

# Άσκηση

$$(\lambda^2 x - 2)(\lambda - 2) + \lambda x - (\lambda - 1)^2 = 2.$$

$$\lambda^3 x - 2\lambda^2 x - 2\lambda + 4 - \lambda x - (\lambda^2 - 2\lambda + 1) = 2$$

$$\lambda^3 x - 2\lambda^2 x - 2\lambda + 4 - \lambda x - \lambda^2 + 2\lambda - 1 = 2$$

$$\lambda^3 x - 2\lambda^2 x - \lambda x = 2 + 2\lambda - 4 + \lambda^2 - 2\lambda + 1$$

$$x(\lambda^3 - 2\lambda^2 - \lambda) = \lambda^2 - 1$$

$$\lambda(\lambda^2 - 2\lambda + 1)x = (\lambda - 1)(\lambda + 1)$$

$$\lambda(\lambda - 1)^2 x = (\lambda - 1)(\lambda + 1)$$

Αν το  $\lambda = 0$  τότε  $0x = -1$  αδυνατη

Αν το  $\lambda = 1$  τότε  $0x = 0$  ταυτοτητα

~~Αν το  $\lambda \neq 0$  και  $\lambda \neq 1$~~

Αν  $\lambda \neq 0$  και  $\lambda \neq 1$

$$\lambda(\lambda - 1)^2 x = (\lambda - 1)(\lambda + 1)$$

$$\lambda(\lambda - 1)^2 \quad \lambda(\lambda - 1)^2$$

$$x = \frac{\lambda + 1}{\lambda(\lambda - 1)} \Leftrightarrow x = \frac{\lambda + 1}{\lambda^2 - \lambda}$$

