

$$8x^5 - x^2 = 0$$

$$x^2 (8x^3 - 1) = 0$$

$$x^2 = 0$$

$$\textcircled{x=0}$$

$$\vee \quad 8x^3 - 1 = 0$$

$$8x^3 = 1$$

$$(2x)^3 = 1^3$$

$$2x = 1$$

$$\textcircled{x = \frac{1}{2}}$$

$$\textcircled{E} \quad 54x^7 = 2x^4$$

$$27x^7 = x^4$$

$$27x^7 - x^4 = 0$$

$$x^4(27x^3 - 1) = 0$$

$$x^4 = 0$$

$$\text{or} \quad 27x^3 - 1 = 0$$

$$27x^3 = 1$$

$$(3x)^3 = 1^3$$

$$3x = 1$$

$$x = \frac{1}{3}$$

$$\underline{\underline{\quad\quad\quad}}$$

$$\textcircled{Q2} \quad x^5 + x^3 = 0$$

$$x^3(x^2 + 1) = 0$$

$$x^3 = 0$$

$$\text{or} \quad x^2 + 1 = 0$$

$$\textcircled{x=0}$$

Answer

Exercícios 11

4. (a) $|2x-1| = |x|$

$$2x-1 = x$$

$$\underline{\underline{x=1}}$$

$$2x-1 = -x$$

$$3x=1$$

$$\underline{\underline{x=\frac{1}{3}}}$$

(b) $|5x-1| - |4x-1| = 0$

$$|5x-1| = |4x-1|$$

$$5x-1 = 4x-1$$

$$\underline{\underline{x=0}}$$

$$5x-1 = -4x+1$$

$$9x = 2$$

$$\underline{\underline{x=\frac{2}{9}}}$$

(c) $\frac{|3x-2|-2}{2} = |x|-1$

$$|3x-2| - \cancel{2} = 2|x| - \cancel{2}$$

$$|3x-2| = 2|x|$$

$$3x-2 = 2x$$

$$\underline{\underline{x=2}}$$

$$3x-2 = -2x$$

$$5x = 2$$

$$\underline{\underline{x=\frac{2}{5}}}$$

12. (a) $\left| \frac{2-x}{x+2} \right| = 3$

$$\frac{2-x}{x+2} = 3$$

$$2-x = 3x+6$$

$$-4 = 4x$$

$$x = -1$$

$$\vee \frac{2-x}{x+2} = -3$$

$$2-x = -3x-6$$

$$2x = -8$$

$$x = -4$$

(b) $\frac{|2x+1|}{|1-2x|} = 3$

$$\left| \frac{2x+1}{1-2x} \right| = 3$$

$$\frac{2x+1}{1-2x} = 3$$

$$2x+1 = 3(1-2x)$$

$$8x = 2 \quad x = \frac{1}{4}$$

$$\vee \frac{2x+1}{1-2x} = -3$$

$$2x+1 = -3(1-2x)$$

$$2x+1 = -3+6x$$

$$-4x = -4$$

$$x = 1$$

$$11. \textcircled{a} |x-3| = 4$$

$$|x|-3=4$$

∴

$$|x|-3=-4$$

$$|x|=7$$

$$|x|=-1$$

$$\textcircled{x=7} \quad \textcircled{x=-7}$$

Абсурт

$$\textcircled{b} |x-1|-5 = 1$$

$$|x-1|-5=1$$

∴

$$|x-1|-5=-1$$

$$|x-1|=6$$

$$|x-1|=4$$

$$x-1=6 \quad \vee \quad x-1=-6$$

$$x-1=4 \quad \vee \quad x-1=-4$$

$$\textcircled{x=7}$$

$$\textcircled{x=-5}$$

$$\textcircled{x=5}$$

$$\textcircled{x=-3}$$

$$\textcircled{3} \left| \left| \overset{\oplus}{|x|+1} \right| - 2 \right| = 1$$

$$\left| |x| + 1 - 2 \right| = 1$$

$$\left| |x| - 1 \right| = 1$$

$$|x| - 1 = 1$$

or

$$|x| - 1 = -1$$

$$|x| = 2$$

$$|x| = 0$$

$$\textcircled{x=2} \quad \textcircled{x=-2}$$

$$\textcircled{x=0}$$

10. (a) $|x^2+1| + |x+1| = x^2+2$

$$\cancel{x^2+1} + \cancel{|x+1|} = \cancel{x^2+2}$$

$$|x| = 0$$

$$\underline{\underline{x=0}}$$

(b) $|x^2-2x+1| = x(x-1)$

$$|(x-1)^2| = x^2-x$$

$$(x-1)^2 = x^2-x$$

$$\cancel{x^2}-2x+1 = \cancel{x^2}-x$$

$$-2x + 1 = -x$$

$$-x = -1$$

$$\boxed{x=1}$$

$$6. \textcircled{a} |x-2| = 2x-3$$

$$\text{при } 2x-3 \geq 0$$

$$x-2 = 2x-3$$

$$\text{и } x-2 = -2x+3$$

$$x-2x = -3+2$$

$$x+2x = 3+2$$

$$-x = -1$$

$$\cancel{x=1}$$

$$3x = 5$$

$$x = \frac{5}{3} \quad \checkmark$$

$$\textcircled{b} |2x-3| - x = -1$$

$$|2x-3| = x-1$$

$$\text{при } x-1 \geq 0$$

$$2x-3 = x-1$$

$$\text{и } 2x-3 = -x+1$$

$$\underline{\underline{x=2}}$$

$$2x+x = 2$$

$$3x = 4$$

$$x = \frac{4}{3}$$

✓

5. (a) $\sqrt{x^2} = 2$

$$|x| = 2$$

$$\underline{\underline{x=2}} \quad \text{or} \quad \underline{\underline{x=-2}}$$

(b) $\sqrt{x^2 - 2x + 1} = |2x - 1|$

$$\sqrt{(x-1)^2} = |2x-1|$$

$$|x-1| = |2x-1|$$

$$x-1 = 2x-1 \quad \text{or} \quad x-1 = -2x+1$$

$$-x = 0$$

$$\boxed{x=0}$$

$$x+2x = 2$$

$$3x = 2$$

$$\boxed{x = \frac{2}{3}}$$

(c) $\sqrt{x^2 + 2} = |x+1|$

$$\sqrt{x^2 + 2}^2 = |x+1|^2$$

$$x^2 + 2 = (x+1)^2$$

$$\cancel{x^2} + 2 = \cancel{x^2} + 2x + 1$$

$$1 = 2x$$

$$x = \frac{1}{2}$$