Ενοτητα 23

26. ⓐ $f(x) = x + \sqrt{x^2+1}$, $x \in R$.

$$f'(x) = 1 + \frac{2x}{2\sqrt{x^2+1}} = \frac{\sqrt{x^2+1}+x}{\sqrt{x^2+1}} > 0$$

$f \nearrow$

Το $\sqrt{x^2+1} + x > 0$ $\forall x \in R$ γιατι

- αν $x > 0$ τοτε προφανως ισχυει.
- αν $x = 0$ τοτε $1 > 0$ που ισχυει.
- αν $x < 0$ τοτε $\sqrt{x^2+1} > -x$

$$\sqrt{x^2+1}^{\,2} > (-x)^2$$

$$x^2+1 > x^2$$

$1 > 0$ που ισχυει.

⑧ $f(x) = x\sqrt{1-x^2}$

x		-1		1	
$1-x^2$	$-$		$+$		$-$

$D_f = [-1, 1]$

$$f'(x) = \sqrt{1-x^2} + x\,\frac{-2x}{2\sqrt{1-x^2}} = \sqrt{1-x^2} - \frac{x^2}{\sqrt{1-x^2}}$$

$$f'(x) = \frac{1-x^2-x^2}{\sqrt{1-x^2}} = \frac{1-2x^2}{\sqrt{1-x^2}}$$

$$f'(x) = 0 \Rightarrow 1-2x^2 = 0 \Rightarrow x^2 = \frac{1}{2}$$

$$x = \pm\frac{\sqrt{2}}{2}$$

x	-1		$-\frac{\sqrt{2}}{2}$		$\frac{\sqrt{2}}{2}$		1
f'		$-$		$+$		$-$	
f'		$\searrow$		$\nearrow$		$\searrow$	

(8) $f(x) = \frac{4}{3} x^{\frac{3}{2}} - x^2 = \frac{4}{3} \sqrt[2]{x^3} - x^2$

$f(x) = \frac{4}{3} x\sqrt{x} - x^2 \quad , x \geq 0$

$f'(x) = \frac{4}{3} \frac{3}{2} x^{\frac{1}{2}} - 2x$

$f'(x) = 2\sqrt{x} - 2x = 2(\sqrt{x} - x)$

$\rightarrow f'(x) = 0 \Rightarrow \sqrt{x} = x$

$x = x^2$

$x(1-x) = 0$

$x = 0 \qquad x = 1$

x	0	1
f'	+	—
f	$\nearrow$	$\searrow$

27. (a) $f(x) = \frac{1}{x^2} - \frac{1}{x-4}$, $x \neq 0$, $x \neq 4$

$$f'(x) = \frac{-2x}{x^4} - \frac{-1}{(x-4)^2}$$

$$f'(x) = \frac{-2}{x3} + \frac{1}{(x-4)^2} = \frac{x^3 - 2(x-4)^2}{x^3(x-4)^2}$$

$$f'(x) = \frac{x^3 - 2(x^2 - 8x + 16)}{x^3(x-4)^2} = \frac{x^3 - 2x^2 + 16x - 32}{x^3(x-4)^2}$$

$$f'(x) = \frac{x^2(x-2) + 16(x-2)}{x^3(x-4)^2} = \frac{(x-2)(x^2+16)}{x^3(x-4)^2}$$

x		0		2		4	
$x-2$	$-$	‖	$-$		$+$	‖	$+$
x^3	$-$	‖	$+$		$+$	‖	$+$
f'	$+$	‖	$-$		$+$	‖	$+$
f	$\nearrow$	‖	$\searrow$		$\nearrow$	‖	$\nearrow$

