

# Евотута 8

2. а)  $|n-3|^{\oplus} = n-3.$

б)  $|4-n|^{\oplus} = 4-n$

в)  $|6-2n|^{\ominus} = -6+2n$

г)  $|\sqrt{3}-\sqrt{2}|^{\oplus} = \sqrt{3}-\sqrt{2}$

д)  $|\sqrt{2}-1|^{\oplus} = \sqrt{2}-1$

е)  $|\sqrt{5}-3|^{\ominus} = -\sqrt{5}+3,$

3. а)  $|n-3|^{\oplus} + |\sqrt{2}-1|^{\oplus} - |\sqrt{3}-2|^{\ominus} =$

$$= n-3 + \sqrt{2}-1 - (-\sqrt{3}+2) =$$

$$= n-3 + \sqrt{2}-1 + \sqrt{3}-2$$

$$= n-6 + \sqrt{3},$$

$$7. \text{ (a) } A = |x-3|^{\oplus} + |x-5|^{\ominus} = \cancel{x-3} - \cancel{x+5} = 2.$$

$$\bullet 3 < x < 5 \Rightarrow 3-3 < x-3 < 5-3 \Rightarrow 0 < x-3 < 2$$

$$\bullet 3 < x < 5 \Rightarrow -2 < x-5 < 0$$

$$\text{(b) } B = |2 - |x-3|^{\oplus}| = |2 - (x-3)| =$$

$$\bullet 3 < x < 5 \Rightarrow 0 < x-3 < 2$$

$$= |2 - x + 3| = |5-x|^{\oplus} = 5-x,$$

$$\bullet 3 < x < 5 \Rightarrow -3 > -x > -5$$

$$2 > 5-x > 0$$

$$8. A = |x-2| - |x-3|$$

$$(a) A = |x-2|^{\oplus} - |x-3|^{\ominus} = x-2 - (-x+3) = 2x-5$$

$$2 \leq x \leq 3 \Rightarrow 0 \leq x-2 \leq 1$$

$$2 \leq x \leq 3 \Rightarrow -1 \leq x-3 \leq 0.$$

$$(b) A = |x-2|^{\ominus} - |x-3|^{\ominus}$$

$$\because x < 2 \Rightarrow x-2 < 2-2 \Rightarrow x-2 < 0$$

$$\because x < 2 \Rightarrow x-3 < 2-3 \Rightarrow x-3 < -1$$

$$A = -x+2 - (-x+3)$$

$$A = -\cancel{x}+2 + \cancel{x}-3$$

$$A = -1$$

$$5. \textcircled{a} |x^2 - 4x + 4| - |-x^2 - 3| =$$

$$= |(\overset{+}{x-2})^2| - |-x^2 - 3| =$$

$$= (x-2)^2 - (-x^2 - 3) =$$

$$= \cancel{x^2} - 4x + 4 - \cancel{x^2} - 3 =$$

$$= -4x + 1$$

$$6. \textcircled{a} |\overset{+}{x-5}| = x-5$$

$$\bullet 5 < x < 10 \Rightarrow 0 < x-5 < 5$$

$$|\overset{-}{x-10}| = -x+10$$

$$\bullet 5 < x < 10 \Rightarrow -5 < x-10 < 0$$

$$\begin{aligned} \textcircled{b} A &= \frac{|x-5|}{x-5} + \frac{|x-10|}{x-10} = \frac{\cancel{x-5}}{\cancel{x-5}} + \frac{-x+10}{x-10} = \\ &= 1 - \frac{\cancel{x-10}}{\cancel{x-10}} = 1 - 1 = 0 \end{aligned}$$

$$9. \quad (a) \quad A = |x+2|^{\oplus} + |x-4|^{\ominus}$$

$$\bullet \quad -1 < x < 3 \Rightarrow -1+2 < x+2 < 3+2$$

$$1 < x+2 < 5$$

$$\bullet \quad -1 < x < 3 \Rightarrow -5 < x-4 < -1$$

$$A = \cancel{x+2} - \cancel{x+4}$$

$$\underline{\underline{A = 6}}$$

$$\underline{a < b < c}$$

$$10. \quad (a) \quad A = |b-a|^{\oplus} + |b-c|^{\ominus} + |c-a|^{\oplus}$$

$$\bullet \quad a < b \Rightarrow 0 < b-a$$

$$\bullet \quad b < c \Rightarrow b-c < 0$$

$$\bullet \quad a < c \Rightarrow 0 < c-a$$

$$A = \cancel{b-a} - \cancel{b+c} + c-a = 2c-2a.$$

11. (a)  $A = |a| + |a+1| - |a-b|$

$a < -1 < b$

- $a < -1$
- $a < -1 \Rightarrow a+1 < -1+1 \Rightarrow a+1 < 0$
- $a < b \Rightarrow a-b < 0$

