10. (B) $\dfrac{9x-1}{9x^2-1} + \dfrac{1}{1-3x} = \dfrac{2}{3x+1}$

$$\frac{9x-1}{(3x-1)(3x+1)} - \frac{1}{3x-1} = \frac{2}{3x+1}$$

→ $(3x-1)(3x+1)=0 \Rightarrow 3x-1=0$ ή $3x+1=0$

$x=\frac{1}{3}$ $\quad x=-\frac{1}{3}$

→ $3x-1=0 \Rightarrow x=\frac{1}{3}$

→ $3x+1=0 \Rightarrow x=-\frac{1}{3}$

Πρπ

$x \neq \frac{1}{3} \quad x \neq -\frac{1}{3}$.

ΕΚΠ: $(3x-1)(3x+1)$.

$(3x-1)(3x+1)\dfrac{9x-1}{(3x-1)(3x+1)} - (3x-1)(3x+1)\dfrac{1}{3x-1} = (3x-1)(3x+1)\dfrac{2}{3x+1}$

$9x-1-(3x+1)=2(3x-1)$

$9x-1-3x-1=6x-2$

$-2=-2$

Ταυτότητα

$x \in \mathbb{R} - \{-\frac{1}{3}, \frac{1}{3}\}$.

9. Εξ 157

(β) $\frac{1}{x+2} - \frac{4}{x^2-4} = \frac{2}{x-2}$.

Βημα 1ο

Παραγοντοποιω ολους τους παρονομαστες.

$\frac{1}{x+2} - \frac{4}{(x-2)(x+2)} = \frac{2}{x-2}$

Βημα 2ο

Παιρνω περιορισμους.

→ $x+2=0 \Rightarrow x=-2$

→ $(x-2)(x+2)=0$

$x-2=0$ ή $x+2=0$

$x=2$ $x=-2$

→ $x-2=0 \Rightarrow x=2$

Προσ

$x \neq -2$

$x \neq 2$

Βημα 30

Πολ/τω παντου με το ΕΚΠ: $(x-2)(x+2)$

$$(x-2)(x+2)\frac{1}{x+2} - (x-2)(x+2)\frac{4}{(x-2)(x+2)} = (x-2)(x+2)\frac{2}{x-2}$$

$$x-2-4 = 2(x+2)$$

$$x-6 = 2x+4$$

$$x-2x = 6+4$$

$$-x = 10$$

$$x = -10$$

9. (δ) $\frac{x-1}{x} - \frac{1}{x+1} + \frac{1}{x^2+x} = 0$

$$\frac{x-1}{x} - \frac{1}{x+1} + \frac{1}{x(x+1)} = 0$$

→ $x = 0$

→ $x+1=0 \Rightarrow x=-1$

→ $x(x+1) = 0$

$x=0$ $\quad x+1=0$

$x=-1$

Πρπ

$x \neq 0$

$x \neq -1$

Εκπ: $x(x+1)$

$$x(x+1)\frac{x-1}{x} - x(x+1)\frac{1}{x+1} + x(x+1)\frac{1}{x(x+1)} = 0$$

$$(x+1)(x-1) - x + 1 = 0$$

$$x^2 - 1 - x + 1 = 0$$

$$x(x-1) = 0$$

$x=0$ ✗ $\quad x=1$ ✓

$a \cdot b = 0$

$a = 0$ ni $b = 0$

$a \cdot b \neq 0$

$a \neq 0$ kan $b \neq 0$

Παραμετρικά 1ου Βαθμού

1. $\lambda x - 3\lambda = \lambda^2 - 3x$

Σκοπός: $(\quad)(\quad)x = (\quad)(\quad)$

$\lambda x + 3x = \lambda^2 + 3\lambda$

$(\lambda+3)x = \lambda(\lambda+3)$

1. Αν $\lambda = -3$ τότε $0x = 0$ Ταυτότητα.
2. Αν $\lambda \neq -3$ τότε $\dfrac{(\lambda+3)x}{\lambda+3} = \dfrac{\lambda(\lambda+3)}{\lambda+3} \Leftrightarrow x = \lambda$

2. $\lambda^2(x+1) = -(-1-\lambda x)$

$\lambda^2 x + \lambda^2 = 1 + \lambda x$

$\lambda^2 x - \lambda x = 1 - \lambda^2$

$(\lambda^2 - \lambda)x = 1 - \lambda^2$

$\lambda(\lambda-1)x = -(\lambda^2-1)$

$$\lambda(\lambda-1)x = -(\lambda-1)(\lambda+1)$$

1. Αν $\lambda=0$ $\quad 0x = 1$ Αδύνατη.

