8.

Ενότητα 8

ⓐ $\lim\limits_{x\to 0} \left(x^2+\eta\mu x-\varepsilon\varphi x\right) =$

$= 0^2 + \eta\mu 0 - \varepsilon\varphi 0 =$

$= 0 + 0 - 0 = 0$

9.

(στ) $\lim\limits_{x\to -\frac{1}{2}} \dfrac{2x^2+x}{4x^2-1} \overset{\left(\frac{0}{0}\right)}{=\!=} \lim\limits_{x\to -\frac{1}{2}} \dfrac{x\cancel{(2x+1)}}{(2x-1)\cancel{(2x+1)}}$

$= \lim\limits_{x\to -\frac{1}{2}} \dfrac{x}{2x-1} = \dfrac{-\frac{1}{2}}{2\left(-\frac{1}{2}\right)-1} = \dfrac{-\frac{1}{2}}{-2} = \dfrac{1}{4}.$

10.

(γ) $\lim\limits_{x\to 1} \dfrac{x^2-2x+1}{x^2-x} \overset{\left(\frac{0}{0}\right)}{=\!=} \lim\limits_{x\to 1} \dfrac{(x-1)\cancel{(x-1)}}{x\cancel{(x-1)}}$

1	−2	1	1
↓	1	−1	
1	−1	0	

$= \lim\limits_{x\to 1} \dfrac{x-1}{x} = \dfrac{0}{1} = 0.$

11. (γ) $\lim\limits_{x\to 3} \dfrac{x^4-81}{x^3-27} \overset{\left(\frac{0}{0}\right)}{=} \lim\limits_{x\to 3} \dfrac{\cancel{(x-3)}(x^3+3x^2+9x+27)}{\cancel{(x-3)}(x^2+3x+9)}$

$$\begin{array}{ccccc|c}
1 & 0 & 0 & 0 & -81 & 3 \\
\downarrow & 3 & 9 & 27 & 81 & \\
\hline
1 & 3 & 9 & 27 & 0 &
\end{array}$$

$$= \frac{27+27+27+27}{9+9+9}$$

$$\begin{array}{cccc|c}
1 & 0 & 0 & -27 & 3 \\
\downarrow & 3 & 9 & 27 & \\
\hline
1 & 3 & 9 & 0 &
\end{array}$$

$$= \frac{4\cdot \cancel{27}}{\cancel{27}} = 4$$

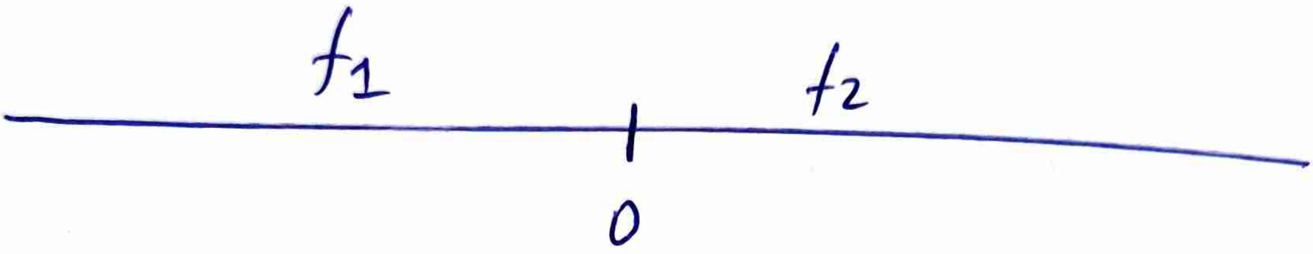
12. (β) $\lim\limits_{x\to 1} \left(\dfrac{1}{1-x} - \dfrac{3}{1-x^3} \right)$

$$\begin{array}{cccc|c}
-1 & 0 & 0 & 1 & 1 \\
\downarrow & -1 & -1 & -1 & \\
\hline
-1 & -1 & -1 & 0 &
\end{array}$$

$$= \lim_{x\to 1} \frac{1}{1-x} - \frac{3}{(x-1)(-x^2-x-1)}$$

$$= \lim_{x\to 1} -\frac{1}{(x-1)} - \frac{3}{(x-1)(-x^2-x-1)} =$$

15. γ) $f(x) = \begin{cases} 2 - \sigma\upsilon\nu x, & x<0 \\ 1, & x=0 \\ \dfrac{\sqrt{x+1}-1}{x}, & x>0 \end{cases}$



