Ενοτητα 28

1. $f(x) = -x^3 + 6x^2 - 9x + 1$

$f'(x) = -3x^2 + 12x - 9$

$f''(x) = -6x + 12$

$\to f''(x) = 0 \Rightarrow -6x + 12 = 0$

$6x = 12$

$x = 2$

x		2
f''	$+$	$-$
f'	$\nearrow$	$\searrow$

Για $x=2$ εχω το μεγιστο

συντελεστη διευθυνσης

2. $f(x) = x^2 + 2\ln x$

x	0		1	
f'		$-$		$+$
f		$\searrow$		$\nearrow$

$f'(x) = 2x + 2\dfrac{1}{x}$

$$f''(x) = 2 - \frac{2}{x^2} = \frac{2x^2-2}{x^2} = 2\,\frac{x^2-1}{x^2}$$

Για $x=1$ εχω τον ελαχιστο ρυθμο μεταβολη.

3. $f(x) = e^{\lambda x} - x \quad , \lambda > 0$

$f'(x) = \lambda e^{\lambda x} - 1$

$\rightarrow f'(x) = 0 \Rightarrow \lambda e^{\lambda x} = 1 \Rightarrow e^{\lambda x} = \frac{1}{\lambda}$

$\lambda x = \ln \frac{1}{\lambda}$

$\lambda x = \ln 1 - \ln \lambda$

$x = -\frac{\ln \lambda}{\lambda}$

x		$-\ln\lambda/\lambda$	
f'	$-$	0	$+$
f	$\searrow$		$\nearrow$

Για $x = -\frac{\ln \lambda}{\lambda}$ εχω ελαχιστη τιμη.

Ελαχιστη τιμη: $f\left(-\frac{\ln \lambda}{\lambda}\right) = e^{\lambda \cdot \left(-\frac{\ln \lambda}{\lambda}\right)} - \left(-\frac{\ln \lambda}{\lambda}\right)$

$= e^{-\ln \lambda} + \frac{\ln \lambda}{\lambda} = \frac{1}{e^{\ln \lambda}} + \frac{\ln \lambda}{\lambda} = \frac{1}{\lambda} + \frac{\ln \lambda}{\lambda}$

$= \frac{1 + \ln \lambda}{\lambda}$ ελαχιστη τιμη.

$$\boxed{g(x) = \frac{1+\ln x}{x}}$$

$$g'(x) = \frac{\frac{1}{x}x - (1+\ln x)}{x^2} = \frac{1-1-\ln x}{x^2} = \frac{-\ln x}{x^2}$$

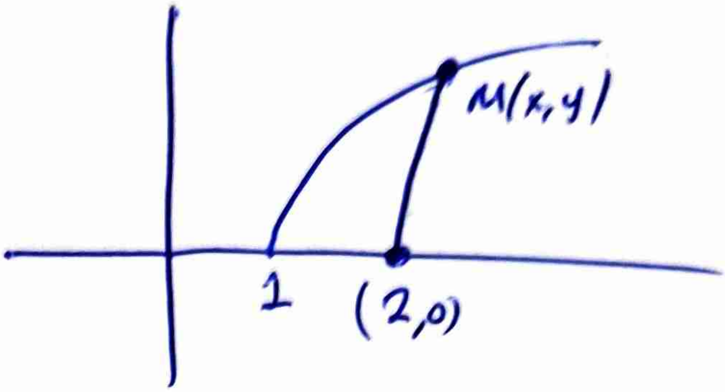
x		1	
g'	$+$		$-$
g	$\nearrow$		$\searrow$

Για $\lambda=1$ η ελαχιστη

τιμη γινεται

μεγιστη.

4. $f(x) = \sqrt{x-1}$



$$d = \sqrt{(x-2)^2 + (y-0)^2}$$

$$d = \sqrt{(x-2)^2 + (\sqrt{x-1})^2}$$

$$d(x) = \sqrt{(x-2)^2 + x - 1}$$

$$\boxed{d(x) = \sqrt{x^2 - 3x + 3}}$$

$$d'(x) = \frac{2x-3}{2\sqrt{x^2-3x+3}}$$

$$d'(x) = 0 \Rightarrow 2x - 3 = 0 \Rightarrow x = \frac{3}{2}.$$

x		$3/2$	
d'	$-$		$+$
d	$\searrow$		$\nearrow$

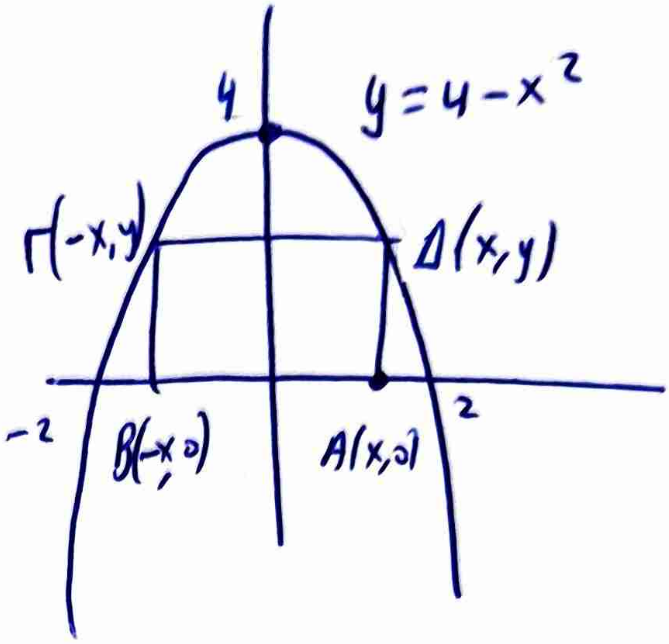
Για $x = \frac{3}{2}$

εχω την

ελαχιστη

αποσταση.

10.



$E = B \cdot U$

$E = 2x \cdot y$

$E = 2x(4 - x^2)$

$E = 8x - 2x^3$

$E(x) = 8x - 2x^3$

$E'(x) = 8 - 8x$

x		1	
E'	$+$		$-$
E	$\nearrow$		$\searrow$

Για $x = 1$

εχω το μεγιστο εμβαδον.