

10.

ESERCIZIO 8

$$\textcircled{B} \lim_{x \rightarrow 2} \frac{x^2 - 2x}{3x^2 - 5x - 2} = \lim_{x \rightarrow 2} \frac{\cancel{x}(x-2)}{(\cancel{x-1})(3x+1)} = \frac{2}{7}$$

$$\begin{array}{ccc} 3 & -5 & -2 \\ \downarrow & 6 & 2 \\ 3 & 1 & 0 \end{array} \quad \textcircled{2}$$

$$\text{11. } \textcircled{B} \lim_{x \rightarrow -1} \frac{x^3 + 1}{x^2 + x + 2} = \lim_{x \rightarrow -1} \frac{\cancel{(x+1)}(x^2 - x + 1)}{\cancel{(x+1)}(x^2 - x + 2)} = \frac{3}{4}$$

$$\begin{array}{cccc} 1 & 0 & 1 & 2 \\ \downarrow & -1 & 1 & -2 \\ 1 & -1 & 2 & 0 \end{array} \quad \textcircled{-1}$$

$$= \frac{3}{4}$$

$$\text{13. } \textcircled{B} \lim_{x \rightarrow 5} \frac{\sqrt{x} - \sqrt{5}}{5 - x} = \lim_{x \rightarrow 5} \frac{(\sqrt{x} - \sqrt{5})(\sqrt{x} + \sqrt{5})}{(5 - x)(\sqrt{x} + \sqrt{5})}$$

$$\begin{aligned} &= \lim_{x \rightarrow 5} \frac{\sqrt{x^2 - 5^2}}{(5 - x)(\sqrt{x} + \sqrt{5})} = \lim_{x \rightarrow 5} \frac{\cancel{x-5}}{-(\cancel{x-5})(\sqrt{x} + \sqrt{5})} \\ &= \frac{1}{-2\sqrt{5}} \end{aligned}$$

20. (B) $\lim_{x \rightarrow -1} \frac{|x^2-1| - x^2 + x + 2}{\sqrt{x+2} - 1}$

x	-1
x^2-1	+ - +

$$\rightarrow \lim_{x \rightarrow -1^-} \frac{|x^2-1| - x^2 + x + 2}{\sqrt{x+2} - 1} = \lim_{x \rightarrow -1^-} \frac{(x+1)(\sqrt{x+2}+1)}{(\sqrt{x+2}-1)(\sqrt{x+2}+1)}$$

$$= \lim_{x \rightarrow -1^-} \frac{\cancel{(x+1)}(\sqrt{x+2}+1)}{\cancel{x+1}} = 2$$

$$\rightarrow \lim_{x \rightarrow -1^+} \frac{|x^2-1| - x^2 + x + 2}{\sqrt{x+2} - 1} = \lim_{x \rightarrow -1^+} \frac{(-2x^2+x+3)(\sqrt{x+2}+1)}{x+1}$$

-2	1	3	(-)
↓	2	-3	
-2	3	0	

$$= \lim_{x \rightarrow -1^+} \frac{\cancel{(-x+1)}(-2x+3)(\sqrt{x+2}+1)}{\cancel{x+1}}$$

$$= 5 \cdot 2 = 10$$

To opio sw unapx

20. (a) $\lim_{x \rightarrow 1} \frac{|x-1| + x^2 - 2x + 1}{x^5 - 1}$

$$\begin{array}{cccccc} 1 & 0 & 0 & 0 & 0 & -1 & \textcircled{1} \\ \downarrow & 1 & 1 & 1 & 1 & 1 & \\ 1 & 1 & 1 & 1 & 1 & 0 & \end{array}$$

x	1	
x-1	-	+

$$\rightarrow \lim_{x \rightarrow 1^-} \frac{|x-1|^{\ominus} + x^2 - 2x + 1}{x^5 - 1} = \lim_{x \rightarrow 1^-} \frac{-(x-1) + (x-1)^2}{(x-1)(x^4 + x^3 + x^2 + x + 1)}$$

$$= \frac{-1 + 0}{5} = -\frac{1}{5}$$

$$\rightarrow \lim_{x \rightarrow 1^+} \frac{|x-1|^{\oplus} + x^2 - 2x + 1}{x^5 - 1} = \lim_{x \rightarrow 1^+} \frac{(x-1) + (x-1)^2}{(x-1)(x^4 + x^3 + x^2 + x + 1)}$$

