34.

ΕΝΟΤΗΤΑ 8

(α) Αν $\lim\limits_{x\to 5} f(x) = 1$

$$\lim_{x\to 2} f(3x-1) \xrightarrow[\substack{x\to 2\\ y\to 5}]{3x-1=y} \lim_{y\to 5} f(y) = 1$$

39. (α) i) $\lim\limits_{x\to -2} (2f(x)+1-x) = 3$

Ψάχνω το $\lim\limits_{x\to -2} f(x)$

Θέτω βοηθητική συνάρτηση $g(x) = 2f(x)+1-x$

Τότε $\boxed{\lim\limits_{x\to -2} g(x) = 3}$

$g(x) - 1 + x = 2f(x)$

$f(x) = \dfrac{g(x)-1+x}{2}$

$$\lim_{x\to -2} f(x) = \lim_{x\to -2} \frac{g(x)-1+x}{2} = \frac{3-1-2}{2} = 0$$

Άρα $\lim\limits_{x\to -2} f(x) = 0$;

(a) iii) $\lim_{x \to -2} \frac{f(x) - 1}{x + 2} = 3$

$$g(x) = \frac{f(x) - 1}{x + 2}$$

$\lim_{x \to -2} g(x) = 3$

$$g(x)(x+2) = f(x) - 1$$

$$f(x) = g(x)(x+2) + 1$$

$$\lim_{x \to -2} f(x) = \lim_{x \to -2} g(x)(x+2) + 1$$

$$\lim_{x \to -2} f(x) = 3(-2+2) + 1$$

$$\lim_{x \to -2} f(x) = 1$$

41. $\lim\limits_{x\to 0} \frac{f(x)}{x} = 1$.

(α) $g(x) = \frac{f(x)}{x}$ αρα $\lim\limits_{x\to 0} g(x) = 1$

$f(x) = x\,g(x)$

$\lim\limits_{x\to 0} f(x) = \lim\limits_{x\to 0} x\,g(x) = 0 \cdot 1 = 0$

$$\lim_{x\to 0} f(x) = 0$$

(β) $$\lim_{x\to 0} \frac{f^2(x) + x\ln x}{f^2(x) + x f(x)} = \lim_{x\to 0} \frac{\frac{f^2(x)}{x^2} + \frac{x\ln x}{x^2}}{\frac{f^2(x)}{x^2} + \frac{x f(x)}{x^2}}$$

$$= \lim_{x\to 0} \frac{\left(\frac{f(x)}{x}\right)^2 + \frac{x}{x}\,\frac{\ln x}{x}}{\left(\frac{f(x)}{x}\right)^2 + \frac{x}{x}\,\frac{f(x)}{x}} = \frac{1^2 + 1}{1^2 + 1} = 1.$$

(8) $\lim_{x\to 0} \frac{f(x^3+x)}{x} = \lim_{x\to 0} \frac{f(x^3+x)}{x^3+x} \cdot \frac{x^3+x}{x}$

$\lim_{x\to 0} \frac{f(x)}{x} = 1$

$= 1 \cdot 1$

$= 1$

$\to \lim_{x\to 0} \frac{f(x^3+x)}{x^3+x} \quad \begin{matrix} x^3+x = y \\ x\to 0 \\ y\to 0 \end{matrix} \quad \lim_{y\to 0} \frac{f(y)}{y} = 1$

$\to \lim_{x\to 0} \frac{x^3+x}{x} = \lim_{x\to 0} \frac{x(x^2+1)}{x} = 1$

Άσκηση

Δίνεται $f(x) = x^5 + x$

1. Νδο η $f(x)$ αντιστρέφεται.

$x_1 < x_2 \Rightarrow x_1^5 < x_2^5$

$x_1 < x_2$ ——— $\oplus$ $f(x_1) < f(x_2)$

$f \nearrow$

2. Μονοτονία αντιστρόφης.

