17. Η εξίσωση $x^2 - \lambda x + \nu = 0$ έχει διπλή

ρίζα $\Rightarrow \Delta = 0 \Rightarrow \lambda^2 - 4\nu = 0$

$\lambda^2 = 4\nu$

Νδο η $x^2 + (\mu+1)x + \frac{\lambda^2}{8} = 0$ έχει δύο ρίζες

$$\Delta = (\mu+1)^2 - 4 \cdot \frac{\lambda^2}{8} = (\mu+1)^2 - \frac{\lambda^2}{2}$$

$$\Delta = (\mu+1)^2 - \frac{4\nu}{2}$$

$$\Delta = \mu^2 + 2\mu + 1 - 2\mu$$

$$\Delta = \mu^2 + 1 > 0$$

15. $x^2 - 2\lambda x + \lambda^2 - \lambda + 1 = 0$

(α) Βρ! το λ ωστε να εχει δυο ρίζες.

Απαιτω $\Delta > 0$.

$$(-2\lambda)^2 - 4(\lambda^2 - \lambda + 1) > 0$$

$$4\lambda^2 - 4(\lambda^2 - \lambda + 1) > 0$$

$$\cancel{\lambda^2} - \cancel{\lambda^2} + \lambda - 1 > 0$$

$$\lambda > 1.$$

Οταν $\lambda \in (1, +\infty)$ τοτε 2 ρίζες.

(β). Απαιτω $\Delta = 0$

$\lambda - 1 = 0$ Εχω διπλη ρίζα.

$\lambda = 1$

(γ). Απαιτω $\Delta \geq 0$

$\lambda \geq 1$.

(δ). Καμια ρίζα Απαιτω $\Delta < 0$ $\lambda < 1$

16. Βρες $\Delta = 0$

$$x^2 - (\lambda - \mu)x - (\mu^2 - \lambda + 1) = 0 \Rightarrow$$

Αφού έχει διπλή ρίζα, θα 'χει ρίζα το 1.

$$1 - (\lambda - \mu) - (\mu^2 - \lambda + 1) = 0 \Rightarrow 1 - \lambda + \mu - \mu^2 + \lambda - 1 = 0 \Rightarrow \mu - \mu^2 = 0$$

$$\mu(1 - \mu) = 0$$

$\mu = 0$ $\mu = 1$

$\Delta = 0$

$$\Delta = (\lambda - \mu)^2 + 4(\mu^2 - \lambda + 1) = 0$$

$$\lambda^2 - 2\lambda\mu + \mu^2 + 4\mu^2 - 4\lambda + 4 = 0$$

$$5\mu^2 + \lambda^2 - 2\lambda\mu - 4\lambda + 4 = 0.$$

1η. Αν $\mu = 0$ $\Rightarrow \lambda^2 - 4\lambda + 4 = 0$

$(\lambda - 2)^2 = 0$

$\lambda = 2$

2η. Αν $\mu = 1$ τότε $5 + \lambda^2 - 2\lambda - 4\lambda + 4 = 0$

$\lambda^2 - 6\lambda + 9 = 0$

$(\lambda - 3)^2 = 0$ $\lambda = 3$

(18) Αν $\alpha \neq -\beta \Rightarrow \alpha+\beta \neq 0$

νδο $2x^2 + 2(\alpha-\beta)x + \alpha^2+\beta^2 = 0$

είναι αδύνατη στο $\mathbb{R}$ όταν $\Delta < 0$

$\Delta = 4(\alpha-\beta)^2 - 4 \cdot 2 \cdot (\alpha^2+\beta^2)$

$\Delta = 4(\alpha^2 - 2\alpha\beta + \beta^2) - 8\alpha^2 - 8\beta^2$

$\Delta = 4\alpha^2 - 8\alpha\beta + 4\beta^2 - 8\alpha^2 - 8\beta^2$

$\Delta = -4\alpha^2 - 8\alpha\beta - 4\beta^2$

$\Delta = -4(\alpha^2 + 2\alpha\beta + \beta^2)$

$\Delta = -4(\alpha+\beta)^2 < 0$

$\oplus$

ισχύει

Άρα $\Delta < 0$

Δείξαμε.

Ασκηση

Αν η εξίσωση $\lambda^4(x-\lambda^2)-8x=8(x-4\lambda)$

είναι ταυτότητα.

