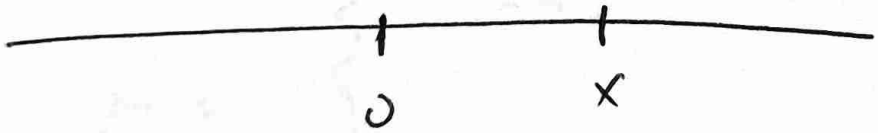
Γεωμετρικά

$$|\alpha - \beta| = d(\alpha, \beta)$$

$$|x| = |x - 0|$$



$A = |x-3|$

1. Αν $x-3 \geq 0$ δηλαδή $x \geq 3$

$$A = |\overset{\oplus}{x-3}| = x-3$$

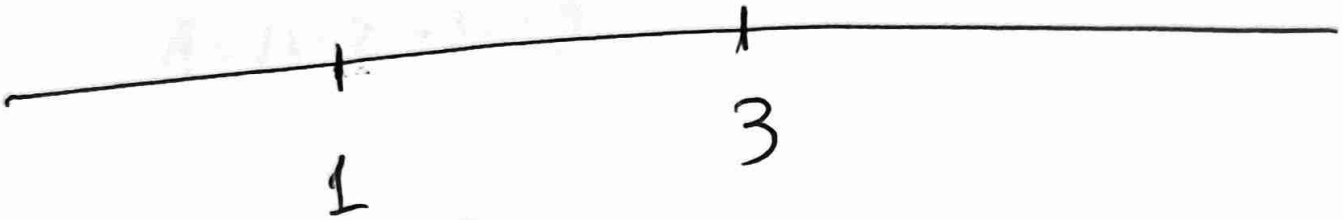
2. Αν $x-3 < 0$ δηλαδή $x < 3$

$$A = |\overset{\ominus}{x-3}| = 3-x .$$

$$A = \begin{cases} 3-x, & x < 3 \\ x-3, & x \geq 3 . \end{cases}$$

$$A = 2|x-1| - 3|x-3|$$

x		1		3	
x-1	−	0	+		+
x-3	−		−	0	+



$x < 1$

$A = 2|x-1| - 3|x-3| = 2(-x+1) - 3(-x+3)$

$A = -2x+2+3x-9 \quad = x-7$

$1 \le x \le 3$

$A = 2|x-1| - 3|x-3|$

$A = 2(x-1) - 3(-x+3)$

$A = 2x-2+3x-9$

$A = 5x-11$

$x > 3$

$A = 2|x-1| - 3|x-3|$ (+) (+)

$A = 2(x-1) - 3(x-3)$

$A = 2x - 2 - 3x + 9$

$A = -x + 7$

$$A = \begin{cases} x-7 & x < 1 \\ 5x-11 & 1 \le x \le 3 \\ -x+7. & x > 3 \end{cases}$$

19. Av $x<2$

(a) $A = |3x-6| - |4-2x| + x$

- $x<2 \Rightarrow 3x<6 \Rightarrow 3x-6<0$ (−)
- $x<2 \Rightarrow -2x>-4 \Rightarrow 4-2x>0$ (+)

$A = -3x+6 -4+2x+x$

$A = -x+x+2$ $\quad \underline{\underline{A=2}}$

(B) $B = 4\left|\frac{x}{2}-1\right| + 8\left|\frac{x}{4}-\frac{1}{2}\right|$.

- $x<2 \Rightarrow \frac{x}{2}<1 \Rightarrow \frac{x}{2}-1<0$ (−)
- $x<2 \Rightarrow \frac{x}{4}<\frac{2}{4} \Rightarrow \frac{x}{4}<\frac{1}{2}$

$\frac{x}{4}-\frac{1}{2}<\frac{1}{2}-\frac{1}{2}$

$\frac{x}{4}-\frac{1}{2}<0$ (−)

$B = 4\left(-\frac{x}{2}+1\right) + 8\left(-\frac{x}{4}+\frac{1}{2}\right)$

$B = -2x+4-2x+4 = -4x+8.$

13. ⓐ $1 < x < 3 \Rightarrow 3 < 3x < 9$

$2 < y < 3 \Rightarrow -2 > -y > -3$

$-3 < -y < -2$ ⊕

$A = |\overset{\oplus}{3x-y}| = 3x-y$

$0 < 3x-y < 7$

ⓑ $B = |\overset{\ominus}{xy-9}| = -xy+9$.

. $1 < x < 3$ } ⊙ $2 < xy < 9$

. $2 < y < 3$ $-7 < xy-9 < 0$

ⓒ $|\overset{\oplus}{x-\frac{1}{y}}| = x-\frac{1}{y}$

⊕

. $1 < x < 3$

. $2 < y < 3 \Rightarrow \frac{1}{2} > \frac{1}{y} > \frac{1}{3} \Rightarrow -\frac{1}{2} < -\frac{1}{y} < -\frac{1}{3}$

$\frac{1}{2} < x-\frac{1}{y} < \frac{8}{3}$

2L. ⓐ $|x| = 1 \Rightarrow x = 1$ ή $x = -1$

Ⓑ $|x| - 3 = 0 \Rightarrow |x| = 3$

$x = 3$ ή $x = -3$

ⓖ $3|x| - 5 = 0$

$3|x| = 5$

$|x| = \frac{5}{3}$

$x = \frac{5}{3}$ ή $x = -\frac{5}{3}$

$|x| = 0$

$x = 0$ ή $x = -0$.

