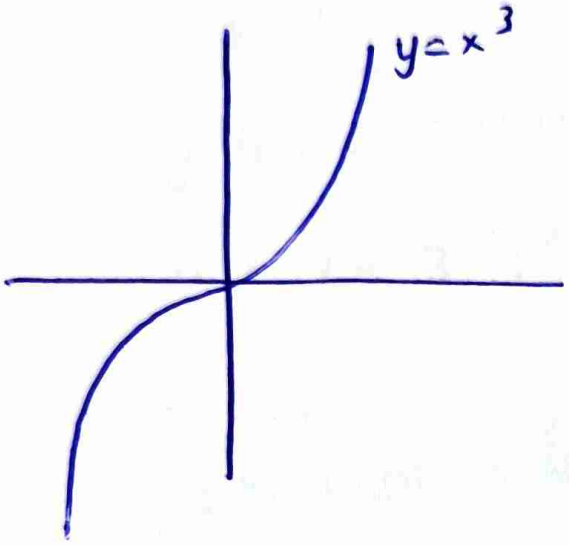
2η Εργασια
Ρυθμος μεταβολη

Ενα υλικο σημειο Μ κινειται κατα μηκος της $y = x^3$, $x > 0$



Η τετμημενη ελαττωνεται κατα 1

$\Rightarrow x'(t) = -1$

Τη χρονικη στιγμη t_1 το Μ διερχεται απο τη θεση $A(1,1)$.

$x(t_1) = 1$

$y(t_1) = 1$

1. Ψαχνω το ρυθμο μεταβολη της τετραγωνικης του Μ δηλαδη ψαχνω το $y'(t_1)$.

$$y = x^3$$

$$y(t) = x^3(t)$$

$$y'(t) = 3x^2(t)\ x'(t)$$

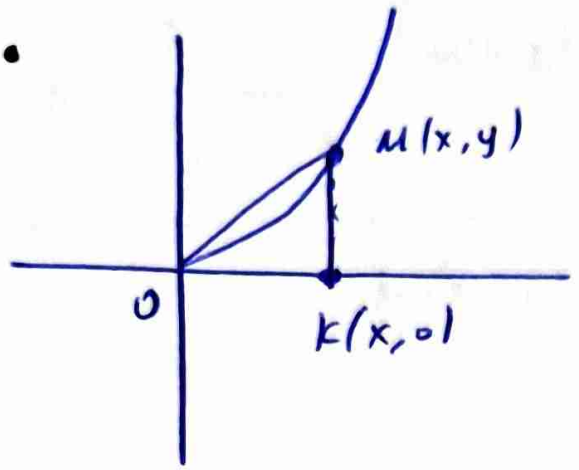
$t = t_1$

$$y'(t_1) = 3x^2(t_1)\ x'(t_1)$$

$$y'(t_1) = 3 \cdot 1^2 (-1)$$

$$y'(t_1) = -3$$

2.



$$(M\hat{O}K) = \frac{B \cdot U}{2}$$

$$E = \frac{x \cdot y}{2}$$

$$E(t) = \frac{1}{2}\, x(t)\, y(t)$$

$$E'(t) = \frac{1}{2}\left(x'(t)\, y(t) + x(t)\, y'(t) \right)$$

$t = t_1$

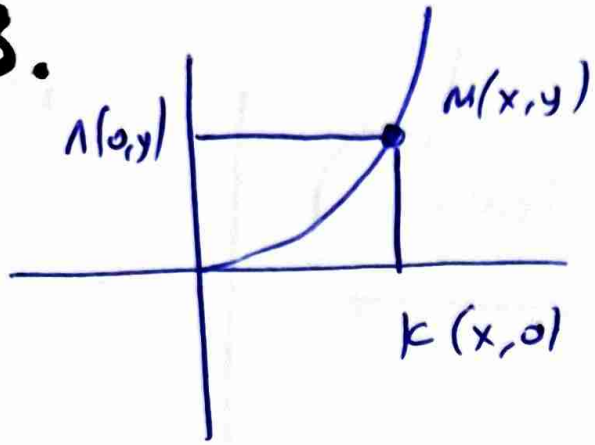
$$E'(t_1) = \frac{1}{2}\left(x'(t_1)\, y(t_1) + x(t_1)\, y'(t_1) \right)$$

$$E'(t_1) = \frac{1}{2}\left(-1 \cdot 1 + 1 \cdot (-3) \right)$$

$$E'(t_1) = \frac{1}{2}\left(-1 - 3 \right)$$

$$E'(t_1) = -2$$

3.



