Ενότητα 27

8. $f(x) = x^3+2x+1$

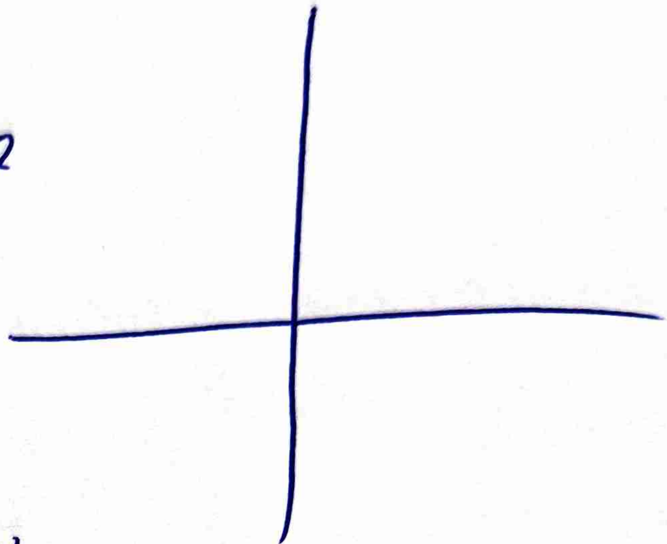
(α) $f'(x) = 3x^2+2>0$

$f \nearrow$

$\lim\limits_{x\to-\infty} f(x) = -\infty$

$\lim\limits_{x\to+\infty} f(x) = +\infty$

$\Big\}\ ΣΤ_f = \mathbb{R}$



. f συνεχής

. $f \nearrow$

$ΣΤ_f = \mathbb{R}$

$\Big\}$ Το $0 \in ΣΤ_f$ άρα $\exists\, \xi$ τ.ω $f(\xi)=0$.

Είναι $\xi<0$ γιατί $f(\xi)<f(0) \Rightarrow 0<1$ ✓

(β) $f(x) = f(-1)$

f 1-1

$\cancel{x=-1}$ $\quad (0,+\infty)$

Αδύνατη.

11. (a) $e^x = 3x$

$$1 = \frac{3x}{e^x}$$

$$\underbrace{\frac{3x}{e^x} - 1}_{f(x)} = 0$$

$$f'(x) = \frac{3e^x - 3xe^x}{e^{2x}} = 3\,\frac{1-x}{e^x}$$

x		1	
f'	$+$	0	$-$
f	$-\infty \nearrow$	θ	$\searrow -1$

$$\lim_{x\to-\infty} f(x) = \lim_{x\to-\infty}\left(3x\,\frac{1}{e^x} - 1\right) = -\infty(+\infty) = -\infty$$

$$f(1) = \frac{3}{e} - 1 > 0$$

$$\lim_{x\to+\infty} f(x) = \lim_{x\to+\infty} \frac{3x}{e^x} - 1 = -1$$

$$\to \lim_{x\to+\infty} \frac{3x}{e^x} = \lim_{x\to+\infty} \frac{3}{e^x} = 0$$

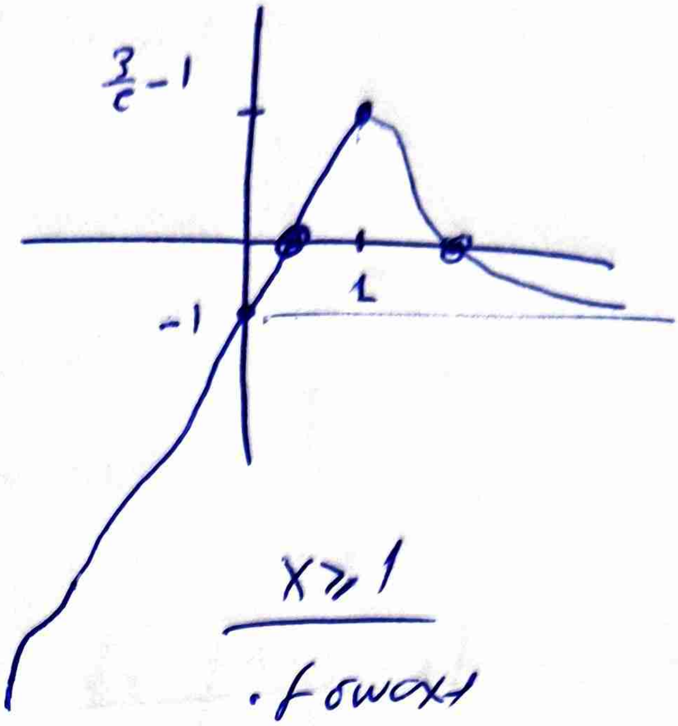
$x<1$

- f συνεχής
- $f \uparrow$
- $\Sigma T f = \left(-\infty, \frac{3}{e} - 1\right]$

Το $0 \in \Sigma T f$

άρα $\exists! \, \xi_1$ τ.ω

$f(\xi_1) = 0$



$x \geq 1$

- f συνεχής
- $f \downarrow$
- $\Sigma T f = \left(-1, \frac{3}{e} - 1\right]$

Το $0 \in \Sigma T f$ άρα

$\exists! \, \xi_2$ τ.ω $f(\xi_2) = 0$.

(B) $x^2 = \cos x$ $\qquad (-\pi, \pi)$

$$\underbrace{x^2 - \cos x}_{f(x)} = 0$$

$f'(x) = 2x + \sin x \qquad f'(0) = 0$

$f''(x) = 2 + \cos x > 0$

x	$-\pi$		0		π
f''		$+$		$+$	
f'		$\nearrow$ $-$	0	$+$ $\nearrow$	
f	π^2+1	$\searrow$	-1	$\nearrow$	π^2+1

$-1 \le \cos x \le 1$

$1 \le 2 + \cos x \le 3$

$$\lim_{x \to -\pi^+} f(x) = (-\pi)^2 - \cos(-\pi) = \pi^2 + 1$$

$f(0) = -1$

$$\lim_{x \to \pi^-} f(x) = \pi^2 - \cos \pi = \pi^2 + 1$$

$\underline{x < 0}$

f συνεχής

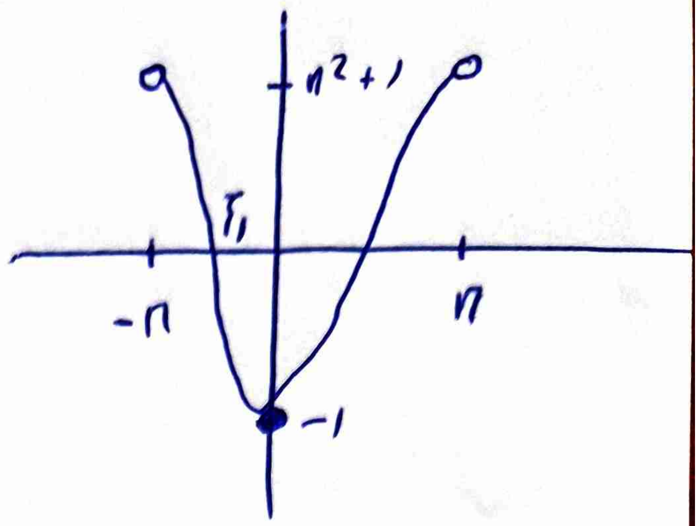
$f \downarrow$

$\Sigma T_f = [-1, n^2+1)$

Το $0 \in \Sigma T_f$ άρα

$\exists! \xi_1$ τ.ω

$f(\xi_1) = 0$



$\underline{x \geq 0}$

f συνεχής

$f \uparrow$ $\quad \Sigma T_f = [-1, n^2+1)$

Το $0 \in \Sigma T_f$.

