Τριωνυμο
$$\alpha x^2 + \beta x + \gamma, \ \alpha \neq 0$$

2 ριζα αντιθ
$$\Delta > 0$$

1 ριζα διπλη
$$\Delta = 0$$

καμια ριζα
$$\Delta < 0$$

Πραγματικες ριζ
$$\Delta \geq 0$$

αντιθετη ριζ
$$\Delta > 0 \qquad S = 0$$

Αντιστροφη ριζ
$$\Delta \geq 0 \qquad P = 1$$

Θετικη ριζ
$$\Delta \geq 0 \quad P > 0 \quad S > 0$$

Αρνητικη ριζ
$$\Delta \geq 0 \quad P > 0 \quad S < 0$$

ομοσημη
$$\Delta \geq 0 \qquad P > 0$$

Ετεροσημη
$$\Delta > 0 \qquad P < 0$$

διατηρη προσημ
$$\Delta < 0$$

Θετικο τριωνυμο
$$\Delta < 0 \qquad \alpha > 0$$

Αρνητικο τριωνυμ
$$\Delta < 0 \qquad \alpha < 0$$

Παραγοντοποιηση
$$\alpha x^2 + \beta x + \gamma = \alpha(x - x_1)(x - x_2) \qquad \Delta > 0$$
$$\alpha x^2 + \beta x + \gamma = \alpha(x - x_1)^2 \qquad \Delta = 0$$

Evotuto 1S

19. $x^2 - 2\lambda x + 1 = 0$

ⓐ $\Delta \geq 0$

$(-2\lambda)^2 - 4 \cdot 1 \cdot 1 \geq 0$

$4\lambda^2 - 4 \geq 0$

$\lambda^2 - 1 \geq 0$

λ		-1	1
$\lambda^2 - 1$	$+$	$-$	$+$

$\lambda \in (-\infty, -1] \cup [1, +\infty)$

ⓑ $x_1^2 x_2 + x_1 x_2^2 = \lambda^2$

$x_1 x_2 (x_1 + x_2) = \lambda^2$

$1 \cdot 2\lambda = \lambda^2$

$2\lambda - \lambda^2 = 0$

$\lambda(2 - \lambda) = 0$

~~$\lambda = 0$~~ $\boxed{\lambda = 2}$ ✓

ⓑ $(x_1 + x_2)^3 + 64\, x_1 x_2 = 0$

$$(2\lambda)^3 + 64 = 0$$

$$(2\lambda)^3 = -64$$

$$(2\lambda)^3 = (-4)^3$$

$$2\lambda = -4$$

$$\boxed{\lambda = -2} \checkmark$$

ⓑ $\sqrt{(x_1+x_2)^2 - 4\lambda + 1} = \lambda + 1$

$$\sqrt{(2\lambda)^2 - 4\lambda + 1} = \lambda + 1$$

$$\sqrt{4\lambda^2 - 4\lambda + 1} = \lambda + 1$$

$$\sqrt{(2\lambda - 1)^2} = \lambda + 1$$

$$|2\lambda - 1| = \lambda + 1$$

при $\lambda + 1 \geqslant 0 \Rightarrow \lambda \geqslant -1$

$2\lambda - 1 = \lambda + 1 \quad \boxed{\lambda = 2} \checkmark$

или

$3\lambda = 0$
~~$\lambda = 0$~~

$2\lambda - 1 = -\lambda - 1$

$$\textcircled{\scriptsize E} \quad \lambda^4 = 5x_1 x_2 + (x_1 + x_2)^2$$

$$\lambda^4 = 5 \cdot 1 + (2\lambda)^2$$

$$\lambda^4 = 5 + 4\lambda^2$$

$$\lambda^4 - 4\lambda^2 - 5 = 0$$

$$(\lambda^2)^2 - 4\lambda^2 - 5 = 0$$

$$\omega^2 - 4\omega - 5 = 0$$

$$\omega = 5 \qquad\qquad \omega = -1$$

$$\lambda^2 = 5 \qquad\qquad \lambda^2 = -1$$

$$\lambda = \pm\sqrt{5} \qquad\qquad Absurd.$$

(20) $X_1 = X_2$

$X_1 + X_2 = 2\lambda$

$X_1 X_2 = 1$

$X_2^2 X_2 = 1$

$X_2^3 = 1^3$

$\boxed{X_2 = 1}$

$\boxed{X_1 = 1}$

$1 + 1 = 2\lambda$

$\underline{2 = 2\lambda}$

$\boxed{\lambda = 1}$

26.

$$\lambda x^2 - 2\lambda x - \lambda + 3 \qquad \underline{\underline{\lambda \neq 0}}$$