$$\Pi = 2x + 2y$$

$$\Pi(t) = 2x(t) + 2y(t)$$

$$\Pi'(t) = 2x'(t) + 2y'(t)$$

$t = t_1$

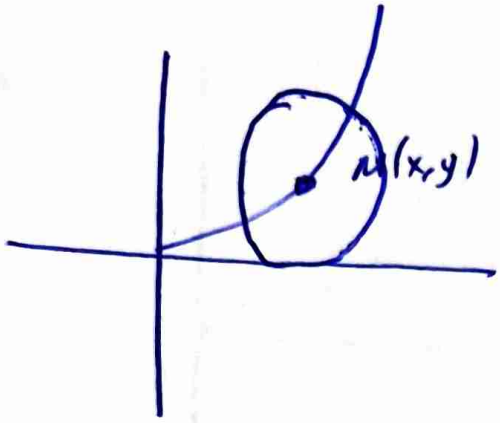
$$\Pi'(t_1) = 2x'(t_1) + 2y'(t_1)$$

$$\Pi'(t_1) = 2(-1) + 2(-3)$$

$$\Pi'(t_1) = -2 - 6$$

$$\Pi'(t_1) = -8$$

4.



$$E = \pi \rho^2$$

$$E = \pi \cdot y^2$$

$$E(t) = \pi \cdot y^2(t)$$

$$E'(t) = 2\pi\, y(t)\, y'(t)$$

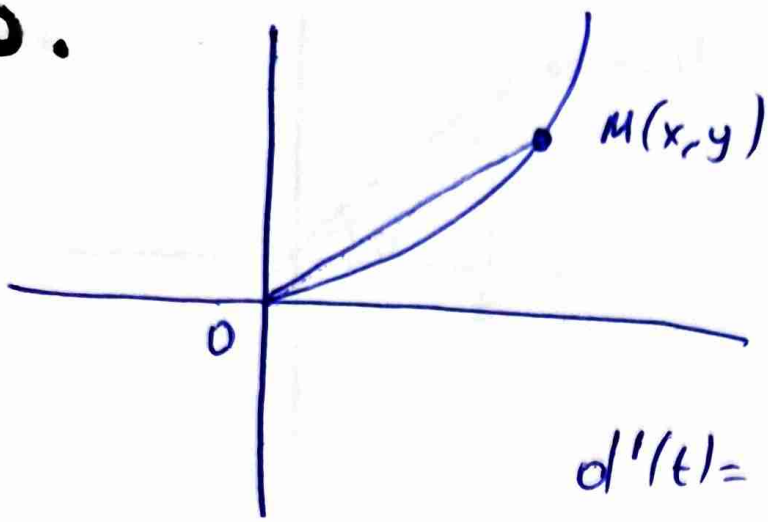
$$t = t_1$$

$$E'(t_1) = 2\pi\, y(t_1)\, y'(t_1)$$

$$E'(t_1) = 2\pi \cdot 1 \cdot (-3)$$

$$E'(t_1) = -6\pi .$$

5.



$$OM = \sqrt{(x-0)^2 + (y-0)^2}$$

$$d(t) = \sqrt{x^2(t) + y^2(t)}$$

$$d'(t) = \frac{2x(t)\,x'(t) + 2y(t)\,y'(t)}{2\sqrt{x^2(t) + y^2(t)}}$$

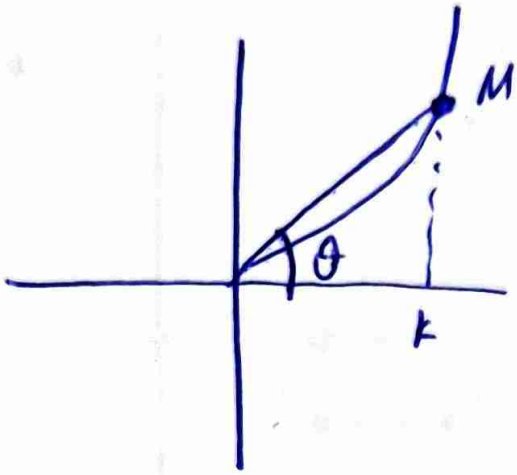
$t = t_1$

$$d'(t_1) = \frac{2 \cdot 1 \cdot (-1) + 2 \cdot 1 \cdot (-3)}{2\sqrt{1^2 + 1^2}}$$

$$d'(t_1) = \frac{-2 - 6}{2\sqrt{2}}$$

$$d'(t_1) = \frac{-8}{2\sqrt{2}} = \frac{-4}{\sqrt{2}}$$

6.



$$\operatorname{tg}\theta = \frac{y}{x}$$

$$\operatorname{tg}\theta(t) = \frac{y(t)}{x(t)}$$

$$\frac{1}{\cos^2\theta(t)}\,\theta'(t) = \frac{y'(t)x(t) - y(t)x'(t)}{x^2(t)}$$

$$\theta'(t) = \frac{y'(t)x(t) - y(t)x'(t)}{x^2(t)}\,\cos^2\theta(t)$$

$$\theta'(t) = \frac{y'(t)x(t) - y(t)x'(t)}{x^2(t)}\cdot\frac{1}{1+\operatorname{tg}^2\theta(t)}$$

