ΕΝΟΤΗΤΑ 17

2. (B) $6x^3 - x^2 - 14x + 5 \;\big|\; 2x - 1$

$-(6x^3 - 3x^2)$ $\qquad = 3x^2 + x - \frac{13}{2}$

$2x^2 - 14x + 5$

$-(2x^2 - x)$

$-13x + 5$

$-(-13x + \frac{13}{2})$

$5 - \frac{13}{2}$

$-\frac{3}{2}$

(Δ) $(x-2)^3 = x^3 - 3x^2 2 + 3x \cdot 2^2 - 2^3$

$= x^3 - 6x^2 + 12x - 8$

$x^4 \;\big|\; x^3 - 6x^2 + 12x - 8$

$-(x^4 - 6x^3 + 12x^2 - 8x)$ $\qquad = x + 6$

$6x^3 - 12x^2 + 8x$

$-(6x^3 - 36x^2 + 72x - 48)$

$24x^2 - 64x + 48.$

43. $P(0) = 0$

$P(2) = -1$

$P(-2) = 3$.

$$P(x) = (x^3 - 4x)\, n(x) + v(x)$$

$$\boxed{P(x) = (x^3 - 4x)\, n(x) + ax^2 + \beta x + \gamma}$$

$P(0) = \underline{\gamma = 0}$

$P(2) = 4a + 2\beta = -1$

$P(-2) = 4a - 2\beta = 3$

$\oplus \quad 8a = 2$

$a = \frac{1}{4}$

$4 \cdot \frac{1}{4} + 2\beta = -1$

$2\beta = -2 \quad \beta = -1.$

5. $P(x) = (x^2-5x)(5x-1) - 3x + 2$

$\Delta(x)$ $\quad$ $\delta(x)$ $\quad$ $\Pi(x)$ $\quad$ $U(x)$

① $\Pi(x) = 5x-1$ $\quad$ $U(x) = -3x+2$.

②

$P(x) = (5x-1)(x^2-5x) - 3x + 2$

$\Delta(x) = \delta(x) \quad \Pi(x) \quad U(x)$

OX /

40. $P(x) = (x^2-1)\Pi(x) + 2x + 1$

$P(0) = 0$

① $P(1) = 3$

② $P(x) = (x^2-x)\Pi(x) + U(x)$

$P(x) = (x^2-x)\Pi(x) + \alpha x + \beta$

$P(0) = \beta = 0$

$P(1) = \alpha + \beta = 3$

$\alpha = 3.$

$U(x) = 3x + 0.$

42. $P(x) = (x-2)\,Q(x) + 10$

$P(2) = 10$

$P(-3) = 5$

$P(x) = (x^2+x-6)\,Q(x) + U(x)$

$P(x) = (x^2+x-6)\,Q(x) + ax+\beta$

$P(2) = 2\alpha + \beta = 10 \Rightarrow -2\alpha - \beta = -10$

(+)

$P(-3) = -3a + \beta = 5$

$-5a = 15$

$a = -3$

$\beta = -4$

45. $P(x)$ 3ου βαθμού $\quad P(0)=3$

$P(1)=4$

$P(x)=(x^2-2x+3)\Pi(x)$

↓ 3ου ↓ 2ου ↓ 1ου

$P(x)=(x^2-2x+3)(\alpha x+\beta)$

$P(0)=3(0+\beta)=3$

$3\beta=3$

$\beta=1$

$P(1)=2(\alpha+\beta)=4$

$2(\alpha+1)=4$

$2\alpha+2=4$

$2\alpha=2$

$\alpha=1$

$P(x)=(x^2-2x+3)(x+1)$

48. $P(x) = x^3 + \alpha x^2 + \beta x - 1$

Το x^2+1

$$\begin{array}{r|l} x^3 + \alpha x^2 + \beta x - 1 & x^2+1 \\ \hline -(x^3 + x) & x + \alpha \\ \hline \alpha x^2 + (\beta-1)x - 1 & \\ -(\alpha x^2 + \alpha) & \\ \hline (\beta-1)x - 1 - \alpha \longrightarrow 0 & \end{array}$$

$\beta - 1 = 0 \Rightarrow \beta = 1$

$-1 - \alpha = 0 \qquad \alpha = -1$

51. $P(x) = \alpha x^3 - 5x^2 + \beta x + 9$

Έχει παράγοντα $(x-3)^2$

α	-5	β	9	(3)
↓	3α	$9\alpha - 15$	$27\alpha + 3\beta - 45$	
α	$3\alpha - 5$	$9\alpha + \beta - 15$	$27\alpha + 3\beta - 36$	

απαιτώ 0

α	$3\alpha - 5$	$9\alpha + \beta - 15$	(3)
↓	3α	$18\alpha - 15$	
α	$6\alpha - 5$	$27\alpha + \beta - 30$	

απαιτώ 0

$$\begin{cases} 27\alpha + 3\beta - 36 = 0 \\ 27\alpha + \beta - 30 = 0 \end{cases}$$

$-27\alpha - 3\beta + 36 = 0$

(+)

$-2\beta + 6 = 0$

$\beta = 3$

$\alpha = 1$

50. $P(x) = \alpha x^3 - (\alpha+\beta)x + \beta - 4$

Το $x^2 - 2x - 3$ παραγοντικά

$\downarrow$

$(x-3)(x+1)$.

$\downarrow \quad \downarrow$

$P(3) = 0 \qquad P(-1) = 0$

$P(3) = 27\alpha - 3(\alpha+\beta) + \beta - 4 = 0$

$27\alpha - 3\alpha - 3\beta + \beta - 4 = 0$

$24\alpha - 2\beta = 4$

$12\alpha - \beta = 2$

$P(-1) = -\alpha + \alpha + \beta + \beta - 4 = 0$

$2\beta = 4$

$\beta = 2$

$\alpha = \frac{1}{3}$.

53 (α) $P(x) = (\lambda^2+\lambda)x^4 + (\lambda+1)x^3 + 3x^2 + \mu - 1$

Έστω διπλή ρίζα του $x+1$

- $\lambda^2+\lambda = 0$

$\lambda(\lambda+1) = 0$

$\lambda = 0$ ή $\lambda = -1$

Αν $\lambda = 0$ $\quad P(x) = x^3 + 3x^2 + \mu - 1$ 3ου ✓

~~Αν $\lambda = -1$~~ $\quad P(x) = 3x^2 + \mu - 2$ 2ου

$P(-1) = 0$

$P(-1) = -1 + 3 + \mu - 1 = 0$

$\mu = -1$

$P(x) = x^3 + 3x^2 - 2$

(B) $P(x) : (x^2+3)$

$$
\begin{array}{r|l}
x^3+3x^2-2 & x^2+3 \\
\hline
-(x^3+3x) & x+3 \\
\hline
3x^2-3x-2 & \\
-(3x^2+9) & \\
\hline
-3x-11 &
\end{array}
$$

$$P(x) = (x^2+3)(x+3) - 3x - 11$$

(8) $P(x) : (x-1)$

1	3	0	-2	(1)
↓	1	4	4	
1	4	4	2	

$\Pi(x) = x^2 + 4x + 4$

$v = 2$.

(9) $Q(x) = P(x) + (x-2)^{10}$

$Q(1) = P(1) + 1 = 2 + 1 = 3$

$Q(0) = P(0) - 2^{10} = -2 - 2^{10}$

Εποχή Μαθημ

17

(46)

(49)

(54)

Ξεκινάω το μαθημ με πολιτικούς εξισώσεις - Ανισώσεις.