17. $x^2 - (2\lambda - 1)x - 2\lambda + 1 = 0$

(a) Απαιτω $\Delta \geq 0$

$$(2\lambda-1)^2 - 4(-2\lambda+1) \geq 0$$

$$4\lambda^2 - 4\lambda + 1 + 8\lambda - 4 \geq 0$$

$$4\lambda^2 + 4\lambda - 3 \geq 0$$

$$\Delta = 16 + 48 = 64$$

$$\lambda = \frac{-4 \pm 8}{8}$$

$\frac{1}{2}$

$-\frac{3}{2}$

λ		$-\frac{3}{2}$		$\frac{1}{2}$	
$4\lambda^2+4\lambda-3$	+	|	−	|	+

$\lambda \in (-\infty, -\frac{3}{2}] \cup [\frac{1}{2}, +\infty)$

(β) $x_1^2 + x_2^2 > 3$

$x_1 + x_2 = 2\lambda - 1$

$x_1 x_2 = 1 - 2\lambda$

$$(x_1+x_2)^2 - 2x_1x_2 > 3$$

$$(2\lambda-1)^2 - 2(1-2\lambda) - 3 > 0$$

$$4\lambda^2 - 4\lambda + 1 - 2 + 4\lambda - 3 > 0$$

$$4\lambda^2 - 4 > 0 \Rightarrow \lambda^2 - 1 > 0$$

λ		-1		1	
λ^2-1	+	|	−	|	+

$\lambda \in (-\infty, -1) \cup (1, +\infty)$

30. (α) $(\lambda+1)x^2+(\lambda+1)+2>0$

Αρκεί $\Delta<0$ και $\lambda+1>0$

$\lambda>-1$

$(\lambda+1)^2-4(\lambda+1)\cdot 2<0$

$\lambda^2+2\lambda+1-8(\lambda+1)<0$

$\lambda^2+2\lambda+1-8\lambda-8<0$

$\lambda^2-6\lambda-7<0$

λ		-1		7
$\lambda^2-6\lambda-7$	$+$	$-$	$-$	$+$

$\lambda\in(-1,7)$

(B) $(\lambda-2)x^2 + (\lambda-2)x - 1 < 0$

Απαιτω $\Delta < 0$ και $\lambda - 2 < 0$

$\lambda < 2$

$(\lambda-2)^2 + 4(\lambda-2) < 0$

$\lambda^2 - 4\lambda + 4 + 4\lambda - 8 < 0$

$\lambda^2 - 4 < 0$

λ		-2		2	
λ^2-4	$+$	0	$-$	0	$+$

$\lambda \in (-2, 2)$

15. B1 $(\lambda-1)x^2-(\lambda-1)x+1=0$

$\lambda \neq 1$

$\Delta = (\lambda-1)^2 - 4(\lambda-1)\lambda$

$\Delta = \lambda^2-2\lambda+1-4\lambda(\lambda-1)$

$\Delta = \lambda^2-2\lambda+1-4\lambda^2+4\lambda$

$\Delta = -3\lambda^2+2\lambda+1$

$\Delta^* = 4+12=16$

$\lambda = \frac{-2\pm 4}{-6}$ → $-\frac{1}{3}$, 1

λ		$-\frac{1}{3}$		1
Δ	$-$		$+$	$-$

Av $\lambda=-\frac{1}{3}$ 1 pilla

Av $\lambda \in (-\infty, -\frac{1}{3}) \cup (1, +\infty)$ Tozc $\Delta<0$ 0 pilla

Av $\lambda \in (-\frac{1}{3}, 1)$ Tozc $\Delta>0$ 2 pilla

18. B) Nšo $y^2 - 2y + 2 > 0$ ✓ (+)

$\Delta = 4 - 8 = -4 < 0$

δ) $3\lambda - 2 < 2\lambda^2$

$-2\lambda^2 + 3\lambda - 2 < 0$ ✓ (−)

$\Delta = 9 - 4(-2)(-2)$

$\Delta = 9 - 16 < 0$

13. $x^2 - 3x + 2 < 0$

(a)

x		1		2	
x^2-3x+2	+		−		+

$x \in (1, 2)$

(B) $\cancel{\frac{5}{2}}$ $\cancel{\frac{1}{123}}$ $\cancel{\sqrt{5}}$ ✓

(j) $k = 789^2 - 3 \cdot 789 + 2 > 0$

(δ) Av $\lambda \in (2, 3)$

$(\lambda - 1)^2 - 3|\lambda - 1| + 2$

$|\overset{+}{\lambda - 1}|^2 - 3|\overset{+}{\lambda - 1}| + 2$

$-$

$2 < \lambda < 3$

$1 < \lambda - 1 < 2$

32. (α) $x^2 - (\lambda+2)x + \lambda^2 + 2$

θετικο.