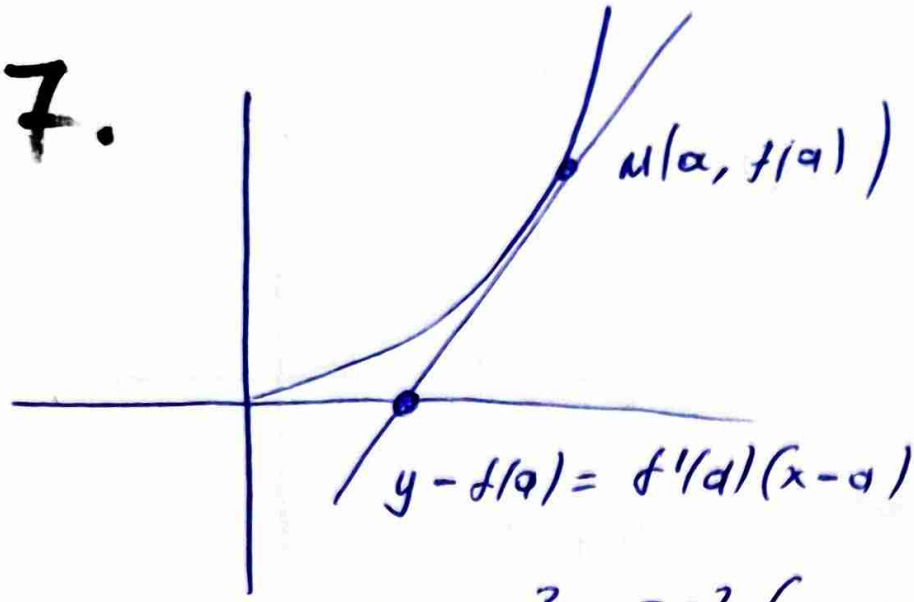
$$\theta'(t) = \frac{y'(t)x(t) - y(t)x'(t)}{x^2(t)}\cdot\frac{1}{1+\left(\frac{y(t)}{x(t)}\right)^2}$$

$t = t_1$

$$\theta'(t_1) = \frac{-3\cdot 1 - 1\cdot(-1)}{1}\cdot\frac{1}{1+1}$$

$$\theta'(t_1) = -2\cdot\frac{1}{2} = -1$$

7.



$f(x) = x^3$

$f'(x) = 3x^2$

$y - f(a) = f'(a)(x - a)$

$y - a^3 = 3a^2(x - a)$

$x'x$

$y = 0$

$-a^3 = 3a^2(x - a)$

$-a^3 = 3a^2x - 3a^3$

$2a^3 = 3a^2x$

$x = \frac{2a}{3}$ ou $x(t) = \frac{2}{3}a(t)$

$x'(t) = \frac{2}{3}a'(t)$

$t = t_1$

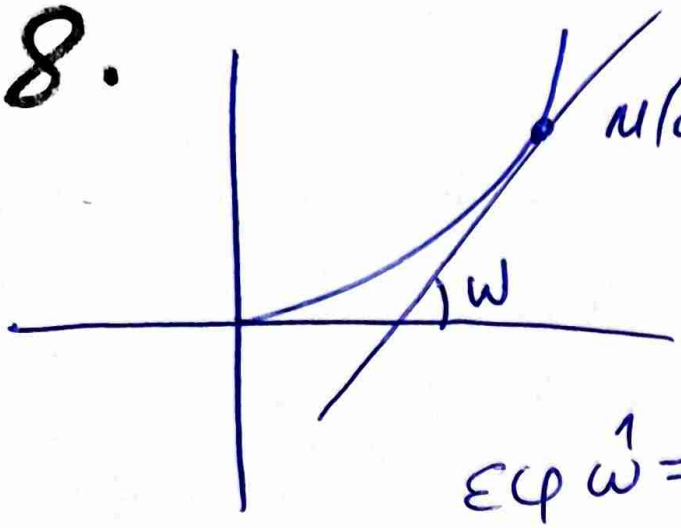
$x'(t_1) = \frac{2}{3}a'(t_1)$

$x'(t_1) = \frac{2}{3}(-1)$

$x'(t_1) = -\frac{2}{3}$

$y - f(a) = f'(a)(x-a)$

8.



$$\varepsilon\varphi\, \hat{w} = f'(a)$$

$$\varepsilon\varphi\, \hat{w} = 3a^2$$

$$\varepsilon\varphi\, w(t) = 3a^2(t)$$

$$\frac{1}{5w^2 w(t)}\, w'(t) = 6a(t)\, a'(t)$$

$$w'(t) = 6a(t)\, a'(t)\, 5w^2 w(t)$$

$$w'(t) = 6a(t)\, a'(t)\, \frac{1}{1+\varepsilon\varphi^2 w(t)}$$

$$w'(t) = 6a(t)\, a'(t)\, \frac{1}{1+(3a^2(t))^2}$$

$t = t_1$

$$w'(t_1) = 6 \cdot 1 \cdot (-1) \cdot \frac{1}{1+9} \quad w'(t_1) = -\frac{3}{5}$$