Ευρεση Τυπου

1. Αν $f: \mathbb{R} \to \mathbb{R}$ και $f(x+1) = (x+1)e^{-x}$

2. Αν $f: \mathbb{R} \to \mathbb{R}$ συνεχής και $x^2 f\left(\frac{1}{x}\right) = ημx$, $\forall x \neq 0$

3. Αν $f: [-1,4] \to \mathbb{R}$ συνεχής και $x^2 + f^2(x) = 3x+4$ $\forall x \in [-1,4]$ και $f(0) = -2$

4. Αν $f: \mathbb{R} \to \mathbb{R}$ συνεχής και $f(0)=1$ και $e^{2x} f^2(x) - 2x^2 e^x f(x) = 2x^2+1$ $\forall x \in \mathbb{R}$.

5. Αν $f: (0,+\infty) \to \mathbb{R}$ παρ/μη και $f(4)=3$ και $2x f'(x) + f(x) = 0$ $\forall x>0$ ν/δο $g(x) = f(x)\sqrt{x}$ σταθερή και να βρεθ τυπο $f(x)$

6. $f: \mathbb{R} \to \mathbb{R}$ συνεχής και $f(0)=0$ παρ/μη $(-\infty,-1) \cup (-1,+\infty)$

$$f'(x) = \begin{cases} -2, & x<-1 \\ 3x^2-1, & x>-1 \end{cases}$$

7. $f: [0,+\infty) \to \mathbb{R}$ και $f'(x) = \frac{1}{2\sqrt{x}} + 3$, $f(4)=11$

8. $f(x) f'(x) = \frac{1}{2}$ $\forall x>0$ $f(1)=1$

9. $f'(x) = 4x^3 e^{-f(x)}$ $\qquad f(1) = \ln 3$

10. $f:\mathbb{R}\to\mathbb{R}$ παρ/μη , $f(x)>0$, $f(1)=e^2$

$f'(x) = 2x\, f(x) \quad \forall x\in\mathbb{R}$

11. $f'(x)\left(e^{f(x)} + e^{-f(x)}\right) = 2$ $\qquad f(0)=0$

12. $(F(x) - e^x)(f(x) - e^x) = 2x^3+2x$ $\qquad$ F παράγουσα

$F(0)=0$

13. $2f(x) + 4x f'(x) + (x^2+9) f''(x) = 0$

$f(0)=0 \qquad f'(0) = \frac{1}{9}$

Νδο $\quad f(x) = \dfrac{x}{x^2+9}$

14. $f:\mathbb{R}\to\mathbb{R}$ συνεχ.

$f(x) = 12x^2 - 2x\int_0^1 f(t)\,dt$

1. $f(x+1)=(x+1)e^{-x}$

Θέτω $x+1=t$

$x=t-1$

$f(t)=t\,e^{-(t-1)}$

$f(t)=t\,e^{-t+1}$

ή $\boxed{f(x)=x\,e^{1-x}}$

2. $x^2 f\left(\frac{1}{x}\right)=ημx \quad \forall x\neq 0$

Θέτω $\frac{1}{x}=t$

$x=\frac{1}{t}$

$\left(\frac{1}{t}\right)^2 f(t)=ημ\frac{1}{t}$

$\frac{1}{t^2}f(t)=ημ\frac{1}{t}$

$f(t)=t^2\,ημ\frac{1}{t}$

ή $f(x)=x^2\,ημ\frac{1}{x}$

$x\neq 0.$

$$f(0)=\lim_{x\to 0} f(x)=\lim_{x\to 0} x^2 ημ\frac{1}{x} = 0$$

$$-1 \le ημ\frac{1}{x} \le 1$$

$$\boxed{-x^2 \le x^2 ημ\frac{1}{x} \le x^2}$$

$$\left.\begin{array}{l} \lim\limits_{x\to 0} -x^2 = 0 \\ \lim\limits_{x\to 0} x^2 = 0 \end{array}\right\} \text{Από κ.π} \lim_{x\to 0} x^2 ημ\frac{1}{x} = 0$$

$$\boxed{f(x)=\begin{cases} x^2 ημ\frac{1}{x}, & x\neq 0 \\ 0, & x=0 \end{cases}}$$