$$\Delta = (\lambda+2)^2 - 4(\lambda^2+2) = \lambda^2 + 4\lambda + 4 - 4\lambda^2 - 8$$

$$\boxed{\Delta = -3\lambda^2 + 4\lambda - 4 < 0}$$

$$\Delta = 16 - 4(-3)(-4)$$

$$\Delta = 16 - 48 < 0$$

(+)

(β) i) $x^2 - (2\lambda-1)x + 2\lambda^2 + 1 > 0$

$$\Delta = (2\lambda-1)^2 - 4(2\lambda^2+1)$$

$$\Delta = 4\lambda^2 - 4\lambda + 1 - 8\lambda^2 - 4$$

(−)

$$\Delta = -4\lambda^2 - 4\lambda - 3 < 0$$

$$\Delta^* = 16 - 4(-4)(-3) < 0$$

Αφου $\Delta < 0$

(+)

11) $x^2 - \lambda x + \lambda^2 - \lambda + 1 > 0$

$\Delta = \lambda^2 - 4(\lambda^2 - \lambda + 1)$

$\Delta = \lambda^2 - 4\lambda^2 + 4\lambda - 4$

(-)

$\Delta = -3\lambda^2 + 4\lambda - 4 < 0$

$\Delta^* = 16 - 4(-3)(-4)$

$\Delta^* = 16 - 48 < 0$

40. $(\lambda-1)x^2 - 2\lambda x + 2\lambda$, $\lambda \neq 1$

(α) $(\lambda-1)x^2 - 2\lambda x + 2\lambda < 0$

Αρκεί $\Delta < 0$ και $\lambda - 1 < 0$

$\lambda < 1$

$4\lambda^2 - 4(\lambda-1)2\lambda < 0$

$\lambda^2 - 2\lambda(\lambda-1) < 0$

$\lambda^2 - 2\lambda^2 + 2\lambda < 0$

$-\lambda^2 + 2\lambda < 0$

λ		0	2
$-\lambda^2+2\lambda$	$-$	$+$	$-$

$\lambda \in (-\infty, 0) \cup (2, +\infty)$

$\lambda \in (-\infty, 0)$

(B) $\Delta > 0$

$\lambda \in (0, 2)$.

$$x_1 + x_2 = \frac{2\lambda}{\lambda - 1}$$

$$x_1 x_2 = \frac{2\lambda}{\lambda - 1}$$

(I) $\frac{1}{x_1} + \frac{1}{x_2} < \lambda$

$$\frac{x_1 + x_2}{x_1 x_2} < \lambda$$

$$\frac{\frac{2\lambda}{\lambda - 1}}{\frac{2\lambda}{\lambda - 1}} < \lambda$$

$$1 < \lambda < 2$$

39.

$$(\lambda-2)x^2 + 2(2\lambda-3)x + 5\lambda-6 = 0$$

$\lambda \neq 2$

α) Αρκεί $\Delta > 0$

$$4(2\lambda-3)^2 - 4(\lambda-2)(5\lambda-6) > 0$$

$$4\lambda^2 - 12\lambda + 9 - (5\lambda^2 - 6\lambda - 10\lambda + 12) > 0$$

$$4\lambda^2 - 12\lambda + 9 - 5\lambda^2 + 6\lambda + 10\lambda - 12 > 0$$

$$-\lambda^2 + 4\lambda - 3 > 0$$

λ		1		3	
$-\lambda^2+4\lambda-3$	−	0	+	0	−

$\lambda \in (1, 3)$

$\lambda \in (1, 2) \cup (2, 3)$.

(B). Απαιτώ $\Delta < 0$ και $\lambda - 2 > 0$

$\lambda > 2$

$\lambda \in (-\infty, 1) \cup (3, +\infty)$

$\lambda \in (3, +\infty)$

16. $x^2 - (3\lambda - 2)x - 3\lambda + 2 = 0$

(α) Διότι $\Delta \geq 0$

$$(3\lambda-2)^2 - 4(-3\lambda+2) \geq 0$$

$$9\lambda^2 - 12\lambda + 4 + 12\lambda - 8 \geq 0$$

$$9\lambda^2 - 4 \geq 0$$

λ		$-\frac{2}{3}$		$\frac{2}{3}$	
$9\lambda^2-4$	+	0	−	0	+

$\lambda \in (-\infty, -\frac{2}{3}] \cup [\frac{2}{3}, +\infty)$.