Βρω το $\lim\limits_{x\to -\pi} f(x) = \lim\limits_{x\to -\pi} (2 - \sigma\upsilon\nu x) = 2-(-1) = 3$

Βρω το $\lim\limits_{x\to 3} f(x) = \lim\limits_{x\to 3} \dfrac{\sqrt{x+1}-1}{x} = \dfrac{\sqrt{4}-1}{3} = \dfrac{2-1}{3} = \dfrac{1}{3}.$

$$\lim_{x\to 0} f(x) = \text{Το όριο δεν υπάρχει!}$$

$$\to \lim_{x\to 0^-} f(x) = \lim_{x\to 0^-} (2-\sigma\upsilon\nu x) = 2-1 = 1$$

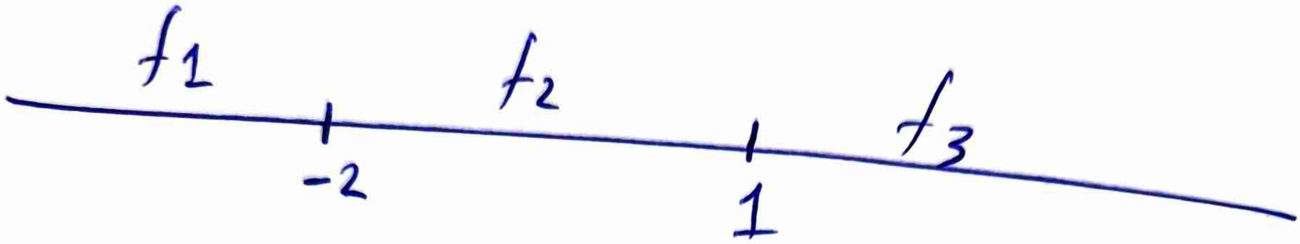
$$\to \lim_{x\to 0^+} f(x) = \lim_{x\to 0^+} \frac{\sqrt{x+1}-1}{x} \overset{\left(\frac{0}{0}\right)}{=}$$

$$= \lim_{x\to 0^+} \frac{(\sqrt{x+1}-1)(\sqrt{x+1}+1)}{x(\sqrt{x+1}+1)}$$

$$= \lim_{x\to 0^+} \frac{\sqrt{x+1}^2-1^2}{x(\sqrt{x+1}+1)} = \lim_{x\to 0^+} \frac{\cancel{x}}{\cancel{x}(\sqrt{x+1}+1)}$$

$$= \frac{1}{2}$$

(δ) $f(x) = \begin{cases} x^3 - 2x, & x < -2 \\ 3x+2, & -2 \le x < 1 \\ x-1, & x \geq 1 \end{cases}$



$$\boxed{\lim_{x \to -2} f(x) = -4}$$

$\to \lim_{x \to -2^-} f(x) = \lim_{x \to -2^-} x^3 - 2x = -8+4 = -4$

$\to \lim_{x \to -2^+} f(x) = \lim_{x \to -2^+} 3x+2 = -4$

$\lim_{x \to 1} f(x)$ Το όριο δεν υπάρχει.

$\to \lim_{x \to 1^-} f(x) = \lim_{x \to 1^-} 3x+2 = 5$

$\to \lim_{x \to 1^+} f(x) = \lim_{x \to 1^+} x-1 = 0$

14. δ) $\lim_{x \to 1} \dfrac{\sqrt{3x-2} - 2x + 1}{x^4 - 1}$

$= \lim_{x \to 1} \dfrac{\sqrt{3x-2} - (2x-1)}{(x^2-1)(x^2+1)}$

$= \lim_{x \to 1} \dfrac{\left(\sqrt{3x-2} - (2x-1)\right)\left(\sqrt{3x-2} + (2x-1)\right)}{(x-1)(x+1)(x^2+1)\left(\sqrt{3x-2} + 2x-1\right)}$

$= \lim_{x \to 1} \dfrac{\sqrt{3x-2}^{\,2} - (2x-1)^2}{\ldots\ldots} =$

$= \lim_{x \to 1} \dfrac{3x-2 - (4x^2-4x+1)}{\ldots\ldots} = \lim_{x \to 1} \dfrac{-4x^2+7x-3}{\ldots\ldots}$

-4	7	-3	1
$\downarrow$	-4	3	
-4	3	0	

$$= \lim_{x \to 1} \frac{\cancel{(x-1)}(-4x+3)}{\cancel{(x-1)}(x+1)(x^2+1)(\sqrt{3x-2}+2x-1)}$$

$$= \frac{-1}{4 \cdot 2} = -\frac{1}{8}.$$

19.