Έστω $f^{-1}(x_1) < f^{-1}(x_2)$

$f \nearrow$

$f(f^{-1}(x_1)) < f(f^{-1}(x_2))$

$x_1 < x_2$

$f^{-1} \nearrow$

3. Να δω f περιττή

$$f(-x) = (-x)^5 + (-x) = -x^5 - x =$$

$$= -(x^5 + x)$$

$$= -f(x)$$

> f περιττή
> $f(-x) = -f(x)$

4. $f(x^5+2x-2) - x^5 = x$

$$f(x^5+2x-2) = x^5 + x$$

$$f(x^5+2x-2) = f(x)$$

$$f: 1-1$$

$$x^5+2x-2 = x$$

$$x^5 + x - 2 = 0$$

$$x^5 + x = 2$$

$$f(x) = f(1)$$

$x = 1$

5. $\left(f^{-1}(x)\right)^5 + f^{-1}(x) < 1 - \ln x$

$$f\left(f^{-1}(x)\right) < 1 - \ln x$$

$$x < 1 - \ln x$$

$$\underbrace{x + \ln x - 1 < 0}_{\varphi(x)}$$

$\varphi(x) < 0$

$\varphi(x) < \varphi(1)$

$\varphi \uparrow$

$0 < x < 1$

Монотон $\varphi(x)$

- $x_1 < x_2 \Rightarrow \ln x_1 < \ln x_2$
- $x_1 < x_2 \Rightarrow x_1 - 1 < x_2 - 1$

(+)

$\varphi(x_1) < \varphi(x_2)$

6. $f^{-1}\left(f(e^{x+1}) - f^{-1}(34) \right) = 0$

$f(x) = x^5 + x$

$f(2) = 34 \Leftrightarrow f^{-1}(34) = 2$

$f^{-1}\left(f(e^{x+1}) - 2 \right) = 0$

$f\left(f^{-1}\left(f(e^{x+1}) - 2 \right) \right) = f(0)$

$f(e^{x+1}) - 2 = 0$

$f(e^{x+1}) = 2$

$f(e^{x+1}) = f(1)$

$f^{-1}(3) - 1$

$e^{x+1} = 1$

$x + 1 = 0$

$x = -1$

7. $f(2x) - f(x) + x = 0$

$f(2x) + x = f(x)$

$f(2x) + 2x - x = f(x)$

$f(2x) + 2x = f(x) + x$

$\boxed{g(x) = f(x) + x}$

$g(2x) = g(x)$

g : 1-1

$2x = x$

$x = 0$

$\equiv$

- $x_1 < x_2 \Rightarrow f(x_1) < f(x_2)$

- $x_1 < x_2$ —— ⊕ —— $g(x_1) < g(x_2)$

$g \nearrow$

8. $f(x) = 2 + f^{-1}(1-x)$

$$\underbrace{f(x) - 2 - f^{-1}(1-x)}_{\varphi(x)} = 0$$

$\varphi(x) = 0$

$\varphi(x) = \varphi(1)$

φ 1-1

$x = 1$

$f(x) = x^5 + x$

$\varphi(1) = f(1) - 2 - f^{-1}(0) =$

$= 2 - 2 - 0 = 0$

$f(0) = 0 \quad (\Rightarrow f^{-1}(0) = 0$

Μονοτονία $\varphi(x)$

(+)

- $x_1 < x_2 \Rightarrow f(x_1) - 2 < f(x_2) - 2$
- $x_1 < x_2 \Rightarrow 1 - x_1 > 1 - x_2 \Rightarrow f^{-1}(1-x_1) > f^{-1}(1-x_2)$

$-f^{-1}(1-x_1) < -f^{-1}(1-x_2)$

$\varphi(x_1) < \varphi(x_2)$

$\varphi \uparrow$

9. $f(x^2) + x = f(x) + 1$ $[0, +\infty)$

Προφανής ρίζα $x = 1$

Για $x > 1$

- $x^2 > x \Rightarrow f(x^2) > f(x) \Rightarrow f(x^2) - f(x) > 0$
- $x > 1 \Rightarrow x - 1 > 0$

