6. (β) $|2x-3| = x-1$

Πρεπει $x-1 \geq 0$

$2x-3 = x-1$ ή $2x-3 = -x+1$

$\boxed{x=2}$ ✓ $3x = 4$

$\boxed{x = \frac{4}{3}}$

(α) $|x-2| = 2x-3$.

πρπ $2x-3 \geq 0$

$x-2 = 2x-3$ και $x-2 = 3-2x$

$\cancel{1 = x}$ $3x = 5$

$\boxed{x = \frac{5}{3}}$ ✓

7. (01) $|x-2| - |3-x| = 1-x$

x		2		3	
$x-2$	$-$	0	$+$		$+$
$3-x$	$+$		$+$	0	$-$

1. Αν $x < 2$ τότε

$\overset{(-)}{|x-2|} - \overset{(+)}{|3-x|} = 1-x$

$-x+2-(3-x) = 1-x$

$-x+2-3+x = 1-x$

$-1 = 1-x$

$\cancel{x = 2}$

2. Αν $2 \le x < 3$ τότε

$\overset{(+)}{|x-2|} - \overset{(+)}{|3-x|} = 1-x$

$x-2-3+x = 1-x$

$2x-5 = 1-x$

$x = 2$ ✓

3. Αν $x \ge 3$ τότε

$\overset{(+)}{|x-2|} - \overset{(-)}{|3-x|} = 1-x$

$x-2-(-3+x) = 1-x$

$x-2+3-x = 1-x$

$1 = 1-x$

$\cancel{x = 0}$

7. (8) $|2x+5| - |x+1| = -3x$

x	$-\frac{5}{2}$	-1	
$2x+5$	$-$ 0 $+$		$+$
$x+1$	$-$	$-$ 0	$+$

• $2x+5=0$

$2x=-5$

$x=-\frac{5}{2}$

1. Αν $x < -\frac{5}{2}$ τοτε

$|\overset{(-)}{2x+5}| - |\overset{(-)}{x+1}| = -3x$

$-2x-5-(-x-1) = -3x$

$-2x-5+x+1 = -3x$

$-x-4 = -3x$

$3x-x = 4$

$2x = 4$

$x = 2$ ✗

2. Αν $-\frac{5}{2} \le x \le -1$ τοτε

$|\overset{(+)}{2x+5}| - |\overset{(-)}{x+1}| = -3x$

$2x+5-(-x-1) = -3x$

$2x+5+x+1 = -3x$

$3x+6 = -3x$

$6x = 6$

$x = 1$ ✗

3. Αν $x > -1$ τοτε

$|\overset{(+)}{2x+5}| - |\overset{(+)}{x+1}| = -3x$

$2x+5-x-1 = -3x$

$3x+3x = -4$

$6x = -4$

$x = -\frac{2}{3}$ ✓

8. (α) $|x-1| = x-1$.

προσπ $x-1 \geq 0 \Rightarrow x \geq 1$

(β) $|2x-1| = 1-2x$

$2x-1 \leq 0$

$2x \leq 1$

$x \leq \frac{1}{2}$

10. (α) $\overset{(+)}{|x^2+1|} + \overset{(+)}{|1x1+1|} = x^2+2$

$x^2+1 + |x|+1 = x^2+2$

$|x| = 0$

$x=0$,

(γ) . $|x^2-2x+1| = x(x-1)$

$\overset{(+)}{|(x-1)^2|} = x(x-1)$

$(x-1)^2 - x(x-1) = 0$

$(x-1)(x-1-x) = 0$

$-(x-1) = 0$

$x=1$

II. a) $|x|-3=4$

$|x|-3=4$ ori $|x|-3=-4$

$|x|=7$ $|x|=-1$

Adevarat.

