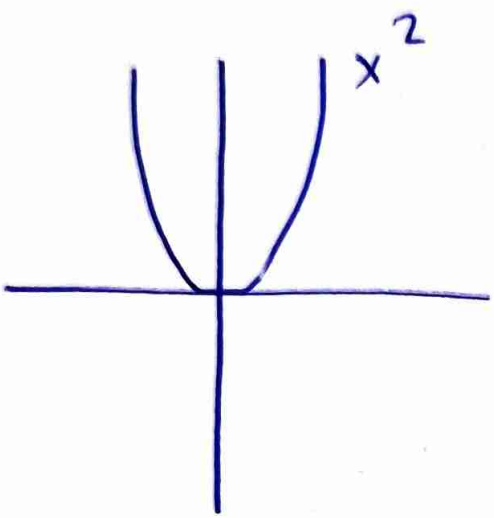
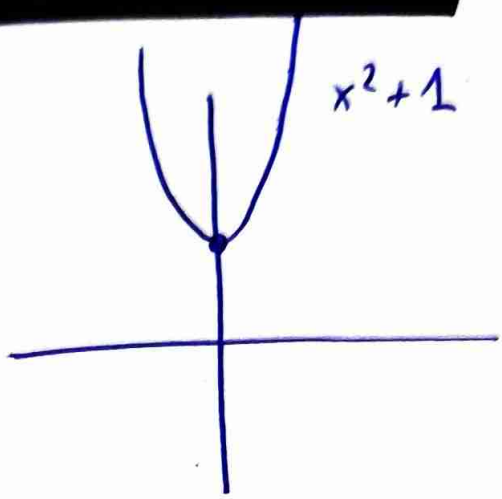
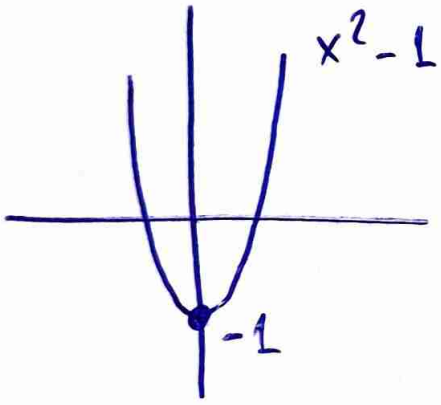
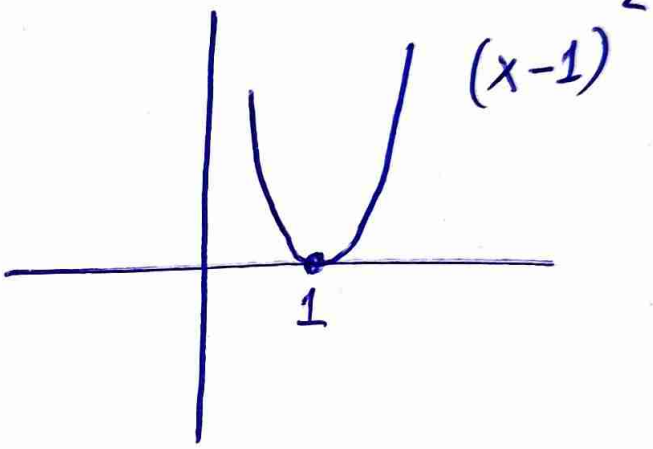
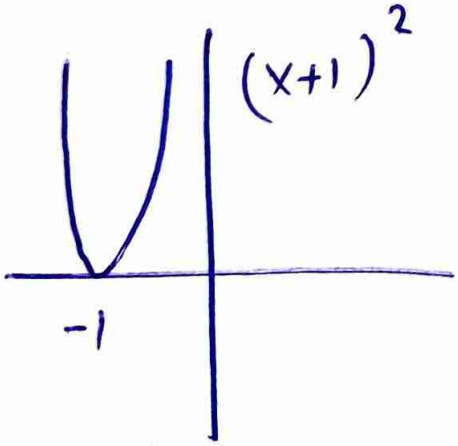
Μετατοπιση καμπυλης

