Ενοτητα 23

2. (β) $f(x) = \frac{1}{x} - \ln x \quad , \quad x > 0$

$f'(x) = -\frac{1}{x^2} - \frac{1}{x} < 0$

$f \downarrow$

(δ) $f(x) = 2x + \sigma\upsilon\nu x - 1$

$f'(x) = 2 - \eta\mu x > 0 \qquad f \nearrow$

$-1 \leq \eta\mu x \leq 1$

$1 \geq -\eta\mu x \geq -1$

$3 \geq 2 - \eta\mu x \geq 1$

(στ) $f(x) = x + \sigma\upsilon\nu x - 1$

$f'(x) = 1 - \eta\mu x \geq 0$

$f \nearrow$

3. (B) $f(x) = -x^2 - 2x + 3$

$$\boxed{f'(x) = -2x - 2}$$

→ $f'(x) = 0$

$-2x - 2 = 0$

$-2x = 2$

$$\boxed{x = -1}$$

x		-1	
f'	$+$	0	$-$
f	$\nearrow$		$\searrow$

Η f γν. αυξ. $(-\infty, -1]$

Η f γν. φθιν. $[-1, +\infty)$

$f(x) \leq f(-1)$

$f(x) \leq 4$

3.

δ) $f(x) = -x^3 + x^2 + x + 3$

$$\boxed{f'(x) = -3x^2 + 2x + 1}$$

$f'(x) = 0$

$-3x^2 + 2x + 1 = 0$

$\Delta = 4 + 12 = 16$

$x = \dfrac{-2 \pm 4}{-6}$ → $-\dfrac{1}{3}$, 1

x		$-\frac{1}{3}$		1	
f'	$-$	0	$+$	0	$-$
f	↘		↗		↘

στ) $f(x) = x^3 - 6x + 1$

$f'(x) = 3x^2 - 6 = 3(x^2 - 2)$

→ $f'(x) = 0 \Rightarrow x^2 - 2 = 0$

$x = \pm\sqrt{2}$

x		$-\sqrt{2}$		$\sqrt{2}$	
f'	$+$		$-$		$+$
f	↗		↘		↗

4. β) $f(x) = x^4 + 32x$

$$f'(x) = 4x^3 + 32 = 4(x^3 + 8)$$

$\rightarrow f'(x) = 0 \Rightarrow x^3 + 8 = 0$

$$x^3 = -8$$

$$x = -2$$

x		-2	
f'	$-$	0	$+$
f	$\searrow$		$\nearrow$

$f(x) \geq f(-2)$

$f(x) \geq -48$

δ) $f(x) = \frac{x^4}{4} - 2x^3 + \frac{11}{2}x^2 - 6x - 1$

$$f'(x) = \frac{1}{4} 4x^3 - 6x^2 + \frac{11}{2} 2x - 6$$

$$f'(x) = x^3 - 6x^2 + 11x - 6$$

1	-6	11	-6	1
$\downarrow$	1	-5	6	
1	-5	6	0	

$$f'(x) = (x-1)(x^2-5x+6)$$

$\rightarrow f'(x)=0 \Rightarrow x-1=0$ ή $x^2-5x+6=0$

$x=1$ $\quad$ $x=2$ $\quad$ $x=3$

x		1		2		3	
$x-1$	$-$	0	$+$		$+$		$+$
x^2-5x+6	$+$		$+$	0	$-$	0	$+$
f'	$-$		$+$		$-$		$+$
f	↘		↗		↘		↗

4. σε $\quad f(x) = x^4 + 2x^2 - 8x + 6$

$$f'(x) = 4x^3 + 4x - 8$$

$$f'(x) = 4(x^3 + x - 2)$$

1	0	1	−2	1
↓	1	1	2	
1	1	2	0	

$$\boxed{f'(x) = 4(x-1)(x^2+x+2)}$$

$\to f'(x) = 0 \Rightarrow x - 1 = 0$ ή $x^2 + x + 2 = 0$

$\Delta < 0$

$x = 1$

x		1	
$x-1$	$-$		$+$
x^2+x+2	$+$		$+$
f'	$-$		$+$
f	↘		↗

$f(x) \geq f(1)$

$f(x) \geq 1$

5. ε $f(x) = \frac{x^5}{5} - \frac{x^3}{3} + 2x$

$$f'(x) = \frac{1}{5} 5x^4 - \frac{1}{3} 3x^2 + 2$$

$$\boxed{f'(x) = x^4 - x^2 + 2} > 0$$

$$\Delta = \beta^2 - 4\alpha\gamma$$

$$\Delta = (-1)^2 - 4 \cdot 2$$

$$\Delta = 1 - 8 = -7$$

εχει

δομη

τριωνυμου !

