Νιοστη ριζα - Τετραγωνικη ριζα

$\sqrt{16} = 4$ γιατι $4^2 = 16$

$\sqrt{64} = 8$ γιατι $8^2 = 64$

$\sqrt[3]{8} = 2$ γιατι $2^3 = 8$

$\sqrt[5]{32} = 2$ γιατι $2^5 = 32$.

Ιδιοτητα Τετραγωνικων ριζων

1. $\sqrt{x} \geq 0$ και $x \geq 0$
2. $\sqrt{x}^2 = x$, $x \geq 0$
3. $\sqrt{x^2} = |x|$, $x \in R$.
4. $\sqrt{a}\sqrt{\beta} = \sqrt{a\beta}$
5. $\frac{\sqrt{a}}{\sqrt{\beta}} = \sqrt{\frac{a}{\beta}}$

Προσοχη: $\sqrt{-2^2} = |-2| = 2$ Λαθος!

$\sqrt{(-2)^2} = |-2| = 2$ Σωστο!

Ιδιοτητες Νιοστων ριζων

1. $\sqrt[\nu]{x} \geq 0$, $x \geq 0$.

2. $\left(\sqrt[\nu]{x}\right)^{\nu} = x$

3. $\sqrt[\nu]{x^{\nu}} = |x|$

4. $\sqrt[\nu]{x}\,\sqrt[\nu]{y} = \sqrt[\nu]{xy}$

5. $\dfrac{\sqrt[\nu]{x}}{\sqrt[\nu]{y}} = \sqrt[\nu]{\dfrac{x}{y}}$

6. $\sqrt[\mu]{x^{\nu}} = x^{\frac{\nu}{\mu}}$

7. $\sqrt[\nu]{\sqrt[\mu]{x}} = \sqrt[\nu\cdot\mu]{x}$

8. $\sqrt[\mu\cdot\cancel{\rho}]{x^{\nu\cdot\cancel{\rho}}} = \sqrt[\mu]{x^{\nu}}$

Ασκηση 1

Εστω $|x-1| \leq 2$ και $|y+1| \leq 1$

(α) Ν.δ.ο. $x \in [-1, 3]$ και $y \in [-2, 0]$

(β) Να απλοποιηθει η παρασταση

$$A = \sqrt{y^2+4y+4} + \sqrt{x^2+2x+1}$$

(γ) Ν.δ.ο. $0 \leq A \leq 6$.

Λυση

(α) $|x-1| \leq 2 \Leftrightarrow -2 \leq x-1 \leq 2 \Leftrightarrow 1-2 \leq x \leq 2+1$

$-1 \leq x \leq 3$

$|y+1| \leq 1 \Leftrightarrow -1 \leq y+1 \leq 1 \Leftrightarrow -1-1 \leq y \leq 1-1$

$-2 \leq y \leq 0$

(β) $A = \sqrt{y^2+4y+4} + \sqrt{x^2+2x+1}$

$A = \sqrt{(y+2)^2} + \sqrt{(x+1)^2}$.

$A = |\overset{(+)}{y+2}| + |\overset{(+)}{x+1}|. = y+2+x+1 = y+x+3$

• $-2 \leq y \leq 0 \Rightarrow 0 \leq y+2 \leq 2$

$A = x+y+3$.

• $-1 \leq x \leq 3 \Rightarrow 0 \leq x+1 \leq 4$.

γ

$$\left.\begin{array}{l} \bullet \quad -1 \le x \le 3 \\ \\ \bullet \quad -2 \le y \le 0 \end{array}\right\} \oplus \quad -3 \le x+y \le 3$$

$$0 \le x+y+3 \le 6$$

$$0 \le A \le 6$$

Άσκηση 2

Νδο $\dfrac{\sqrt{3}}{\sqrt{5}-\sqrt{3}} + \dfrac{\sqrt{5}}{\sqrt{5}+\sqrt{3}} = 4$.

Λύση

Είναι $\dfrac{\sqrt{3}}{\sqrt{5}-\sqrt{3}} + \dfrac{\sqrt{5}}{\sqrt{5}+\sqrt{3}} =$

$$= \frac{\sqrt{3}(\sqrt{5}+\sqrt{3})}{(\sqrt{5}-\sqrt{3})(\sqrt{5}+\sqrt{3})} + \frac{\sqrt{5}(\sqrt{5}-\sqrt{3})}{(\sqrt{5}+\sqrt{3})(\sqrt{5}-\sqrt{3})} =$$

$$= \frac{\sqrt{15}+\sqrt{9}}{\sqrt{5}^2-\sqrt{3}^2} + \frac{\sqrt{25}-\sqrt{15}}{\sqrt{5}^2-\sqrt{3}^2} =$$

$$= \frac{\sqrt{15}+3}{5-3} + \frac{5-\sqrt{15}}{5-3} =$$

$$= \frac{\sqrt{15}+3}{2} + \frac{5-\sqrt{15}}{2} = \frac{\cancel{\sqrt{15}}+3+5-\cancel{\sqrt{15}}}{2} = \frac{8}{2} = 4 .$$

Ασκηση 3

Νδο $\sqrt[3]{2+\sqrt{3}} \cdot \sqrt[3]{2+\sqrt{2+\sqrt{3}}} \cdot \sqrt[3]{2-\sqrt{2+\sqrt{3}}} = 1$.