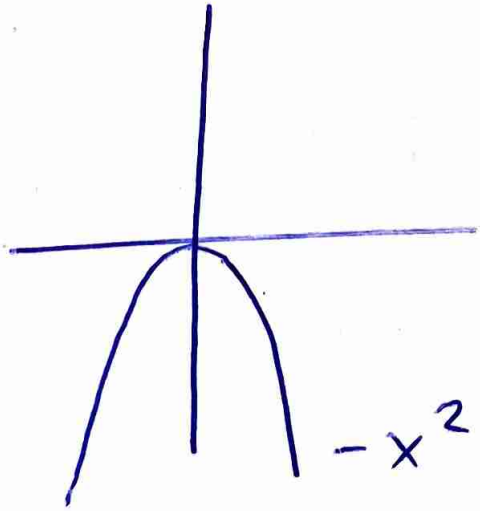




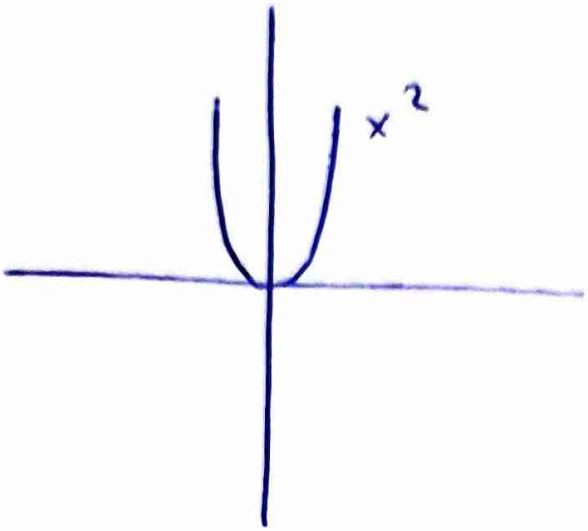


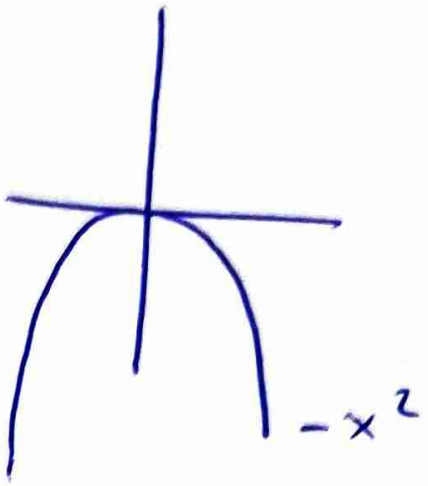


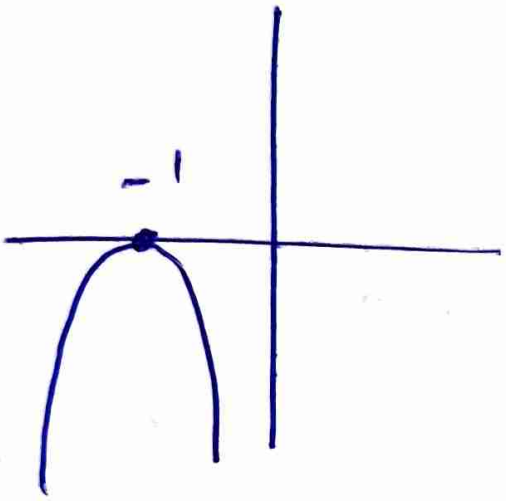




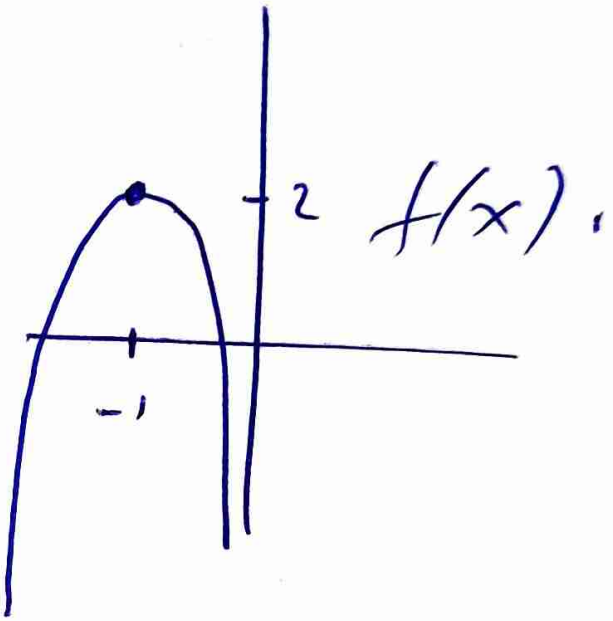
$f(x) = -(x+1)^2 + 2$



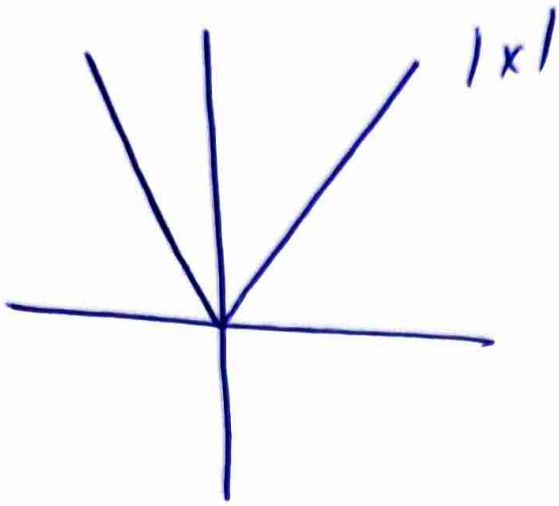