3. $x^2 + f^2(x) = 3x+4$

$$f^2(x) = -x^2+3x+4$$

$$f^2(x) = \sqrt{-x^2+3x+4}^{\,2}$$

$$|f(x)| = |\overset{\oplus}{\sqrt{-x^2+3x+4}}|$$

$$|\overset{\ominus}{f(x)}| = \sqrt{-x^2+3x+4}$$

[illegible] $f(x)$

$$f(x) = 0$$

$$|f(x)| = 0$$

$$\sqrt{-x^2+3x+4} = 0$$

$$-x^2+3x+4 = 0$$

$x = -1$ $\quad$ $x = 4$

x		-1		4	
$f(x)$	///	0	$-$	0	///

$$f(0) = -2$$

$$-f(x) = \sqrt{-x^2+3x+4}$$

$$f(x) = -\sqrt{-x^2+3x+4}$$

4. $e^{2x} f^2(x) - 2x^2 e^x f(x) = 2x^2+1$

$$\underbrace{e^{2x} f^2(x) - 2x^2 e^x f(x) + x^4} = x^4 + 2x^2 + 1$$

$$\left(e^x f(x) - x^2\right)^2 = \left(x^2+1\right)^2$$

$$\left|\underbrace{e^x f(x) - x^2}_{g(x)}\right| = |x^2+1|$$

$$|g(x)| = x^2+1$$

Ρίζα $g(x)$

$g(x)=0$

$|g(x)|=0$

$x^2+1=0$

Αδύνατο

$g(x) \neq 0$

$g(x)>0$ ή $g(x)<0$

$g(0)=1$

$g(x)>0$

$g(0) = e^0 f(0) - 0^2 = 1$

$g(x) = x^2+1$

$e^x f(x) - x^2 = x^2+1$

$e^x f(x) = 2x^2+1$

$$f(x) = \frac{2x^2+1}{e^x}$$

5. $2x f'(x) + f(x) = 0$

Νδο $g(x) = f(x)\sqrt{x}$ σταθερή

Αρκεί νδο $g'(x) = 0$

$$g'(x) = f'(x)\sqrt{x} + f(x)\frac{1}{2\sqrt{x}}$$

$$g'(x) = \frac{f'(x)\sqrt{x}\,2\sqrt{x}}{2\sqrt{x}} + \frac{f(x)}{2\sqrt{x}}$$

$$g'(x) = \frac{2f'(x)x + f(x)}{2\sqrt{x}} = \frac{0}{2\sqrt{x}} = 0$$

Αφού $g'(x) = 0 \Rightarrow g(x) = c$

$$f(x)\sqrt{x} = c$$

$$\underline{x=4}$$

$$f(4)\sqrt{4} = c$$

$$3 \cdot 2 = c$$

$$c = 6$$

$$f(x) = \frac{6}{\sqrt{x}}$$

6. $$f'(x) = \begin{cases} -2, & x < -1 \\ 3x^2-1, & x > -1 \end{cases}$$

$\underline{x < -1}$

$f'(x) = -2$

$f'(x) = (-2x)'$

$$\boxed{f(x) = -2x + C_1}$$

$\underline{x > -1}$

$f'(x) = 3x^2-1$

$f'(x) = (x^3-x)'$

$f(x) = x^3 - x + C_2$

$f(0) = 0^3 - 0 + C_2$

$0 = C_2$

$\underline{\underline{f(x) = x^3 - x}}$

$$f(x) = \begin{cases} -2x+C_1, & x < -1 \\ \text{συνεχή!} & \\ x^3 - x, & x \geq -1 \end{cases}$$

$$\lim_{x \to -1^-} f(x) = \lim_{x \to -1^-} (-2x + C_1) = -2(-1) + C_1 = \underline{\underline{C_1 + 2}}$$

$$\lim_{x \to -1^+} f(x) = \lim_{x \to -1^+} x^3 - x = -1 + 1 = \underline{\underline{0}}$$

