12. ⓐ $\int_0^1 \frac{x}{x^2+9}\,dx = \frac{1}{2}\int_0^1 \frac{2x}{x^2+9}\,dx$ **Ενότητα 34**

$= \frac{1}{2}\left(\ln|x^2+9|\right)_0^1 =$

$= \frac{1}{2}\left(\ln 10 - \ln 9\right)$

ⓑ $\int_0^1 \frac{x}{\sqrt{x^2+4}}\,dx = \left(\sqrt{x^2+4}\right)_0^1 = \sqrt{5} - 2$

ⓒ $\int_{\frac{\pi}{6}}^{\frac{\pi}{4}} \sigma\varphi x = \int_{\frac{\pi}{6}}^{\frac{\pi}{4}} \frac{\sigma\upsilon\nu x}{\eta\mu x}\,dx = \left(\ln|\eta\mu x|\right)_{\frac{\pi}{6}}^{\frac{\pi}{4}} =$

$= \ln\frac{\sqrt{2}}{2} - \ln\frac{1}{2} = \ln\sqrt{2}$

ⓓ $\int_0^{\pi} \frac{\eta\mu x}{\sigma\upsilon\nu^2 x}\,dx \quad \frac{\sigma\upsilon\nu x = t}{-\eta\mu x\,dx = dt} \quad -\int_1^{-1} \frac{1}{t^2}\,dt =$

$= -\left(-\frac{1}{t}\right)_1^{-1} = \left(\frac{1}{t}\right)_1^{-1} = -1 - 1 = -2$

ⓔ $\int_0^1 \frac{x}{e^{x^2}}\,dx = \int_0^1 x e^{-x^2}\,dx = -\frac{1}{2}\int_0^1 -2x e^{-x^2} =$

$= -\frac{1}{2}\left(e^{-x^2}\right)_0^1 = -\frac{1}{2}\left(e^{-1} - 1\right)$

(στ) $\int_0^1 \frac{1}{1+e^{-x}}\,dx = \int_0^1 \frac{1}{1+\frac{1}{e^x}}\,dx = \int_0^1 \frac{e^x}{e^x+1}\,dx$

$= \left(\ln(e^x+1)\right)_0^1 = \ln(e+1) - \ln 2$

15. $f(x) = \begin{cases} \frac{1}{x}, & x>1 \\ 2x-1, & x \le 1 \end{cases}$

(α) $\lim_{x\to 1^-} f(x) = 1$, $\lim_{x\to 1^+} f(x) = 1$ $\Big\}$ $\lim_{x\to 1} f(x) = 1$ και $f(1) = 1$

Συνεχής στο 1!

Η f συνεχής $(-\infty, 1)$ και $(1, +\infty)$ ως π.σ.σ

Άρα f συνεχής στο $\mathbb{R}$.

(β) $\int_{-1}^{2} f(x)\,dx = \int_{-1}^{1} f(x)\,dx + \int_1^2 f(x)\,dx$

$= \int_{-1}^{1} 2x-1\,dx + \int_1^2 \frac{1}{x}\,dx$

$= (x^2)_{-1}^{1} - (x)_{-1}^{1} + (\ln x)_1^2 =$

$= -2 + \ln 2$

13. (α) $\int_0^{\pi} \frac{ημx}{2+συνx}\,dx = \left(-\ln|2+συνx|\right)_0^{\pi} =$

$= -\left(\ln 1 - \ln 3\right) = \ln 3$

(β) $\int_0^1 \frac{x+1}{x^2+2x+3}\,dx = \frac{1}{2}\int_0^1 \frac{2x+2}{x^2+2x+3}\,dx =$

$= \frac{1}{2}\left(\ln|x^2+2x+3|\right)_0^1 = \frac{1}{2}\ln 6 - \ln 3 = \frac{1}{2}\ln 2$

(γ) $\int_0^1 (2x+1)(x^2+x+1)^2\,dx = \int_1^3 t^2\,dt =$

$x^2+x+1 = t$

$(2x+1)\,dx = dt$

$= \frac{1}{3}\left(t^3\right)_1^3 = \frac{26}{3}$

(δ) $\int_0^{\pi} συνx\, ημx\,dx = \int_0^0 t\,dt = 0$

$ημx = t$

$συνx\,dx = dt$

(ε) $\int_1^e \frac{1+\ln x}{x}\,dx = \int_1^2 t\,dt = \frac{1}{2}\left(t^2\right)_1^2$

