27. ⓑ $f(x) = \ln\left(\sqrt{x^2+1} - x\right)$ $x \in \mathbb{R}$

$$f'(x) = \frac{\frac{2x}{2\sqrt{x^2+1}} - 1}{\left(\sqrt{x^2+1} - x\right)\sqrt{x^2+1}}$$

$$f'(x) = \frac{x - \sqrt{x^2+1}}{\left(\sqrt{x^2+1} - x\right)\sqrt{x^2+1}} \quad (-) \quad < 0$$

f ↓

27. (B) $f(x) = \ln \frac{x}{1-x}$

$x \in (0,1)$

x	0		1
x	$-0+$	$+$	$+$
$1-x$	$+$	$+0-$	
$\frac{x}{1-x}$	$-$	$+$	$-$

$$f'(x) = \frac{1}{\frac{x}{1-x}} \left(\frac{x}{1-x}\right)'$$

$$f'(x) = \frac{1-x}{x} \cdot \frac{1-x-x}{(1-x)^2}$$

$$\boxed{f'(x) = \frac{1-2x}{x(1-x)}}$$

$f'(x) = 0 \Rightarrow 1-2x=0 \Rightarrow x = \frac{1}{2}$

x	0		$1/2$		1
f'	$+$		0	$-$	
f	↗		max	↘	

27. (a) $f(x) = \frac{1}{x^2} - \frac{1}{x-4}$

$x \neq 4$
$x \neq 0$

$$f'(x) = \frac{-2x}{x^4} - \frac{-1}{(x-4)^2}$$

$$f'(x) = -\frac{2}{x^3} + \frac{1}{(x-4)^2}$$

$$f'(x) = \frac{x^3 - 2(x-4)^2}{x^3(x-4)^2}$$

$$f'(x) = \frac{x^3 - 2(x^2 - 8x + 16)}{x^3(x-4)^2}$$

$$f'(x) = \frac{x^3 - 2x^2 + 16x - 32}{x^3(x-4)^2}$$

$$f'(x) = \frac{(x-2)(x^2+16)}{x^3(x-4)^2}.$$

x		0	2	4	
$x-2$	$-$	$-$	0	$+$	$+$
x^3	$-$	0	$+$	$+$	$+$
f'	$+$	$-$	$+$	$+$	
f	↗	↘	↗	↗	

26. ⑧ $f(x) = \dfrac{x}{\sqrt{x^2+1}}$

$$f'(x) = \frac{\sqrt{x^2+1}' - x \cdot \frac{2x}{2\sqrt{x^2+1}}}{x^2+1}$$

$$f'(x) = \frac{\sqrt{x^2+1} - \frac{x^2}{\sqrt{x^2+1}}}{x^2+1}$$

$$f'(x) = \frac{x^2+1-x^2}{(x^2+1)\sqrt{x^2+1}} = \frac{1}{(x^2+1)\sqrt{x^2+1}}$$

$f'(x) > 0$

f

26. (B) $f(x) = x\sqrt{1-x^2}$ $\quad A_f = [-1,1]$

$$f'(x) = \sqrt{1-x^2} + x\frac{-2x}{2\sqrt{1-x^2}}$$

$$f'(x) = \sqrt{1-x^2} - \frac{x^2}{\sqrt{1-x^2}}$$

$$f'(x) = \frac{\sqrt{1-x^2}^2 - x^2}{\sqrt{1-x^2}} = \frac{1-2x^2}{\sqrt{1-x^2}}$$

$f'(x) = 0 \Rightarrow 1-2x^2 = 0$

$2x^2 = 1$

$x = \pm\frac{\sqrt{2}}{2}$

x	-1	$-\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$	1
f'	$-$	0	$+$ 0	$-$
f	$\searrow$		$\nearrow$	$\searrow$

