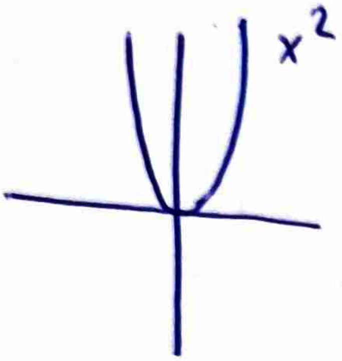
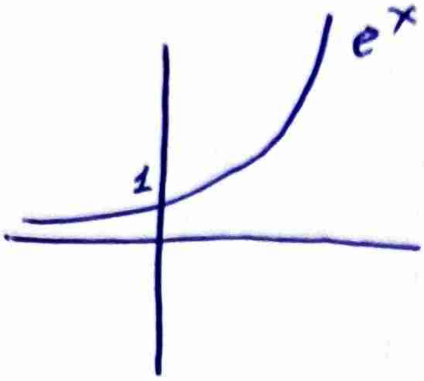
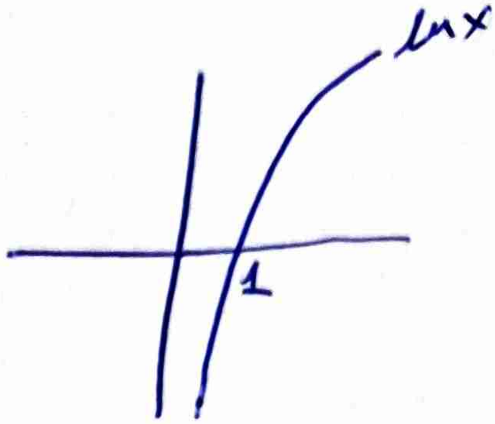
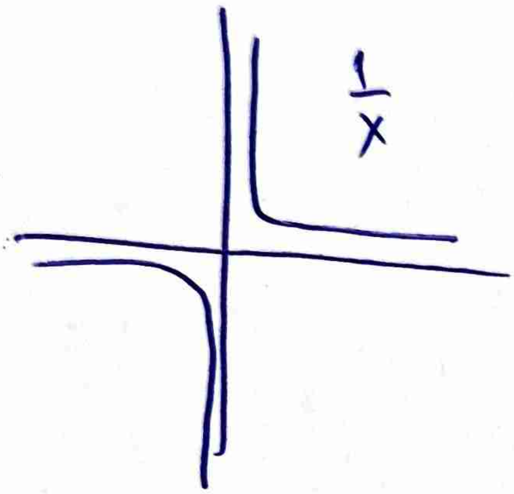
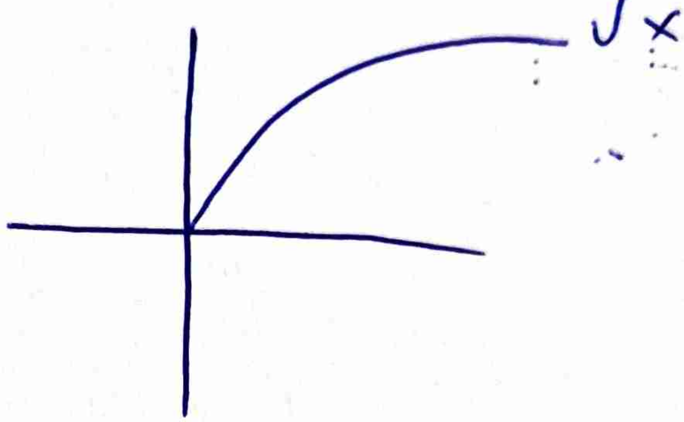
Βασικα σχηματα

