14. $f(x) = x^2 - \alpha x - 1$

$g(x) = x + \beta$

Ενότητα 23

α) Έχει ρίζα το 1 $\Rightarrow$ $f(1) = 0$

Έχει ρίζα το −1 $\Rightarrow$ $g(-1) = 0$

$f(1) = 1 - \alpha - 1 = 0$

$-\alpha = 0$

$\boxed{\alpha = 0}$

$-1 + \beta = 0$

$\boxed{\beta = 1}$

$f(x) = x^2 - 1$

$g(x) = x + 1$

β) $f(x) < g(x)$

$x^2 - 1 < x + 1$

$x^2 - 1 - x - 1 < 0$

$x^2 - x - 2 < 0$

x		-1		2
$x^2 - x - 2$	$+$	0	$-$	0 $+$

$x \in (-1, 2)$.

15. $f(x) = \dfrac{x^2 - x}{3x^2 - x - 2}$

(α) Πρεπει $3x^2 - x - 2 \neq 0$

$$A_f = R - \left\{-\frac{2}{3}, 1\right\}.$$

(β) $f(x) = \dfrac{x(x-1)}{3(x-1)(x+\frac{2}{3})} = \dfrac{x}{3x+2}$

(γ) $f(x) = \dfrac{1}{4}$

$$\frac{x}{3x+2} = \frac{1}{4}$$

$$4x = 3x + 2$$

$$x = 2$$

18. $f(x) = \begin{cases} 2x, & x < 1 \\ x - 1, & x \geq 1 \end{cases}$

(a) $f(\sqrt{2}) = \sqrt{2} - 1$

(B) $f\left(\frac{\pi}{4}\right) = 2 \cdot \frac{\pi}{4} = \frac{\pi}{2}$

(8) $f(a^2+1) = a^2 + 1 - 1 = a^2$

(8) $f\left(\frac{1}{a}\right) = 2 \cdot \frac{1}{a}$.

$a > 1 \Rightarrow 0 < \frac{1}{a} < 1$

19. $f(x) = \begin{cases} \frac{1}{x-2}, & x < 2 \\ x^2 - 3, & x \geq 2 \end{cases}$

(a) $f(\sqrt{3}) = \frac{1}{\sqrt{3}-2} = \frac{\sqrt{3}+2}{1} = \sqrt{3} + 2$

(B) $f(a^2+2) = a^2 + 2 - 3 = a^2 - 1$

(8) $f(f(2)) = 1^{2021} \Rightarrow f(1) = 1^{2021}$

$-1 = 1^{2021}$

$\rightarrow f(2) = 1$

$1 = -1$

20. $f(x) = \begin{cases} 4x-3, & x \le 2 \\ 3x+2, & x > 2 \end{cases}$

$$f(x) = 5$$

$x \le 2$

$f(x) = 5$

$4x - 3 = 5$

$4x = 8$

$x = 2$ ✓

$x > 2$

$f(x) = 5$

$3x + 2 = 5$

$3x = 3$

~~$x = 1$~~

9. (α) $f(x) = x - 1$

Λύσωσυ $f(x) > 0$

$x - 1 > 0$

$x > 1$

(β) $f(x) < 0$

$|x-1| - 2 < 0$

$|x-1| < 2$

$-2 < x - 1 < 2$

$-1 < x < 3$

(γ) $f(x) > 0$

$2x^2 - x - 1 > 0$

x		$\frac{1}{2}$		1
$2x^2-x-1$	$+$	$\|$	$-$	$+$

$x \in (-\infty, \frac{1}{2}) \cup (1, +\infty)$

Εν ΤΟΥΤΟΙΣ

25

1. (α) $A_f = [-2, 5]$

$\Sigma T_f = (-2, 3]$.

(β) $f(1) = 3 \qquad f(5) = 1$

(γ) $A(-1, 0) \qquad B(3, 0)$

(δ) i) $f(x) > 0 \qquad x \in (-1, 3) \cup \{5\}$.

ii) $f(x) < 0 \qquad x \in [-2, -1) \cup (3, 5)$.