$$25. \textcircled{a} \quad x^3 + (3x-1)^3 + (1-4x)^3 = 0.$$

$$\rightarrow x + 3x - 1 + 1 - 4x = 0$$

$$0 = 0$$

$$3x(3x-1)(1-4x) = 0$$

$$3x = 0$$

$$\textcircled{x=0}$$

or

$$3x - 1 = 0$$

$$3x = 1$$

$$\textcircled{x = \frac{1}{3}}$$

or

$$1 - 4x = 0$$

$$1 = 4x$$

$$\textcircled{x = \frac{1}{4}}$$

24. (a)  $x^2(x-2) + 2x(x-2) + x-2 = 0$

$$(x-2)(x^2 + 2x + 1) = 0$$

$$(x-2)(x+1)^2 = 0$$

$$x-2=0$$

$$x=2$$

$$\text{or } (x+1)^2 = 0$$

$$x=-1$$

(b)  $(2x-1)^2 = 2x-1$

$$(2x-1)^2 - 2x + 1 = 0$$

$$(2x-1)^2 - (2x-1) = 0$$

$$(2x-1)(2x-1-1) = 0$$

$$2x-1=0$$

$$2x=1$$

$$x = \frac{1}{2}$$

$$\text{or } 2x-2=0$$

$$2x=2$$

$$x=1$$

$$7. \textcircled{B} \quad x^2 - 2x(x-1) = x$$

$$x^2 - 2x^2 + 2x = x$$

$$-x^2 + 2x - x = 0$$

$$-x^2 + x = 0$$

$$x(-x+1) = 0$$

$$\textcircled{x=0}$$

$$-x+1 = 0$$

$$\textcircled{x=1}$$

$$\textcircled{E} (x^2-9)(x-1) = (x^2-1)(x-3)$$

$$(x-3)(x+3)(x-1) = (x-1)(x+1)(x-3)$$

$$(x-3)(x+3)(x-1) - (x-1)(x+1)(x-3) = 0$$

$$(x-3)(x-1) \left( x+3 - (x+1) \right) = 0$$

$$(x-3)(x-1) \left( \cancel{x+3} - \cancel{x-1} \right) = 0$$

$$2(x-3)(x-1) = 0$$

$$x-3=0$$

$$x=3$$

or

$$x-1=0$$

$$x=1$$

$$(7) \quad x(x-1)^2 = x^2 - 2x + 1$$

$$x(x-1)^2 = (x-1)^2$$

$$x(x-1)^2 - (x-1)^2 = 0$$

$$(x-1)^2(x-1) = 0$$

$$(x-1)^2 = 0$$

$$\text{or } x-1 = 0$$

$$x=1$$

$$x=1$$

$$(8) \quad x^3 - 3x^2 = (2x-1)(x-3)$$

$$x^2(x-3) - (2x-1)(x-3) = 0$$

$$(x-3)(x^2 - 2x + 1) = 0$$

$$x-3 = 0$$

$$\text{or } x^2 - 2x + 1 = 0$$

$$x=3$$

$$(x-1)^2 = 0$$

$$x=1$$

6. (α)  $x^2 = x$

$$x^2 - x = 0$$

$$x(x-1) = 0$$

$$x=0$$

$$\vee x-1=0$$

$$x=1$$

(β)  $x^3 = 2x^2$

$$x^3 - 2x^2 = 0$$

$$x^2(x-2) = 0$$

$$x^2 = 0$$

$\vee$

$$x-2=0$$

$$x=0$$

$$x=2$$

(γ)  $x^3 = x$

$$x^3 - x = 0$$

$$x(x^2-1) = 0$$

$$x=0$$

$\vee$

$$x^2-1=0$$

$$(x-1)(x+1) = 0$$

$$x=1$$

$$x=-1$$

$$\textcircled{52} \quad -x^2 + x = 0$$

$$x(-x+1) = 0$$

$$\textcircled{x=0}$$

or

$$-x+1=0$$

$$\textcircled{x=1}$$



9. (20)  $\frac{x}{x^2-1} = \frac{1}{x+1}$

$$\frac{x}{(x-1)(x+1)} = \frac{1}{x+1}$$

EKAD  $(x-1)(x+1)$ .

$\rightarrow (x-1)(x+1) = 0$

$x-1=0$      $x+1=0$

$x=1$

$x=-1$

$x \neq 1$   
 $x \neq -1$

$\rightarrow x+1=0$

$x=-1$

~~$x = x-1$~~

$0 = -1$

Answer.

$$9. \quad \textcircled{3} \quad \frac{1}{x^2+x} = \frac{x}{x+1}$$

$$\frac{1}{x(x+1)} = \frac{x}{x+1}$$

$$\rightarrow x(x+1) = 0$$

$$\textcircled{x=0}$$

u'

$$x+1=0$$

$$\textcircled{x=-1}$$

$$\begin{aligned} x &\neq 0 \\ x &\neq -1 \end{aligned}$$

$$\rightarrow x+1=0$$

$$\underline{\underline{x=-1}}$$

$$E(\cap) = x(x+1)$$

$$1 = x^2$$

$$\textcircled{x=1}$$

~~$$x=1$$~~

3.  $\sum_{r=1}^n 156$

$$\textcircled{E} \quad \frac{2(x-2)}{3} - \frac{1-3(x-2)}{2} = \frac{1}{6}x$$

$$2 \cdot \frac{2(x-2)}{3} - 3 \left( \frac{1-3(x-2)}{2} \right) = x$$

$$4x - 4 - 3 \left( \frac{1-3x+6}{2} \right) = x$$

$$4x - 4 - 3 + 9x - 18 = x$$

$$13x - 25 = x$$

$$13x - x = 25$$

$$12x = 25$$

$$x = \frac{25}{12}$$

# Άσκηση

$$2\lambda^2 x - \lambda^2 (\lambda^2 x - 1) = -2\lambda (\lambda x - 1)$$

$$2\lambda^2 x - \lambda^4 x + \lambda^2 = -2\lambda^2 x + 2\lambda$$

$$2\lambda^2 x - \lambda^4 x + 2\lambda^2 x = -\lambda^2 + 2\lambda$$

$$x \cdot (2\lambda^2 - \lambda^4 + 2\lambda^2) = -\lambda^2 + 2\lambda$$

$$\lambda \cdot (2\lambda - \lambda^3 + 2\lambda) \cdot x = \lambda \cdot (-\lambda + 2)$$

$$\lambda(4\lambda - \lambda^3) x = \lambda(2 - \lambda)$$

$$\lambda^2(4 - \lambda^2) x = \lambda(2 - \lambda)$$

$$\lambda^2(2 - \lambda)(2 + \lambda) x = \lambda(2 - \lambda)$$

1. Αν το  $\lambda$  είναι  $2$ , τότε  $0x = 0$  Τάωωωωωωωω.

2. Αν το  $\lambda$  είναι  $-2$ , τότε  $0x = -8$  Αδύνατο.

