12. ⓐ $f(x)=x^2$, $x\in R$

$g(x)=\ln x$, $x>0$

ΕΝΟΤΗΤΑ 3

$(g\circ f)(x)=g(f(x))=\ln x^2$

$x\in D_f$ και $f(x)\in D_g$

$x\in R$ $\quad x^2>0$ $\quad D_{g\circ f}=R^*$

$x\neq 0$

ⓑ $f(x)=\sigma\upsilon\nu x$ $\quad x\in R$

$g(x)=\sqrt{1-x^2}$ $\quad x\in[-1,1]$

$(g\circ f)(x)=g(f(x))=\sqrt{1-\sigma\upsilon\nu^2 x}=\sqrt{\eta\mu^2 x}$

$=|\eta\mu x|$

$x\in D_f$ $\quad$ και $\quad f(x)\in D_g$

$x\in R$ $\quad -1\le \sigma\upsilon\nu x\le 1$

που ισχύει

$D_{g\circ f}=R$.

13. ⓐ $f(2x-3) = 4x^2 - 14x + 13$

$2x - 3 = t$

$2x = t + 3$

$x = \frac{t+3}{2}$

$$f(t) = 4 \cdot \left(\frac{t+3}{2}\right)^2 - 14 \cdot \frac{t+3}{2} + 13$$

$$f(t) = \cancel{4} \cdot \frac{(t+3)^2}{\cancel{4}} - 7(t+3) + 13$$

$$f(t) = t^2 + 6t + 9 - 7t - 21 + 13$$

$$f(t) = t^2 - t + 1$$

∴ $f(x) = x^2 - x + 1$

① $f(1+\ln x) = x + 2\ln x + 3$

$1+\ln x = t$

$\ln x = t-1$

$x = e^{t-1}$

$f(t) = e^{t-1} + 2\ln e^{t-1} + 3$

$f(t) = e^{t-1} + 2(t-1) + 3$

$f(t) = e^{t-1} + 2t + 1$

$f(x) = e^{x-1} + 2x + 1$

14. (01) $(g \circ f)(x) = g(f(x)) = x - \ln x + 1$

$f(x) = \ln x - 1$

Θέτω $f(x) = t$

$\ln x - 1 = t$

$\ln x = t + 1$

$x = e^{t+1}$

$g(t) = e^{t+1} - \ln e^{t+1} + 1$

$g(t) = e^{t+1} - (t+1) + 1$

$g(t) = e^{t+1} - t$

$g(x) = e^{x+1} - x$

(B) $(g \circ f) = g(H(x)) = x - \ln x + 1$

$g(x) = 2x - 5$

Χτ. Τω

$g(f(x)) = 2f(x) - 5$ $\Big\}$ (=) $2H(x) - 5 = x - \ln x + 1$

$g(H(x)) = x - \ln x + 1$

$2H(x) = x - \ln x + 6$

$$f(x) = \frac{x - \ln x + 6}{2}$$

15. ⓐ $(gof) = g(f(x)) = 9x^2 - 24x + 1$

$g(x) = 3x - 1$

Χτιζω

$g(f(x)) = 3f(x) - 1$ } (=)

$g(f(x)) = 9x^2 - 24x + 1$ }

$3f(x) - 1 = 9x^2 - 24x + 1$

$3f(x) = 9x^2 - 24x + 2$

$$f(x) = \frac{9x^2 - 24x + 2}{3}$$

(B) $g(f(x)) = 2x - 1$

$$g(x) = \frac{x}{x-2}$$

Χειλω

$$g(f(x)) = \frac{f(x)}{f(x)-2} \Bigg\} (=)$$

$$g(f(x)) = 2x - 1$$

$$\frac{f(x)}{f(x)-2} = 2x - 1$$

$$f(x) = (2x-1)(f(x)-2)$$

$$f(x) = 2x f(x) - 4x - f(x) + 2$$

$$2f(x) - 2x f(x) = 2 - 4x \rightarrow f(x)(1-x) = 1-2x$$

$$f(x) - x f(x) = 1 - 2x$$

$$f(x) = \frac{1-2x}{1-x}$$

13. Ενότητα

(7) $$\lim_{x\to 1} \frac{\sqrt{3x^2+1} - 2}{x^2-5x+4} =$$

$$= \lim_{x\to 1} \frac{\left(\sqrt{3x^2+1}-2\right)\left(\sqrt{3x^2+1}+2\right)}{(x-4)(x-1)\left(\sqrt{3x^2+1}+2\right)}$$

