

# κλασματική Εξίσωση

Εξοτ. 10

$$2 - \frac{x^2 + 7x}{x^2 - 1} = \frac{2x - 1}{x + 1} + \frac{3}{1 - x}$$

$$2 - \frac{x^2 + 7x}{(x - 1)(x + 1)} = \frac{2x - 1}{x + 1} - \frac{3}{x - 1}$$

πρσν  $(x - 1)(x + 1) \neq 0$

$x \neq 1$

$x \neq -1$

$x + 1 \neq 0$

$\Rightarrow x \neq -1$

$x - 1 \neq 0$

$\Rightarrow x \neq 1$

$\in \mathbb{M} \exists (x - 1)(x + 1)$

$$2(x - 1)(x + 1) - (x^2 + 7x) = (x - 1)(2x - 1) - 3(x + 1)$$

$$2(x^2 - 1) - x^2 - 7x = 2x^2 - x - 2x + 1 - 3x - 3$$

$$\cancel{2x^2} - 2 - x^2 - 7x = \cancel{2x^2} - 3x - 3x - 2$$

$$-x^2 - 7x - \cancel{2} = -6x - \cancel{2}$$

$$-x^2 - x = 0$$

$$-x(x+1) = 0$$

$$-x = 0$$

$$x = 0$$

$$x+1 = 0$$

$$x = -1$$

24. (B)  $(3x-2)^2 = (3x-2)(x-1)$

$$(3x-2)^2 - (3x-2)(x-1) = 0$$

$$(3x-2)(3x-2-x+1) = 0$$

$$(3x-2)(2x-1) = 0$$

$$3x-2=0$$

$$\vee 2x-1=0$$

$$x = \frac{2}{3}$$

$$x = \frac{1}{2}$$

(8)  $x(x-2) - (x+1)(2-x) = 0$

$$x(x-2) + (x+1)(x-2) = 0$$

$$(x-2)(x+(x+1)) = 0$$

$$x-2 = 0$$

$$\vee x+(x+1) = 0$$

$$x = 2$$

$$2x+1 = 0$$

$$x = -\frac{1}{2}$$

$$(52) \quad (x+1)^2 + x^2 - 1 = 0$$

$$(x+1)^2 + (x-1)(x+1) = 0$$

$$(x+1) (x+1 + x-1) = 0$$

$$x+1=0 \quad \vee \quad 2x=0$$

$$\underline{\underline{x=-1}}$$

$$\underline{\underline{x=0}}$$

$$(5) \quad x^3 - x^2 - x + 1 = 0$$

$$x^2(x-1) - (x-1) = 0$$

$$(x-1)(x^2-1) = 0$$

$$x-1=0$$

$\vee$

$$x^2-1=0$$

$$x^2 = 1^2$$

$$\underline{\underline{x=1}}$$

$$\underline{\underline{x=1}}$$

$$\underline{\underline{x=-1}}$$

$$(i) \quad x^3 - 7x + 6 = 0$$

$$x^3 - x - 6x + 6 = 0$$

$$x(x^2 - 1) - 6(x - 1) = 0$$

$$x(x-1)(x+1) - 6(x-1) = 0$$

$$(x-1)(x(x+1) - 6) = 0$$

$$x-1=0$$

$$\vee \quad x^2 + x - 6 = 0$$

$$x = 1$$

$$x = -3$$

$$x = 2$$

# Ενότητα 11

3. α)  $|-x| + |3x| - 4\left|\frac{x}{2}\right| = 2$

$$|x| + 3|x| - 4 \cdot \frac{1}{2}|x| = 2$$

$$4|x| - 2|x| = 2$$

$$2|x| = 2$$

$$|x| = 1$$

$$= x = 1 \quad x = -1$$

β)  $\left|\frac{2x}{3}\right| - |-2x| = -1$

$$\frac{2}{3}|x| - |2x| = -1$$

$$\frac{2}{3}|x| - 2|x| = -1$$

$$2|x| - 6|x| = -3$$

$$-4|x| = -3$$

$$|x| = \frac{3}{4}$$

$$x = \frac{3}{4}$$

$$x = -\frac{3}{4}$$

$$\frac{\text{Εξίσωση}}{|2x|}$$

$$|2||x|$$

$$2|x|$$

$$\textcircled{7} \quad |2x-1| + 2|1-2x| = |4x-2| + 1$$

$$|2x-1| + 2|2x-1| = 2|2x-1| + 1$$

$$3|2x-1| - 2|2x-1| = 1$$

$$|2x-1| = 1$$

