Τετραγωνική Ρίζα

1. $x^2 = \theta \Leftrightarrow x = \sqrt{\theta}$

όπου $x \geq 0$ και $\theta \geq 0$

2. $\sqrt{x}^2 = x$, $x \geq 0$

3. $\sqrt{x^2} = |x|$, $x \in \mathbb{R}$.

4. $\sqrt{x}\sqrt{y} = \sqrt{x \cdot y}$

5. $\frac{\sqrt{x}}{\sqrt{y}} = \sqrt{\frac{x}{y}}$,

Ασκησεις - Σειρα

1. Να υπολογιστουν οι παρακατω

(α) $\sqrt{49} - \sqrt{1} + \sqrt{121} - \sqrt{225}$

$= 7 - 1 + 11 - 15$

$= 6 - 4 = 2.$

(β) $\sqrt{9 - 5\sqrt{3 - \sqrt{4}}} =$

$= \sqrt{9 - 5\sqrt{3-2}} = \sqrt{9 - 5\sqrt{1}}$

$= \sqrt{9 - 5} = \sqrt{4} = 2.$

2. (a) $\sqrt{(n-3)^2} + \sqrt{(n-4)^2} =$

$= |\overset{(+)}{n-3}| + |\overset{(-)}{n-4}| = n-3-n+4 = 1.$

(B) $\sqrt{\frac{x^2}{9}} + \sqrt{x^2-2x+1} =$

$= \sqrt{\left(\frac{x}{3}\right)^2} + \sqrt{(x-1)^2}$

$= \left|\frac{x}{3}\right| + |x-1|$

3.

(α) $\sqrt{8} - \sqrt{27} - \sqrt{18} + \sqrt{12} - \sqrt{200}$

$= \sqrt{2}\sqrt{4} - \sqrt{3}\sqrt{9} - \sqrt{2}\sqrt{9} + \sqrt{3}\sqrt{4} - \sqrt{2}\sqrt{100}$

$= 2\sqrt{2} - 3\sqrt{3} - 3\sqrt{2} + 2\sqrt{3} - 10\sqrt{2}$

$= -11\sqrt{2} - \sqrt{3}.$

(β) $\dfrac{\sqrt{2}\sqrt{12}}{\sqrt{6}} = \dfrac{\sqrt{2}\sqrt{3}\sqrt{4}}{\sqrt{2}\sqrt{3}} = \sqrt{4} = 2.$

(γ) $\sqrt{7}\sqrt{3-\sqrt{2}}\sqrt{3+\sqrt{2}} =$

$= \sqrt{7 \cdot (3-\sqrt{2})(3+\sqrt{2})}$

$= \sqrt{7 \cdot (3^2 - \sqrt{2}^2)} = \sqrt{7 \cdot (9-2)} =$

$= \sqrt{77} = \sqrt{49} = 7.$

4. Ρητοποίηση

(α) $\frac{15}{\sqrt{5}} = \frac{15\sqrt{5}}{\sqrt{5}\sqrt{5}} = \frac{15\sqrt{5}}{\sqrt{25}} = \frac{15\sqrt{5}}{5} = 3\sqrt{5}$.

(β) $\frac{6}{\sqrt{7}-\sqrt{3}} = \frac{6(\sqrt{7}+\sqrt{3})}{(\sqrt{7}-\sqrt{3})(\sqrt{7}+\sqrt{3})}$

$= \frac{6(\sqrt{7}+\sqrt{3})}{\sqrt{7}^2-\sqrt{3}^2} = \frac{6(\sqrt{7}+\sqrt{3})}{7-3}$

$= \frac{6(\sqrt{7}+\sqrt{3})}{4}$

5. ⑧ $\frac{\sqrt{5}}{\sqrt{7}-\sqrt{5}} + \frac{\sqrt{7}}{\sqrt{7}+\sqrt{5}} = 6.$

$$\frac{\sqrt{5}(\sqrt{7}+\sqrt{5})}{(\sqrt{7}-\sqrt{5})(\sqrt{7}+\sqrt{5})} + \frac{\sqrt{7}(\sqrt{7}-\sqrt{5})}{(\sqrt{7}+\sqrt{5})(\sqrt{7}-\sqrt{5})} = 6$$

$$\frac{\sqrt{5}(\sqrt{7}+\sqrt{5})}{\sqrt{7}^2-\sqrt{5}^2} + \frac{\sqrt{7}(\sqrt{7}-\sqrt{5})}{\sqrt{7}^2-\sqrt{5}^2} = 6$$

$$\frac{\sqrt{35}+\sqrt{25}}{7-5} + \frac{\sqrt{49}-\sqrt{35}}{7-5} = 6$$

$$\frac{\sqrt{35}+5}{2} + \frac{7-\sqrt{35}}{2} = 6$$

$$\frac{\sqrt{35}+5+7-\sqrt{35}}{2} = 6 \quad \rightarrow \quad \frac{12}{2} = 6 \quad \checkmark$$