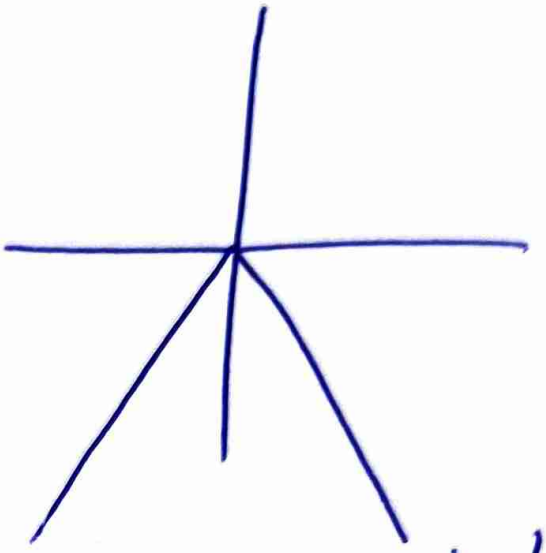


$-(x+1)^2$

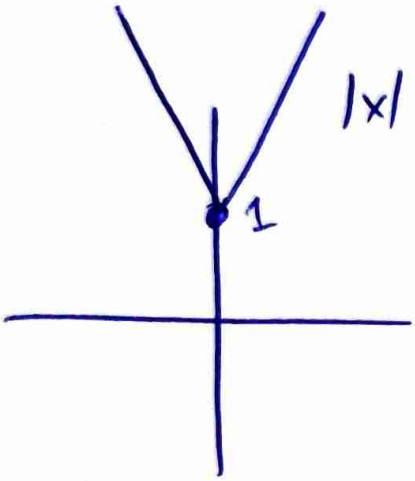


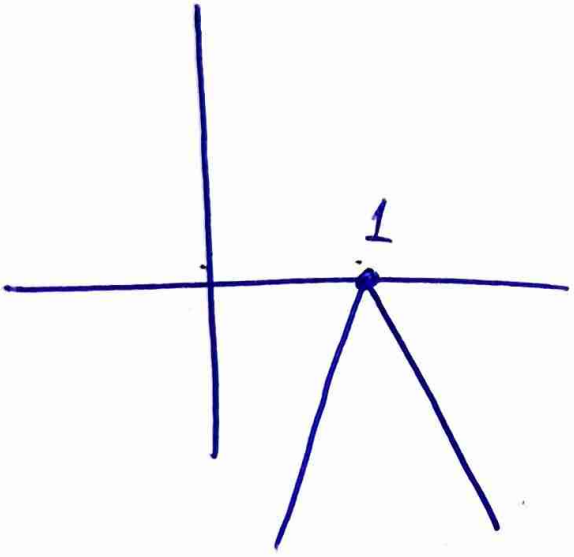
$f(x)$.





$-|x|$





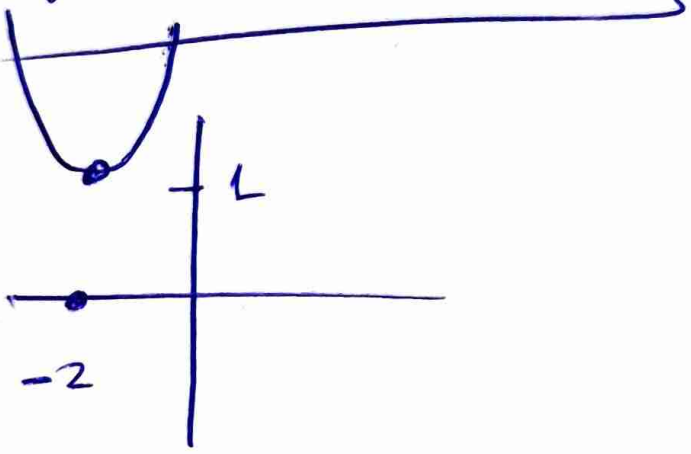
$-|x-1|$

SOS εφαρμογή

1. $f(x) = x^2 + 4x + 5$