Λυση

Ειναι $\sqrt[3]{(2+\sqrt{3})\left(2+\sqrt{2+\sqrt{3}}\right)\left(2-\sqrt{2+\sqrt{3}}\right)} =$

$$= \sqrt[3]{(2+\sqrt{3})\left(2^2-\sqrt{2+\sqrt{3}}^{\,2}\right)} =$$

$$= \sqrt[3]{(2+\sqrt{3})\left(4-(2+\sqrt{3})\right)} =$$

$$= \sqrt[3]{(2+\sqrt{3})(2-\sqrt{3})} =$$

$$= \sqrt[3]{2^2-\sqrt{3}^2} =$$

$$= \sqrt[3]{4-3} = \sqrt[3]{1} = 1.$$

Άσκηση 4

Ν.δ.ο $\sqrt[12]{\sqrt{2}+1}\;\sqrt[3]{(\sqrt{2}+1)^2}\;\sqrt[4]{(\sqrt{2}-1)^3}=1$.

Λύση

Είναι $\sqrt[12]{\sqrt{2}+1}\;\sqrt[12]{(\sqrt{2}+1)^8}\;\sqrt[12]{(\sqrt{2}-1)^9}=$

$$=\sqrt[12]{(\sqrt{2}+1)(\sqrt{2}+1)^8(\sqrt{2}-1)^9}=$$

$$=\sqrt[12]{(\sqrt{2}+1)^9(\sqrt{2}-1)^9}=$$

$$=\sqrt[12]{[(\sqrt{2}+1)(\sqrt{2}-1)]^9}=$$

$$=\sqrt[12]{(\sqrt{2}^2-1^2)^9}=$$

$$=\sqrt[12]{(2-1)^9}=\sqrt[12]{1^9}=1.$$

Ασκηση 5

(α) Να βρεί τα αναπτύγματα

i) $(3+2\sqrt{7})^2$ ii) $(3-2\sqrt{7})^2$

(β) νδο $\sqrt{37+12\sqrt{7}}-\sqrt{37-12\sqrt{7}}=6$

(γ) νδο ο αριθμός $\left(\sqrt{\frac{2}{3}}+\sqrt{\frac{3}{2}}\right)^2$ είναι ρητός.

(δ) Αν α θετικός ρητός νδο $\left(\sqrt{a}+\frac{1}{\sqrt{a}}\right)^2$ ρητός.

Λύση

(α) i) $(3+2\sqrt{7})^2 = 3^2+2\cdot3\cdot2\sqrt{7}+(2\sqrt{7})^2 =$

$= 9+12\sqrt{7}+4\cdot7 =$

$= 9+12\sqrt{7}+28 = 37+12\sqrt{7}$.

ii) $(3-2\sqrt{7})^2 = 3^2-2\cdot3\cdot2\sqrt{7}+(2\sqrt{7})^2 =$

$= 9-12\sqrt{7}+28 = 37-12\sqrt{7}$.

(β) $\sqrt{37+12\sqrt{7}}-\sqrt{37-12\sqrt{7}} = \sqrt{(3+2\sqrt{7})^2}-\sqrt{(3-2\sqrt{7})^2}$

$= |3\overset{\oplus}{+}2\sqrt{7}| - |3\overset{\ominus}{-}2\sqrt{7}| = 3+2\sqrt{7}+3-2\sqrt{7} = 6$

(γ) $\left(\sqrt{\frac{2}{3}}+\sqrt{\frac{3}{2}}\right)^2=\sqrt{\frac{2}{3}}^{\,2}+2\sqrt{\frac{2}{3}}\sqrt{\frac{3}{2}}+\sqrt{\frac{3}{2}}^{\,2}$

$=\frac{2}{3}+2\sqrt{\cancel{\frac{2}{3}}\cancel{\frac{3}{2}}}+\frac{3}{2}=$

$=\frac{2}{3}+2+\frac{3}{2}=\frac{4}{6}+\frac{12}{6}+\frac{9}{6}=$

$=\frac{25}{6}.$

(δ) $\left(\sqrt{a}+\frac{1}{\sqrt{a}}\right)^2=\sqrt{a}^{\,2}+2\cancel{\sqrt{a}}\frac{1}{\cancel{\sqrt{a}}}+\left(\frac{1}{\sqrt{a}}\right)^2=$

$=a+2+\frac{1}{a}=\frac{a^2}{a}+\frac{2a}{a}+\frac{1}{a}$

$=\frac{a^2+2a+1}{a}=\frac{(a+1)^2}{a}.$

Ασκηση 6

ⓐ Νδο $\sqrt[5]{a^2}\sqrt[4]{a^3}\sqrt{a} = \sqrt[20]{a^{33}}$

ⓑ Νδο $\dfrac{\sqrt[4]{a^3}\sqrt[3]{a}}{\sqrt[6]{a^5}} = \sqrt[4]{a}$

ⓒ Νδο $\sqrt{2\sqrt[3]{2\sqrt{2}}} = \sqrt[4]{2^3}$

ⓓ Νδο $\sqrt{5\sqrt[3]{5\sqrt[4]{25}}} = \sqrt[4]{5^3}$.