Ενότητα 8

26 Β) $|x-1| < |x-3|$

$$|x-1|^2 < |x-3|^2$$

$$(x-1)^2 < (x-3)^2$$

$$x^2 - 2x + 1 < x^2 - 6x + 9$$

$$6x - 2x < 8$$

$$4x < 8$$

$$x < 2$$

Γ) $d(2x, 1) > d(2x, 3)$

$$|2x-1| > |2x-3|$$

$$|2x-1|^2 > |2x-3|^2$$

$$4x^2 - 4x + 1 > 4x^2 - 12x + 9$$

$$8x > 8$$

$x > 1$

26. (8) $x^2 < 9$

$x^2 < 3^2$

$|x| < |3|$ (+)

$|x| < 3$

$-3 < x < 3$

$|x|^2 = |x^2| = x^2$

44. ⓐ $\left|\frac{1}{x}-2\right|<1$

$$\left|\frac{1-2x}{x}\right|<1$$

$$\frac{|1-2x|}{|x|}<1$$

$$|1-2x|<|x|$$

$$|1-2x|^2<|x|^2$$

$$1-4x+4x^2<x^2$$

$$1-4x+3x^2<0$$

$$3x^2-4x+1<0$$

$$\Delta=16-12=4$$

$$x=\frac{4\pm 2}{6}$$ ① $\frac{1}{3}$

x		$1/3$		1	
$3x^2-4x+1$	$+$	0	$-$	0	$+$

$x\in\left(\frac{1}{3},1\right)$.

46. $\left|\frac{3a+1}{a+3}\right| < 1$

$$\frac{|3a+1|}{|a+3|} < 1$$

$$|3a+1| < |a+3|$$

$$|3a+1|^2 < |a+3|^2$$

$$9a^2 + 6a + 1 < a^2 + 6a + 9$$

$$8a^2 < 8$$

$$a^2 < 1$$

$$a^2 < 1^2$$

$$|a| < |1| \quad (+)$$

$$|a| < 1$$

47. $d(a,0) < 1$

$$|a-0| < 1$$

$$|a| < 1$$

$$\boxed{-1 < a < 1}$$

ⓐ Ndo $\quad |2-|a-1|| = a+1$

$$\rightarrow |2-\underset{\ominus}{|a-1|}| = |2-(-a+1)| =$$

• $-1 < a < 1 \Rightarrow -2 < a-1 < 0$

$$= |2+a-1| = |\underset{\oplus}{1+a}| = a+1$$

• $-1 < a < 1 \Rightarrow 0 < a+1 < 2$

(B) i) $A = |\overset{\ominus}{2a-2}| - |\overset{\oplus}{3a+4}|$

• $-1 < a < 1 \Rightarrow -2 < 2a < 2 \Rightarrow -4 < 2a-2 < 0$

• $-1 < a < 1 \Rightarrow -3 < 3a < 3 \Rightarrow 1 < 3a+4 < 7$

$A = -2a+2 - (3a+4)$

$A = -2a+2-3a-4$

$A = -5a-2$

ii) $|\overset{\ominus}{a^2-1}| = 1-a^2.$

• $-1 < a < 1 \Rightarrow |a| < 1 \Rightarrow |a|^2 < 1^2 \Rightarrow a^2 < 1$

$a^2 - 1 < 0$

49. $A = \dfrac{x^2-9}{|x|-3}$

(α) Πρεπι $|x|-3 \neq 0$

$\rightarrow |x|-3=0 \Rightarrow |x|=3$

$x=3$ ή $x=-3$

Για να ορίζεται πρεπι $x\neq 3, x\neq -3$

(Β) $A = \dfrac{x^2-9}{|x|-3} = \dfrac{|x|^2-9}{|x|-3} = \dfrac{(|x|-3)(|x|+3)}{|x|-3}$

$$A = |x|+3$$

(Γ) $A<7 \Rightarrow |x|+3<7 \Rightarrow |x|<4$

$$-4<x<4$$

Επόμενο μάθημα

Ημερομηνία μαθήματος : 30/10/26

Ώρα μαθήματος : 18:30.

Ασκήσεις

Ενδιάμεσοι 9

(2) α γ

(3)

(4)

(12) α, β.

(17) α.

παρατηρήσεις