Ενότητα 20

6. (γ) $\frac{3x+2}{2x+1} \leq 0$

Πρέπει $2x+1 \neq 0$

$x \neq -\frac{1}{2}$

→ $3x+2=0$

$x=-\frac{2}{3}$

x	$-\frac{2}{3}$		$-\frac{1}{2}$	
$3x+2$	$-$	0 $+$		$+$
$2x+1$	$-$	$-$	$\nexists$	$+$
$P(x)$	$+$	$-$	$\nexists$	$+$

$x \in [-\frac{2}{3}, -\frac{1}{2})$

(δ) $\frac{x}{3-x} \leq 0$

x		0		3
x	$-$	$+$		$+$
$3-x$	$+$	$+$	$\nexists$	$-$
$P(x)$	$-$	$+$	$\nexists$	$-$

$x \in (-\infty, 0] \cup (3, +\infty)$

8. Ⓑ $\dfrac{x}{x^2-x+1} < 1$

$\Delta < 0$

$x < x^2 - x + 1$

$0 < x^2 - 2x + 1$

x		1	
x^2-2x+1	$+$	0	$+$

$x \in (-\infty, 1) \cup (1, +\infty)$

9. Ⓖ $\dfrac{7x^2-x-6}{9-x^2} \leq 0$

	-3		$-\frac{6}{7}$		1		3	
$7x^2-x-1$	$+$	$\|$	$+$	0	$-$	0	$+$	$+$
$9-x^2$	$-$	$\|$	$+$	$+$	$+$	$+$	$\|$	$-$
$P(x)$	$-$	$\|$	$+$	$-$	$+$	$+$	$\|$	$-$

$\Delta = 169$

$x = \dfrac{1 \pm 13}{14}$ → ① , $-\frac{6}{7}$

$x \in (-\infty, -3) \cup [-\frac{6}{7}, 1] \cup (3, +\infty)$.

7. (B) $\frac{x+2}{3x+2} \leq 1$

$$\frac{x+2}{3x+2} - 1 \leq 0$$

$$\frac{x+2}{3x+2} - \frac{3x+2}{3x+2} \leq 0$$

$$\frac{x+2-3x-2}{3x+2} \leq 0$$

$$\frac{-2x}{3x+2} \leq 0$$

x		$-\frac{2}{3}$		0	
$-2x$	$+$	‖	$+$		$-$
$3x+2$	$-$	‖	$+$		$+$
$P(x)$	$-$	‖	$+$		$-$

$x \in (-\infty, -\frac{2}{3}) \cup [0, +\infty)$

(Y) $\frac{x-1}{x} < \frac{3}{4}$

$$\frac{x-1}{x} - \frac{3}{4} < 0$$

$$\frac{4x-4}{4x} - \frac{3x}{4x} < 0$$

$$\frac{x-4}{4x} < 0$$

x		0		4	
$x-4$	$-$		$-$		$+$
$4x$	$-$	‖	$+$		$+$
$P(x)$	$+$	‖	$-$		$+$

$x \in (0, 4)$.

10. ① $\frac{x^2-2}{x^2-1} < \frac{2}{3}$

$$\frac{x^2-2}{x^2-1} - \frac{2}{3} < 0$$

$$\frac{3(x^2-2)}{3(x^2-1)} - \frac{2(x^2-1)}{3(x^2-1)} < 0$$

$$\frac{x^2-4}{3(x^2-1)} < 0$$

x	-2	-1	1	2
x^2-4	$+\ 0\ -$	$\|\|$	$-$	$-\ 0\ +$
x^2-1	$+\ +\ 0\ -$	$\|\|$	$0\ +$	$+$
$p(x)$	$+\ -$	$\|\|\ +$	$-$	$+$