Ενα τριωνυμο εχει σταθερο προσημο

οταν $\Delta < 0$

$$(-2\lambda)^2 - 4\lambda(-\lambda+3) \leq 0$$

$$4\lambda^2 + 4\lambda^2 - 12\lambda \leq 0$$

$$8\lambda^2 - 12\lambda \leq 0$$

$$2\lambda^2 - 3\lambda \leq 0$$

$$\lambda(2\lambda - 3) \leq 0$$

$$\downarrow \qquad \downarrow$$
$$0 \qquad \lambda = \frac{3}{2}.$$

λ		0	$\frac{3}{2}$	
$2\lambda^2 - 3\lambda$	+		−	+

$$\lambda \in \left(0, \frac{3}{2}\right)$$

36. $\quad x^2 - 2\lambda x - \lambda + 2 = 0$

(a)

$$\Delta \geq 0$$

$$4\lambda^2 - 4(-\lambda + 2) \geq 0$$

$$\lambda^2 - (-\lambda + 2) \geq 0$$

$$\lambda^2 + \lambda - 2 \geq 0$$

λ		-2		1	
$\lambda^2 + \lambda - 2$	$+$		$-$		$+$

$$\lambda \in (-\infty, -2] \cup [1, +\infty),$$

(B) $\quad x_1^2 x_2 + x_1 x_2^2 > 0$

$$x_1 x_2 (x_1 + x_2) > 0$$

$$x_1 + x_2 = 2\lambda$$

$$x_1 x_2 = -\lambda + 2$$

$$(-\lambda + 2) \, 2\lambda > 0$$

$$-2\lambda^2 + 4\lambda > 0$$

λ		0		2	
$-2\lambda^2 + 4\lambda$	$-$		$+$		$-$

$$\lambda \in (0, 2] \,\, ,$$

① Βρd λ ωστε $x^2 - 2\lambda x - \lambda + 2 > 0$

ΘΕΤΙΚΩ.

Δ < 0

$\lambda^2 + \lambda - 2 < 0$

$\lambda \in (-2, 1)$

α > 0

1 > 0 ✓

27. $(\lambda-1)x^2 - 2(\lambda-1)x - \lambda$, $\lambda \neq 0$

αρνητικό .

$\boxed{\Delta < 0}$

$\boxed{\lambda - 1 < 0}$

$$\lambda < 1$$
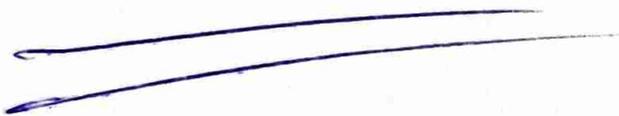
$B^2 - 4\alpha\gamma \leq 0$

$4(\lambda-1)^2 - 4(\lambda-1)(-\lambda) \leq 0$

$4(\lambda^2 - 2\lambda + 1) + 4\lambda(\lambda-1) \leq 0$

$\lambda^2 - 2\lambda + 1 + \lambda^2 - \lambda \leq 0$

$2\lambda^2 - 3\lambda + 1 \leq 0$

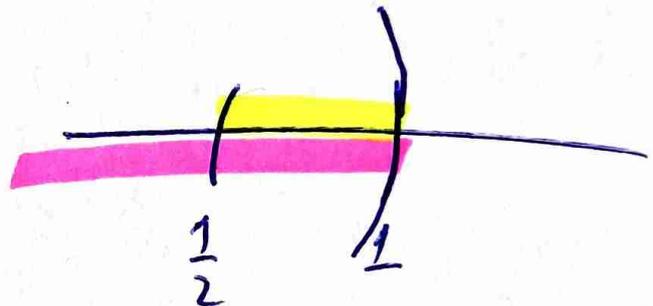| λ		$\frac{1}{2}$	1
$2\lambda^2 - 3\lambda + 1$	$+$	$-$	$+$

$\Delta = 9 - 8 = 1$

$\lambda = \dfrac{3 \pm 1}{4} \begin{cases} ① \\ \frac{1}{2} \end{cases}$
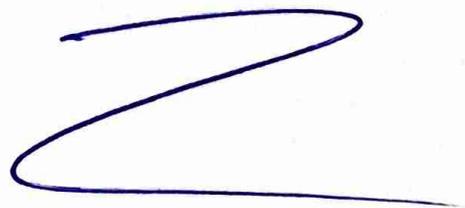
$\lambda \in \left(\dfrac{1}{2}, 1\right)$

