10. (δ) $\dfrac{x^2-1}{x^2-x} = 1 + \dfrac{1}{x}$

$\dfrac{(x-1)\cdot(x+1)}{x\cdot(x-1)} = 1 + \dfrac{1}{x}$

$x\cdot(x-1)$

ΕΚΠ $x(x-1)$.

- $x = 0$.
- $x-1=0 \Rightarrow x=1$.

Πρέπει $x \neq 0,\ x \neq 1$

$x\cdot(x-1)\cdot\dfrac{(x-1)\cdot(x+1)}{x\cdot(x-1)} = x\cdot(x-1)\cdot 1 + x\cdot(x-1)\cdot\dfrac{1}{x}$

$(x-1)\cdot(x+1) = x\cdot(x-1)\cdot(x-1)$

$(x-1)\cdot(x+1) = x^2 - x\cdot(x-1)$

$x^2 - 1 = x^2 - 2x + x$

$x^2 - 1 - x^2 + 2x - x$

$x - 1 = 0$

$x = 1$.

Αδύνατη.

10. (α) $\frac{x+2}{x^2-4} + \frac{2}{x^2-4x+4} = 0.$

$\Leftrightarrow \frac{x+2}{(x-2)(x+2)} + \frac{2}{(x-2)^2} = 0$

Περιορισμοί:

- $x-2 \neq 0 \Leftrightarrow x \neq 2$
- $x+2 \neq 0 \Leftrightarrow x \neq -2$
- $x-2 \neq 0 \Leftrightarrow x \neq 2$

$\Leftrightarrow$

ΕΚΠ $= (x-2)^2$ $\left(\frac{1}{x-2} + \frac{2}{(x-2)^2} = 0 \Leftrightarrow\right.$

$\frac{x-2}{(x-2)^2} + \frac{2}{(x-2)^2} = 0$

$\frac{1}{(x-2)(x-2)^2} +$

$(x-2)^2 \frac{1}{(x-2)} + (x-2)^2 \frac{2}{(x-2)^2} = 0$

$x - x + 2 = 0$

$x = 0$

εξίσωση

$$\frac{15}{x-2} - \frac{4}{x+2} = \frac{5}{x^2-4}.$$

$$\frac{15}{x-2} - \frac{4}{x+2} = \frac{5}{(x-2)\cdot(x+2)}$$

Περιορισμοί

$x-2=0$

$\Rightarrow x=2$

$x+2=0$

$\Rightarrow x=-2$

$x \neq 2$

$x \neq -2$

$$\frac{(x-2)\cdot(x+2)\cdot 15}{x-2} - \frac{(x-2)\cdot(x+2)\cdot 4}{x+2} = \frac{(x-2)\cdot(x+2)\cdot 5}{(x-2)\cdot(x+2)}$$

$$(x+2)\cdot 15 - (x-2)\cdot 4 = 5$$

$$15x + 30 - (4x - 8) = 5$$

$$15x + 30 - 4x + 8 = 5$$

$$11x = 5 - 8 - 30$$

$$11x = -33$$

$$\frac{11x}{11} = \frac{-33}{11}$$

$$x = -3$$

10. (β) $2 - \frac{x^2+7x}{x^2-1} = \frac{2x-1}{x+1} + \frac{3}{1-x}$

$$2 - \frac{x(x+7)}{(x-1)(x+1)} = \frac{2x-1}{x+1} + \frac{3}{1-x}$$

$$\frac{x(x+7)}{(x-1)(x+1)} - \frac{2x-1}{x+1} + \frac{3}{x-1} = -2$$

$(x-1)(x+1)$ $(x-1)(x+1)$ $(x-1)(x+1)$ $(x-1)(x+1)$

ΕΚΠ = $(x-1)(x+1)$

$x \neq 1$

$x \neq -1$

$x-1=0$

$x=1$

$x+1=0$

$x=-1$

$x(x+7) - (x-1)(2x-1) + (x+1)3 = -2(x-1)(x+1)$

$x^2 + 7x - (2x^2 - x - 2x + 1) + 3x + 3 = -2(x^2-1)$

$x^2 + 7x - 2x^2 + x + 2x - 1 + 3x + 3 = -2x^2 + 2$

$x^2 + 7x - 2x^2 + x + 2x + 3x + 2x^2 = 1 - 3 + 2$

$13x = 0$

$x = 0$

extra

$(\lambda^2 x - 2)(\lambda - 2) + \lambda x - (\lambda - 1)^2 = 2$.