$$= \frac{1}{5}$$

To solve the limit

$$19. \quad (a) \quad \lim_{x \rightarrow 2} \frac{|x-1|^{\oplus} - |x-3|^{\ominus}}{x^2-4} = \lim_{x \rightarrow 2} \frac{x-1 - (-x+3)}{x^2-4}$$

$$= \lim_{x \rightarrow 2} \frac{x-1+x-3}{x^2-4} = \lim_{x \rightarrow 2} \frac{2x-4}{x^2-4} =$$

$$= \lim_{x \rightarrow 2} \frac{2\cancel{(x-2)}}{\cancel{(x-2)}(x+2)} = \lim_{x \rightarrow 2} \frac{2}{x+2} = \frac{1}{2}.$$

$$(8) \quad \lim_{x \rightarrow -1^+} \frac{|x+1|^0 + x^2 - x - 2}{x^3 + 1}$$

x	-1
$x+1$	$- \quad +$

$$= \lim_{x \rightarrow -1^+} \frac{|x+1|^{\oplus} + x^2 - x - 2}{x^3 + 1} =$$

$$= \lim_{x \rightarrow -1^+} \frac{x+1 + x^2 - x - 2}{x^3 + 1} = \lim_{x \rightarrow -1^+} \frac{x^2 - 1}{x^3 + 1}$$

$$= \lim_{x \rightarrow -1} \frac{(x-1)\cancel{(x+1)}}{\cancel{(x+1)}(x^2-x+1)} = \frac{-2}{3}.$$

$$13. \textcircled{n} \lim_{x \rightarrow 0} \frac{x}{1 - \sqrt{3x+1}} = \lim_{x \rightarrow 0} \frac{x(1 + \sqrt{3x+1})}{(1 - \sqrt{3x+1})(1 + \sqrt{3x+1})}$$

$$= \lim_{x \rightarrow 0} \frac{x(1 + \sqrt{3x+1})}{1^2 - \cancel{\sqrt{3x+1}}^2} = \lim_{x \rightarrow 0} \frac{x(1 + \sqrt{3x+1})}{1 - 3x - \cancel{1}}$$

$$= \lim_{x \rightarrow 0} \frac{\cancel{x}(1 + \sqrt{3x+1})}{-3\cancel{x}} = \frac{2}{-3}$$

$$13. \quad \textcircled{8} \quad \lim_{x \rightarrow 1} \frac{\sqrt{x+3} - 2}{x^2 - 1} =$$

$$= \lim_{x \rightarrow 1} \frac{(\sqrt{x+3} - 2)(\sqrt{x+3} + 2)}{(x-1)(x+1)(\sqrt{x+3} + 2)}$$

$$= \lim_{x \rightarrow 1} \frac{\cancel{\sqrt{x+3}} - 2^2}{(x-1)(x+1)(\sqrt{x+3} + 2)}$$

$$= \lim_{x \rightarrow 1} \frac{\cancel{x-1}}{(\cancel{x-1})(x+1)(\sqrt{x+3} + 2)} = \frac{1}{8}$$

$$13. \textcircled{7} \lim_{x \rightarrow 1} \frac{\sqrt{3x^2+1} - 2}{x^2 - 5x + 4} =$$

$$= \lim_{x \rightarrow 1} \frac{(\sqrt{3x^2+1} - 2)(\sqrt{3x^2+1} + 2)}{(x-4)(x-1)(\sqrt{3x^2+1} + 2)} =$$

$$= \lim_{x \rightarrow 1} \frac{\sqrt{3x^2+1}^2 - 2^2}{(x-4)(x-1)(\sqrt{3x^2+1} + 2)}$$

$$= \lim_{x \rightarrow 1} \frac{3x^2 - 3}{(x-4)(x-1)(\sqrt{3x^2+1} + 2)} =$$

$$= \lim_{x \rightarrow 1} \frac{3\cancel{(x-1)}(x+1)}{(x-4)\cancel{(x-1)}(\sqrt{3x^2+1} + 2)} = \frac{6}{-3 \cdot 4} = -\frac{1}{2}.$$

$$13. \quad (8) \quad \lim_{x \rightarrow 2} \frac{x-2}{\sqrt{x+2}-2} = \lim_{x \rightarrow 2} \frac{(x-2)(\sqrt{x+2}+2)}{(\sqrt{x+2}-2)(\sqrt{x+2}+2)}$$