Άρα $\exists! \xi_2$

τ.ω $f(\xi_2) = 0$

1) $x^2 = 2\ln x + 2$

$$\underbrace{x^2 - 2\ln x - 2}_{f(x)} = 0 \quad , x > 0$$

$$f'(x) = 2x - \frac{2}{x} = \frac{2x^2 - 2}{x} = 2\,\frac{x^2 - 1}{x}$$

x	0		1		
f'		$-$		$+$	
f	$+\infty$	$\searrow$	-1	$\nearrow$	$+\infty$

$$\lim_{x \to 0^+} f(x) = \lim_{x \to 0^+} \left(x^2 - 2\ln x + 2\right) = +\infty$$

$$f(1) = -1$$

$$\lim_{x \to +\infty} f(x) = \lim_{x \to +\infty} \left(x^2 - 2\ln x - 2\right)$$

$$= \lim_{x \to +\infty} x^2 \left(1 - 2\cancel{\frac{\ln x}{x^2}} - \cancel{\frac{2}{x^2}}\right) = +\infty$$

$$\rightarrow \lim_{x \to +\infty} \frac{\ln x}{x^2} = \lim_{x \to +\infty} \frac{\frac{1}{x}}{2x} = 0$$

$x<1$

f συνεχής

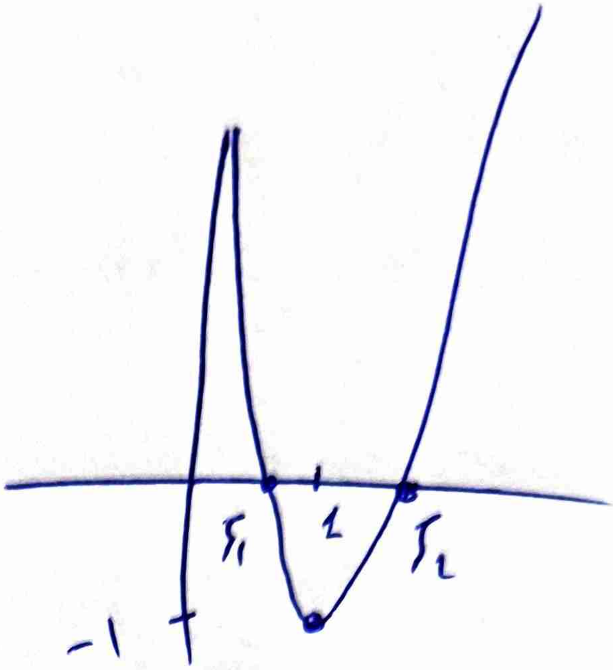
$f \downarrow$

$f(\Delta_1) = [-1, +\infty)$

Το $0 \in f(\Delta_1)$

άρα $\exists! \xi_1$ τ.ω

$f(\xi_1) = 0$



$x>1$

f συνεχής

$f \uparrow$

$f(\Delta_2) = [-1, +\infty)$

Το $0 \in f(\Delta_2)$

άρα $\exists! \xi_2$

τ.ω $f(\xi_2) = 0$.

12. (a) $f(x)=e^x$ και $g(x)=\frac{1}{x}$

$$f(x)=g(x)$$

$$e^x=\frac{1}{x}$$

$$e^x-\frac{1}{x}=0 \quad , x\neq 0$$

$$h(x)=0 \quad , x\neq 0$$

$$h'(x)=e^x+\frac{1}{x^2}>0 \qquad h \nearrow$$

x	$-\infty$		0		$+\infty$
$h(x)$	0	$\nearrow$	$+\infty$ ‖ $-\infty$	$\nearrow$	$+\infty$

$$\lim_{x\to -\infty} h(x)=\lim_{x\to -\infty} e^x-\frac{1}{x}=0$$

$$\lim_{x\to 0^-} h(x)=+\infty$$

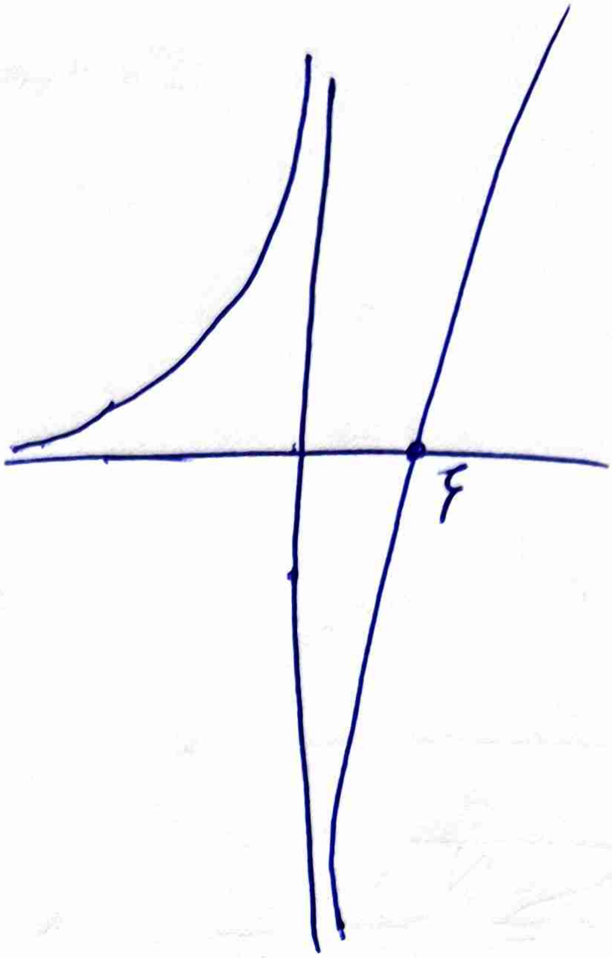
$$\lim_{x\to 0^+} h(x)=-\infty$$

$$\lim_{x\to +\infty} h(x)=\lim_{x\to +\infty} e^x-\frac{1}{x}=+\infty$$

$x<0$

$\Sigma T_f = (0, +\infty)$

Δεν έχει ρίζα
εδώ



$x>0$

• f συνεχής

• $f \nearrow$

• $\Sigma T_f = \mathbb{R}$

το $0 \in \Sigma T_f$

άρα $\exists!\ \xi$ τ.ω. $h(\xi) = 0$.

(B) $f(x) = e^{x-1}$

$g(x) = x\ln x + 1$

$\underline{\underline{x > 0}}$

$f(x) = g(x)$

$e^{x-1} = x\ln x + 1$

$$\underbrace{e^{x-1} - x\ln x - 1}_{h(x)} = 0$$

$h'(x) = e^{x-1} - \ln x - 1$

- $e^x \geqslant x + 1$

$e^{x-1} \geqslant x - 1 + 1$

$e^{x-1} \geqslant x$

- $\ln x \leq x - 1$

$-\ln x \geqslant 1 - x$

$\oplus$

$e^{x-1} - \ln x \geqslant x + 1 - x$

$e^{x-1} - \ln x - 1 \geqslant 0$

$h'(x) \geqslant 0$ $\quad h \nearrow$

$$\lim_{x\to 0^+} h(x) = \lim_{x\to 0^+} e^{x-1} - \cancelto{0}{x\ln x} - 1 = e^{-1}-1$$

$$\to \lim_{x\to 0^+} x\ln x = \lim_{x\to 0^+} \frac{\ln x}{\frac{1}{x}} = \lim_{x\to 0^+} \frac{\frac{1}{x}}{-\frac{1}{x^2}} = 0$$