(B) $f(x) = \ln\left(\frac{x}{x-1}\right)$

x		0		1	
x	$-$		$+$		$+$
$x-1$	$-$		$-$		$+$
$\frac{x}{x-1}$	$+$		$-$		$+$

$x \in (-\infty, 0) \cup (1, +\infty)$

$$f'(x) = \frac{1}{\frac{x}{x-1}} \cdot \frac{x-1-x}{(x-1)^2}$$

$$f'(x) = \frac{x-1}{x} \cdot \frac{-1}{(x-1)^2} = \frac{-1}{x(x-1)}$$

x		0		1	
f'	$-$				$-$
f	$\searrow$				$\searrow$

27. (γ) $f(x) = \ln\left(\sqrt{x^2+1} - x\right) \qquad D_f = \mathbb{R}$

$$f'(x) = \frac{1}{\sqrt{x^2+1} - x} \cdot \left(\frac{2x}{2\sqrt{x^2+1}} - 1\right)$$

$$f'(x) = \frac{1}{\cancel{\sqrt{x^2+1} - x}} \left(\frac{\cancel{x - \sqrt{x^2+1}}}{\sqrt{x^2+1}} \right)$$

$$f'(x) = -\frac{1}{\sqrt{x^2+1}} < 0$$

$f \searrow$

29. (B) $f(x) = \begin{cases} \dfrac{x\ln x}{x-1}, & 0<x\neq 1 \\ 1, & x=1 \end{cases}$

$$f'(x) = \frac{(\ln x+1)(x-1) - x\ln x}{(x-1)^2} = \frac{\cancel{x\ln x} - \ln x + x - 1 - \cancel{x\ln x}}{(x-1)^2}$$

$$f'(x) = \frac{-\ln x + x - 1}{(x-1)^2} \geq 0 \quad f \nearrow$$

- $\ln x \leq x-1$

$0 \leq x - 1 - \ln x$

$$\lim_{x\to 1} f(x) = \lim_{x\to 1} \frac{x\ln x}{x-1} = \lim_{x\to 1} \frac{\ln x+1}{1} = 1.$$

Σωστό το 1.

16. $f(x) = e^x - x - 3\,\text{συν}\,x$

(α) Νδο $\dfrac{e^x - x}{\text{συν}\,x} = 3$ έχει 1 ρίζα $\left(0, \frac{\pi}{2}\right)$

$e^x - x = 3\,\text{συν}\,x$

$e^x - x - 3\,\text{συν}\,x = 0$

$f(x) = 0$

$f(0) = 1 - 3 = -2$

$f\left(\frac{\pi}{2}\right) = e^{\pi/2} - \frac{\pi}{2} - 0 > 0$

- $e^x \geq x + 1$

$e^{\pi/2} \geq \frac{\pi}{2} + 1$

$e^{\pi/2} - \frac{\pi}{2} \geq 1$

$\underline{e^{\pi/2} - \frac{\pi}{2} > 0}$

<u>Μονοτονία</u>

$f'(x) = e^x - 1 + 3\,\text{ημ}\,x$

$\underline{f'(0) = 0}$

- $x > 0 \Rightarrow e^x > e^0$

$e^x > 1$

$e^x - 1 > 0$

- $x > 0 \Rightarrow \text{ημ}\,x > 0$

$f'(x) > 0$

$f \nearrow$

$f(0)\,f\left(\frac{\pi}{2}\right) < 0$ Bolzano $\exists\, x_0 \in \left(0, \frac{\pi}{2}\right)$

τ.ω $f(x_0) = 0$

(B) $\lim_{x \to x_0} \frac{1}{f(x)}$

$\lim_{x \to x_0^-} \frac{1}{f(x)} = -\infty$

Το όριο δεν υπάρχει.