$$= \lim_{x \rightarrow 2} \frac{(x-2)(\sqrt{x+2}+2)}{\cancel{\sqrt{x+2}-2}^2} = \lim_{x \rightarrow 2} \frac{\cancel{(x-2)}(\sqrt{x+2}+2)}{\cancel{x-2}}$$

$$= \lim_{x \rightarrow 2} \sqrt{x+2} + 2 = 4$$

$$(20) \quad \lim_{x \rightarrow 3} \frac{\sqrt{x^2+2} - \sqrt{11}}{x^2 - 5x + 6} =$$

$$= \lim_{x \rightarrow 3} \frac{(\sqrt{x^2+2} - \sqrt{11})(\sqrt{x^2+2} + \sqrt{11})}{(x-2)(x-3)(\sqrt{x^2+2} + \sqrt{11})}$$

$$= \lim_{x \rightarrow 3} \frac{\cancel{\sqrt{x^2+2}}^2 - \cancel{\sqrt{11}}^2}{(x-2)(x-3)(\sqrt{x^2+2} + \sqrt{11})}$$

$$= \lim_{x \rightarrow 3} \frac{x^2 - 11}{(x-2)(x-3)(\sqrt{x^2+2} + \sqrt{11})} = \lim_{x \rightarrow 3} \frac{\cancel{(x-3)}(x+3)}{(x-2)\cancel{(x-3)}(\sqrt{x^2+2} + \sqrt{11})} = \frac{6}{2\sqrt{11}}$$

$$13. \quad \textcircled{e} \quad \lim_{x \rightarrow 2} \frac{x^2 - 2x}{\sqrt{x^2 + 5} - 3} = \lim_{x \rightarrow 2} \frac{x(x-2)(\sqrt{x^2 + 5} + 3)}{(\sqrt{x^2 + 5} - 3)(\sqrt{x^2 + 5} + 3)}$$

$$= \lim_{x \rightarrow 2} \frac{x(x-2)(\sqrt{x^2 + 5} + 3)}{\cancel{\sqrt{x^2 + 5}}^2 - 3^2} = \lim_{x \rightarrow 2} \frac{x(x-2)(\sqrt{x^2 + 5} + 3)}{x^2 - 4}$$

$$= \lim_{x \rightarrow 2} \frac{x \cancel{(x-2)} (\sqrt{x^2 + 5} + 3)}{\cancel{(x-2)}(x+2)} = \lim_{x \rightarrow 2} \frac{x(\sqrt{x^2 + 5} + 3)}{x+2}$$

$$= \frac{2 \cdot 6}{4} = 3.$$

9.

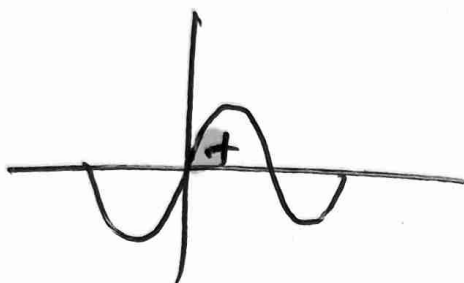
$$\textcircled{a} \lim_{x \rightarrow 0} \frac{x+1+\sqrt{x}}{x} = \lim_{x \rightarrow 0^+} (x+1+\sqrt{x}) \frac{1}{x}$$

$$= 1 \cdot (+\infty) = +\infty.$$

Answer
9

$$\textcircled{b} \lim_{x \rightarrow 0} \frac{1+\sqrt{x}}{4+x} = \lim_{x \rightarrow 0} (1+\sqrt{x}) \frac{1}{4+x} = 1 \cdot (+\infty)$$

$$= +\infty$$



$$\textcircled{c} \lim_{x \rightarrow 0} \left(\frac{2}{x} - \frac{2}{\sqrt{x}} \right) = \lim_{x \rightarrow 0} \frac{2\sqrt{x} - 2x}{x\sqrt{x}} =$$

$$= \lim_{x \rightarrow 0} \frac{2(\sqrt{x} - x)}{x\sqrt{x}} = \lim_{x \rightarrow 0} 2 \cdot \lim_{x \rightarrow 0} \frac{\sqrt{x} - x}{x\sqrt{x}}$$