$$\lim_{x\to+\infty} h(x) = \lim_{x\to+\infty} e^{x-1} - x\ln x - 1 =$$

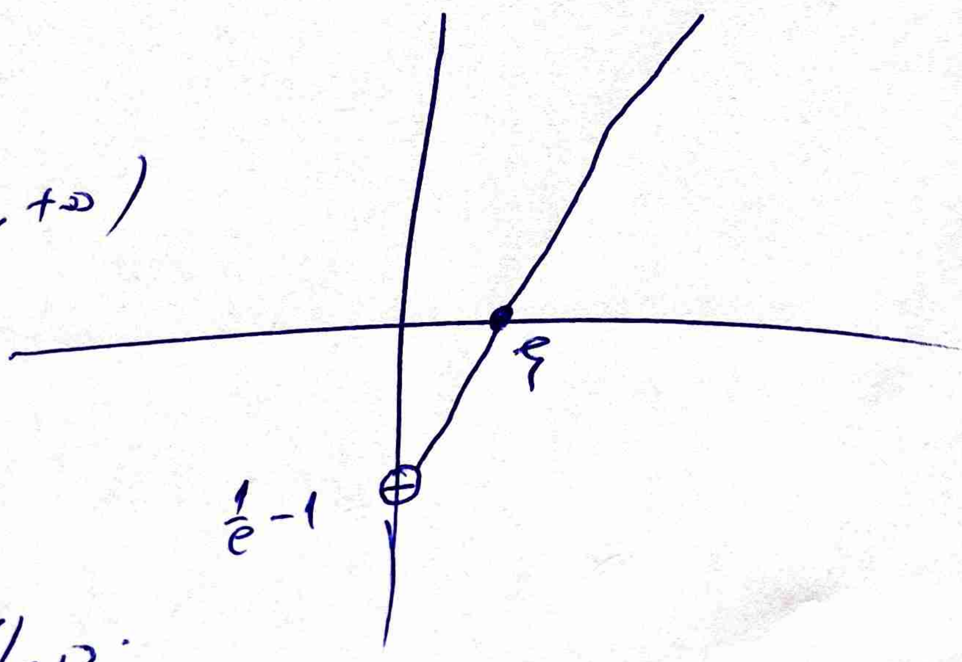
$$= \lim_{x\to+\infty} e^{x-1}\left(1 - \cancelto{0}{\frac{x\ln x}{e^{x-1}}} - \cancel{\frac{1}{e^{x-1}}}\right) = +\infty$$

$$\to \lim_{x\to+\infty} \frac{x\ln x}{e^{x-1}} = \lim_{x\to+\infty} \frac{\ln x + 1}{e^{x-1}} = \lim_{x\to+\infty} \frac{1}{xe^{x-1}} = 0.$$

- f συνεχ
- $\Sigma T_f = \left(\frac{1}{e}-1, +\infty\right)$
- $\nearrow f$

το $0 \in \Sigma T_f$

άρα $\exists\, \xi$ τω $h(\xi)=0$.



24. $f(x) = x^5 + 10x^2 + x - 1.$

$f'(x) = 5x^4 + 20x + 1$

$f''(x) = 20x^3 + 20 = 20(x^3+1)$

$\longrightarrow x^3+1=0 \Rightarrow x=-1.$

x		-1	
f''	$-$		$+$
f'	$+\infty \searrow$	-14	$\nearrow +\infty$
f			

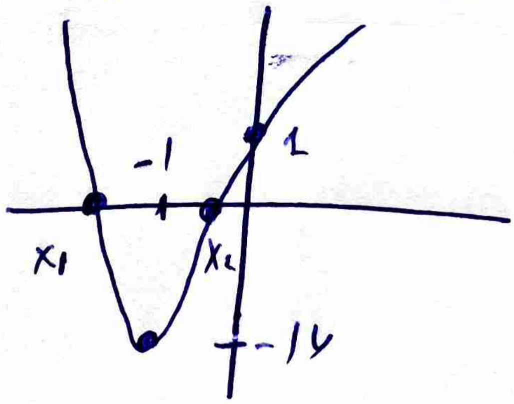
$f'(x) \geq f'(-1)$

$f'(x) \geq -14$

$$\lim_{x\to-\infty} f'(x) = +\infty$$

$$\lim_{x\to+\infty} f'(x) = +\infty$$

$C_{f'}$



$\underline{x<-1}$

- f' συνεχής
- $f' \downarrow$
- $ΣΤ_{f'} = [-14, +\infty)$

Το $0 \in ΣΤ_{f'}$ άρα $\exists!\ x_1$ τ.ω $f(x_1)=0$

$\underline{x>-1}$

- f' συνεχής
- $f' \uparrow$
- $ΣΤ_{f'} = [-14, +\infty)$

Το $0 \in ΣΤ_{f'}$ άρα $\exists!\ x_2$ τ.ω $f(x_2)$

x		x_1		-1		x_2	
f''	$-$		$-$	0	$+$		$+$
f'	$\searrow$ $+$	0	$-$ $\searrow$		$-$ $\nearrow$	0	$+$ $\nearrow$
f	$\nearrow$		$\searrow$		$\searrow$		$\nearrow$

23. Νδο η $f(x) = e^x + x^2 - 2x + 3$ έχει μοναδικό ακρότατο.

$f'(x) = e^x + 2x - 2$

$f''(x) = e^x + 2 > 0$

x	$-\infty$ $\quad$ $+\infty$
f''	$+$
f'	$\nearrow$
f	

(1η απομίμηση)

$\Sigma T_{f'}$

$\left.\begin{array}{l} \lim\limits_{x \to -\infty} f'(x) = -\infty \\ \lim\limits_{x \to +\infty} f'(x) = +\infty \end{array}\right\} \Sigma T_{f'} = \mathbb{R}.$

- f' συνεχής
- $f' \nearrow$
- $\Sigma T_{f'} = \mathbb{R}$

Το $0 \in \Sigma T_{f'}$ άρα $\exists! \, x_0$ τ.ω $f'(x_0) = 0$.

x		x_0	
f''	$+$		$+$
f'	$\nearrow -$	0	$+ \nearrow$
f	$\searrow$		$\nearrow$

extra

Νδο $f(x_0) = x_0^2 - 4x_0 + 5$.

Γνωρίζω ότι $f(x_0) = e^{x_0} + x_0^2 - 2x_0 + 3$

$f(x_0) = 2 - 2x_0 + x_0^2 - 2x_0 + 3$

όμως $f'(x_0) = 0$

$e^{x_0} + 2x_0 - 2 = 0$

$\boxed{f(x_0) = x_0^2 - 4x_0 + 5}$

$e^{x_0} = 2 - 2x_0$

2. (B) $f(x) = \dfrac{1}{x^2+1}, \ x \in \mathbb{R}$

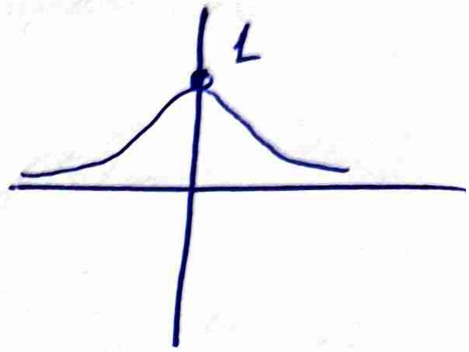
Ενοτητα 32

Μονοτονια - ακροτατα - Συνολο Τιμων

$$f'(x) = \frac{-2x}{(x^2+1)^2}$$

x	$-\infty$	0	$+\infty$
f'	$+$		$-$
f	$0 \nearrow$	1	$\searrow 0$

$$\lim_{x\to-\infty} f(x) = \lim_{x\to-\infty} \frac{1}{x^2+1} = 0$$



$$f(0) = 1$$

$$\lim_{x\to+\infty} f(x) = \lim_{x\to+\infty} \frac{1}{x^2+1} = 0$$

$$\Sigma T_f = (0, 1].$$

Κυρτοτητα - Σημεια καμπης

$$f''(x) = \frac{-2(x^2+1)^2 - (-2x)\,2(x^2+1)\,2x}{(x^2+1)^4}$$

$$f''(x) = \frac{-2(x^2+1) + 8x^2}{(x^2+1)^3} = \frac{6x^2-2}{(x^2+1)^3}$$

$\rightarrow 6x^2 - 2 = 0 \rightarrow x^2 = \frac{1}{3}$ $\quad x = \pm \frac{\sqrt{3}}{3}$