$\lim_{x \to x_0^+} \frac{1}{f(x)} = +\infty$

x		x_0	
$f(x)$	↗ $-$		$+$ ↗

18. $f'(x) < 0$

ב נכון $f(3) - \cos 2 < f(2) - \cos 3$

$f(3) + \cos 3 < f(2) + \cos 2$

$\boxed{g(x) = f(x) + \cos x}$

$g(3) < g(2)$

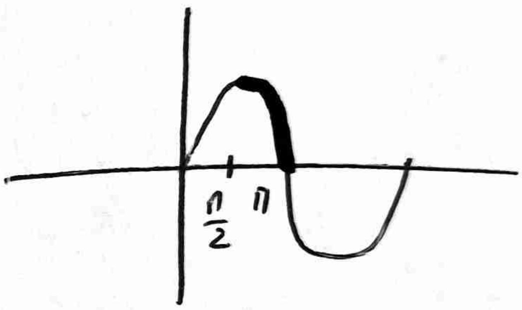
$g \downarrow$

$3 > 2$ ✓

$g'(x) = f'(x) - \sin x < 0$

$\ominus \qquad \oplus$

$g \downarrow$



ד $f(2) - \cos 2 < f(1) - \cos 3$

$f(2) + \cos 3 < f(1) + \cos 2$

$\boxed{g(x) = f(x-1) + \cos x}$

$g(3) < g(2)$

$g \downarrow$

$3 > 2$

$g'(x) = f'(x-1) - \sin x$

$g'(x) < 0$

$g \downarrow$

19. (γ) $f(e) < \ln \frac{1+e^2}{5^{1-e}}$ $f(2) = e\ln 5$

$f(e) < \ln(1+e^2) - \ln 5^{1-e}$

$f(e) < \ln(1+e^2) - (1-e)\ln 5$

$f(e) < \ln(1+e^2) - \ln 5 + e\ln 5$

$f(e) < \ln(1+e^2) - \ln 5 + f(2)$

$f(e) - \ln(1+e^2) < f(2) - \ln(2^2+1)$

$\boxed{g(x) = f(x) - \ln(x^2+1)}$

$g(e) < g(2)$

$g \searrow$

$e > 2$

$g'(x) = f'(x) - \frac{2x}{x^2+1} < 0$ $g \searrow$.

20. (B) $3^{x-1}+4^{x-1}=5^{x-1}$

$$3^{x}\cdot 3^{-1}+4^{x}\cdot 4^{-1}=5^{x}\cdot 5^{-1}$$

$$\frac{1}{3}3^{x}+\frac{1}{4}4^{x}=\frac{1}{5}5^{x}$$

$$\frac{1}{3}\left(\frac{3}{5}\right)^{x}+\frac{1}{4}\left(\frac{4}{5}\right)^{x}=\frac{1}{5}$$

$$\underbrace{\frac{1}{3}\left(\frac{3}{5}\right)^{x}+\frac{1}{4}\left(\frac{4}{5}\right)^{x}-\frac{1}{5}}_{f(x)}=0$$

$f(x)=0$

$f(x)=f(3)$

f 8 1-1

$x=3$

$\bullet\ x_1<x_2 \Rightarrow \left(\frac{3}{5}\right)^{x_1}>\left(\frac{3}{5}\right)^{x_1}$

$$\left.\begin{aligned}&\frac{1}{3}\cdot\left(\frac{3}{5}\right)^{x_1}>\frac{1}{3}\left(\frac{3}{5}\right)^{x_2}\\ &\frac{1}{4}\left(\frac{4}{5}\right)^{x_1}>\frac{1}{4}\left(\frac{4}{5}\right)^{x_2}\end{aligned}\right\}+$$

$\bullet\ x_1<x_2 \rightarrow \frac{1}{4}\left(\frac{4}{5}\right)^{x_1}>\frac{1}{4}\left(\frac{4}{5}\right)^{x_2}$

$f\downarrow$

21. $2x\,\eta\mu x + 2\sigma\upsilon\nu x = \pi$ $\qquad [-\frac{\pi}{2}, \frac{\pi}{2}]$.

$$\underbrace{2x\,\eta\mu x + 2\sigma\upsilon\nu x - \pi}_{f(x)} = 0$$

Προφανείς ρίζες: $x = -\frac{\pi}{2}$ $\qquad x = \frac{\pi}{2}$.