$$= 2 \cdot \lim_{x \rightarrow 0} \frac{(\sqrt{x} - x)(\sqrt{x} + x)}{x\sqrt{x}(\sqrt{x} + x)} =$$

$$= 2 \lim_{x \rightarrow 0} \frac{\sqrt{x}^2 - x^2}{x\sqrt{x}(\sqrt{x} + x)} = 2 \lim_{x \rightarrow 0} \frac{x(1-x)}{x\sqrt{x}(\sqrt{x} + x)}$$

$$= 2 \cdot \lim_{x \rightarrow 0^+} \frac{1-x}{\sqrt{x}(\sqrt{x}+x)} =$$

$$= 2 \lim_{x \rightarrow 0^+} (1-x) \cdot \frac{1}{\underbrace{\sqrt{x}}_{(+)} \underbrace{(\sqrt{x}+x)}_{(+)}} = 2 \cdot 1 \cdot (+\infty) = +\infty$$

$$8. \textcircled{B} \lim_{x \rightarrow -\infty} \frac{|x^3 - 2x + 3| + x - 1}{x^4 + 2x + 2} =$$

$$\rightarrow \lim_{x \rightarrow -\infty} x^3 - 2x + 3 = \lim_{x \rightarrow -\infty} x^3 = -\infty$$

$$= \lim_{x \rightarrow -\infty} \frac{-x^3 + 2x - 3 + x - 1}{x^4 + 2x + 2} =$$

$$= \lim_{x \rightarrow -\infty} \frac{-x^3}{x^4} = \lim_{x \rightarrow -\infty} -\frac{1}{x} = 0.$$

$$2. \textcircled{a} i) \lim_{x \rightarrow +\infty} f(x) = +\infty. \quad \text{ii) } \lim_{x \rightarrow +\infty} \frac{1}{f(x)} = \frac{1}{+\infty} = 0$$

$$\textcircled{B} i) \lim_{x \rightarrow +\infty} f(x) = 1. \quad \text{ii) } \lim_{x \rightarrow +\infty} \frac{1}{f(x) - 1} = \frac{1}{0} = \infty.$$

$$f(x) > 1$$

$$f(x) - 1 > 0$$

$$3. \textcircled{a} \lim_{x \rightarrow -\infty} f(x) = 0$$

$$\textcircled{b} \lim_{x \rightarrow -\infty} \frac{1}{f(x)} = +\infty$$

④

$$\textcircled{c} \lim_{x \rightarrow 0} f(x) = +\infty.$$

$$\textcircled{d} \lim_{x \rightarrow 0} \frac{nx}{f(x)} = \lim_{x \rightarrow 0} nx \cdot \frac{1}{f(x)} = 0 \cdot \frac{1}{\infty} = 0 \cdot 0 = 0.$$

$$\textcircled{e} \lim_{x \rightarrow 1} \frac{1}{f(x)} = \frac{1}{0^+} = +\infty$$

$$\textcircled{f} \lim_{x \rightarrow 2} \frac{1}{f(x)-1} = \frac{1}{0^-} = -\infty$$

$$f(x) < 1 \Rightarrow f(x) - 1 < 0$$

$$\textcircled{g} \lim_{x \rightarrow 3} \frac{\ln(3-x)}{f(x)} = \lim_{x \rightarrow 3} \ln(3-x) \cdot \frac{1}{f(x)}$$

$$= -\infty.$$

$$\rightarrow \lim_{x \rightarrow 3^-} \ln(3-x) \cdot \frac{1}{f(x)} = -\infty \cdot (+\infty) = -\infty$$

$$\rightarrow \lim_{x \rightarrow 3^+} \ln(3-x) \cdot \frac{1}{f(x)} = -\infty(-\infty) = +\infty$$

To solve the next problem.