2. Αν $\lambda=1$ $\quad 0x = 0$ Ταυτότητα.

3. Αν $\lambda \neq 0, \lambda \neq 1$ $\quad x = \dfrac{-(\lambda-1)(\lambda+1)}{\lambda(\lambda-1)}$ (=)

$$x = -\frac{\lambda+1}{\lambda}$$

3. $\lambda^2(\lambda x-\lambda+2)-\lambda(x+1)=0$

$\lambda^3 x-\lambda^2+2\lambda-\lambda x-\lambda=0$

$\lambda^3 x-\lambda x=\lambda^2-2\lambda+\lambda$

$(\lambda^3-\lambda)x=\lambda^2-\lambda$

$\lambda(\lambda^2-1)x=\lambda(\lambda-1)$

$$\lambda(\lambda-1)(\lambda+1)x=\lambda(\lambda-1)$$

1. Αν $\lambda=0$ τότε $0x=0$ Ταυτότητα

2. Αν $\lambda=1$ τότε $0x=0$ Ταυτότητα

3. Αν $\lambda=-1$ τότε $0x=2$ Αδύνατη.

4. Αν $\lambda\neq 0, \lambda\neq 1, \lambda\neq -1$ τότε

$$x=\frac{\cancel{\lambda(\lambda-1)}}{\cancel{\lambda(\lambda-1)}(\lambda+1)} \Leftrightarrow x=\frac{1}{\lambda+1}$$

Εξ 167

1. (α) $|x|-2=0$

$|x|=2$

$x=2$ $\quad$ $x=-2$

(β) $2|x|-3=0 \Leftrightarrow \frac{2|x|}{2}=\frac{3}{2} \Leftrightarrow$

$|x|=\frac{3}{2} \Leftrightarrow x=\frac{3}{2}$ ή $x=-\frac{3}{2}$ ✓

(γ) $2-5|x|=0 \cdot 2=5|x| \Leftrightarrow \frac{2}{5}=|x|$

$x=\frac{2}{5}$ ή $x=-\frac{2}{5}$

(δ) $2(|x|-1)=|x|-2 \cdot 2|x|-2=|x|-2$

$2|x|-|x|=-2+2 \Leftrightarrow$

$|x|=0 \Leftrightarrow x=0$

1. (ε) $\frac{|x|}{2} - \frac{|x|}{3} = 1$.

$6 \cdot \frac{|x|}{2} - 6 \cdot \frac{|x|}{3} = 6 \cdot 1$

$3|x| - 2|x| = 6$

$|x| = 6$

$x = 6$ и $x = -6$

(бε) $\frac{|x|-2}{3} + |x| = -1$.

$3 \cdot \frac{|x|-2}{3} + 3|x| = 3 \cdot (-1)$

$|x| - 2 + 3|x| = -3$

$4|x| = -1$

$x = -3$ ~~и~~ $x = 3$

Абсциса.

2. (α) $|x-1|-2=0$

$|x-1|=2$

$x-1=2$ ή $x-1=-2$

$x=1$ $x=-1$ ✓

(β) $3-2|(3x,1)|=0$

$3-2|3x-1|=0$

$\frac{2|3x-1|}{2}=\frac{3}{2}$

$|3x-1|=\frac{3}{2}$

$3x-1=\frac{3}{2}$ ή $3x-1=-\frac{3}{2}$

$3x=\frac{3}{2}+1$ $6x-2=-3$

$3x=\frac{5}{2}$ $6x=-1$

$6x=5$ $x=-\frac{1}{6}$.

$x=\frac{5}{6}$

(ε) $\frac{|x-2|-1}{2}=-1$.