2. (α) $A(-1, 1) \qquad B(3, 2)$

(β) i) $f(x) > g(x) \qquad x \in (-1, 3)$

ii) $f(x) < g(x) \qquad x \in (-2, -1) \cup (3, 4)$.

(γ) 2.

4. (α) $f(x) = 3x + \lambda$ $\quad M(-1, 2)$

$$2 = 3(-1) + \lambda$$

$$2 = -3 + \lambda$$

$$\lambda = 5$$

(β) $f(x) = \lambda^2 x^2 - \lambda x \longrightarrow M(1, 2)$

$$2 = \lambda^2 \cdot 1^2 - \lambda \cdot 1$$

$$2 = \lambda^2 - \lambda$$

$$\lambda^2 - \lambda - 2 = 0$$

$\lambda = 2$ $\quad$ $\lambda = -1$

(γ) $f(x) = 3x - 5 \longrightarrow M(\lambda, 2\lambda - 1)$

$$2\lambda - 1 = 3\lambda - 5$$

$$4 = \lambda$$

3. $f(x) = \frac{x^2-3}{x-1}$ $A_f = \mathbb{R} - \{1\}$

(a) $A(2,1)$

$f(2) = \frac{2^2-3}{2-1} = 1$ ✓

(b) $B(3,4)$

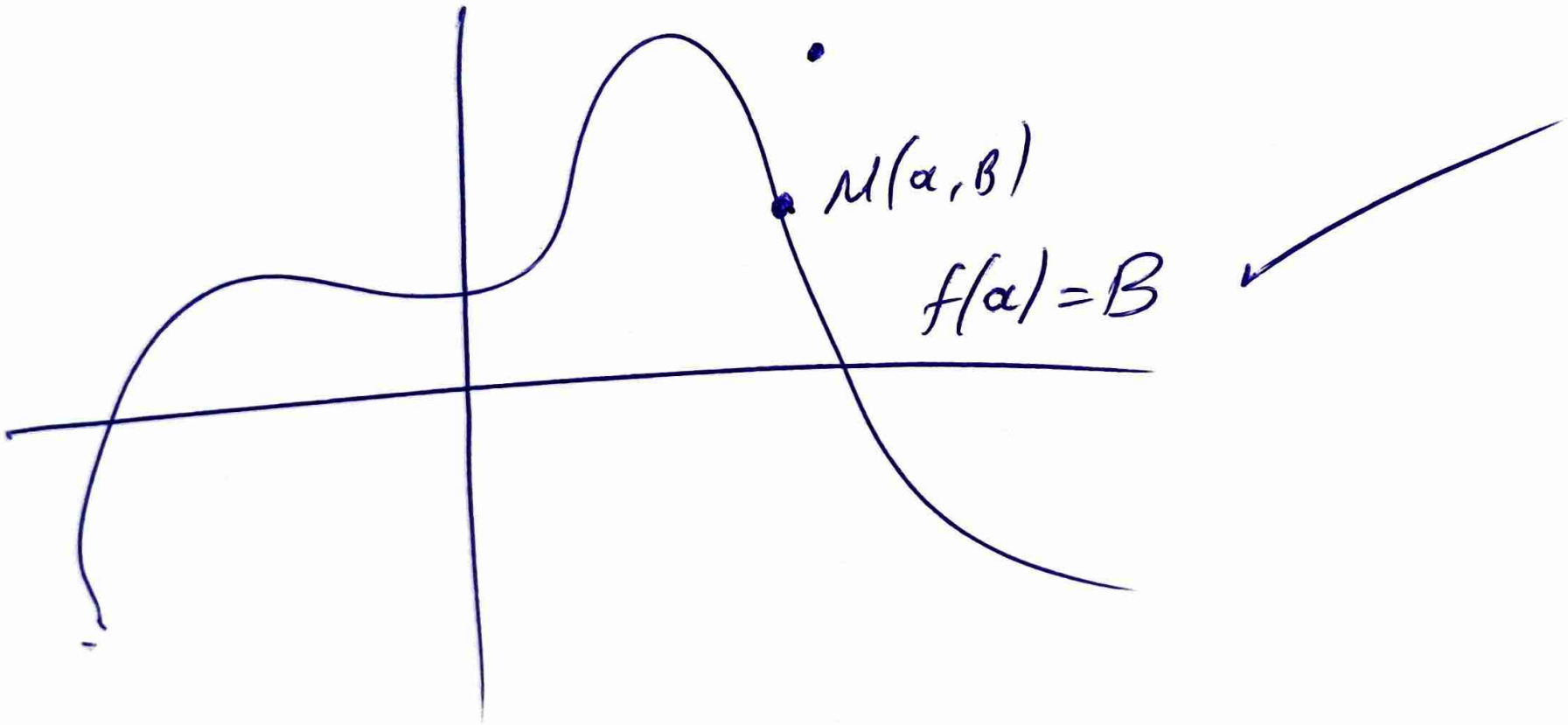
$f(3) = \frac{3^2-3}{3-1} = \frac{9-3}{2} = \frac{6}{2} = 3 \neq 4$