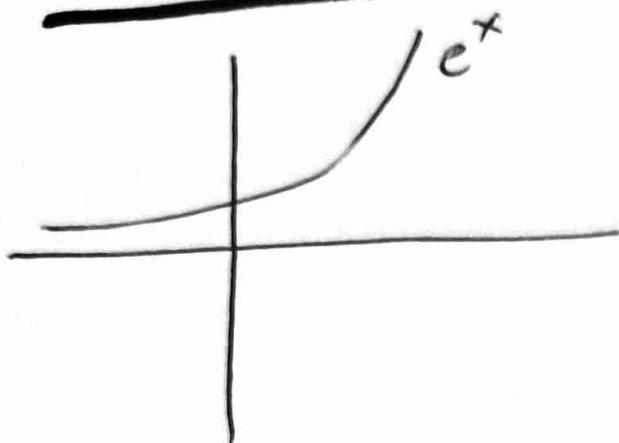
$$(n) \lim_{x \rightarrow +\infty} f(f(x)) = \lim_{u \rightarrow -\infty} f(u) = 0.$$

$$\text{DET } f(x) = u$$

$$x \rightarrow +\infty$$

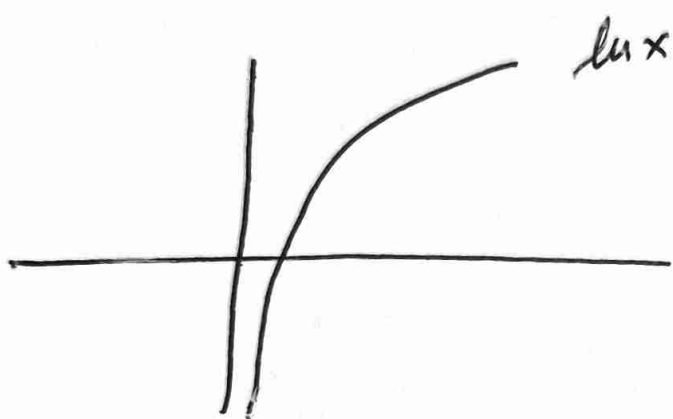
$$u \rightarrow -\infty$$

Μεγαλι προβοχι



$$\lim_{x \rightarrow -\infty} e^x = 0$$

$$\lim_{x \rightarrow +\infty} e^x = +\infty$$



$$\lim_{x \rightarrow 0^+} \ln x = -\infty$$

$$\lim_{x \rightarrow +\infty} \ln x = +\infty$$

Μητροικω κανονα

$$e^{-\infty} = 0$$

$$e^{+\infty} = +\infty$$

$$\ln 0 = -\infty$$

$$\ln +\infty = +\infty$$

$$10. \textcircled{d}. \lim_{x \rightarrow \infty} \sqrt{9x^2 + x + 1} + 3x$$

$$= \lim_{x \rightarrow \infty} \frac{(\sqrt{9x^2 + x + 1} + 3x)(\sqrt{9x^2 + x + 1} - 3x)}{\sqrt{9x^2 + x + 1} - 3x}$$

$$= \lim_{x \rightarrow \infty} \frac{\sqrt{9x^2 + x + 1}^2 - (3x)^2}{\sqrt{9x^2 + x + 1} - 3x}$$

$$= \lim_{x \rightarrow \infty} \frac{\cancel{9x^2} + x + 1 - \cancel{9x^2}}{\sqrt{9x^2 + x + 1} - 3x} = \lim_{x \rightarrow \infty} \frac{x + 1}{\sqrt{9x^2 + x + 1} - 3x}$$

$$= \lim_{x \rightarrow \infty} \frac{x(1 + \frac{1}{x})}{\sqrt{x^2(9 + \frac{1}{x} + \frac{1}{x^2})} - 3x} = \lim_{x \rightarrow \infty} \frac{x(1 + \frac{1}{x})}{\sqrt{x^2} \sqrt{9 + \frac{1}{x} + \frac{1}{x^2}} - 3x}$$

$$= \lim_{x \rightarrow \infty} \frac{x(1 + \frac{1}{x})}{-x \sqrt{9 + \frac{1}{x} + \frac{1}{x^2}} - 3x} = \frac{1}{-3 - 3} = -\frac{1}{6}$$

10. (B) $\lim_{x \rightarrow +\infty} (\sqrt{4x^2+1} - 2x) \xrightarrow{+\infty - \infty};$

$$= \lim_{x \rightarrow +\infty} \frac{(\sqrt{4x^2+1} - 2x)(\sqrt{4x^2+1} + 2x)}{\sqrt{4x^2+1} + 2x}$$

$$= \lim_{x \rightarrow +\infty} \frac{\cancel{\sqrt{4x^2+1}}^2 - (2x)^2}{\sqrt{4x^2+1} + 2x}$$

$$= \lim_{x \rightarrow +\infty} \frac{\cancel{4x^2+1} - \cancel{4x^2}}{\sqrt{4x^2+1} + 2x}$$