$|x-2|-1=-2$

$|x-2|=-1$

$x-2=-1$

3. ⓐ $|-x|+|3x|-4\left|\frac{x}{2}\right|=2$.

$$|-x|+|3x|-4\frac{|x|}{|2|}=2$$

$$|x|+3|x|-2|x|=2$$

$$2|x|=2$$

$$|x|=1$$

$x=1$ $\quad$ $x=-1$

ⓑ $|2x-1|+2|1-2x|=|4x-2|+1$.

$$|2x-1|+|2-4x|=|4x-2|+1$$

$$|2x-1|+|4x-2|-|4x-2|=1$$

$$|2x-1|=1$$

$2x-1=1 \Rightarrow 2x=1+1 \Leftrightarrow \frac{2x}{2}=\frac{2}{2}$

$x=1$ $\quad$ ή

$2x-1=-1 \Rightarrow 2x=-1+1 \Leftrightarrow 2x=0$

$x=0$

Α

2. (B) $|5x-1|-4=0$.

$|5x-1|=4$

$5x-1=4$ și $5x-1=-4$

$\frac{5x}{5}=\frac{5}{5}$ și $\frac{5x}{5}=\frac{-3}{5}$

$x=1$ $\quad x=-\frac{3}{5}$

(8) $2(|x-3|-1)=3|x-3|-2$.

$2(|x-3|-1)=3|x-3|-2$

$2|x-3|-2=3|x-3|-2$

$2|x-3|-3|x-3|=0$

$-|x-3|=0$

$x-3=0$ $\quad x=3$.

(52) $\frac{|x-1|}{2}-\frac{|x-1|-1}{4}=1$.

$4\cdot\frac{|x-1|}{2}-4\cdot\frac{|x-1|-1}{4}=4\cdot 1$

$2\cdot|x-1|-(|x-1|-1)=4$

$2|x-1|-|x-1|+1=4$

$|x-1|=3$

$x-1=3$ și $x-1=-3$

$x=4$ $\quad x=-2$.

11. ⓔ $\left| \left| |x|+1 \right| - 2 \right| = 1$

$$\left| |x|+1-2 \right| = 1$$

$$\left| |x|-1 \right| = 1$$

$|x|-1=1$ ili $|x|-1=-1$

$|x|=2$ $|x|=0$

$x=2$ ili $x=-2$ $x=0$.

3. ⑦ $\left|\frac{2x}{3}\right| - |-2x| = -1$.

$\frac{|2x|}{|3|} - |-2x| = -1$

$3\frac{|2x|}{3} - 3|-2x| = -1^3$

$|2x| - 3|-2x| = -3$

$2|x| - 6|x| = -3$

$-4|x| = -3$

$4|x| = 3$

$|x| = \frac{3}{4}$

$x = \frac{3}{4}$ $x = -\frac{3}{4}$.

⑧ $|6x-2| - |3-9x| = -1$.

$2|3x-1| - 3|3x-1| = -1$

$-1|3x-1| = -1$

$-1 \cdot 3|x-1| = -1$

$\frac{-3|x-1|}{-3} = \frac{-1}{-3}$

$|x-1| = \frac{1}{3}$

$x-1 = \frac{1}{3}$

$x = \frac{1}{3} + 1$

$x = \frac{4}{3}$

$x-1 = -\frac{1}{3}$

$x = -\frac{1}{3} + 1$

$x = \frac{2}{3}$

✓

⑨ $\frac{|3x-6|}{4} - \left|1 - \frac{x}{2}\right| = 1$.

$\frac{3|x-2|}{4} - \left|\frac{2}{2} - \frac{x}{2}\right| = 1$

$\frac{3|x-2|}{4} - \left|\frac{2-x}{2}\right| = 1$

$\frac{3|x-2|}{4} - \frac{|2-x|}{|2|} = 1$

$4\frac{3|x-2|}{4} - 4\frac{|2-x|}{2} = 4 \cdot 1$

$3|x-2| - 2|2-x| = 4$

$3|x-2| - 2|x-2| = 4$

$|x-2| = 4$

$x-2 = 4$ $x-2 = -4$

$x = 6$ $x = -2$

10. ① $|x^2-2x+1| = x(x-1)$.

$|(x-1)^2| = x^2-x$ (①)

$(x-1)^2 = x^2-x$

~~$x^2-2x+1=x^2-x$~~

$-x+1=$

$-x=-1$

$-x \quad -x$

$x=1$ ✓

10. (B) $\left| |x-2| + 3 \right| = 5$.

(+)

$|x-2| + 3 = 5$

$|x-2| = 2$

$x-2 = 2$ ve $x-2 = -2$

$x = 4$ $\qquad$ $x = 0$

11. (8) $\left| |2x-1| - 5 \right| = 5$.