$\oplus$

$$f(x^2) - f(x) + x - 1 > 0$$

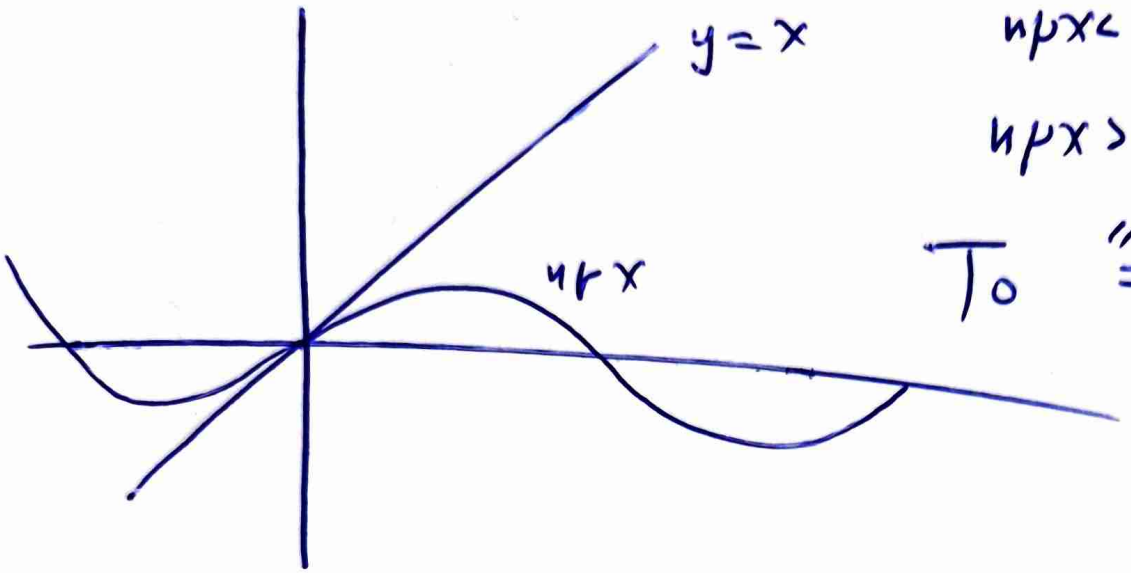
$$f(x^2) + x > f(x) + 1$$

Για $0 \leq x < 1$

- $x^2 < x \Rightarrow f(x^2) < f(x)$
- $x < 1$ $\oplus$

$$f(x^2) + x < f(x) + 1$$

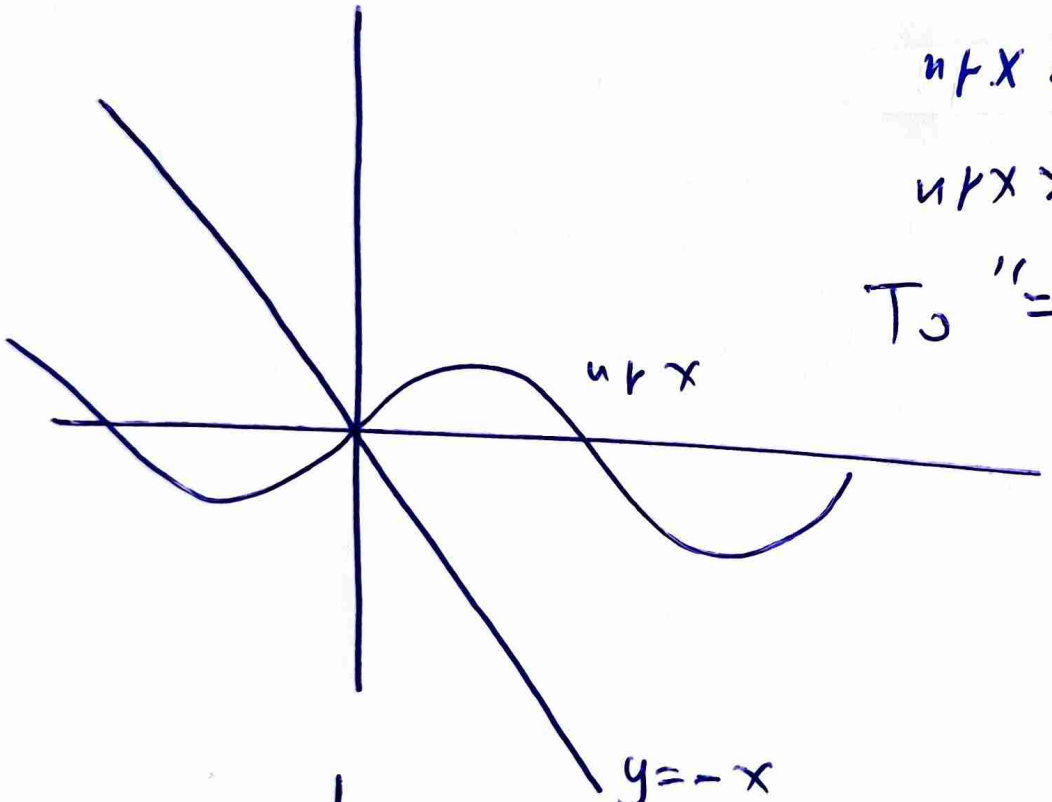
SOS Ιδιοτητα



$\eta\mu x < x \quad , x > 0$

$\eta\mu x > x \quad , x < 0$

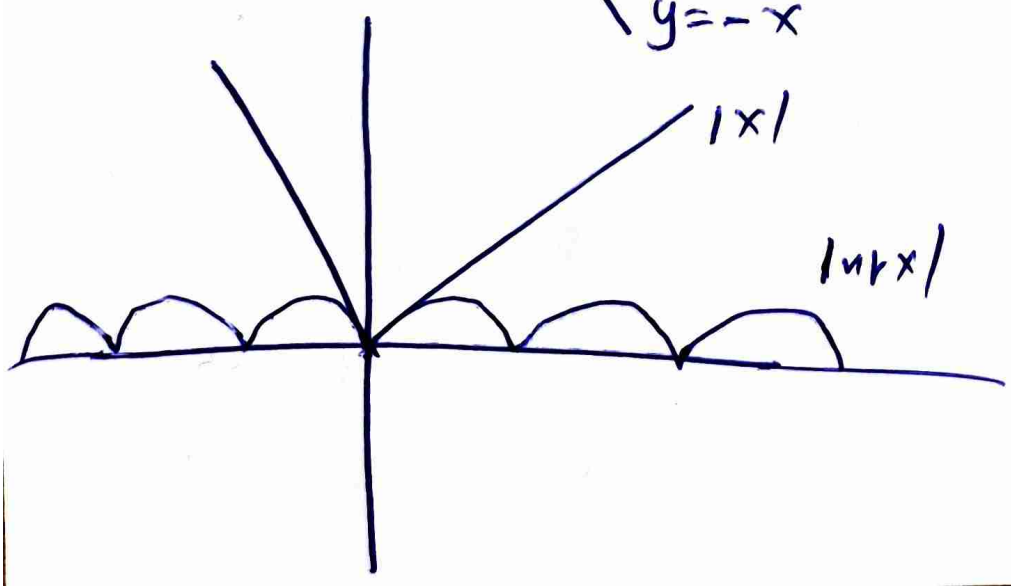
Το "=" για $x = 0$.



$\eta\mu x < -x \quad , x < 0$

$\eta\mu x > -x \quad , x > 0$

Το "=" για $x = 0$.



$|\eta\mu x| \leq |x|$

Το "=" για $x = 0$

45. ⓐ $f(x) = \ln(|x| - |\ln x|)$

Πρεπει $|x| - |\ln x| > 0$

$|x| > |\ln x|$

$x \neq 0$

$D_f = \mathbb{R}^*$

Σωστο 8

ⓑ $f(x) = \dfrac{x}{x^2 - \ln^2 x}$

πρεπει $x^2 - \ln^2 x \neq 0$

$x^2 \neq \ln^2 x$

$|x| \neq |\ln x|$

$x \neq 0$

$D_f = \mathbb{R}^*$

47. ⓐ $\ln(x^2-1)+1=x^2$

$\ln(x^2-1)=x^2-1$

$\ln x = x$

То "="

при $x=0$

$x^2-1=0$

$x=\pm 1$

ⓑ $e^x+x^3=1+\ln(e^x+x^3-1)$

$e^x+x^3-1=\ln(e^x+x^3-1)$

То $\ln x = x$