$$= \lim_{x \rightarrow +\infty} \frac{1}{\sqrt{4x^2+1} + 2x}$$

$$= 0$$

$$9. \quad \textcircled{B} \quad \lim_{x \rightarrow -\infty} \sqrt{x^2 - 5x + 2} = \sqrt{+\infty} = +\infty$$

$$\textcircled{B} \quad \lim_{x \rightarrow -\infty} (\sqrt{x^2 + 1} - x) = \sqrt{+\infty} - (-\infty) = +\infty + \infty = +\infty$$

$$\textcircled{20} \quad \lim_{x \rightarrow -\infty} \frac{x+1}{\sqrt{x^2+1}} =$$

$$= \lim_{x \rightarrow -\infty} \frac{x \left(1 + \frac{1}{x}\right)}{\sqrt{x^2 \left(1 + \frac{1}{x^2}\right)}} =$$

$$= \lim_{x \rightarrow -\infty} \frac{x \left(1 + \frac{1}{x}\right)}{\sqrt{x^2} \sqrt{1 + \frac{1}{x^2}}} =$$

$$= \lim_{x \rightarrow -\infty} \frac{x \left(1 + \frac{1}{x}\right)}{|x| \sqrt{1 + \frac{1}{x^2}}} = \lim_{x \rightarrow -\infty} \frac{\cancel{x} \left(1 + \frac{1}{x}\right)}{-\cancel{x} \sqrt{1 + \frac{1}{x^2}}}$$

$$= \frac{1+0}{-\sqrt{1+0}} = -1$$

Τριγωνομετρικά όρια

Βασικά όρια

$$\lim_{x \rightarrow 0} \frac{\eta\mu x}{x} = 1$$

$$\lim_{x \rightarrow 0} \frac{\sigma\omega x - 1}{x} = 0$$

extra

$$\lim_{x \rightarrow x_0} \frac{\eta\mu f(x)}{f(x)} = 1 \quad \text{για } \lim_{x \rightarrow x_0} f(x) = 0.$$

$$\underline{\underline{\eta \cdot x}} \quad \lim_{x \rightarrow 1} \frac{\eta\mu(x-1)}{x-1} = 1$$

$$\lim_{x \rightarrow x_0} \frac{\sigma\omega f(x) - 1}{f(x)} = 0 \quad \text{για } \lim_{x \rightarrow x_0} f(x) = 0.$$

$\eta \cdot x$

$$\lim_{x \rightarrow -\infty} \frac{\sigma\omega e^x - 1}{e^x} = 0 \quad \text{για } \lim_{x \rightarrow -\infty} e^x = 0.$$

$$\lim_{x \rightarrow x_0} f(x) = 0$$

$$\lim_{x \rightarrow x_0} \frac{u(x)}{f(x)} = \lim_{x \rightarrow x_0} \frac{u(x)}{f(x)} = \lim_{t \rightarrow 0} \frac{u(t)}{t} = 1.$$

$$1. \quad \lim_{x \rightarrow 0} x^2 \eta \frac{1}{x} \stackrel{\eta \rightarrow \infty}{=} 0$$

$$-1 \leq \eta \frac{1}{x} \leq 1$$

$$\boxed{-x^2 \leq x^2 \eta \frac{1}{x} \leq x^2}$$

$$\lim_{x \rightarrow 0} -x^2 = 0 \quad \left. \vphantom{\lim_{x \rightarrow 0} -x^2 = 0} \right\} \begin{array}{l} \text{Από το κριτήριο} \\ \text{παραβολών} \end{array}$$

$$\lim_{x \rightarrow 0} x^2 = 0$$

$$\lim_{x \rightarrow 0} x^2 \eta \frac{1}{x} = 0$$

$$2. \quad \lim_{x \rightarrow 0} x \eta \frac{1}{x} \stackrel{\eta \rightarrow \infty}{=} 0$$

