Άσκηση

1. $3(x-2) - 2(1+3x) = -2(x-4) - x - 16$

$3x - 6 - 2 - 6x = -2x + 8 - x - 16$

$-3x - 8 = -3x - 8$

$0 = 0$ Άπειρα.

2. $\lambda(\lambda x + 6) = \lambda^2 - 9 \cdot (-1 - x)$.

$\lambda^2 x + 6\lambda = \lambda^2 + 9 + 9x$

$\lambda^2 x - 9x = \lambda^2 - 6\lambda + 9$.

$(\lambda^2 - 9)x = \lambda^2 - 6\lambda + 9$

$(\lambda - 3)(\lambda + 3)\, x = (\lambda - 3)^2$

1. Αν $\lambda = 3$ τότε $0x = 0$ Άπειρα
2. Αν $\lambda = -3$ τότε $0x = 36$
3. Αν $\lambda \neq 3$, $\lambda \neq -3$ τότε $x = \dfrac{(\lambda-3)^{\cancel{2}}}{(\lambda-3)(\lambda+3)}$

3. $\dfrac{x+5}{x-1} = \dfrac{x+6}{x} - \dfrac{2(x+3)}{x^2+x}$

$\dfrac{x+5}{x-1} = \dfrac{x+6}{x} - \dfrac{2(x+3)}{x(x+1)}$

- $x-1=0 \Rightarrow \boxed{x=1}$
- $\boxed{x=0}$
- $x(x+1)=0$

$\boxed{x=0}$ ή $x+1=0$

$\boxed{x=-1}$

Πρη

$x \neq 0$

$x \neq 1$

$x \neq -1.$

εκπ: $x(x-1)(x+1)$

$x(x-1)(x+1)\dfrac{x+5}{x-1} = x(x-1)(x+1)\dfrac{x+6}{x} - x(x-1)(x+1)\dfrac{2(x+3)}{x(x+1)}$

$(x+1)(x+5) = (x-1)(x+1)(x+6) - 2(x-1)(x+3)$

$x^2+5x+x+5 = (x^2-1)(x+6) - 2(x^2+3x-x-3)$

$x^2+6x+5 = x^3+6x^2-x-6-2x^2-6x+2x+6$

$x^2+6x+5 = x^3+4x^2-2x^2-5x$

$x^2+6x+5 = x^3+2x^2-5x$

$-x^3-x^2+11x+5=0$

Έχει γίνει λάθος στην σκέψη

4. $(x-1)(x+3) - (2x-1)^2 = (1-x)(x-4)$

$x^2+3x-x-3 - (4x^2-4x+1) = x-4-x^2+4x$

$x^2+2x-3-4x^2+4x-1 = 5x-x^2-4$

$-3x^2+6x-4 = 5x-x^2-4$

$-3x^2+6x-5x+x^2 = 0$

$-2x^2+x = 0$

$x(-2x+1) = 0$

$x=0$

$-2x+1=0$

$2x=1$

$x=\frac{1}{2}$

5. $3x(x-3)+(x-3)^2+9-x^2=0$

$3x(x-3)+(x-3)^2+(3-x)(3+x)=0$

$3x(x-3)+(x-3)^2-(x-3)(x+3)=0$

$(x-3)(3x+x-3-x-3)=0$

$(x-3)(3x-6)=0$

$x-3=0$ or $3x-6=0$

$x=3$ $3x=6$

$x=2$

6. $(x^2-9)(2x+1) = (x+3)(2x+1)^2$

$(x-3)(x+3)(2x+1) = (x+3)(2x+1)^2$

$(x-3)(x+3)(2x+1) - (x+3)(2x+1)(2x+1) = 0$

$(x+3)(2x+1)(x-3-2x-1) = 0$

$(x+3)(2x+1)(-x-4) = 0$

~~$(2x^2+x+6x+3)(-x-4)=0$~~

~~$2x^2+7x+3$~~

$x+3=0$ ή $2x+1=0$ ή $-x-4=0$

$x=-3$ $x=-\frac{1}{2}$ $x=-4$

Περ

- $x+3=0 \Leftrightarrow x=-3$
- $2x+1=0 \Leftrightarrow 2x=-1$ $x=\frac{1}{2}$
- $-x-4=0 \Leftrightarrow -x=4 \Leftrightarrow x=-4$

7. $(x+1)^3 + x^2 - 1 = 0$

$(x+1)^3 + x^2 - 1 = 0$

$(x+1)^3 + (x-1)(x+1) = 0$

$(x+1)\left[x-1+(x+1)^2\right] = 0$

$x+1=0$ ni $x-1+(x+1)^2=0$

$x=-1$

$x-1+x^2+2x+1=0$

$3x+x^2=0$

$x(3+x)=0$

$3+x=0$

$x=-3$

8. $(x+1)(2-x)(x+3) + (x+1)(2-x)(5-2x) = (x+1)(x-2)$.

$(x+1)\cdot(2-x)\cdot(x+3+5-2x) - (x+1)(x-2) = 0$,

$(x+1)(2-x)(8-x) - (x+1)(x-2) = 0$.