$x=7$ $x=-7$

b) $||x-1|-5|=1$

$|x-1|-5=1$ ori $|x-1|-5=-1$

$|x-1|=6$ $|x-1|=4$

$x-1=6$ ori $x-1=-6$ $x-1=4$ ori $x-1=-4$

$x=7$ $x=-5$ $x=5$ $x=-3$

(E) $\left| \left| |x| + 1 \right| - 2 \right| = 1$

$\left| |x| + 1 - 2 \right| = 1$

$\left| |x| - 1 \right| = 1$

$|x| - 1 = 1$ ve $|x| - 1 = -1$

$|x| = 2$ $|x| = 0$

$x = 2$ $x = -2$ $x = 0$

12. (a) $\left|\frac{2-x}{x+2}\right| = 3$

$$\frac{|2-x|}{|x+2|} = 3 \quad \Rightarrow |2-x| = 3|x+2|$$

$2-x = 3(x+2)$ ni $2-x = -3(x+2)$

$2-x = 3x+6$ $\quad 2-x = -3x-6$

$-4 = 4x$ $\quad 3x-x = -2-6$

$x = -1$ $\quad 2x = -8$

$x = -4$

13. (a) $|x-1|\,|x-2| - |x-2| = 0$

$|x-2|\,(|x-1|-1) = 0$

$|x-2| = 0$ ni $|x-1|-1 = 0$

$x = 2$ $\quad |x-1| = 1$

$x-1 = 1$ ni $x-1 = -1$

$x = 2$ $\quad x = 0$

(γ) $|x-1| \cdot |x+1| = 3$

$|(x-1)(x+1)| = 3$

$x^2-1 = 3$ ή $x^2-1 = -3$

$x^2 = 4$ $x^2 = -2$

Αδύνατον.

$x=2$ $x=-2$

14. (α) $x^2 - 2\sqrt{x^2} = 0$

$x^2 - 2|x| = 0$

$|x|^2 - 2|x| = 0$

$|x|(|x|-2) = 0$

$|x| = 0$ ή $|x|-2 = 0$

$x=0$ $|x| = 2$

$x=2$ ή $x=-2$

(γ) $|x|^3 - 5x^2 = 0$

$|x|^3 - 5|x|^2 = 0$

$|x|^2(|x| - 5) = 0$

$|x|^2 = 0$ ή $|x| - 5 = 0$

$|x| = 5$

$x = 0$ $x = 5$ $x = -5$

(ε) · $|x|^3 - 2x^2 - 9|x| + 18 = 0$

$|x|^3 - 2|x|^2 - 9|x| + 18 = 0$

$|x|^2(|x| - 2) - 9(|x| - 2) = 0$

$(|x| - 2)(|x|^2 - 9) = 0$

$|x| - 2 = 0$ ή $|x|^2 - 9 = 0$

$|x| = 2$ $|x|^2 = 9$

$|x| = 3$ ή $|x| = -3$ Αδύνατη

$x = 2$ $x = -2$ $x = 3$ $x = -3$.

Εξίσωση 2ου Βαθμού

$$\alpha x^2 + \beta x + \gamma = 0, \quad \alpha \neq 0$$

Τότε $\Delta = \beta^2 - 4\alpha\gamma$

1. Αν $\Delta > 0$ τότε η εξίσωση έχει δυο ρίζες πραγματικές και άνισους.

$$x_{1,2} = \frac{-\beta \pm \sqrt{\Delta}}{2\alpha}.$$

2. Αν $\Delta = 0$ τότε η εξίσωση έχει διπλή ρίζα. $x = -\frac{\beta}{2\alpha}$.

3. Αν $\Delta < 0$ τότε η εξίσωση είναι αδύνατη στο $\mathbb{R}$.

1. (γ) $2x^2 - x - 1 = 0$ $\quad a=2,\ \beta=-1,\ \gamma=-1$

$$\Delta = \beta^2 - 4\alpha\gamma = 1 - 4\cdot 2(-1) = 1 - 8(-1) = 1 + 8 = 9$$

$$x_{1,2} = \frac{-\beta \pm \sqrt{\Delta}}{2\alpha} = \frac{1 \pm \sqrt{9}}{4} \begin{cases} \frac{1+3}{4} = 1 \\ \frac{1-3}{4} = -\frac{1}{2} \end{cases}$$

(δ2) $3x(2-3x) = 1 \Rightarrow 6x - 9x^2 - 1 = 0$ $\quad a=-9,\ \beta=6,\ \gamma=-1$

$$\Delta = \beta^2 - 4\alpha\gamma = 36 - 4\cdot(-9)(-1) = 36 - 36 = 0$$

$$x = -\frac{\beta}{2\alpha} = -\frac{6}{-18} = +\frac{1}{3}$$

(θ) $x^2 - 3x + 4 = 0$, $\quad a=1,\ \beta=-3,\ \gamma=4$

$$\Delta = \beta^2 - 4\alpha\gamma = 9 - 16 = -7$$ Αδύνατη

2. (б) $-3x^2+1=0$

2ой способ

$\Delta = \beta^2 - 4ac = -4(-3)\cdot 1 =$

$= 12 \quad \Delta > 0 \Rightarrow$

1ый способ $-3x^2 = -1$

$x^2 = \frac{1}{3}$

$x = \pm\sqrt{\frac{1}{3}}$

$x_{1,2} = \frac{\pm\sqrt{\Delta}}{2a} \Rightarrow x_{1,2} = \frac{\pm\sqrt{12}}{2\cdot(-3)} \Rightarrow$