<u>Τσεκ</u>

$$f\left(\frac{\pi}{2}\right) = \cancel{2}\cdot\frac{\pi}{\cancel{2}}\,\eta\mu\frac{\pi}{2} + \cancel{2\sigma\upsilon\nu\frac{\pi}{2}} - \pi = \pi - \pi = 0$$

$$f\left(-\frac{\pi}{2}\right) = 2\left(-\frac{\pi}{2}\right)\eta\mu\left(-\frac{\pi}{2}\right) + \cancel{2\sigma\upsilon\nu\left(-\frac{\pi}{2}\right)} - \pi = 0.$$

$$f'(x) = \cancel{2\eta\mu x} + 2x\,\sigma\upsilon\nu x \cancel{- 2\eta\mu x}$$

$$f'(x) = 2x\,\underset{+}{\sigma\upsilon\nu x}$$

x	$-\frac{\pi}{2}$	0	$\frac{\pi}{2}$
f'	$-$		$+$
f	↘		↗

Άρα η f έχει 2 διαστήματα μονοτονίας συνεπώς έχει το πολύ 2 λύσεις και αφού βρήκα ήδη 2 λύσεις αυτές είναι,

22. $x^4 = 4^x$ $\quad (0,+\infty)$

Προφανείς ρίζες: $x=2$, $x=4$

$$4\ln x = x\ln 4$$

$$\frac{\ln x}{x} = \frac{\ln 4}{4}$$

$$\boxed{g(x) = \frac{\ln x}{x}}$$

$$g'(x) = \frac{\frac{1}{x}x - \ln x}{x^2} = \frac{1-\ln x}{x^2}$$

x		e	
g'	$+$	0	$-$
g	$\nearrow$		$\searrow$

Έχω 2 διαστήματα μονοτονίας άρα το πολύ 2 ρίζες και αφού ήδη βρήκα 2

Αυτές είναι!.

23. (Β) $3^x = 2x+1$ προφανείς ρίζες • $x=1$ • $x=0$

$$1 = \frac{2x+1}{3^x}$$

$$g(x) = \frac{2x+1}{3^x} - 1$$

$$g'(x) = \frac{2\cdot \cancel{3^x} - (2x+1)\cancel{3^x}\ln 3}{(3^x)^{\cancel{2}}}$$