x	$-\frac{\sqrt{3}}{3}$	$\frac{\sqrt{3}}{3}$	
f''	$+$	$-$	$+$
f	$\cup$	$\cap$	$\cup$

$$f\left(\frac{\sqrt{3}}{3}\right)=\frac{1}{\left(\frac{\sqrt{3}}{3}\right)^2+1}=\frac{1}{\frac{3}{9}+1}$$

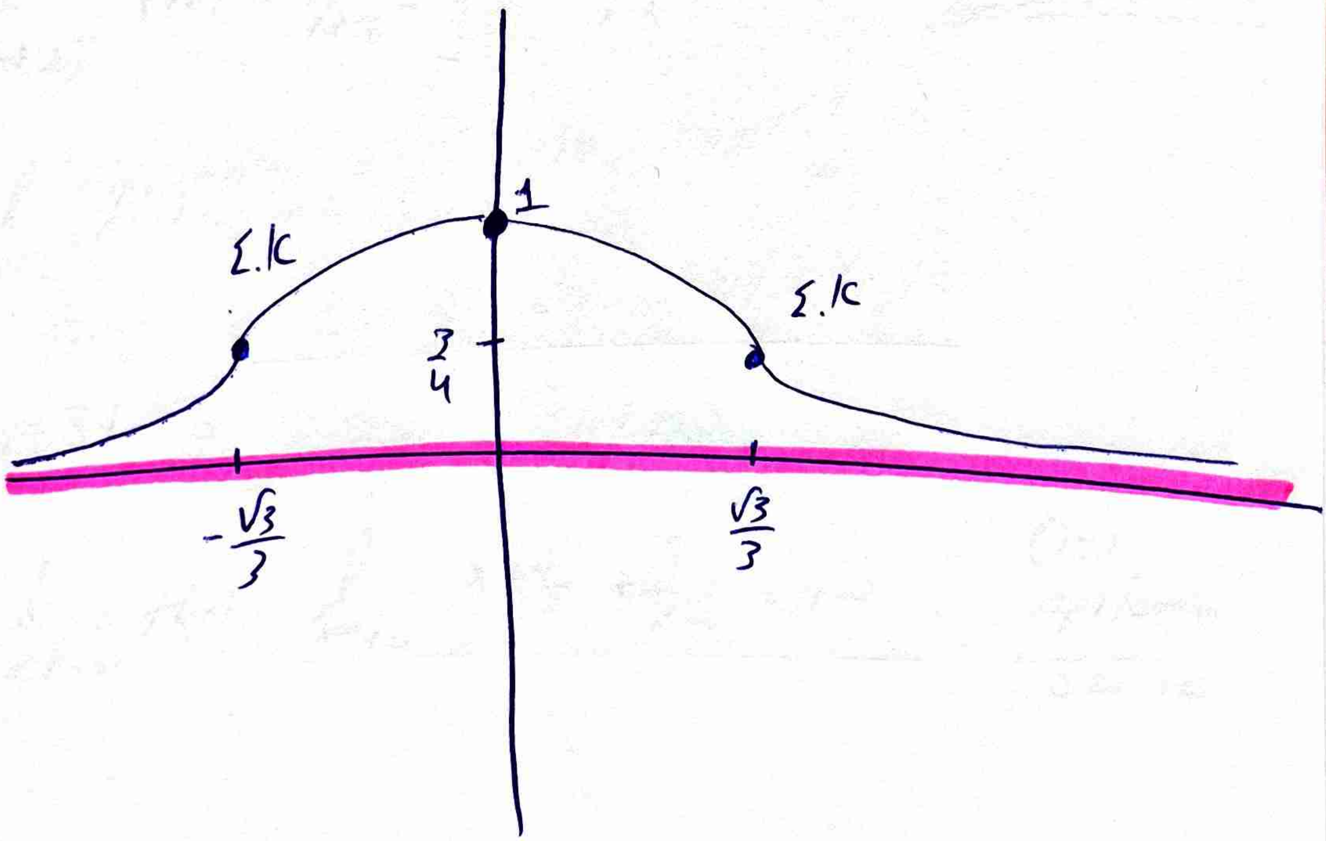
$$=\frac{9}{12}=\frac{3}{4}$$

Ασυμπτωτη

Αφου $D_f=\mathbb{R}$ δεν εχει κατακορυφη.

Αφου $\lim\limits_{x\to-\infty} f(x)=0$ και $\lim\limits_{x\to+\infty} f(x)=0$.

$\varepsilon : y=0$ φιλοντας $\pm\infty$



2. (8) $f(x) = x+2+\dfrac{1}{x-2}, \quad x \neq 2$

$$f'(x) = 1 + \frac{-1}{(x-2)^2} = \frac{(x-2)^2-1}{(x-2)^2} = \frac{x^2-4x+3}{(x-2)^2}$$

x		1		2		3	
f'	$+$		$-$	‖	$-$		$+$
f	$-\infty \nearrow$	2	$\searrow -\infty$	‖	$+\infty \searrow$	6	$\nearrow +\infty$

$$\lim_{x\to-\infty} f(x) = \lim_{x\to-\infty} x+2+\frac{1}{x-2} = -\infty$$ Όχι οριζόντια στο $-\infty$

$f(1) = 2$

κατακόρυφη

$$\lim_{x\to2^-} f(x) = \lim_{x\to2^-} x+2+\frac{1}{x-2} = -\infty$$ $\varepsilon: x=2$

$$\lim_{x\to2^+} f(x) = +\infty$$

$f(3) = 6$

$$\lim_{x\to+\infty} f(x) = \lim_{x\to+\infty} x+2+\frac{1}{x-2} = +\infty$$ Όχι οριζόντια στο $+\infty$

