Сходится 10

$$tg^2x = \frac{\varepsilon tg^2x}{1+\varepsilon tg^2x}$$

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(B) $$\frac{\cos 3x}{\cos x} + \frac{\varepsilon tgx}{1+\varepsilon tg^2x} = \sin x$$

$$\frac{\cos 3x}{\cos x} + \frac{\varepsilon tg^2x}{1+\varepsilon tg^2x} \cdot \frac{1}{\varepsilon tgx} = \sin x$$

$$\frac{\cos 3x}{\cos x} + \cos^2 x \cdot \frac{1}{\varepsilon tgx} = \sin x$$

$$\frac{\cos 3x}{\cos x} + \cos^2 x \cdot \frac{1}{\frac{\sin x}{\cos x}} = \frac{\sin x}{\cos x}$$

$$\frac{\cos 3x}{\cos x} + \cos^2 x \frac{\cos x}{\sin x} = \frac{\sin x}{\cos x}$$

$$\frac{\cos 3x}{\cos x} + \cos x \sin x = \frac{\sin x}{\cos x}$$

$$\cos 3x + \sin^2 x \cos x = \cos x$$

$$\cos x \left(\cos^2 x + \sin^2 x \right) = \cos x$$

$$\cos x \cdot \underline{1} = \cos x$$

$$\cos x = \cos x$$

22. (B) $\sigma\upsilon\nu x + \sqrt{3}\,\eta\mu x = 2$

$\Rightarrow \sigma\upsilon\nu x = 2 - \sqrt{3}\,\eta\mu x$

$\eta\mu^2 x + \sigma\upsilon\nu^2 x = 1$

$\eta\mu^2 x + (2 - \sqrt{3}\,\eta\mu x)^2 = 1$

$\eta\mu^2 x + 4 - 4\sqrt{3}\,\eta\mu x + 3\eta\mu^2 x = 1$

$4\eta\mu^2 x - 4\sqrt{3}\,\eta\mu x + 3 = 0$

Θέτω $\eta\mu x = t$.

$4t^2 - 4\sqrt{3}\,t + 3 = 0$

$\Delta = (-4\sqrt{3})^2 - 4 \cdot 4 \cdot 3$

$\Delta = 48 - 48 = 0$

$t = \dfrac{4\sqrt{3}}{8} = \dfrac{\sqrt{3}}{2}$

$\eta\mu x = \dfrac{\sqrt{3}}{2}$

$$\sin x = 2 - \sqrt{3} \cdot \frac{\sqrt{3}}{2}$$

$$\sin x = 2 - \frac{3}{2}$$

$$\boxed{\sin x = \frac{1}{2}}$$

$$\operatorname{tg} x = \frac{\sin x}{\cos x} = \frac{\frac{\sqrt{3}}{2}}{\frac{1}{2}} = \sqrt{3}$$

$$\boxed{\operatorname{tg} x = \sqrt{3}}$$

$$\boxed{\operatorname{ctg} x = \frac{1}{\sqrt{3}}}$$

23. (B) $\varepsilon\varphi\omega = \frac{2k}{k+5}$ $\quad \sigma\varphi\omega = \frac{4k}{5-k}$

$$\varepsilon\varphi\omega \cdot \sigma\varphi\omega = 1$$

$$\frac{2k}{k+5} \cdot \frac{4k}{5-k} = 1$$

$$\frac{8k^2}{(k+5)(5-k)} = 1$$

$$8k^2 = (k+5)(5-k)$$

$$8k^2 = 5k - k^2 + 25 - 5k$$

$$k^2 + 8k^2 = 25$$

$$9k^2 = 25$$

$$k^2 = \frac{25}{9}$$

$k = \frac{5}{3}$ $\quad k = -\frac{5}{3}$

26. $\eta\mu x - \sigma\upsilon\nu x = \frac{1}{2}$

(α) $\eta\mu x\, \sigma\upsilon\nu x$

$$(\eta\mu x - \sigma\upsilon\nu x)^2 = \left(\frac{1}{2}\right)^2$$

$$\eta\mu^2 x - 2\eta\mu x\, \sigma\upsilon\nu x + \sigma\upsilon\nu^2 x = \frac{1}{4}$$