Ενότητα 23

26. (α) $f(x) = x + \sqrt{x^2+1}$, $x \in \mathbb{R}$

$$f'(x) = 1 + \frac{2x}{2\sqrt{x^2+1}}$$

$$f'(x) = 1 + \frac{x}{\sqrt{x^2+1}}$$

$$f'(x) = \frac{\sqrt{x^2+1} + x}{\sqrt{x^2+1}} > 0$$

$f \nearrow$

$\longrightarrow f'(x) = 0 \Rightarrow \sqrt{x^2+1} + x = 0$

$$\sqrt{x^2+1} = -x$$

$x > 0$

Αδύνατον

$x \leq 0$

$\sqrt{x^2+1}^2 = (-x)^2$

$x^2 + 1 = x^2$

$1 = 0$

Άτοπο

$\longrightarrow \sqrt{x^2+1} + x \neq 0$

28. $f(x) = \begin{cases} \pi + x, & -\pi < x \leq 0 \\ x^2 \ln x, & x > 0 \end{cases}$

$$\lim_{x \to 0^-} f(x) = \lim_{x \to 0^-} \pi + x = 0$$

$$\lim_{x \to 0^+} f(x) = \lim_{x \to 0^+} x^2 \ln x = \lim_{x \to 0^+} \frac{\ln x}{\frac{1}{x^2}} =$$

$$= \lim_{x \to 0^+} \frac{\frac{1}{x}}{\frac{-2x}{x^4}} = \lim_{x \to 0^+} -\frac{x^4}{-2x^2} =$$

$$= \lim_{x \to 0^+} \frac{x^2}{2} = 0 .$$

Очов $\lim_{x \to 0^-} f(x) = \lim_{x \to 0^+} f(x) = f(0)$

Функ е нр 0!

$$\lim_{x\to 0^-} \frac{f(x)-f(0)}{x-0} = \lim_{x\to 0^-} \frac{\sin x - 0}{x-0} = 1$$

$$\lim_{x\to 0^+} \frac{f(x)-f(0)}{x-0} = \lim_{x\to 0^+} \frac{x^2 \ln x}{x} =$$

$$= \lim_{x\to 0^+} x \ln x = \lim_{x\to 0^+} \frac{\ln x}{\frac{1}{x}} =$$

$$= \lim_{x\to 0^+} \frac{\frac{1}{x}}{-\frac{1}{x^2}} = \lim_{x\to 0^+} -\frac{x^2}{x} = 0$$

Ідж сіқаи пар/ма ғоб 0.

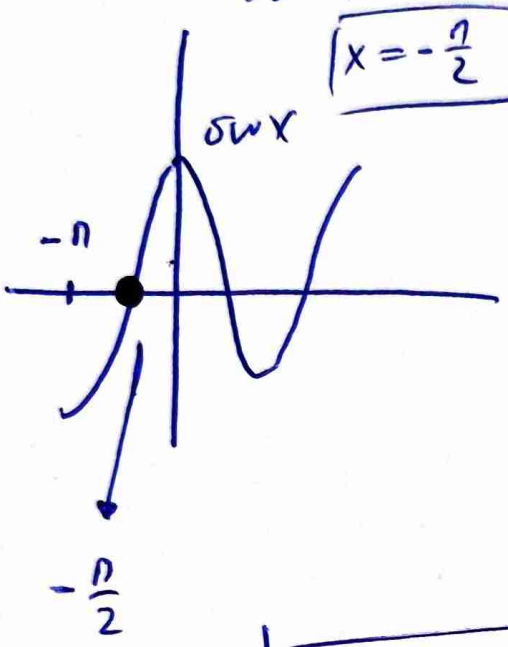
$-\pi < x \leq 0$

$f_1(x) = \sin x$

$f_1'(x) = \cos x$

$\longrightarrow f_1'(x) = 0$

$\boxed{x = -\frac{\pi}{2}}$



$x > 0$

$f_2(x) = x^2 \ln x$

$f_2'(x) = 2x \ln x + x$

$f_2'(x) = x(2\ln x + 1)$

$\longrightarrow 2\ln x + 1 = 0$

$\ln x = -\frac{1}{2}$

$x = e^{-\frac{1}{2}} = \frac{1}{\sqrt{e}} = \frac{\sqrt{e}}{e}$

x	$-\pi$	$-\frac{\pi}{2}$	0	$\sqrt{e}/e$
f_1'	$-$	0 $+$	//	//
f_2'	//	//	$-$	$+$
f'	$-$	$+$	$-$	$+$
f	$\searrow$	$\nearrow$	$\searrow$	$\nearrow$

