10. (α) $x^3-21x+20>0$

ΕΝΟΤΗΤΑ 19

	1	0	-21	20	(1)
↓		1	1	-20	
	1	1	-20	0	

$(x-1)(x^2+x-20)>0$

(1) (4) (-5)

x		-5		1		4	
x-1	-		-	0	+		+
x^2+x-20	+	0	-		-	0	+
P(x)	-		+		-		+

$x \in (-5,1) \cup (4,+\infty)$.

(β) $-x^3+3x+2<0$

	-1	0	3	2	(-1)
↓		1	-1	-2	
	-1	1	2	0	

$(x+1)(-x^2+x+2)<0$

(-1) (-2) (1)

x		-2		-1		1	
x+1	-		-	0	+		+
$-x^2+x+2$	-	0	+		+	0	-
P(x)	+		-		+		-

$x \in (-2,-1) \cup (1,+\infty)$

11. ⓐ $3x^3+3x^2+10 > x^4+15x$

$$-x^4+3x^3+3x^2-15x+10>0$$

-1	3	3	-15	10	(2)
↓	-2	2	10	-10	
-1	1	5	-5	0	

$$(x-2)(-x^3+x^2+5x-5)>0$$

	-1	1	5	-5 (1)
	↓	-1	0	5
	-1	0	5	0

$$(x-2)(x-1)(-x^2+5)>0$$

(2) (1) $\pm\sqrt{5}$

x	$-\sqrt{5}$		1		2		$\sqrt{5}$
$x-2$	$-$	$-$	$-$	$-$	0	$+$	$+$
$x-1$	$-$	$-$	0	$+$	$+$	$+$	$+$
$-x^2+5$	$-$	0	$+$	$+$	$+$	0	$-$
$P(x)$	$-$	$+$	$-$	$+$			$-$

$x \in (-\sqrt{5}, 1) \cup (2, \sqrt{5})$

(B) $4x^4 - 8x^3 + 5x^2 - x \geq 0$

$x(4x^3 - 8x^2 + 5x - 1) \geq 0$

	4	−8	5	−1	(1)
↓		4	−4	1	
	4	−4	1	0	

$x(x-1)(4x^2 - 4x + 1) \geq 0$

0 (1) $\left(\frac{1}{2}\right)$

x		0		$\frac{1}{2}$		1	
x	−	0	+	+	+	+	
x−1	−		−		−	0	+
$4x^2-4x+1$	+		+	0	+		+
P(x)	+	‖	−	‖	−	0	+

$x \in (-\infty, 0] \cup [1, +\infty) \cup \left\{\frac{1}{2}\right\}$.