$$A = -a - a - 1 - (-a+b)$$

$$A = -2a - 1 + a - b$$

$$A = -a - 1 - b$$

$$12. \quad \textcircled{a} \quad A = |a+2|^{\oplus} - |a-1|^{\ominus}$$

$$\bullet -2 < a < 1 \Rightarrow 0 < a+2 < 3$$

$$\bullet -2 < a < 1 \Rightarrow -3 < a-1 < 0$$

$$A = a+2 - (-a+1)$$

$$A = a+2 + a-1$$

$$\underline{A = 2a+1}$$

$$\textcircled{b} \quad B = |a+3|^{\oplus} - |a-2|^{\ominus}$$

$$\bullet -2 < a < 1 \Rightarrow 1 < a+3 < 4$$

$$\bullet -2 < a < 1 \Rightarrow -4 < a-2 < -1$$

$$B = a+3 - (-a+2)$$

$$B = a+3 + a-2 = 2a+1$$

$$\textcircled{1} \quad \Gamma = \frac{2a+5}{-} - \frac{3a-4}{+}$$

$$\bullet -2 < a < 1 \Rightarrow -4 < 2a < 2 \Rightarrow 1 < 2a+5 < 7$$

$$\bullet -2 < a < 1 \Rightarrow -6 < 3a < 3$$

$$-10 < 3a-4 < -1$$

$$\Gamma = 2a+5 - (-3a+4)$$

$$\Gamma = 2a+5 + 3a-4$$

$$\Gamma = 5a+1$$



$$|x| = 0 \quad (\Rightarrow) \quad x = 0 \quad \vee \quad x = -0$$

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$$1. \quad |x| = 8$$

$$\underline{\underline{x = 8}} \quad \vee \quad \underline{\underline{x = -8}}$$

$$2. \quad |2x - 1| = 5$$

$$2x - 1 = 5 \quad \vee$$

$$2x = 6$$

$$\underline{\underline{x = 3}}$$

$$2x - 1 = -5$$

$$2x = -4$$

$$\underline{\underline{x = -2}}$$

$$3. \quad |5x - 8| = -5$$

Answer!

$$4. \quad |5x - 4| = 0$$

$$5x - 4 = 0$$

$$5x = 4$$

$$\underline{\underline{x = \frac{4}{5}}}$$

$$6. \quad | |x-1| - 4 | = 5$$

$$|x-1| - 4 = 5$$

и

$$|x-1| - 4 = -5$$

$$|x-1| = 9$$

$$|x-1| = -1$$

Ответ!

$$x-1=9$$

и

$$x-1=-9$$

$$x=10$$

$$x=-8$$

$$|x| = |y| \Rightarrow x = y \text{ u } x = -y$$

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$$1. |x-1| = |2x-3|$$

$$x-1 = 2x-3$$

u

$$x-1 = -2x+3$$

$$-x = -2$$

$$\boxed{x=2}$$

$$x+2x = 4$$

$$3x = 4$$

$$\boxed{x = \frac{4}{3}}$$

$$2. ||x|-1| = |3-|x||$$

$$|x|-1 = 3-|x|$$

$$\text{u } |x|-1 = -3+|x|$$

$$2|x| = 4$$

$$-1 = -3$$

Atorol,

$$|x| = 2$$

$$\boxed{x=2}$$

$$\boxed{x=-2}$$

$$3. |x| \leq \theta \quad (\Rightarrow) \quad -\theta \leq x \leq \theta$$

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$$1. |x| \leq 4 \quad (\Rightarrow) \quad -4 \leq x \leq 4$$
$$x \in [-4, 4]$$

$$2. |2x-1| \leq 3$$

$$-3 \leq 2x-1 \leq 3$$

$$1-3 \leq 2x \leq 1+3$$

$$-2 \leq 2x \leq 4$$

$$-1 \leq x \leq 2 \quad x \in [-1, 2]$$

$$3. \quad ||x-1|-1| \leq 2$$

$$-2 \leq |x-1|-1 \leq 2$$

$$-1 \leq |x-1| \leq 3$$

$$-1 \leq |x-1| \quad \text{konu} \quad |x-1| \leq 3$$

$$x \in \mathbb{R}.$$

$$-3 \leq x-1 \leq 3$$

$$-2 \leq x \leq 4$$

$$x \in [-2, 4].$$



$$\text{konu} \ni x \in [-2, 4]$$

$$4. \quad |x - 2025| \leq 0$$

$$\Rightarrow |x - 2025| = 0$$

$$x - 2025 = 0$$

$$\underline{\underline{x = 2025}}$$

$$5. \quad |x - 2025| < 0$$

A Solution.

