7. (α) $\lambda x^2 - 2x - \lambda = 0$, $\lambda \neq 0$

$\alpha = \lambda$

$\beta = -2$

$\gamma = -\lambda$

$\Delta = \beta^2 - 4\alpha\gamma$

$\Delta = (-2)^2 - 4 \cdot \lambda \cdot (-\lambda)$

$\Delta = 4 + 4\lambda^2 > 0$ 2 ρίζες άνισες

(β) $\lambda x^2 + (\lambda - 1)x - 1 = 0$ $\lambda \neq 0$

$\Delta = (\lambda - 1)^2 - 4\lambda(-1)$

$\Delta = \lambda^2 - 2\lambda + 1 + 4\lambda$

$\Delta = \lambda^2 + 2\lambda + 1$

$\Delta = (\lambda + 1)^2 \geq 0$ πραγμ. ρίζες.

8. (a) $x^2 - 2\lambda x + \lambda^2 - 1 = 0$

$\alpha = 1 \qquad \beta = -2\lambda \qquad \gamma = \lambda^2 - 1$

$\Delta = (-2\lambda)^2 - 4 \cdot 1 \cdot (\lambda^2 - 1)$

$\Delta = 4\lambda^2 - 4(\lambda^2 - 1)$

$\Delta = 4\lambda^2 - 4\lambda^2 + 4$

$\Delta = 4$

$$x_{1,2} = \frac{-\beta \pm \sqrt{\Delta}}{2a} = \frac{-(-2\lambda) \pm \sqrt{4}}{2} = \frac{2\lambda \pm 2}{2}$$

$x_{1,2} = \lambda \pm 1$

$\lambda + 1$

$\lambda - 1$

10. $x^2-(2\lambda-1)x+1-2\lambda=0$

Έχει δύο ρίζες.

$\longrightarrow \Delta=0$

$$[-(2\lambda-1)]^2-4\cdot 1\cdot(1-2\lambda)=0$$

$$(2\lambda-1)^2-4(1-2\lambda)=0$$

$$4\lambda^2-4\lambda+1-4+8\lambda=0$$

$$4\lambda^2+4\lambda-3=0$$

$$\Delta=16-4\cdot 4(-3)$$

$$\Delta=16+48=64.$$

$$\lambda=\frac{-4\pm 8}{8}$$

$\frac{1}{2}$

$\frac{-12}{8}$

13. $\lambda^2 x^2 - 2\lambda^3 x + \lambda^4 - 4 = 0$, $\lambda \neq 0$

$$\Delta = (-2\lambda^3)^2 - 4 \cdot \lambda^2 (\lambda^4 - 4)$$

$$\Delta = 4\lambda^6 - 4\lambda^2(\lambda^4 - 4)$$

$$\Delta = 4\lambda^6 - 4\lambda^6 + 16\lambda^2$$

$$\Delta = 16\lambda^2 > 0$$

$$x_{1,2} = \frac{2\lambda^3 \pm 4\lambda}{2\lambda^2} = \frac{\lambda^3 \pm 2\lambda}{\lambda^2} = \frac{\lambda^2 \pm 2}{\lambda}$$

$$x = \frac{\lambda^2 + 2}{\lambda}.$$

$$x = \frac{\lambda^2 - 2}{\lambda}$$

14. (α) $x^2 - 2x + \lambda = 0$

$$\Delta = (-2)^2 - 4 \cdot 1 \cdot \lambda$$

$$\Delta = 4 - 4\lambda$$

1. Αν $4 - 4\lambda > 0 \Rightarrow 4 > 4\lambda \rightarrow 1 > \lambda$. τοτε $\Delta > 0$ και εχω δυο ριζες.

