Επιλογή 21

1. (Β) $\sqrt{x^2-9}=4$ πρεπει $x^2-9\geq 0$

$x^2-9=16$

$x^2=25$

$x=5$ ✓ $x=-5$ ✓

2. (Δ) $\sqrt{25-x^2}=\sqrt{2x+1}$

$2x+1\geq 0$

$25-x^2\geq 0$

$25-x^2=2x+1$

$-x^2-2x+24=0$

$\Delta=4+4\cdot 24=100$

$x=\frac{2\pm 10}{-2}$ → -6 ✗, 4 ✓

3. (B) $x+\sqrt{x}-2=0$

$x \geq 0$

$2-x \geq 0$

$\sqrt{x} = 2-x$

$x = (2-x)^2$

$x = 4-4x+x^2$

$0 = x^2-5x+4$

$x=1$ ✓ $\quad$ ~~$x=4$~~

(E) $\sqrt{x-1} = x-3$

$x-3 \geq 0$

$x-1 \geq 0$

$x-1 = (x-3)^2$

$x-1 = x^2-6x+9$

$x^2-7x+10 = 0$

~~$x=2$~~ $\quad$ $x=5$ ✓

4. ① $\sqrt{1-x} + x = 2x+5$

$1-x \geq 0$

$x+5 \geq 0$

$$\sqrt{1-x} = x+5$$

$$1-x = (x+5)^2$$

$$1-x = x^2+10x+25$$

$$x^2+11x+24 = 0$$

$\Delta = 25$

$$x = \frac{-11 \pm 5}{2}$$

-3 ✓

-8 ✗

8. Ⓑ $\sqrt{x+3} + \sqrt{3x-2} = 7$

$x+3 \geq 0$

$3x-2 \geq 0$

$$\left(\sqrt{x+3} + \sqrt{3x-2}\right)^2 = 7^2$$

$$x+3 + 2\sqrt{x+3}\sqrt{3x-2} + 3x-2 = 49$$

$$4x+1+2\sqrt{x+3}\sqrt{3x-2} = 49$$

$$2\sqrt{x+3}\sqrt{3x-2} = 48-4x$$

52 $\sqrt{x-1} = \sqrt{x+2} - \sqrt{3x-5}$

$\sqrt{x-1} + \sqrt{3x-5} = \sqrt{x+2}$

$x-1 \geq 0$

$x+2 \geq 0$

$3x-5 \geq 0$

$x-1 + 2\sqrt{x-1}\sqrt{3x-5} + 3x-5 = x+2$

$\sqrt{x+2} - \sqrt{3x-5} \geq 0$

$3x + 2\sqrt{x-1}\sqrt{3x-5} = 8$

$11 - 3x \geq 0$

$2\sqrt{x-1}\sqrt{3x-5} = 8 - 3x$

$4(x-1)(3x-5) = (8-3x)^2$

$4(3x^2 - 5x - 3x + 5) = 64 - 48x + 9x^2$

$12x^2 - 32x + 20 = 64 - 48x + 9x^2$

$3x^2 + 16x - 44 = 0$

$x = \frac{-16 \pm 28}{6}$

2

$-\frac{44}{6}$

$$\sqrt{x+3}\ \sqrt{3x-2} = 24-2x$$

$24-2x \geq 0$

$$(x+3)(3x-2) = (24-2x)^2$$

$$3x^2 - 2x + 9x - 6 = 576 - 96x + 4x^2$$

$$-x^2 + 103x - 582 = 0$$ ✓

$$x = \frac{-103 \pm 91}{-2}$$

6

~~97~~

Ответ

Ενότητα 23

15. (α) $4^x - 32 \cdot 2^x > 0$

$(2^x)^2 - 32 \cdot 2^x > 0$

$2^x = t$

$t^2 - 32t > 0$

$t(t-32) > 0$

↓ ↓

(0) (32)

t		0		32	
$t^2 - 32t$	+		−		+

$t < 0$ ή $t > 32$

$2^x < 0$ — $2^x > 2^5$

Αδύνατη — $x > 5$

21. (01) $\dfrac{e^x-1}{e^x+2} > 0$

e^x+2 (+)

$\Rightarrow e^x - 1 > 0$

$e^x > 1$

$e^x > e^0$

$x > 0$

27. (a) $16^x - 8^x - 2^{x+2} + 4 = 0$

$(2^x)^4 - (2^x)^3 - 2^x \cdot 2^2 + 4 = 0$

$2^x = \lambda$

$\lambda^4 - \lambda^3 - 4\lambda + 4 = 0$

$\lambda^3(\lambda-1) - 4(\lambda-1) = 0$

$(\lambda-1)(\lambda^3-4) = 0$

$\lambda - 1 = 0$; $\lambda^3 - 4 = 0$

$\lambda = 1$; $\lambda^3 = 4$

$2^x = 1$

$2^x = 2^0$

$x = 0$

ii) $(2^x)^3 = 4$

$2^{3x} = 2^2$

$3x = 2$

$x = \frac{2}{3}$

Exercise 20

6. (a) $\frac{x-3}{x+1} > 0$

x		-1		3
x-3	-		-	0 +
x+1	-	0	+	+
~	+	‖	-	+

$x \in (-\infty, -1) \cup (3, +\infty)$.

(b) $\frac{x}{x-1} > 0$

x		0		1	
x	-	0	+		+
x-1	-		-	0	+
P(x)	+	‖	-	‖	+

$x \in (-\infty, 0) \cup (1, +\infty)$.

7. (a) $\frac{5x-2}{2x-1} > 3$

$\frac{5x-2}{2x-1} - 3 > 0$

$\frac{5x-2}{2x-1} - \frac{3(2x-1)}{2x-1} > 0$

$\frac{5x-2-6x+3}{2x-1} > 0$

$\frac{-x+3}{2x-1} > 0$

x		$\frac{1}{2}$		3	
-x+3	+		+	0	-
2x-1	-	0	+		+
P(x)	-	‖	+	‖	-

$x \in (\frac{1}{2}, 3)$.