если при

$x=0$

$e^x+x^3-1=0$

$\varphi(x)$

$\varphi(x)=0$

$\varphi(x)=\varphi(0)$

$\varphi\uparrow-1$

$x=0$

$x_1<x_2 \Rightarrow e^{x_1}<e^{x_2}$

$x_1<x_2 \Rightarrow x_1^3-1<x_2^3-1$

$+$

$\varphi\uparrow$

46. $|\eta\mu x| = |\sigma\upsilon\nu x|$

$\eta\mu x = \sigma\upsilon\nu x$

$x = -\frac{\pi}{4}$

ή $\eta\mu x = -\sigma\upsilon\nu x$

$x = \frac{\pi}{4}$

Έχω άλλο μία;
Δεν βρω.

$|\eta\mu x| = |\sigma\upsilon\nu x|$

$|-\eta\mu(\frac{\pi}{2}+x)| = |\sigma\upsilon\nu x|$

$|\eta\mu(\frac{\pi}{2}+x)| = |\sigma\upsilon\nu x|$

Γνωρίζω ότι $|\eta\mu x| = |x| \Leftrightarrow x = 0$

$\Rightarrow |\eta\mu(\frac{\pi}{2}+x)| = |\sigma\upsilon\nu x|$

$\frac{\pi}{2}+x = 0$

$x = -\frac{\pi}{2}$

$$\lim_{x \to 0} \frac{\ln x}{\ln x - x} =$$

$$= \lim_{x \to 0^+} \frac{\ln x}{\ln x - x} =$$

$$= \lim_{x \to 0^+} \ln x \cdot \frac{1}{\ln x - x}$$

$$= -\infty \cdot (-\infty) = +\infty$$

$\forall x > 0 \quad \ln x < x$

$\ln x - x < 0$