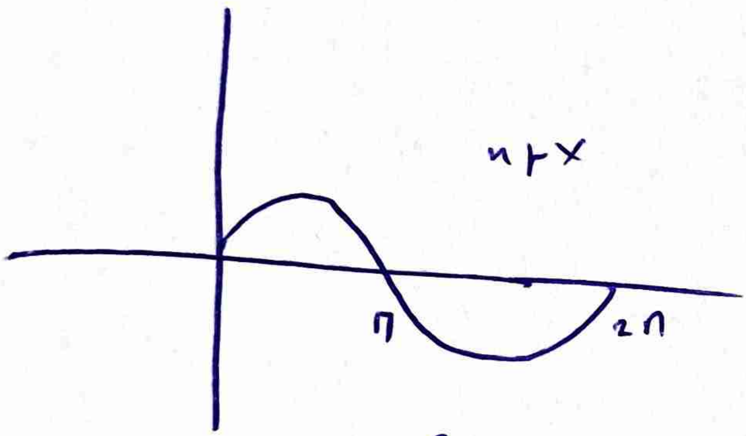


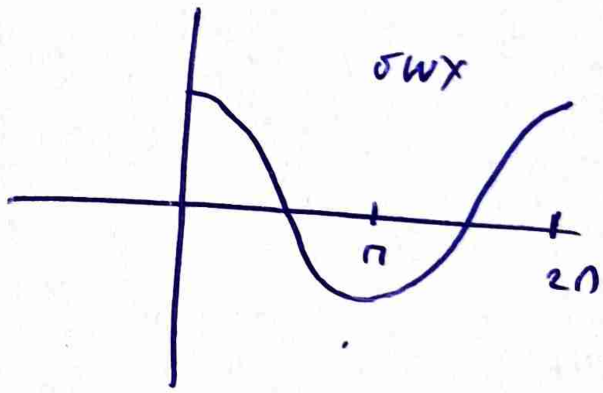


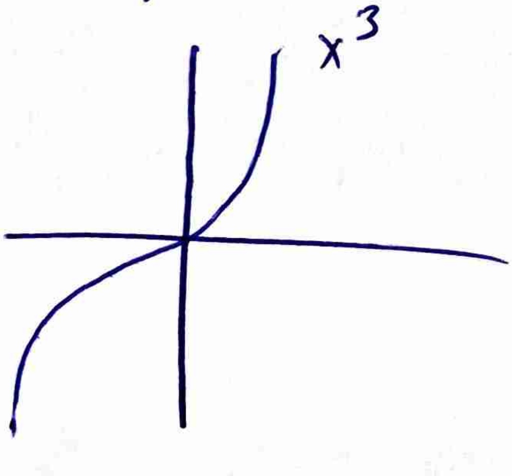












3. (B) $f(x) = xe^{\frac{1}{x}}, x \neq 0$

Ενοτητα 32

Μονοτονια

$$f'(x) = e^{\frac{1}{x}} + x\, e^{\frac{1}{x}}\left(-\frac{1}{x^2}\right)$$

$$f'(x) = e^{\frac{1}{x}}\left(1 - \frac{1}{x}\right) = e^{\frac{1}{x}}\,\frac{x-1}{x}$$

$$\boxed{f'(x) = e^{\frac{1}{x}} \cdot \frac{x-1}{x}}$$

x		0	1	
$x-1$	$-$		$-$	0 $+$
x	$-$		$+$	$+$
f'	$+$		$-$	$+$
f	$\nearrow$		$\searrow$	$\nearrow$

$\longrightarrow f'(x) = 0 \Rightarrow \frac{x-1}{x} = 0 \Rightarrow x = 1$

Συνολο Τιμων

$$\lim_{x \to -\infty} f(x) = \lim_{x \to -\infty} xe^{\frac{1}{x}} = -\infty \cdot 1 = -\infty$$