$C_1 + 2 = 0 \qquad \underline{\underline{C_1 = -2}}$

$$f(x) = \begin{cases} -2x - 2, & x < -1 \\ x^3 - x, & x \geq -1 \end{cases}$$

7. $f'(x) = \frac{1}{2\sqrt{x}} + 3$

$f'(x) = (\sqrt{x} + 3x)'$

$f(x) = \sqrt{x} + 3x + C$

$\underline{x=4}$

$f(4) = \sqrt{4} + 3\cdot 4 + C$

$15 = 2 + 12 + C$

$\underline{\underline{C=1}}$

$f(x) = \sqrt{x} + 3x + 1$

8. $f(x)\,f'(x) = \frac{1}{2}$

$2f(x)\,f'(x) = 1$

$(f^2(x))' = (x)'$

$f^2(x) = x + C \longrightarrow f^2(x) = x.$

$\underline{x=1}$

$f^2(1) = 1 + C$

$1 = 1 + C \qquad C=0$

$f^2(x) = x$

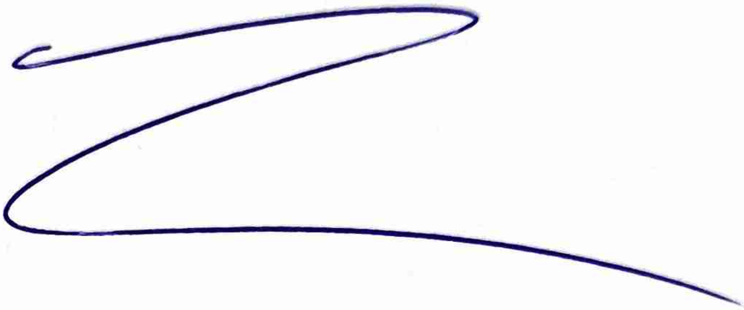
$f^2(x) = \sqrt{x}^2$

$|f(x)| = |\sqrt{x}|$

$|f(x)| = \sqrt{x}$

$f(x) = \sqrt{x}$

P,Td $f(x)$



$f(x) = 0$

$|f(x)| = 0$

$\sqrt{x} = 0$

$x = 0$

x		0	
$f(x)$	//////	0	+

$f(1) = 1$

9. $f'(x) = 4x^3 e^{-f(x)}$

$f'(x) = 4x^3 \dfrac{1}{e^{f(x)}}$

$f'(x) e^{f(x)} = 4x^3$

$\left(e^{f(x)}\right)' = (x^4)'$

$e^{f(x)} = x^4 + c \longrightarrow e^{f(x)} = x^4 + 2$

$\underline{x=1}$

$f(x) = \ln(x^4 + e)$

$e^{f(1)} = 1 + c$

$e^{\ln 3} = 1 + c$

$3 = 1 + c$

$c = 2$

10. $f'(x) = 2x f(x)$

$\dfrac{f'(x)}{f(x)} = 2x$

$\left(\ln f(x)\right)' = (x^2)'$

$\ln f(x) = x^2 + c$

$\underline{x=1}$

$\ln f(1) = 1 + c$

$\ln e^2 = 1 + c$

$\underline{c = 1}$

$\ln f(x) = x^2 + 1$

$f(x) = e^{x^2+1}$

Σχόλιο

$$f(x)\,f'(x) = \left(\frac{1}{2} f^2(x)\right)'$$

$$e^{f(x)}\,f'(x) = \left(e^{f(x)}\right)'$$

$$\frac{f'(x)}{f(x)} = \left(\ln f(x)\right)'$$

$$\frac{f^2(x)}{f(x)} = \left(-\frac{1}{f(x)}\right)'$$

$$f'(x)\,\text{συν}\, f(x) = \left(\text{ημ}\, f(x)\right)'$$

11. $f'(x)\left(e^{f(x)}+e^{-f(x)}\right)=2$

$$f'(x)e^{f(x)}+f'(x)e^{-f(x)}=2$$

$$\left[e^{f(x)}-e^{-f(x)}\right]'=(2x)'$$

$$e^{f(x)}-e^{-f(x)}=2x+c$$

$\underline{x=0}$

$$e^{f(0)}-e^{-f(0)}=c$$

$$e^{0}-e^{0}=c \quad \Rightarrow c=0$$

$$e^{f(x)}-e^{-f(x)}=2x$$

$$e^{f(x)}-\frac{1}{e^{f(x)}}=2x$$

$$e^{2f(x)}-1=2xe^{f(x)}$$

$$e^{2f(x)}-2xe^{f(x)}=1$$

$$e^{2f(x)}-2xe^{f(x)}+x^2=x^2+1$$

$$\left(e^{f(x)}-x\right)^2=\sqrt{x^2+1}^{\,2}$$

$$\left|e^{f(x)}-x\right| = \left|\overset{+}{\sqrt{x^2+1}}\right|$$

$$\left|\overset{+}{g(x)}\right| = \sqrt{x^2+1}$$

$g(x)=0$

$|g(x)|=0$

$\sqrt{x^2+1}=0$

Άτοπο!

