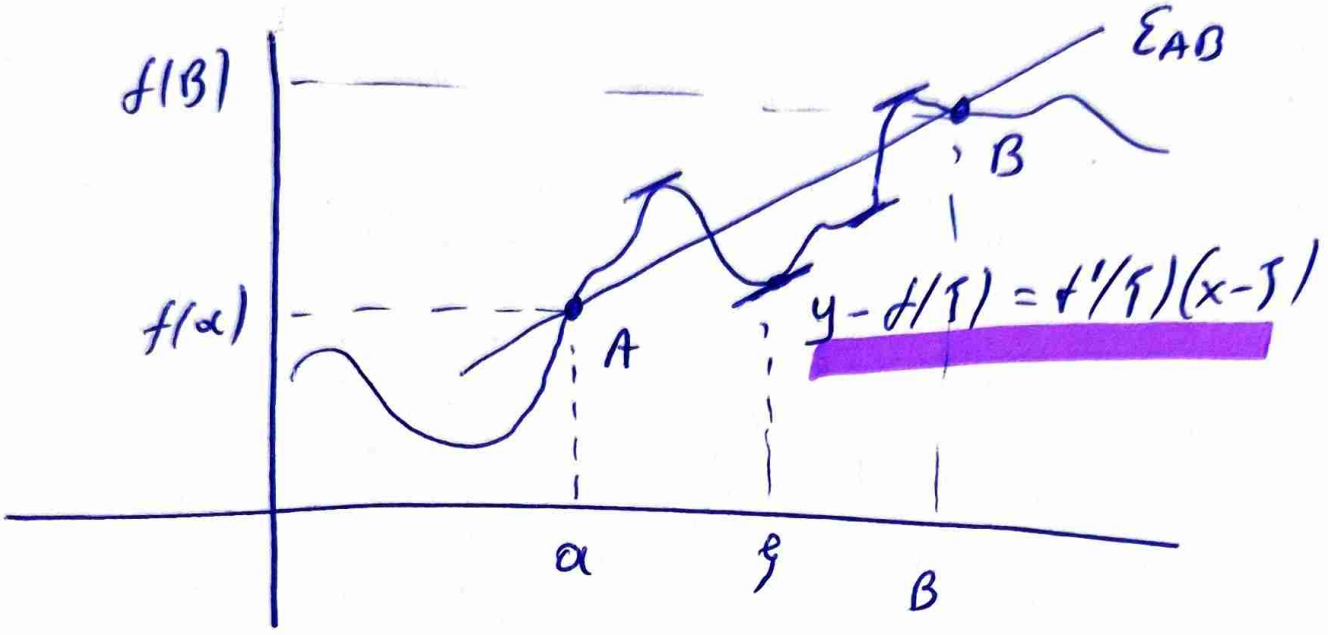
Θεώρημα Μέσης Τιμής



Παρατηρώ ότι $y - f(\xi) = f'(\xi)(x-\xi) \parallel \varepsilon_{AB}$

άρα $f'(\xi) = \lambda_{AB}$

$$f'(\xi) = \frac{y_B - y_A}{x_B - x_A}$$

$$f'(\xi) = \frac{f(\beta) - f(\alpha)}{\beta - \alpha}.$$

Θ. Μ. Τ

1. f συνεχής $[\alpha, \beta]$

2. f παραγωγίσιμη (α, β)

$$\exists\, \xi \in (\alpha, \beta) \text{ τ.ω } f'(\xi) = \frac{f(\beta) - f(\alpha)}{\beta - \alpha}.$$

ΘΜΤ και ανισότητες

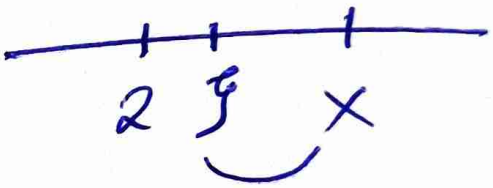
Έστω f είναι δυο φορά παραγωγίσιμη και $f'\nearrow$

f συν+παρ
f' συν+παρ
f''