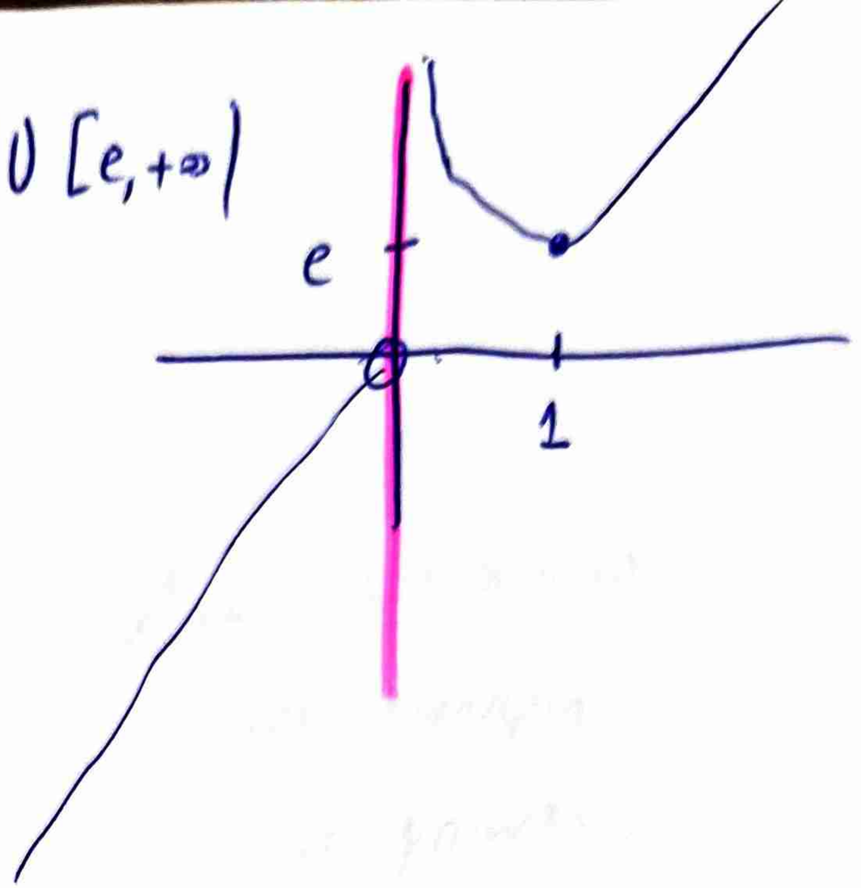
$$\lim_{x \to 0^-} f(x) = \lim_{x \to 0^-} xe^{\frac{1}{x}} = 0 \cdot e^{-\infty} = 0 \cdot 0 = 0$$

$$\lim_{x \to 0^+} f(x) = \lim_{x \to 0^+} \frac{e^{\frac{1}{x}}}{\frac{1}{x}} = \lim_{x \to 0^+} \frac{e^{\frac{1}{x}}\cancel{\left(-\frac{1}{x^2}\right)}}{\cancel{-\frac{1}{x^2}}} = +\infty$$

$f(1) = e$

$\lim_{x \to +\infty} f(x) = +\infty$.

$\Sigma T_f = (-\infty, 0) \cup [e, +\infty)$



Κυρτότητα.

$$f''(x) = e^{\frac{1}{x}}\left(-\frac{1}{x^2}\right)\left(\frac{x-1}{x}\right) + e^{\frac{1}{x}}\,\frac{x-x+1}{x^2}$$

$$f''(x) = -\frac{1}{x^2}\,e^{\frac{1}{x}}\,\frac{x-1}{x} + e^{\frac{1}{x}}\,\frac{1}{x^2}$$

$$f''(x) = \frac{1}{x^2}\,e^{\frac{1}{x}}\left(-\frac{x-1}{x} + 1\right) = \frac{1}{x^2}\,e^{\frac{1}{x}}\left(\frac{x-x+1}{x}\right)$$

$$f''(x) = \frac{1}{x^2}\,e^{\frac{1}{x}}\,\frac{1}{x}$$

x		0	
f''	$-$		$+$
f	$\frown$		$\smile$

Ασυμπτωτη

$$\lim_{x\to 0^-} f(x) = 0$$

$$\lim_{x\to 0^+} f(x) = +\infty$$ Αρα εβ $x=0$ κατακορυφη ασυμπτωτη.

$$\lim_{x\to -\infty} f(x) = -\infty$$ δεν εχει οριζοντια στο $-\infty$

$$\lim_{x\to +\infty} f(x) = +\infty$$ » » » » $+\infty$

$$\lim_{x\to -\infty} \frac{f(x)}{x} = \lim_{x\to -\infty} \frac{\cancel{x} e^{\frac{1}{x}}}{\cancel{x}} = 1$$

$$\lim_{x\to -\infty} f(x) - x = \lim_{x\to -\infty} \left(x e^{\frac{1}{x}} - x\right) =$$

$$= \lim_{x\to -\infty} x\left(e^{\frac{1}{x}} - 1\right) = \lim_{x\to -\infty} \frac{e^{\frac{1}{x}} - 1}{\frac{1}{x}}$$