$g(x)>0$ ή $g(x)<0$

$g(0)=e^{f(0)}-0=e^0=1$

$g(x)>0$

$g(x)=\sqrt{x^2+1}$

$e^{f(x)}-x=\sqrt{x^2+1}$

$e^{f(x)}=\sqrt{x^2+1}+x$

$f(x)=\left(\ln \sqrt{x^2+1}+x\right)$

12. $(F(x) - e^x)(f(x) - e^x) = 2x^3 + 2x$

$$\left[\frac{1}{2}(F(x) - e^x)^2\right]' = \left(\frac{2}{4}x^4 + x^2\right)'$$

$$\frac{1}{2}(F(x) - e^x)^2 = \frac{1}{2}x^4 + x^2 + C$$

$\underline{x = 0}$

$$\frac{1}{2}(F(0) - 1)^2 = C$$

$$\frac{1}{2}(0 - 1)^2 = C \qquad C = \frac{1}{2}$$

$$\frac{1}{2}(F(x) - e^x)^2 = \frac{1}{2}x^4 + x^2 + \frac{1}{2}$$

$$(F(x) - e^x)^2 = x^4 + 2x^2 + 1$$

$$(F(x) - e^x)^2 = (x^2 + 1)^2$$

$$|F(x) - e^x| = |x^2 + 1|$$

$+$

$$|g(x)| = x^2 + 1$$

$g(x)=0$

$|g(x)|=0$

$x^2+1=0$

Αδύνατη

$g(x)\neq 0$

$g(x)>0$ ή $g(x)<0$

$g(0)=F(0)-e^0=-1$

$g(x)<0$

$$|g(x)| = x^2+1$$

$$-g(x) = x^2+1$$

$$g(x) = -x^2-1$$

$$F(x)-e^x = -x^2-1$$

$$F(x) = e^x-x^2-1$$

$$f(x) = e^x-2x$$

13. $2f(x) + 4x\, f'(x) + (x^2+9)f''(x) = 0$

Νδο $f(x) = \dfrac{x}{x^2+9}$

$$f(x)(x^2+9) = x$$

$$\underbrace{f(x)(x^2+9) - x}_{g(x)} = 0$$

$$g'(x) = f'(x)(x^2+9) + f(x)2x - 1$$

$$g''(x) = f''(x)(x^2+9) + f'(x)2x + f'(x)2x + f(x)2$$

$$g''(x) = f''(x)(x^2+9) + 4xf'(x) + 2f(x) = 0$$

$$g''(x) = 0$$

$$g'(x) = c$$

$$f'(x)(x^2+9) + f(x)2x - 1 = c$$

$\underline{x=0}$

$$f'(0)\cdot 9 - 1 = c$$

$$c = 0$$

$$\underbrace{f'(x)(x^2+9) + 2x f(x)}_{} - 1 = 0$$

$$\left(f(x)(x^2+9) - x \right)' = 0$$

$$f(x)(x^2+9) - x = c$$

$x = 0$

$$f(0) \cdot 9 - 0 = c$$

$$c = 0$$

$$f(x)(x^2+9) - x = 0$$

$$f(x) = \frac{x}{x^2+9}$$

14. $f(x) = 12x^2 - 2x \underbrace{\int_0^1 f(t)\,dt}_{k}$

$f(x) = 12x^2 - 2x \cdot k$

$\int_0^1 f(x)\,dx = \int_0^1 12x^2 - 2xk\,dx$

$k = 12\int_0^1 x^2\,dx - 2k\int_0^1 x\,dx$

$k = 12 \cdot \frac{1}{3}\left(x^3\right)\Big|_0^1 - 2k\,\frac{1}{2}\left(x^2\right)\Big|_0^1$

$k = 4 - k$

$2k = 4$

$k = 2$

$f(x) = 12x^2 - 4x$