$(\ )(\ )\, x = (\ )(\ )$ Εκουμ.

$\lambda^3 x - 2\lambda^2 x - 2\lambda + 4 + \lambda x - (\lambda^2 - 2\lambda + 1) = 2$

$\lambda^3 x - 2\lambda^2 x + \lambda x = 2\lambda - 4 + \lambda^2 - 2\lambda + 1 + 2$

$(\lambda^3 - 2\lambda^2 + \lambda)\, x = \lambda^2 - 1$.

$\lambda(\lambda^2 - 2\lambda + 1)\, x = (\lambda - 1)(\lambda + 1)$

$\boxed{\lambda(\lambda-1)^2 x = (\lambda - 1)(\lambda + 1)}$

1. Αν $\lambda = 0$ τοτε $0x = -1$ Αδυνατη
2. Αν $\lambda = 1$ τοτε $0x = 0$ Αορισ.
3. Αν $\lambda \neq 0, \lambda \neq 1$ τοτε $\frac{\lambda(\lambda-1)}{\lambda(\lambda-1)} x = \frac{(\lambda-1)(\lambda+1)}{\lambda(\lambda-1)}$

$x = \frac{\lambda+1}{\lambda}$

25. (B) $8x^3 + (x-2)^3 - (3x-2)^3 = 0$

$8x^3 + (x-2)^3 + (2-3x)^3 = 0$

$(2x)^3 + (x-2)^3 + (2-3x)^3 = 0$

$\rightarrow 2x + x - 2 + 2 - 3x = 0$

$0 = 0$

Δουλειά το κόλπο.

3. $2x \cdot (x-2)(2-3x) = 0.$

$2x = 0$ ή $x - 2 = 0$ ή $2 - 3x = 0$

$x = 0$ $\quad$ $x = 2$ $\quad$ $x = \frac{2}{3}$

24. (d) $(3x-2)^2 = (3x-2)(x-1)$

$(3x-2)^2 - (3x-2)(x-1) = 0$

$(3x-2)(3x-2-x+1) = 0 \Rightarrow (3x-2)(2x-1) = 0$

$x = \frac{2}{3}$ $x = \frac{1}{2}$

(e) $x(x-2) - (x+1)(2-x) = 0$

$x(x-2) + (x+1)(x-2) = 0$

$(x-2)(x+x+1) = 0$

$(x-2)(2x+1) = 0$

$x-2=0$ ou $2x+1=0$

$x=2$ $x=-\frac{1}{2}$

(29) $(x+1)^2 + x^2 - 1 = 0$.

$(x+1)^2 + (x-1)(x+1) = 0$

$(x+1)(x+1+x-1) = 0$

$(x+1)\,2x = 0$

$x=-1$ $x=0$

(h) $x^3 - x^2 - x + 1 = 0$

$x^2(x-1) + (-x+1) = 0$

$x^2(x-1) - (x-1) = 0$

$(x-1)(x^2-1) = 0$

$x-1=0$ ou $x^2-1=0$

$x=1$ $(x-1)(x+1)=0$

$x-1=0$ ou $x+1=0$

$x=1$ $x=-1$

(i) $x^3 - 7x + 6 = 0$

$x^3 - x - 6x + 6 = 0$

$x(x^2-1) - 6(x-1) = 0$

$x(x-1)(x+1) - 6(x-1) = 0$

$(x-1)(x(x+1) - 6) = 0$

$x=1$ $x^2+x-6=0$

$x=-3$ $x=2$

Σελ 181

1 (α) $3x^2+4x+1=0$

$\Delta = \beta^2 - 4\alpha\gamma = 4^2 - 4\cdot 3\cdot 1 = 16-12 = 4$

$x_{1,2} = \frac{-\beta \pm \sqrt{\Delta}}{2\cdot a} = \frac{-4 \pm \sqrt{4}}{2\cdot 3} = \frac{-4 \pm 2}{6} \Rightarrow x_1 = \frac{-2}{6}$

$x_2 = \frac{-6}{6} = -1$

(β) $x^2-2x+1=0$

$\Delta = \beta^2 - 4\alpha\gamma = (-2)^2 - 4\cdot 1\cdot 1 = 4-4 = 0$

$x = \frac{-\beta}{2\cdot a} = \frac{-(-2)}{2\cdot 1} = \frac{2}{2} = 1$

(γ) $x^2-x+2=0$

$\Delta = \beta^2 - 4\alpha\gamma = -1 - 4\cdot 1\cdot 2 = -9$ Αδύνατη

1. (β) $x^2-3x+2=0$.