$f \nearrow$

6. (5) $f(x) = x^4 - \frac{4}{3}x^3 - 2x^2 + 4x - 1$

$$f'(x) = 4x^3 - \frac{4}{3}3x^2 - 2\cdot 2x + 4$$

$$f'(x) = 4x^3 - 4x^2 - 4x + 4$$

$$f'(x) = 4(x^3 - x^2 - x + 1)$$

1	-1	-1	1	1
↓	1	0	-1	
1	0	-1	0	

$$\boxed{f'(x) = 4(x-1)(x^2-1)}$$

→ $f'(x) = 0 \Rightarrow x - 1 = 0$ ή $x^2 - 1 = 0$

$x = 1$

$x^2 = 1$

$x = 1$ $x = -1$

x		-1		1	
$x-1$	$-$		$-$	0	$+$
x^2-1	$+$	0	$-$	0	$+$
f'	$-$		$+$		$-$
f	↘		↗		↘

6. B $f(x) = -\frac{4x^3}{3} + 2x^2 - x - 4$

$$f'(x) = -\frac{4}{3}\, 3x^2 + 2 \cdot 2x - 1$$

$$f'(x) = -4x^2 + 4x - 1$$

$$f'(x) = -\left(4x^2 - 4x + 1\right)$$

$$f'(x) = -(2x-1)^2 \leq 0$$

fd.

6. 02 $f(x) = (x+1)^5 \cdot (x-2)^4$

$$f'(x) = 5(x+1)^4 (x-2)^4 + (x+1)^5 \cdot 4(x-2)^3$$

$$f'(x) = (x+1)^4 (x-2)^3 \left[5(x-2) + 4(x+1)\right]$$

$$\boxed{f'(x) = (x+1)^4 (x-2)^3 (9x-6)}$$

$\rightarrow$ $f'(x) = 0 \Rightarrow x+1=0$ ή $x-2=0$

$x=-1$ $\quad$ $x=2$

ή $9x-6=0$

$x = \frac{2}{3}$

x		-1		$\frac{2}{3}$		2	
$(x+1)^4$	$+$	0	$+$		$+$		$+$
$(x-2)^3$	$-$		$-$		$-$	0	$+$
$9x-6$	$-$		$-$	0	$+$		$+$
f'	$+$		$+$		$-$		$+$
f	$\nearrow$		$\nearrow$		$\searrow$		$\nearrow$

5. γ) $f(x) = \frac{1}{5}x^5 + \frac{1}{2}x^2 - 1$

$f'(x) = \frac{1}{\cancel{5}}\cancel{5}x^4 + \frac{1}{\cancel{2}}\cancel{2}x$

$f'(x) = x^4 + x$.

$\boxed{f'(x) = x(x^3+1)}$

$\rightarrow f'(x) = 0 \Rightarrow x = 0$ ή $x^3+1=0$

$x^3 = -1$

$x = -1$

x		-1		0	
x	$-$		$-$	0	$+$
x^3+1	$-$	0	$+$		$+$
f'	$+$		$-$		$+$
f	↗		↘		↗

7. σ2 $f(x) = x + \frac{4}{x^2}$, $x \neq 0$

$$f'(x) = 1 + \frac{-4 \cdot 2x}{x^4}$$

$$f'(x) = 1 - \frac{8}{x^3}$$

$$f'(x) = \frac{x^3}{x^3} - \frac{8}{x^3}$$

$$\boxed{f'(x) = \frac{x^3 - 8}{x^3}}$$

$\rightarrow f'(x) = 0 \Rightarrow \frac{x^3-8}{x^3} = 0 \Rightarrow x^3 - 8 = 0$

$x^3 = 8$

$x = 2$

x		0		2	
x^3-8	$-$		$-$	0	$+$
x^3	$-$	0	$+$		$+$
f'	$+$		$-$		$+$
f	↗		↘		↗

7. (B) $f(x) = \frac{x}{x^2+1}$ $D_f = \mathbb{R}$

$$f'(x) = \frac{x^2+1 - x \cdot 2x}{(x^2+1)^2}$$

$$f'(x) = \frac{x^2+1-2x^2}{(x^2+1)^2} \Rightarrow \boxed{f'(x) = \frac{1-x^2}{(x^2+1)^2}}$$

$\longrightarrow f'(x) = 0 \Rightarrow \frac{1-x^2}{(x^2+1)^2} = 0$

$1 - x^2 = 0$

$x^2 = 1$ $x=1$ $x=-1$

x		-1		1	
$1-x^2$	$-$	0	$+$	0	$-$
$(x^2+1)^2$	$+$		$+$		$+$
f'	$-$		$+$		$-$
f	$\searrow$		$\nearrow$		$\searrow$

7. γ) $f(x) = \dfrac{x}{x^2-1}$ $\quad A_f = \mathbb{R} - \{1, -1\}$.

$$f'(x) = \frac{x^2-1 - x \cdot 2x}{(x^2-1)^2} = \frac{x^2-1-2x^2}{(x^2-1)^2}$$

$$\boxed{f'(x) = \frac{-1-x^2}{(x^2-1)^2} < 0}$$

x	-1	1
$f'(x)$	$-$ ‖ $-$ ‖ $-$	
$f(x)$	↘ ‖ ↘ ‖ ↘	

Η f γν. φθίνουσα στο

$(-\infty, -1)$ και $(-1, 1)$ και $(1, +\infty)$.