$1+\ln x = t$

$\frac{1}{x}\,dx = dt$

$= \frac{3}{2}.$

(στ)

$$\int_0^{\pi/6} \frac{1-\eta\mu x}{\sigma\upsilon\nu x}\,dx = \int_0^{\pi/6} \frac{(1-\eta\mu x)(1+\eta\mu x)}{\sigma\upsilon\nu x\,(1+\eta\mu x)}\,dx$$

$$= \int_0^{\pi/6} \frac{1-\eta\mu^2 x}{\sigma\upsilon\nu x(1+\eta\mu x)}\,dx = \int_0^{\pi/6} \frac{\sigma\upsilon\nu^2 x}{\sigma\upsilon\nu x\,(1+\eta\mu x)}\,dx$$

$$= \int_0^{\pi/6} \frac{\sigma\upsilon\nu x}{1+\eta\mu x}\,dx = \left(\ln|1+\eta\mu x| \right)_0^{\pi/6} =$$

$$= \ln\frac{3}{2}$$

Ενότητα 35

12. (α) $\int_1^2 f(x-1)\,dx \;\overset{x-1=t}{\underset{dx=dt}{=\!=}}\;$

$$\int_0^1 f(t)\,dt = \int_0^1 6t^5-2t+1\,d = \ldots$$

(β) $\int_0^{\pi/2} \sigma\upsilon\nu x\, f(\eta\mu x)\,dx \;\overset{\eta\mu x=t}{\underset{\sigma\upsilon\nu x\,dx=dt}{=\!=}}\; \int_0^1 f(t)\,dt = \ldots$

(γ) $\int_e^1 \frac{f(\ln x)}{x}\,dx \;\overset{\ln x=t}{\underset{\frac{1}{x}dx=dt}{=\!=}}\; \int_1^0 f(t)\,dt = \ldots$

(δ) $\int_{\ln 2}^{\ln 3} e^x f(e^x-2)\,dx \;\overset{e^x-2=t}{\underset{e^x dx=dt}{=\!=}}\; \int_0^1 f(t)\,dt = \ldots$

13. (α) ν.δ.ο. $\int_0^1 f(3x)\,dx = \frac{1}{3}\int_0^3 f(x)\,dx$

$\rightarrow \int_0^1 f(3x)\,dx \;\overset{3x=t}{\underset{3dx=dt}{=\!=}}\; \int_0^3 \frac{1}{3} f(t)\,dt$ ✓

$dx = \frac{1}{3}dt$

(β) $2\int_1^3 x\,f(x^2)\,dx = \int_1^9 f(x)\,dx$

$\rightarrow 2\int_1^3 x\,f(x^2)\,dx \;\overset{x^2=t}{\underset{2x\,dx=dt}{=\!=}}\; \int_1^9 f(t)\,dt$

14. $f(x) = x^4 - x^3 + \lambda x^2 - \lambda x$

$$\int_0^1 x f(x^2)\, dx = 0$$

$x^2 = t$

$2x\, dx = dt$

$x\, dx = \frac{1}{2} dt$

$$\frac{1}{2}\int_0^1 f(t)\, dt = 0$$

$$\int_0^1 f(x)\, dx = 0$$

$$\int_0^1 x^4 - x^3 + \lambda x^2 - \lambda x \, dx = 0$$

$$\frac{1}{5}(x^5)\Big|_0^1 - \frac{1}{4}(x^4)\Big|_0^1 + \frac{1}{3}\lambda(x^3)\Big|_0^1 - \frac{1}{2}\lambda(x^2)\Big|_0^1 = 0$$

$$\frac{1}{5} - \frac{1}{4} + \frac{\lambda}{3} - \frac{\lambda}{2} = 0$$

$$-\frac{1}{20} - \frac{\lambda}{6} \Rightarrow -\frac{\lambda}{6} = \frac{1}{20} \Rightarrow \lambda = -\frac{6}{20}$$

10. (a) $E = \int_0^1 |f(x) - g(x)|\, dx$

Ενότητα 36

$E = \int_0^1 |e^x - x|\, dx = \int_0^1 e^x - x\, dx$ (+)

