Άσκηση 1

$$A = \frac{x^2 - 4|x|}{|x| - 4}$$

(α) Για να ορίζεται η A πρέπει

$$|x| - 4 \neq 0$$

$$\rightarrow |x| - 4 = 0 \rightarrow |x| = 4 \Rightarrow x = 4$$

$$x = -4$$

Πρέπει $x \neq 4$, $x \neq -4$

(β) $$A = \frac{|x|^2 - 4|x|}{|x| - 4} = \frac{|x|(|x| - 4)}{|x| - 4}$$

$$A = |x|$$

$$x^2 = |x|^2$$

(γ) $A < 6$

$$|x| < 6$$

$$-6 < x < 6 \longrightarrow x \in (-6, -4) \cup (-4, 4) \cup (4, 6).$$

Άσκηση 2

(α) $|3x-2| = 7$

$3x-2 = 7$ ή $3x-2 = -7$

$3x = 9$ $3x = -5$

$x = 3$ $x = -\frac{5}{3}$

(β) $|x+1| = |2x-5|$

$x+1 = 2x-5$ ή $x+1 = -2x+5$

$x-2x = -5-1$ $x+2x = 5-1$

$-x = -6$ $3x = 4$

$x = 6$ $x = \frac{4}{3}$

(8) $|x-4|-3=0$

$|x-4|=3$

$x-4=3$ i $x-4=-3$

$x=7$ $x=1$.

Ασκηση 3

(B) $x\,y$

- $1<x<3$
- $2<y<5$

$\Rightarrow$ $2<xy<15$

(α) $x+y$

- $1<x<3$
- $2<y<5$

$\Rightarrow$ $3<x+y<8$.

① $2x-y$

• $1<x<3 \Rightarrow 2<2x<6 \Rightarrow 6>2x>2$

• $2<y<5 \Rightarrow -2>-y>-5$ ⊥ ⊕

$$4>2x-y>-3$$

② $\frac{x}{y}$

• $1<x<3 \Rightarrow 3>x>1$

• $2<y<5 \Rightarrow \frac{1}{2}>\frac{1}{y}>\frac{1}{5}$ } ⊙ $\frac{3}{2}>\frac{x}{y}>\frac{1}{5}$

Άσκηση 4

α) $\frac{x^2-9}{x^2+6x+9} = \frac{(x-3)(x+3)}{(x+3)^2} = \frac{x-3}{x+3}$

πρέπει $x^2+6x+9 \neq 0$

$(x+3)^2 \neq 0$

$x \neq -3$

(В) $\frac{x^3-4x}{x^2-2x} = \frac{x(x^2-4)}{x(x-2)} = \frac{(x-2)(x+2)}{x-2} = x+2$

при $x^2-2x \neq 0$

$x(x-2) \neq 0$

$x \neq 0 \quad x \neq 2$

(Г) $\frac{x^2-4x+4}{x^2-4} = \frac{(x-2)^2}{(x-2)(x+2)} = \frac{x-2}{x+2}$

при $x^2-4 \neq 0$

$(x-2)(x+2) \neq 0$

$x \neq 2 \quad x \neq -2$

Άσκηση 5

(α) $|\overset{(+)}{x+3}| + |\overset{(-)}{x-1}| = x+3-x+1 = 4$

• $-3 < x < -1 \Rightarrow 0 < x+3 < 2$

• $-3 < x < -1 \Rightarrow -4 < x-1 < -2$

(β) $|\overset{(-)}{2x-1}| - |\overset{(-)}{x+1}| = -2x+1-(-x-1) \overset{(*)}{=}$