(β) $x_1^2 x_2 + x_1 x_2^2 < -9$

$x_1 + x_2 = 3\lambda - 2$

$x_1 x_2 = 2 - 3\lambda$

$x_1 x_2 (x_1 + x_2) < -9$

$(2-3\lambda)(3\lambda-2) < -9$

$6\lambda - 4 - 9\lambda^2 + 6\lambda + 9 < 0$

$-9\lambda^2 + 12\lambda + 5 < 0$

$\Delta = 144 + 180 = 324$

$-\frac{1}{3}$ $-\frac{5}{3}$

λ		$-\frac{5}{3}$		$-\frac{1}{3}$	
$-9\lambda^2+12\lambda+5$	−	0	+	0	−

$\lambda \in (-\infty, -\frac{5}{3}) \cup (-\frac{1}{3}, +\infty)$

Ενότητα 19

14. ⓐ $x^2-(2\lambda-1)x-2\lambda+1=0$

Αρκεί $\Delta>0$

$$B^2-4\alpha\beta>0$$

$$(2\lambda-1)^2-4\cdot(-2\lambda+1)>0$$

$$4\lambda^2-4\lambda+1+8\lambda-4>0$$

$$4\lambda^2+4\lambda-3>0$$

$$\Delta=16+16\cdot 3=64$$

$$\lambda=\frac{-4\pm 8}{8} \quad \left(\frac{1}{2}\right) \quad \left(-\frac{3}{2}\right)$$

λ		$-\frac{3}{2}$		$\frac{1}{2}$	
$4\lambda^2+4\lambda-3$	+		−		+

$$\lambda\in\left(-\infty,-\frac{3}{2}\right)\cup\left(\frac{1}{2},+\infty\right).$$

(13) $(\lambda+1)x^2 - 2\lambda x = \lambda + 1$

$(\lambda+1)x^2 - 2\lambda x - (\lambda+1) = 0$

Алгоритм $\Delta > 0$

$4\lambda^2 + 4(\lambda+1)(\lambda+1) > 0$

$\lambda^2 + (\lambda+1)^2 > 0$

$\lambda^2 + \lambda^2 + 2\lambda + 1 > 0$

$2\lambda^2 + 2\lambda + 1 > 0$

$\Delta < 0$

λ	
$2\lambda^2+2\lambda+1$	$+$

$\lambda \in (-\infty, +\infty)$.

29. ⓐ $\lambda x^2 - (\lambda-1)x + \lambda - 1 > 0, \lambda \neq 0$

Αρκεί $\Delta < 0$ και $\lambda > 0$

$(\lambda-1)^2 - 4\lambda(\lambda-1) < 0$ $\quad \underline{\lambda > 0}$

$\lambda^2 - 2\lambda + 1 - 4\lambda^2 + 4\lambda < 0$

$-3\lambda^2 + 2\lambda + 1 < 0$

λ		$-\frac{1}{3}$		1
$-3\lambda^2+2\lambda+1$	$-$	$+$	$+$	$-$

$\lambda \in (-\infty, -\frac{1}{3}) \cup (1, +\infty)$

20. (δ) $x^4 - 8x^2 - 9 > 0$

$(x^2)^2 - 8x^2 - 9 > 0$

$x^2 = \omega$

$\omega^2 - 8\omega - 9 > 0$

ω		-1		9
$\omega^2-8\omega-9$	$+$	$-$		$+$

$\omega \in (-\infty, -1) \cup (9, +\infty)$

$\omega < -1$ ή $\omega > 9$

$x^2 < -1$ ή $x^2 > 9$

Αδύνατη

$x^2 > 3^2$

$|x| > |3|$ (+)

$|x| > 3$

$x > 3$ ή $x < -3$

$x \in (-\infty, -3) \cup (3, +\infty)$

(σ2) $x - 8\sqrt{x} - 9 > 0$, $x > 0$

$(\sqrt{x})^2 - 8\sqrt{x} - 9 > 0$

$\omega = \sqrt{x}$

$\omega^2 - 8\omega - 9 > 0$

ω		-1		9
$\omega^2 - 8\omega - 9$	$+$	$\|$	$-$	$+$

$\omega \in (-\infty, -1) \cup (9, +\infty)$

$\omega < -1$ ή $\omega > 9$

$\sqrt{x} < -1$ $\sqrt{x} > 9$

Αδύνατη $x > 81$

Спорсо Майора

19

(15) α

(18) αγ

(19)

(37).

(20) αβγ.