$$1 - 2\eta\mu x\, \sigma\upsilon\nu x = \frac{1}{4}$$

$$-2\eta\mu x\, \sigma\upsilon\nu x = \frac{1}{4} - 1$$

$$-2\eta\mu x\, \sigma\upsilon\nu x = -\frac{3}{4}$$

$$\eta\mu x\, \sigma\upsilon\nu x = \frac{3}{8}$$

(β) $\dfrac{1}{\eta\mu x} - \dfrac{1}{\sigma\upsilon\nu x} = \dfrac{\sigma\upsilon\nu x}{\eta\mu x\, \sigma\upsilon\nu x} - \dfrac{\eta\mu x}{\eta\mu x\, \sigma\upsilon\nu x}$

$$= \frac{\sigma\upsilon\nu x - \eta\mu x}{\eta\mu x\, \sigma\upsilon\nu x} = \frac{-\frac{1}{2}}{\frac{3}{8}} = -\frac{8}{6}$$

(8) $\left| \operatorname{ctg} x - \operatorname{tg} x \right| = \left| \frac{\cos x}{\sin x} - \frac{\sin x}{\cos x} \right|$

$= \left| \frac{\cos^2 x}{\sin x \cos x} - \frac{\sin^2 x}{\sin x \cos x} \right|$

$= \left| \frac{\cos^2 x - \sin^2 x}{\sin x \cos x} \right| = \left| \frac{(\cos x - \sin x)(\cos x + \sin x)}{\sin x \cos x} \right|$

$= \frac{|\cos x - \sin x| \; |\cos x + \sin x|}{|\sin x \cos x|} = \frac{\frac{1}{2} \cdot |\cos x + \sin x|}{\frac{3}{8}}$

$= \frac{\frac{1}{2} \cdot \frac{\sqrt{7}}{2}}{\frac{3}{8}}$

$\rightarrow (\cos x + \sin x)^2 = \cos^2 x + 2 \cos x \sin x + \sin^2 x =$

$= 1 + 2 \cdot \frac{3}{8} = 1 + \frac{6}{8} = \frac{14}{8} = \frac{7}{4}$

$\cos x + \sin x = \frac{\sqrt{7}}{2}$

(d) $\eta\rho^3 x - \sigma\omega^3 x =$

$= (\eta\rho x - \sigma\omega x)(\eta\rho^2 x + \eta\rho x\,\sigma\omega x + \sigma\omega^2 x)$

$= \frac{1}{2}\left(1 + \frac{3}{8}\right) = \frac{1}{2} + \frac{3}{16} = \frac{11}{16}$.

29. Αν $0 \le x < \frac{\pi}{2}$

Νδο $\dfrac{\sqrt{1-\sigma\upsilon\nu x} - \sqrt{1+\sigma\upsilon\nu x}}{\sqrt{1-\sigma\upsilon\nu x} + \sqrt{1+\sigma\upsilon\nu x}} = \dfrac{\varepsilon\varphi x - 1}{\sigma\upsilon\nu x}$.

$$\frac{\left(\sqrt{1-\sigma\upsilon\nu x} - \sqrt{1+\sigma\upsilon\nu x}\right)\left(\sqrt{1-\sigma\upsilon\nu x} - \sqrt{1+\sigma\upsilon\nu x}\right)}{\left(\sqrt{1-\sigma\upsilon\nu x} + \sqrt{1+\sigma\upsilon\nu x}\right)\left(\sqrt{1-\sigma\upsilon\nu x} - \sqrt{1+\sigma\upsilon\nu x}\right)} =$$

$$= \frac{\left(\sqrt{1-\sigma\upsilon\nu x} - \sqrt{1+\sigma\upsilon\nu x}\right)^2}{\sqrt{1-\sigma\upsilon\nu x}^2 - \sqrt{1+\sigma\upsilon\nu x}^2} =$$

$$= \frac{\sqrt{1-\sigma\upsilon\nu x}^2 - 2\sqrt{1-\sigma\upsilon\nu x}\sqrt{1+\sigma\upsilon\nu x} + \sqrt{1+\sigma\upsilon\nu x}^2}{1 - \sigma\upsilon\nu x - 1 - \sigma\upsilon\nu x}$$