• $-3 < x < -1 \Rightarrow -6 < 2x < -2 \Rightarrow -7 < 2x-1 < -3$

• $-3 < x < -1 \Rightarrow -2 < x+1 < 0$

$\overset{(*)}{=} -2x+1+x+1 = -x+2$

(γ) $|\overset{(-)}{3x+2}| - |\overset{(-)}{x-4}| = -3x-2-(-x+4) = -2x-6$

• $-3 < x < -1 \Rightarrow -9 < 3x < -3 \Rightarrow -7 < 3x+2 < -1$

• $-3 < x < -1 \Rightarrow -7 < x-4 < -5$

Άσκηση 6

(α) Να δείξετε ότι $x^2+4y^2 \geq 4|xy|$

$|x|^2+4|y|^2 \geq 4|x||y|$

$|x|^2+4|y|^2-4|x||y| \geq 0$

$(|x|-2|y|)^2 \geq 0$ ✓

(β) Να δείξετε ότι $\frac{2|a|}{a^2+1} \leq 1$

$2|a| \leq a^2+1$

$2|a| \leq |a|^2+1$

$0 \leq |a|^2-2|a|+1$

$0 \leq (|a|-1)^2$

✓

Ασκηση 7

(a) $|x-1| \leq 3$

$-3 \leq x-1 \leq 3$

$1-3 \leq x \leq 3+1$

$-2 \leq x \leq 4$ ή $x \in [-2, 4]$.

(β) $|2x+3| > 5$

$2x+3 > 5$ ή $2x+3 < -5$

$2x > 2$ $2x < -8$

$x > 1$ $x < -4$

$x \in (-\infty, -4) \cup (1, +\infty)$.

(β) $|x+2| < |x-1|$

$|x+2|^2 < |x-1|^2$

$(x+2)^2 < (x-1)^2$

$x^2+4x+4 < x^2-2x+1.$

$4x+2x < 1-4$

$6x < -3$

$x < -\frac{1}{2}$

$x \in (-\infty, -\frac{1}{2})$

Άσκηση 8

(α) $(3x-2)^2 - x(x-1)(x+1) =$

$= 9x^2 - 12x + 4 - x(x^2-1)$

$= 9x^2 - 12x + 4 - x^3 + x$

$= -x^3 + 9x^2 - 11x + 4$

(β) $2-(x+1)^2-(2x-1)(x-2)=$

$=2-(x^2+2x+1)-(2x^2-4x-x+2)$

$=2-x^2-2x-1-(2x^2-5x+2)$

$=1-x^2-2x-2x^2+5x-2$

$=-3x^2+3x-1$

(γ) $(x^2-x+1)^2-x^2(x^2+2)=$

$=(x^2-x)^2+2(x^2+x)\cdot 1+1^2-x^4-2x^2$

$=x^4-2x^3+x^2+2x^2+2x+1-x^4-2x^2$

$=-2x^3+x^2+2x+1$

Άσκηση 9

(α) $x^2+y^2-4x+6y+13=0$

$x^2-4x+4+y^2+6y+9=0$

$(x-2)^2+(y+3)^2=0$

$x-2=0$ $\quad$ $y+3=0$

$x=2$ $\quad$ $y=-3$

(β) $2x^2+2y^2-8x+4y+10=0$

$x^2+y^2-4x+2y+5=0$

$x^2-4x+4+y^2+2y+1=0$

$(x-2)^2+(y+1)^2=0$

$x-2=0$ και $y+1=0$

$x=2$ $\quad$ $y=-1$.

Άσκηση 10

$$d(a,0) < 1$$

$$|a-0| < 1$$

$$|a| < 1 \Rightarrow -1 < a < 1$$

(01) $A = |2 - |a-1|| = |2-(-a+1)| =$

• $-1 < a < 1 \Rightarrow -2 < a-1 < 0$

$= |2+a-1| = |a+1| = a+1$

• $-1 < a < 1 \rightarrow 0 < a+1 < 2$

(13) $|2a-3| + |a+2| = -2a+3+a+2 = -a+5$

• $-1 < a < 1 \Rightarrow -2 < 2a < 2 \Rightarrow -5 < 2a-3 < -1$

• $-1 < a < 1 \Rightarrow 1 < a+2 < 3$

(β) $|a^2-1| \overset{(5)}{=} 1-a^2$

• $-1<a<1 \Rightarrow |a|<1 \Rightarrow |a|^2<1^2 \Rightarrow a^2<1$

$a^2-1<0$

Άσκηση 11

(α) $$\frac{(x^{-2}y^3)^{-1}(x^2y^{-1})^2}{x^3y^{-2}} =$$

$$= \frac{x^2y^{-3}\,x^4y^{-2}}{x^3y^{-2}} = \frac{x^6y^{-5}}{x^3y^{-2}} = x^3y^{-3}$$