- $e^x \geq x + 1$

  $e^x - x \geq 1$

$= \int_0^1 e^x dx - \int_0^1 x\, dx$

$= (e^x)_0^1 - \frac{1}{2}(x^2)_0^1$

$= e - 1 - \frac{1}{2} = e - \frac{3}{2}$

12. (a) $y - f(1) = f'(1)(x-1)$

$f(x) = x^3$

$f'(x) = 3x^2$

$y - 1 = 3(x-1)$

$\boxed{y = 3x - 2}$

$f(x) = 3x - 2 \quad \Rightarrow \quad x^3 - 3x + 2 = 0$

1	0	-3	2	(1)
↓	1	1	-2	
1	1	-2	0	

$(x-1)(x^2 + x - 2) = 0$

$x = 1$, $x = -2$, $x = 1$

(B) $\int_{-2}^{1} |f(x) - (3x-2)|\, dx =$

$= \int_{-2}^{1} |x^3 - 3x + 2| \, dx = \int_{-2}^{1} x^3 - 3x + 2 \, dx$

(+)

x	-2		1	
x − 1	−	−	0	+
x^2+x-2	+ 0	−	0	+
f(x)	−	+		+

$= \frac{1}{4}(x^4)_{-2}^{1} - \frac{3}{2}(x^2)_{-2}^{1}$

$+ 2\,(x)_{-2}^{1} =$

$= \infty$

13. ⓐ $f(x) = g(x) \Leftrightarrow \frac{1}{x} = 1 \Rightarrow x = 1$

$$E = \int_1^2 |f(x) - g(x)|\, dx = \int_1^2 \left|\frac{1}{x} - 1\right| dx$$

$$= \int_1^2 \left|\frac{1-x}{x}\right| dx = \int_1^2 \frac{x-1}{x}\, dx =$$

$$= \int_1^2 1 - \frac{1}{x}\, dx = (x)_1^2 - (\ln x)_1^2 =$$

$$= 1 - \ln 2$$

ⓑ $f(x) = g(x) \Rightarrow \frac{1}{x} = x \Rightarrow x^2 = 1 \Rightarrow x = \pm 1$

$$E = \int_1^2 |f(x)|\, dx = \int_1^2 \left|\frac{1}{x} - x\right| dx = \int_1^2 \left|\frac{1-x^2}{x}\right| dx$$

x		-1		0		1	
$1-x^2$	$-$	0	$+$		$+$	0	$-$
x	$-$		$-$	0	$+$		$+$
$P(x)$	$+$		$-$		$+$		$-$

② $$= \int_1^2 \frac{x^2-1}{x}\, dx =$$

$$= \int_1^2 x - \frac{1}{x}\, dx =$$

$$= \frac{1}{2}(x^2)_1^2 - (\ln x)_1^2$$

$$= \frac{3}{2} - \ln 2$$

16. $y - f(1) = f'(1)(x-1)$

$f(x) = \frac{\ln x}{x}$

$y - 0 = 1(x-1)$

$f'(x) = \frac{1-\ln x}{x^2}$

$y = x - 1$

$f''(x) = \frac{-\frac{1}{x}x^2 - 2x(1-\ln x)}{x^4}$

$E = \int_1^2 |f(x) - (x-1)| \, dx =$

$= \int_1^2 \left| \frac{\ln x}{x} - (x-1) \right| dx$ = (−) (*)

$f''(x) = \frac{-x - 2x(1-\ln x)}{x^4}$

$\varphi(x) = \frac{\ln x}{x} - x + 1$

$\varphi'(x) = \frac{1-\ln x}{x^2} - 1 = \frac{1-\ln x - x^2}{x^2}$

$f''(x) = \frac{-1-2+2\ln x}{x^3}$

$h(x) = 1 - \ln x - x^2$

$f''(x) = \frac{2\ln x - 3}{x^3}$

$h'(x) = -\frac{1}{x} - 2x < 0$

x		1	
h'	−		−
h	↘ +	0	− ↘
φ'	+		−
φ	↗		↘

$\varphi(x) \leq \varphi(1)$

$\varphi(x) \leq 0$

Αν $x > 1$ το $\ln x > 0$

Άτοπο

$$= \int_1^2 x - 1 - \frac{\ln x}{x} \, dx =$$

$$= \int_1^2 x \, dx - \int_1^2 1 \, dx - \int_1^2 \frac{\ln x}{x} \, dx$$

$$= \frac{1}{2} \left(x^2\right)_1^2 - \left(x\right)_1^2 - \int_1^2 \frac{\ln x}{x} \, dx$$

$$\frac{3}{2} - 1 - \int_0^{\ln 2} t \cdot dt$$

$\ln x = t$

$\frac{1}{x} dx = dt$

$$\frac{1}{2} - \frac{1}{2} \left(t^2\right)_0^{\ln 2} = \frac{1}{2} - \frac{1}{2} \ln^2 2.$$