×

(c) $\Gamma(1,2)$

Το $1 \notin D_f$.

Вашки архи



6. $f(x) = \begin{cases} x^2 - \alpha, & x \le 1 \\ \frac{\alpha}{x-1} + \beta x, & x > 1 \end{cases}$

$A(-1, 0)$ $B(2, 5)$

$f(-1) = 0$ $f(2) = 5$

$\rightarrow f(-1) = (-1)^2 - \alpha = 1 - \alpha$

$0 = 1 - \alpha$

$\alpha = 1$

$\rightarrow f(2) = \frac{\alpha}{2-1} + \beta \cdot 2$

$5 = \alpha + 2\beta$

$5 = 1 + 2\beta$

$4 = 2\beta$ $\beta = 2$

5. $f(x) = \alpha x^3 - (\alpha+1)x^2 - \beta x - 2$

$A(1,-2) \quad B(-1,3)$

Αφού η $f(x)$ διέρχεται από το $A(1,-2)$

$\Longrightarrow f(1) = -2$

$f(1) = \alpha \cdot 1^3 - (\alpha+1)1^2 - \beta \cdot 1 - 2$

$-2 = \alpha - (\alpha+1) - \beta - 2$

$-\cancel{2} = \cancel{\alpha} - \cancel{\alpha} - 1 - \beta - \cancel{2}$