$x \in (-2,-1) \cup (1,2)$

1. (B) $\frac{3x^2+1}{x-1} + \frac{4}{x-x^2} = \frac{x^2-3x+2}{x}$

$$\frac{3x^2+1}{x-1} - \frac{4}{x(x-1)} = \frac{x^2-3x+2}{x}$$

- $x-1=0$

$\boxed{x=1}$

- $x(x-1)=0$

$x=0$ $x=1$

- $x=0$

Προϋ $x \neq 0$, $x \neq 1$

ΕΚΠ: $x(x-1)$

$$x(x-1)\frac{3x^2+1}{x-1} - x(x-1)\frac{4}{x(x-1)} = \frac{x^2-3x+1}{x}x(x-1)$$

$$x(3x^2+1) - 4 = (x-1)(x^2-3x+2)$$

$$3x^3 + x - 4 = x^3 - 3x^2 + 2x - x^2 + 3x - 2$$

$$2x^3 + 4x^2 - 4x - 2 = 0$$

$$x^3 + 2x^2 - 2x - 1 = 0.$$

1	2	-2	-1	(1)
↓	1	3	1	
1	3	1	0	

$$(x-1)(x^2+3x+1) = 0$$

$x-1=0$

~~$x=1$~~

și. $x^2+3x+1=0$

$\Delta = 5$

$$x = \frac{-3 \pm \sqrt{5}}{2}$$

2. ⑧ $\eta\mu^6 x - 4\eta\mu^3 x + 3 = 0$

$$(\eta\mu^3 x)^2 - 4\eta\mu^3 x + 3 = 0$$

$$\eta\mu^3 x = t$$

$$t^2 - 4t + 3 = 0$$

$t = 1$ $\qquad$ $t = 3$

$\eta\mu^3 x = 1$ $\qquad$ $\eta\mu^3 x = 3$

$\eta\mu x = 1$ $\qquad$ $\eta\mu x = \sqrt[3]{3} > 1$

$\eta\mu x = \eta\mu\frac{\pi}{2}$ $\qquad$ Αδύνατη

$x = 2k\pi + \frac{\pi}{2}$

$x = 2k\pi + \pi - \frac{\pi}{2}$

2. $2\operatorname{ctg}^3 x + \frac{1}{\sin^2 x} = 0$

$$\sin^2 x = \frac{1}{1+\operatorname{ctg}^2 x}$$

$$2\operatorname{ctg}^3 x + \frac{1}{\frac{1}{1+\operatorname{ctg}^2 x}} = 0$$

$$2\operatorname{ctg}^3 x + 1 + \operatorname{ctg}^2 x = 0$$

$\operatorname{ctg} x = t$

$$2t^3 + t^2 + 1 = 0$$

2	1	0	1	(-1)
↓	-2	1	-1	
2	-1	1	0	

$(t+1)(2t^2 - t + 1) = 0$

$\Delta < 0$

$t = -1$

$\operatorname{ctg} x = -1$

$\operatorname{ctg} x = -\operatorname{ctg}\frac{\pi}{4}$

$\operatorname{ctg} x = \operatorname{ctg}\left(-\frac{\pi}{4}\right)$

$x = \pi k - \frac{\pi}{4}$

Εναλλακτικά 23

15. (B) $e^{2x} - e \cdot e^x < 0$

$(e^x)^2 - e \cdot e^x < 0$

$e^x = t$

$e^x (e^x - e) < 0$

$t(t-e) < 0$

$t^2 - et < 0$

t		0		e
t^2-et	$+$		$-$	$+$

$0 < t < e$

$0 < e^x < e$

$0 < e^x < e^1$

✓

$e^x < e^1$

$x < 1$

21. (B) $\frac{2^x+3}{2^x+1} \leq 2$

$2^x = t$

Ayo $2^x > 0$

$t > 0$

$\frac{t+3}{t+1} \leq 2$

(+)

$t+3 \leq 2(t+1)$

$t+3 \leq 2t+2$

$0 \leq 2t-t+2-3$

$0 \leq t-1$

$1 \leq t$

$1 \leq 2^x$

$2^0 \leq 2^x$

$0 \leq x$

21. ⑴ $\dfrac{e^x - e}{e^x - 1} > 0$ $\qquad e^x = t$

$$\frac{t-e}{t-1} > 0$$

t		1		e	
$t-e$	$-$	$-$	$-$	0	$+$
$t-1$	$-$	0	$+$	$+$	$+$
$p(t)$	$+$	$\|\|$	$-$	0	$+$

$t < 1$ ili $t > e$

$e^x < 1$ ili $e^x > e$

$e^x < e^0$ $\qquad e^x > e^1$

$x < 0$ $\qquad x > 1$

$x \in (-\infty, 0) \cup (1, +\infty)$.

29. $27^x + 2 \cdot 9^x - 3 < 0$

$$(3^x)^3 + 2 \cdot (3^x)^2 - 3 < 0$$

$$\boxed{3^x = u}$$

$$u^3 + 2u^2 - 3 < 0$$

1	2	0	-3	(1)
↓	1	3	3	
1	3	3	0	

$$(u-1)(u^2+3u+3) < 0$$

(1) $\quad \Delta < 0$

u		1	
$u-1$	$-$	0	$+$
u^2+3u+3	$+$		$+$
$P(u)$	$-$		$+$

$$u < 1$$

$$3^x < 1$$

$$3^x < 3^0$$

$$x < 0$$

(5) $9^x - 4 \cdot 3^x + 3 > 0$

$(3^2)^x - 4 \cdot 3^x + 3 > 0$

$(3^x)^2 - 4 \cdot 3^x + 3 > 0$

$3^x = t$

$t^2 - 4t + 3 > 0$

t		1		3
t^2-4t+3	$+$	$-$		$+$

$t < 1$ ili $t > 3$

$3^x < 1$ $\quad$ $3^x > 3$

$3^x < 3^0$ $\quad$ $3^x > 3^1$

$x < 0$ $\quad$ $x > 1$

Εποχω Μαθημα

23

(12) α γ δ

(13) α β γ

(14) α β

(15) α γ

(21) α.

(27) α β

20

(1) α

(2) α β

(6) α β

(7) α δ

(8) α γ

(9) α β

(10) α β