5.

Ενότητα 9

$A = 2 - \sqrt{3}$

$B = 2 + \sqrt{3}$

(α) $AB = (2-\sqrt{3})(2+\sqrt{3}) = 2^2 - \sqrt{3}^2 =$

$= 4 - 3 = 1$

(β) $A^2 + B^3 = (2-\sqrt{3})^2 + (2+\sqrt{3})^3 =$

$= 4 - 4\sqrt{3} + 3 + 8 + 3 \cdot 4\sqrt{3} + 3 \cdot 2 \cdot 3 + \sqrt{27}$

$= 7 - 4\sqrt{3} + 8 + 12\sqrt{3} + 18 + \sqrt{9}\sqrt{3}$

$= 15 + 8\sqrt{3} + 18 + 3\sqrt{3} =$

$= 33 + 11\sqrt{3}.$

6. $x = 2\sqrt{3} - 1$

$y = 2\sqrt{3} + 1$

$x^2 - xy = (2\sqrt{3} - 1)^2 - (2\sqrt{3} - 1)(2\sqrt{3} + 1)$

$= (2\sqrt{3})^2 - 2 \cdot 2\sqrt{3} \cdot 1 + 1^2 - \left((2\sqrt{3})^2 - 1^2 \right)$

$= 12 - 4\sqrt{3} + 1 - (12 - 1)$

$= 13 - 4\sqrt{3} - 11$

$\rightarrow 2 - 4\sqrt{3}.$

12. ⑧ $\sqrt{3}\sqrt{2\sqrt{2}-\sqrt{5}}\sqrt{2\sqrt{2}+\sqrt{5}} = 3$

$$\sqrt{3\cdot(2\sqrt{2}-\sqrt{5})(2\sqrt{2}+\sqrt{5})} = 3$$

$$\sqrt{3\left[(2\sqrt{2})^2-(\sqrt{5})^2\right]} = 3$$

$$\sqrt{3\cdot(8-5)} = 3$$

$$\sqrt{9} = 3 \checkmark$$

17. ⑧ $\dfrac{3}{3-2\sqrt{2}} - \dfrac{2\sqrt{2}}{3+2\sqrt{2}} = 17$

$$\frac{3(3+2\sqrt{2})}{(3-2\sqrt{2})(3+2\sqrt{2})} - \frac{2\sqrt{2}(3-2\sqrt{2})}{(3+2\sqrt{2})(3-2\sqrt{2})} = 17$$

$$\frac{9+6\sqrt{2}}{3^2-(2\sqrt{2})^2} - \frac{6\sqrt{2}-8}{3^2-(2\sqrt{2})^2} = 17$$

$$\frac{9+6\sqrt{2}}{9-8} - \frac{6\sqrt{2}-8}{9-8} = 17$$

$$\frac{9+6\sqrt{2}}{1} - \frac{6\sqrt{2}-8}{1} = 17$$

$$9+6\sqrt{2}-6\sqrt{2}+8 = 17$$

$$17 = 17$$

Рητολοιηοι

13. ⑧ $\frac{3}{\sqrt{8}} = \frac{3\sqrt{8}}{\sqrt{8}\sqrt{8}} = \frac{3\sqrt{8}}{\sqrt{64}} =$

$= \frac{3\sqrt{8}}{8}$

15. ⑧ $\frac{\sqrt{2}}{\sqrt{50}-\sqrt{12}} = \frac{\sqrt{2}}{\sqrt{2}\sqrt{25}-\sqrt{3}\sqrt{4}}$

$= \frac{\sqrt{2}}{5\sqrt{2}-2\sqrt{3}} = \frac{\sqrt{2}(5\sqrt{2}+2\sqrt{3})}{(5\sqrt{2}-2\sqrt{3})(5\sqrt{2}+2\sqrt{3})}$

$= \frac{\sqrt{2}(5\sqrt{2}+2\sqrt{3})}{(5\sqrt{2})^2-(2\sqrt{3})^2} = \frac{5\sqrt{4}+2\sqrt{6}}{50-12}$

$= \frac{10+2\sqrt{6}}{38}.$

V - οστή ρίζα

$\sqrt[2]{36} = 6$ γιατί $6^2 = 36$

$\sqrt[3]{27} = 3$ γιατί $3^3 = 27$

$\sqrt[5]{32} = 2$ γιατί $2^5 = 32$

Ισχύουν οι ίδιες ιδιότητες

- $\sqrt[\nu]{a^\nu} = |a|$
- $\sqrt[\nu]{a}^\nu = a.$

- $\sqrt[v]{a}\ \sqrt[v]{\beta} = \sqrt[v]{a \cdot \beta}$
- $\dfrac{\sqrt[v]{a}}{\sqrt[v]{\beta}} = \sqrt[v]{\dfrac{a}{\beta}}$
- $\sqrt[v]{a^{\mu}} = a^{\frac{\mu}{v}}$