(β) $$\frac{(x^{-1}y^2)^3}{(x^2y^{-1})^{-2}} = \frac{x^{-3}y^6}{x^{-4}y^2} = xy^4$$

Άσκηση 12

(α) $-2 \leq x < 5 \Rightarrow x \in [-2, 5)$

(β) $x < -1$ ή $x \geq 3 \Rightarrow x \in (-\infty, -1) \cup [3, +\infty)$

(γ) $|x-2| \leq 4 \Rightarrow -4 \leq x-2 \leq 4$

$-2 \leq x \leq 6$

$x \in [-2, 6]$

(δ) $|x+1| > 3$

$x+1 > 3$ ή $x+1 < -3$

$x > 2$ $\quad$ $x < -4$

$x \in (-\infty, -4) \cup (2, +\infty)$

Asmen 16

$$A = \left(\frac{51}{50} + \frac{50}{51}\right)^2 - \left(\frac{51}{50} - \frac{50}{51}\right)^2$$

$$A = \left(\frac{51}{50} + \frac{50}{51} + \frac{51}{50} - \frac{50}{51}\right)\left(\frac{51}{50} + \frac{50}{51} - \frac{51}{50} + \frac{50}{51}\right)$$

$$A = 2 \cdot \frac{51}{50} \cdot 2\frac{50}{51} = 4.$$

Asmen 17

Av $a < -1 < b$

(a) $|a| + |a+1| - |a-b| = -a - a - 1 - (-a+b)$

$= -2a - 1 + a - b$

$= -a - b - 1$

- $a < -1 \Rightarrow a < 0$
- $a < -1 \Rightarrow a + 1 < 0$
- $a < b \Rightarrow a - b < 0$

(B) $|b-a| - |a+1| + |b+1|$ (+) (−) (+)

$= b-a-(-a-1)+b+1$

$= b-a+a+1+b+1$

$= 2b+2$

- $a < b \Rightarrow 0 < b-a$
- $a < -1 \Rightarrow a+1 < 0$
- $b > -1 \Rightarrow b+1 > 0$

Άσκηση 18

(α) $x^2 < 16 \Rightarrow x^2 < 4^2$

$|x| < |4|$

$|x| < 4$

$-4 < x < 4$

(β) $x^2 > 4 \Rightarrow x^2 > 2^2 \Rightarrow |x| > |2| \Rightarrow |x| > 2$

$x > 2$ ή $x < -2$

(δ) $(x-2)^3 \geq 0$

$x-2 \geq 0$

$x \geq 2$

Άσκηση 23

(α) $x^2+y^2-2x-4y+5=0$

$x^2-2x+1+y^2-4y+4=0$

$(x-1)^2+(y-2)^2=0$

$x-1=0$ και $y-2=0$

$x=1$ $y=2$

(β) $x^2+4y^2-6x+8y+13=0$.