29. ⓐ $f(x) = \begin{cases} \dfrac{xe^x}{e^x-1}, & x \neq 0 \\ 1, & x = 0 \end{cases}$

$$f'(x) = \frac{(xe^x)'(e^x-1) - xe^x(e^x-1)'}{(e^x-1)^2}$$

$$f'(x) = \frac{(e^x+xe^x)(e^x-1) - e^x xe^x}{(e^x-1)^2}$$

$$f'(x) = \frac{e^{2x} - e^x + xe^{2x} - e^x - xe^{2x}}{(e^x-1)^2}$$

$$f'(x) = \frac{e^{2x} - 2e^x}{(e^x-1)^2} = \frac{e^x(e^x-2)}{(e^x-1)^2}$$

x		0		$\ln 2$	
f'	$-$	‖	$-$	0	$+$
f	$\searrow$	‖	$\searrow$	min	$\nearrow$

29. (B) $f(x) = \begin{cases} \frac{x \ln x}{x-1}, & x \in (0,1) \cup (1,+\infty) \\ 1, & x = 1 \end{cases}$

$$f'(x) = \frac{(x \ln x)'(x-1) - x \ln x}{(x-1)^2}$$

$$f'(x) = \frac{(\ln x + 1)(x-1) - x \ln x}{(x-1)^2}$$

$$f'(x) = \frac{x \ln x - \ln x + x - 1 - x \ln x}{(x-1)^2}$$

$$f'(x) = \frac{x - \ln x - 1}{(x-1)^2} > 0 \quad f \nearrow$$

• $\ln x < x - 1 \Rightarrow x - \ln x - 1 > 0$

Έκδοση 24

9. (α) $e^{x^2} + \ln(x^2+1) = 1$

$f(x) = e^x + \ln(x+1)$

$x > -1$

$f'(x) = e^x + \frac{1}{x+1} > 0$

$f \nearrow$

$f(x^2) = f(0)$

$f \overset{1-1}{=} -1$

$x^2 = 0$

$x = 0$

(β) Αντίστοιχα $\ln\frac{x^2+1}{x+1} > e^x - e^{x^2}$

x	0	1
x^2-x	$+$	$-$ $+$

$\ln(x^2+1) - \ln(x+1) > e^x - e^{x^2}$

$x \in (-\infty, 0) \cup (1, +\infty)$

$e^{x^2} + \ln(x^2+1) > e^x + \ln(x+1)$

$x \in (-1, 1)$

$f(x^2) > f(x) \longrightarrow x^2 > x$

$x^2 - x > 0$

(8). $e^{x^2-1} + \ln x = e^{x-1}$

$e^{x^2-1} + 2\ln x = e^{x-1} + \ln x$

$e^{x^2-1} + \ln x^2 = e^{x-1} + \ln x$

$f(x^2-1) = f(x-1)$

$f(\square) = f(\square)$

$x^2-1 = x-1$

$x^2 = x$

~~$x=0$~~ $\quad x=1$

$x^2-1 > -1$ ✓

при $x-1 > -1$

$x > 0$

13. $f(x) = \begin{cases} x\ln x + a, & x > 0 \\ \sin x + \ln(1+a^2), & x \le 0 \end{cases}$

Συνεχώς στο 0.