18. $f(x) = \sqrt{x-1}\ , \ x \geq 1$

(a) $y - f(x_0) = f'(x_0)(x - x_0) \longrightarrow (0,0)$

$0 - f(x_0) = f'(x_0)(0 - x_0)$

$-\sqrt{x-1} = \dfrac{1}{2\sqrt{x-1}}(-x)$

$\sqrt{x-1} = \dfrac{x}{2\sqrt{x-1}} \Rightarrow 2(x-1) = x$

$2x - 2 = x$

$\boxed{x = 2}$

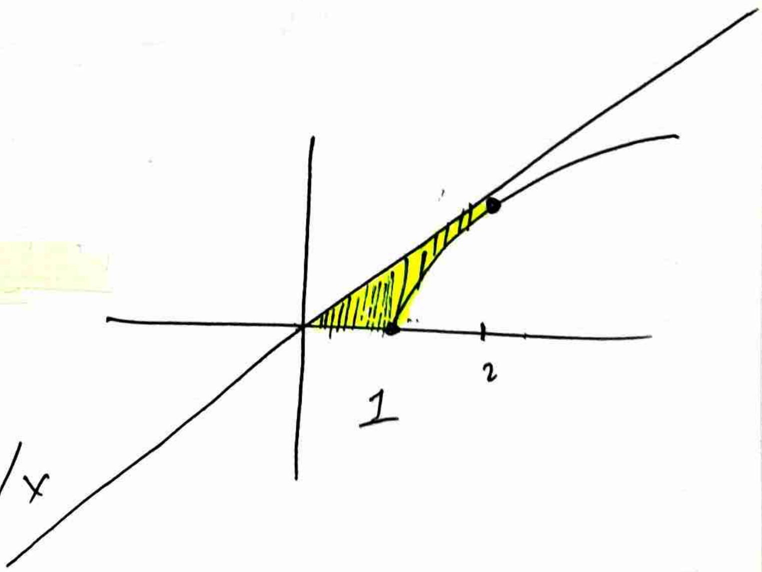
$y - f(2) = f'(2)(x-2)$

$y - 1 = \frac{1}{2}(x-2)$

$\boxed{y = \frac{1}{2}x}$

(B) $\in = \int_0^1 \frac{1}{2}x\, dx$

$+ \int_1^2 \frac{1}{2}x - \sqrt{x-1}\, dx$

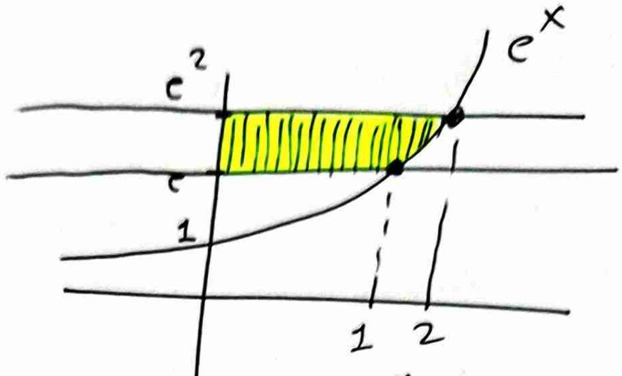


$\longrightarrow \int_1^2 \sqrt{x-1}\, dx \ \underline{\underline{\ \sqrt{x-1} = t\ }}\ \int_0^1 2t^2\, dt = \omega$

$x - 1 = t^2$

$dx = 2t\, dt$

19.