$f(x) = x^2 + 4x + 4 + 1$

$f(x) = (x+2)^2 + 1$



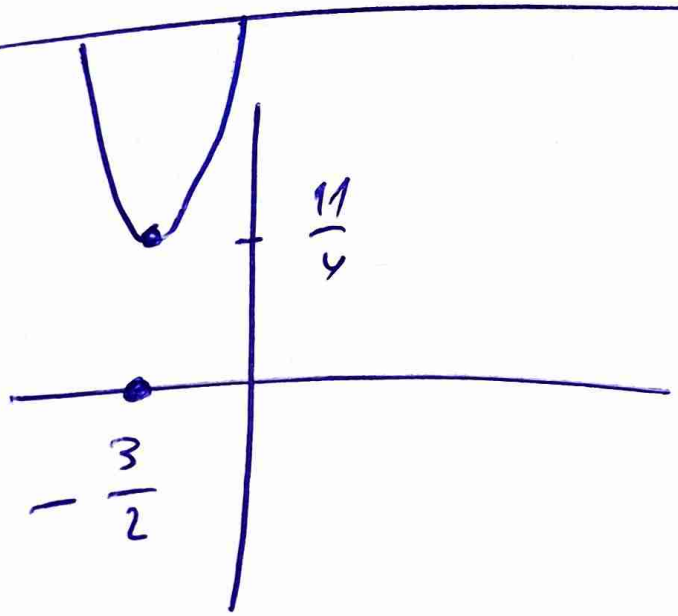
2. $f(x) = x^2 + 3x + 5$

$$f(x) = x^2 + 2 \cdot \frac{3}{2} \cdot x + 5$$

$$f(x) = x^2 + 2 \cdot \frac{3}{2} \cdot x + \left(\frac{3}{2}\right)^2 - \left(\frac{3}{2}\right)^2 + 5$$