3. Αν το  $\lambda \neq 2$  και  $\lambda \neq -2$  τότε:

$\lambda \neq 0$

$$\frac{\lambda^2(2 - \lambda)(\cancel{\lambda + 2}) x = \cancel{\lambda}(2 - \lambda)}{\cancel{\lambda^2(2 - \lambda)(\lambda + 2)}} \quad \frac{\lambda(2 - \lambda)}{\lambda(2 - \lambda)(\lambda + 2)}$$

$$\frac{\lambda^2(2 + \lambda)}{\lambda} \Rightarrow \frac{2\lambda^2 + \lambda^3}{\lambda}$$

$$x = \frac{1}{\lambda(\lambda + 2)}$$

4. Αν  $\lambda = 0$  τότε  $0x = 0$  Τάωωωωωωωω

# Абхыш

$$a^3(x-1) - 6a(x+a) = 3a(x-3a)$$

$$x a^3 - a^3 - 6ax - 6a^2 = 3ax - 9a^2$$

$$x a^3 - a^3 - 6ax - 3ax = +6a^2 - 9a^2$$

$$x a^3 - 6ax - 3ax = 6a^2 - 9a^2 + a^3$$

$$x a^3 - 9ax = -3a^2 + a^3$$

$$x(a-9a) = a^2(-3+a)$$

$$x(8a) = a^2(-3+a)$$

~~8a=0~~  
~~a=0~~

$$8a = 0$$

$$a = 0$$

$$a^3 x - 6ax - 3ax = a^3 + 6a^2 - 9a^2$$

$$(a^3 - 6a - 3a) x = a^3 - 3a^2$$

$$(a^3 - 9a) x = a^3 - 3a^2$$

$$a(a^2 - 9) x = a^2(a - 3)$$

$$a(a-3)(a+3) x = a^2(a-3)$$

$$\lambda(\lambda-3)(\lambda+3)x = \lambda^2(\lambda-3)$$

1.  $\lambda = 0$  ~~200~~

$0x = 0$   
ταυτοτητα

2.  $\lambda - 3 = 0$       $3(3-3)(3+3)x = 3^2(3-3)$

$\lambda = 3$       $0x = 0$   
ταυτοτητα

3.  $\lambda + 3 = 0$   
 $\lambda = -3$

$-3(-3+3)(-3+3)x = -3^2(-3-3)$

$-3(-6)(0)x = 0 \cdot 9(-6)$

$0x = -54$

Αδυνα

4. Av  $\lambda \neq 0$   $\lambda \neq 3$   $\lambda \neq -3$

$$\lambda(\lambda-3)(\lambda+3)x = \lambda^2(\lambda-3)$$

$$\frac{\lambda(\lambda-3)(\lambda+3)x}{\lambda(\lambda-3)(\lambda+3)} = \frac{\lambda^2(\lambda-3)}{\lambda(\lambda-3)(\lambda+3)}$$