$|2x-1| - 5 = 5$ ve $|2x-1| - 5 = -5$

$|2x-1| = 10$ $\qquad$ $|2x-1| = 0$

$2x-1 = 10$ ve $2x-1 = -10$ $\qquad$ $2x-1 = 0$

$2x = 11$ $\qquad$ $2x = -9$ $\qquad$ $x = \frac{1}{2}$

$x = \frac{11}{2}$ $\qquad$ $x = -\frac{9}{2}$ ~~ve~~

~~$|-5| = 5 \Rightarrow 5 = 5$~~

10. ⓐ $|\overset{+}{x^2+1}| + |\overset{+}{|x|+1}| = x^2+2$

$x^2+1 + |x|+1 = x^2+2$

$|x| + 2 = 2$

$|x| = 0$

$x = 0$.

11. ⓐ $||x|-3| = 4$.

$|x|-3 = 4$ или $|x|-3 = -4$

$|x| = 7$ или $|x| = -1$

Абсолютная!

$x = 7$ $x = -7$.

3. (ε) $1 + \frac{|2x-3|-3}{2} = \frac{|6x-9|-3}{6}$

$6 \cdot 1 + 6 \cdot \frac{|2x-3|-3}{2} = 6 \cdot \frac{|6x-9|-3}{6}$

$6 + 3(|2x-3|-3) = |6x-9|-3$

$6 + 3|2x-3| - 9 = |6x-9|-3$

$= 3|2x-3| - 3$

$6 + 3|2x-3| - 9$

~~$3|2x-3|$~~

$3|2x-3| - 3|2x-3| = -3 + 9 - 6$

$0 = 0$ Δορισμός!

4. (B) $|2x-3| = |3x-2|$

$2x-3 = 3x-2$

$2x-3x = 1$

$-x = 1$

$x = -1$

ή $2x-3 = -3x+2$

$5x = 5$

$x = 1$

(Δ) $|x-4| - 3|x| = 0$

$|x-4| = 3|x|$

$x-4 = 3x$

$x-3x = 4$

$-2x = 4$

$x = -2$

ή $x-4 = -3x$

$x+3x = 4$

$4x = 4$

$x = 1$

(Θ2) $\dfrac{|2x-1|}{3} = \dfrac{|x-2|}{2}$

$6 \cdot \dfrac{|2x-1|}{3} = 6 \cdot \dfrac{|x-2|}{2}$

$2|2x-1| = 3|x-2|$

$|4x-2| = |3x-6|$

$4x-2 = 3x-6$

$4x-3x = -6+2$

$x = -4$ ✓

ή $4x-2 = -3x+6$

$7x = 8$

$x = \dfrac{8}{7}$ ✓

11. ⓘ $|3|x|-2| = 2$

$3|x|-2=2$

$\frac{3|x|}{3} = \frac{4}{3}$

$|x| = \frac{4}{3}$

$x = \frac{4}{3}$ ή $x = -\frac{4}{3}$

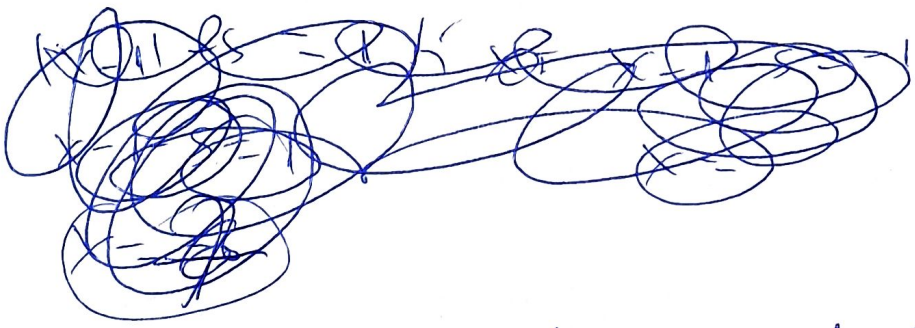
✓

$3|x|-2=-2$

$3|x| = 0$

$x=0$ δεκτή

ⓘ $||x-1|-5| = 1$.