$0 = -1 - \beta$

$\beta = -1$

Αφού η $f(x)$ διέρχεται από το

$B(-1,3) \Longrightarrow f(-1) = 3$

$f(-1) = \alpha \cdot (-1)^3 - (\alpha+1)(-1)^2 - \beta(-1) - 2$

$3 = -\alpha - (\alpha+1) + \beta - 2$

$$3 = -a - a - 1 - 1 - 2$$

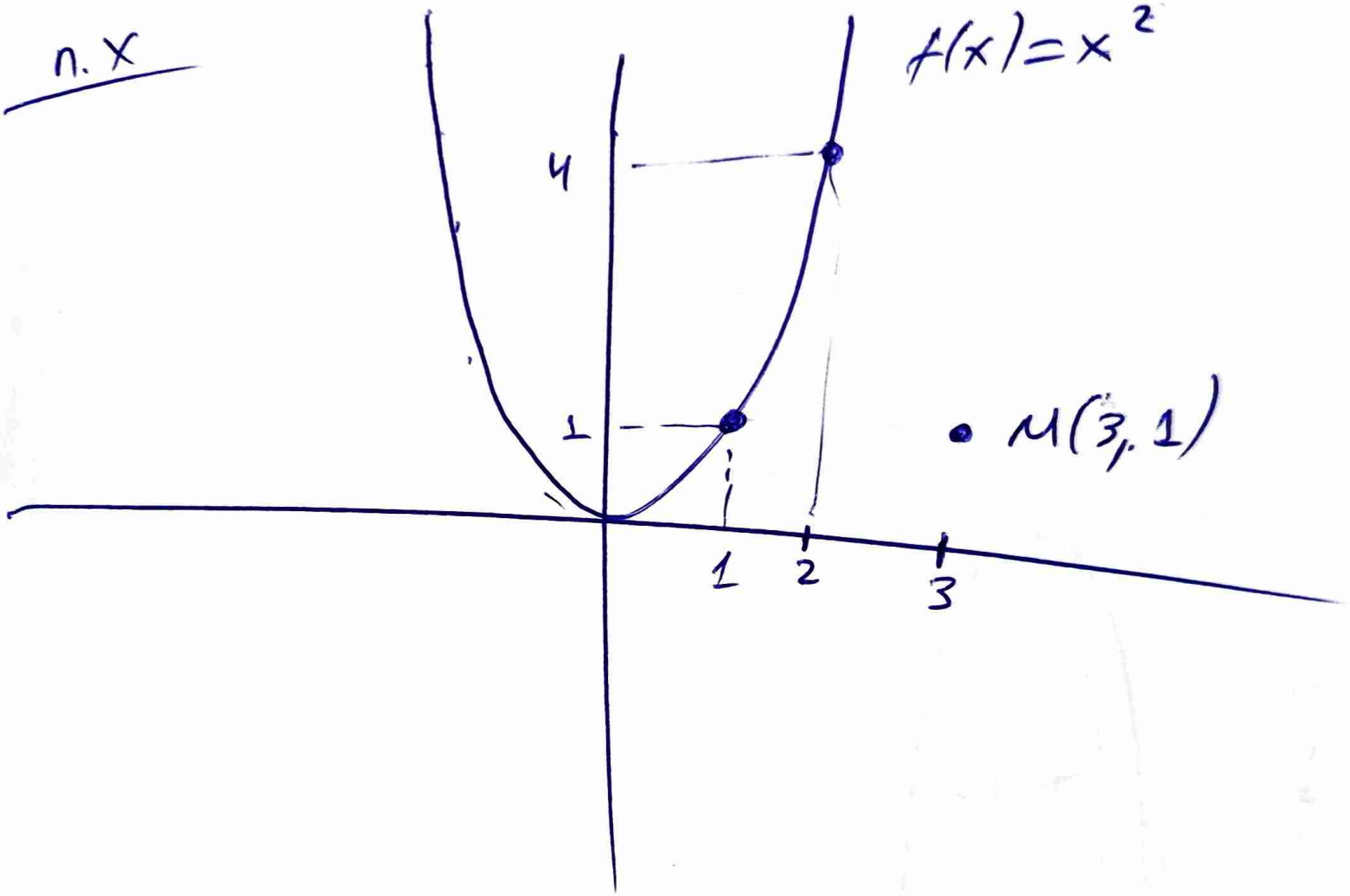
$$3 = -2a - 4$$

$$2a = -4 - 3$$

$$2a = -7$$

$$a = -\frac{7}{2}$$

n. X



10. $f(x) = x^2 - 3$

$g(x) = 5x - 9$

(a) $f(x) = g(x)$

$x^2 - 3 = 5x - 9$

$x^2 - 5x + 6 = 0$

$x = 2$ $x = 3$

$A(2, 1)$ $B(3, 6)$

(B). $f(x) > g(x)$

$x^2 - 3 > 5x - 9$

$x^2 - 5x + 6 > 0$

x		2		3
$x^2 - 5x + 6$	$+$	0	$-$	0 $+$

$x \in (-\infty, 2) \cup (3, +\infty)$.

Εξαρτα Μαθημα

Τεσαρτι 1/4

25.

16

17

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