Θέτω $\frac{1}{x} = t$

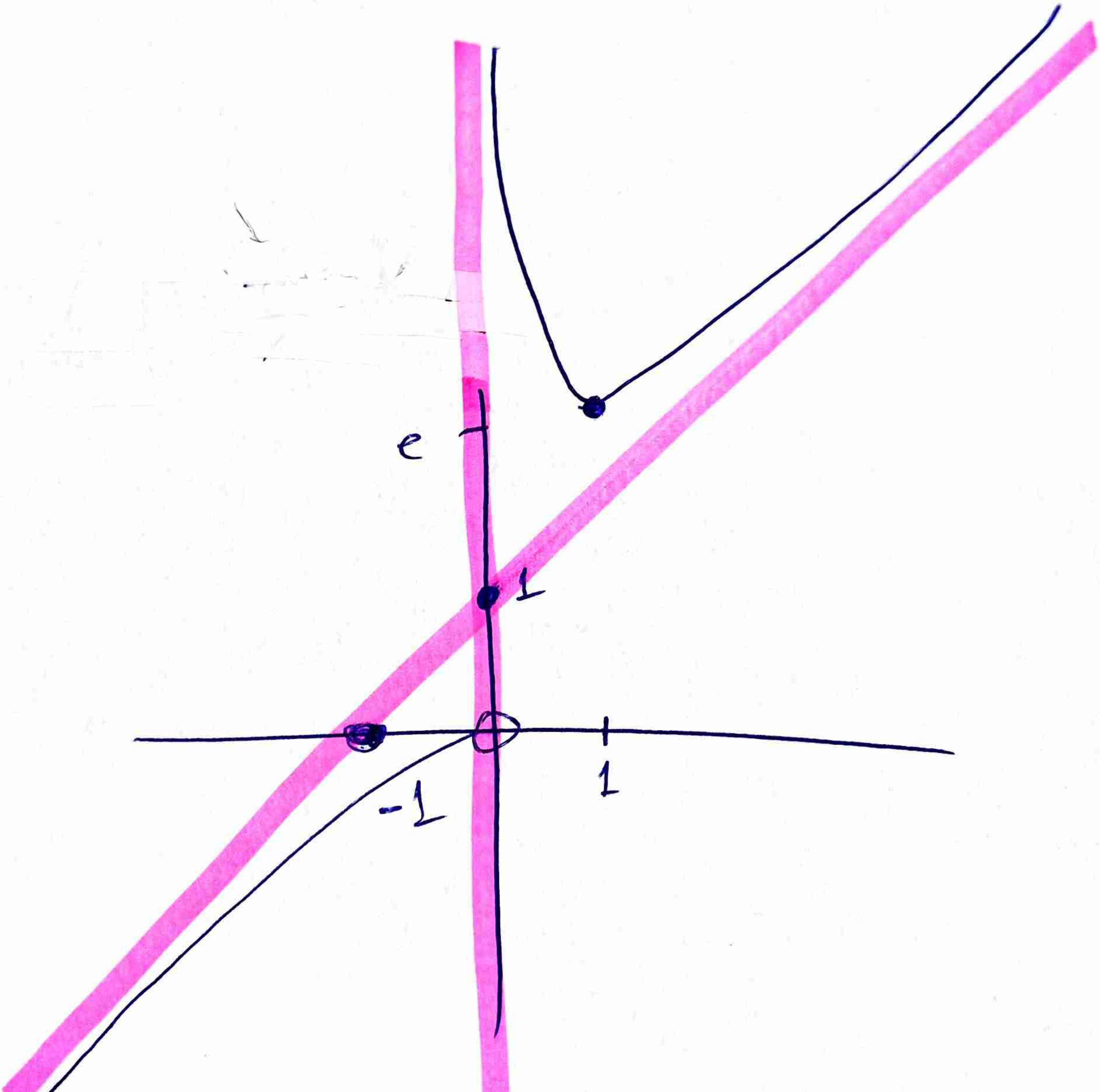
όταν $x \to \infty$

$t \to 0$

$$y = x + 1$$

$$\pm\infty$$

$$\lim_{t \to 0} \frac{e^t - 1}{t} = \lim_{t \to 0} \frac{e^t}{1} = 1.$$



28.

Συνέχεια 35

$f(x) = \frac{1}{\ln x}, \ x > 1$

(a) $f'(x) = \frac{-\frac{1}{x}}{\ln^2 x} < 0 \quad f \downarrow.$

$y = \frac{1}{\ln x} \Rightarrow \ln x = \frac{1}{y} \quad \underline{\underline{y \neq 0}}$

