15.

(a) $\quad x^2 - (\lambda+1)x + \lambda^2 - 1 = 0$
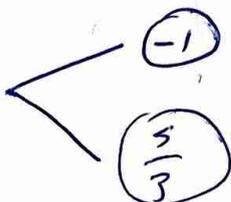
$$\Delta = (\lambda+1)^2 - 4(\lambda^2-1)$$

$$\Delta = \lambda^2 + 2\lambda + 1 - 4\lambda^2 + 4$$

$$\Delta = -3\lambda^2 + 2\lambda + 5$$

$$\Delta^* = 4 + 4 \cdot 3 \cdot 5 = 64$$

$$\lambda = \frac{-2 \pm 8}{-6} \quad \begin{cases} -1 \\ \frac{5}{3} \end{cases}$$

λ		-1		$\frac{5}{3}$	
Δ	$-$	$\circ$	$+$	$\circ$	$-$

Αν $\quad \lambda \in (-\infty, -1) \cup (\frac{5}{3}, +\infty)$, τοτε $\quad \Delta < 0$

καμία λυση.

Αν $\lambda \in (-1, \frac{5}{3})$ τοτε

$\Delta > 0 \quad 2$ ριζ√

Αν $\lambda = -1 \qquad \lambda = \frac{5}{3} \qquad \Delta = 0 \qquad 1$ διπλη ριζα

37. $x^2 - 2\lambda x + \lambda - 2 = 0$

(a) $\Delta = (2\lambda)^2 - 4(\lambda - 2)$

$\Delta = \boxed{4\lambda^2 - 4\lambda + 8} \quad (f)$

$\Delta^* = 16 - 4 \cdot 4 \cdot 8 < 0$

Άρα $\Delta > 0$ Έχω 2 ρίζμ.

(B) $x_1^2 + x_2^2 > 6$

$(x_1 + x_2)^2 - 2x_1 x_2 > 6$

$(2\lambda)^2 - 2(\lambda - 2) > 6$

$4\lambda^2 - 2(\lambda - 2) > 6$

$2\lambda^2 - (\lambda - 2) > 3$

$2\lambda^2 - \lambda + 2 - 3 > 0$

$2\lambda^2 - \lambda - 1 > 0$

$\begin{cases} x_1 + x_2 = 2\lambda \\ x_1 x_2 = \lambda - 2 \end{cases}$

$\Delta = 1 + 8 = 9$

$\lambda = \dfrac{1 \pm 3}{4} \quad \begin{array}{l} \text{①} \\ \boxed{-\frac{1}{2}} \end{array}$

λ		$-\frac{1}{2}$	1	
$2\lambda^2 - \lambda - 1$	$+$	$-$	$+$

$x \in (-\infty, -\frac{1}{2}) \cup (1, +\infty)$

① $\Delta \geq 0$ ✓ $P > 0$ $S > 0$

$\quad\quad\quad\quad\quad\quad\quad\quad\quad a - 2 > 0 \quad\quad\quad 2a > 0$

$\quad\quad\quad\quad\quad\quad\quad\quad\quad a > 2 \quad\quad\quad\quad a > 0$

Əεaκd $_3$ $a \in (2, +\infty) \Rightarrow a = 3$

② $x^2 - |x| < 6$

$|x|^2 - |x| - 6 < 0$

$t^2 - t - 6 < 0$

t		-2		3	
$t^2 - t - 6$	$+$		$-$		$+$

$-2 < t < 3$

$-2 < |x| < 3$

$|x| > -2 \quad\quad\quad$ κay $\quad\quad\quad |x| < 3$

✓ $\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad x \in (-3, 3)$

20.

ⓐ $x^2 - 5|x| + 4 < 0$

$|x|^2 - 5|x| + 4 < 0$

$$\boxed{|x| = t}$$

$t^2 - 5t + 4 < 0$

t		1	4	
$t^2 - 5t + 4$	+	-		+

$t \in (1, 4)$

$1 < t < 4$

$1 < |x| < 4$

$|x| > 1$ $\qquad \underline{\underline{kay}}$ $\qquad |x| < 4$

$x > 1$ ú $x < -1$ $\qquad\qquad\qquad x \in (-4, 4)$

$x \in (-\infty, -1) \cup (1, +\infty)$

$x \in (-4, -1) \cup (1, 4)$.
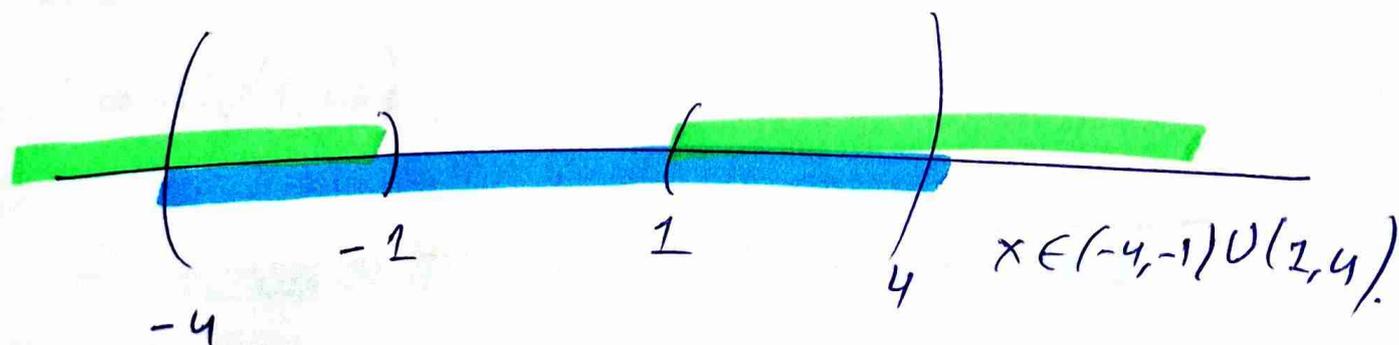
(i) $x^4 - 5x^2 + 4 < 0$