ⓓ $|x| + 2 = 0$

$|x| = -2$ Άδικα.

$|-x| = |x|$

ⓔ $2|x| - 5|-x| + 6 = 0$

$2|x| - 5|x| + 6 = 0$

$-3|x| + 6 = 0$

$-3|x| = -6$

$|x| = 2$

$x = 2$

ή $x = -2$

(52) $|2x| - \left|\frac{x}{2}\right| - 1 = 0.$

$|2| \cdot |x| - \frac{|x|}{|2|} - 1 = 0$

$2|x| - \frac{|x|}{2} - 1 = 0$

$4|x| - |x| - 2 = 0$

$3|x| = 2$

$|x| = \frac{2}{3}$ $\quad x = \frac{2}{3}$ ou $x = -\frac{2}{3}$.

$$|\alpha \cdot \beta| = |\alpha| \cdot |\beta|$$

$$\left|\frac{\alpha}{\beta}\right| = \frac{|\alpha|}{|\beta|}.$$

(7) $x^2 - 2|x| = 0$

$|x|^2 - 2|x| = 0$

$|x|(|x| - 2) = 0$

$x^2 = |x|^2 = |x^2|$

$|x| = 0$ or $|x| - 2 = 0$

$x = 0$ $|x| = 2$

$x = 2$ or $x = -2$

22.

(a) $|x-1| = 5$

$x-1 = 5$ ή $x-1 = -5$

$x = 6$ $x = -4$

(β) $|x-2| - 1 = 0$

$|x-2| = 1$

$x-2 = 1$ ή $x-2 = -1$

$x = 3$ $x = 1$

(γ) $3|x-2| - 2|2-x| - 5 = 0$

$3|x-2| - 2|x-2| - 5 = 0$

$|x-2| = 5$

$x-2 = 5$ $x-2 = -5$

$x = 7$ $x = -3$

(δ) $|3x-6| - |4-2x| = 1$

$|3(x-2)| - |2(2-x)| = 1$

$3|x-2| - 2|2-x| = 1$

$3|x-2| - 2|x-2| = 1$

$\therefore |x-2| = 1$

$x-2 = 1$ $\qquad$ $x-2 = -1$

$x = 3$ $\qquad$ $x = 1$

23. ⓐ $|2x-1| = |x-2|$

$|x| = |y|$

$x = y$ ή $x = -y$

$2x-1 = x-2$ ή $2x-1 = -x+2$

$x = -1$ $\qquad$ $3x = 3$

$x = 1$

ⓑ $|3x-2| - |2x-3| = 0$.

$|3x-2| = |2x-3|$

$3x-2 = 2x-3$ ή $3x-2 = -2x+3$

$x = -1$ $\qquad$ $5x = 5$

$x = 1$

ⓓ $3|x-2| - 2|x| = 0$

$3|x-2| = 2|x|$

$|3||x-2| = |2||x|$

$|3x-6| = |2x|$

$3x-6 = 2x$ ή $3x-6 = -2x$

$x = 6$ $\qquad$ $5x = 6$ $\qquad$ $x = \frac{6}{5}$

24. (α) $|x| < 5$

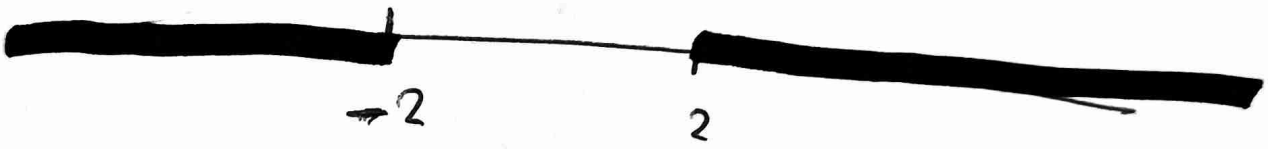
$-5 < x < 5$ $\quad x \in (-5, 5)$.

(β) $|x| \geq 2 \Rightarrow x \geq 2$ ή $x \leq -2$

$x \in (-\infty, -2] \cup [2, +\infty)$

$|x| \geq \theta$

$x \geq \theta$ ή $x \leq -\theta$



(γ) $3|x| - 2 < 0$

$|x| < \theta$

$-\theta < x < \theta$

$3|x| < 2$

$|x| < \frac{2}{3}$

$-\frac{2}{3} < x < \frac{2}{3}$ $\quad x \in \left(-\frac{2}{3}, \frac{2}{3}\right)$

(8) $2-3|x| \leq 0$

$-3|x| \leq -2$

$|x| \geq \frac{2}{3}$

$x \geq \frac{2}{3}$ 或 $x \leq -\frac{2}{3}$

$x \in (-\infty, -\frac{2}{3}] \cup [\frac{2}{3}, +\infty)$.