Λυση

ⓐ Είναι $\sqrt[5]{a^2}\sqrt[4]{a^3}\sqrt{a} =$

$= \sqrt[20]{a^8}\sqrt[20]{a^{15}}\sqrt[20]{a^{10}} =$

$= \sqrt[20]{a^8 a^{15} a^{10}} = \sqrt[20]{a^{33}}$

(β) $\dfrac{\sqrt[4]{a^3}\,\sqrt[3]{a}}{\sqrt[6]{a^5}} = \dfrac{\sqrt[12]{a^9}\,\sqrt[12]{a^4}}{\sqrt[12]{a^{10}}} = \sqrt[12]{\dfrac{a^9 a^4}{a^{10}}} =$

$= \sqrt[12]{\dfrac{a^{13}}{a^{10}}} = \sqrt[12]{a^3} = \sqrt[4]{a}$

(γ) Είναι $\sqrt{2\cdot\sqrt[3]{2\sqrt{2}}} = \sqrt{2\sqrt[3]{\sqrt{2^2\cdot 2}}} =$

$= \sqrt{2\sqrt[6]{2^3}} = \sqrt{2\sqrt{2}} =$

$= \sqrt{\sqrt{2^2\cdot 2}} = \sqrt[4]{2^3}$

Προσοχη

$\sqrt[3]{2\sqrt{2}} = \sqrt[3]{\sqrt{2^2\cdot 2}} = \sqrt[6]{2^3} = \sqrt{2}$

(δ). $\sqrt{5\sqrt[3]{5\sqrt[4]{25}}} =$

$\sqrt{5\sqrt[3]{\sqrt[4]{5^4\cdot 5^2}}} = \sqrt{5\sqrt[12]{5^6}}$

$= \sqrt{5\sqrt{5}} = \sqrt{\sqrt{5^2 5}} = \sqrt[4]{5^3}.$

Ασκηση 7

Να γινει αντιστοιχιση

$\frac{15}{\sqrt{3}}$	$\frac{5(\sqrt{5}+1)}{2}$
$\frac{2\sqrt{3}}{\sqrt{75}}$	$5\sqrt{3}$
$\frac{10}{\sqrt{5}-1}$	$\frac{2}{5}$
$\frac{\sqrt{3}-1}{\sqrt{3}+1}$	$\frac{2\sqrt{3}-7}{5}$
$\frac{6}{\sqrt{7}+\sqrt{5}}$	$2-\sqrt{3}$
	$\sqrt{5}+\sqrt{7}$
	$3(\sqrt{7}-\sqrt{5})$.

$$\rightarrow \frac{15}{\sqrt{3}} = \frac{15\sqrt{3}}{\sqrt{3}\sqrt{3}} = \frac{15\sqrt{3}}{\sqrt{9}} = \frac{15\sqrt{3}}{3} = 5\sqrt{3}.$$

$$\rightarrow \frac{2\sqrt{3}}{\sqrt{75}} = \frac{2\sqrt{3}\sqrt{75}}{\sqrt{75}\sqrt{75}} = \frac{2\sqrt{3}\sqrt{75}}{\cancel{\sqrt{75}^2}} = \frac{2\sqrt{3}\sqrt{3}\sqrt{25}}{75} =$$

$$= \frac{2\cdot 3\cdot 5}{75} = \frac{30}{75} = \frac{6}{15} = \frac{2}{5}.$$

$$\rightarrow \frac{10}{\sqrt{5}-1} = \frac{10(\sqrt{5}+1)}{(\sqrt{5}-1)(\sqrt{5}+1)} = \frac{10(\sqrt{5}+1)}{\sqrt{5}^2-1^2} = \frac{10(\sqrt{5}+1)}{4}$$

$$= \frac{5(\sqrt{5}+1)}{2}.$$

$$\rightarrow \frac{\sqrt{3}-1}{\sqrt{3}+1} = \frac{(\sqrt{3}-1)(\sqrt{3}-1)}{(\sqrt{3}+1)(\sqrt{3}-1)} = \frac{(\sqrt{3}-1)^2}{\sqrt{3}^2-1^2} =$$

$$= \frac{\cancel{\sqrt{3}^2}-2\sqrt{3}+1}{2} = \frac{4-2\sqrt{3}}{2} = 2-\sqrt{3}.$$

$$\rightarrow \frac{6}{\sqrt{7}+\sqrt{5}} = \frac{6(\sqrt{7}-\sqrt{5})}{(\sqrt{7}+\sqrt{5})(\sqrt{7}-\sqrt{5})} = \frac{6(\sqrt{7}-\sqrt{5})}{\sqrt{7}^2-\sqrt{5}^2} =$$

$$= \frac{6(\sqrt{7}-\sqrt{5})}{2} = 3(\sqrt{7}-\sqrt{5}).$$