$x^2 = t$

$t^2 - 5t + 4 < 0$

t		1	4	
$t^2 - 5t + 4$	$+$	$-$	$+$	

$$t \in (1, 4)$$

$$1 < t < 4$$

$$1 < x^2 < 4$$

$$1^2 < x^2 < 2^2$$

$$|1|^2 < |x| < |2|$$

$$1 < |x| < 2$$

$|x| > 1$ $\qquad$ και $\qquad$ $|x| < 2$

$x > 1$ ή $x < -1$ $\qquad\qquad\qquad -2 < x < 2$

$x \in (-\infty, -1) \cup (1, +\infty)$
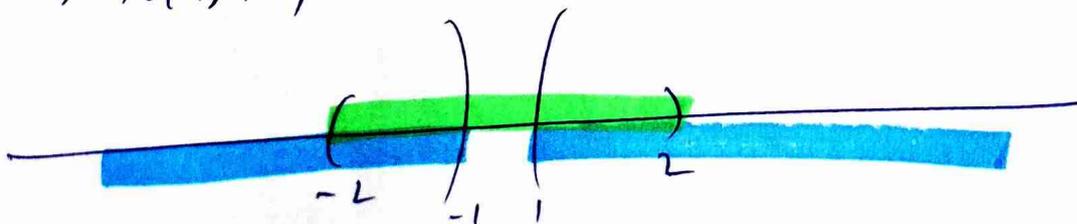
Ⓑ $x^2 - 8\sqrt{x^2} - 9 > 0$

$|x|^2 - 8|x| - 9 > 0$

$|x| = t$

$t^2 - 8t - 9 > 0$

		-1	9
t			
$t^2 - 8t - 9$	$+$	$-$	$+$

$t \in (-\infty, -1) \cup (9, +\infty)$

$t < -1$ ن $t > 9$

$|x| < -1$ ن $|x| > 9$

Ⓐ قمجاى $x > 9$ ن $x < -9$

$x \in (-\infty, -9) \cup (9, +\infty)$

ل

18. (α) Νδο $x^2 - x + 1 > 0$

$\Delta = 1 - 4 < 0$ αρα Θετικο τριωνυμ.

(β) $\lambda^2 > \lambda - 2$
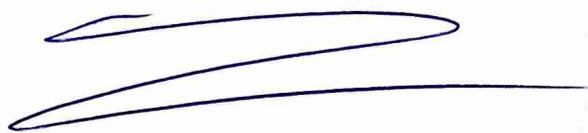
$\lambda^2 - \lambda + 2 > 0$

$\Delta = 1 - 8 < 0$

Θετικο τριωνυμ.

19. (α) $x^2 - \lambda x + \lambda - 3 = 0$

$\Delta = \lambda^2 - 4(\lambda - 3) = \boxed{\lambda^2 - 4\lambda + 12 \quad (+)}$

$\Delta^* = 16 - 48 < 0$

Αρα αιμος

$\Delta > 0$

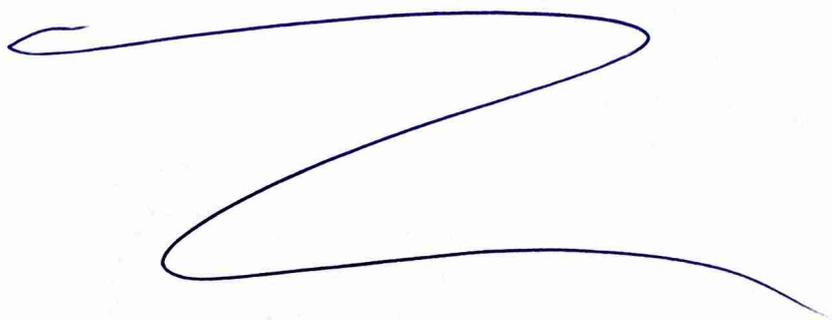2 ριζ

Ⓑ $x^2 + (3\lambda + 1) \cdot x + \lambda^2 - 1 = 0$

$\Delta = (3\lambda + 1)^2 - 4(\lambda^2 - 1)$

$\Delta = 9\lambda^2 + 6\lambda + 1 - 4\lambda^2 + 4$

$\Delta = 5\lambda^2 + 6\lambda + 5 \quad +.$

$\Delta^* = 36 - 4 \cdot 5 \cdot 5 < 0$

2 pitol

Z

Τεστ των παρασκευων

1312126

1. Αποδειται του Vieta

$$x_1 + x_2 = \frac{-\beta + \sqrt{\Delta}}{2a} + \frac{-\beta - \sqrt{\Delta}}{2a} = \frac{-2\beta}{2a} = -\frac{\beta}{a}$$

$$x_1 x_2 = \frac{-\beta + \sqrt{\Delta}}{2a} \cdot \frac{-\beta - \sqrt{\Delta}}{2a} = \frac{\beta^2 - \sqrt{\Delta}^2}{4a^2} = \frac{\beta^2 - \Delta}{4a^2}$$

$$= \frac{\beta^2 - (\beta^2 - 4a\gamma)}{4a^2} = \frac{\beta^2 - \beta^2 + 4a\gamma}{4a^2} = \frac{4a\gamma}{4a^2}$$

$$= \frac{\gamma}{a}$$

Ασκήσεις

1o

9. κλασματικη επιλυση

12. παραμετρικη 1ου Βαθμου

14

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21 .

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17   39
26 .
29   40 .
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37 .