$$x = \frac{\lambda^2}{\lambda(\lambda+3)}$$

$$x = \frac{\lambda}{\lambda+3}$$

# Εξισώσεις με απόλυτα Τιμή.

1.  $|2x+4|=8$

$$2x+4=8$$

$$2x=4$$

$$x=2$$

ή

$$2x+4=-8$$

$$2x=-8-4$$

$$2x=-12$$

$$x=-6$$

2.  $|x+4|=|2x-2|$

$$x+4=2x-2$$

$$x-2x=-2-4$$

$$-x=-6$$

$$x=6$$

ή

$$x+4=-2x+2$$

$$x+2x=2-4$$

$$3x=-2$$

$$x=-\frac{2}{3}$$

3.  $5 - d(2x, -2) = 7 - |3x+3|$

$$5 - |2x+2| = 7 - |3x+3|$$

$$5 - 2|x+1| = 7 - 3|x+1|$$

$$-2|x+1| + 3|x+1| = 7-5$$

$$|x+1| = 2 \rightarrow$$

$$x+1=2$$

$$x+1=-2$$

$$x=1$$

$$x=-3$$

$$4. \quad \frac{d(x,3)}{2} + \frac{|6-2x|}{3} = 8 - \frac{|3-x|}{6}$$

$$\frac{|x-3|}{2} + \frac{2|3-x|}{3} = 8 - \frac{|x-3|}{6}$$

$$\frac{|x-3|}{2} + \frac{2|x-3|}{3} = 8 - \frac{|x-3|}{6}$$

$$3|x-3| + 4|x-3| = 48 - |x-3|$$

$$7|x-3| + |x-3| = 48$$

$$8|x-3| = 48$$

$$|x-3| = 6$$

$$x-3=6$$

∴

$$x-3=-6$$

$$x=9$$

$$x=-3$$



$$5. \quad |x-4| \cdot |x+3| = |x-2| \cdot |x-6|$$

$$|(x-4)(x+3)| = |(x-2)(x-6)|$$

$$|x^2 + 3x - 4x - 12| = |x^2 - 6x - 2x + 12|$$

$$|x^2 - x - 12| = |x^2 - 8x + 12|$$

$$x^2 - x - 12 = x^2 - 8x + 12$$

$$8x - x = 12 + 12$$

$$7x = 24$$

$$x = \frac{24}{7}$$

$$x^2 - x - 12 = -x^2 + 8x + 12$$

$$2x^2 - 9x = 0$$

$$x(2x - 9) = 0$$

$$x = 0$$

$$2x - 9 = 0$$

$$2x = 9$$

$$x = \frac{9}{2}$$

$$6. \quad |x-4| \cdot |x+3| = |x-2| \cdot |x+3|$$

$$|x+3| (|x-4| - |x-2|) = 0$$

$$|x+3| = 0$$

$$x+3 = 0$$

$$x = -3$$

$$|x-4| - |x-2| = 0$$

$$|x-4| = |x-2|$$

$$x-4 = x-2$$

Aduna

$$x-4 = -x+2$$

$$2x = 6$$

$$x = 3$$

$$7. \frac{|x-1|}{4} - 2x = \frac{|2x-2|}{2} - (x+1)$$

$$\frac{|x-1|}{4} - 2x = \frac{|x-1|}{1} - x - 1$$

$$\frac{|x-1|}{4} = x - 1 + |x-1|$$

$$|x-1| = 4x - 4 + 4|x-1|$$

Прини  $|x-1| - 4|x-1| = 4x - 4$

$$-3|x-1| = 4x - 4$$

$$|x-1| = \frac{4x-4}{-3}$$

и

прини  $\frac{4x-4}{-3} \geq 0$

$$4x - 4 \leq 0$$

$$4x \leq 4$$

$$x \leq 1$$

$$x-1 = \frac{4x-4}{-3}$$

$$-3x+3 = 4x-4$$

$$-7x = -7$$

$$x = 1$$

$$x-1 = -\frac{4x-4}{-3}$$

$$-3x+3 = -4x+4$$

$$x = 1$$

$$8. \quad |x^2 - 2x - 3| + |9 - x^2| = 0.$$

atau

$$x^2 - 2x - 3 = 0$$

$$\boxed{x=3} \quad \boxed{x=-1}$$

atau  $9 - x^2 = 0$

$$\boxed{x=3} \quad \boxed{x=-3}$$

$$\underline{\underline{x=3}}$$

9.

$$5 + \sqrt{x^2 - 6x + 9} = 3x$$

$$5 + \sqrt{(x-3)^2} = 3x$$

$$|x-3| = 3x - 5$$



atau  $3x - 5 \geq 0$

$$\Rightarrow 3x \geq 5 \Rightarrow x \geq \frac{5}{3}$$

$$x - 3 = 3x - 5$$

↳

$$x - 3 = -3x + 5$$

$$x - 3x = -5 + 3$$

$$x + 3x = 5 + 3$$

$$4x = 8$$

$$-2x = -2$$

~~$$\boxed{x=1}$$~~

~~$$\boxed{x = \frac{1}{2}}$$~~

Answer.

