Αναγωγή στο 1ο τεταρτημόριο

1. $\eta\mu(\kappa\cdot 360+\omega)=\eta\mu\omega$

   $\sigma\upsilon\nu(\kappa\cdot 360+\omega)=\sigma\upsilon\nu\omega$

   $\varepsilon\varphi(\kappa\cdot 360+\omega)=\varepsilon\varphi\omega$

   $\sigma\varphi(\kappa\cdot 360+\omega)=\sigma\varphi\omega$

2. $\eta\mu(-\omega)=-\eta\mu\omega$

   $\sigma\upsilon\nu(-\omega)=\sigma\upsilon\nu\omega$

   $\varepsilon\varphi(-\omega)=-\varepsilon\varphi\omega$

   $\sigma\varphi(-\omega)=-\sigma\varphi\omega$

3. $\eta\mu(180-\omega)=\eta\mu\omega$

   $\sigma\upsilon\nu(180-\omega)=-\sigma\upsilon\nu\omega$

   $\varepsilon\varphi(180-\omega)=-\varepsilon\varphi\omega$

   $\sigma\varphi(180-\omega)=-\sigma\varphi\omega$

4. $\eta\mu(180+\omega)=-\eta\mu\omega$

   $\sigma\upsilon\nu(180+\omega)=-\sigma\upsilon\nu\omega$

   $\varepsilon\varphi(180+\omega)=\varepsilon\varphi\omega$

   $\sigma\varphi(180+\omega)=\sigma\varphi\omega$

5. $\eta\mu(90-\omega)=\sigma\upsilon\nu\omega$

   $\sigma\upsilon\nu(90-\omega)=\eta\mu\omega$

   $\varepsilon\varphi(90-\omega)=\varepsilon\varphi\omega$

   $\sigma\varphi(90-\omega)=\sigma\varphi\omega$

6. $\eta\mu(90+\omega)=\sigma\upsilon\nu\omega$

   $\sigma\upsilon\nu(90+\omega)=-\eta\mu\omega$

   $\varepsilon\varphi(90+\omega)=-\sigma\varphi\omega$

   $\sigma\varphi(90+\omega)=-\varepsilon\varphi\omega$

Τριγωνομετρικός πίνακας

Βασικών γωνιών

Μοίρες	0	30	45	60	90	180	270	360
rad	0	$\pi/6$	$\pi/4$	$\pi/3$	$\pi/2$	π	$3\pi/2$	2π
ημω	0	$1/2$	$\sqrt{2}/2$	$\sqrt{3}/2$	1	0	-1	0
συνω	1	$\sqrt{3}/2$	$\sqrt{2}/2$	$1/2$	0	-1	0	1
εφω	0	$\sqrt{3}/3$	1	$\sqrt{3}$	-	0	-	0
σφω	-	$\sqrt{3}$	1	$\sqrt{3}/3$	0	-	0	-

Τριγωνομετρικές Εξισώσεις

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1. $\eta\mu x = \eta\mu\theta \iff x = 2k\pi + \theta$ ή $x = 2k\pi + \pi - \theta$

2. $\sigma\upsilon\nu x = \sigma\upsilon\nu\theta \iff x = 2k\pi + \theta$ ή $x = 2k\pi - \theta$

3. $\varepsilon\varphi x = \varepsilon\varphi\theta \iff x = k\pi + \theta$

4. $\sigma\varphi x = \sigma\varphi\theta \iff x = k\pi + \theta$.