$\Delta=\beta^2-4\alpha\gamma \Leftrightarrow \Delta=9-8 \Leftrightarrow \Delta=1$

$x_{1,2}=\frac{-\beta\pm\sqrt{\Delta}}{2\alpha} \Leftrightarrow x_{1,2}=\frac{3\pm1}{2}$ → $x_1=2$, $x_2=1$

(ε) $x^2+9=6x$

$x^2+9-6x=0 \Leftrightarrow (x-3)^2=0$

$x=3$

(γ) $x(1-2x)=1$. $-2x^2+x-1=0$

$x-2x^2-1=0 \Leftrightarrow$

$\Delta=\beta^2-4\alpha\gamma \Leftrightarrow \Delta=(-2)^2-4\cdot1\cdot1 \Leftrightarrow \Delta=-7$

αδύνατη

$1=4+4=8$.

$\Delta=1-4\cdot(-2)(-1)=1-8=-7$

Άβαση

4. (α) $x^2 - 2(x-1) = 2x - 1$

$x^2 - 2x + 2 - 2x + 1 = 0$

$x^2 - 4x + 3 = 0$

$\Delta = b^2 - 4ac = (4)^2 - 4 \cdot 1 \cdot 3 = 16 - 12 = 4$

$x_{1,2} = \frac{-b \pm \sqrt{\Delta}}{2a} = \frac{4 \pm 2}{2} = \begin{cases} 3 \\ 1 \end{cases}$

(β) $(2x-1)^2 - 3(x-1) = 1$.

$4x^2 - 4x + 1 - 3x + 3 - 1 = 0$

$4x^2 - 4x - 3x + 1 + 3 - 1 = 0$

$4x^2 - 7x + 3 = 0$

$\Delta = b^2 - 4ac = 49 - 4 \cdot 4 \cdot 3 = 49 - 48 = 1$

$x_{1,2} = \frac{-b + \sqrt{\Delta}}{2a} = \frac{7 \pm 1}{8} = \begin{cases} 1 \\ \frac{6}{8} \end{cases}$

1. (ა) $2x^2 - x - 1 = 0$ $\quad \alpha = 2$

$\beta = -1$

$\gamma = -1$

$D = (-1)^2 - 4 \cdot 2 \cdot (-1)$

$D = 1 + 8 = 9.$

$x = \frac{-(-1) \pm \sqrt{9}}{2 \cdot 2} = \frac{1 \pm 3}{4}$ → (1), $-\frac{2}{4}$.

(ბ) $3x(2 - 3x) = 1$

$6x - 9x^2 = 1$

$6x - 9x - 1 = 0$

$-9x^2 + 6x - 1 = 0$

$D = b^2 - 4ac \Rightarrow 9^2 - 4 \cdot 6 \cdot 1 = 0$

$81 - 24 = 57$

$D = 0$

$x_{1,2} = \frac{-(-9) \pm \sqrt{57}}{2 \cdot 6}$ → $\frac{9 + \sqrt{57}}{12}$, $\frac{9 - \sqrt{57}}{12}$

$x = \frac{-6}{-18} = \frac{1}{3}$

(გ) $x^2 - 3x + 4 = 0$

$D = b^2 - 4ac \Rightarrow 3^2 - 4 \cdot 1 \cdot 4 = 0$

$9 - 16 = -4 < 0$ ამოხსნა არ აქვს.

4. ε) $(x-1)^3 = x^3 - 7 \Leftrightarrow$

$x^3 - 2x^2 + 2x - 1 = x^3 - 7 \Leftrightarrow$

$x^3 - 2x^2 + 2x - 1 - x^3 + 7 = 0 \Leftrightarrow$

$-2x^2 + 2x + 6 = 0 \Leftrightarrow$

$\Delta = b^2 - 4ay \Leftrightarrow \Delta = 4 - (-2)(6) \Leftrightarrow$

$\Delta = 4 - 48$

$\Delta = -44$ αδύνατη

β) $(x-1)^2 = 3x(x-2) - 1$.