$e^{\ln x} = e^{\frac{1}{y}}$

$x = e^{\frac{1}{y}}$

$f^{-1}(x) = e^{\frac{1}{x}}$
$x \in (0, +\infty)$

Αφού $x > 1$ τότε $e^{\frac{1}{y}} > 1$

$\frac{1}{y} > 0 \Rightarrow \underline{\underline{y > 0}}$

(B) $$I = \int_e^{e^2} \frac{1}{\ln x}\,dx + \int_1^{\frac{1}{2}} e^{\frac{1}{x}}\,dx$$

$$I = \int_e^{e^2} f(x)\,dx + \int_1^{\frac{1}{2}} f^{-1}(x)\,dx$$

$$\longrightarrow \int_1^{e^2} f^{-1}(x)\,dx \;\xrightarrow[x = f(t)]{f^{-1}(x) = t}$$

$$dx = f'(t)\,dt$$

$$\int_e^{e^2} t\, f'(t)\,dt = \int_e^{e^2} x f'(x)\,dx$$

$$I = \int_e^{e^2} f(x)\,dx + \int_e^{e^2} x f'(x)\,dx$$

$$I = \int_e^{e^2} f(x) + x f'(x)\,dx = \left(x f(x)\right)_e^{e^2}$$

$$e^2 f(e^2) - e f(e)$$

30. (θ) $\displaystyle\int_{-2}^{2} \frac{e^x-1}{e^x+1}\,dx$

$$f(-x) = \frac{e^{-x}-1}{e^{-x}+1} = \frac{\frac{1}{e^x}-1}{\frac{1}{e^x}+1} = \frac{1-e^x}{1+e^x} =$$

$$= -\frac{e^x-1}{1+e^x} = -f(x)$$

f περιττή

Άρα $\displaystyle\int_{-2}^{2} \frac{e^x-1}{e^x+1}\,dx = 0.$

f περιττή : $f(-x) = -f(x)$

$$I = \int_{-a}^{a} f(x)\,dx \xlongequal[dx=-dt]{x=-t} \int_{a}^{-a} -f(-t)\,dt$$

$$I = -\int_{a}^{-a} -f(t)\,dt = \int_{a}^{-a} f(t)\,dt$$

$$I = -\int_{-a}^{a} f(t)\,dt \Leftrightarrow I = -I$$

$$2I = 0 \quad I = 0$$

34.

(a) $f(x) + f(1-x) = 2$. Ισχυει.

$$I = \int_0^1 f(x)\,dx \xrightarrow[dx = -dt]{x = 1-t}$$

$$I = \int_1^0 -f(1-t)\,dt = -\int_1^0 f(1-t)\,dt$$

$$I = \int_0^1 f(1-x)\,dx = \int_0^1 2 - f(x)\,dx$$

$$I = \int_0^1 2\,dx - \int_0^1 f(x)\,dx$$

$$I = 2\,(x)\Big|_0^1 - I$$

$$2I = 2$$

$$I = 1.$$

46. $f(1) = f'(0) = f'(1) = 0 \qquad f(0) = 1$

$$I = \int_0^1 \frac{f''(x) - f(x)}{e^x}\, dx$$

$$I = \int_0^1 \left(f''(x) - f(x)\right) e^{-x}\, dx$$

$$I = \int_0^1 f''(x)\, e^{-x} - f(x)\, e^{-x}\, dx$$

$$I = \int_0^1 f''(x)\, e^{-x}\, dx - \int_0^1 f(x) e^{-x}\, dx$$

$$I = \left(f'(x)\, e^{-x}\right)\Big|_0^1 + \int_0^1 f'(x)\, e^{-x}\, dx - \int_0^1 f(x) e^{-x}\, dx$$

$$I = f'(1)e^{-1} - f'(0) + \left(f(x)\, e^{-x}\right)\Big|_0^1 + \cancel{\int_0^1 f(x) e^{-x}\, dx} \; \cancel{- \int_0^1 f(x) e^{-x}\, dx}$$

$$I = 0 - 0 + f(1)e^{-1} - f(0)$$

$$I = -1$$

50. (B) $f(x) = e^x - \int_0^1 e^{1-x} f(x)\, dx$

$$\boxed{\text{Θέτω } \int_0^1 e^{1-x} f(x) = k}$$

$$f(x) = e^x - k$$

$$e^{1-x} f(x) = e^x e^{1-x} - k e^{1-x}$$

$$e^{1-x} f(x) = e - k e^{1-x}$$

$$\underbrace{\int_0^1 e^{1-x} f(x)\, dx}_{k} = \int_0^1 e - k e^{1-x}\, dx$$

$$k = \int_0^1 e\, dx - \int_0^1 k e^{1-x}\, dx$$

$$k = e\, (x)_0^1 - k \int_0^1 e^{1-x}\, dx$$

$$k = e + k \cdot \left(e^{1-x}\right)_0^1 \qquad k = 1$$

$$k = e + k(1 - e) \Rightarrow \cancel{k} = e + \cancel{k} - ek$$

35. (γ) $I(\alpha) = \int_1^{\alpha} \frac{\ln t}{t^2}\, dt$

Βρείτε $\lim_{a \to +\infty} I(a)$.