10.  $x + |x+3| - |4-x| = 0$

x	-3	4
x+3	- ⊖ +	+ ⊕
4-x	+ ⊕	- ⊖

1. Av  $x < -3$  T02c

$$x + |x+3| - |4-x| = 0$$

$$x + (-x-3) - (4-x) = 0$$

$$x - x - 3 - 4 + x = 0$$

$$\boxed{x = 7}$$

2. Av  $-3 \leq x \leq 4$  T02c

$$x + |x+3| - |4-x| = 0$$

$$x + x + 3 - (4-x) = 0$$

$$2x + 3 - 4 + x = 0$$

$$3x - 1 = 0$$

$$\boxed{x = \frac{1}{3}}$$

30. A v  $x > 4$  T02C

$$x + |x+3|^{\oplus} - |4-x|^{\ominus} = 0$$

$$x + x + 3 - (-4 + x) = 0$$

$$2x + 3 + 4 - x = 0$$

$$x + 7 = 0$$

~~$$x = -7$$~~

$$11. \quad |1 - |3 - 2x|| = 6$$

$$1 - |3 - 2x| = 6$$

$$-|3 - 2x| = 6 - 1$$

$$-|3 - 2x| = 5$$

$$|3 - 2x| = -5$$

Answer

$$\vee \quad 1 - |3 - 2x| = -6$$

$$-|3 - 2x| = -6 - 1$$

$$-|3 - 2x| = -7$$

$$|3 - 2x| = 7$$

$$3 - 2x = 7 \quad \vee \quad 3 - 2x = -7$$

$$-2x = 7 - 3$$

$$-2x = 4$$

$$\underline{\underline{x = -2}}$$

$$-2x = -7 - 3$$

$$-2x = -10$$

$$\underline{\underline{x = 5}}$$

$$12. \quad d(4, d(x, 0)) = d(d(x, 0), -3)$$

$$d(4, |x-0|) = d(|x-0|, -3)$$

$$d(4, |x|) = d(|x|, -3)$$

$$|4 - |x|| = ||x| + 3|$$

$$4 - |x| = |x| + 3$$

$$\text{or } \cancel{4 - |x|} = \cancel{-|x| - 3}$$

$$-2|x| = -1$$

Admitted.

$$|x| = \frac{1}{2}$$

$$\textcircled{x = \frac{1}{2}} \quad \text{or} \quad \textcircled{x = -\frac{1}{2}}$$

$$13. \quad |2x^3| - |x|^3 - 4x^2 = 0$$

$$2|x^3| - |x|^3 - 4|x|^2 = 0$$

$$2|x|^3 - |x|^3 - 4|x|^2 = 0$$

$$|x|^3 - 4|x|^2 = 0$$

$$|x|^2 (|x| - 4) = 0$$

$$|x|^2 = 0$$

$$x = 0$$

$$\therefore |x| - 4 = 0$$

$$|x| = 4$$

$$x = 4$$

$$x = -4$$

$$14. \quad \left| \frac{x-3}{x-2} \right| + 1 - \frac{x-1}{|x-2|} = 0.$$

$$\frac{|x-3|}{|x-2|} + 1 - \frac{x-1}{|x-2|} = 0$$

$$|x-3| + |x-2| - (x-1) = 0$$

$$|x-3| + |x-2| = x-1.$$

$$\text{RPN } x-1 \geq 0$$

$$x \geq 1$$



$$|x-3| + |x-2| = x-1$$

$$x > 1$$

x	2	3
x-3	-	+
x-2	-	+

1. Av  $x < 2$  TOLL

$$\ominus \quad \ominus$$
$$|x-3| + |x-2| = x-1$$

$$-x+3 - x+2 = x-1$$

$$-2x+5 = x-1$$

$$-3x = -6$$

$$\cancel{x=2}$$

2. Av  $2 \leq x \leq 3$  TOLL

$$\ominus \quad \oplus$$
$$|x-3| + |x-2| = x-1$$

$$\cancel{x+3} + \cancel{x-2} = x-1$$

$$1 = x-1$$

$$x=2$$

3. Av  $x > 3$  TOLL

$$\oplus \quad \oplus$$
$$|x-3| + |x-2| = x-1$$

$$x-3 + x-2 = x-1$$

$$x=4$$

# Επορω Μαθημα

1. Προχωράμε στην  $x^v = 0$ .

## Άσκηση

Να λυθούν οι εξισώσεις

$$\textcircled{\alpha} \cdot 2x - \frac{3-2x}{6} = 1 - \frac{5-x}{4}$$

$$\textcircled{\beta} \cdot 3(x-2) - 2(1+3x) = -2(x-4) - x - 16$$

$$\textcircled{\gamma} \cdot 2(2x+6) = 2^2 - 9(-1-x)$$

$$\textcircled{\delta} \cdot \frac{x+5}{x-1} = \frac{x+6}{x} - \frac{2(x+3)}{x^2+x}$$

$$\textcircled{\epsilon} \cdot (x-1)(x+3) - (2x-1)^2 = (1-x)(x-4)$$

$$\textcircled{\zeta} \cdot 3x(x-3) + (x-3)^2 + 9 - x^2 = 0.$$