$\Rightarrow x_{1,2} = \frac{\pm 2\sqrt{3}}{-6} \Rightarrow$

$x_1 = -\frac{2\sqrt{3}}{-6} \Rightarrow x_1 = \frac{\sqrt{3}}{3}$

$x_2 = \frac{+2\sqrt{3}}{-6} \Rightarrow x_2 = -\frac{\sqrt{3}}{3}$

$x_2 = \pm\frac{\sqrt{3}}{3}$

(в2) $\frac{x}{4} = \frac{x^2}{2}$

$\Rightarrow$

$\frac{x}{4} = \frac{2x^2}{4} \Rightarrow x = 2x^2 \Rightarrow -2x^2 + x = 0$

$\Rightarrow x(1-2x) = 0$

1) $x = 0$

2) $1 - 2x = 0$

$2x = 1$

$x = \frac{1}{2}$

✓

3. (α) $\frac{1}{2}x^2 = x + 2 \Rightarrow \frac{x^2}{2} = x + 2 \Rightarrow x^2 = 2x + 4 \Rightarrow$

$\Rightarrow x^2 - 2x - 4 = 0$

$\Delta = \beta^2 - 4\alpha\gamma \Rightarrow \Delta = 4 + 16 \Rightarrow \Delta = 20$

$x_{1,2} = \frac{-\beta \pm \sqrt{\Delta}}{2\alpha} \Rightarrow x_{1,2} = \frac{2 \pm 2\sqrt{5}}{2} =$ $x_1 = \frac{2+2\sqrt{5}}{2}$, $x_2 = \frac{2-2\sqrt{5}}{2}$ ✓

(β) $x \cdot x + x\frac{3}{x} = x1 \Rightarrow x^2 + 3 = x \Rightarrow x^2 - x + 3 = 0$

$\Delta = \beta^2 - 4\alpha\gamma \Rightarrow \Delta = 1 - 12 \Rightarrow \Delta = -11 \Rightarrow$ ΑΔΥΝΑΤΗ ✓

4. (в) $(x-1)^2 = 3x(x-2) - 1$

$x^2 - 2x + 1 = 3x^2 - 6x - 1$

$x^2 - 2x + 1 - (3x^2 - 6x - 1) = 0$

$x^2 - 2x + 1 - 3x^2 + 6x + 1 = 0$

$-2x^2 + 4x + 2 = 0$

$a = -2$

$b = 4$

$\gamma = 2$

$\Delta = b^2 - 4a\gamma$

$\Delta = 16 + 16$

$\Delta = 32$

$x_{1,2} = \frac{-b \pm \sqrt{\Delta}}{2a} = \frac{-4 \pm 4\sqrt{2}}{-4} =$

$1^{\circ}\ \frac{-4 + 4\sqrt{2}}{-4} = \frac{-4(-1+\sqrt{2})}{-4} = -(-1+\sqrt{2}) = 1 - \sqrt{2}$

$2 = \frac{-4 - 4\sqrt{2}}{-4} = \frac{-4(1+\sqrt{2})}{-4} = 1 + \sqrt{2}$

(б) $x^2 - \frac{2x-1}{6} = x - \frac{x^2}{3}$.

$6x^2 - 2x + 1 = 6x - 2x^2$

$6x^2 - 2x + 1 - 6x + 2x^2 = 0$

$8x^2 - 8x + 1 = 0$

$a = 8$

$b = -8$

$\gamma = 1$

$\Delta = b^2 - 4a\gamma$

$\Delta = 64 - 32$

$\Delta = 32$

$x_{1,2} = \frac{-b \pm \sqrt{\Delta}}{2a} = \frac{8 \pm 4\sqrt{2}}{16} = \frac{2 \pm \sqrt{2}}{8}$

$x = \frac{2+\sqrt{2}}{8}$

$x_2 = \frac{2-\sqrt{2}}{8}$

Επομαο Μαθημα

Σελ 182

1. α β δ ε ζ η
2. α β δ ε
3. α β δ ε
4. α γ ε στ.