$$f''(x) = -\frac{-2(x-2)}{(x-2)^4}$$

$$f'(x) = 1 - \frac{1}{(x-2)^2}$$

$$f''(x) = 2\,\frac{1}{(x-2)^3}$$

x		2	
f''	$-$		$+$
f	$\cap$		$\cup$

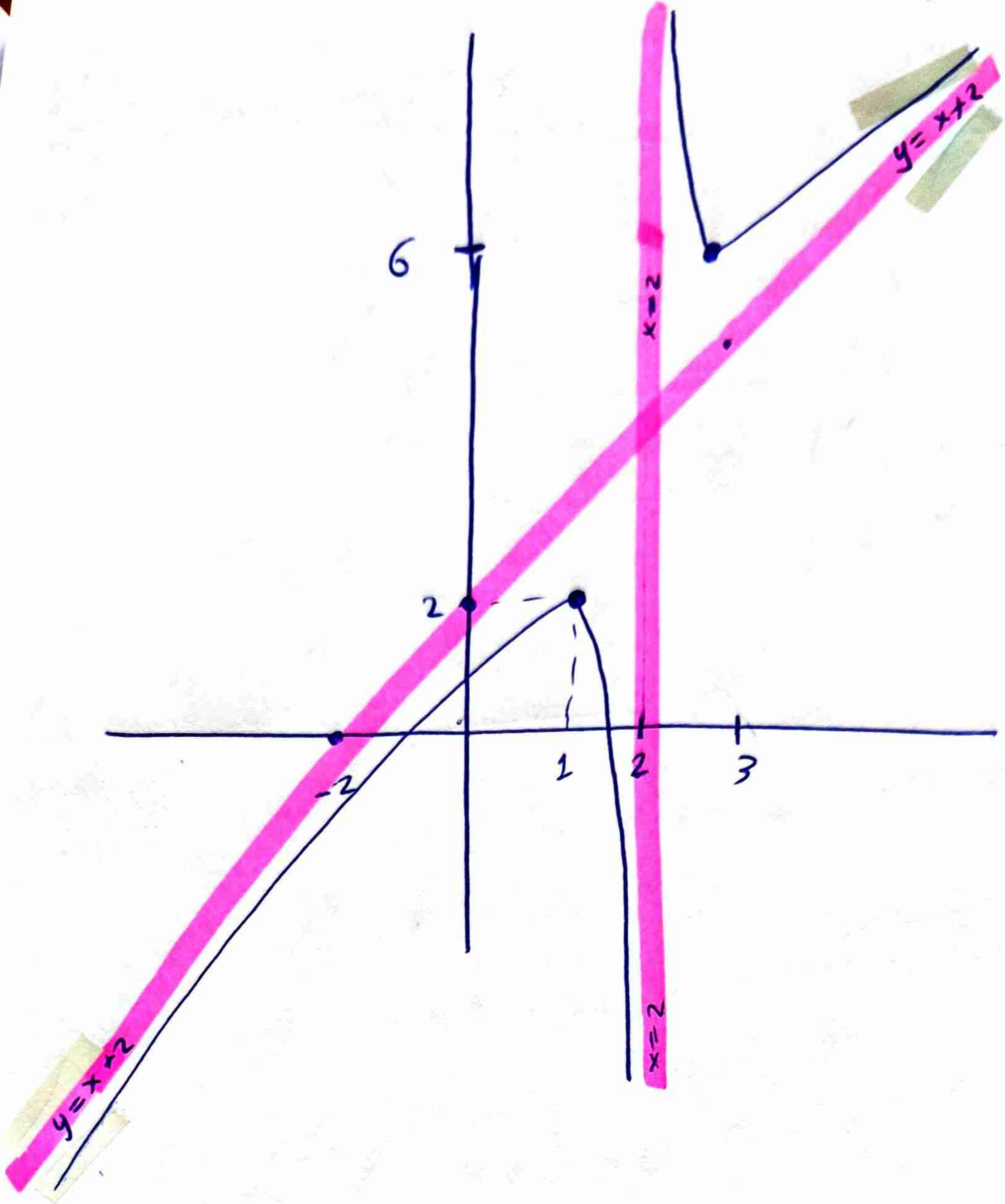
$$\lim_{x\to+\infty} \frac{f(x)}{x} = \lim_{x\to+\infty} \frac{x+2+\frac{1}{x-2}}{x} = \lim_{x\to+\infty} \frac{(x-2)(x+2)+1}{x(x-2)}$$

$$= \lim_{x\to+\infty} \frac{x^2}{x^2} = 1$$

$$\lim_{x\to+\infty} f(x) - x = \lim_{x\to+\infty} x+2+\frac{1}{x-2} - x = 2$$

$$\varepsilon : y = x + 2$$

πλάγια $\begin{matrix} +\infty \\ -\infty \end{matrix}$



6
2
-2
1
2
3
x=2
x=2
y=x+2
y=x+2

3. (B) $f(x) = x e^{\frac{1}{x}} \quad , x \neq 0$

$f'(x) = e^{\frac{1}{x}} + x\, e^{\frac{1}{x}} \left(-\frac{1}{x^2}\right)$

$f'(x) = e^{\frac{1}{x}}\left(1 - \frac{1}{x}\right) = e^{\frac{1}{x}} \cdot \frac{x-1}{x}$

x		0		1	
$x-1$	$-$	||	$-$		$+$
x	$-$	||	$+$		$+$
f'	$+$	||	$-$		$+$
f	$-\infty \nearrow 0$	||	$+\infty \searrow e$		$\nearrow +\infty$

$\to f'(x) = 0 \Rightarrow \underline{\underline{x = 1}}$

$\lim\limits_{x \to -\infty} f(x) = \lim\limits_{x \to -\infty} \left(x e^{\frac{1}{x}}\right) = -\infty \cdot 1 = -\infty$ όχι οριζόντια.

$\lim\limits_{x \to +\infty} f(x) = \lim\limits_{x \to +\infty} x e^{\frac{1}{x}} = +\infty$ όχι οριζόντια

$\lim\limits_{x \to 0^-} f(x) = \lim\limits_{x \to 0^-} x e^{\frac{1}{x}} = 0$

$\lim\limits_{x \to 0^+} f(x) = \lim\limits_{x \to 0^+} x e^{\frac{1}{x}} = \lim\limits_{x \to 0^+} \frac{e^{\frac{1}{x}}}{\frac{1}{x}} =$

$= \lim\limits_{x \to 0^+} \frac{e^{\frac{1}{x}} \cancel{\left(-\frac{1}{x^2}\right)}}{\cancel{-\frac{1}{x^2}}} = +\infty$

$f(1) = e$

Ε: $x = 0$

κατακόρυφη

$$\lim_{x\to+\infty} \frac{f(x)}{x} = \lim_{x\to+\infty} \frac{\cancel{x} e^{\frac{1}{x}}}{\cancel{x}} = 1$$

$$\lim_{x\to+\infty} f(x)-x = \lim_{x\to+\infty} x e^{\frac{1}{x}} - x =$$

$$= \lim_{x\to+\infty} x\left(e^{\frac{1}{x}} - 1\right) \overset{\frac{1}{x}=t}{\underset{\substack{x=\frac{1}{t} \\ x\to+\infty \\ t\to 0}}{=\!=\!=}}$$

$$= \lim_{t\to 0} \frac{e^t - 1}{t} = \lim_{t\to 0} \frac{e^t}{1} = 1$$

$y = x+1$ πλάγια σε $+\infty$, $-\infty$.

$$\lim_{x\to-\infty} \frac{f(x)}{x} = \lim_{x\to-\infty} \frac{\cancel{x} e^{\frac{1}{x}}}{\cancel{x}} = 1$$

$$\lim_{x\to-\infty} f(x)-x = \lim_{x\to-\infty} x\left(e^{\frac{1}{x}} - 1\right) = 1.$$

$$f'(x) = \frac{x-1}{x} \cdot e^{\frac{1}{x}}$$

$$f''(x) = \frac{(x-1)e^{\frac{1}{x}}}{x}$$

$$f''(x) = \frac{\left[e^{\frac{1}{x}} + (x-1)\, e^{\frac{1}{x}}\left(-\frac{1}{x^2}\right)\right] x - (x-1)\, e^{\frac{1}{x}}}{x^2}$$