Πανελληνια 2025
Β' Θεμα.

Δινεται $f(x) = x^3 - 6x^2 + 9x - 3$

Νδο η εξισωση $f(x) = 0$ εχει

3 θετικες ριζες.

Λυση

Συνολο Τιμων

$D_f = \mathbb{R}$.

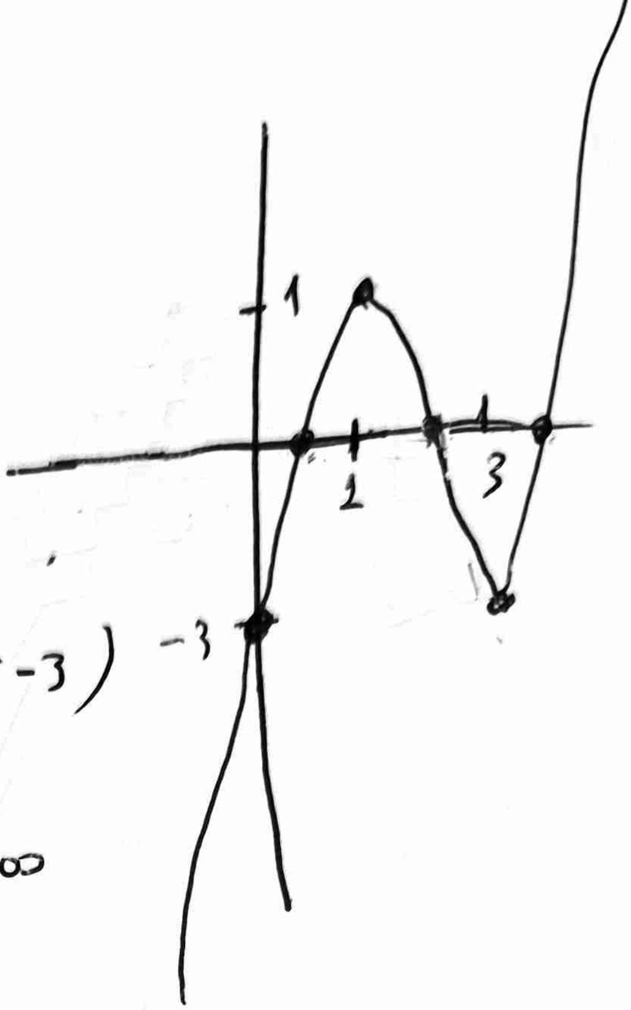
$f'(x) = 3x^2 - 12x + 9 = 3(x^2 - 4x + 3)$

$\rightarrow f'(x) = 0 \Rightarrow x^2 - 4x + 3 = 0$

$x = 1$ $\quad$ $x = 3$

x		1		3	
f'	+	0	−	0	+
f	↗		↘		↗

x	$-\infty$ 1 3 $+\infty$		
$f(x)$	$-\infty \nearrow 1$	$\searrow -3$	$\nearrow$



$$\lim_{x\to-\infty} f(x) = \lim_{x\to-\infty} (x^3 - 6x^2 + 9x - 3)$$

$$= \lim_{x\to-\infty} x^3 = -\infty$$

$f(1) = 1$

$f(3) = -3$

$\lim_{x\to+\infty} f(x) = +\infty.$

$x < 1$

- f συνεχής
- $f \nearrow$
- $ΣΤ_f = (-\infty, 1)$

Το $0 \in ΣΤ_f$
άρα $\exists x_1$ τ.ω
$f(x_1) = 0$

$1 \leq x \leq 3$

- f συνεχής
- $f \downarrow$
- $ΣΤ_f = [-3, 1]$

Το $0 \in ΣΤ_f$
άρα $\exists x_2$ τ.ω
$f(x_2) = 0.$

$x > 3$

- f συνεχής
- $f \nearrow$
- $ΣΤ_f = [-3, +\infty)$

Το $0 \in ΣΤ_f$
άρα $\exists x_3$
τ.ω $f(x_3) = 0$

Έστω $x_1 \leq 0$

$f(x_1) \leq f(0)$

$0 \leq -3$

Άτοπο!.

$x_1 > 0$

Επομενω Μαθημα

23

2 α γ ε

3 α γ ε

4 α γ ε

5 α β δ

6 α γ ς

7 α δ ε.