$x^2-6x+9+4y^2+8y+4$

$(x-3)^2+4(y+1)^2=0$

$x-3=0$ $y+1=0$

$x=3$ $y=-1$

Άσκηση 24

(γ) $x^2 - y$

- $2 \leq x < 3 \Rightarrow 4 \leq x^2 < 9$ (+)
- $4 \leq y \leq 5 \Rightarrow -4 \geq -y \geq -5 \Rightarrow -5 \leq -y \leq -4$

$$-1 \leq x^2 - y < 5$$

(δ) $\frac{y}{x}$

- $2 \leq x < 3 \Rightarrow \frac{1}{2} \geq \frac{1}{x} > \frac{1}{3}$
- $4 \leq y \leq 5 \Rightarrow 5 \geq y \geq 4$ (·)

$$\frac{5}{2} \geq \frac{y}{x} \geq \frac{4}{3}$$

M.1

$A = |x-1| - 2x$

- Αν $x-1 \geq 0 \Rightarrow x \geq 1$ τότε

$A = \overset{\oplus}{|x-1|} - 2x = x-1-2x$

$$\boxed{A = -x-1}$$

- Αν $x-1 < 0 \Rightarrow x < 1$ τότε

$A = \overset{\ominus}{|x-1|} - 2x$

$A = -x+1-2x$

$$\boxed{A = -3x+1}$$

M.2

$$A = |x-1| - |x-2|$$

x		1		2	
$x-1$	$-$	0	$+$		$+$
$x-2$	$-$		$-$	0	$+$

- $x < 1$

$$A = |\overset{(-)}{x-1}| - |\overset{(-)}{x-2}| = -x+1-(-x+2) = -x+1+x-2$$

$$= -1$$

- $1 \leq x \leq 2$

$$A = |\overset{(+)}{x-1}| - |\overset{(-)}{x-2}| = x-1-(-x+2) =$$

$$= x-1+x-2 = 2x-3$$

- $x > 2$

$$A = |\overset{(+)}{x-1}| - |\overset{(+)}{x-2}| = x-1-(x-2) = x-1-x+2$$

$$= 1$$

Μ.3

Αν $|\alpha| \leq 1$ και $|\beta| \leq 2$

τότε $|2\alpha - 3\beta + 1| \leq 9$.

Τριγωνική Ανισότητα

- $|2\alpha - 3\beta + 1| \leq |2\alpha| + |-3\beta| + |1|$

$|2\alpha - 3\beta + 1| \leq 2|\alpha| + 3|\beta| + 1 \leq 9$

- $|2\alpha - 3\beta + 1| \leq 9$

- $|\alpha| \leq 1 \Rightarrow 2|\alpha| \leq 2$
- $|\beta| \leq 2 \Rightarrow 3|\beta| \leq 6$

} (+) $2|\alpha| + 3|\beta| \leq 8$

$2|\alpha| + 3|\beta| + 1 \leq 9$

1. $|\alpha \cdot \beta| = |\alpha||\beta|$

Απόδειξη

$|\alpha\beta| = |\alpha||\beta|$

$|\alpha\beta|^2 = (|\alpha||\beta|)^2$

$(\alpha\beta)^2 = |\alpha|^2 |\beta|^2$

$\alpha^2\beta^2 = \alpha^2\beta^2$ που ισχύει.

2. $|\alpha+\beta| \leq |\alpha| + |\beta|$

Απόδειξη

$|\alpha+\beta| \leq |\alpha| + |\beta|$

$|\alpha+\beta|^2 \leq (|\alpha| + |\beta|)^2$

$(\alpha+\beta)^2 \leq |\alpha|^2 + 2|\alpha||\beta| + |\beta|^2$

$\alpha^2 + 2\alpha\beta + \beta^2 \leq \alpha^2 + 2|\alpha\beta| + \beta^2$

$2\alpha\beta \leq 2|\alpha\beta| \Rightarrow \alpha\beta \leq |\alpha\beta|$ που ισχύει.

Γνωστό $|x| \geq x \quad \forall x \in \mathbb{R}$

$$|\alpha\beta| \geq \alpha\beta$$

Πότε ισχύει το "=";

$$|\alpha\beta| = \alpha\beta$$

όταν $\alpha\beta \geq 0$

ή $\alpha\beta < 0$

$\alpha \cdot \beta = 0$

α, β ετερόσημοι

$\alpha = 0$ ή $\beta = 0$