(1) $$\lim_{x\to -1^+} \frac{|x+1| + x^2 - x - 2}{x^3+1} =$$

x	-1	
x+1	− 0	+

$$= \lim_{x\to -1^+} \frac{|x+1| + x^2 - x - 2}{x^3+1}$$

$$= \lim_{x\to -1^+} \frac{\cancel{x}+1 + x^2 - \cancel{x} - 2}{x^3+1} = \lim_{x\to -1^+} \frac{x^2-1}{x^3+1}$$

$$= \lim_{x\to -1^+} \frac{(x-1)\cancel{(x+1)}}{\cancel{(x+1)}(x^2-x+1)} = \frac{-2}{3}.$$

20. (B) $\lim\limits_{x \to -1} \dfrac{|x^2-1| - x^2+x+2}{\sqrt{x+2}-1} \quad \left(\frac{0}{0}\right)$

x	-1 1
x^2-1	+ | − | +

Το όριο δεν υπάρχει.

$$\to \lim_{x \to -1^-} \frac{|x^2-1| - x^2+x+2}{\sqrt{x+2}-1} = \lim_{x\to -1^-} \frac{\cancel{x^2}-1-\cancel{x^2}+x+2}{\sqrt{x+2}-1}$$

$$= \lim_{x\to -1^-} \frac{x+1}{\sqrt{x+2}-1} = \lim_{x\to -1^-} \frac{(x+1)(\sqrt{x+2}+1)}{(\sqrt{x+2}-1)(\sqrt{x+2}+1)}$$

$$= \lim_{x\to -1^-} \frac{(x+1)(\sqrt{x+2}+1)}{\cancel{\sqrt{x+2}}^2-1^2} =$$

$$= \lim_{x\to -1^-} \frac{\cancel{(x+1)}(\sqrt{x+2}+1)}{\cancel{x+1}} = 2$$

$$\rightarrow \lim_{x \to -1^+} \frac{\overset{-}{|x^2-1|} - x^2 + x + 2}{\sqrt{x+2} - 1}$$

$$= \lim_{x \to -1^+} \frac{-x^2 + 1 - x^2 + x + 2}{\sqrt{x+2} - 1}$$

$$= \lim_{x \to -1^+} \frac{-2x^2 + x + 3}{\sqrt{x+2} + 1}$$

-2	1	3	-1
$\downarrow$	2	-3	
-2	3	0	

$$= \lim_{x \to -1^+} \frac{(x+1)(-2x+3)(\sqrt{x+2} - 1)}{(\sqrt{x+2} + 1)(\sqrt{x+2} - 1)}$$

$$= \lim_{x \to -1^+} \frac{(x+1)(-2x+3)(\sqrt{x+2} - 1)}{\cancel{\sqrt{x+2}}^{\cancel{2}} - 1^2}$$

$$= \lim_{x \to -1^+} \frac{\cancel{(x+1)}(-2x+3)(\sqrt{x+2} - 1)}{\cancel{x+1}} = 5 \cdot 0$$

Ορια με απολυτες Τιμες

Ενοτητα 8

20. ① $\lim_{x\to 3} \dfrac{x^2-9-\sqrt{x^2-6x+9}}{|x-1|-2}$

$$= \lim_{x\to 3} \frac{x^2-9-\sqrt{(x-3)^2}}{|x-1|-2} = \lim_{x\to 3} \frac{x^2-9-|x-3|}{|x-1|-2}$$

$$= \lim_{x\to 3} \frac{x^2-9-|x-3|}{x-1-2} = \lim_{x\to 3} \frac{x^2-9-|x-3|}{x-3}$$

x		3	
$x-3$	$-$	0	$+$

$$\rightarrow \lim_{x\to 3^-} \frac{x^2-9-\overset{\ominus}{|x-3|}}{x-3} = \lim_{x\to 3^-} \frac{x^2-9-(-x+3)}{x-3}$$

$$= \lim_{x\to 3^-} \frac{x^2-9+x-3}{x-3} = \lim_{x\to 3^-} \frac{x^2+x-12}{x-3}$$