$(x+1)(2-x)(8-x) + (x+1)(2-x) = 0$

$(x+1)(2-x)(8-x+1) = 0$

$(x+1)(2-x)(9-x) = 0$

$x+1=0$ atau $2-x=0$ atau $9-x=0$

$x=-1$ $-x=-2$ $-x=-9$

$x=2$ $x=9$

9. $\frac{4}{x+2} + \frac{3}{x-2} = \frac{3x^2-8}{x^2-4}$

$$\frac{4}{x+2} + \frac{3}{x-2} = \frac{3x^2-8}{(x-2)(x+2)}$$

Περιορισμοί
$x \neq -2$
$x \neq 2$

$x+2=0$
$x=-2$

$x-2=0$
$x=2$

$$(x-2)(x+2)\frac{4}{x+2} + (x-2)(x+2)\frac{3}{x-2} = (x-2)(x+2)\frac{3x^2-8}{(x-2)(x+2)}$$

$$4(x-2)+3(x+2) = 3x^2-8$$

$$4x-8+3x+6-3x^2+8=0$$

$$7x+6-3x^2=0$$

$$-3x^2+7x+6=0.$$

$$\Delta = 49+12\cdot 6$$

$$\Delta = 49+72 = 121$$

$$x = \frac{-7 \pm 11}{-6}$$

$-\frac{2}{3}$

3

10. $1 - \frac{x+2}{x-2} = \frac{x-10}{x^2-2x} - \frac{x+2}{x}$ ΕΚΠ $x(x-2)$

$$1 - \frac{x+2}{x-2} = \frac{x-10}{x(x-2)} - \frac{x+2}{x}$$

Περ

- $x-2 \neq 0 \Leftrightarrow x \neq 2$
- ~~$x(x-2) \neq 0$~~
- $x \neq 0$
- $x \neq 0$
- $x-2 \neq 0 \Leftrightarrow x \neq 2$

$$x(x-2) - x(x-2)\frac{x+2}{x-2} - x(x-2)\frac{x-10}{x(x-2)} - x(x-2)\frac{x+2}{x}$$

$$x(x-2) - x(x+2) - (x-10) + (x-2)(x+2)$$

$$x^2 - 2x - x^2 - 2x - x + 10 + (x^2 - 4)$$

$$-5x + 10 + x^2 - 4 \Leftrightarrow$$

$$x^2 - 5x + 6 = 0$$

~~$x=2$~~ $x=3$

11. $\frac{x+5}{x+1} = \frac{x+6}{x} - \frac{2(x+3)}{x^2+x}$

$\frac{x+5}{x+1} = \frac{x+6}{x} - \frac{2x+6}{x\cdot(x+1)}$

- $x+1=0$
  $x=-1$
- $x=0$

Πρέπει $x \neq 0, x \neq -1$

- $x(x+1)=0$
  $x=0$ ή $x+1=0$
  $x=-1$

$x\cdot(x+1)\cdot\frac{x+5}{x+1} = x(x+1)\cdot\frac{x+6}{x} - x\cdot(x+1)\cdot\frac{2x+6}{x\cdot(x+1)}$

$x\cdot(x+5) = (x+1)\cdot(x+6) - x\cdot(2x+6)$

$x^2+5x = x^2+6+x+6-2x-6$

$x^2+5x-x^2-6-x-6+2x+6=0$

$6x-6=0$

$\frac{6x}{6} = \frac{6}{6}$

$x=1$

12. $4-\lambda|\lambda-2x| = -\lambda^2 x$

$4-\lambda^2+2x\lambda = -\lambda^2 x$

$2x\lambda+\lambda^2 x = -4+\lambda^2$

$x(2\lambda+\lambda^2) = (\lambda-2)(\lambda+2)$

$\boxed{x(\lambda+2)\lambda = (\lambda-2)(\lambda+2)} \Rightarrow$

→ Αν το $\lambda+2=0$ → Αν το $\lambda=0$

$\lambda=-2$

$0x=0$ $\quad$ $0x=-4$

Αορισμ $\quad$ Αδυνα

→ Αν το $\lambda\neq-2$ $\lambda\neq0$

$\Rightarrow \dfrac{x(\lambda+2)\lambda}{(\lambda+2)\lambda} = \dfrac{(\lambda-2)(\lambda+2)}{(\lambda+2)\lambda}$

$$x = \frac{\lambda-2}{\lambda}$$

✓

13. $\lambda^2(x+1) = -(-1-\lambda x)$.

$\lambda^2 x + \lambda^2 = 1 + \lambda x$

$\lambda^2 x - \lambda x = 1 - \lambda^2$

$\boxed{x \cdot (\lambda^2 - \lambda) = 1 - \lambda^2}$

$x \cdot (\lambda^2 - \lambda) = -(1 - \lambda^2)$

$x \cdot (\lambda^2 - \lambda) = \lambda^2 - 1$

$x \cdot (\lambda^2 - \lambda) = (\lambda - 1) \cdot (\lambda + 1)$

$\boxed{\lambda(\lambda - 1)\, x = (\lambda - 1)(\lambda + 1)}$

1. Αν $\lambda = 1$, τότε $0x = 0$ ταυτότητα.
2. Αν $\lambda = 0$ τότε $0x = -1$ Αδύνατη
3. Αν $\lambda \neq 1$ και $\lambda \neq 0$ τότε

$$\frac{\lambda(\lambda-1)\, x}{\lambda(\lambda-1)} = \frac{(\lambda-1)(\lambda+1)}{\lambda(\lambda-1)}.$$

$$x = \frac{\lambda + 1}{\lambda}.$$

Ασκησεις για το σπιτι

1. $\frac{x+1}{2x-3} + \frac{x-3}{2x+3} = 1 - \frac{2(x-9)}{4x^2-9}$

2. $\frac{15}{x-2} - \frac{4}{x+2} = \frac{5}{x^2-4}$

3. $\frac{2|5-3x|-1}{9} = 1$.

4. $\frac{|x-2|}{2} = \frac{11}{20} - \frac{|6-3x|}{5}$.

5. $d(3x,-1) = 5$

6. $d(2x,5) = d(x,-1)$

7. $||x|-3| = 1$

8. $d(d(x,-1),5) = 4$

9. $(x^2+2)^2 = x[(x+1)^3 - (3x^2-x+1)]$

10. $2x(x^2-12) - 4(2x-1) = 4$