κοινή

$\lambda \in \left(\dfrac{1}{2}, 1\right)$

29. ⑥ $(3\lambda-1)x^2 - (3\lambda-1)x + \lambda > 0$

Βρες λ ωστε να αληθευει

ΘΕΤΙΚΟ

$3\lambda - 1 > 0$

$\lambda > \dfrac{1}{3}$

$\Delta < 0$

$(3\lambda-1)^2 - 4(3\lambda-1)\lambda < 0$

$9\lambda^2 - 6\lambda + 1 - 4\lambda(3\lambda-1) < 0$

$9\lambda^2 - 6\lambda + 1 - 12\lambda^2 + 4\lambda < 0$

$-3\lambda^2 - 2\lambda + 1 < 0$

$\Delta = 4 + 12 = 16$

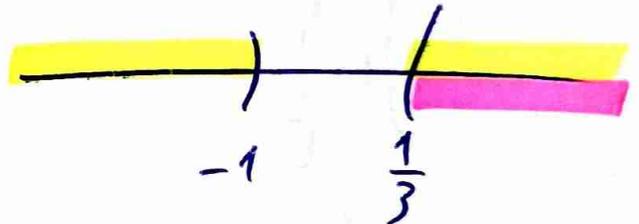$\lambda = \dfrac{2 \pm 4}{-6} \left< \begin{array}{c} -1 \\ \frac{1}{3} \end{array} \right.$

λ		-1		$1/3$	
$-3\lambda^2 - 2\lambda + 1$	$-$		$+$		$-$

$-1 \qquad \dfrac{1}{3}$

κοινα

$\lambda \in \left(\dfrac{1}{3}, +\infty\right)$

$\lambda \in (-\infty, -1) \cup \left(\dfrac{1}{3}, +\infty\right)$

28. $(2\lambda-1)x^2 - 2\lambda x + 2\lambda - 1, \quad \lambda \neq \frac{1}{2}$

ΘΕΤΙΚΟ.

$\Delta < 0$

$4\lambda^2 - 4(2\lambda-1)^2 < 0$

$\lambda^2 - (2\lambda-1)^2 < 0$

$\lambda^2 - (4\lambda^2 - 4\lambda + 1) < 0$

$\lambda^2 - 4\lambda^2 + 4\lambda - 1 < 0$

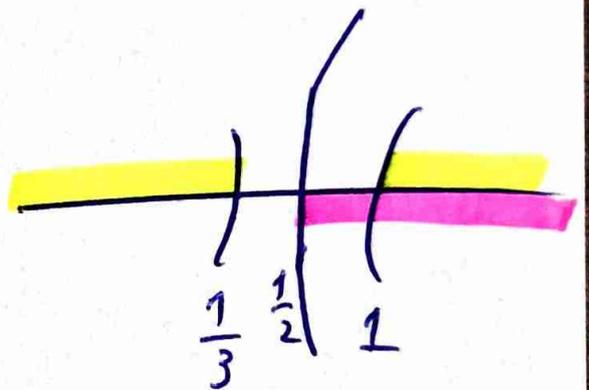$-3\lambda^2 + 4\lambda - 1 < 0$

$\Delta = 16 - 12 = 4$

$\lambda = \dfrac{-4 \pm 2}{-6} < \begin{array}{l} \frac{1}{3} \\ 1 \end{array}$

$2\lambda - 1 > 0$

$\lambda > \frac{1}{2}$

κοινη

$\lambda \in (1, +\infty)$

λ		$\frac{1}{3}$	1	
$-3\lambda^2 + 4\lambda - 1$	$-$		$+$	$-$

$\lambda \in (-\infty, \frac{1}{3}) \cup (1, +\infty)$

$$x^4 - 5x^2 + 4 > 0$$

$$x^2 = \omega$$

$$\omega^2 - 5\omega + 4 > 0$$

ω	1	4	
$\omega^2 - 5\omega + 4$	+	−	+

$$\omega \in (-\infty, 1) \cup (4, +\infty)$$

$$\omega < 1 \quad \text{ó} \quad \omega > 4$$

$$x^2 < 1 \qquad\qquad x^2 > 4$$

$$x^2 - 1 < 0 \qquad\qquad x^2 - 4 > 0$$

x	−1	1	
$x^2 - 1$	+	−	+

x	−2	2	
$x^2 - 4$	+	−	+

$$x \in (-1, 1) \qquad\qquad x \in (-\infty, -2) \cup (2, +\infty)$$

$$x \in (-\infty, -2) \cup (-1, 1) \cup (2, +\infty)$$

Επομεν Μαθημα

19

(14)

(16)

(17) .

(29) a .

(30) .

(32)

(39)

(40)