$$= \frac{1-\cos x - 2\sqrt{(1-\cos x)(1+\cos x)} + 1+\cos x}{-2\cos x}$$

$$= \frac{2-2\sqrt{1-\cos^2 x}}{-2\cos x} =$$

$$= \frac{2-2\sqrt{\sin^2 x}}{-2\cos x} =$$

$$= \frac{2-2|\sin x|}{-2\cos x} \overset{(+)}{=} \frac{2-2\sin x}{-2\cos x} =$$

$$= \frac{2(1-\sin x)}{-2\cos x} = \frac{\sin x - 1}{\cos x}$$

$$\eta\mu(-x) = -\eta\mu x$$

$$\sigma\upsilon\nu(-x) = \sigma\upsilon\nu x$$

$$\varepsilon\varphi(-x) = -\varepsilon\varphi x$$

$$\sigma\varphi(-x) = -\sigma\varphi x$$

$$\eta\mu(\pi - x) = \eta\mu x$$

$$\sigma\upsilon\nu(\pi - x) = -\sigma\upsilon\nu x$$

$$\varepsilon\varphi(\pi - x) = -\varepsilon\varphi x$$

$$\sigma\varphi(\pi - x) = -\sigma\varphi x$$

$$\eta\mu(\pi + x) = -\eta\mu x$$

$$\sigma\upsilon\nu(\pi + x) = -\sigma\upsilon\nu x$$

$$\varepsilon\varphi(\pi + x) = \varepsilon\varphi x$$

$$\sigma\varphi(\pi + x) = \sigma\varphi x$$

$$\eta\mu\left(\frac{\pi}{2} - x\right) = \sigma\upsilon\nu x$$

$$\sigma\upsilon\nu\left(\frac{\pi}{2} - x\right) = \eta\mu x$$

$$\varepsilon\varphi\left(\frac{\pi}{2} - x\right) = \sigma\varphi x$$

$$\sigma\varphi\left(\frac{\pi}{2} - x\right) = \varepsilon\varphi x$$

Ενότητα 11

11. $$\frac{\eta\mu(2\pi+\theta)\cdot\sigma\varphi\left(\frac{3\pi}{2}-\theta\right)\sigma\upsilon\nu(3\pi+\theta)}{\sigma\upsilon\nu\left(\frac{3\pi}{2}+\theta\right)\cdot\varepsilon\varphi(\pi+\theta)\sigma\upsilon\nu\left(\frac{\pi}{2}-\theta\right)}.$$

- $\eta\mu(2\pi+\theta) = \eta\mu\theta$
- $\sigma\varphi\left(\frac{3\pi}{2}-\theta\right) = \sigma\varphi\left(\frac{2\pi}{2}+\frac{\pi}{2}-\theta\right) =$

  $= \sigma\varphi\left(\pi+\frac{\pi}{2}-\theta\right) = \sigma\varphi\left(\frac{\pi}{2}-\theta\right) = \varepsilon\varphi\theta$
- $\sigma\upsilon\nu(3\pi+\theta) = \sigma\upsilon\nu(2\pi+\pi+\theta) = \sigma\upsilon\nu(\pi+\theta)$

  $= -\sigma\upsilon\nu\theta$
- $\sigma\upsilon\nu\left(\frac{3\pi}{2}+\theta\right) = \sigma\upsilon\nu\left(\frac{2\pi}{2}+\frac{\pi}{2}+\theta\right) = \sigma\upsilon\nu\left(\pi+\frac{\pi}{2}+\theta\right).$

  $= -\sigma\upsilon\nu\left(\frac{\pi}{2}+\theta\right) = -\sigma\upsilon\nu\left(\frac{\pi}{2}-(-\theta)\right) =$

  $= -\eta\mu(-\theta) = +\eta\mu\theta.$

• $\varepsilon\varphi(\pi+\theta) = \varepsilon\varphi\theta$

• $\sigma\omega\left(\frac{\pi}{2}-\theta\right) = \eta\mu\theta$

Αρ4 $\dfrac{\eta\mu\theta \quad \varepsilon\varphi\theta \quad \theta\pi\eta \quad (-\sigma\omega\theta)}{\eta\mu\theta \quad \varepsilon\varphi\theta \quad \theta\pi\eta \quad \eta\mu\theta}$

Αρ2 $-\sigma\varphi\theta$

Спорсо Майнр

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(6)

(8) α γ

(9) α

(10) α

(11)

(12) α γ .