2 διαστήματα μονοτονίας

$$g'(x) = \frac{2 - (2x+1)\ln 3}{3^x}$$

$\rightarrow$ $g'(x) = 0 \Rightarrow 2 - \ln 3(2x+1) = 0$

$$2 = \ln 3(2x+1)$$

$$2 = 2\ln 3\, x + \ln 3$$

$$\frac{2 - \ln 3}{2\ln 3} = x.$$

$$\xi = x$$

x		ξ	
g'	$+$	0	$-$
g	$\nearrow$		$\searrow$

27. $f(x) = \ln x - x \quad , x>0$

(α) $f'(x) = \frac{1}{x} - 1 = \frac{1-x}{x}$

x	0		1	
f'		$+$	0	$-$
f		$\nearrow$		$\searrow$

(β) $e\,\eta\mu x < e^{\eta\mu x} \qquad (0,\pi)$

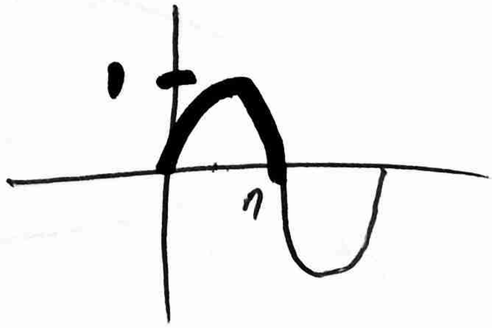
$\ln e\eta\mu x < \ln e^{\eta\mu x}$

$\ln e + \ln \eta\mu x < \eta\mu x$

$\ln(\eta\mu x) - \eta\mu x < -\ln e$

$f(\eta\mu x) < -1$

$f(\eta\mu x) < f(1)$



$\forall x<1$ η $f \nearrow$

$\eta\mu x < 1$

$x \in (0, \frac{\pi}{2}) \cup (\frac{\pi}{2}, \pi)$

1) $\ln \dfrac{x^2+3}{2x^2+1} = 2-x^2$

$$\ln(x^2+3) - \ln(2x^2+1) = 2-x^2$$

$$\ln(x^2+3) - (x^2+3) = \ln(2x^2+1) - (2x^2+1)$$

$$f(x^2+3) = f(2x^2+1)$$

$\left.\begin{array}{l} \bullet\ x^2+3>1 \\ \bullet\ 2x^2+1>1 \end{array}\right\}\ \forall x>1 \quad \nearrow f$

$$x^2+3 = 2x^2+1$$

$$2 = x^2$$

$$x = \pm\sqrt{2}$$

|

29. $f''(x)<0$

$f \ \sigma\omega + nap$

$f' \ \sigma\omega + nap.$

f''

$f'(1)=0$

(a)

x		1	
f''	$-$		$-$
f'	$\searrow +$	0	$- \searrow$
f	$\nearrow$		$\searrow$

$x<1 \Rightarrow f'(x)>f'(1)$

$f'(x)>0$

$x>1 \Rightarrow f'(x)<f'(1)$

$f'(x)<0$

(B) i) $f(2-x)=f(3)$ $\quad (-\infty,0)$

• $\underline{3>1}$

• $x<0 \Rightarrow -x>0 \Rightarrow \underline{2-x>2}$

$\forall x>1$ η $f \searrow$

$2-x=3$

$\underline{-1=x}$

11) $f(e^{2-x}) = f\left(\frac{1}{x-1}\right)$ $\qquad (1,2]$

• $1<x\leq 2 \Rightarrow -1>-x\geq -2 \Rightarrow 1>2-x\geq 0$

$$e^{1}>e^{2-x}\geq e^{0}$$

$$e>e^{2-x}\geq 1$$

• $1<x\leq 2 \Rightarrow 0<x-1\leq 1$

$$\frac{1}{x-1}\geq 1$$

$\forall x>1$ n $f\cancel{\ }$.

$$e^{2-x}=\frac{1}{x-1} \Leftrightarrow \frac{1}{e^{x-2}}=\frac{1}{x-1}$$

$x-1=e^{x-2}$

• $e^{x}\geq x+1 \Rightarrow e^{x-2}\geq x-2+1$

$$e^{x-2}\geq -1+x$$

To "=" $x-2=0$

$x=2$

(γ) $f(e^{x-2}) = f(\frac{1}{x})$ $(1,2)$

• $1 < x < 2 \Rightarrow -1 < x-2 < 0$

$e^{-1} < e^{x-2} < 1$

• $1 < x < 2 \Rightarrow 1 > \frac{1}{x} > \frac{1}{2}$

$\forall x < 1$. $f \nearrow$

$e^{x-2} = \frac{1}{x}$

$\underbrace{e^{x-2} - \frac{1}{x}}_{g(x)} = 0$

$g(1) = e^{-1} - 1 < 0$
$g(2) = 1 - \frac{1}{2} > 0$ $\Big\}$ Bolzano $\exists \xi \in (1,2)$

τ.ω $g(\xi) = 0$.

$g'(x) = e^{x-2} + \frac{1}{x^2} > 0$ $g \nearrow$

30. $f(1)=g(1)$

$f'(x) > g'(x) \quad \forall x \in \mathbb{R}.$

Εστω $f(x) < g(x)$

$$\underbrace{f(x) - g(x)}_{h(x)} < 0$$

$h(x) < 0$

$h(x) < h(1)$

$h \nearrow$

$x < 1$

$h'(x) = f'(x) - g'(x) > 0$

$h \nearrow$

Αν $x<1$ C_f κατω C_g

Αν $x>1$ C_f πανω C_g

Αν $x=1$ τεμνονται.