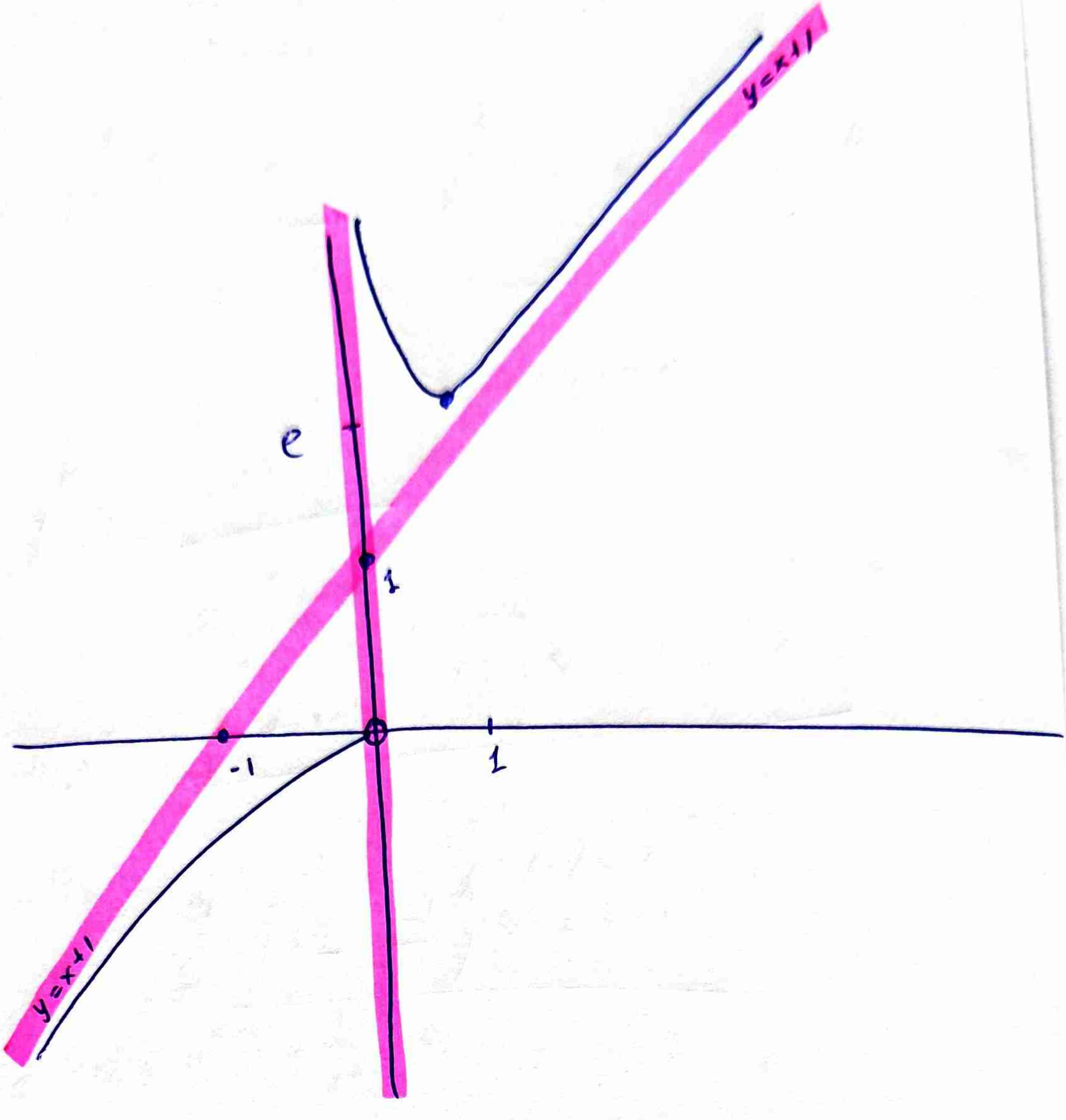
$$f''(x) = e^{\frac{1}{x}} \frac{\left(1 - \frac{x-1}{x^2}\right) x - x + 1}{x^2}$$

$$f''(x) = e^{\frac{1}{x}} \frac{x - \frac{x-1}{x} - x + 1}{x^2}$$

$$f''(x) = e^{\frac{1}{x}} \frac{\cancel{x^2} - \cancel{x} + 1 - \cancel{x^2} + \cancel{x}}{x^3}$$

$$f''(x) = e^{\frac{1}{x}} \frac{1}{x^3}$$

x		0	
f''	$-$		$+$
f	$\curvearrowright$		$\curvearrowleft$


y=x+1
e
1
-1
1
y=x+1

3. 8) $f(x) = e^{x-1} - x\ln x \ , \ x > 0$

$f'(x) = e^{x-1} - \ln x - 1 \geq 0 \qquad f \nearrow$

- $e^x \geq x+1$ $\qquad$ $\ln x \leq x-1$

$e^{x-1} \geq x-1+1$ $\qquad$ $-\ln x \geq 1-x$

$e^{x-1} \geq x$ $\qquad$ (+)

$e^{x-1} - \ln x \geq 1$

$e^{x-1} - \ln x - 1 \geq 0$

$f''(x) = e^{x-1} - \dfrac{1}{x} \qquad f''(1) = 0$

$f'''(x) = e^{x-1} + \dfrac{1}{x^2} > 0$

x	0		1	
f'''		+		+
f''		↗ −	0	+ ↗
f				

$$\lim_{x\to 0^+} f(x) = \lim_{x\to 0^+} \left(e^{x-1} - x\ln x\right) = \frac{1}{e}$$

$$\to \lim_{x\to 0^+} x\ln x = \lim_{x\to 0^+} \frac{\ln x}{\frac{1}{x}} = \lim_{x\to 0^+} \frac{\frac{1}{x}}{-\frac{1}{x^2}} = 0$$

$$\lim_{x\to +\infty} f(x) = \lim_{x\to +\infty} e^{x-1} - x\ln x =$$

$$= \lim_{x\to +\infty} e^{x-1}\left(1 - \frac{x\ln x}{e^{x-1}}\right) = +\infty.$$

$$\to \lim_{x\to +\infty} \frac{x\ln x}{e^{x-1}} = \lim_{x\to +\infty} \frac{\ln x + \cancel{x}\frac{1}{\cancel{x}}}{e^{x-1}} =$$

$$= \lim_{x\to +\infty} \frac{\ln x + 1}{e^{x-1}} = \lim_{x\to +\infty} \frac{1}{x\, e^{x-1}} = 0$$