Να λυθεί η εξίσωση

$$\lambda^2(x^3+3)(x^3-3)-(2x^3+2)^2=-5.$$

$$\lambda^4 x-\lambda^6-8x=8x-32\lambda$$

$$\lambda^4 x-8x-8x=\lambda^6-32\lambda$$

$$\lambda^4 x-16x=\lambda^6-32\lambda$$

$$(\lambda^4-16)x=\lambda^6-32\lambda$$

$$((\lambda^2)^2-(2^2)^2)x=\lambda^6-32\lambda$$

$$(\lambda^2-2^2)(\lambda^2+2^2)x=\lambda^6-32\lambda$$

$$(\lambda+2)(\lambda-2)(\lambda^2+4)x=\lambda^6-32\lambda$$

Αν $\lambda+2=0 \Rightarrow \lambda=-2$ τότε

$$0x=(-2)^6+32\cdot 2$$

$$0x=2^6+64$$

2^6+64 ⊕

άρα Αδύνατη

AV $\lambda - 2 = 0 \Rightarrow \lambda = 2$

$0x = 2^6 - 32 \cdot 2$

$0x = 64 - 64$

$0x = 0$

Tautologia

$2^2 (x^3+3)(x^3-3) - (2x^3+1)^2 = -5$

$4(x^6 - 9) - 4x^6 - 4x^3 - 1 = -5$

$4x^6 - 36 - 4x^6 - 4x^3 - 1 = -5$

$-4x^3 = 37 - 5$

$\frac{-4x^3}{-4} = \frac{32}{-4}$

$x^3 = -8$

$x^3 = (-2)^3$

$x = -2$

Δοκιμή

$$\alpha^{10} + \beta^6 + 65 = 2\left(a^5 - (2\beta)^3\right).$$

(α) Βρείτε α, β.

(β) $\left(|3x+3| - \alpha\right)^4 = (2\alpha - 3\beta)^{4/3}$

Λύση

α) $\alpha^{10} + \beta^6 + 65 = 2(a^5 - (2\beta)^3) \Rightarrow$

$\Rightarrow \alpha^{10} + \beta^6 + 65 = 2(a^5 - 8\beta^3) \Rightarrow$

$\Rightarrow \alpha^{10} + \beta^6 + 65 = 2a^5 - 16\beta^3 \Rightarrow$

$\Rightarrow \alpha^{10} - 2a^5 + \beta^6 + 16\beta^3 + 65 = 0 \Rightarrow$

$\Rightarrow \alpha^{10} - 2a^5 + 1 + \beta^6 + 16\beta^3 + 64 = 0 \Rightarrow$

$\Rightarrow (a^5 - 1)^2 + (\beta^3 + 8)^2 = 0 \Rightarrow$

$\Rightarrow a^5 - 1 = 0 \Rightarrow$ και $\beta^3 + 8 = 0 \Rightarrow$

$\Rightarrow a^5 = 1 \Rightarrow$ $\Rightarrow \beta^3 = -8 \Rightarrow$

$\Rightarrow \boxed{\alpha = 1}$ $\Rightarrow \beta^3 = (-2)^3 \Rightarrow$

$\Rightarrow \boxed{\beta = -2}$

β) $(|\beta x+3|-\alpha)^4 = (2\alpha-3\beta)^{4/3} \Rightarrow$

$\Rightarrow (|-2x+3|-2)^4 = (2+6)^{4/3} \Rightarrow$

$\Rightarrow (|-2x+3|-2)^4 = \sqrt[3]{8^4} \Rightarrow$

$\Rightarrow (|-2x+3|-2)^4 = \sqrt[3]{4096} \Rightarrow$

$\Rightarrow (|-2x+3|-2)^4 = 16 \Rightarrow$

$\Rightarrow |-2x+3|-2 = \sqrt[4]{16} \Rightarrow$

$\Rightarrow |-2x+3| = 3 \Rightarrow -2x+3 = 3 \Rightarrow$ ή $-2x+3 = -3 \Rightarrow$

$\Rightarrow \frac{-2x}{-2} = \frac{0}{-2} \Rightarrow$ $\Rightarrow \frac{-2x}{-2} = \frac{-6}{-2} \Rightarrow$

$\Rightarrow \boxed{x=0}$ $\Rightarrow \boxed{x=3}$

14. (B) $x^2 - x - \lambda + 1 = 0$.