1	1	−12	3
↓	3	12	
1	4	0	

$$= \lim_{x\to 3^-} \frac{\cancel{(x-3)}(x-4)}{\cancel{x-3}}$$

$$= -1$$

$$\rightarrow \lim_{x\to 3^+} \frac{x^2-9-\overset{\oplus}{|x-3|}}{x-3} = \lim_{x\to 3^+} \frac{x^2-9-x+3}{x-3}$$

$$= \lim_{x\to 3^+} \frac{x^2-x-6}{x-3} = \lim_{x\to 3^+} \frac{\cancel{(x-3)}(x+2)}{\cancel{x-3}}$$

$$= 5$$

Το οριο δεν υπαρχει

13.

σε $\lim\limits_{x \to 3} \dfrac{\sqrt{x^2+2} - \sqrt{11}}{x^2 - 5x + 6}$

$$= \lim_{x \to 3} \frac{\left(\sqrt{x^2+2} - \sqrt{11}\right)\left(\sqrt{x^2+2} + \sqrt{11}\right)}{\left(x^2 - 5x + 6\right)\left(\sqrt{x^2+2} + \sqrt{11}\right)}$$

$$= \lim_{x \to 3} \frac{\cancel{\sqrt{x^2+2}}^{\cancel{2}} - \cancel{\sqrt{11}}^{\cancel{2}}}{(x-2)(x-3)\left(\sqrt{x^2+2} + \sqrt{11}\right)} = \lim_{x \to 3} \frac{x^2+2-11}{(x-2)(x-3)\left(\sqrt{x^2+2} + \sqrt{11}\right)}$$

$$= \lim_{x \to 3} \frac{\cancel{(x-3)}(x+3)}{(x-2)\cancel{(x-3)}\left(\sqrt{x^2+2} + \sqrt{11}\right)}$$

$$= \frac{6}{1 \cdot 2\sqrt{11}} = \frac{3}{\sqrt{11}}$$

$$= \lim_{x \to 1} \frac{-(-x^2-x-1)-3}{(x-1)(-x^2-x-1)},$$

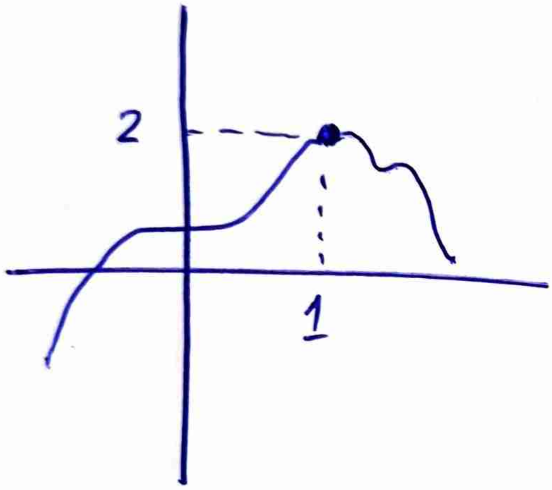
$$= \lim_{x \to 1} \frac{x^2+x+1-3}{(x-1)(-x^2-x-1)} = \lim_{x \to 1} \frac{x^2+x-2}{(x-1)(-x^2-x-1)}$$

$$= \lim_{x \to 1} \frac{(x+2)\cancel{(x-1)}}{\cancel{(x-1)}(-x^2-x-1)} = \frac{3}{-3}$$

$$= -1,$$

SOS: Ποτε μια συναρτηση $f(x)$ ειναι συνεχης στο x_0;

Οταν $\lim_{x \to x_0} f(x) = f(x_0)$.