25. ⑧ $\sqrt[3]{2+\sqrt{3}} \cdot \sqrt[3]{2+\sqrt{2+\sqrt{3}}} \cdot \sqrt[3]{2-\sqrt{2+\sqrt{3}}} = 1$

$$\sqrt[3]{(2+\sqrt{3})\left(2+\sqrt{2+\sqrt{3}}\right)\left(2-\sqrt{2+\sqrt{3}}\right)} = 1$$

$$\sqrt[3]{(2+\sqrt{3})\left(2^2-\left(\sqrt{2+\sqrt{3}}\right)^2\right)} = 1$$

$$\sqrt[3]{(2+\sqrt{3})(4-(2+\sqrt{3}))} = 1$$

$$\sqrt[3]{(2+\sqrt{3})(2-\sqrt{3})} = 1$$

$$\sqrt[3]{2^2-\sqrt{3}^2} = 1$$

$$\sqrt[3]{1} = \underline{1}$$

29. ⑧ NSO $\sqrt[3]{9} \cdot \sqrt{3} \cdot \sqrt[4]{27} \cdot \sqrt[12]{3} = 9$

$$9^{\frac{1}{3}} \cdot 3^{\frac{1}{2}} \cdot 27^{\frac{1}{4}} \cdot 3^{\frac{1}{12}} = 9$$

$$(3^2)^{\frac{1}{3}} \; 3^{\frac{1}{2}} \cdot (3^3)^{\frac{1}{4}} \cdot 3^{\frac{1}{12}} = 9$$

$$3^{\frac{2}{3}} \cdot 3^{\frac{1}{2}} \cdot 3^{\frac{3}{4}} \cdot 3^{\frac{1}{12}} = 9$$

$$3^{\frac{2}{3}+\frac{1}{2}+\frac{3}{4}+\frac{1}{12}} = 9$$

$$3^{\frac{8}{12}+\frac{6}{12}+\frac{9}{12}+\frac{1}{12}} = 9$$

$$3^{\frac{24}{12}} = 9$$

$$3^2 = 9 \quad \checkmark$$

29. (02) $\dfrac{\sqrt{8}\ \sqrt[3]{4}}{\sqrt[6]{2}} = 4$

$$\frac{8^{\frac{1}{2}} \cdot 4^{\frac{1}{3}}}{2^{1/6}} = 4$$

$$\frac{(2^3)^{\frac{1}{2}} \cdot (2^2)^{\frac{1}{3}}}{2^{\frac{1}{6}}} = 4$$

$$\frac{2^{\frac{3}{2}} \cdot 2^{\frac{2}{3}}}{2^{\frac{1}{6}}} = 4$$

$$\frac{2^{\frac{3}{2}+\frac{2}{3}}}{2^{\frac{1}{6}}} = 4 \qquad \frac{2^{\frac{13}{6}}}{2^{\frac{1}{6}}} = 4$$

$$2^{\frac{12}{6}} = 2^2 = 4$$

37. (B) $B = \dfrac{\sqrt{x^2+4x+4}}{x+2} - \dfrac{9x^2-1}{\sqrt{1-6x+9x^2}}$

$d(3x, -1) < 2$

$|3x+1| < 2$

$-2 < 3x+1 < 2$

$-3 < 3x < 1$

$$-1 < x < \frac{1}{3}$$

Tumi : $d(a,b) = |a-b|$

$$B = \frac{\sqrt{(x+2)^2}}{x+2} - \frac{9x^2-1}{\sqrt{(1-3x)^2}}$$

$$B = \frac{|x+2|}{x+2} - \frac{(3x-1)(3x+1)}{|1-3x|}$$

(+) (+)

- $-1 < x < \frac{1}{3} \Rightarrow 1 < x+2 < \frac{7}{3}$

- $-1 < x < \frac{1}{3} \Rightarrow 3 > -3x > -1$

$4 > 1-3x > 0$

$$B = \frac{x+2}{x+2} + \frac{(3x-1)(3x+1)}{3x-1} = 1+3x+1$$

$= 3x+2$