$$\Delta = B^2 - 4\alpha\gamma$$

$$\Delta = (-1)^2 - 4 \cdot 1(-\lambda + 1)$$

$$\Delta = 1 - 4(-\lambda + 1)$$

$$\Delta = 1 + 4\lambda - 4$$

$$\boxed{\Delta = 4\lambda - 3}$$

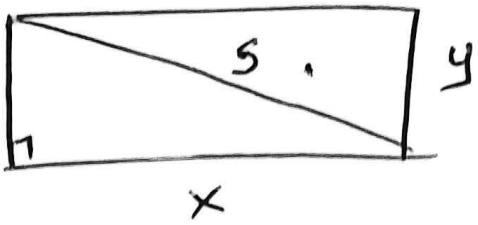
1. Αν $4\lambda - 3 > 0 \Rightarrow 4\lambda > 3 \Rightarrow \lambda > \frac{3}{4}$ $(\Delta > 0)$
   τότε έχω 2 ρίζες.

2. Αν $4\lambda - 3 < 0 \Rightarrow \lambda < \frac{3}{4}$ $(\Delta < 0$.
   τότε καμία ρίζα.

3. Αν $\lambda = \frac{3}{4}$ τότε $\Delta = 0$

1 ρίζα διπλή.

12.



$\square = 14$

$2x + 2y = 14$

$x + y = 7$ $\quad y = 7 - x$

$x^2 + y^2 = 5^2$

$x^2 + (7-x)^2 = 25$

$x^2 + 49 - 14x + x^2 = 25$

$2x^2 - 14x + 24 = 0$

$x^2 - 7x + 12 = 0$

$x = 2$ $\quad x = 5$

$y = 5$ $\quad y = 2$

9. (a) $x^2 - (\lambda-3)x - \lambda + 2 = 0$.

$$\Delta = (\lambda-3)^2 - 4(2-\lambda)$$

$$\Delta = \lambda^2 - 6\lambda + 9 - 8 + 4\lambda$$

$$\Delta = \lambda^2 - 2\lambda + 1$$

$$\Delta = (\lambda-1)^2 \geq 0$$

1. Durum: $\lambda = 1$ iken $\Delta = 0$

$$x = \frac{\lambda-3}{2} = \frac{1-3}{2} = \frac{-2}{2} = \boxed{-1}$$.

2. Durum: $\lambda \neq 1$ iken $\Delta > 0$

$$x = \frac{\lambda-3 \pm \lambda-1}{2}$$

$$\frac{2\lambda-4}{2} = \boxed{\lambda-2}$$,

$$\frac{-2}{2} = \boxed{-1}$$.

(B) $\rho_1 = \lambda - 2$.

$\rho_2 = -1$

$|\rho_1 - \rho_2| = 2$.

$|\lambda - 2 + 1| = 2$

$|\lambda - 1| = 2$

$\lambda - 1 = 2$

$\lambda = 3$

$\lambda - 1 = -2$

$\lambda = -1$

Λύση

$$(\lambda+1)x^2 + 2(\lambda-1)x + \lambda - 3 = 0 \quad .$$

1) Αν $\lambda = -1$ τότε $2(-1-1)x + (-1) - 3 = 0$

$$-4x - 4 = 0$$

$$-4x = 4$$

$$\boxed{x = -1}$$

Αν $\lambda \neq -1$ τότε

$$\Delta = \beta^2 - 4\alpha\gamma = (2(\lambda-1))^2 - 4(\lambda+1)(\lambda-3) =$$

$$= 4(\lambda-1)^2 - 4(\lambda+1)(\lambda-3) = 4(\lambda^2 - 2\lambda + 1 - (\lambda+1)(\lambda-3))$$

$$= 4(\lambda^2 - 2\lambda + 1 - (\lambda^2 - 3 - 2\lambda)) =$$

$$= 4(\cancel{\lambda^2} - \cancel{2\lambda} + 1 - \cancel{\lambda^2} + \cancel{2\lambda} + 3) =$$

$$= 4 \cdot 4 = 16$$

$\Delta > 0 \Longrightarrow x_{1,2} = \dfrac{-2(\lambda-1) \pm \sqrt{16}}{2(\lambda+1)} \Rightarrow x_{1,2} = \dfrac{-2\lambda + 2 \pm 4}{2\lambda + 2} \Rightarrow$

$\Rightarrow$

$$x_1 = \frac{-2\lambda + 2 + 4}{2\lambda + 2} = \frac{-2(\lambda - 3)}{2(\lambda+1)} = -\frac{\lambda-3}{\lambda+1}$$

$$x_2 = \frac{-2\lambda + 2 - 4}{2(\lambda+1)} = \frac{-2(\lambda+1)}{2(\lambda+1)} = -1$$

8. (B) $\lambda^2 x^2 - 3\lambda x - 4 = 0 \qquad \lambda \neq 0$

$\Delta = B^2 - 4\alpha\gamma$

$\Delta = (-3\lambda)^2 - 4 \cdot \lambda^2 \cdot (-4)$.