$$f(x) = \left(x + \frac{3}{2}\right)^2 + 5 - \frac{9}{4}$$

$$\boxed{f(x) = \left(x + \frac{3}{2}\right)^2 + \frac{11}{4}}$$



3. $f(x) = 3x^2 - 8x + 13$

$f(x) = 3\left(x^2 - \frac{8}{3}x + \frac{13}{3}\right)$

$f(x) = 3\left(x^2 - 2\cdot\frac{8}{6}x + \frac{13}{3}\right)$

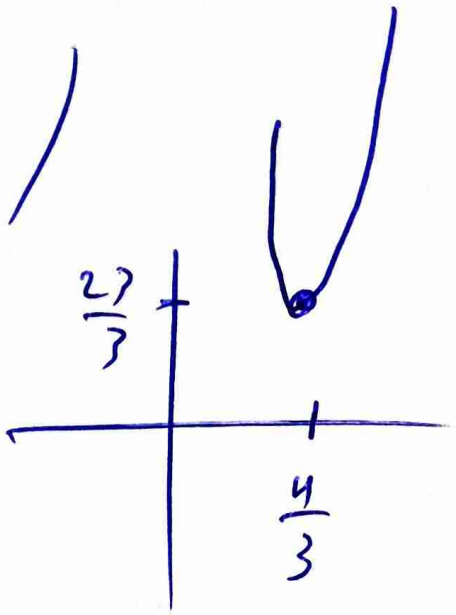
$f(x) = 3\left(x^2 - 2\cdot\frac{4}{3}x + \frac{13}{3}\right)$

$f(x) = 3\left(x^2 - 2\cdot\frac{4}{3}\cdot x + \left(\frac{4}{3}\right)^2 - \left(\frac{4}{3}\right)^2 + \frac{13}{3}\right)$

$f(x) = 3\left(\left(x - \frac{4}{3}\right)^2 + \frac{13}{3} - \frac{16}{9}\right)$

$f(x) = 3\left(\left(x - \frac{4}{3}\right)^2 + \frac{23}{9}\right)$

$f(x) = 3\left(x - \frac{4}{3}\right)^2 + \frac{23}{3}$



3. (B) Νδο η $f(x)=\frac{6x}{x^2+9}$ έχει ελάχιστο για $x=-3$

Ενότητα 6

Αρκεί νδο $f(x) \geq f(-3)$

$$\frac{6x}{x^2+9} \geq \frac{-18}{18}$$

$$\frac{6x}{x^2+9} \geq -1 \Rightarrow 6x \geq -x^2-9$$

$$x^2+6x+9 \geq 0$$

$$(x+3)^2 \geq 0$$ ✓

Αρκεί νδο $f(x) \leq f(3)$

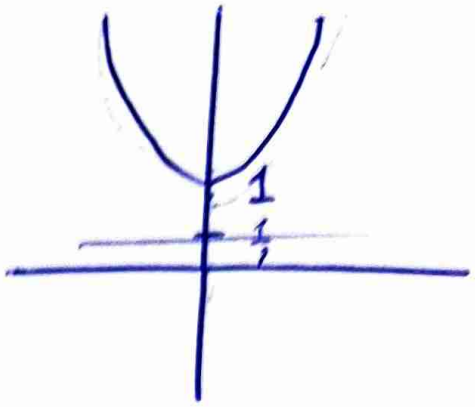
$$\frac{6x}{x^2+9} \leq \frac{18}{18}$$

$$\frac{6x}{x^2+9} \leq 1 \Rightarrow 6x \leq x^2+9$$

$$0 \leq x^2-6x+9$$

$$0 \leq (x-3)^2$$ ✓

5. $f(x) = \sqrt{x^2+1}$



(α) $A(0,1) \quad 0 \in$

(β) $x^2 \geq 0 \Rightarrow x^2+1 \geq 1$

$\sqrt{x^2+1} \geq \sqrt{1}$

$f(x) \geq 1$

$\rightarrow f(x) = 1 \Rightarrow \sqrt{x^2+1} = 1 \Rightarrow x^2+1 = 1$

$x^2 = 0$

$x = 0$.