$$2x-1 = 1$$

u

$$2x-1 = -1$$

$$2x = 2$$

$$2x = 0$$

$$\textcircled{x=1}$$

$$\textcircled{x=0}$$

$$\textcircled{8} \quad |6x-2| - |3-9x| = -1$$

$$2|3x-1| - 3|1-3x| = -1$$

$$2|3x-1| - 3|3x-1| = -1$$

$$-|3x-1| = -1$$

$$|3x-1| = 1$$

$$3x-1 = 1$$

u

$$3x-1 = -1$$

$$3x = 2$$

$$3x = 0$$

$$\textcircled{x = \frac{2}{3}}$$

$$\textcircled{x=0}$$

$$\textcircled{E} \quad 1 + \frac{|2x-3|-3}{2} = \frac{|6x-9|-3}{6}$$

$$6 + 3(|2x-3|-3) = |6x-9|-3$$

$$6 + 3\cancel{|2x-3|} - 9 = 3\cancel{|2x-3|} - 3$$

$$-3 = -3$$

Answer  $x \in \mathbb{R}.$

$$\textcircled{52} \quad \frac{|3x-6|}{4} - \left| 1 - \frac{x}{2} \right| = 1$$

$$\frac{3|x-2|}{4} - \left| \frac{2-x}{2} \right| = 1$$

$$\frac{3|x-2|}{4} - \frac{|2-x|}{2} = 1$$

$$\frac{3|x-2|}{4} - \frac{|x-2|}{2} = 1$$

$$3|x-2| - 2|x-2| = 4$$

$$|x-2| = 4$$

$$x-2=4$$

$$x-2=-4$$

$$\textcircled{x=6}$$

$$\textcircled{x=-2}$$

2. (a)  $|x-1|-2=0$

$$|x-1|=2$$

$$x-1=2$$

$$x=3$$

or

$$x-1=-2$$

$$x=-1$$

(b)  $|5x-1|=4=0$

$$|5x-1|=4$$

$$5x-1=4$$

$$5x=5$$

$$x=1$$

or

$$5x-1=-4$$

$$5x=-3$$

$$x=-\frac{3}{5}$$

(c)  $3-2|3x-1|=0$

$$3-2|3x-1|=0$$

$$3=2|3x-1|$$

$$|3x-1|=\frac{3}{2}$$

$$3x-1=\frac{3}{2}$$

$$3x=\frac{5}{2}$$

or

$$x=\frac{5}{6}$$

$$3x-1=-\frac{3}{2}$$

$$3x=-\frac{1}{2}$$

$$x=-\frac{1}{6}$$

$$\textcircled{d} \quad 2(|x-3|-1) = 3|x-3|-2$$

$$2|x-3| - \cancel{2} = 3|x-3| - \cancel{2}$$

$$2|x-3| - 3|x-3| = 0$$

$$-|x-3| = 0$$

$$\textcircled{x=3}$$

$$\textcircled{e} \quad \frac{|x-2|-1}{2} = -1$$

$$|x-2|-1 = -2$$

$$|x-2| = -1$$

Answer.

$$(2) \quad \frac{|x-1|}{2} - \frac{|x-1|-1}{4} = 1$$

$$2|x-1| - (|x-1|-1) = 4$$

$$2|x-1| - |x-1| + 1 = 4$$

$$|x-1| = 3$$

$$x-1=3$$

$$(x=4)$$

or

$$x-1=-3$$

$$(x=-2)$$

$$4. \textcircled{B} |2x-3| = |3x-2|$$

$$2x-3 = 3x-2$$

$$\vee 2x-3 = -3x+2$$

$$-x = 1$$

$$\textcircled{x = -1}$$

$$5x = 5$$

$$\textcircled{x = 1}$$

$$\textcircled{8} |x-4| - 3|x| = 0$$

$$|x-4| = 3|x|$$

$$|x-4| = |3x|$$

$$x-4 = 3x$$

$\vee$

$$x-4 = -3x$$

$$-2x = 4$$

$$\textcircled{x = -2}$$

$$4x = 4$$

$$\textcircled{x = 1}$$

$$\textcircled{52} \frac{|2x-1|}{3} = \frac{|x-2|}{2}$$

$$2|2x-1| = 3|x-2|$$

$$2(2x-1) = 3(x-2)$$

$$\vee 2(2x-1) = -3(x-2)$$

$$\textcircled{x = -4}$$

$$7x = 8$$

$$\textcircled{x = \frac{8}{7}}$$

5.