$$\lim_{x\to 0^+} f(x) = \lim_{x\to 0^+} (x\ln x + a) = a$$

$$\longrightarrow \lim_{x\to 0^+} x\ln x = \lim_{x\to 0^+} \frac{\ln x}{\frac{1}{x}} = \lim_{x\to 0^+} \frac{\frac{1}{x}}{-\frac{1}{x^2}}$$

$$= \lim_{x\to 0^+} -\frac{x^2}{x} = 0$$

$$\lim_{x\to 0^-} f(x) = \lim_{x\to 0^-} \sin x + \ln(1+a^2) =$$

$$\ln(1+a^2)$$

$$\ln(1+a^2) = a.$$

$$g(a) = 0$$

$$g(a) = g(0)$$

$$g \text{ 1-1}$$

$$a = 0.$$

$$g(x) = \ln(1+x^2) - x$$

$$g'(x) = \frac{2x}{x^2+1} - 1$$

$$g'(x) = \frac{2x - x^2 - 1}{x^2+1}$$

$$g'(x) = -\frac{x^2 - 2x + 1}{x^2+1}$$

$$g'(x) = -\frac{(x-1)^2}{x^2+1} \leq 0$$

$g \searrow$

41. $f(x) = x^3 - 6\ln x$

$y - f(x_0) = f'(x_0)(x - x_0) \longrightarrow A(0;4)$

$4 - f(x) = f'(x)(0 - x)$

$4 - (x^3 - 6\ln x) = \left(3x^2 - \frac{6}{x}\right)(-x)$

$4 - x^3 + 6\ln x = -3x^3 + 6$

$2x^3 + 6\ln x - 2 = 0$

$\boxed{x^3 + 3\ln x - 1 = 0}$

проціанрівн

$x = 1$

$g(x) = x^3 + 3\ln x - 1$

$g'(x) = 3x^2 + \frac{3}{x} > 0$

g ↗

$y - f(1) = f'(1)(x - 1)$

42. $f(x) = \eta\rho x \qquad x \in [0, \pi]$

$y - f(x_0) = f'(x_0)(x - x_0) \longrightarrow A\left(\frac{\pi}{2}, \frac{\pi}{2}\right)$

$\frac{\pi}{2} - \eta\rho x = \sigma\omega x\left(\frac{\pi}{2} - x\right)$

$\pi - 2\eta\rho x = \sigma\omega x\ (\pi - 2x)$

$[0, \pi]$

$\pi - 2\eta\rho x - \sigma\omega x\,(\pi - 2x) = 0.$

$g(x)$

$g(0) = \pi - \pi = 0$

$g(\pi) = \pi - (-1)(-\pi) = \pi - \pi = 0$

Το $x=0$ και $x=\pi$

Εχω ρίζα διπλή;

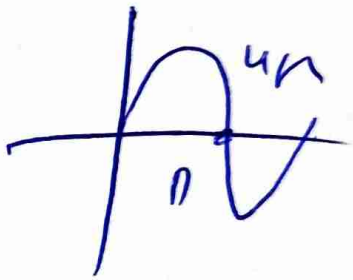
$$g(x) = \pi - 2\eta\mu x - \sigma\upsilon\nu x(\pi - 2x)$$

$$g'(x) = -2\sigma\upsilon\nu x - \left[(-\eta\mu x)(\pi - 2x) + \sigma\upsilon\nu x\,(-2)\right]$$

$$g'(x) = -2\sigma\upsilon\nu x - \left[-\pi\eta\mu x + 2x\eta\mu x - 2\sigma\upsilon\nu x\right]$$

$$g'(x) = -2\sigma\upsilon\nu x + \pi\eta\mu x - 2x\eta\mu x + 2\sigma\upsilon\nu x$$

$$g'(x) = \overset{\oplus}{\eta\mu x}\,(\pi - 2x).$$



x	0	$\frac{\pi}{2}$	π
g'	$+$	0	$-$
g	↗		↘

Άρα η $f(x)$ έχει δύο διαστήματα μονοτονίας εκ των οποίων 2 ίσως αρά έχει δύο.