2. Αν $\lambda > 1$ τοτε $\Delta < 0$ αδυνατη χαμενα ριζα

3. Αν $\lambda = 1$ τοτε εχω 1 διπλη ριζα

Έστω ότι έχω μια εξίσωση 2ου βαθμού

$$\alpha x^2 + \beta x + \gamma = 0 \quad , \quad \alpha \neq 0$$

και έχω δύο ρίζες x_1, x_2 πραγματικές και ανίσες.

$$x_1 = \frac{-\beta + \sqrt{\Delta}}{2\alpha} \qquad x_2 = \frac{-\beta - \sqrt{\Delta}}{2\alpha}$$

$$x_1 + x_2 = \frac{-\beta + \sqrt{\Delta}}{2\alpha} + \frac{-\beta - \sqrt{\Delta}}{2\alpha} = \frac{-\beta + \sqrt{\Delta} - \beta - \sqrt{\Delta}}{2\alpha} = \frac{-2\beta}{2\alpha}$$

$$\boxed{x_1 + x_2 = -\frac{\beta}{\alpha}} \quad (S)$$

$$\boxed{x_1 \cdot x_2 = \frac{\gamma}{\alpha}} \quad (P)$$

$$x_1 \cdot x_2 = \frac{-\beta + \sqrt{\Delta}}{2\alpha} \cdot \frac{-\beta - \sqrt{\Delta}}{2\alpha} = \frac{(-\beta + \sqrt{\Delta})(-\beta - \sqrt{\Delta})}{4\alpha^2} =$$

$$= \frac{\beta^2 - \sqrt{\Delta}^2}{4\alpha^2} = \frac{\beta^2 - \Delta}{4\alpha^2} = \frac{\beta^2 - (\beta^2 - 4\alpha\gamma)}{4\alpha^2}$$

$$= \frac{\beta^2 - \beta^2 + 4\alpha\gamma}{4\alpha^2} = \frac{4\alpha\gamma}{4\alpha^2} = \frac{\gamma}{\alpha}$$

п. X

$$x^2 + x - 1 = 0.$$

$$\Delta = 1 + 4 = 5 > 0 \quad \text{2 ρίζες.}$$

Τότε κατά το $x_1 + x_2$ και $x_1 \cdot x_2$;

$$x_1 + x_2 = -\frac{\beta}{\alpha} = -\frac{1}{1} = -1.$$

$$x_1 \cdot x_2 = \frac{\gamma}{\alpha} = \frac{-1}{1} = -1$$

Έστω $\alpha x^2 + \beta x + \gamma = 0$ $\quad \alpha \neq 0$

$$x^2 + \frac{\beta}{\alpha} x + \frac{\gamma}{\alpha} = 0$$

$$x^2 - \left(-\frac{\beta}{\alpha}\right) x + \frac{\gamma}{\alpha} = 0$$

$$x^2 - Sx + P = 0 .$$

π.χ

Έστω 5, 7 δυο ρίζες.

$$x^2 - (5+7)x + 35 = 0$$

$$\boxed{x^2 - 12x + 35 = 0}$$

Εξ 19 ② ③

$$x^2 - 2x - 1 = 0$$

$$S = x_1 + x_2 = -\frac{B}{\alpha} = -\frac{-2}{1} = 2$$

$$P = x_1 \cdot x_2 = \frac{\gamma}{\alpha} = \frac{-1}{1} = -1$$

(α) $x_1^2 + x_2^2 = (x_1 + x_2)^2 - 2x_1x_2 = 2^2 - 2\cdot(-1) = 4 + 2 = 6$

$$\alpha^2 + \beta^2 = (\alpha+\beta)^2 - 2\alpha\beta$$

(β) $\frac{x_1}{x_2} + \frac{x_2}{x_1} = \frac{x_1^2}{x_1x_2} + \frac{x_2^2}{x_1x_2} = \frac{x_1^2 + x_2^2}{x_1x_2} = \frac{6}{-1}$

$= -6$

(γ) $x_1^3 x_2 + x_2^3 x_1 = x_1x_2(x_1^2 + x_2^2) = -1 \cdot 6 = -6$.