1. Νδο $f'(x) > \dfrac{f(x)}{x-2}$ $\forall x>2$. αν $f(2)=0$

Η $f(x)$ συνεχής $[2,x]$

Η $f(x)$ παρ/μη $(2,x)$



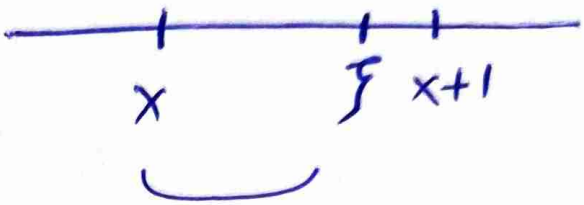
$$f'(\xi) = \frac{f(x)-f(2)}{x-2} = \frac{f(x)-0}{x-2} = \frac{f(x)}{x-2}$$

$$\xi < x \xrightarrow{f'\uparrow} f'(\xi) < f'(x)$$

$$\boxed{\frac{f(x)}{x-2} < f'(x)}$$

2. Να $f'(x) < f(x+1) - f(x)$

$\forall x \in \mathbb{R}$



$$f'(\xi) = \frac{f(x+1) - f(x)}{x+1 - x}$$

$$f'(\xi) = f(x+1) - f(x)$$

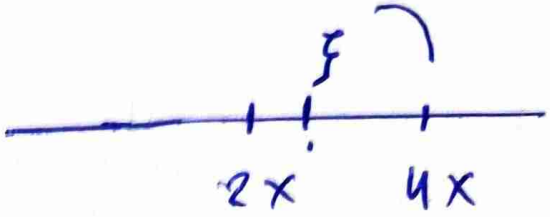
$$x < \xi \xrightarrow{f'\uparrow} f'(x) < f'(\xi)$$

$$\boxed{f'(x) < f(x+1) - f(x)}$$

3. Να $f(4x) - f(2x) < 2x f'(4x)$

$\forall x > 0$

$$f'(\xi) = \frac{f(4x) - f(2x)}{4x - 2x}$$



$$f'(\xi) = \frac{f(4x) - f(2x)}{2x}$$

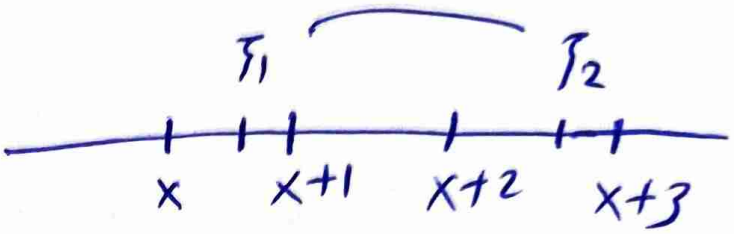
$\xi < 4x \xrightarrow{f'\uparrow} f'(\xi) < f'(4x)$

$$\frac{f(4x) - f(2x)}{2x} < f'(4x)$$

$$\boxed{f(4x) - f(2x) < 2x f'(4x)}$$

4. Να δ.ο. $f(x+1)+f(x+2) < f(x)+f(x+3)$

$\forall x \in \mathbb{R}$



$$f'(\xi_1) = \frac{f(x+1)-f(x)}{x+1-x} = \frac{f(x+1)-f(x)}{1} = f(x+1)-f(x)$$

$$f'(\xi_2) = \frac{f(x+3)-f(x+2)}{x+3-(x+2)} = f(x+3)-f(x+2)$$

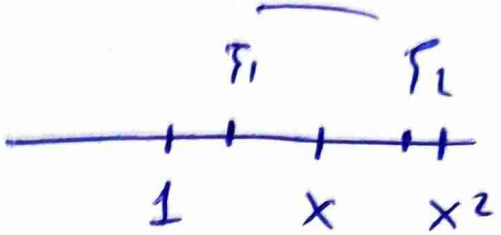
$\xi_1 < \xi_2 \xrightarrow{f'\uparrow} f'(\xi_1) < f'(\xi_2)$