Енотия 20

1. (B) $\frac{3x^2+1}{x-1} + \frac{4}{x-x^2} = \frac{x^2-3x+2}{x}$

$$\frac{3x^2+1}{x-1} + \frac{4}{x(1-x)} = \frac{x^2-3x+2}{x}$$

• $x-1=0$ $\quad$ • $x(1-x)=0$ $\quad$ • $x=0$

$x=1$ $\quad$ $x=0$ $\quad$ $x=1$

Прэпи $\quad x \neq 0 \quad x \neq 1$

$$\frac{3x^2+1}{x-1} - \frac{4}{x(x-1)} = \frac{x^2-3x+2}{x}$$

$$x(x-1)\frac{3x^2+1}{x-1} - x(x-1)\frac{4}{x(x-1)} = x(x-1)\frac{x^2-3x+2}{x}$$

$$x(3x^2+1) - 4 = (x-1)(x^2-3x+2)$$

$$3x^3 + x - 4 = x^3 - 3x^2 + 2x - x^2 + 3x - 2$$

$$2x^3 - 4x + 4x^2 - 2 = 0$$

$$x^3 + 2x^2 - 2x - 1 = 0$$

1	2	-2	-1	(1)
↓	1	3	1	
1	3	1	0	

$$(x-1)(x^2+3x+1) = 0$$

$x - 1 = 0$; $x^2 + 3x + 1 = 0$

~~$x = 1$~~

$\Delta = 9 - 4 = 5$

$$x = \frac{-3 \pm \sqrt{5}}{2}$$

2. (B) $2\sin^4 x - 5\sin^3 x + 5\sin x - 2 = 0$

$\underline{\sin x = t}$

$$2t^4 - 5t^3 + 5t - 2 = 0$$

	2	-5	0	5	-2	(1)
↓		2	-3	-3	2	
	2	-3	-3	2	0	

$$(t-1)(2t^3 - 3t^2 - 3t + 2) = 0$$

$t = 1$ ή

$$2t^3 - 3t^2 - 3t + 2 = 0$$

	2	-3	-3	2	(-1)
↓		-2	5	-2	
	2	-5	2	0	

$$(t+1)(2t^2 - 5t + 2) = 0$$

$t = -1$

$\Delta = 25 - 16 = 9$

$$t = \frac{5 \pm 3}{4}$$

$t = 2$ $\quad t = \frac{1}{2}$

$t = 1$ ч $t = -1$ ч $t = 2$ ч $t = \frac{1}{2}$

$\sin x = 1$ $\sin x = -1$ $\sin x = 2$ $\sin x = \frac{1}{2}$

$\sin x = \sin 0$ $\sin x = \sin \pi$ Асмыз $\sin x = \sin \frac{\pi}{3}$

$x = 2k\pi$ $x = 2k\pi \pm \pi$ $x = 2k\pi \pm \frac{\pi}{3}$

1. ⑧ $2\operatorname{ctg}^3 x + \frac{1}{\sin^2 x} = 0$

$$\sin^2 x = \frac{1}{1+\operatorname{ctg}^2 x}$$

$$2\operatorname{ctg}^3 x + \frac{1}{\frac{1}{1+\operatorname{ctg}^2 x}} = 0$$

$$2\operatorname{ctg}^3 x + 1 + \operatorname{ctg}^2 x = 0$$

$$\operatorname{ctg} x = u$$

$$2u^3 + u^2 + 1 = 0$$

2	1	0	1	(-1)
↓	-2	1	-1	
2	-1	1	0	

$$(u+1)(2u^2 - u + 1) = 0$$

$\Delta < 0$

$$u + 1 = 0$$

$$u = -1$$

$$\operatorname{ctg} x = -1$$

$$\operatorname{ctg} x = -\operatorname{ctg}\frac{\pi}{4}$$

$$\operatorname{ctg} x = \operatorname{ctg}\left(-\frac{\pi}{4}\right)$$

$$x = \pi k - \frac{\pi}{4}$$

9. ① $\frac{7x^2-x-6}{9-x^2} \le 0$

① $\left(-\frac{6}{7}\right)$ ③ (-3)

$\Delta = 169$

$x = \frac{1 \pm 13}{14}$

x		-3		$-\frac{6}{7}$		1		3	
$9-x^2$	$-$	$\|$	$+$	$+$	$+$	$+$	$+$	$\|$	$-$
$7x^2-x-6$	$+$	$\|$	$+$	0	$-$	0	$+$	$\|$	$+$
$P(x)$	$-$	$\|$	$+$	$-$	$+$	$+$	$+$	$\|$	$-$

$x \in (-\infty, -3) \cup \left[-\frac{6}{7}, 1\right] \cup (3, +\infty)$

10. ② $\frac{x^2-2}{x^2-1} < \frac{2}{3}$

$\frac{x^2-2}{x^2-1} - \frac{2}{3} < 0 \Rightarrow \frac{3(x^2-2) - 2(x^2-1)}{3(x^2-1)} < 0$

$\frac{x^2-4}{3(x^2-1)} < 0$

x		-2		-1		1		2	
x^2-4	$+$	0	$-$	$\|$	$-$	$\|$	$-$	0	$+$
x^2-1	$+$	$\|$	$+$	0	$-$	0	$+$	$\|$	$+$
$P(x)$	$+$	$\|$	$-$	$\|$	$+$	$\|$	$-$	$\|$	$+$

$x \in [-2, -1) \cup (1, 2]$.

6. ⓔ $\frac{3x+2}{2x+1} \leq 0$

x		$-\frac{2}{3}$		$-\frac{1}{2}$	
$3x+2$	$-$	0	$+$		$+$
$2x+1$	$-$		$-$	0	$+$
$P(x)$	$+$		$-$		$+$

• $3x+2=0$

$3x=-2$

$x=-\frac{2}{3}$

• $2x+1=0$

$x=-\frac{1}{2}$

$x \in [-\frac{2}{3}, -\frac{1}{2})$

ⓕ $\frac{x}{3-x} \leq 0$

x		0		3	
x	$-$	0	$+$		$+$
$3-x$	$+$		$+$	0	$-$
$P(x)$	$-$	0	$+$		$-$

$x \in (-\infty, 0] \cup (3, +\infty)$

7. (B) $\frac{x+2}{3x+2} \leq 1$

$$\frac{x+2}{3x+2} - 1 \leq 0$$

$$\frac{x+2}{3x+2} - \frac{3x+2}{3x+2} \leq 0$$

$$\frac{-2x}{3x+2} \leq 0$$

x		$-\frac{2}{3}$		0
$-2x$	$+$	$+$	ϕ	$-$
$3x+2$	$-$	$\|$	$+$	$+$
$P(x)$	$-$	$\|$	$+$	$-$

$x \in (-\infty, -\frac{2}{3}) \cup [0, +\infty)$.

(8) $\frac{x-1}{x} < \frac{3}{4}$

$$\frac{x-1}{x} - \frac{3}{4} < 0$$

$$\frac{4(x-1)}{4x} - \frac{3x}{4x} < 0$$

$$\Rightarrow \frac{4x-4-3x}{4x} < 0$$

$$\frac{x-4}{4x} < 0$$

x		0		4
$x-4$	$-$	$-$	ϕ	$+$
$4x$	$-$	ϕ	$+$	$+$
$P(x)$	$+$	$\|$	$-$	$+$

$x \in (0, 4)$.

17. ⓐ $\dfrac{5x^3+1}{x^2-1} + \dfrac{3}{1-x} > 1$

$$\frac{5x^3+1}{(x-1)(x+1)} - \frac{3}{x-1} - 1 > 0$$

$$\frac{5x^3+1}{(x-1)(x+1)} - \frac{3(x+1)}{(x-1)(x+1)} - \frac{(x-1)(x+1)}{(x-1)(x+1)} > 0$$

$$\frac{5x^3+1-3x-3-(x^2-1)}{x^2-1} > 0$$

$$\frac{5x^3-x^2-3x-1}{x^2-1} > 0$$

$$\frac{(x-1)(5x^2+4x+1)}{(x-1)(x+1)} > 0$$

5	−1	−3	−1	①
↓	5	4	1	
5	4	1	0	

$$\frac{5x^2+4x+1}{x+1} > 0 \quad (+)$$

$x+1 > 0$

$x > -1$

Εποχαο Μαθημα

19

(7) α γ ε

(8) α β γ

(9) α γ.

(21)

(22)

(23).

20

(1) α

(2) α γ

(6) α β

(7) α δ

(9) α β

(10) α β

(17) β