(δ) $(x_1 - x_2)^2 = x_1^2 - 2x_1x_2 + x_2^2 = x_1^2 + x_2^2 - 2x_1x_2$

$= 6 - 2(-1) = 8$.

5. $x^2 - 3x + 1 = 0$

$$S = x_1 + x_2 = -\frac{-3}{1} = 3$$

$$P = x_1 x_2 = \frac{\gamma}{a_1} = \frac{1}{1} = 1$$

(a) $\sqrt{\frac{x_1}{x_2}} + \sqrt{\frac{x_2}{x_1}} = \sqrt{7}$

$$\longrightarrow \left(\sqrt{\frac{x_1}{x_2}} + \sqrt{\frac{x_2}{x_1}}\right)^2 = \frac{x_1}{x_2} + 2\sqrt{\frac{x_1}{x_2}}\sqrt{\frac{x_1}{x_1}} + \frac{x_2}{x_1} =$$

$$= \frac{x_1}{x_2} + \frac{x_2}{x_1} + 2\sqrt{\frac{x_1}{x_2}\cdot\frac{x_2}{x_1}} = \frac{x_1^2 + x_2^2}{x_1 x_2} + 2 \cdot 1 =$$

$$= \frac{(x_1+x_2)^2 - 2x_1x_2}{x_1 x_2} + 2 = \frac{3^2 - 2\cdot 1}{1} = 7 .$$

(B) $\frac{1}{x_1 - 2} + \frac{1}{x_2 - 2} = \frac{x_2 - 2}{(x_1-2)(x_2-2)} + \frac{x_1 - 2}{(x_1-2)(x_2-2)}$

$$= \frac{x_1 + x_2 - 4}{(x_1-2)(x_2-2)} = \frac{3-4}{x_1x_2 - 2x_1 - 2x_2 + 4} = \frac{-1}{1 - 2(x_1+x_2)+4}$$

$$= \frac{-1}{-1} = 1$$

⑦ $\frac{2}{x_1^2} + \frac{2}{x_2^2} = \frac{2x_2^2 + 2x_1^2}{x_1^2 x_2^2} =$

$= \frac{2(x_1^2 + x_2^2)}{(x_1 x_2)^2} = \frac{2 \cdot 7}{1^2} = 14$

⑧ $(3x_1 - x_2)(3x_2 - x_1) =$

$= 9x_1x_2 - 3x_1^2 - 3x_2^2 + x_1x_2 =$

$= 9 \cdot 1 - 3(x_1^2 + x_2^2) + 1 =$

$= 10 - 3 \cdot 7 = 10 - 21 = -11.$

Άσκηση 1

x_1, x_2 ρίζες του

$a=1, \beta=-3, \gamma=1$

Έστω $x^2-3x+1=0$

(α) $x_1+x_2 = -\frac{\beta}{a} = -\frac{-3}{1} = 3$

(β) $x_1 \cdot x_2 = \frac{\gamma}{a} = \frac{1}{1} = 1$

(γ) $x_1^2+x_2^2 = (x_1+x_2)^2 - 2x_1 \cdot x_2 = 3^2 - 2 \cdot 1 = 9-2 = 7$

(δ) $x_1^3+x_2^3 = (x_1+x_2)\cdot(x_1^2 - x_1x_2 + x_2^2) = 3\cdot(7-1) = 18$

(ε) $\frac{1}{x_1}+\frac{1}{x_2} = \frac{x_2}{x_1x_2}+\frac{x_1}{x_1x_2} = \frac{x_1+x_2}{x_1x_2} = \frac{3}{1} = 3$ ✓

(στ) $\frac{x_1}{x_2}+\frac{x_2}{x_1} = \frac{x_1^2}{x_1 \cdot x_2}+\frac{x_2^2}{x_1 \cdot x_2} = \frac{x_1^2+x_2^2}{x_1 \cdot x_2} = \frac{7}{1} = 7$ ✓