$f(x+1)-f(x) < f(x+3)-f(x+2)$

~~$f(x+1)+f(x+2) < f(x)+f(x+3)$~~

5. Να δ.ο $(x+1)f(x) < xf(1) + f(x^2)$

$\forall x > 1$



$$f'(\xi_1) = \frac{f(x) - f(1)}{x-1}$$

$$f'(\xi_2) = \frac{f(x^2) - f(x)}{x^2 - x}$$

$$\xi_1 < \xi_2 \xrightarrow{f'\uparrow} f'(\xi_1) < f'(\xi_2)$$

$$\frac{f(x) - f(1)}{x-1} < \frac{f(x^2) - f(x)}{x(x-1)}$$

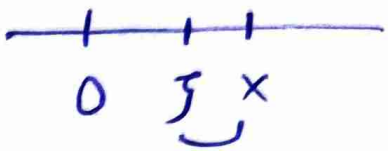
$$xf(x) - xf(1) < f(x^2) - f(x)$$

$$xf(x) + f(x) < xf(1) + f(x^2)$$

$$\boxed{(x+1)f(x) < xf(1) + f(x^2).}$$

6. Αν $f'(x) > 1$ για $x > 0$ και $f(0) = 0$ τότε

$x < f(x)$ και $f(x) < x f'(x)$



$$f'(\xi) = \frac{f(x) - f(0)}{x - 0} = \frac{f(x)}{x}$$

$$\xi < x \Rightarrow f'(\xi) < f'(x) \Rightarrow \frac{f(x)}{x} < f'(x)$$

$$f(x) < x f'(x)$$

Γνωρίζω μονοτονία για την $f'(x)$

$$f'(x) > 1 \quad \forall x > 0$$

$$f'(\xi) > 1$$

$$\frac{f(x)}{x} > 1$$

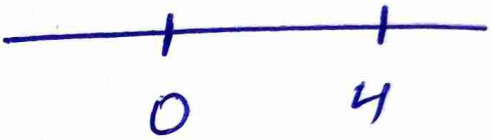
$$f(x) > x$$

Άσκηση

$f(0)=2$ και $f'(x)\le 3$

Ν.δ.ο $f(4)\le 14$.

Γνωρίζω ανισότητα για την $f'(x)$



$$f'(\xi)=\frac{f(4)-f(0)}{4-0}$$

$$\boxed{f'(\xi)=\frac{f(4)-2}{4}}$$

Όμως $f'(x)\le 3$

$f'(\xi)\le 3$

$\frac{f(4)-2}{4}\le 3$

$\Rightarrow f(4)-2\le 12$

$f(4)\le 14$

Παν22ανιά 2019

- $f(x) = (x-1)\ln(x^2-2x+2) - x + 2$
- $f'(x) \geq -1 \quad \forall x \in \mathbb{R}$

Νδο $f(\lambda+\frac{1}{2}) + \lambda \geq (\lambda-1)\ln(\lambda^2-2\lambda+2) + \frac{3}{2}$.

$$2 + f(\lambda+\tfrac{1}{2}) - \tfrac{3}{2} \geq (\lambda-1)\ln(\lambda^2-2\lambda+2) - \lambda + 2$$

$$\boxed{\frac{1}{2} + f(\lambda+\tfrac{1}{2}) \geq f(\lambda)}$$



$$f'(\xi) = \frac{f(\lambda+\frac{1}{2}) - f(\lambda)}{\lambda+\frac{1}{2}-\lambda}$$

$$f'(\xi) = \frac{f(\lambda+\frac{1}{2}) - f(\lambda)}{\frac{1}{2}} = 2f(\lambda+\tfrac{1}{2}) - 2f(\lambda)$$