$x^2 - 2x + 1 = 3x^2 - 6x - 1 \Leftrightarrow$

$x^2 - 2x + 1 - 3x^2 + 6x + 1 = 0$

$-2x^2 + 4 + 2$

$\Delta = b^2 - 4ay \Leftrightarrow 16 - 16 \Leftrightarrow \Delta = 0$

$x = \frac{-b}{2a} \Leftrightarrow x = \frac{-4}{4} \Leftrightarrow x = -1$

Επίλυση εξίσωσης 2ου Βαθμού

$$\alpha x^2 + \beta x + \gamma = 0 \qquad \alpha \neq 0$$

Διακρίνουσα : $\Delta = \beta^2 - 4\alpha\gamma$.

1. Αν $\Delta > 0$ τότε έχω δύο ρίζες.

$$x_{1,2} = \frac{-\beta \pm \sqrt{\Delta}}{2\alpha}$$

2. Αν $\Delta = 0$ τότε έχω διπλή ρίζα.

$$x = \frac{-\beta}{2\alpha}$$

3. Αν $\Delta < 0$ αδύνατη.

2. (a) $x^2 - 81 = 0$

$(x-9)(x+9) = 0$

$x - 9 = 0$ ó $x + 9 = 0$

$x = 9$ $x = -9$

(b) $x^2 - 2 = 0$

$(x - \sqrt{2})(x + \sqrt{2}) = 0$

$x - \sqrt{2} = 0$ ó $x + \sqrt{2} = 0$

$x = \sqrt{2}$ $x = -\sqrt{2}$

(c) $-3x^2 + 1 = 0$

$-(\sqrt{3}x - 1)(\sqrt{3}x + 1)$

$\sqrt{3}x - 1 = 0$ ó $\sqrt{3}x + 1 = 0$

$\sqrt{3}x = 1$ ó $\sqrt{3}x = -1$

$x = \frac{1}{\sqrt{3}}$ $x = -\frac{1}{\sqrt{3}}$

(d) $x^2 + 3 = 0$

$(x - \sqrt{3})(x + \sqrt{3}) = 0$ Adalah.

$x - \sqrt{3} = 0$ ó $x + \sqrt{3} = 0$

$x = \sqrt{3}$ $x = -\sqrt{3}$

(e) $x^2 = 3x$

$x^2 - 3x = 0$ $x(x-3) = 0$

$(x - \sqrt{3}x)(x + \sqrt{3}x) = 0$

$x - \sqrt{3}x = 0$ $x = 0$

$\sqrt{3}x = -x$

$x = \frac{-x}{\sqrt{3}}$

(f) $\frac{x}{4} = \frac{x^2}{2}$.

$\frac{x}{4} - \frac{x^2}{2} = 0$

$x - 2x^2 = 0$

$(x - \sqrt{2}x)(x + \sqrt{2}x) = 0$

$x - \sqrt{2}x = 0$

$-\sqrt{2}x = -x$

$-\sqrt{2} = \frac{-x}{x}$

$-\sqrt{2}$

$2-(2x-1)^2 = 7x - 2x(1-x)$

$2-\{(2x)^2 - 2x2\cdot 1 + 1\} = 7x - 2x + 2x^2$

$2-(4x^2 - 4x + 1) = 7x - 2x + 2x^2$

$2 - 4x^2 + 4x - 1 = 7x - 2x + 2x^2$

$2 - 4x^2 + 4x - 1 - 7x + 2x - 2x^2 =$

$-6x^2 - x - 1 = 0$

$\Delta = B^2 - 4ac = 1 - 4\cdot(-6)\cdot(-1) = 1 + 24 = 25$

$x_{1,2} = \frac{-B \pm \sqrt{\Delta}}{2\cdot a} = \frac{1 \pm \sqrt{25}}{2\cdot(-6)} = \frac{1 \pm 5}{-12}$ $x_1 = \frac{6}{-12} = \frac{3}{-6}$

$x_2 = \frac{-1-5}{-12} = \frac{-6}{-12}$

$= \frac{3}{6}$

Επομαω Μαθημα

Παλια	Νεα
Σελ 156	(5) αγε.
(1) αγε	(6). αγε στ.
(2) αγ	(7) ΟΛΗ .
(3) αγε7	(13)
(9) αγεστ	(14) .
(11)	(24) αγε7θ
(12)	(25) α.