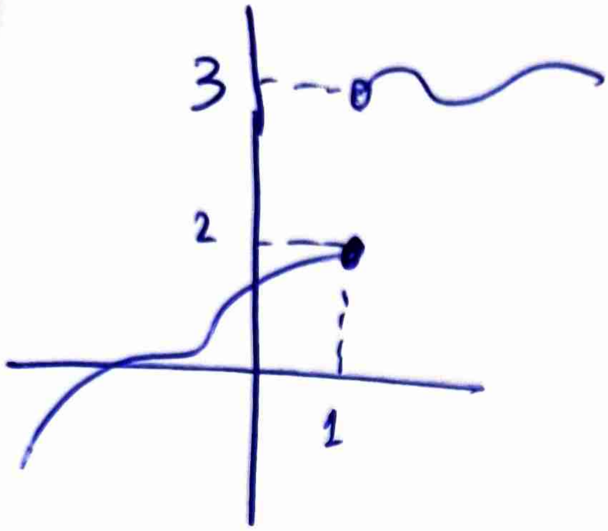


$$\left.\begin{aligned} \lim_{x \to 1^-} f(x) = 2 \\ \lim_{x \to 1^+} f(x) = 2 \end{aligned}\right\} \lim_{x \to 1} f(x) = 2$$

$f(1) = 2$

Παρατηρω οτι $f(1) = \lim_{x \to 1} f(x)$

Συνεχης στο 1.

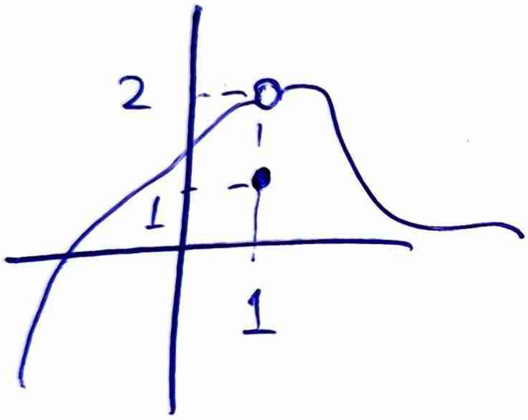


ΟΧΙ Συνεχής

Το $\lim_{x\to 1} f(x)$ δεν υπάρχει

$\to \lim_{x\to 1^-} f(x)=2$

$\to \lim_{x\to 1^+} f(x)=3$



ΟΧΙ Συνεχής

$\lim_{x\to 1} f(x)=2$

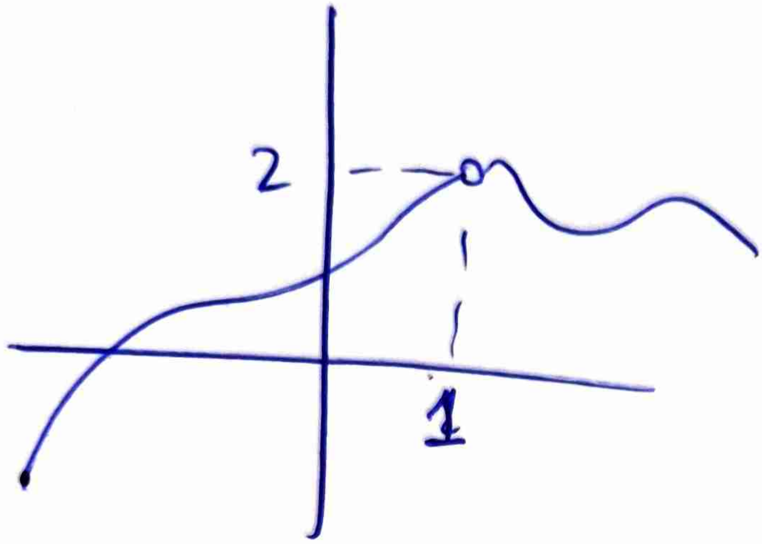
$f(1)=1$

Άρα $f(1)\neq \lim_{x\to 1} f(x)$

η $f(x)$ δεν είναι συνεχής

στο 1,

Παγιδα



$$\lim_{x \to 1} f(x) = 2.$$

$$D_f = R - \{1\}$$

Αφου το 1 δεν ανηκει στο πεδιο ορισμου δεν εξεταζουμε τη συνεχεια εδω (στο 1 δηλ)!

17.