Ομως $f'(x) \geq -1 \quad \forall x$

$$f'(\beta) \geq -1$$

$$2f\left(\lambda+\frac{1}{2}\right) - 2f(\lambda) \geq -1$$

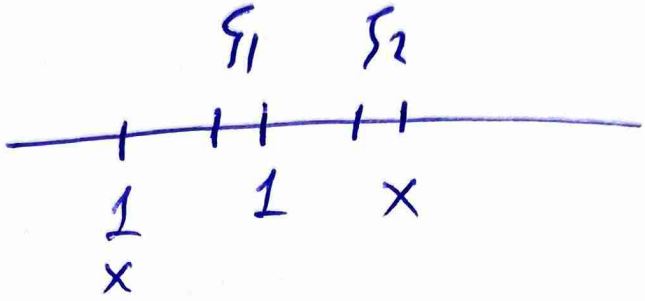
$$f\left(\lambda+\frac{1}{2}\right) - f(\lambda) \geq -\frac{1}{2}$$

$$\boxed{\frac{1}{2} + f\left(\lambda+\frac{1}{2}\right) \geq f(\lambda)}$$

7. Νδο $(x+1)G(1) < G(x) + G(\frac{1}{x})x$

αν $x>1$ και $f'(x)>0$ όπου

$G(x)$ παράγουσα της $f(x)$



G

f

f'

f''

$$G'(\xi_1) = \frac{G(1) - G(\frac{1}{x})}{1 - \frac{1}{x}}$$

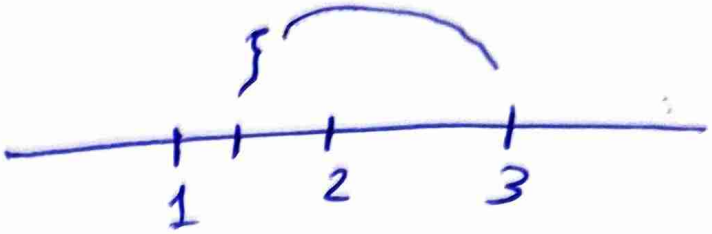
$$G'(\xi_2) = \frac{G(x) - G(1)}{x - 1}$$

$$\xi_1 < \xi_2 \xrightarrow[f']{G'} G'(\xi_1) < G'(\xi_2)$$

$f'(x)>0$

$$\frac{G(1) - G(\frac{1}{x})}{1 - \frac{1}{x}} < \frac{G(x) - G(1)}{x - 1}$$

8. Αν $f'(x)>0$ τότε $G(2)<f(3)+G(1)$



$$G'(\xi)=\frac{G(2)-G(1)}{2-1}$$

$$G'(\xi)=G(2)-G(1)$$

G

f

f'

$$\xi<3 \xrightarrow[f\uparrow]{G'\uparrow} G'(\xi)<G'(3)$$

$f'(x)>0$

$$G(2)-G(1)<f(3)$$

$$G(2)<f(3)+G(1)$$

$$\frac{G(1) - G\left(\frac{1}{x}\right)}{\frac{x-1}{x}} < \frac{G(x) - G(1)}{x-1}$$

$$\frac{xG(1) - xG\left(\frac{1}{x}\right)}{x-1} < \frac{G(x) - G(1)}{x-1}$$

$$xG(1) - xG\left(\frac{1}{x}\right) < G(x) - G(1)$$

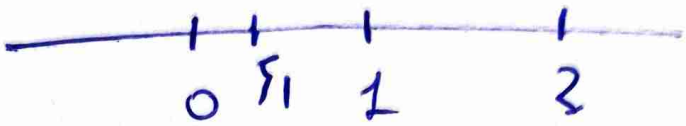
$$xG(1) + G(1) < G(x) + xG\left(\frac{1}{x}\right)$$

$$\boxed{(x+1)\,G(1) < G(x) + xG\left(\frac{1}{x}\right)}$$

Άσκηση

$f(0)=1 \qquad f(1)=3 \qquad f(2)=5$

Να δειχθεί ότι $\exists \xi \in (0,2)$ τ.ω $f''(\xi)=0$.



$$f'(\xi_1)=\frac{f(1)-f(0)}{1-0}=\frac{3-1}{1}=2$$

$$f'(\xi_2)=\frac{f(2)-f(1)}{2-1}=\frac{5-3}{1}=2$$

$f'(\xi_1)=f'(\xi_2)$

Rolle

$\exists \xi \in (\xi_1,\xi_2)$ τ.ω $f''(\xi)=0$