(γ) (+)

(δ) $g(x) = |f(x)-1| = f(x)-1$

• $f(x) \geq 1 \Rightarrow f(x)-1 \geq 0$

(δ) i) $f(x)-1 = 0$

$f(x) = 1$

$x = 0$

ii) $f(x) = \frac{1}{3}$

Αδύνατη

iii) $f(x) + x^2 = 1 \Rightarrow f(x) - 1 + x^2 = 0$

προφανώς για $x = 0$

Αν $x \neq 0$

• $f(x) > 1 \Rightarrow f(x)-1 > 0$

• $x^2 > 0$ (+)

$f(x)-1+x^2 > 0$

7. ⓐ $f(x) = |x-2|$

• $|x-2| \geq 0 \Rightarrow f(x) \geq 0$

$\rightarrow f(x) = 0 \Rightarrow |x-2| = 0$ $\quad A(2,0)$ O.E

$x = 2$

ⓑ $f(x) = 3 - 2|x-1|$

• $|x-1| \geq 0 \Rightarrow -2|x-1| \leq 0 \cdot (-2)$

$-2|x-1| \leq 0$

$3 - 2|x-1| \leq 3$

$f(x) \leq 3$

$\rightarrow f(x) = 3$

~~$3 - 2|x-1| = 3$~~

$-2|x-1| = 0$

$|x-1| = 0$

$x = 1$

$A(1,3)$

O.M.

⑥ $f(x) = 3x^2 - 1$

• $x^2 \geq 0 \Rightarrow 3x^2 \geq 0 \Rightarrow 3x^2 - 1 \geq -1$

$f(x) \geq -1$

$\rightarrow f(x) = -1$

$3x^2 - 1 = -1$

$3x^2 = 0$

$x = 0$

$A(0, -1)$ O.E.

⑥ $f(x) = 5 - 2x^4$

• $x^4 \geq 0 \Rightarrow -2x^4 \leq 0$

$5 - 2x^4 \leq 5$

$f(x) \leq 5$

$\rightarrow f(x) = 5$

~~$5 - 2x^4 = 5$~~

$-2x^4 = 0$

$x = 0$

$A(0, 5)$

O.M.

(02) $f(x) = x^2(x-1)^4$

- $x^2 \geq 0$
- $(x-1)^4 \geq 0$

$\Big\}$ $x^2(x-1)^4 \geq 0$

$f(x) \geq 0$

$\rightarrow f(x) = 0$ $\quad A(0,0)$

$x^2(x-1)^4 = 0$ $\quad B(1,0)$

$x^2 = 0$ ó $(x-1)^4 = 0$ $\quad$ O. $\in$.

$x = 0$ $\quad$ $x = 1$

(11) $f(x) = 2 - 3\sqrt{x-1}$

- $\sqrt{x-1} \geq 0 \Rightarrow -3\sqrt{x-1} \leq 0$

$2 - 3\sqrt{x-1} \leq 2$

$A(1,2)$ O.M. $\quad f(x) \leq 2$

$\rightarrow f(x) = 2$

$2 - 3\sqrt{x-1} = 2 \quad -3\sqrt{x-1} = 0$

$x = 1$

⑨ $f(x) = \sqrt{x^2+1}$

• $x^2 \geq 0 \Rightarrow x^2+1 \geq 1 \Rightarrow \sqrt{x^2+1} \geq 1$

$f(x) \geq 1$

$\rightarrow f(x) = 1$

$\sqrt{x^2+1} = 1$

$x^2+1 = 1$

$x^2 = 0$

$x = 0$

$A(0,1)$

O. E.

8. $f: (0, +\infty) \longrightarrow \mathbb{R}$

$$f(x) = \frac{x^2+5x+1}{x}$$

(a) $$f(x) = \frac{x^2}{x} + \frac{5x}{x} + \frac{1}{x}$$

$$f(x) = x + \frac{1}{x} + 5$$

- $x + \frac{1}{x} \geq 2 \qquad \forall x > 0$

$$x + \frac{1}{x} + 5 \geq 7$$

$$f(x) \geq 7.$$

$\longrightarrow f(x) = 7$

$$x + \frac{1}{x} + 5 = 7$$

$$x + \frac{1}{x} - 2 = 0$$

$$x^2 + 1 - 2x = 0$$

$$(x-1)^2 \geq 0 \qquad \underline{\underline{x = 1}}$$

(B) Νδο $f(2021) + \frac{3}{7} f(2022) > 10$.