Ενότητα 4

$f(x) = \ln x \; , \; x > 0$

$g(x) = \sqrt{2-x} - 1 \; , \; D_g = (-\infty, 2]$

Έστω $f(x) < g(x)$

$\ln x < \sqrt{2-x} - 1$

$\underbrace{\ln x - \sqrt{2-x} + 1}_{h(x)} < 0$

$h(x) < 0$

$h(x) < h(1)$

$h \nearrow$

$0 < x < 1$

Μονοτονία $h(x)$

- $x_1 < x_2 \Rightarrow \ln x_1 < \ln x_2$
- $x_1 < x_2 \Rightarrow 2 - x_1 > 2 - x_2$

$\sqrt{2-x_1} > \sqrt{2-x_2}$

$-\sqrt{2-x_1} < -\sqrt{2-x_2}$

$+$

$h \nearrow$

Όταν $x \in (0,1)$ η $f(x) < g(x)$

Όταν $x > 1$ η $f(x) > g(x)$

Όταν $x = 1$ $f(x) = g(x)$

19.

$$f(x) = \left(\frac{3}{5}\right)^x + \left(\frac{4}{5}\right)^x - 1$$

ⓐ

$$x_1 < x_2 \Rightarrow \left(\frac{3}{5}\right)^{x_1} > \left(\frac{3}{5}\right)^{x_2}$$

$$x_1 < x_2 \Rightarrow \left(\frac{4}{5}\right)^{x_1} > \left(\frac{4}{5}\right)^{x_2}$$

$\Big\}\ (+)$

$f \downarrow$

ⓑ

$$3^x + 4^x > 5^x$$

$$\frac{3^x}{5^x} + \frac{4^x}{5^x} > 1$$

$$\underbrace{\left(\frac{3}{5}\right)^x + \left(\frac{4}{5}\right)^x - 1}_{f(x)} > 0$$

$$f(x) > 0$$

$$f(x) > f(2)$$

$f \downarrow$

$\underline{x < 2}$

10.

$$f(x) = \frac{x-1}{x+1}$$

$$D_f = \mathbb{R} - \{-1\}$$

Εύρεση 3

$$g(x) = \frac{1}{x}$$

$$D_g = \mathbb{R}^*$$

ⓐ $(f \circ g)(x) = f(g(x)) = \dfrac{\frac{1}{x} - 1}{\frac{1}{x} + 1} = \dfrac{1-x}{1+x}$

$x \in D_g$ και $g(x) \in D_f$

$\underline{x \neq 0}$

$\frac{1}{x} \neq -1$

$1 \neq -x$

$\underline{x \neq -1}$

$$D_{f \circ g} = \mathbb{R} - \{0, -1\},$$

(B) $(f \circ f)(x) = f\Big(f(x)\Big) = \dfrac{\dfrac{x-1}{x+1} - 1}{\dfrac{x-1}{x+1} + 1} =$

$= \dfrac{x-1-(x+1)}{x-1+x+1} = \dfrac{-2}{2x} = -\dfrac{1}{x}$

$\boxed{x \in D_f \quad \text{και} \quad f(x) \in D_f}$

$\underline{\underline{x \neq -1}}$

$\dfrac{x-1}{x+1} \neq -1$

$x-1 \neq -(x+1)$

$x-1 \neq -x-1$

$2x \neq 0$

$\underline{\underline{x \neq 0}}$

$D_{f \circ f} = \mathbb{R} - \{-1, 0\}$

① $\left(g \circ \frac{1}{f}\right)(x) = g\left(\frac{1}{f}(x)\right) = \dfrac{1}{\frac{x+1}{x-1}} = \dfrac{x-1}{x+1}$

$x \in D_{1/f}$ και $\left(\frac{1}{f}\right)(x) \in D_g$

$\underline{\underline{x \neq 0}}$ $\underline{\underline{x \neq -1}}$

$\dfrac{x+1}{x-1} \neq 0$

$x+1 \neq 0$

$x \neq -1$

$D_{g \circ \frac{1}{f}} = \mathbb{R} - \{0, -1\}.$

$\left(\frac{1}{f}\right)(x) = \dfrac{1}{f(x)} = \dfrac{1}{\frac{x-1}{x+1}} = \dfrac{x+1}{x-1}$

$x \in D_f$ και $f(x) \in D_h$

$x \neq -1$ $\dfrac{x-1}{x+1} \neq -1$

$x-1 \neq -(x+1)$

$\underline{x \neq 0}$

$D_{1/f} = \mathbb{R} - \{-1, 0\}.$

Επομαο Μαθημα

Τεταρτη 10:30.

Ενοτητα 8

(8) β γ

(9) α β γ

(10) α β

(11) α β

(12) α

(13) α β γ δ

(14) α β

(15) α β

(20) α

(19) α β

Ενοτητα 4

(20) α β

(21)

Παλιες

(18) Ενοτητα

(19) 7