$$I(a) = \int_1^{a} \frac{\ln t}{t^2}\, dt = \int_1^{a} \frac{1}{t^2} \ln t \, dt$$

$$I(a) = \int_1^{a} \left(-\frac{1}{t}\right)' \ln t \, dt$$

$$I(a) = \left(-\frac{\ln t}{t}\right)_1^{a} + \int_1^{a} \frac{1}{t} \cdot \frac{1}{t}\, dt$$

$$I(a) = -\left(\frac{\ln a}{a}\right) - \left(\frac{1}{t}\right)_1^{a}$$

$$I(a) = -\frac{\ln a}{a} - \left(\frac{1}{a} - 1\right)$$

$$I(a) = -\frac{\ln a}{a} - \frac{1}{a} + 1.$$

$$\lim_{a\to+\infty} I(a) = \lim_{a\to+\infty} -\cancelto{0}{\frac{\ln a}{a}} - \cancelto{0}{\frac{1}{a}} + 1.$$

$$1.$$

$$\to \lim_{a\to+\infty} \frac{\ln a}{a} = \lim_{a\to+\infty} \frac{1}{a} = 0$$

25. (γ) $\int_0^{\frac{\pi}{6}} \eta\mu^2 x \; \sigma\upsilon\nu^3 x \; dx =$

$$= \int_0^{\pi/6} \eta\mu^2 x \cdot \sigma\upsilon\nu x \cdot \sigma\upsilon\nu^2 x \; dx =$$

$$= \int_0^{\pi/6} \eta\mu^2 x \; \sigma\upsilon\nu x \; (1-\eta\mu^2 x) \; dx$$

Έστω $\eta\mu x = t$

$\sigma\upsilon\nu x \, dx = dt$

$$= \int_0^{\frac{1}{2}} t(1-t^2) \; dt = \dots$$

26. B $\displaystyle\int_0^{\frac{\pi}{4}} \frac{1}{\sigma\upsilon\nu x}\,dx =$

$$= \int_0^{\pi/4} \frac{\eta\mu^2 x + \sigma\upsilon\nu^2 x}{\sigma\upsilon\nu x}\,dx =$$

$$= \underbrace{\int_0^{\pi/4} \frac{\eta\mu^2 x}{\sigma\upsilon\nu x}\,dx}_{I} + \int_0^{\pi/4} \frac{\sigma\upsilon\nu^2 x}{\sigma\upsilon\nu x}\,dx$$

$$= I + \int_0^{\pi/4} \sigma\upsilon\nu x\,dx$$

$$= I + (\eta\mu x)_0^{\pi/4} = I + \frac{\sqrt{2}}{2}$$

$$I = \int_0^{\pi/4} \frac{\eta\mu^2 x}{\sigma\upsilon\nu x}\,dx = \int_0^{\pi/4} \sigma\upsilon\nu x\,\frac{\eta\mu^2 x}{\sigma\upsilon\nu^2 x}\,dx$$

$$= \int_0^{\pi/4} \sigma\upsilon\nu x\,\frac{\eta\mu^2 x}{1-\eta\mu^2 x}\,dx \;\xrightarrow[\sigma\upsilon\nu x\,dx = dt.]{\eta\mu x = t}$$

$$= \int_0^{\sqrt{2}/2} \frac{t^2}{1-t^2}\, dt = \int_0^{\sqrt{2}/2} \frac{-(1-t^2)+1}{1-t^2}\, dt$$

$$\begin{array}{r|l} t^2 & 1-t^2 \\ \hline -(t^2-1) & -1 \\ \hline 1 & \end{array}$$

$$= \int_0^{\sqrt{2}/2} -1 + \frac{1}{1-t^2}\, dt$$

$$= \int_0^{\sqrt{2}/2} -1\, dt + \int_0^{\sqrt{2}/2} \frac{1}{1-t^2}\, dt$$

A, B.