$$= \lim_{x\to 1} \frac{\left(\sqrt{3x^2+1}\right)^2 - 2^2}{(x-4)(x-1)\left(\sqrt{3x^2+1}+2\right)}$$

$$= \lim_{x\to 1} \frac{3x^2-3}{(x-4)(x-1)\left(\sqrt{3x^2+1}+2\right)}$$

$$= \lim_{x\to 1} \frac{3(x-1)(x+1)}{(x-4)(x-1)\left(\sqrt{3x^2+1}+2\right)}$$

$$= \frac{6}{-3\cdot 4} = -\frac{6}{12} = -\frac{1}{2}$$

9. ΕΥΟΤΗΤΑ 9

(α) $\lim\limits_{x\to 0} \dfrac{x+1+\sqrt{x}}{x}$

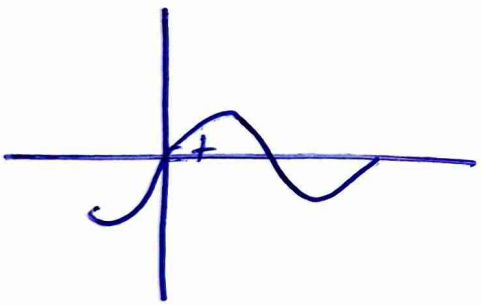
$= \lim\limits_{x\to 0^+} \dfrac{x+1+\sqrt{x}}{x} =$

$= \lim\limits_{x\to 0^+} (x+1+\sqrt{x})\ \dfrac{1}{x} = 1\cdot(+\infty) = +\infty,$

(+)

(β) $\lim\limits_{x\to 0} \dfrac{1+\sqrt{x}}{\eta\mu x} = \lim\limits_{x\to 0^+} (1+\sqrt{x})\ \dfrac{1}{\eta\mu x} =$

$= 1\cdot(+\infty) = +\infty$



(γ) $\lim\limits_{x\to 0} \left(\dfrac{2}{x} - \dfrac{2}{\sqrt{x}}\right) = \lim\limits_{x\to 0^+} \dfrac{2\sqrt{x}-2x}{x\sqrt{x}} =$

$= \lim\limits_{x\to 0^+} \dfrac{2\sqrt{x}\sqrt{x} - 2x\sqrt{x}}{x\sqrt{x}\sqrt{x}} = \lim\limits_{x\to 0^+} \dfrac{2x-2x\sqrt{x}}{x^2}$

$= \lim\limits_{x\to 0^+} \dfrac{2-2\sqrt{x}}{x} = \lim\limits_{x\to 0^+} (2-\sqrt{x})\ \dfrac{1}{x} = 2\cdot(+\infty) = +\infty$

7. (B) $\lim_{x\to+\infty} \frac{x^2}{x+1} - \frac{2x^2+3}{x}$

Εντοπτα 10

$$\lim_{x\to+\infty} \frac{x^3 - (x+1)(2x^2+3)}{x(x+1)} =$$

$$= \lim_{x\to+\infty} \frac{x^3 - (2x^3+3x+2x^2+3)}{x^2+x} =$$

$$= \lim_{x\to+\infty} \frac{-x^3}{x^2} = \lim_{x\to+\infty} -x = -\infty.$$

8. (a) $\lim_{x\to+\infty} |x^3 - x^2 - 1| - |-x^2+3| - 2x^3$

(+) (−)

$$= \lim_{x\to+\infty} x^3 - x^2 - 1 - (x^2 - 3) - 2x^3$$

$$= \lim_{x\to+\infty} -x^3 = -\infty$$

Ενότητα 8

Τριγωνομετρικά όρια

$$\lim_{x \to 0} \frac{\eta\mu x}{x} = 1$$

$$\lim_{x \to 0} \frac{\sigma\upsilon\nu x - 1}{x} = 0$$

25. (α) $\lim\limits_{x \to 0} \left(\frac{2\eta\mu x}{x} + x \right) = \lim\limits_{x \to 0} \left(2 \frac{\eta\mu x}{x} + x \right)$

$= 2 \cdot 1 + 0 = 2$

(β) $\lim\limits_{x \to 0} \left(\frac{\sigma\upsilon\nu x - 1}{x} - \eta\mu x \right) = 0 - 0 = 0.$