$|x-1|-5=1$ ή $|x-1|-5=-1$

$|x-1| = 6$

$|x-1| = 4$

$x-1=6$ ή $x-1=-6$

$x=7$ $x=-5$

$x-1=4$ ή $x-1=-4$

$x=5$ $x=-3$

5. (B) $\sqrt{x^2-6x+9} = 1$

$\sqrt{(x-3)^2} = 1$

$|x-3| = 1$

$x-3 = 1$ ou $x-3 = -1$

$x = 4$ $\qquad$ $x = 2$

(δ) $\sqrt{25x^2-10x+1} - |3x-5| = 0 \iff \sqrt{(5x-1)^2} = |3x-5|$

$|5x-1| = |3x-5|$

$\sqrt{(5x-10x)(5x+10x)+1} - |3x-5| = 0$

$\sqrt{5x^2-10^2+1} - |3x-5| = 0$

$\sqrt{5\cdot(x^2-2)^2+1} - |3x-5| = 0$

(ε) $\sqrt{4x^2+1} = |2x-1|$

$\sqrt{(4x^2+1)} = |2x-1|^2$

$4x^2+1 = (2x-1)^2$

$4x^2+1 = (2x)^2 - 2\cdot 2x\cdot 1 + 1^2$

$4x^2+1 = 4x^2 - 4x + 1$

$4x^2+1-4x^2+4x-1 = 0$

$4x = 0$

$x = 0$

11. (52) $|||x|-2|-3|=1$,

$||x|-2|-3=1$ ή $||x|-2|-3=-1$

$||x|-2|=4$ $\quad$ $||x|-2|=3$

$x=5$

~~$x-2=4$ ή $x-2=-4$~~

~~$x=6$ ή $x=-2$~~

$||x|-2|=4$ ή $||x|-2|=3$

$|x|-2=4$ ή $|x|-2=-4$ $\quad$ $|x|-2=3$ ή $|x|-2=-3$

$|x|=6$ $\quad$ $|x|=-2$ $\quad$ $|x|=5$ $\quad$ $|x|=-1$

Αδύνατη $\quad$ Αδύνατη

$x=6$ $\quad$ $x=-6$ $\quad$ $x=5$ $\quad$ $x=-5$

4. (α) $|2x-1| = |x| \Leftrightarrow$ [illegible]

$2x-1 = x$ ή $2x-1 = -x$

$x = 1$ $3x = 1$ $x = \frac{1}{3}$

(γ) $|5x-1| - |4x-1| = 0$.

$|5x-1| = |4x-1|$ ✓

$5x-1 = 4x-1 \Leftrightarrow 1x = 0 \Leftrightarrow x = 0$

ή

$5x-1 = -4x+1 \Leftrightarrow 9x = 2 \Leftrightarrow x = \frac{2}{9}$

(ε) $\frac{|3x-2| - 2}{2} = |x| - 1$.

$|3x-2| - 2 = 2|x| - 2 \Leftrightarrow$ ✓

$3x-2 = 2x \Leftrightarrow x = -2$

ή

$3x-2 = -2x \Leftrightarrow 5x = 2 \Leftrightarrow x = \frac{2}{5}$

5. ⓐ $\sqrt{x^2} = 2$

$|x| = 2$ ή $x = -2$

$x = 2$

ⓑ $\sqrt{x^2 - 2x + 1} = |2x - 1|$

$\sqrt{(x-1)^2} = |2x-1|$ ή $x - 1 = -2x + 1$

$|x-1| = |2x-1|$

$x - 2x = -1 + 1$

$-x = 0$

$x = 0$

$3x = 2$

$\frac{3x}{3} = \frac{2}{3}$

$x = \frac{2}{3}$ ✓

ⓒ $\sqrt{x^2+2} = |x+1|$.

$|x+2| = |x+1|$ ή $x + 2 = -x - 2$

$x + 2 = x + 1$

$2x = 0$

$0x = -1$ Αδύνατη

$x = -2$

$\sqrt{x^2+2} = |x+1|$.

$\left(\sqrt{x^2+2}\right)^2 = |x+1|^2$

$x^2 + 2 = (x+1)^2$

$x^2 + 2 = x^2 + 2x + 1$

$2 - 1 = 2x$

$1 = 2x$

$x = \frac{1}{2}$

Εποραω Μαθημα

Σελ 156

(1) αγε.

(2) αγ.

(3) αγετ.

(9) αγεσε.

(11)

(12).