32. $f: \mathbb{R} \to \mathbb{R}$ παρ/μη $f(0)=0$

$f(x) + f'(x) > e^{-x}$ $\forall x \in \mathbb{R}$.

(a) Νδο $f(x) < \frac{x}{e^x}$ $\forall x < 0$

$\underbrace{e^x f(x) - x}_{g(x)} < 0$ $\Rightarrow$ $g(x) < 0$ $\forall x < 0$ αυτό θέλω να δείξω

$g'(x) = e^x f(x) + e^x f'(x) - 1$

$g'(x) = e^x \left(f(x) + f'(x)\right) - 1 > 0$

$g \nearrow$

Ισχύει ότι $f(x) + f'(x) > e^{-x}$

$f(x) + f'(x) > \frac{1}{e^x}$

$e^x(f(x) + f'(x)) > 1$

$e^x (f(x) + f'(x)) - 1 > 0$

Τελος

Αν $x < 0$

$g(x) < g(0)$

$g(x) < 0$ ✓

Ⓑ $\lim_{x \to -\infty} f(x)$

$$f(x) < \frac{x}{e^x} \quad \forall x < 0$$

$$\lim_{x \to -\infty} f(x) < \lim_{x \to -\infty} \frac{x}{e^x}$$

$$\lim_{x \to -\infty} f(x) < \lim_{x \to -\infty} x \cdot \frac{1}{e^x}$$

$$\lim_{x \to -\infty} f(x) < (-\infty)(+\infty)$$

$$\lim_{x \to -\infty} f(x) < -\infty$$

$$\underline{\underline{\lim_{x \to -\infty} f(x) = -\infty}}$$

33. $f:(0,+\infty) \longrightarrow \mathbb{R}$ $\quad f(1)=0$

f גזירה

$x^2 f'(x) > 1 - x f(x) \quad \forall x>0$

(a) הוכח $\quad f(x) < \frac{\ln x}{x} \quad \forall x \in (0,1)$

$$\underbrace{x f(x) - \ln x}_{g(x)} < 0 \quad \Rightarrow \quad \forall x \in (0,1) \;\; g(x)<0$$

$$g'(x) = f(x) + x f'(x) - \frac{1}{x}$$

$$g'(x) = \frac{x f(x) + x^2 f'(x) - 1}{x} > 0$$

$g \nearrow$

לכן

אם $\quad x<1 \Rightarrow g(x) < g(1)$

$g(x) < 0$ ✓

(B)