• Γνωρίζω ότι $f(x) \geq 7 \quad \forall x > 0$.

$\longrightarrow f(2021) > 7 \Rightarrow 7f(2021) > 49$ (+)

$\longrightarrow f(2022) > 7 \Rightarrow 3f(2022) > 21$

$7f(2021) + 3f(2022) > 70$

$7f(2021) + 3f(2022) > 70$

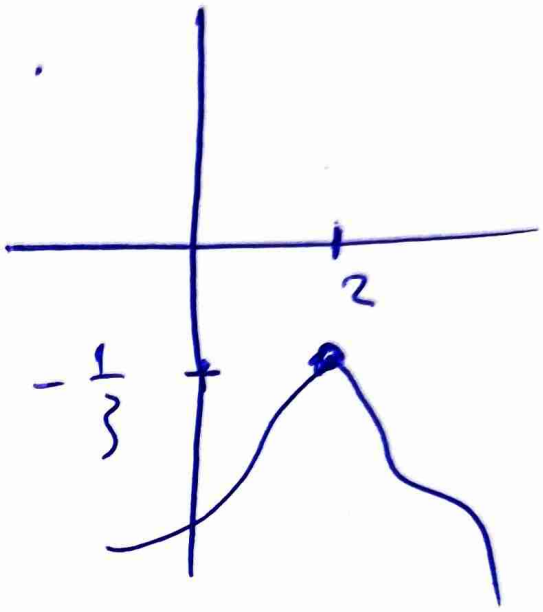
$f(2021) + \frac{3}{7} f(2022) > 10$

9. $3f(x)+1 \le 0$

$$f(x) \le -\frac{1}{3}$$

$f(2) = -\frac{1}{3}$.

(α) Μεγίστη Τιμή $-\frac{1}{3}$.



(β) $f(x) = 0$

Άδειστη

αφού $f(x) \le -\frac{1}{3}$

(γ) $f(x) = (x-2)^2 - \frac{1}{3}$

$0 = (x-2)^2 - \frac{1}{3} - f(x)$ Πρόχειρα

$x = 2$

Για $x \neq 2$

$(x-2)^2 - \frac{1}{3} - f(x) > 0$

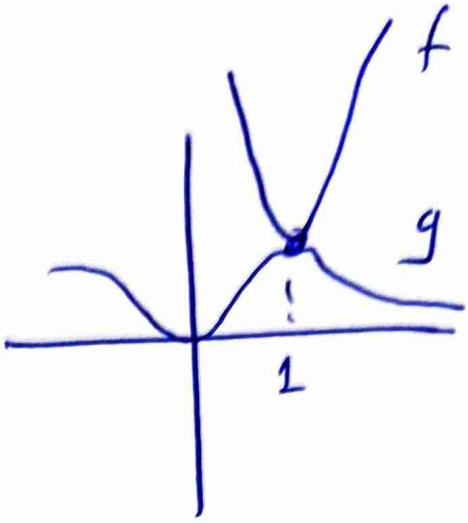
- $(x-2)^2 > 0$
- $f(x) < -\frac{1}{3} \Rightarrow -f(x) > \frac{1}{3}$

$-f(x) - \frac{1}{3} > 0$

(+)

11. $f(x) = |x-1|+1$

$$g(x) = \frac{2x^2}{x^4+1}$$



(α) Από σχήμα $f(x) \geq 1$ και $g(x) \leq 1$

(β) $$|x-1|+1 = \frac{2x^2}{x^4+1}$$

$$f(x) = g(x)$$

$$x = 1$$