$$|x| \geq 0 \quad \Leftrightarrow x \geq 0 \quad \vee \quad x \leq -0$$


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$$1. \quad |x| \geq 2 \quad \Rightarrow \quad x \geq 2 \quad \vee \quad x \leq -2$$

$$x \in (-\infty, -2] \cup [2, +\infty).$$

$$2. \quad \left| \frac{x}{2} - 1 \right| \geq 1$$

$$\frac{x}{2} - 1 \geq 1$$

$$x - 2 \geq 2$$

$$x \geq 4$$

$\vee$

$$\frac{x}{2} - 1 \leq -1$$

$$x - 2 \leq -2$$

$$x \leq 0$$

$$x \in (-\infty, 0] \cup [4, +\infty).$$

$$3. |x-1| > -5$$

$$\underline{\underline{x \in \mathbb{R} .}}$$

$$4. |x-1| \geq 0$$

$$x \in \mathbb{R} .$$

$$5. |x-1| > 0$$

$$x \in (-\infty, 1) \cup (1, +\infty)$$

$$6. \left| |x-1| - 1 \right| \geq 2$$

$$|x-1| - 1 \geq 2$$

∨

$$|x-1| - 1 \leq -2$$

$$|x-1| \geq 3$$

$$|x-1| \leq -1$$

Ad absurdum .

$$x-1 \geq 3 \quad \vee \quad x-1 \leq -3$$

$$x \geq 4$$

$$x \leq 0$$

$$x \in (-\infty, 0] \cup [4, +\infty) .$$



# Assume

1.  $|x-1|=4$

2.  $|2x-5|=|5-x|$

3.  $||x|-1|=1$

4.  $|x-3| \leq 5$

5.  $|x+2| \geq 3$

6.  $||x|-1| \leq 4$

7.  $|2|x|-1| = ||x|-1|$

$|x|=0 \Rightarrow x=0 \vee x=-0$

$|x|=|y| \Rightarrow x=y \vee x=-y$

$|x| \leq 0 \Rightarrow 0 \leq x \leq 0$

$|x| \geq 0 \Rightarrow x \geq 0$

$\vee x \leq -0$

1.  $|x-1| = 4$

$$x-1 = 4$$

$$\boxed{x=5}$$

u'

$$x-1 = -4$$

$$\boxed{x=-3}$$

2.  $|2x-5| = |5-x|$

$$2x-5 = 5-x$$

u'

$$2x-5 = -5+x$$

$$3x = 10$$

$$\boxed{x = \frac{10}{3}}$$

$$\underline{\underline{x=0}}$$

3.  $||x|-1| = 1$

$$|x|-1 = 1$$

u'

$$|x|-1 = -1$$

$$|x| = 2$$

$$|x| = 0$$

$$\underline{\underline{x=2}}$$

u'

$$\underline{\underline{x=-2}}$$

$$\underline{\underline{x=0}}$$

$$4. |x-3| \leq 5.$$

$$-5 \leq x-3 \leq 5$$

$$\underline{\underline{-2 \leq x \leq 8}}$$

$$5. |x+2| \geq 3.$$

$$x+2 \geq 3 \quad \vee \quad x+2 \leq -3$$

$$\underline{\underline{x \geq 1}}$$

$$\underline{\underline{x \leq -5}}$$

$$x \in (-\infty, -5] \cup [1, +\infty).$$

$$6. \quad | |x| - 1 | \leq 4$$

$$-4 \leq |x| - 1 \leq 4$$

$$-3 \leq |x| \leq 5$$

$$|x| \geq -3$$

και

$$|x| \leq 5$$

Αρα ισχύει

$$x \in \mathbb{R}.$$

$$-5 \leq x \leq 5$$

$$x \in [-5, 5]$$

$$7. \quad |2|x|-1| = |x|-1|$$

$$2|x|-1 = |x|-1 \quad \vee \quad 2|x|-1 = -|x|+1$$

$$|x| = 0$$

$$(x=0)$$

$$3|x| = 2$$

$$|x| = \frac{2}{3}$$

$$(x = \frac{2}{3}) \quad \vee \quad (x = -\frac{2}{3})$$

Εποραιο Μαθημα

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Ενοτητα 8

(13)

(21)

(22)

(23)

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(25)

(26)

(19)