$e^x = e$

$x = 1$

$$E = \int_0^1 e^2 - e \, dx + \int_1^2 e^x - e \, dx$$

$$E = (e^2 - e)\,(x)\Big|_0^1 + (e^x)\Big|_1^2 - e(x)\Big|_1^2$$

$$E = e^2 - e + e^2 - e - e$$

$$E = 2e^2 - 3e.$$

6. $f(x) = \sqrt{x^2+1}$

Ενότητα 37

α) $\lim_{x \to +\infty} (f(x) - x) = \lim_{x \to +\infty} \sqrt{x^2+1} - x$

$= \lim_{x \to +\infty} \dfrac{1}{\sqrt{x^2+1} + x} = 0$ ✓

Νδο $f(x) > x \Rightarrow \sqrt{x^2+1} > x$

Αν $x \geq 0$

$\sqrt{x^2+1}^2 > x^2 \Rightarrow x^2 + 1 > x^2$

$1 > 0$

Αν $x < 0$

Ισχύει αυτόματα η αλλη

β) νδο $\int_0^1 f(x)\,dx > \frac{1}{2}$

$f(x) > x$

$\int_0^1 f(x)\,dx > \int_0^1 x\,dx$

$\int_0^1 f(x)\,dx > \frac{1}{2}.$

(γ) $\int_1^2 \frac{f(x)}{x}\,dx > \frac{2}{3}\left(2\sqrt{2}-1\right)$

$f(x) > x$

$\frac{f(x)}{\sqrt{x}} > \frac{x}{\sqrt{x}}$

$\int_1^2 \frac{f(x)}{\sqrt{x}}\,dx > \int_1^2 \frac{x}{\sqrt{x}}\,dx.$

$\longrightarrow \int_1^2 \frac{x}{\sqrt{x}}\,dx \; \overset{\sqrt{x}=t}{\underset{x=t^2}{=\!=}} \; \int_1^{\sqrt{2}} \frac{t^2}{\cancel{t}}\, 2\cancel{t}\,dt$

$dx = 2t\,dt$

$= 2\int_1^{\sqrt{2}} t^2\,dt = \frac{2}{3}\left(t^3\right)_1^{\sqrt{2}}$

$= \frac{2}{3}\left(\sqrt{2}^3 - 1\right) = \frac{2}{3}\left(2\sqrt{2}-1\right).$

7. $f(x) = e^{x^2}$

ⓐ Νδο $f(x) \geq x^2+1 \Rightarrow e^{x^2} \geq x^2+1$

Αφου $e^x \geq x+1 \Rightarrow e^{x^2} \geq x^2+1$ ✓

ⓑ Νδο $\int_0^2 f(x)\,dx > \frac{14}{3}$

$f(x) \geq x^2+1$

$\int_0^2 f(x)\,dx > \int_0^2 x^2+1\,dx$

$\int_0^2 f(x)\,dx > \frac{1}{3}\left(x^3\right)_0^2 + (x)_0^2$

$\int_0^2 f(x)\,dx > \frac{8}{3} + 2 = \frac{14}{3}$ ✓

Προετοιμασια Διαγωνισματος 8/3/26

Εξεταστεα υλη

25 εως 32

Μπαρλα

Θεωρια

1.38	2.9	3.19
1.39	2.10	3.20
1.40	2.11	3.21
1.41	2.12	3.22
1.42		
1.43		
1.44		
1.45		
1.46		
1.47		
1.48		
1.49		

Ασκηση sos 1

Η $f(x) = ax^4 + \beta x^3$ παρουσιάζει τοπικό ακρότατο στο 1 το -1. Βρείτε a, β.

- Αφού η f παρουσιάζει ακρότατο στο 1.
- Το 1 εσωτερικό του $\mathbb{R}$.
- Η f παρ/μη στο 1.

Από Fermat $f'(1)=0$

Επίσης $f(1) = -1$.

Άρα $f'(x) = 4ax^3 + 3\beta x^2$

$$\begin{cases} f'(1) = 4a + 3\beta = 0 \\ f(1) = a + \beta = -1 \end{cases} \Rightarrow \begin{cases} 4a + 3\beta = 0 \\ -3a - 3\beta = +3 \end{cases} \Leftrightarrow$$

$a = 3$

$\beta = -4$

Ασκηση SOS 2

Εστω $f:(0,+\infty) \to \mathbb{R}$ παραγωγισιμη ωστε

$$(x-1)f(x) \geq e^{x-1} - 1 \quad \forall x>0$$

Νδο $f(1)=1$

Απο ανισοτητα σε ισοτητα $\Rightarrow$ Fermat

Αφου $(x-1)f(x) \geq e^{x-1} - 1$

$$\underbrace{(x-1)f(x) - e^{x-1} + 1}_{\varphi(x)} \geq 0$$

$\varphi(x) \geq 0$

$\varphi(x) \geq \varphi(1)$

Εχω ελαχιστο στο 1

Ακροτατο στο 1

φ παρ/μη στο 1

Το 1 εσωτερικο του $(0,+\infty)$

$\Rightarrow$ Fermat $\varphi'(1) = 0$.