$\Delta = 9\lambda^2 + 16\lambda^2$

$\Delta = 25\lambda^2 > 0$ άρα έχει δύο ρίζες άνισες.

$$x_{1,2} = \frac{-B \pm \sqrt{\Delta}}{2\alpha}$$

$$x_{1,2} = \frac{-(-3\lambda) \pm 5\lambda}{2\lambda^2} = \frac{3\lambda \pm 5\lambda}{2\lambda^2} = \begin{cases} \frac{8\lambda}{2\lambda^2} = \frac{4}{\lambda} \\ \frac{-2\lambda}{2\lambda^2} = -\frac{1}{\lambda} \end{cases}$$

(8) $\lambda x^2 - 2x - \lambda + 2 = 0$ $\quad \lambda \neq 0$.

$\Delta = (-2)^2 - 4\lambda(-\lambda+2)$

$\Delta = 4 + 4\lambda^2 - 8\lambda$

$\Delta = 4(\lambda^2 - 2\lambda + 1)$

$\Delta = 4(\lambda-1)^2 \geq 0$

1. Αν $\lambda = 1$ τοτε $\Delta = 0$ αρα

έχω διπλη ριζα

$$x = -\frac{B}{2a} = -\frac{-2}{2\lambda} = \frac{1}{\lambda} = \frac{1}{1} = 1$$

$x = 1$

2. Αν $\lambda \neq 1$ τοτε $\Delta > 0$ αρα

έχω δυο διαφορα ριζες.

$$x = \frac{-(-2) \pm 2(\lambda-1)}{2\lambda} = \frac{2 \pm 2(\lambda-1)}{2\lambda}$$

$\frac{2+2(\lambda-1)}{2\lambda}$

$\frac{2-2(\lambda-1)}{2\lambda}$

$x_1 = 1$ $\qquad x_2 = \frac{4-2\lambda}{2\lambda} = \frac{2-\lambda}{\lambda}$.

Άσκηση

$$\lambda x^2 - (\lambda-2)x - 2 = 0$$

1η: Αν $\lambda = 0$ τότε $2x - 2 = 0 \Rightarrow 2x = 2 \Rightarrow x = 1$

2η: Αν $\lambda \neq 0$ τότε $\lambda x^2 - (\lambda-2)x - 2 = 0$ $\quad a = \lambda,\ \beta = -(\lambda-2),\ \gamma = -2$

$$\Delta = \beta^2 - 4\alpha\gamma = [-(\lambda-2)]^2 - 4(\lambda(-2)) = (\lambda-2)^2 + 8\lambda =$$
$$= (\lambda^2 - 4\lambda + 4) + 8\lambda = \lambda^2 + 4\lambda + 4 = (\lambda+2)^2$$

$$\Delta = (\lambda+2)^2 \geq 0$$

(α) Αν $\Delta = 0$ τότε $\lambda = -2$

$$x = -\frac{\beta}{2\alpha} = \frac{-(\lambda-2)}{2\lambda} = \frac{2-\lambda}{2\lambda} = \frac{4}{-4} = -1$$

(β) Αν $\Delta > 0$ τότε έχω δύο ρίζες

$$x = \frac{\lambda - 2 \pm \lambda + 2}{2\lambda}$$

$$\frac{2\lambda}{2\lambda} = 1$$

$$\frac{-4}{2\lambda} = -\frac{2}{\lambda}$$

Άσκηση

Να λυθεί η εξίσωση.