Άσκηση 2

$x^2+5x-4=0$

(α) $x_1+x_2=-\frac{\beta}{\alpha}=-\frac{5}{1}$ $\boxed{-5}$

(β) $x_1x_2=\frac{\gamma}{\alpha}=\frac{-4}{1}=\boxed{-4}$

(γ) $x_1^2+x_2^2=(x_1+x_2)^2-2x_1x_2=25-2\cdot(-4)=$

$=25+8=\boxed{33}$

(δ) $x_1^3x_2+x_1x_2^3=x_1x_2(x_1^2+x_2^2)=-4(33)=$

$=132.$

(ε) $(x_1-x_2)^2=$ [illegible]

$x_1^2-2x_1x_2+x_2^2=33-2(-4)=33+8=\boxed{41}$

(ϛ) $\frac{1}{x_1^2}+\frac{1}{x_2^2}=\frac{x_2^2}{x_1^2x_2^2}+\frac{x_1^2}{x_2^2x_1^2}=\frac{x_2^2+x_1^2}{x_2^2x_1^2}=$

$=\frac{33}{16}$

Άσκηση 3

$x^2+4x+2=0$

(α) $x_1+x_2 = S = -\frac{\beta}{\alpha} = -\frac{4}{1} = -4$

(β) $x_1 x_2 = P = \frac{\gamma}{\alpha} = \frac{2}{1} = 2$

(γ) $x_1^2+x_2^2 = (x_1+x_2)^2 - 2x_1x_2 = (-4)^2 - 2\cdot 2 = 16-4 = 12$

(δ) $\frac{1}{x_1^2} + \frac{1}{x_2^2} = x_2^2 + x_1^2 = 12$

$\frac{x_2^2+x_1^2}{x_1^2 x_2^2} = \frac{12}{(x_1x_2)^2} = \frac{12}{4} = 3$

(ε) $\frac{1}{x_1^3} + \frac{1}{x_2^3} = \frac{x_2^3+x_1^3}{x_1^3 x_2^3} = \frac{(x_2+x_1)(x_2^2-x_2x_1+x_1^2)}{2^3} =$

$= \frac{-4\,(12-2)}{8} = -\frac{40}{8} = -5$

(στ) $\frac{x_1}{x_2^2} + \frac{x_2}{x_1^2} = \frac{x_1^3+x_2^3}{x_1^2 x_2^2} =$

$= \frac{-40}{12}$

Άσκηση 4

$2x^2+3x-4=0$.

(α) $x_1+x_2=-\frac{\beta}{\alpha}=-\frac{3}{2}$

(β) $x_1\cdot x_2=\frac{\gamma}{\alpha}=\frac{-4}{2}=-2$

(δ) $\sqrt{x_1^2+x_2^2}=\sqrt{(x_1+x_2)^2-2x_1x_2}=\sqrt{\left(-\frac{3}{2}\right)^2-2\cdot(-2)}=\sqrt{\frac{9}{4}+4}$.

(ε) $(x_1+1)(x_2+1)=x_1x_2+x_1+x_2+1=-2+\left(-\frac{3}{2}\right)+1$

$-4-3+2=5$

(ε) $(2x_1-3)(2x_2-3)=2x_1\cdot 2x_2-6x_1-6x_2+9$

$2(x_1+x_2)-6(x_1+x_2)+9=2\left(-\frac{3}{2}\right)-6\left(-\frac{3}{2}\right)+9=-3+9+9=15$

(στ) $(x_1^2-x_1x_2)(x_2x_1-x_2^2)=\left[x_1^2-(-2)\right]\left[(-2)-x_2^2\right]$

$(x_1^2+2)(-2-x_2^2)=-2x_1^2+[x_1(-x_2)]^2-4-2x_2^2=-2x_1^2-(-2)^2-4-2x_2^2$

$-2x_1^2-4-2x_2^2=-2x_1^2-2x_2^2=2(x_1^2+x_2^2)=-2$

Επαναληψη Μαθημα

Για μετα τα χριστουγεννα.

Σελ 183	Σελ 192
19	1
20	2
21.	4.