$\varphi'(x) = f(x) + (x-1) f''(x) - e^{x-1}$

$\varphi'(1) = f(1) - 1 = 0$

$f(1) = 1.$

Ασκηση SOS 3

Εστω $f: R \to R$ παρ/μη με $f'(0)=1$

Αν $f(f(x)) + e^{f(x)} = e^x - x$

Νδο η f δεν εχει ακροτατα

Εστω οτι η f εχει ακροτατο στο x_0. Η f παρ/μη στο x_0 και το x_0 εσωτερικο του R.

Fermat $f'(x_0) = 0$

$f'(f(x)) f'(x) + f'(x) e^{f(x)} = e^x - 1$

$\underline{x = x_0}$

$f'(f(x_0)) f'(x_0) + f'(x_0) e^{f(x_0)} = e^{x_0} - 1$

$0 + 0 = e^{x_0} - 1 \Rightarrow e^{x_0} = 1$

$\underline{x_0 = 0}$

Αρα $f'(0) = 0$ Ατοπο! γιατι $f'(0) = 1$

Αρα δεν εχει ακροτατα.

Σχόλιο

$$\left(f^2(x)\right)' = 2f(x)\, f'(x)$$

$$\left(e^{f(x)}\right)' = e^{f(x)}\, f'(x)$$

$$\left(\ln f(x)\right)' = \frac{1}{f(x)}\, f'(x)$$

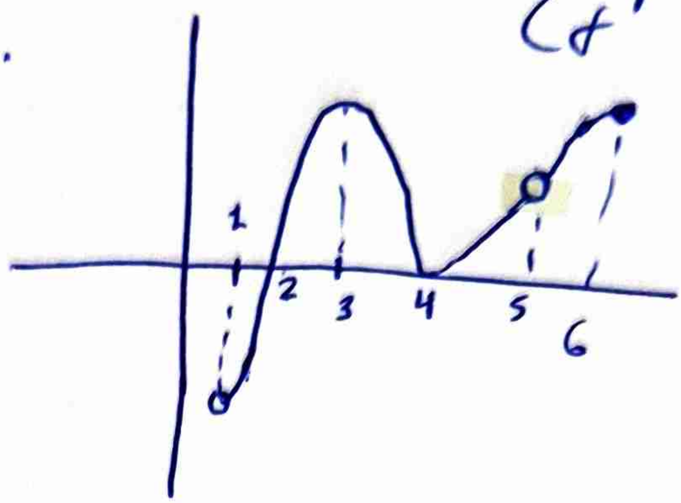
$$\left(f(g(x))\right)' = f'(g(x))\, g'(x)$$

$$\left(\text{ημ}\, f(x)\right)' = \text{συν}\, f(x) \cdot f'(x)$$

$$\left(\frac{1}{f(x)}\right)' = -\frac{1}{f^2(x)}\, f'(x).$$

Ασκηση SOS 4

f : [1,6] → ℝ συνεχής.



1. κρίσιμα σημεία

- Είναι τα εσωτερικά σημεία που η f(x) δεν είναι παραγωγίσιμη.

  ~~1~~, (5) → κρίσιμο σημείο.

  ↳ δεν είναι εσωτερικό.

- Ρίζα της f'(x) πάντα εσωτερικά.

  (2), (4) κρίσιμα σημεία.

2. Πιθανές θέσεις ακροτάτων

κρίσιμα σημεία : 2, 4, 5

Άκρα κλειστά : 1, 6.

3. Μονοτονια – κυρτοτητα

x		2	
f'	$-$		$+$
f	↘		↗

x		3		4	
f'	↗		↘		↗
f	∪		∩		∪

4. Ευρεση οριων

(α) $\lim_{x \to 4} \frac{1}{f'(x)} = +\infty$ γιατι η $f'(x) > 0$ κοντα στο 4

(β) $\lim_{x \to 2} \frac{1}{f'(x)}$

- $\lim_{x \to 2^-} \frac{1}{f'(x)} = -\infty$
- $\lim_{x \to 2^+} \frac{1}{f'(x)} = +\infty$

Δεν υπαρχει.