$$(\lambda-3)x^2 + 2\lambda x + \lambda + 3 = 0.$$

- Αν $\lambda = 3$ τότε $6x + 6 = 0 \Rightarrow 6x = -6 \Rightarrow x = -1$

- Αν $\lambda \neq 3$ τότε $\Delta = \beta^2 - 4\alpha\gamma \Rightarrow \Delta = 4\lambda^2 - 4(\lambda-3)(\lambda+3) \Rightarrow$

$$\Rightarrow \Delta = 4\lambda^2 - 4(\lambda^2 - 9) \Rightarrow$$

$$\Rightarrow \Delta = 4\lambda^2 - 4\lambda^2 + 36 \Rightarrow \Delta = 36$$

$$x_{1,2} = \frac{-\beta \pm \sqrt{\Delta}}{2\alpha} \Rightarrow x_{1,2} = \frac{-2\lambda \pm 6}{2\lambda - 6} \Rightarrow$$

$$\Rightarrow \begin{cases} x_1 = \dfrac{-2\lambda + 6}{2\lambda - 6} \Rightarrow x_1 = \dfrac{2(3-\lambda)}{2(\lambda-3)} \Rightarrow x_1 = -\dfrac{2(3-\lambda)}{2(\lambda-3)} \Rightarrow \\ \Rightarrow \boxed{x_1 = -1} \\ x_2 = \dfrac{-2\lambda - 6}{2\lambda - 6} \Rightarrow x_2 = \dfrac{-2(\lambda+3)}{2(\lambda-3)} \Rightarrow \boxed{x_2 = -\dfrac{\lambda+3}{\lambda-3}} \end{cases}$$

Άσκηση

$$(\lambda-2)x^2 - 2(\lambda+1) + \lambda + 4 = 0 .$$

1. Αν $\lambda = 2$ τότε $-6x + 6 = 0$

$$\boxed{x = 1}$$

2. Αν $\lambda \neq 2$ τότε

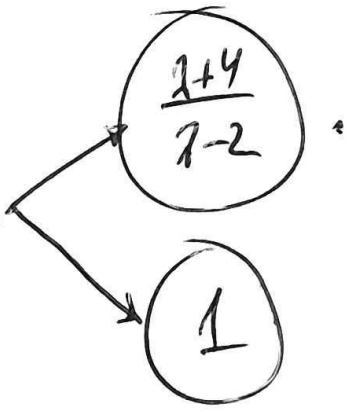
$$\Delta = 4(\lambda+1)^2 - 4(\lambda-2)(\lambda+4)$$

$$\Delta = 4\left[\lambda^2 + 2\lambda + 1 - (\lambda^2 + 4\lambda - 2\lambda - 8)\right] .$$

$$\Delta = 4(\cancel{\lambda^2} + 2\lambda + 1 - \cancel{\lambda^2} - 4\lambda + 2\lambda + 8)$$

$\Delta = 36 > 0$ δύο ρίζες.

$$X = \frac{2(\lambda+1) \pm 6}{2(\lambda-2)} = \frac{\lambda+1 \pm 3}{\lambda-2}$$



7. (8) $\lambda x^2 + 2x - \lambda - 2 = 0$

$\alpha = \lambda \qquad \beta = 2 \qquad \gamma = -\lambda - 2$

$\Delta = 2^2 - 4 \cdot \lambda \cdot (-\lambda - 2)$

$\Delta = 4 + 4\lambda^2 + 8\lambda$

$\Delta = 4(\lambda^2 + 2\lambda + 1)$

$\Delta = 4(\lambda + 1)^2 \geq 0$ ✓ 2

6. Σελ 181

$$\lambda x^2 - (\lambda-1)x - 1 = 0 \qquad \lambda \neq 0.$$

(α). Έχει ρίζα το -2.

Όπου x το -2.

$$\lambda(-2)^2 - (\lambda-1)(-2) - 1 = 0$$

$$4\lambda + 2(\lambda-1) - 1 = 0$$

$$6\lambda - 3 = 0$$

$$\lambda = \frac{1}{2}$$

$\alpha = \lambda$

$\beta = -(\lambda-1)$

$\gamma = -1$

(β)

$$\Delta = \beta^2 - 4\alpha\gamma.$$

$$\Delta = [-(\lambda-1)]^2 - 4 \cdot \lambda \cdot (-1)$$

$$\Delta = (\lambda-1)^2 + 4\lambda = \lambda^2 - 2\lambda + 1 + 4\lambda = \lambda^2 + 2\lambda + 1$$

$$\Delta = (\lambda+1)^2 \geq 0 \quad \checkmark$$

Εξισώσεις με απόλυτα Τιμή.

$|x| = \theta \Leftrightarrow x = \theta$ ή $x = -\theta$

1. $|2x+4| = 8$

$2x+4 = 8$ ή $2x+4 = -8$.