(3)

$$\sqrt{x^2 - 6x + 9} = 1$$

$$\sqrt{(x-3)^2} = 1$$

$$|x-3| = 1$$

$$x-3 = 1$$

$$x = 4$$

or

$$x-3 = -1$$

$$x = 2$$

$$(8) \sqrt{25x^2 - 10x + 1} = |3x - 5|$$

$$\sqrt{(5x-1)^2} = |3x-5|$$

$$|5x-1| = |3x-5|$$

$$5x-1 = 3x-5$$

$$2x = -4$$

$$x = -2$$

$$5x-1 = -3x+5$$

$$8x = 6$$

$$x = \frac{6}{8}$$

$$(52) \sqrt{4x^2+1} = |2x-1|$$

$$\sqrt{4x^2+1}^2 = |2x-1|^2$$

$$4x^2+1 = (2x-1)^2$$

$$\cancel{4x^2+1} = \cancel{4x^2-4x+1}$$

$$0 = -4x$$

$$x = 0$$

# ΕΥΟΤΥΤΑ 12

11. (a)  $|3x-1|^3 - 8 = 0$

$$|3x-1|^3 = 8$$

$$|3x-1|^3 = 2^3$$

$$|3x-1| = 2$$

$$3x-1 = 2$$

$$3x = 3$$

$$x = 1$$

ή

$$3x-1 = -2$$

$$3x = -1$$

$$x = -\frac{1}{3}$$

(b)  $(x-2)^6 - 32 d(x, 2) = 0$

$$(x-2)^6 - 32 |x-2| = 0$$

$$|x-2|^6 - 32 |x-2| = 0$$

$$|x-2| (|x-2|^5 - 32) = 0$$

$$|x-2| = 0 \quad \text{ή} \quad |x-2|^5 - 32 = 0$$

$$x = 2$$

$$|x-2|^5 = 2^5$$

$$|x-2| = 2$$

$$x = 4$$

$$x-2 = 2$$

ή

$$x-2 = -2$$

$$x = 0$$

$$\textcircled{8} \left( |x-1| - 2 \right)^5 - 1 = 0$$

$$\left( |x-1| - 2 \right)^5 = 1^5$$

$$|x-1| - 2 = 1$$

$$|x-1| = 3$$

$$x-1 = 3$$

$$\textcircled{x=4}$$

↯

$$x-1 = -3$$

$$\textcircled{x=-2}$$

$$\textcircled{9} \left( |x-3| - 1 \right)^4 - 81 = 0$$

$$\left( |x-3| - 1 \right)^4 = 3^4$$

$$|x-3| - 1 = 3$$

$$\text{↯} \quad |x-3| - 1 = -3$$

$$|x-3| = 4$$

$$|x-3| = -2$$

Answer!

$$x-3 = 4 \quad \text{↯} \quad x-3 = -4$$

$$\textcircled{x=7}$$

$$\textcircled{x=-1}$$

1 2. (a)  $8x^3 - 1 = 0$

$$8x^3 = 1$$

$$(2x)^3 = 1^3$$

$$2x = 1$$

$$x = \frac{1}{2}$$

(b)  $64x^6 - 1 = 0$

$$64x^6 = 1$$

$$(2x)^6 = 1^6$$

$$2x = 1 \quad \vee \quad 2x = -1$$

$$x = \frac{1}{2}$$

$$x = -\frac{1}{2}$$

(c)  $27x^3 + 1 = 0$

$$27x^3 = -1$$

$$(3x)^3 = (-1)^3$$

$$3x = -1$$

$$x = -\frac{1}{3}$$

$$\textcircled{8} \quad 4x^2 - 9 = 0$$

$$4x^2 = 9$$

$$(2x)^2 = 3^2$$

$$2x = 3$$

$$\vee \quad 2x = -3$$

$$x = \frac{3}{2}$$

$$x = -\frac{3}{2}$$

$$\textcircled{9} \quad 32x^5 - 243 = 0$$

$$32x^5 = 243$$

$$(2x)^5 = (3)^5$$

$$2x = 3$$

$$x = \frac{3}{2}$$

$$\textcircled{10} \quad 16x^4 - 625 = 0$$

$$16x^4 = 625$$

$$(2x)^4 = 5^4$$

$$2x = 5$$

$$x = \frac{5}{2}$$

$$2x = -5$$

$$x = -\frac{5}{2}$$

# Εποραιο Μαθημα

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$$\frac{11}{\text{1024}}$$

$$\frac{10}{11, 12, 24}$$

$$\frac{12}{1}$$