26. ⓐ $\lim\limits_{x\to 0} \dfrac{x}{\sin x \cdot \cos x} =$

$$= \lim_{x\to 0} \frac{\frac{x}{x}}{\frac{\sin x \cos x}{x}} = \lim_{x\to 0} \frac{1}{\frac{\sin x}{x} \cdot \cos x}$$

$$= \frac{1}{1 \cdot 1} = \underline{1}.$$

ⓑ $\lim\limits_{x\to 0} \dfrac{x}{\operatorname{tg} x} = \lim\limits_{x\to 0} \dfrac{x}{\frac{\sin x}{\cos x}} = \lim\limits_{x\to 0} \dfrac{x \cos x}{\sin x}$

$$= \lim_{x\to 0} \frac{\frac{x\cos x}{x}}{\frac{\sin x}{x}} = \lim_{x\to 0} \frac{\cos x}{\frac{\sin x}{x}} = \frac{1}{1}$$

$$= 1$$

(8) $\lim_{x \to 0} \frac{\operatorname{tg} x \cdot \operatorname{ctg} x}{x^2} = \lim_{x \to 0} \frac{\operatorname{tg} x \cdot \frac{\operatorname{tg} x}{\operatorname{ctg} x}}{x^2} =$

$$= \lim_{x \to 0} \frac{\frac{\operatorname{tg}^2 x}{\operatorname{ctg} x}}{\frac{x^2}{1}} = \lim_{x \to 0} \frac{\operatorname{tg}^2 x}{x^2 \operatorname{ctg} x}$$

$$= \lim_{x \to 0} \frac{\operatorname{tg}^2 x}{x^2} \cdot \frac{1}{\operatorname{ctg} x} = 1 \cdot \frac{1}{1} = 1$$

(9) $\lim_{x \to 0} \frac{1 - \cos^2 x}{x^2} = \lim_{x \to 0} \frac{\operatorname{tg}^2 x}{x^2} = 1$

(10) $\lim_{x \to 0} \frac{\operatorname{tg} x \cos x - \operatorname{tg} x}{x^2} = \lim_{x \to 0} \frac{\operatorname{tg} x (\cos x - 1)}{x^2}$

$$= \lim_{x \to 0} \frac{\operatorname{tg} x}{x} \cdot \frac{\cos x - 1}{x} = 1 \cdot 0 = 0.$$

SOS

1. $\lim\limits_{x \to 0} x^2 \eta\mu \frac{1}{x}$

Όταν απαιτείται το ημίτονο ή το συνημίτονο τους το όριο δεν θα υπάρχει.

$$-1 \leq \eta\mu \frac{1}{x} \leq 1$$

$$\boxed{-x^2 \leq x^2 \eta\mu \frac{1}{x} \leq x^2}$$

- $\lim\limits_{x \to 0} -x^2 = 0$
- $\lim\limits_{x \to 0} x^2 = 0$

} Από κριτήριο παρεμβολής

$$\lim_{x \to 0} x^2 \eta\mu \frac{1}{x} = 0.$$

Κριτήριο παρεμβολής

Όταν $g(x) \le f(x) \le h(x)$

και $\lim\limits_{x \to x_0} g(x) = \ell$

$\lim\limits_{x \to x_0} h(x) = \ell$

τότε $\lim\limits_{x \to x_0} f(x) = \ell$.

2. $\lim_{x \to 0} x^3 \sigma\upsilon\nu \frac{1}{x}$ $\frac{\sigma\upsilon\nu\, 0}{\text{χαοκμ}}$

$$-1 \le \sigma\upsilon\nu \frac{1}{x} \le 1$$

$$\left|\sigma\upsilon\nu \frac{1}{x}\right| \le 1.$$

$$|x^3| \cdot \left|\sigma\upsilon\nu \frac{1}{x}\right| \le 1 \cdot |x^3|$$

$$\left|x^3 \sigma\upsilon\nu \frac{1}{x}\right| \le |x^3|$$

$$\boxed{-|x^3| \le x^3 \sigma\upsilon\nu \frac{1}{x} \le |x^3|}$$

- $\lim_{x \to 0} -|x^3| = 0$
- $\lim_{x \to 0} |x^3| = 0$

} Από κ. π

$$\lim_{x \to 0} x^3 \sigma\upsilon\nu \frac{1}{x} = 0.$$

Εποπτεία Μαθημάτων

Τετάρτη 10:30 - 12:30

Ενότητα 10

(9) α γ ε

(10) α γ

(11) όλη

(12) α γ.

Ενότητα 8

(25) δ

(26) στ