Άσκηση SOS 5

Απόδειξη ανισότητας

ΝΔΟ $e^x + \sigma\upsilon\nu x > 2 + x - \frac{x^3}{6} \quad \forall x > 0$

$$\underbrace{e^x + \sigma\upsilon\nu x - 2 - x + \frac{x^3}{6}}_{\varphi(x)} > 0$$

$\varphi'(x) = e^x - \eta\mu x - 1 + \frac{3}{6}x^2$

$\varphi'(x) = e^x - \eta\mu x - 1 + \frac{1}{2}x^2$ $\qquad \varphi'(0) = 0$

$\varphi''(x) = e^x - \sigma\upsilon\nu x + x$ $\qquad \varphi''(0) = 0$

$\varphi'''(x) = e^x + 1 + \eta\mu x > 0$ γιατί $\eta\mu x > -1$

x		0
φ'''	+	+
φ''	↗ − 0	+ ↗
φ'	↘ +	+ ↗
φ	↗	↗

$x > 0 \Rightarrow \varphi''(x) > \varphi''(0) \Rightarrow \varphi''(x) > 0$

$x > 0 \Rightarrow \varphi'(x) > \varphi'(0) \Rightarrow \varphi'(x) > 0$

$x > 0 \Rightarrow \varphi(x) > \varphi(0) \Rightarrow \varphi(x) > 0$ ✓

Ασκηση SOS 6

Δινεται $f(x) = e^{x-1} - \ln x + 1 \ , \ x > 0$

1. Συνολο Τιμων

$f'(x) = e^{x-1} - \frac{1}{x}$ $f'(1) = 0$

$f''(x) = e^{x-1} + \frac{1}{x^2} > 0$

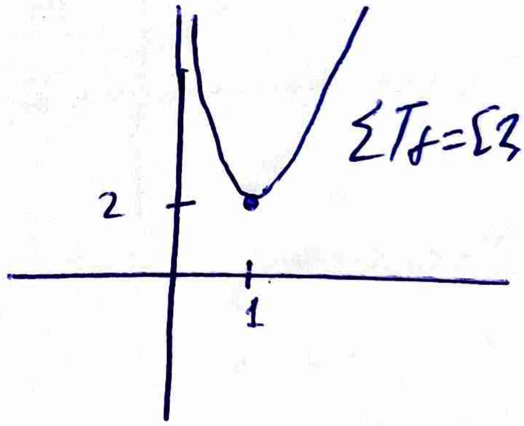
x	0		1	
f''		+		+
f'		↗ −	0	+ ↗
f		↘		↗

$x < 1 \Rightarrow f'(x) < f'(1) \Rightarrow f'(x) < 0$

$x > 1 \Rightarrow f'(x) > f'(1) \Rightarrow f'(x) > 0$

$f(x) \geq f(1)$

$f(x) \geq 2$



$ΣΤ_f = [2, +\infty)$

$\lim_{x \to 0^+} f(x) = +\infty$

$\lim_{x \to +\infty} f(x) = \lim_{x \to +\infty} (e^{x-1} - \ln x + 1)$ $\xrightarrow[\text{κ.Π το } e^{x-1}]{+\infty - \infty}$

$= \lim_{x \to +\infty} e^{x-1} \left(1 - \frac{\ln x}{e^{x-1}} + 1\right) = +\infty$

$\rightarrow \lim_{x \to +\infty} \frac{\ln x}{e^{x-1}} = \lim_{x \to +\infty} \frac{\frac{1}{x}}{e^{x-1}} = \lim_{x \to +\infty} \frac{1}{x e^{x-1}} = 0$

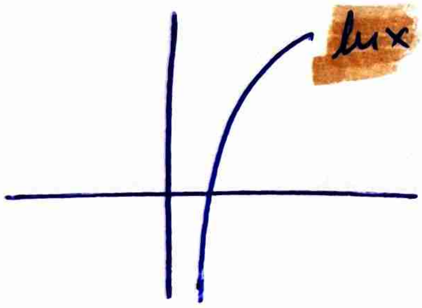
Σχόλιο

- $\lim_{x \to 0} x^2 \ln x \overset{0 \cdot \infty}{=\!=} \lim_{x \to 0} \frac{\ln x}{\frac{1}{x^2}} = \lim_{x \to 0} \frac{\frac{1}{x}}{-\frac{2x}{x^4}} = 0$