$2x = 4$ $\quad$ $2x = -8-4$

$2x = -12$

$x = 2$ $\quad$ $x = -6$

$|x| = |y| \Leftrightarrow x = y$ ή $x = -y$

2. $|x+4| = |2x-2|$

$x+4 = 2x-2$ ή $x+4 = -2x+2$

$x-2x = -2-4$ $\quad$ $x+2x = 2-4$

$-x = -6$ $\quad$ $3x = -2$

$x = 6$ $\quad$ $x = -\frac{2}{3}$

$d(x,y) = |x-y|$

3. $5 - d(2x,-2) = 7 - |3x+3|$

$5 - |2x+2| = 7 - |3x+3|$.

$5 - 2|x+1| = 7 - 3|x+1|$

$-2|x+1| + 3|x+1| = 7-5$

$|x+1| = 2 \rightarrow$ $x+1 = 2$ $\quad x = 1$

$x+1 = -2$ $\quad x = -3$

4. $\frac{d(x,3)}{2} + \frac{|6-2x|}{3} = 8 - \frac{|3-x|}{6}$.

$|x| = |-x|$

$$\frac{|x-3|}{2} + \frac{2|3-x|}{3} = 8 - \frac{|x-3|}{6}$$

$$\frac{|x-3|}{2} + \frac{2|x-3|}{3} = 8 - \frac{|x-3|}{6}$$

$$3|x-3| + 4|x-3| = 48 - |x-3|$$

$$7|x-3| + |x-3| = 48$$

$$8|x-3| = 48.$$

$$|x-3| = 6$$

$x-3=6$ ó $x-3=-6$

$x=-3$

$x=9$

5. $|x-4| \cdot |x+3| = |x-2| \cdot |x-6|$

$$|(x-4)(x+3)| = |(x-2)(x-6)|$$

$$|x^2+3x-4x-12| = |x^2-6x-2x+12|$$

$$|x^2-x-12| = |x^2-8x+12|$$

$x^2 - x - 12 = x^2 - 8x + 12$ ή $x^2 - x - 12 = -x^2 + 8x + 12$

$8x - x = 12 + 12$ $\qquad$ $2x^2 - 9x = 0$

$7x = 24$ $\qquad$ $x(2x-9) = 0$

$x = \frac{24}{7}$

$x = 0$ ή $2x - 9 = 0$

$2x = 9$

$x = \frac{9}{2}$

6. $|x-4| \cdot |x+3| = |x-2| \, |x+3|$

$$|x+3| \left(|x-4| - |x-2| \right) = 0$$

$|x+3| = 0$ ή $|x-4| - |x-2| = 0$

$x + 3 = 0$ $\qquad$ $|x-4| = |x-2|$

$x = -3$

$x - 4 = x - 2$ ή $x - 4 = -x + 2$

Αδύνατη $\qquad$ $2x = 6$

$x = 3$

7. $\frac{|x-1|}{4} - 2x = \frac{|2x-2|}{2} - (x+1)$.

$$\frac{|x-1|}{4} - 2x = \frac{2|x-1|}{2} - x - 1$$

$$\frac{|x-1|}{4} = x - 1 + |x-1|$$

$$|x-1| = 4x - 4 + 4|x-1|$$

Прим $|x-1| - 4|x-1| = 4x - 4$

$$-3|x-1| = 4x - 4$$

$$|x-1| = \frac{4x-4}{-3}$$

при $\frac{4x-4}{-3} \geq 0$

$4x - 4 \leq 0$

$4x \leq 4$

$x \leq 1$.

$x - 1 = \frac{4x-4}{-3}$

$-3x + 3 = 4x - 4$

$-7x = -7$

$x = 1$

и $x - 1 = -\frac{4x-4}{-3}$

$-3x + 3 = -4x + 4$

$x = 1$

8. $|x^2-2x-3|+|9-x^2|=0$.

πρπι $x^2-2x-3=0$ και $9-x^2=0$

$x=3$ $x=-1$ $x=3$ $x=-3$

$x=3$

9. $5+\sqrt{x^2-6x+9}=3x$

$5+\sqrt{(x-3)^2}=3x$

$|x-3|=3x-5$

$\Rightarrow 3x\geq 5 \Rightarrow x\geq \frac{5}{3}$

πρπι $3x-5\geq 0$

$x-3=3x-5$ ή $x-3=-3x+5$

$x-3x=-5+3$ $x+3x=5+3$

$-2x=-2$ $4x=8$

$x=1$ $x=\frac{1}{2}$

Δεκτή.