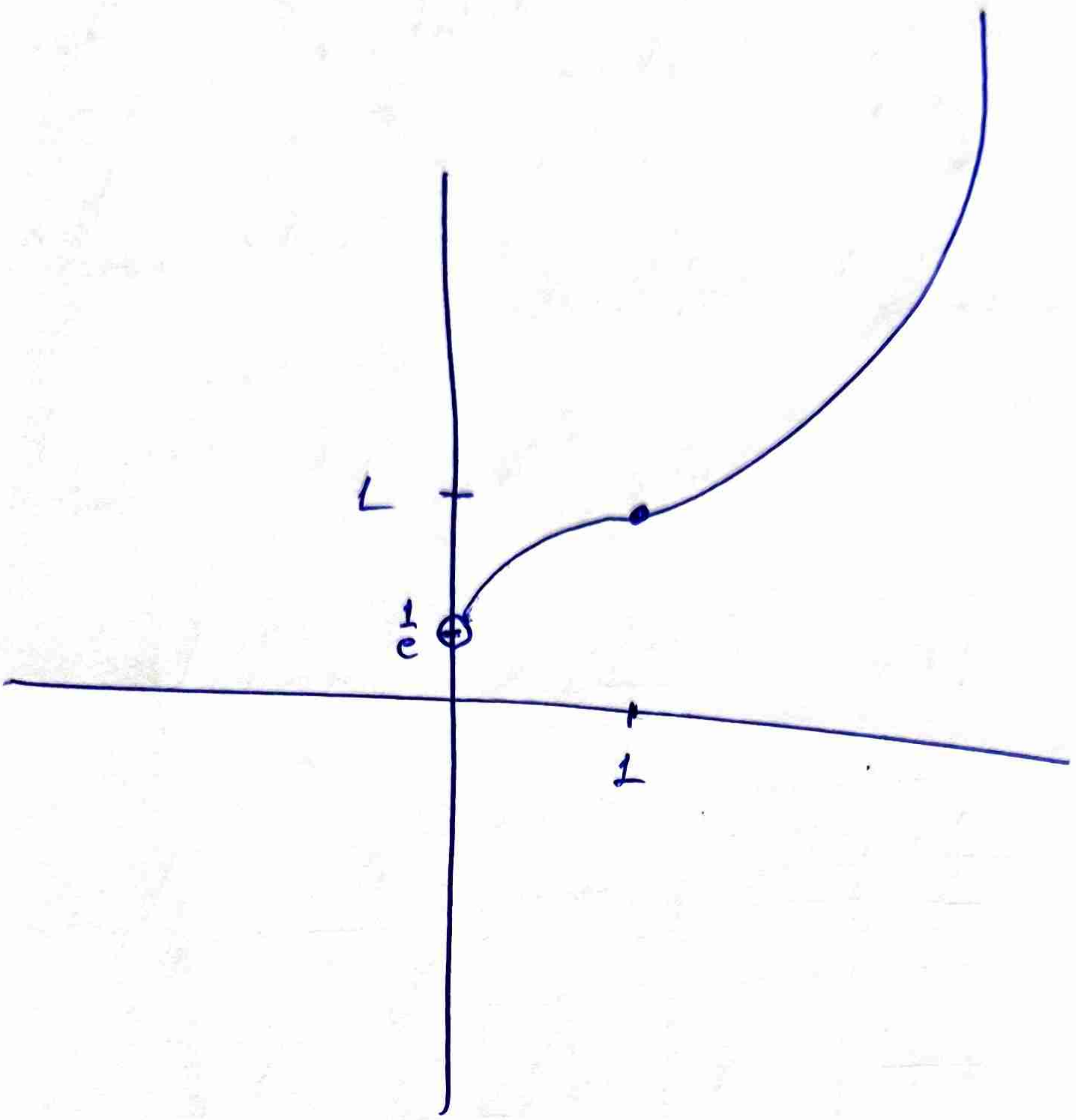
$$\lim_{x\to +\infty} \frac{f(x)}{x} = \lim_{x\to +\infty} \frac{e^{x-1} - x\ln x}{x} =$$

$$= \lim_{x\to +\infty} \frac{e^{x-1} - \ln x - 1}{1} = \lim_{x\to +\infty} e^{x-1}\left(1 - \frac{\ln x}{e^{x-1}} - 1\right.$$

$$= +\infty$$

Δεν έχει πλάγιες

$\ln(x^2+1)$, $D_f = R$



4. (B) $f(x) = x + \ln(x^2+1)$, $D_f = \mathbb{R}$

$$f'(x) = 1 + \frac{2x}{x^2+1} = \frac{x^2+1+2x}{x^2+1} = \frac{(x+1)^2}{x^2+1} \geq 0$$

$f \nearrow$

$$f''(x) = \frac{2(x^2+1) - 2x \cdot 2x}{(x^2+1)^2} = \frac{2x^2+2-4x^2}{(x^2+1)^2}$$

$$f''(x) = \frac{2-2x^2}{(x^2+1)^2} = 2\,\frac{1-x^2}{(x^2+1)^2}$$

x		-1		1	
f''	$-$		$+$		$-$
f	$\frown$		$\smile$		$\frown$.

$$\lim_{x\to-\infty} f(x) = \lim_{x\to-\infty} x\left(1 + \frac{\ln(x^2+1)}{x}\right) = -\infty ,$$

$$\rightarrow \lim_{x\to-\infty} \frac{\ln(x^2+1)}{x} = \lim_{x\to-\infty} \frac{\frac{2x}{x^2+1}}{1} = 0$$

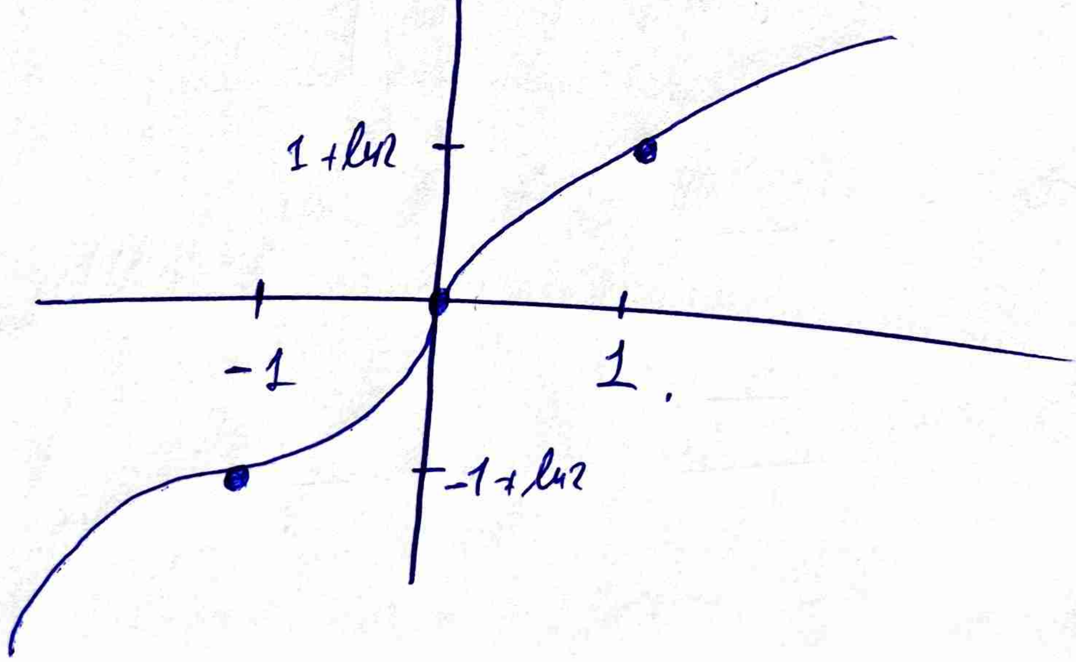
$$\lim_{x\to+\infty} f(x) = +\infty .$$

$$\lim_{x\to+\infty} \frac{f(x)}{x} = \lim_{x\to+\infty} \frac{x+\ln(x^2+1)}{x} = \lim_{x\to+\infty} 1 + \frac{2x}{x^2+1} = 1$$

$$\lim_{x\to+\infty} f(x) - x = \lim_{x\to+\infty} x+\ln(x^2+1) - x = +\infty .$$

$$\lim_{x\to-\infty} \frac{f(x)}{x} = \lim_{x\to-\infty} \frac{x+\ln(x^2+1)}{x} = 1$$

$$\lim_{x\to-\infty} f(x) - x = +\infty .$$



Επομενο Μαθημα

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Προετοιμασια Διαγωνισματος

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Θεωρια

1.1	1.22	3.4
1.3	1.28	3.5
1.11	1.31	3.6
1.12	1.34	3.7
1.13	1.36	3.9
1.14	1.40	3.10
1.15	1.42	3.16
1.18	1.43	3.18
1.21	1.44	3.19.
Ορισμοι.		Αποδειξη.