- $\lim_{x \to +\infty} (x - \ln x) \overset{+\infty - \infty}{=\!=} \lim_{x \to +\infty} x\left(1 - \frac{\ln x}{x}\right) = +\infty$

$\rightarrow \lim_{x \to +\infty} \frac{\ln x}{x} = \lim_{x \to +\infty} \frac{1}{x} = 0$

Σχόλιο

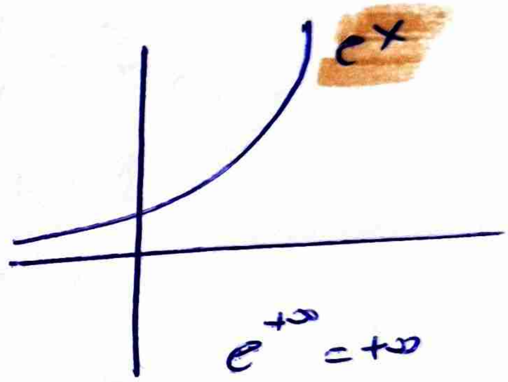


$\ln(+\infty) = +\infty$

$\ln 0 = -\infty$

$\ln e = 1$

$\ln 1 = 0$

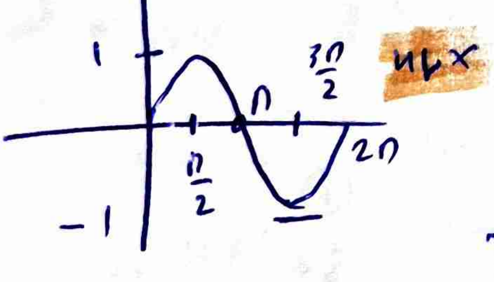


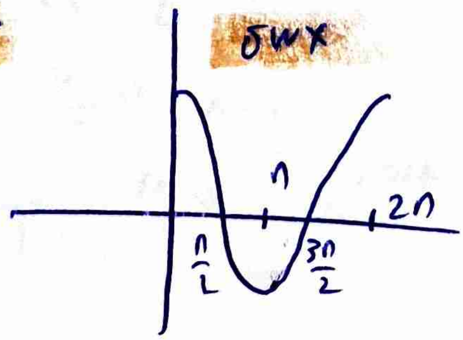
$e^{+\infty} = +\infty$

$e^{-\infty} = 0$

$e^0 = 1$

$e^1 = e$.





2. Να λυθεί η εξίσωση $f(f(x^2+1)-1)=2$

$$f(f(x^2+1)-1)=2$$

Γνωρίζω ότι το $A(1,2)$ είναι ολικό ελάχιστο

άρα ΜΟΝΟ το $f(1)=2$

$$f(\underbrace{f(x^2+1)-1}_{1})=2$$

$$\Rightarrow f(x^2+1)-1=1$$

$$f(\underbrace{x^2+1}_{1})=2$$

$$x^2+1=1$$

$$x^2=0$$

$$x=0$$

3. Να λυθεί η εξίσωση $f(x)+f(x^{2026})=4$

- $f(x)\geq 2$ Το "=" $x=1$

$\oplus$

- $f(x^{2026})\geq 2$ Το "=" $x^{2026}=1 \Rightarrow x=1$ ή $x=-1$

$f(x)+f(x^{2026})\geq 4$ Το "=" για $x=1$

4. Βρείτε $\lim_{x\to 1} \frac{\ln x}{(x-1)(f(x)-2)}$

Είναι $\lim_{x\to 1} \frac{\ln x}{x-1} \cdot \frac{1}{f(x)-2} = 1 \cdot (+\infty) = +\infty$

(+) καθώς $f(x) \geq 2$

$\rightarrow \lim_{x\to 1} \frac{\ln x}{x-1} = \lim_{x\to 1} \frac{1}{x} = 1$

5. Να λυθεί η ανίσωση $f(f(x)) > e + \ln\frac{e}{2}$

$f(f(x)) > e + \ln e - \ln 2$

$f(f(x)) > e + 1 - \ln 2$

$f(f(x)) > f(2)$

- $f(x) \geq 2$
- $2 \geq 2$

$\Big\}$ $\forall x > 2$ и $f \nearrow$

Άρα $f(x) > 2$

$x \neq 1$