10. $x + |x+3| - |4-x| = 0$

x		-3		4
$x+3$	$-$	0	$+$	$+$
$4-x$	$+$		$+$	0 $-$

1o. Av $x < -3$ TOZ.

$$x + |x+3| - |4-x| = 0$$

$$x + (-x-3) - (4-x) = 0$$

$$x - x - 3 - 4 + x = 0$$

$$\boxed{x = 7}$$

2o. Av $-3 \leq x \leq 4$ TOZC

$$x + |x+3| - |4-x| = 0$$

$$x + x + 3 - (4-x) = 0$$

$$2x + 3 - 4 + x = 0$$

$$3x - 1 = 0$$

$$x = \frac{1}{3}$$

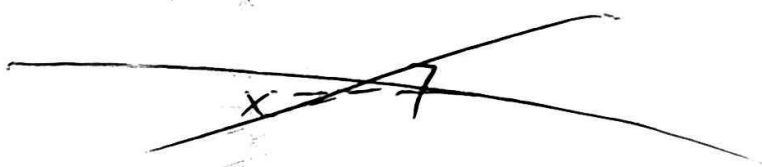
30. A ∨ $x > 4$ TOZC

$x + |x+3| - |4-x| = 0$ (+) (−)

$x + x + 3 - (-4+x) = 0$

$2x + 3 + 4 - x = 0$

$x + 7 = 0$



11. $\left| 1 - |3-2x| \right| = 6$.

$1 - |3-2x| = 6$ ή $1 - |3-2x| = -6$

$-|3-2x| = 6-1$ $\quad -|3-2x| = -6-1$

$-|3-2x| = 5$ $\quad -|3-2x| = -7$

$|3-2x| = -5$ $\quad |3-2x| = 7$

Αδύνατη

$3-2x = 7$ ή $3-2x = -7$

$-2x = 7-3$ $\quad -2x = -7-3$

$-2x = 4$ $\quad -2x = -10$

$x = -2$ $\quad x = 5$

12. $d(4, d(x,0)) = d(d(x,0), -3)$.

$d(4, |x-0|) = d(|x-0|, -3)$

$d(4, |x|) = d(|x|, -3)$

$|4-|x|| = ||x|+3|$

$4-|x| = |x|+3$ ili ~~$4-|x| = -|x|-3$~~

Apsurd.

$-2|x| = -1$

$|x| = \frac{1}{2}$

$x = \frac{1}{2}$ i $x = -\frac{1}{2}$

13. $|2x^3| - |x|^3 - 4x^2 = 0$

$2|x^3| - |x|^3 - 4|x|^2 = 0$

$2|x|^3 - |x|^3 - 4|x|^2 = 0$

$|x|^3 - 4|x|^2 = 0$

$|x|^2(|x| - 4) = 0$

$|x|^2 = 0$ и $|x| - 4 = 0$

$x = 0$ $|x| = 4$

$x = 4$ $x = -4$

14. $\left|\frac{x-3}{x-2}\right| + 1 - \frac{x-1}{|x-2|} = 0.$

$\frac{|x-3|}{|x-2|} + 1 - \frac{x-1}{|x-2|} = 0$

$|x-3| + |x-2| - (x-1) = 0$

$|x-3| + |x-2| = x - 1.$

при $x - 1 \geq 0$

$x \geq 1.$

$|x-3| + |x-2| = x-1$

$x \geq 1$

x		2		?	
x-3	−		−	0	+
x-2	−	0	+		+

1. Av $x < 2$ tozc

$\ominus \quad \ominus$
$|x-3| + |x-2| = x-1$

$-x+3 - x+2 = x-1$

$-2x+5 = x-1$

$-3x = -6$

$x = 2$

2. Av $2 \leq x \leq 3$ tozc

$\ominus \quad \oplus$
$|x-3| + |x-2| = x-1$

$-x+3 + x-2 = x-1$

$1 = x-1$

$x = 2$

3. Av $x > 3$ tozc

$\oplus \quad \oplus$
$|x-3| + |x-2| = x-1$

$x-3 + x-2 = x-1$

$x = 4$

Σελ 182

Εποχές Μαθημ

(7) α, β, δ

(8) α, γ

(10)

(11)

(13)

(14) α.