14. (B) $3\cos^2 x - \sin^2 x - 3 = 0$

$3(1-\sin^2 x) - \sin^2 x - 3 = 0$

$3 - 3\sin^2 x - \sin^2 x - 3 = 0$

$-4\sin^2 x = 0$

$\sin^2 x = 0$

$\sin x = 0$

$\sin x = \sin 0$

$x = 2k\pi + 0$

$x = 2k\pi + \pi - 0$

6. (B) $(2\cos x+1)\cdot(\operatorname{ctg} x+1)=0$

$2\cos x+1=0$ ili $\operatorname{ctg} x+1=0$

$\cos x=-\frac{1}{2}$ $\operatorname{ctg} x=-1$

$\cos x=-\cos\frac{\pi}{3}$ $\operatorname{ctg} x=-\operatorname{ctg}\frac{\pi}{4}$

$\cos x=\cos(\pi-\frac{\pi}{3})$ $\operatorname{ctg} x=\operatorname{ctg}(-\frac{\pi}{4})$

$x=2k\pi-\frac{\pi}{4}$

$x=2k\pi+\pi-\frac{\pi}{3}$ ili $x=2k\pi-\pi+\frac{\pi}{3}$

$x=2k\pi+\frac{2\pi}{3}$ $x=2k\pi-\frac{2\pi}{3}$

12. ⑧ $\operatorname{ctg}\pi x + \operatorname{ctg}\left(2\pi x - \frac{\pi}{3}\right) = 0$

$$\operatorname{ctg}(\pi x) = -\operatorname{ctg}\left(2\pi x - \frac{\pi}{3}\right).$$

$$\operatorname{ctg}(\pi x) = \operatorname{ctg}\left(-2\pi x + \frac{\pi}{3}\right)$$

$$\pi x = k\pi - 2\pi x + \frac{\pi}{3}$$

$$3\pi x = k\pi + \frac{\pi}{3}$$

$$3x = k + \frac{1}{3}$$

$$x = \frac{k}{3} + \frac{1}{9}$$

12. (α) $\sin 2x = \sin\left(2x + \frac{\pi}{3}\right)$.

$2x = 2\pi k + 2x + \frac{\pi}{3}$ и $2x = 2\pi k + \pi - 2x - \frac{\pi}{3}$

Дальше

$4x = 2\pi k + \pi - \frac{\pi}{3}$

$x = \frac{\pi k}{2} + \frac{\pi}{4} - \frac{\pi}{12}$

$x = \frac{\pi k}{2} + \frac{3\pi - \pi}{12}$

$x = \frac{\pi k}{2} + \frac{2\pi}{12}$

$x = \frac{\pi k}{2} + \frac{\pi}{6}$

5. ⑤

$$\cos x = -\frac{\sqrt{3}}{2}$$

$$\cos x = -\cos\frac{\pi}{6}$$

$$\cos x = \cos\left(\pi - \frac{\pi}{6}\right)$$

$$x = 2k\pi + \pi - \frac{\pi}{6} \quad \text{또는} \quad x = 2k\pi - \pi + \frac{\pi}{6}.$$

$$x = 2k\pi + \frac{5\pi}{6}$$

$$x = 2k\pi - \frac{5\pi}{6}$$

11. ⑤

$$2\cos \pi x + \sqrt{3} = 0$$

$$\cos \pi x = -\frac{\sqrt{3}}{2}$$

$$\cos \pi x = -\cos\frac{\pi}{6}$$

$$\cos \pi x = \cos\left(\pi - \frac{\pi}{6}\right)$$

$$\cos \pi x = \cos\left(\frac{5\pi}{6}\right).$$

$$\pi x = 2k\pi + \frac{5\pi}{6} \quad \text{또는} \quad \pi x = 2k\pi - \frac{5\pi}{6}$$