$$-1 \leq \eta \frac{1}{x} \leq 1$$

$$|\eta \frac{1}{x}| \leq 1$$

$$|x| \cdot |\eta \frac{1}{x}| \leq |x|$$

$$|x \eta \frac{1}{x}| \leq |x|$$

$$-|x| \leq x \eta \frac{1}{x} \leq |x|$$

$$\lim_{x \rightarrow 0} -|x| = 0$$

$$\lim_{x \rightarrow 0} |x| = 0$$

} And k. 0

$$\lim_{x \rightarrow 0} x \cdot \frac{1}{x} = 0$$

3. $\lim_{x \rightarrow +\infty} x \cdot \frac{1}{x} = \frac{\infty}{\infty}$

$$= \lim_{x \rightarrow +\infty} \frac{x}{\frac{1}{x}}$$

$$= \lim_{x \rightarrow +\infty} \frac{x \cdot \frac{1}{x}}{\frac{1}{x}} =$$

$$\frac{\frac{1}{x} = t}{x \rightarrow +\infty} \lim_{t \rightarrow 0} \frac{x \cdot t}{t} = 1$$

$$28. \textcircled{B} \lim_{x \rightarrow +\infty} \operatorname{arctg} \left(\frac{x}{x^2+1} \right) = \operatorname{arctg} 0 = 0$$

$$\rightarrow \lim_{x \rightarrow +\infty} \frac{x}{x^2+1} = \lim_{x \rightarrow +\infty} \frac{\cancel{x}}{x^2} = 0$$

$$29. \textcircled{B} \lim_{x \rightarrow -\infty} \left(\sqrt{x^2+1} + x \right) \operatorname{arctg} x \stackrel{\frac{\infty}{\infty}}{=} \quad ,$$

$$-1 \leq \operatorname{arctg} x \leq 1$$

$$|\operatorname{arctg} x| \leq 1$$

$$|\sqrt{x^2+1} + x| |\operatorname{arctg} x| \leq 1 \cdot |\sqrt{x^2+1} + x|$$

$$\left| \left(\sqrt{x^2+1} + x \right) \operatorname{arctg} x \right| \leq |\sqrt{x^2+1} + x|$$

$$-|\sqrt{x^2+1} + x| \leq \left(\sqrt{x^2+1} + x \right) \operatorname{arctg} x \leq |\sqrt{x^2+1} + x|$$

$$\begin{aligned} \rightarrow \lim_{x \rightarrow -\infty} \left(\sqrt{x^2+1} + x \right) &= \lim_{x \rightarrow -\infty} \frac{\left(\sqrt{x^2+1} + x \right) \left(\sqrt{x^2+1} - x \right)}{\sqrt{x^2+1} - x} \\ &= \lim_{x \rightarrow -\infty} \frac{\cancel{\sqrt{x^2+1}}^2 - x^2}{\sqrt{x^2+1} - x} = \lim_{x \rightarrow -\infty} \frac{1}{\sqrt{x^2+1} - x} = 0 \end{aligned}$$

$$\lim_{x \rightarrow -\infty} -|\sqrt{x^2+1} - x| = 0$$

$$\lim_{x \rightarrow \infty} |\sqrt{x^2+1} - x| = 0$$

Ans k. n

$$\lim_{x \rightarrow \infty} (\sqrt{x^2+1} + x) \text{ n } x = 0.$$

Μεθοδο 1

$$\lim_{x \rightarrow x_0} g(x) \text{ υπ } f(x)$$

1. Αν το ημετορο ανηριζεται τοτε
πρεπει να χτισω.

$$-1 \leq \text{υπ } f(x) \leq 1$$

αηλο
χτισω.

χτισω με
αηολυτα

Εξαρτωται αηο το αν
γνωριζα το προσηρο του
 $g(x)$

2. Αν το ημετορο μηδωνιζεται
τοτε του δινω αηω αν θελω

$$\lim_{x \rightarrow x_0} g(x) \text{ υπ } f(x) = \lim_{x \rightarrow x_0} g(x) \frac{\text{υπ } f(x)}{f(x)} f(x)$$

1.

Διαγωνισμα

Μαθηματικων

12/10/25

Θεωρια

3

4

6

1.1 2.3

1.3 2.5

1.5

1.6

1.7

1.10

1.11

1.12

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17 28

18 29

19 31

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25.

+ 2 Διαγωνισματα.

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10

Γενικη

συνολικη

ορια.

Επορρω

Μαθιμα

Τεταρτη

7/10

9-11:30,

9

(5) α β γ δ

(6) α γ δ σ τ

(7) α β γ δ

(10) α .

10

(4)

(5) α β γ ε

(6) α β δ ε σ τ .

(7) α β γ .

(8) α .

(9) α γ ε

(10) α γ .