$$f(x) < \frac{\ln x}{x}$$

$$\lim_{x \to 0^+} f(x) < \lim_{x \to 0^+} \frac{\ln x}{x}$$

$$\lim_{x \to 0^+} f(x) < \lim_{x \to 0^+} \ln x \;\frac{1}{x}$$

$$\lim_{x \to 0^+} f(x) < -\infty \; (+\infty)$$

$$\lim_{x \to 0^+} f(x) < -\infty$$

$$\lim_{x \to 0^+} f(x) = -\infty$$

34. $f:(0,+\infty)\to\mathbb{R}$ $\quad f(1)=0$

$\forall x>0 \quad x f'(x)<f(x)+x^2\,\text{συν}\,x$

Νδο $\quad f(x)<x\,\text{ημ}\,x \quad \forall x>1$

$$\underbrace{f(x)-x\,\text{ημ}\,x}_{g(x)}<0$$

$$g'(x)=f'(x)-\text{ημ}\,x-x\,\text{συν}\,x \;\;;$$

$$x f'(x)<f(x)+x^2\,\text{συν}\,x$$

$$x f'(x)-f(x)<x^2\,\text{συν}\,x$$

$$\frac{x f'(x)-f(x)}{x^2}<\text{συν}\,x$$

$$\frac{x f'(x)-f(x)}{x^2}-\text{συν}\,x<0 \,.$$

$$\Big(\underbrace{\frac{f(x)}{x} - nrx}_{h(x)}\Big)' < 0$$

$$h(x) \downarrow$$

$$\forall x > n$$

$$h(x) < h(n)$$

$$\frac{f(x)}{x} - nr\,x < \frac{f(n)}{n}.$$

$$\frac{f(x)}{x} - nr\,x < 0$$

$$f(x) < x\,nr\,x$$

37. $f(x) = e^{-x} + x - 1$

(a) $f(x) < f(x^2) + \ln x$

$f(x) < f(x^2) + \ln \frac{x^2}{x}$

$f(x) < f(x^2) + \ln x^2 - \ln x$

$f(x) + \ln x < f(x^2) + \ln x^2$

$$\boxed{g(x) = f(x) + \ln x \quad x>0}$$

$g(x) < g(x^2)$

$g \nearrow$

$x < x^2$

$\underline{\underline{1 < x}}$

$f'(x) = -e^{-x} + 1$

x		0	
f'	$-$	0	$+$
f	$\searrow$		$\nearrow$

$g'(x) = f'(x) + \frac{1}{x} > 0$

$g \nearrow$

$\forall x > 0.$

(B) $e^{x-1} - 1 + f(x^5) = f(x^3)$ $[1,+\infty)$.

Προφανής ρίζα $\underline{\underline{x=1}}$

$$e^{x-1} - 1 + f(x^5) - f(x^3) = 0$$

- $x>1 \Rightarrow x-1>0 \Rightarrow e^{x-1}>1 \Rightarrow e^{x-1}-1>0$ (+)
- $x^3<x^5 \Rightarrow f(x^3)<f(x^5) \Rightarrow f(x^5)-f(x^3)>0$

$$e^{x-1}-1+f(x^5)-f(x^3)>0$$

$\forall x>1$

42. $f(x) = ημx \quad x \in [0, π]$

Βρες εφαπτομένη που διέρχεται $A\left(\frac{π}{2}, \frac{π}{2}\right)$

$$y - f(x_0) = f'(x_0)(x - x_0)$$

$$\frac{π}{2} - ημx = συνx\left(\frac{π}{2} - x\right)$$

$$\boxed{\underbrace{\frac{π}{2} - ημx - \frac{π}{2} συνx + x συνx}_{g(x)} = 0. \quad x \in [0, π].}$$

$$g(0) = \frac{π}{2} - \cancel{ημ0} - \frac{π}{2} συν0 - \cancel{0 συν0} = 0$$

$$g(π) = \frac{π}{2} - \cancel{ημπ} - \frac{π}{2} συνπ + π συνπ =$$

$$= \frac{π}{2} + \frac{π}{2} - π = 0.$$

Προφανείς ρίζες

$x = 0 \qquad x = π.$

$$g'(x) = \cancel{-\sigma\upsilon\nu x} + \frac{\pi}{2}\eta\mu x + \cancel{\sigma\upsilon\nu x} - x\,\eta\mu x$$

$$g'(x) = \eta\mu x\left(\frac{\pi}{2} - x\right)$$

x	0		$\frac{\pi}{2}$		π
$g'(x)$		$+$		$-$	
$g(x)$		$\nearrow$		$\searrow$	

Δυο διαστηματα

μονοτονιας αρα

εχω 2 ριζες το πολυ.

Εχω βρει ηδη δυο.

$$y - f(0) = f'(0)(x-0)$$

$$y - f(\pi) = f'(\pi)(x-\pi).$$

Επομενο Μαθημα

24

4

5

8

9

11

13

14 α γ

15 α β.

17

18 α γ

19 α β

20 α

23 α

24

25

26

28.