$$x = 2k + \frac{5}{6}, \quad x = 2k - \frac{5}{6}$$

Ενοτητα 13

1. (δ) $\sigma\varphi x = \sigma\varphi \frac{2\pi}{11}$

$$x = k\pi + \frac{2\pi}{11}$$

2. (Β) $2\sigma\upsilon\nu x = \sqrt{3}$

$$\sigma\upsilon\nu x = \frac{\sqrt{3}}{2}$$

$$\sigma\upsilon\nu x = \sigma\upsilon\nu \frac{\pi}{6}$$

$$x = 2k\pi + \frac{\pi}{6} \quad \text{ή} \quad x = 2k\pi - \frac{\pi}{6}$$

8. ⓐ $\tan x \cdot \cos x + \tan x = 1 + \cos x$

$\tan x \cos x + \tan x - 1 - \cos x = 0$

$\tan x (\cos x + 1) - (\cos x + 1) = 0$

$(\cos x + 1)(\tan x - 1) = 0.$

$\cos x + 1 = 0$ ve $\tan x - 1 = 0$

$\cos x = -1$

$\cos x = -\cos 0$

$\cos x = \cos(\pi - 0)$

$\cos x = \cos \pi$

$x = 2k\pi + \pi$

$x = 2k\pi - \pi$

$\tan x = 1$

$\tan x = \tan \frac{\pi}{2}$

$x = k\pi + \frac{\pi}{2}$

$x = 2k\pi + \pi - \frac{\pi}{2}$

$x = 2k\pi + \frac{\pi}{2}$

11. (8) $\sqrt{3}\operatorname{ctg}\left(\pi x+\frac{\pi}{4}\right)+1=0$

$$\sqrt{3}\operatorname{ctg}\left(\pi x+\frac{\pi}{4}\right)=-1$$

$$\operatorname{ctg}\left(\pi x+\frac{\pi}{4}\right)=-\frac{1}{\sqrt{3}}$$

$$\operatorname{ctg}\left(\pi x+\frac{\pi}{4}\right)=-\frac{\sqrt{3}}{3}$$

$$\operatorname{ctg}\left(\pi x+\frac{\pi}{4}\right)=-\operatorname{ctg}\frac{\pi}{6}$$

$$\operatorname{ctg}\left(\pi x+\frac{\pi}{4}\right)=\operatorname{ctg}\left(-\frac{\pi}{6}\right).$$

$$\pi x+\frac{\pi}{4}=k\pi-\frac{\pi}{6}$$

$$\pi x=k\pi-\frac{\pi}{6}-\frac{\pi}{4}$$

$$x=k-\frac{1}{6}-\frac{1}{4}$$

$$x=k-\frac{2}{12}-\frac{3}{12}$$

$$x=k-\frac{5}{1\cdot 2}$$

4. (B) $\cos x = -\cos\frac{\pi}{9}$

$\cos(\pi - x) = -\cos x$

$\cos x = \cos\left(\pi - \frac{\pi}{9}\right)$.

$x = 2k\pi + \pi - \frac{\pi}{9}$ ii $x = 2k\pi - \pi + \frac{\pi}{9}$

$x = 2k\pi + \frac{8\pi}{9}$ $\quad$ $x = 2k\pi - \frac{8\pi}{9}$

(A) $\tan x + \tan\frac{\pi}{10} = 0$

$\tan(-x) = -\tan x$

$\tan x = -\tan\frac{\pi}{10}$

$\tan x = \tan\left(-\frac{\pi}{10}\right)$

$x = k\pi - \frac{\pi}{10}$

3. (г) $\sqrt{3}\,\varepsilon\varphi x - 1 = 0$

$$\sqrt{3}\,\varepsilon\varphi x = 1$$

$$\varepsilon\varphi x = \frac{1}{\sqrt{3}}$$

$$\varepsilon\varphi x = \frac{\sqrt{3}}{3}$$

$$\varepsilon\varphi x = \varepsilon\varphi \frac{\pi}{6}$$

$$\boxed{x = \pi k + \frac{\pi}{6}.}$$

4. (б) $\sigma\varphi x + \sigma\varphi \frac{\pi}{8} = 0$

$\sigma\varphi(-x) = -\sigma\varphi x$

$$\sigma\varphi x = -\sigma\varphi \frac{\pi}{8}$$

$$\sigma\varphi x = \sigma\varphi\left(-\frac{\pi}{8}\right)$$

$$x = \pi k - \frac{\pi}{8}$$

Ελληνικά Μαθήματα

13

(1) α

(2) α

(3) β.

(4) α.

(5) γ

(6) α.

(12) βγ.