3.5 Νδο αν η f παρ/μη στο x_0

Τοτε ειναι και συνεχ/ση στο x_0

Αποδειξη

$$f(x) - f(x_0) = f(x) - f(x_0)$$

$$f(x) - f(x_0) = \frac{f(x) - f(x_0)}{x - x_0} (x - x_0)$$

$$\lim_{x \to x_0} [f(x) - f(x_0)] = \lim_{x \to x_0} \left(\frac{f(x) - f(x_0)}{x - x_0} \cdot (x - x_0) \right)$$

$$\lim_{x \to x_0} [f(x) - f(x_0)] = f'(x_0) \cdot 0$$

$$\lim_{x \to x_0} [f(x) - f(x_0)] = 0$$

$$\lim_{x \to x_0} f(x) = f(x_0)$$

Συνεχης στο x_0

3.6 ΝΔΟ $(c)' = 0$

Απόδειξη

Έστω $f(x) = c$

$$f'(x_0) = \lim_{x \to x_0} \frac{f(x) - f(x_0)}{x - x_0} = \lim_{x \to x_0} \frac{c - c}{x - x_0} = 0$$

3.7 ΝΔΟ $(x)' = 1$

Απόδειξη

Έστω $f(x) = x$

$$f'(x_0) = \lim_{x \to x_0} \frac{f(x) - f(x_0)}{x - x_0} = \lim_{x \to x_0} \frac{\cancel{x - x_0}}{\cancel{x - x_0}} = 1$$

3.9 ΝΔΟ $(\sqrt{x})' = \frac{1}{2\sqrt{x}}$

Απόδειξη

Έστω $f(x) = \sqrt{x}$

$$f'(x_0) = \lim_{x \to x_0} \frac{f(x) - f(x_0)}{x - x_0} = \lim_{x \to x_0} \frac{\sqrt{x} - \sqrt{x_0}}{x - x_0}$$

$$= \lim_{x \to x_0} \frac{(\sqrt{x} - \sqrt{x_0})(\sqrt{x} + \sqrt{x_0})}{(x - x_0)(\sqrt{x} + \sqrt{x_0})} = \lim_{x \to x_0} \frac{\sqrt{x}^2 - \sqrt{x_0}^2}{(x - x_0)(\sqrt{x} + \sqrt{x_0})} = \lim_{x \to x_0} \frac{\cancel{x - x_0}}{\cancel{(x - x_0)}(\sqrt{x} + \sqrt{x_0})} = \frac{1}{2\sqrt{x_0}}$$

3.10 Νδο $(f+g)'(x_0) = f'(x_0) + g'(x_0)$

Απόδειξη

Έστω $h(x) = f(x) + g(x)$

$$h'(x_0) = \lim_{x \to x_0} \frac{h(x) - h(x_0)}{x - x_0} = \lim_{x \to x_0} \frac{f(x) + g(x) - (f(x_0) + g(x_0))}{x - x_0}$$

$$= \lim_{x \to x_0} \frac{f(x) + g(x) - f(x_0) - g(x_0)}{x - x_0}$$

$$= \lim_{x \to x_0} \frac{f(x) - f(x_0)}{x - x_0} + \frac{g(x) - g(x_0)}{x - x_0}$$

$$= f'(x_0) + g'(x_0)$$

3.16 Έστω f ορισμένη στο Δ

Αν η f συνεχής στο Δ και αν

$f'(x)=0$ $\forall x$ εσωτερικό του Δ.

Τότε νδο $f(x)$ σταθερή.

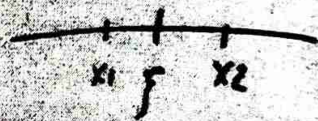
Απόδειξη

Αρκεί νδο $\forall x_1, x_2 \in \Delta$ η $f(x_1)=f(x_2)$

$\longrightarrow$ $x_1 = x_2 \Rightarrow f(x_1)=f(x_2)$

$\longrightarrow$ $x_1 < x_2$

$$f'(\xi) = \frac{f(x_2)-f(x_1)}{x_2-x_1}$$



όμως $f'(x)=0$ άρα $f'(\xi)=0$

$$\frac{f(x_2)-f(x_1)}{x_2-x_1}=0 \Rightarrow f(x_2)-f(x_1)=0 \Rightarrow f(x_1)=f(x_2)$$

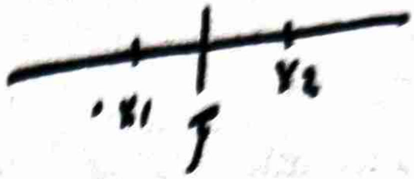
$\longrightarrow$ $x_1 > x_2$

όμοια.

3.18 Αν $f'(x)>0$ νδο $f \nearrow$

Απόδειξη

Έστω [redacted]



$$f'(\xi)=\frac{f(x_2)-f(x_1)}{x_2-x_1}$$

όμως $f'(x)>0$

$f'(\xi)>0$

$$\frac{f(x_2)-f(x_1)}{\underset{\oplus}{x_2-x_1}}>0$$

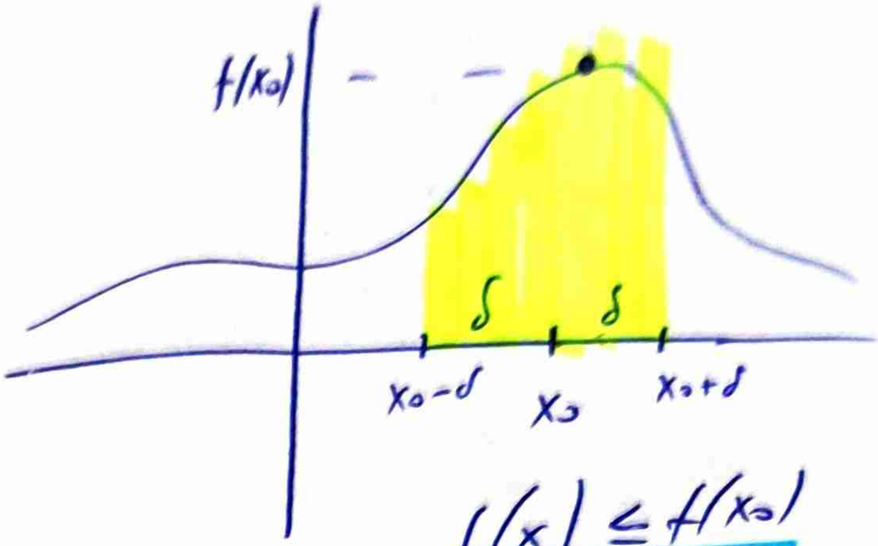
$\Rightarrow f(x_2)-f(x_1)>0$

$\underline{f(x_2)>f(x_1)}$

$f \nearrow$

3.19 (Fermat)

$A(x_0, f(x_0))$ Τοπ. μέγιστο



$f(x) \leq f(x_0)$

$$\lim_{x \to x_0^-} \frac{f(x) - f(x_0)}{x - x_0} \geq 0$$

$$\lim_{x \to x_0^+} \frac{f(x) - f(x_0)}{x - x_0} \leq 0$$

Γνωρίζω ότι η f παραγωγίσιμη στο x_0

$$\lim_{x \to x_0^-} f(x) = \lim_{x \to 0^+} f(x)$$

$$\Rightarrow \lim_{x \to x_0} f(x) = 0$$

$$f'(x_0) = 0.$$